

Leptonic D decays at BESIII

Bai-Cian Ke

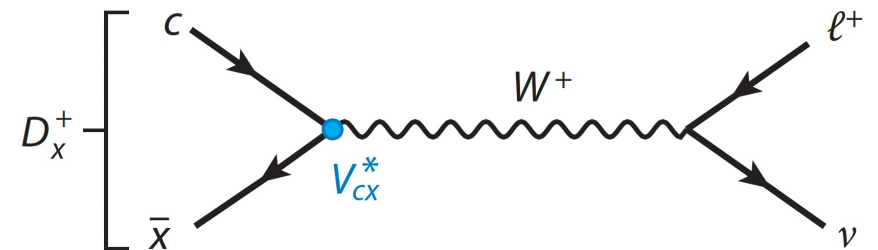
Zhengzhou University



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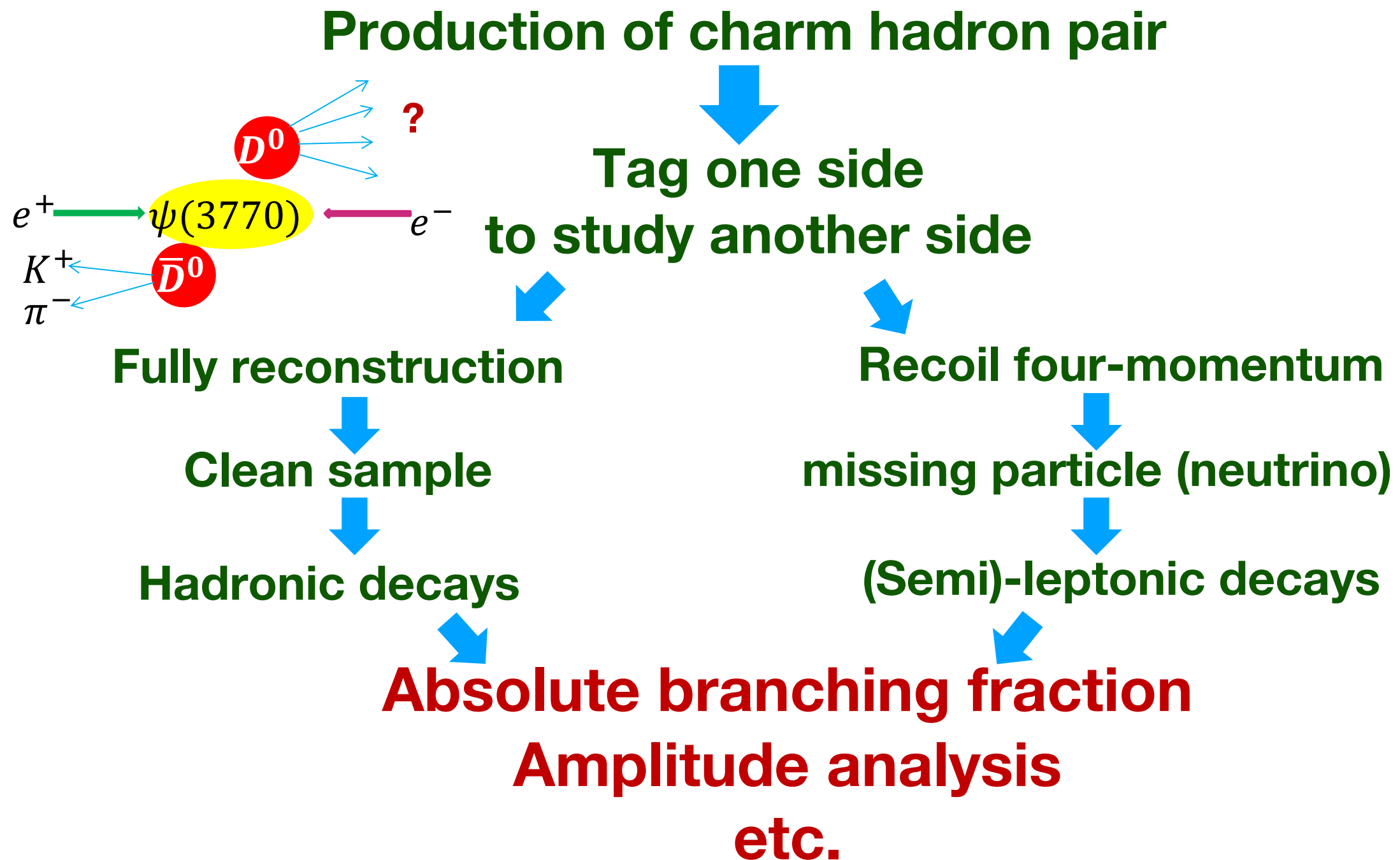
Outline

- BESIII dataset
- Pure leptonic decays
- Radiative correlation
- Summary

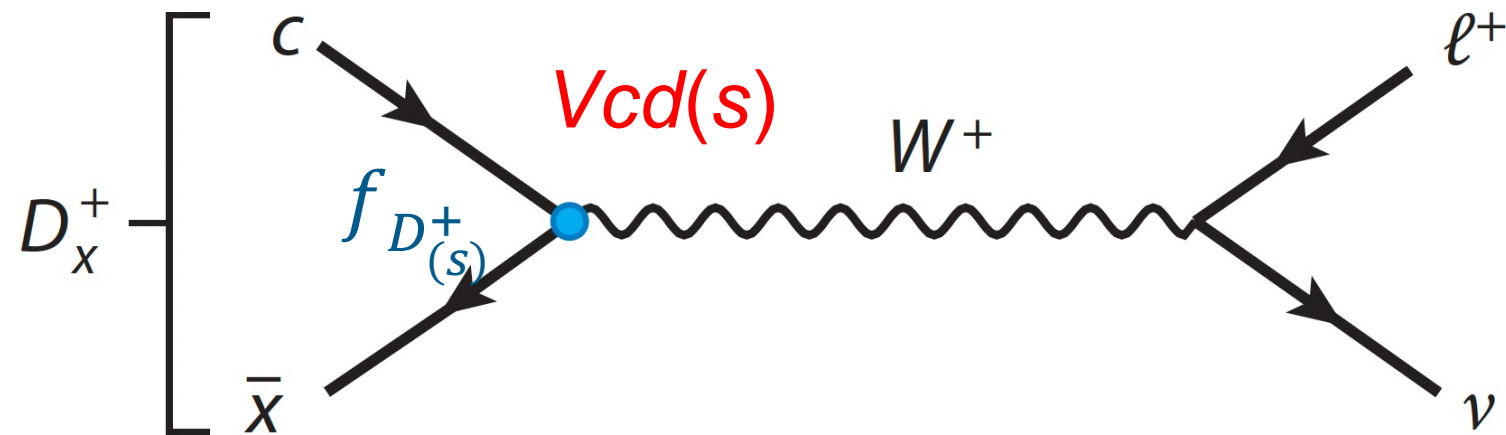


BESIII Data Taken near Threshold

- 20.3 fb⁻¹ at E_{cm} 3.773 GeV: $e^+e^- \rightarrow \psi(3770) \rightarrow D\bar{D}$
- 7.33 fb⁻¹ at E_{cm} 4.128 - 4.226 GeV: $e^+e^- \rightarrow D_s D_s^*$



Pure leptonic D decay



$$J^P = 0^- \quad \Gamma(D_{(s)}^+ \rightarrow l^+ \nu) = \frac{G_F^2 f_{D(s)^+}^2}{8\pi} |V_{cd(s)}|^2 m_l^2 m_{D(s)^+} \left(1 - \frac{m_l^2}{m_{D(s)^+}^2}\right)^2$$

$$J^P = 1^- \quad \Gamma(D_{(s)}^{*+} \rightarrow l^+ \nu) = \frac{G_F^2 f_{D(s)^{*+}}^2}{8\pi} |V_{cd(s)}|^2 m_l^2 m_{D(s)^{*+}} \left(1 - \frac{m_l^2}{m_{D(s)^{*+}}^2}\right)^2 \left(1 + \frac{m_l^2}{m_{D(s)^{*+}}^2}\right)$$

Decay constant $f_{D(s)^+}$:

Calibrate Lattice QCD

CKM matrix element $|V_{cd(s)}|$:

Test the unitarity of CKM matrix

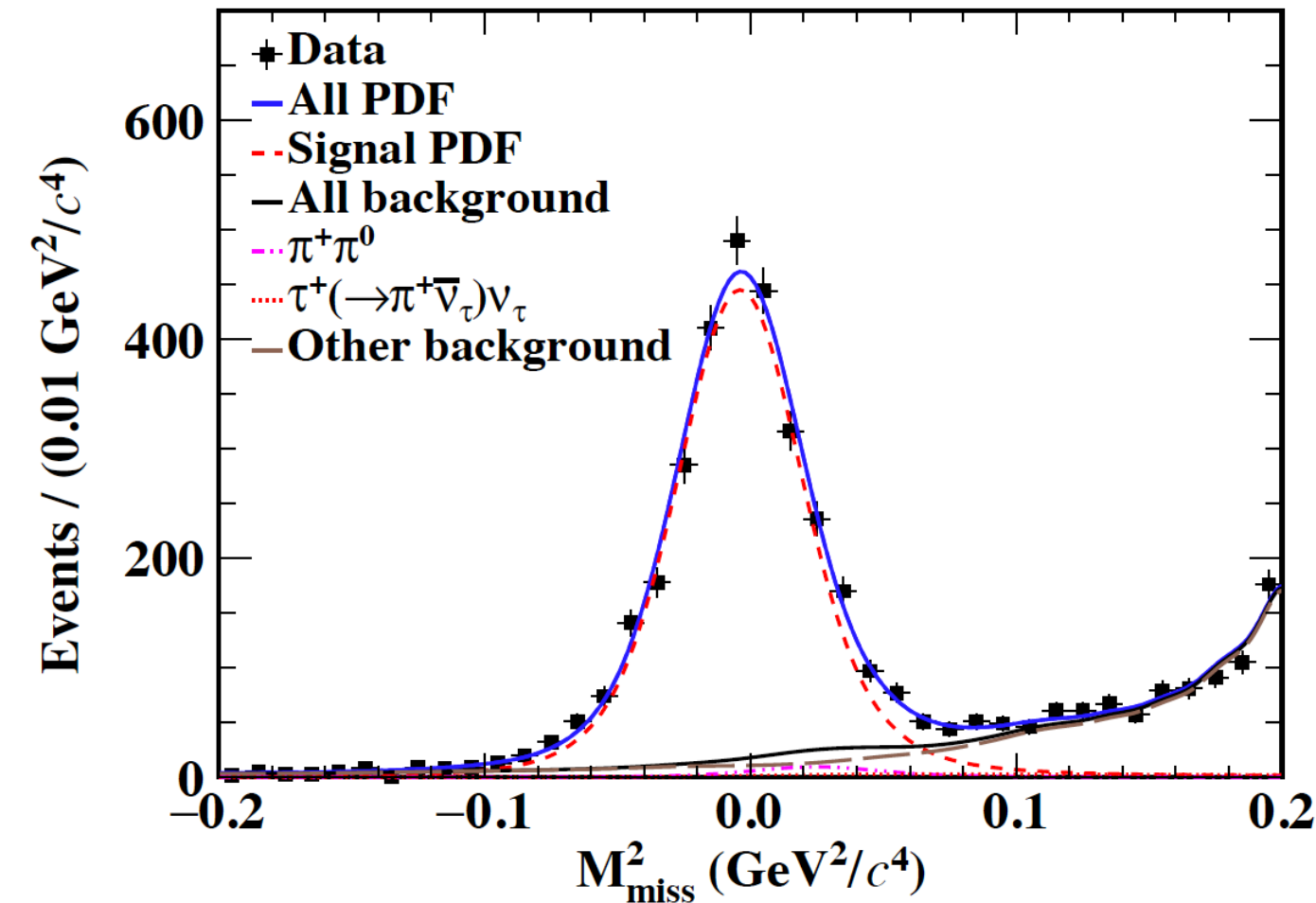
Lepton flavor universality

$e^+ \nu_e : \mu^+ \nu_\mu : \tau^+ \nu_\tau$

$D^+ \ 10^{-5} : 1 : 2.67$

$D_s^+ \ 10^{-5} : 1 : 9.75$

$$D^+ \rightarrow \mu^+ \nu_\mu$$



**Short-distance electroweak
correction increases BF by 1.8%
[PRD98,074512, NPB196,83]**

**Long-distance electroweak
correction [inner bremsstrahlung
and virtual photon] reduce BF by
2.5% with 0.6% uncertainty
[PRD98,074512]**

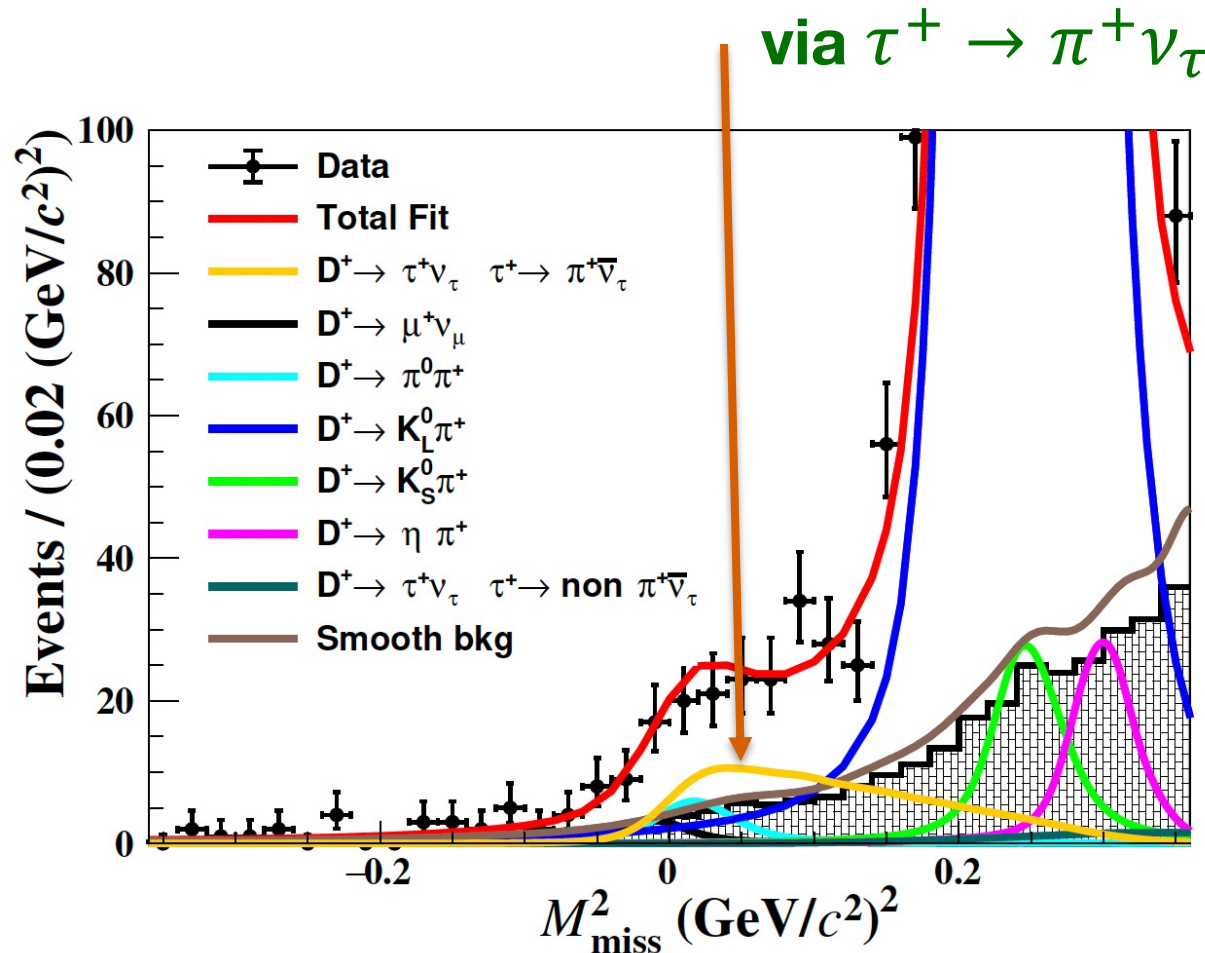
$$\mathcal{B}(D^+ \rightarrow \mu^+ \nu_\mu) = (4.034 \pm 0.080 \pm 0.040) \times 10^{-4}$$

$$f_{D^+} |V_{cd}| = (48.02 \pm 0.48_{\text{stat}} \pm 0.24_{\text{syst}} \pm 0.12_{\text{input}} \pm 0.15_{\text{EM}}) \text{ MeV}$$

Using the 20.3 fb⁻¹ full dataset

Precision: 1.2%

$$D^+ \rightarrow \tau^+ \nu_\tau$$



JHEP01(2025)89

The peaking-background shapes of $D^+ \rightarrow \pi^0 \pi^+$ and $D^+ \rightarrow K^0_L \pi^+$ are extracted from data-based control samples.

$$\mathcal{B}(D^+ \rightarrow \tau^+ \nu_\tau) = (9.9 \pm 1.1 \pm 0.5) \times 10^{-4}$$

$$R_{\tau/\mu} = \Gamma(D^+ \rightarrow \tau^+ \nu_\tau) / \Gamma(D^+ \rightarrow \mu^+ \nu_\mu) = 2.49 \pm 0.31$$

$$f_{D^+} = (204 \pm 11_{\text{stat}} \pm 5_{\text{syst}} \pm 1_{\text{input}}) \text{ MeV} \quad \textbf{Precision: } \sim 6\%$$

$$|V_{cd}| = (0.216 \pm 0.012_{\text{stat}} \pm 0.006_{\text{syst}} \pm 0.001_{\text{input}})$$

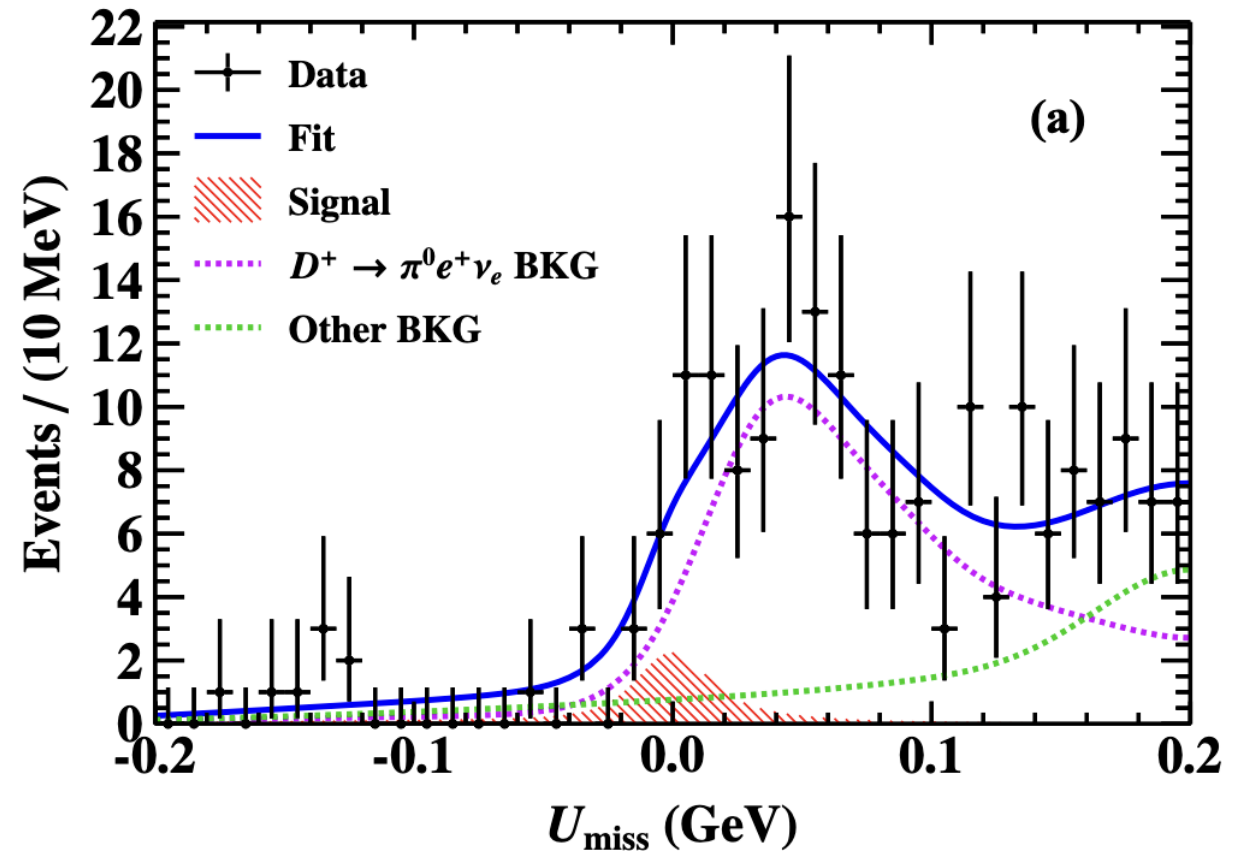
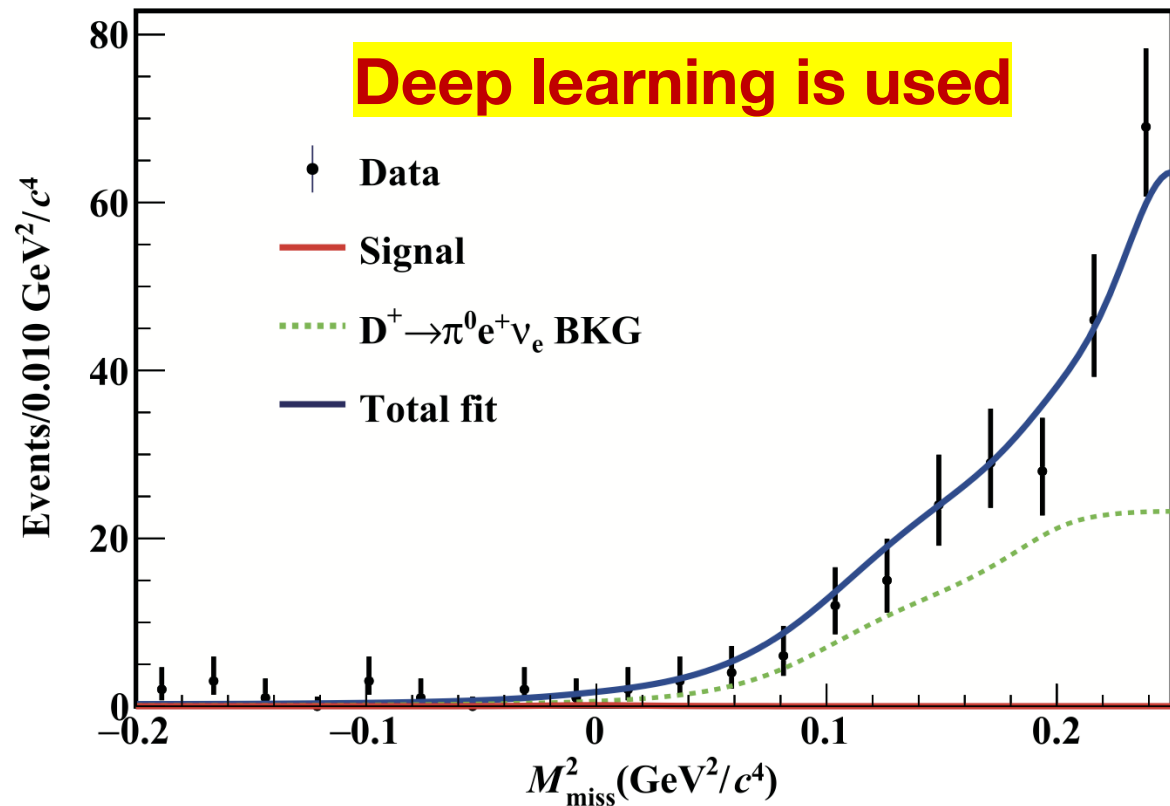
Unfortunately $D^+ \rightarrow \tau^+ \nu_\tau$ can't contribute to D^+ decay constant measurement

To be updated using the 20 fb^{-1} full dataset

$$D^+ \rightarrow e^+ \nu_e \text{ and } D^+ \rightarrow \gamma e^+ \nu_e$$

CPC 49, 063001 (2025)

arXiv:2503.16070
Accepted by CPC



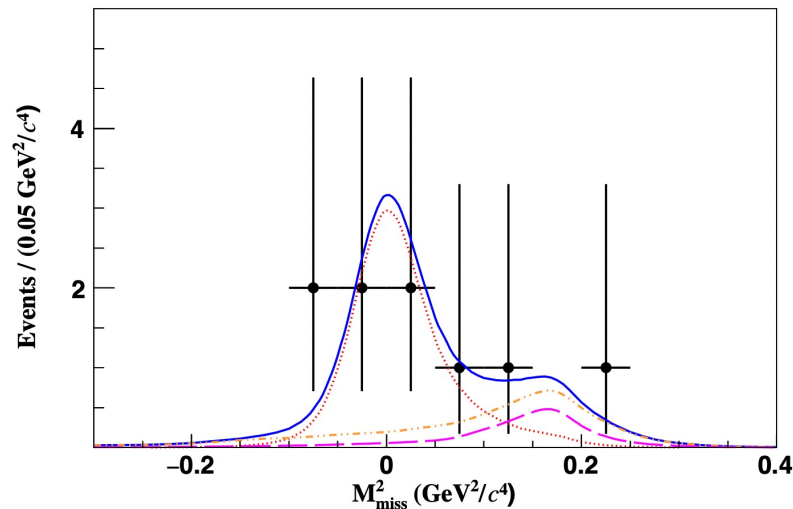
$$\mathcal{B}(D^+ \rightarrow e^+ \nu_e) < 9.7 \times 10^{-7} \text{ @90\% C. L.}$$

$$\mathcal{B}(D^+ \rightarrow \gamma e^+ \nu_e) < 1.2 \times 10^{-5} \text{ @90\% C. L.}$$

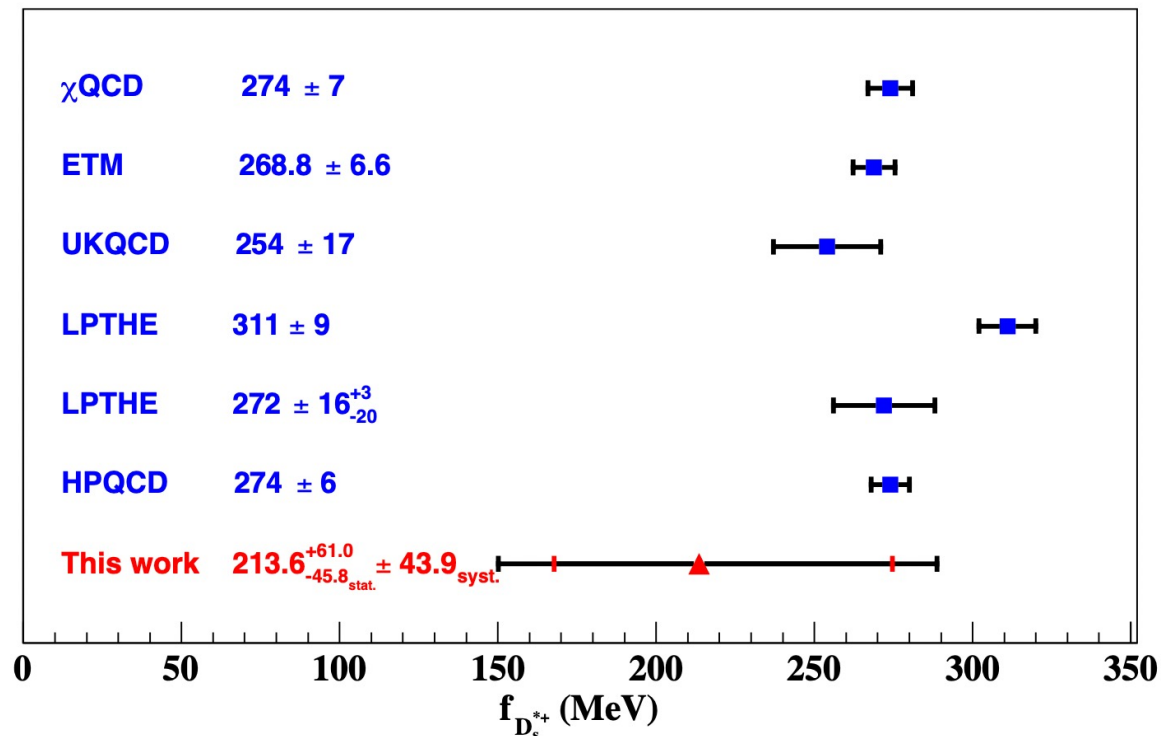
$D^+ \rightarrow e^+ \nu_e$ is helicity suppression. Predicted BF $< 10^{-8}$
 $D^+ \rightarrow \gamma e^+ \nu_e$ mitigates this suppression. Predicted BF $10^{-5} - 10^{-3}$

First experimental study of $D_s^{*+} \rightarrow e^+ \nu$

Phys. Rev. Lett. 131, 14180(2023)



First experimental result on $f_{D_s^{*+}}$

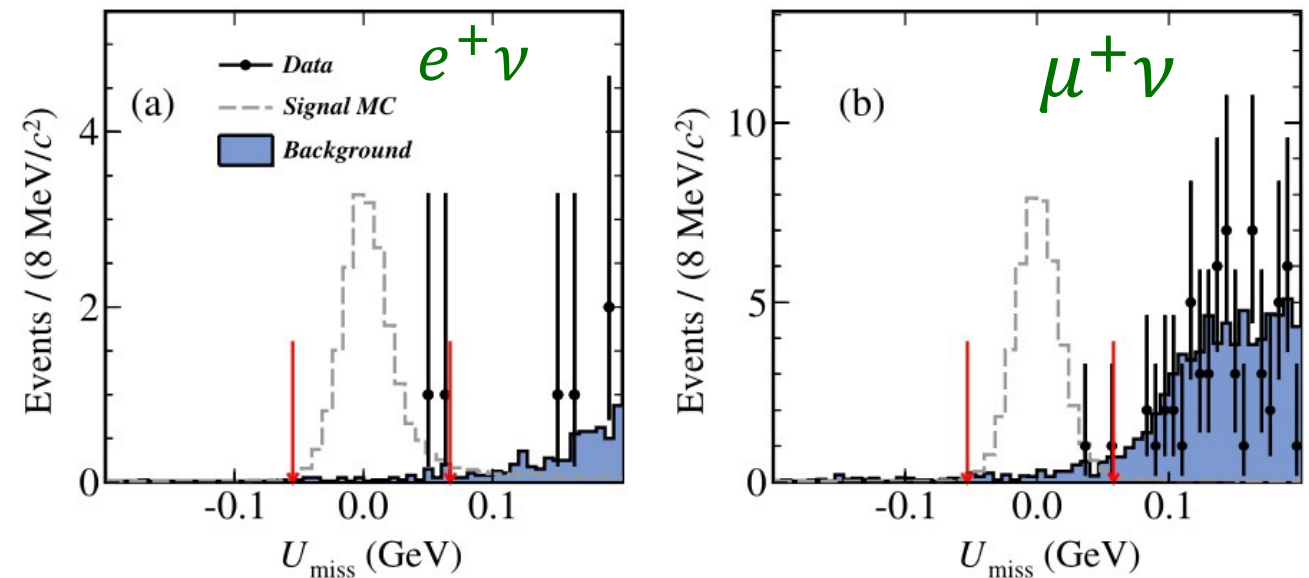


$$\mathcal{B}(D_s^{*+} \rightarrow e^+ \nu_e) = (2.1^{+1.2}_{-0.9} \pm 0.2) \times 10^{-5}$$

with significance 2.9σ

Search for $D^{*+} \rightarrow l^+ \nu$

Phys. Rev. D 110, 012003(2024)



No significant signal observed

$$\mathcal{B}(D^{*+} \rightarrow e^+ \nu_e) < 1.1 \times 10^{-5} \text{ @ 90\% C.L.}$$

$$\mathcal{B}(D^{*+} \rightarrow \mu^+ \nu_\mu) < 4.3 \times 10^{-6} \text{ @ 90\% C.L.}$$

$D^+ \rightarrow \mu^+ \nu_\mu$ 中的辐射修正

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问题回顾

- 赝标介子衰变常数和CKM矩阵元的乘积可以从纯轻衰变的最低阶分支比中提取，即下式中的 $\Gamma^{(0)}$

$$\begin{aligned}\Gamma_{D^+ \rightarrow \ell^+ \nu_\ell} &= \Gamma_{D^+ \rightarrow \ell^+ \nu_\ell}^{(0)} \left[1 + \frac{\alpha}{\pi} C_p \right] \\ &= \frac{G_F^2 f_{D^+}^2 m_{D^+}^3 |V_{cd}|^2 \mu_\ell^2 (1 - \mu_\ell^2)^2}{8\pi} \left[1 + \frac{\alpha}{\pi} C_p \right]\end{aligned}$$

修正因子

- 然而，实验测量中纯轻衰变 $D^+ \rightarrow \mu^+ \nu_\mu$ 和辐射衰变 $D^+ \rightarrow \gamma \mu^+ \nu_\mu$ 无法区分
- $D^+ \rightarrow \gamma \mu^+ \nu_\mu$ 本底需要被减除

辐射修正包含下面三种过程的修正：短程(1)和长程(2)电弱修正、结构依赖的辐射修正(3)

参考文献：PDG review、[PRD 98, 074512 \(2018\)](#)

问题回顾

- 之前实验上辐射修正处理方法:
- 长程：PHOTOS包
- 结构依赖贡献(+1.0%)：实验上减去了1.0%

This correction accounts for tree-level radiative processes in which the D meson decays into a real photon and an off-shell vector meson, which subsequently decays weakly to a charged lepton and neutrino. It is estimated using Eq. (12) of [Burdman et al.](#) with the CLEO-c [cut on the photon energy](#), which is typical of all the measurements. [\[PDG review\]](#)

- 短程贡献(+1.8%)：需要减去1.8%,实验上未处理 [PDG review减去了1.8%]

LARGE m_W, m_Z BEHAVIOUR OF THE $O(\alpha)$ CORRECTIONS TO SEMILEPTONIC PROCESSES MEDIATED BY W

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Received 17 August 1981

Using the current algebra formulation of radiative corrections and working in the framework of the $SU(2)_L \times U(1) \times SU(3)_c$ theory, we derive a theorem that governs the large m_W, m_Z behaviour of the $O(\alpha)$ corrections to general semileptonic processes mediated by W. The leading asymptotic dependence is logarithmic with a universal coefficient not affected by the strong interactions. As a byproduct, we obtain the leading asymptotic effect induced perturbatively by the strong interactions, which is of $O(\ln \ln (m_W/\Lambda))$.

问题回顾

- PRL的referee认为之前处理方式有争议
- 尤其是对于长程贡献的估计:PHOTOS不能估计特定的辐射修正

I find **controversial the way the Section 72.3.1** treats the radiative corrections for D decays. When I referred to the PDG review previously I kept in mind more the charged kaon decay where high precision measurements already have done long time ago and the theory for radiative corrections is well established. **PHOTOS cannot be a replacement for the explicit radiative correction treatment for $|V_{cd}|$ and f_{D^+} extractions.**

[\[https://hnbcs3.ihep.ac.cn/HyperNews/get/paper769/24.html\]](https://hnbcs3.ihep.ac.cn/HyperNews/get/paper769/24.html)

我们的做法

一、结构依赖相关部分

- 引用BESIII最新测量的 $D^+ \rightarrow \gamma e^+ \nu_e$ 分支比 [[arXiv:2503.16070.](#)]
- 估计其贡献，在对数据的拟合中固定该背景事例数
- 具体结果： $D^+ \rightarrow \gamma e^+ \nu_e$ 的贡献事例数为 6.6 ± 4.6 ，造成0.17%的系统误差

给出了 $D^+ \rightarrow \mu^+ \nu_\mu$ 的分支比: $\mathcal{B}(D^+ \rightarrow \mu^+ \nu_\mu) = (4.034 \pm 0.080 \pm 0.040) \times 10^{-4}$

提取 $f_{D^+} |V_{cd}|$ 时，我们对所测分支比进行了长程和短程电弱修正

二、长程和短程电弱修正:根据[Fermilab Lattice and MILC Collaborations](#)处理

- 短程: 减去1.8% [[Sirlin](#)]
- 长程: 增加2.5% [[Kinoshita's work](#)]
- 由于介子结构不确定性造成的电弱修正系统误差为0.6% [[PRD 98, 074512 \(2018\)](#)]

PDG Sec.7.3.1

To obtain the purely leptonic rates $\Gamma^{(0)}(D^+ \rightarrow \mu^+(\tau^+)\nu)$, we subtract the radiative contributions as in Sec. 72.2.1, but use numerical values for the corrections appropriate for D mesons. First, we reduce both the $\mu^+\nu$ and $\tau^+\nu$ branching fractions in Table 72.2 by 1.8%, which is the universal short-distance electroweak contribution of Sirlin [4] evaluated using the D -meson mass for the factorization scale. We do not adjust the experimental rates by the universal long-distance correction [5]. This is because QED bremsstrahlung contributions have already been subtracted at leading-log order from the measurements in Table 72.2 using Monte-Carlo estimates computed with PHOTOS [12]. The $\mu^+\nu$ rates should also be reduced by the 1% estimate of the structure-dependent contributions from Dobrescu and Kronfeld [17]. This correction accounts for tree-level radiative processes in which the D meson decays into a real photon and an off-shell vector meson, which subsequently decays weakly to a charged lepton and neutrino. It is estimated using Eq. (12) of Burdman *et al.* [7] with the CLEO-c cut on the photon energy from Ref. [74], which is typical of all the measurements. We do not need to apply the structure-dependent correction to the $\mu^+\nu$ branching fractions in Table 72.2, however, because the experiments have already included it in their quoted results. Therefore, in summary, we reduce both the $D^+ \rightarrow \mu^+\nu$ and the $D^+ \rightarrow \tau^+\nu$ rates by 1.8% to account for radiative corrections. It is worth noting, however, that

[4] A. Sirlin, *Nucl. Phys.* **B196**, 83 (1982).

[5] T. Kinoshita, *Phys. Rev. Lett.* **2**, 477 (1959).

LQCD辐射修正处理方法

The products of decay constants times CKM factors from the Particle Data Group [67],

$$(f_{D^+}|V_{cd}|)_{\text{expt}} = 45.91(1.05) \text{ MeV}, \quad (7.31)$$

$$(f_{D_s^+}|V_{cs}|)_{\text{expt}} = 250.9(4.0) \text{ MeV}, \quad (7.32)$$

are obtained by averaging the experimentally-measured decay rates into electron and muon final states. The value for $f_{D^+}|V_{cd}|$ in Eq. (7.31) includes the correction from structure-dependent bremsstrahlung effects that lowers the $D^+ \rightarrow \mu^+ \nu_\mu$ rate by $\sim 1\%$ [113,114]. Other electroweak corrections, however, are not accounted for in the PDG averages shown above. The electroweak contributions to leptonic pion and kaon decays are estimated to be about one or two percent [115,116], and the uncertainties in these corrections lead to $\sim 0.1\%$ uncertainties in $|V_{us}|/|V_{ud}|$ and $|V_{us}|$. Now that the errors on f_D and f_{D_s} are well below half a percent, electroweak corrections must also be included when extracting $|V_{cd}|$ and $|V_{cs}|$ from leptonic D -meson decays.

We take the estimate of the electroweak corrections to the leptonic $D_{(s)}^+$ -meson decay rates from our earlier work [23], which includes all contributions that are included for pion and kaon decays. We first adjust the experimental decay rates quoted in the PDG by the known long- and short-distance electroweak corrections [117,118]. The former lowers the D^+ - and D_s -meson leptonic decay rates by about 2.5%, while the latter increases them by about 1.8%, such that the net effect is a slight decrease in the rates by less than a percent. We then include a 0.6% uncertainty to account for unknown electromagnetic corrections that depend upon the mesons' structure. This estimate is based on calculations of the structure-dependent electromagnetic corrections to pion and kaon decays [115,119,120], but allowing for much larger coefficients than for the light pseudoscalar mesons.

[117] T. Kinoshita, Phys. Rev. Lett. **2**, 477 (1959).

[118] A. Sirlin, Nucl. Phys. **B196**, 83 (1982).

LQCD辐射修正处理方法

Nuclear Physics B196 (1982) 83–92
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LARGE m_W, m_Z BEHAVIOUR OF THE $O(\alpha)$ CORRECTIONS TO SEMILEPTONIC PROCESSES MEDIATED BY W

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Using the current algebra formulation of radiative corrections and working in the framework of the $SU(2)_L \times U(1) \times SU(3)_c$ theory, we derive a theorem that governs the large m_W, m_Z behaviour of the $O(\alpha)$ corrections to general semileptonic processes mediated by W. The leading asymptotic dependence is logarithmic with a universal coefficient not affected by the strong interactions. As a byproduct, we obtain the leading asymptotic effect induced perturbatively by the strong interactions, which is of $O(\ln \ln (m_W/\Lambda))$.

LQCD辐射修正处理方法

RADIATIVE CORRECTIONS TO π - e DECAY*

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(Received May 11, 1959)

As is well known, the ratio of probabilities for π - e and π - μ decays is given by¹

$$R_0 = \left(\frac{m_e}{m_\mu} \right)^2 \left(\frac{m_\pi^2 - m_e^2}{m_\pi^2 - m_\mu^2} \right)^2 = 1.28 \times 10^{-4}, \quad (1)$$

neglecting electromagnetic corrections, if the decay interaction is assumed to be

$$ig\bar{\psi}_l \gamma_\mu a \psi_\nu (\partial \varphi_\pi / \partial x_\mu), \quad (2)$$

where $a = (1 + i\gamma_5)/2$ and l represents either muon or electron. This interaction is consistent² with

the hypothesis of universal V - A interaction of Fermi couplings.³ Recent experiments⁴ support these assumptions strongly. The new measurements are becoming sufficiently accurate to justify a calculation of the effect of radiative corrections. This problem has recently been studied by Berman,⁵ who has found surprisingly large corrections to π - e decay. In this note it is attempted to understand the reason why the radiative corrections are so large. We are also interested to see whether the pion decay agrees with the recently conjectured "theorem"⁶ that the radiative correction to the total probability of a decay process is finite in the limit where the

LQCD辐射修正处理方法

The former value is the most precise measurement in the context of the standard model, while the latter does not use any standard model assumptions. In both cases the additional systematic errors due to the D^+ lifetime measurement and the error on $|V_{cd}| = |V_{us}|$ are negligible.

The data have already been corrected for final state radiation of the muon, as our Monte Carlo simulation incorporates this effect [31]. There is however, another process where the $D^+ \rightarrow \gamma D^{*+} \rightarrow \gamma \mu^+ \nu$, where the D^{*+} is a virtual vector or axial-vector meson. The $D^{*+} \rightarrow \mu^+ \nu$ transition is not helicity-suppressed, so the factor α for radiation is compensated by a relative factor $(M_{D^+}/m_\mu)^2$. Using Eq. (12) of Burdman *et al.* [32] and imposing the 250 MeV photon cut, we find that the radiative rate is approximately 1%, to which we assign a $\pm 1\%$ systematic error. This is essentially the same calculation done by Dobrescu and Kronfeld for $D_s^+ \rightarrow \mu^+ \nu$ decays [6]. (The results shown above for the branching fractions and f_{D^+} are all radiatively corrected; the branching fractions have been reduced by 1%.)



[31] FSR: PHOTOS



[32] 结构依赖部分

PHYSICAL REVIEW D

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Radiative leptonic decays of heavy mesons

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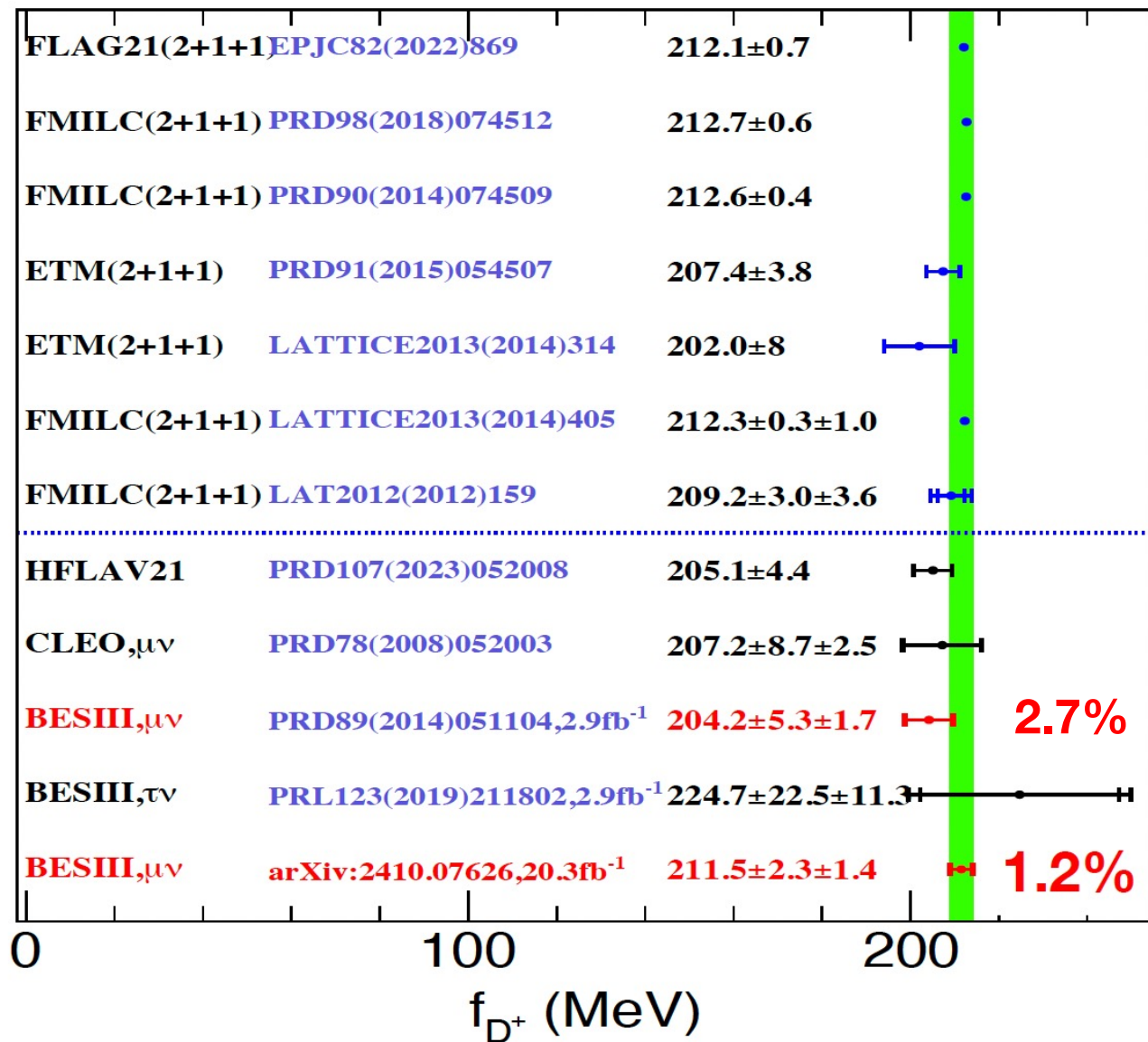
We compute the photon spectrum and the rate for the decays $B(D) \rightarrow l \nu \gamma$. These photonic modes constitute a potentially large background for the purely leptonic decays which are used to extract the heavy meson decay constants. While the rate for $D \rightarrow l \nu \gamma$ is small, the radiative decay in the B meson case could be of comparable magnitude or even larger than $B \rightarrow \mu \nu$. This would affect the determination of f_B if the τ channel cannot be identified. We obtain theoretical estimates for the photonic rates and discuss their possible experimental implications.

PACS number(s): 13.40.Hq, 13.20.Fc, 13.20.He

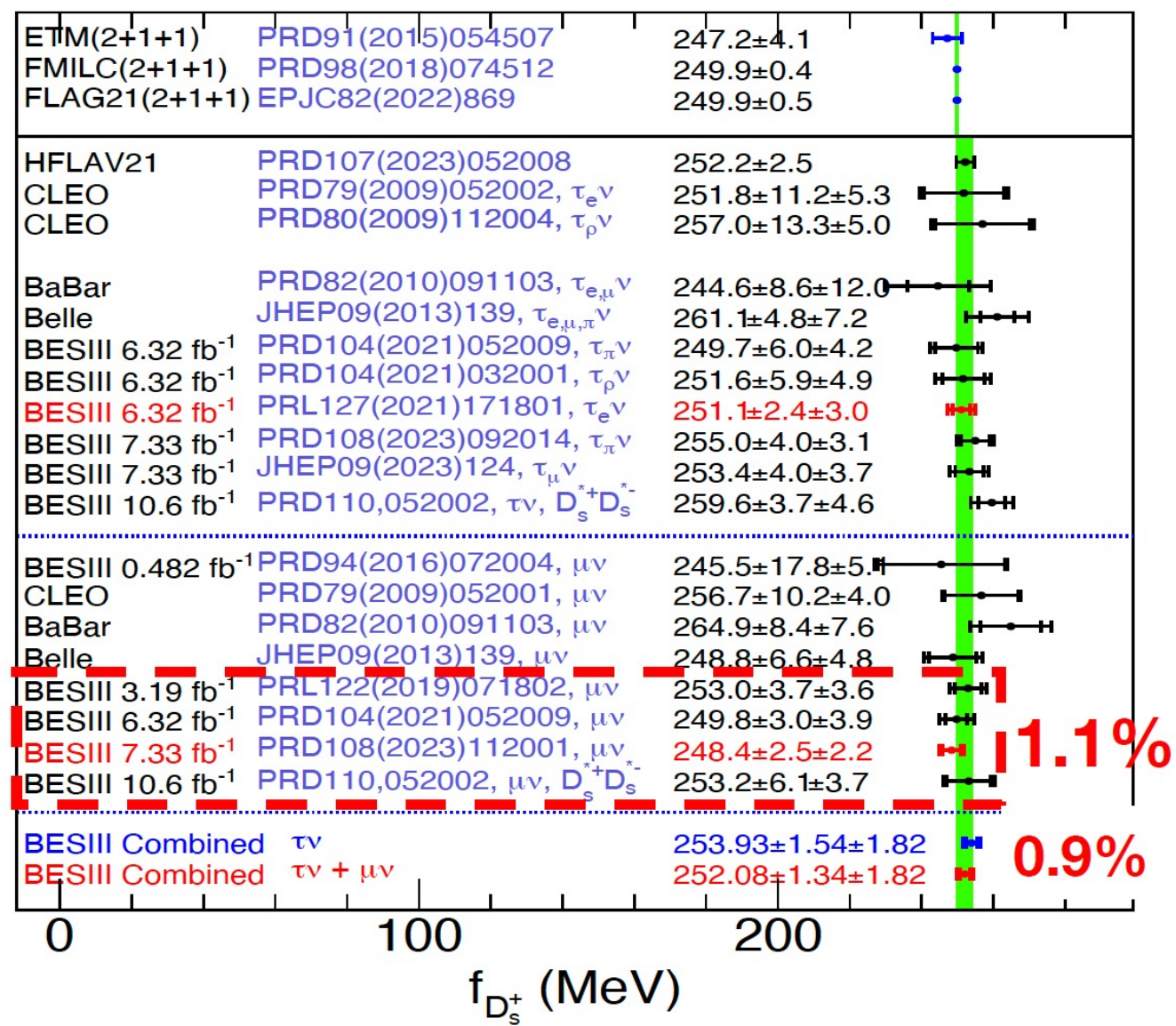
In this paper we investigate the decay modes $B^-(D^-) \rightarrow l^- \nu_l \gamma$. There are two types of contributions: internal bremsstrahlung (IB) and structure-dependent (SD) photon emission [6]. As is known, the IB contributions are still helicity suppressed. On the other hand, the SD contributions are reduced by the electromagnetic coupling constant α but they are *not* suppressed by the charged lepton mass. Therefore, what in principle could be regarded as a mere radiative correction to the purely leptonic decays has the potential to be of comparable magnitude and in some cases even much larger. In

Comparison of decay constant

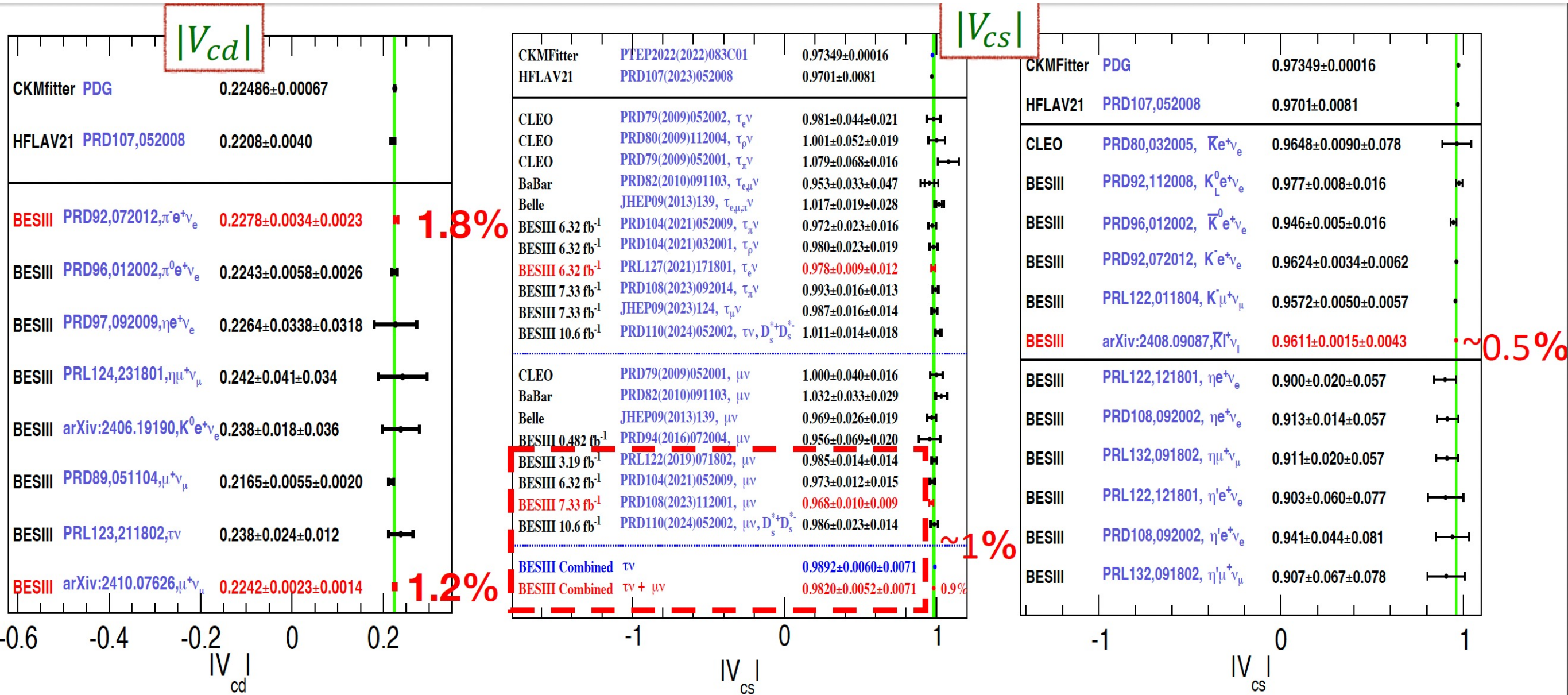
$$f_{D^+}$$



$$f_{D_s^+}$$



Comparison of $|V_{cd(s)}|$



Both pure- and semi-leptonic decays contribute

Thanks for your attention

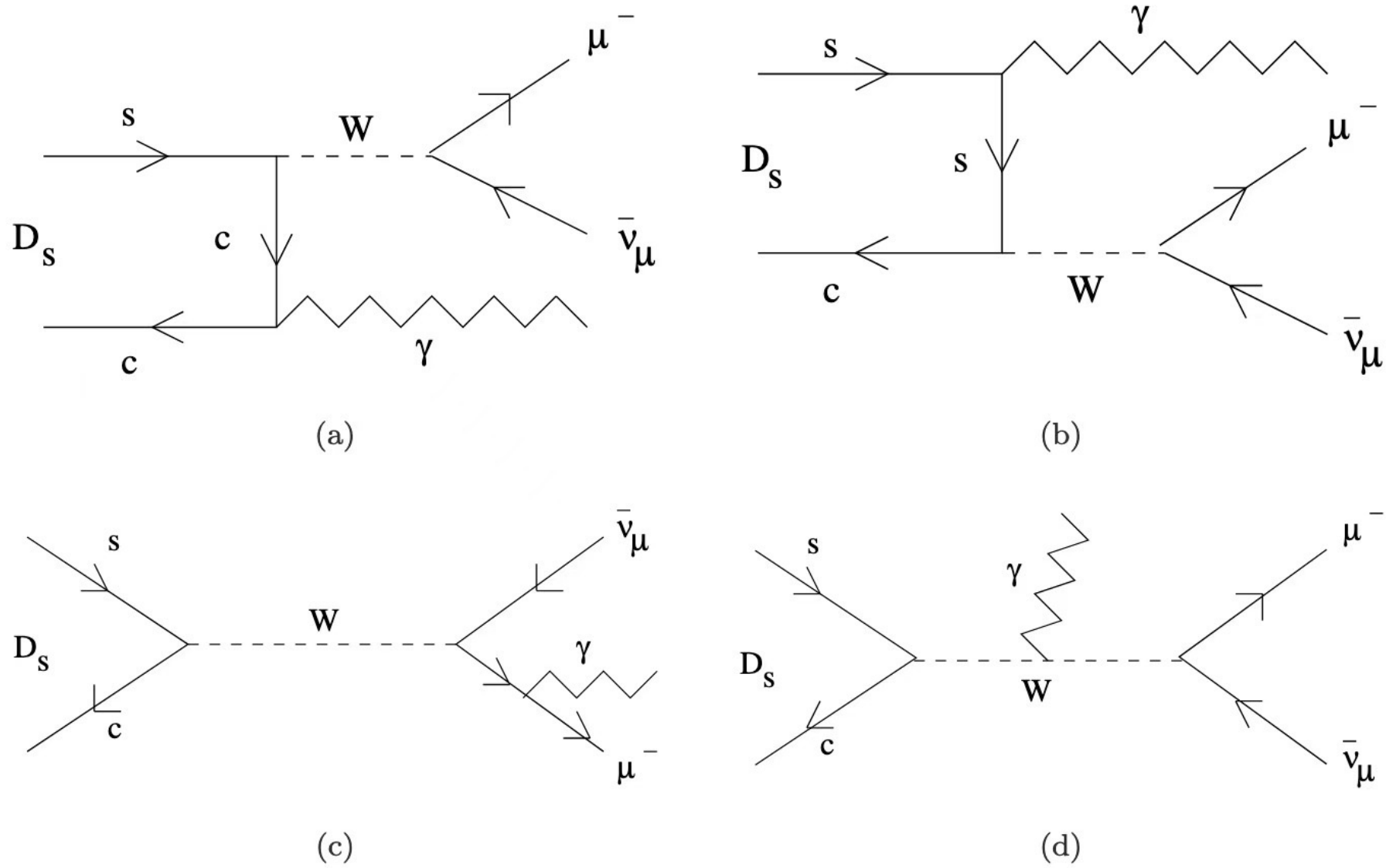


Fig. 1. Feynmann diagrams for the decay $D_s \rightarrow \mu \nu_\mu \gamma$.