粲介子半轻衰变的格点研究

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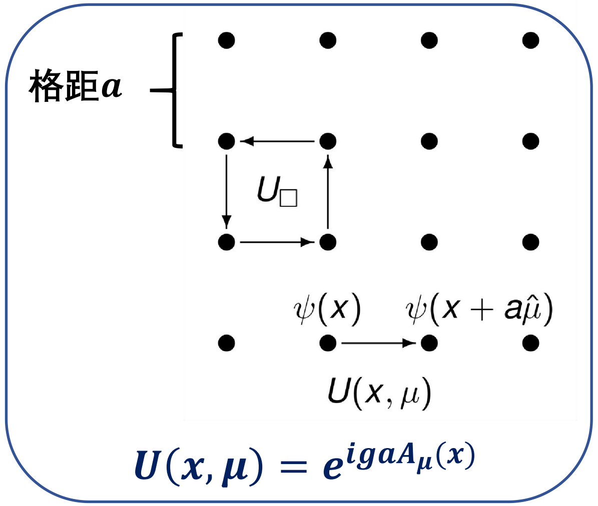
报告提纲

- **格点QCD**
- **粲介子半轻衰变研究**
  - $D \to Kl\nu$ T. Shen, ..., ZL et al., 2503.01219, PoS LAT2024 (2025) 294
  - $D_s \to \phi l\nu$ G. Fan, Y. Meng, ZL et al., in progress

格点QCD (1973, Wilson; 1979, Creutz) 用数值模拟研究QCD的非微扰性质



- 4维闵氏时空 → 4维欧氏空间（$\tau = it$）
- $\langle O\rangle = \dfrac{\int DA_\mu D\bar{\psi}D\psi O[A,\bar{\psi},\psi]e^{-\int \mathcal{L}_{QCD}\,d^4x}}{\int DA_\mu D\bar{\psi}D\psi e^{-\int \mathcal{L}_{QCD}\,d^4x}}$，$\mathcal{L}_{QCD} = \bar{\psi}M[A]\psi + \mathcal{L}_G$

$$M = \gamma\cdot D + m_q$$

- $\langle O\rangle = \dfrac{\int DU_\mu O[U,M^{-1}[U]]\mathrm{Det}[M[U]]e^{-S_G}}{\int DU_\mu \mathrm{Det}[M[U]]e^{-S_G}} \sim \dfrac{\int dx\, f(x)\,\rho(x)}{\int dx\,\rho(x)}$ ➡ $\sim \dfrac{1}{N}\sum_{n=1}^{N} f(x_n)$

离散的x_n按$\rho(x_n)$分布

上式为带权重$\mathrm{Det}[M[U]]e^{-S_G}$的平均，类似Boltzmann系综平均

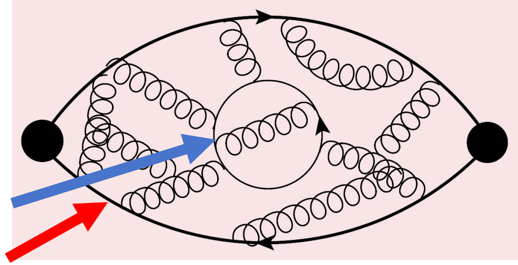
- 在有限体积4维超立方格子上，自由度个数可数，路径积分具有良好定义
- 巨大高维积分，无法直接计算；用重点抽样按权重分布产生U（组态）
- 路径积分变为对组态的统计平均：$\langle O\rangle=\frac{1}{N}\sum_{i=1}^{N} O_i$

  N有限，统计误差$\sim 1/\sqrt{N}$

输入若干实验测量值，确定自由参数：
格距 a 和夸克质量 m_q。预言其他结果

海夸克效应 $\mathrm{Det}[M[U]]$

价夸克传播子 $M^{-1}[U]$



介子两点关联函数

费米子格点作用量

Naive fermions（简单费米子作用量）

标量场作用量中含二次求导

- 考虑自由的夸克场

$$S = \int d^4x \bar{\psi}(\gamma_\mu \partial_\mu + m)\psi$$

费米子作用量中含一次求导

- 简单（naive）地格点离散化给出：

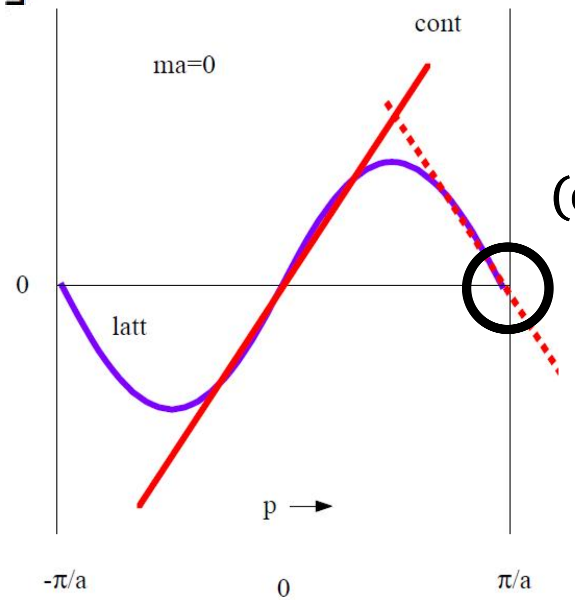
$$S^{naive} = a^4 \sum_x \left[\bar{\psi}_x \sum_\mu \gamma_\mu \frac{\psi_{x+a\hat{\mu}} - \psi_{x-a\hat{\mu}}}{2a} + m\bar{\psi}_x \psi_x \right]$$

- 变换到动量空间，得到狄拉克算符（夸克传播子G_{naive}的逆）

$$G^{-1}_{naive} = i\gamma_\mu \frac{\sin(p_\mu a)}{a} + m$$

- $a \to 0$时，$\sin(p_\mu a) \to p_\mu a$，因此有

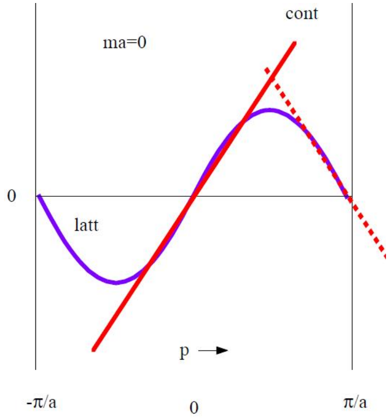
$$G^{-1}_{naive} = i\gamma_\mu \frac{\sin(p_\mu a)}{a} + m = i\gamma_\mu p_\mu + m + \mathcal{O}(a^2) = G^{-1}_{cont} + \mathcal{O}(a^2)$$



加倍子
(doubler)

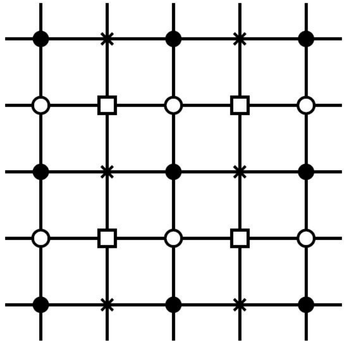
Wilson费米子（加上二次导数项：Wilson项）

- 若不牺牲局域性和连续极限，就得接受加倍子或牺牲手征对称性
- 连续极限下，Wilson项消失
- Wilson项破坏手征对称性，通过抬高加倍子质量，使其和低能区物理脱耦
- $m = 0$时，pion的质量不等于零



Staggered费米子（Kogut和Susskind，1975）：

- 通过一个变换将费米子作用量在自旋空间对角化，只留下一个自旋分量；
- 将16个加倍子分成4个taste，每个有4个自旋；
- 每个加倍子分布在4 维的小hypercubic格点上（布里渊区缩小一半）。
- 一种物理的味道对应四种taste。有taste symmetry breaking。
- 保留部分手征对称性$U(1)$。模拟计算快。



overlap费米子

Ginsparg & Wilson, 1982

- overlap费米子的Dirac算符M满足Ginsparg-Wilson关系： $\gamma_5 M + M\gamma_5 = \frac{a}{r_0} M\gamma_5 M$

a为格距，r_0是一个常实数

- 以一种温和的方式破坏手征对称性：$\gamma_5 M + M\gamma_5 = 0$
- Ginsparg-Wilson关系可以改写为

$$\left(1 - \frac{a}{2r_0}M\right)\gamma_5 M + M\gamma_5\left(1 - \frac{a}{2r_0}M\right) = 0$$

因此overlap费米子作用量满足如下$U_V(n_f)\otimes U_A(n_f)$手征对称性

$$\psi \to \exp\left(-i\theta\gamma_5\left(1 - \frac{a}{2r_0}M\right)\right)\psi, \quad \bar{\psi} \to \bar{\psi}\exp\left(-i\theta\left(1 - \frac{a}{2r_0}M\right)\gamma_5\right)$$

费米子作用量

- Domain wall fermion，近似满足 Ginsparg-Wilson 关系
- Wilson twisted mass fermion，加入 twisted mass 项，避免数值模拟时出现 exceptional configuration
- Highly improved staggered fermion (HISQ)
- ……

- Many fermion lattice actions:
  - All go back to the continuum action as $a \to 0$.
  - Preserve local gauge symmetry.
  - Each has advantages and disadvantages.

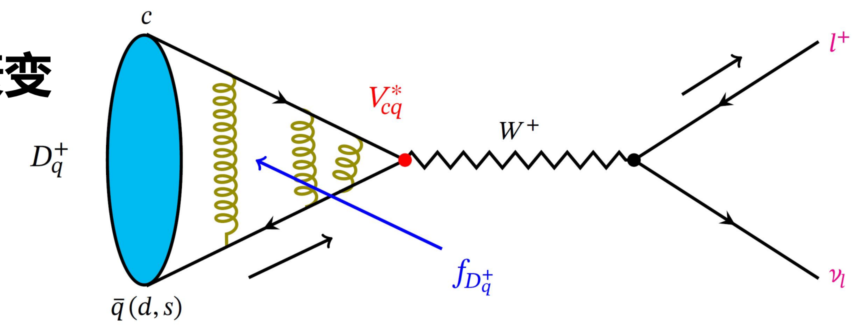
- staggered fermion 模拟速度快，但有4种 taste 和 taste symmetry breaking，且不方便用于研究重子
- Wilson clover fermion 耗时中等，但破坏手征对称性
- Wilson twisted mass fermion 破坏同位旋对称性
- Domain wall fermion 和 overlap fermion 有良好的手征对称性，但数值模拟非常昂贵

粲强子含轻衰变与LQCD

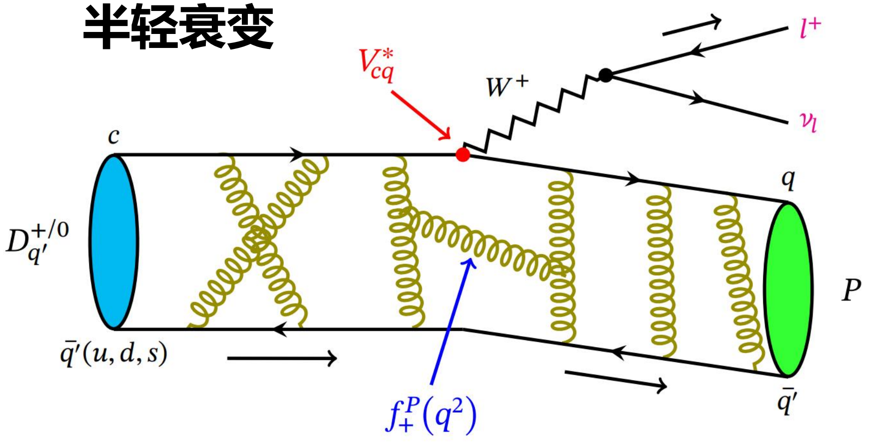
- LQCD can calculate form factors and meson decay constants appearing in weak decays of hadrons
- Combined with experiments, they can give us CKM matrix elements
- Test the SM (is the CKM matrix unitary?)
- Or use V_{ab} from elsewhere to compare QCD/SM results with experiments

$\mathbf{V_{ud}}$	$\mathbf{V_{us}}$	$\mathbf{V_{ub}}$
$\pi \to \ell\nu$	$K \to \ell\nu$	$B \to \pi\ell\nu$
	$K \to \pi\ell\nu$	
$\mathbf{V_{cd}}$	$\mathbf{V_{cs}}$	$\mathbf{V_{cb}}$
$D \to \ell\nu$	$D_s \to \ell\nu$	$B \to D\ell\nu$
$D \to \pi\ell\nu$	$D \to K\ell\nu$	$B \to D^*\ell\nu$
$\mathbf{V_{td}}$	$\mathbf{V_{ts}}$	$\mathbf{V_{tb}}$
$B_d \leftrightarrow \overline{B}_d$	$B_s \leftrightarrow \overline{B}_s$	

纯轻衰变



半轻衰变



courtesy arXiv:2103.00908

CKM矩阵元 LQCD计算

例如: $$\Gamma(P \to \ell\nu) = \frac{G_F^2|V_{q_1q_2}|^2}{8\pi} f_P^2 m_\ell^2 M_P \left(1 - \frac{m_\ell^2}{M_P^2}\right)^2$$

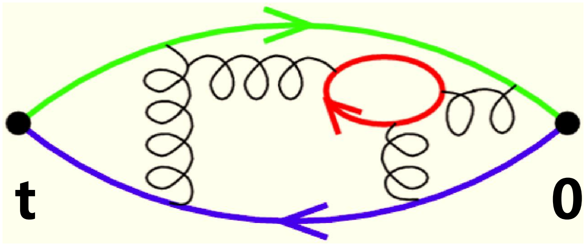
实验测量

强子矩阵元的格点计算

$$C(t) = \langle\Omega|O(t)O^\dagger(0)|\Omega\rangle \xrightarrow{t\to\infty} |\langle\Omega|O|P\rangle|^2 e^{-m_P t} \equiv A e^{-m_P t}$$

- 介子衰变常数从两点函数抽取，例如$O = \bar{q}\gamma_0\gamma_5 c$

$$\langle 0|\bar{q}(0)\gamma_\mu\gamma_5 c(0)|P(p)\rangle = i f_P p_\mu, \quad q = d, s$$

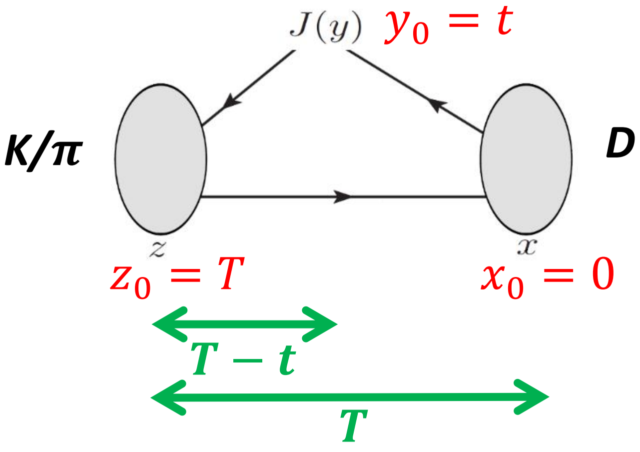


两点关联函数

- 结合两点和三点函数可抽取半轻过程强子矩阵元（形状因子）：

$$C_3(\vec{p},\vec{p}\,',T,t) = \sum_{\vec{z}}\sum_{\vec{y}} \langle 0|O_P(\vec{z},T)J(\vec{y},t)O_D^\dagger(\vec{x},0)|0\rangle\, e^{-i\vec{p}\cdot\vec{z}} e^{i\vec{q}\cdot\vec{y}}$$

$$\xrightarrow[(T-t)\to\infty]{t\to\infty,\ T\to\infty} \langle 0|O_P|P\rangle\langle P|J|D\rangle\langle D|O_D^\dagger|0\rangle e^{-m_D t} e^{-m_P(T-t)}$$



三点关联函数

- 算符的重正化常数
  - 由于离散效应，格子上的局域(轴)矢量流 $\bar{q}\gamma_\mu c$ ($\bar{q}\gamma_\mu\gamma_5 c$) 需要归一化常数 $Z_{V,A}$
  - 标量及张量流算符随能标跑动，$Z_{S,T}$
- 使用手征格点费米子有时可避免重正化常数的计算（$Z_S Z_m = 1$）
  - PCAC：$(m_q + m_c)\langle 0|\bar{q}(0)\gamma_5 c(0)|P(p)\rangle = f_P m_{PS}^2$
- PCVC： $\langle K|S|D\rangle = f_0^{D\to K}(q^2)\dfrac{M_D^2 - M_K^2}{m_c - m_s}$

粲介子半轻衰变

- $D \to \pi l \nu$， $D \to K l \nu$可用于确定 $|V_{cd}|$和$|V_{cs}|$

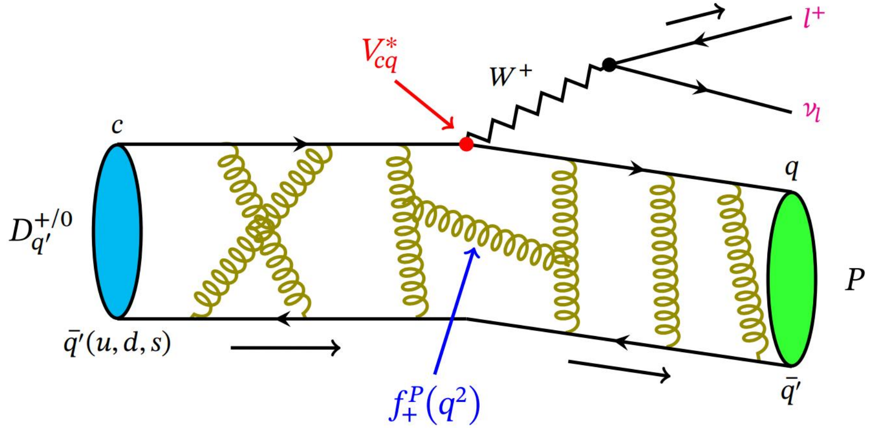
$$\frac{d\Gamma(D \to K\ell\nu)}{dq^2} = (\text{known})\,|\mathbf{p}_K|^3\,|V_{cs}|^2\,|f_+^{D\to K}(q^2)|^2$$

- 非微扰输入量：形状因子$f_{+/0}(q^2)$

$$\langle K|V^\mu|D\rangle = f_+(q^2)\left(p_D^\mu + p_K^\mu - \frac{m_D^2 - m_K^2}{q^2}q^\mu\right) + f_0(q^2)\frac{m_D^2 - m_K^2}{q^2}q^\mu$$

$$f_+(0) = f_0(0)$$

对于$l = e, \mu$, 形状因子f_0对衰变宽度的贡献较小（正比于m_l^2）



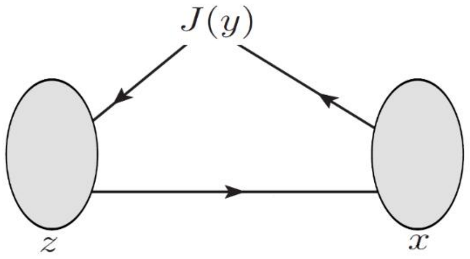
courtesy arXiv:2103.00908

- 标量流形状因子 $\langle K|S|D\rangle = f_0^{D\to K}(q^2)\frac{M_D^2 - M_K^2}{m_c - m_s}$ (手征格点费米子有$Z_S Z_m = 1$, 无需计算重整化常数)

- 初末态强子四动量：p、p'

  $q^2 = (p - p')^2$，格点计算中3-动量取分立值

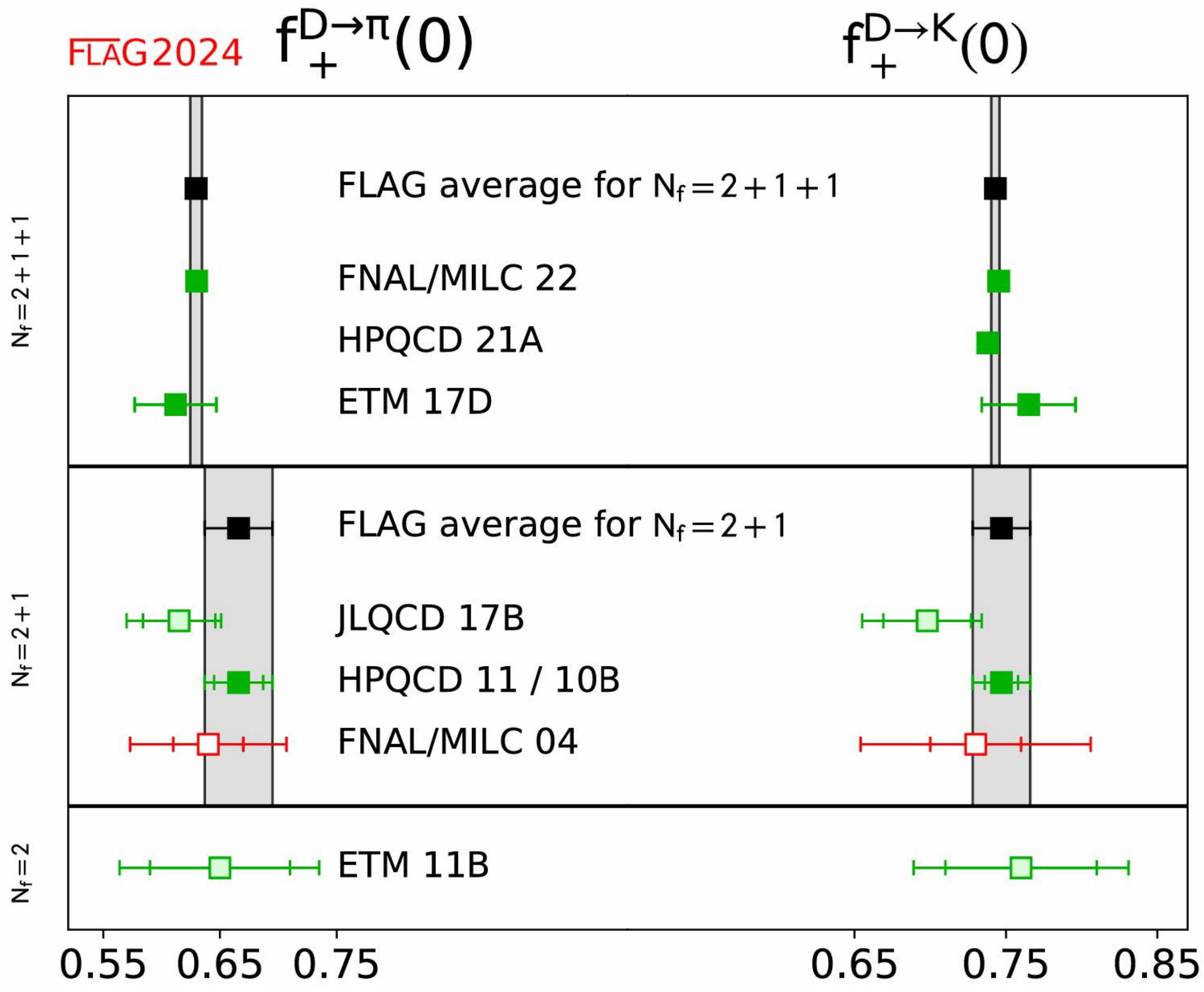
K/π



D

三点关联函数

$f_+(q^2 = 0)$ for $D \to \pi/K$



FLAG Review 2024, arXiv:2411.04268
Results up to 2024.4.30

2+1+1味

- $f_+^{D\pi}(0) = 0.6296(50)$ [ETM 17D: PRD96 (2017) 054514, 1706.03017;
- FNAL/MILC 22: 2212.12648, PRD107.094516]
- $f_+^{DK}(0) = 0.7430(27)$ [ETM 17D: PRD96 (2017) 054514, 1706.03017. HPQCD 21A: PRD104 (2021) 034505, 2104.09883. FNAL/MILC 22: 2212.12648, PRD107.094516]

2+1味

- JLQCD 17: LAT2017 [1711.11235]
- FNAL/MILC/HPQCD 04: 一个格距，m_π大于500 MeV
- $f_+^{D\pi}(0) = 0.666(29)$ [HPQCD 11, PRD84,114505, 1109.1501]
- $f_+^{DK}(0) = 0.747(19)$ [HPQCD 10B, PRD82,114506, 1008.4562]

$D\rightarrow\pi/K,\ D_s\rightarrow K$形状因子

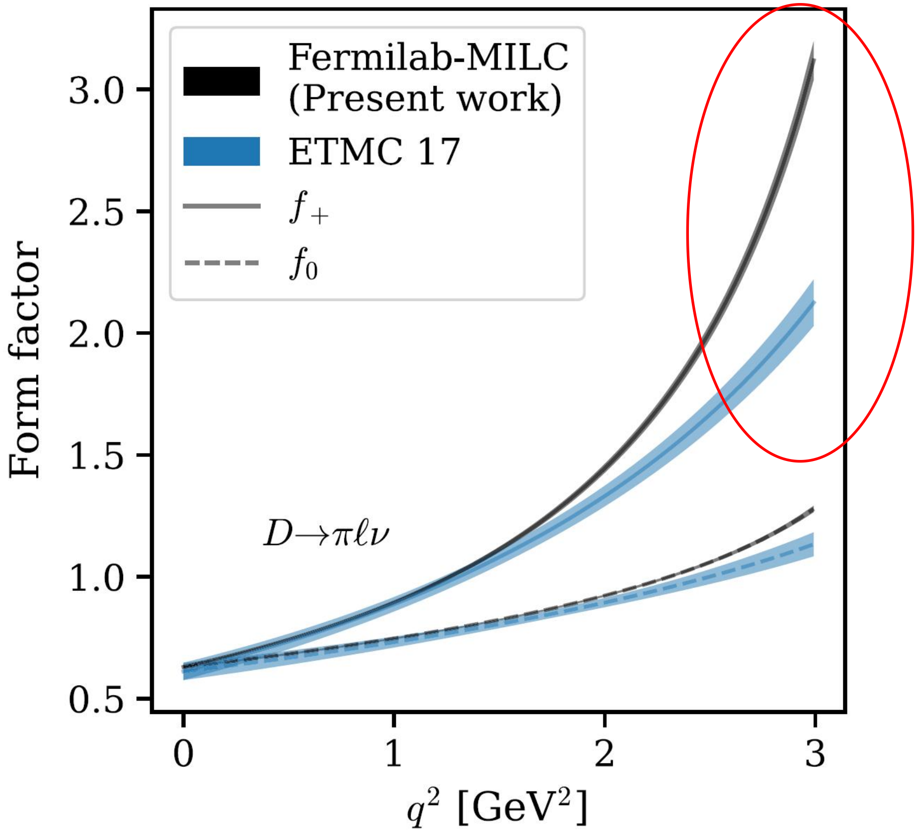
Fermilab/MILC, 2+1+1味HISQ

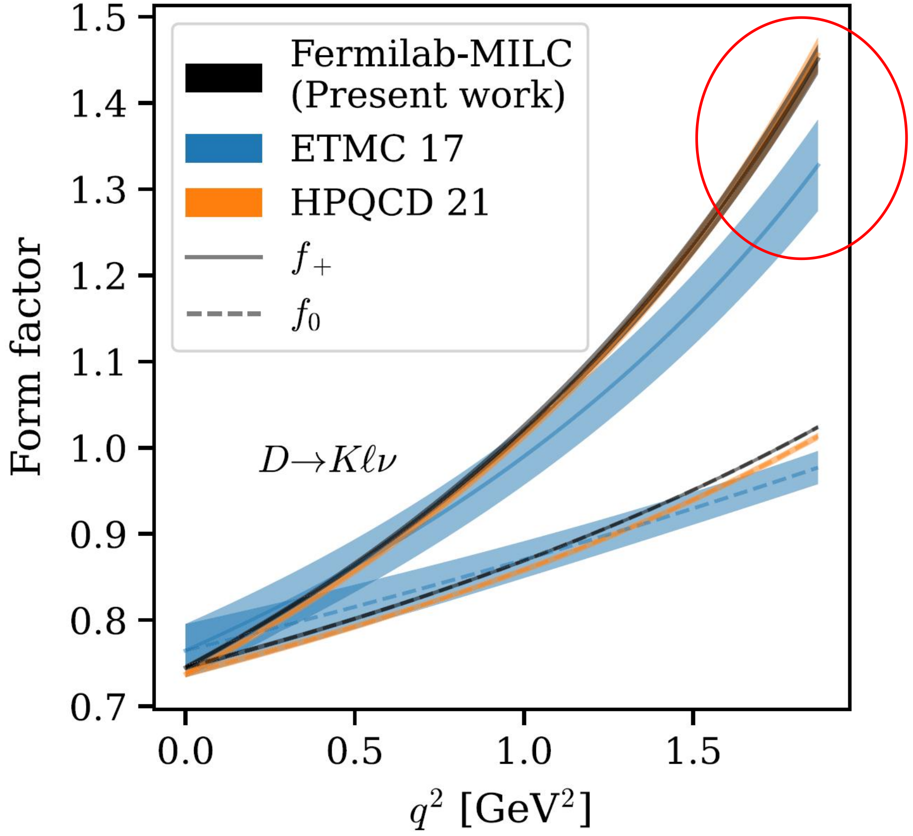
A. Bazavov et al., 2212.12648, PRD107.094516

- 四个格距：~0.12 fm - 0.04 fm
- $\mathcal{O}(1000)$组态数，多次测量/组态
- 两或三个体积@两个格距
- 物理轻夸克质量@三个格距

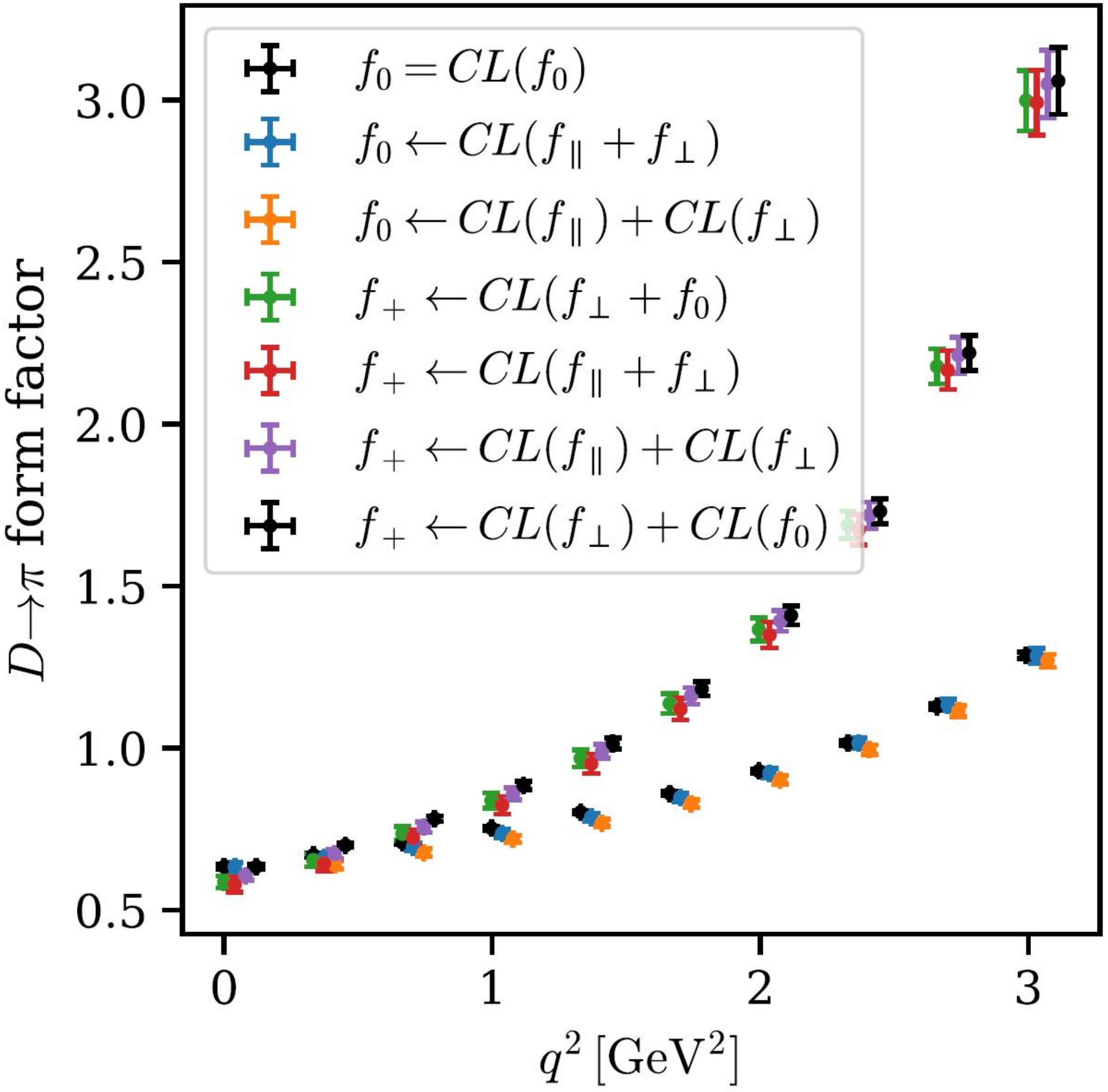
$$f_+^{D\pi}(0)=0.6300(51)$$
$$f_+^{DK}(0)=0.7452(31)$$
$$f_+^{D_sK}(0)=0.6307(20)$$

Not included: systematic uncertainties associated with QED, isospin, and electroweak corrections





形状因子不同抽取方式的差别



$$\langle K|V^\mu|D\rangle = f_+(q^2)\left(p_D^\mu + p_K^\mu - \frac{m_D^2 - m_K^2}{q^2}q^\mu\right) + f_0(q^2)\frac{m_D^2 - m_K^2}{q^2}q^\mu$$

$$\langle L|\mathcal{V}^\mu|H\rangle \equiv \sqrt{2M_H}[v^\mu f_\parallel(q^2) + p_\perp^\mu f_\perp(q^2)]$$

k^μ为初态介子四动量，$v^\mu = k^\mu/M_H$

p^μ为末态介子四动量，$p_\perp^\mu = p^\mu - (p\cdot v)v^\mu$

$$f_+^{\text{alt}}(q^2) = \frac{1}{\sqrt{2M_H}}[f_\parallel(q^2) + (M_H - E_L)f_\perp(q^2)]$$

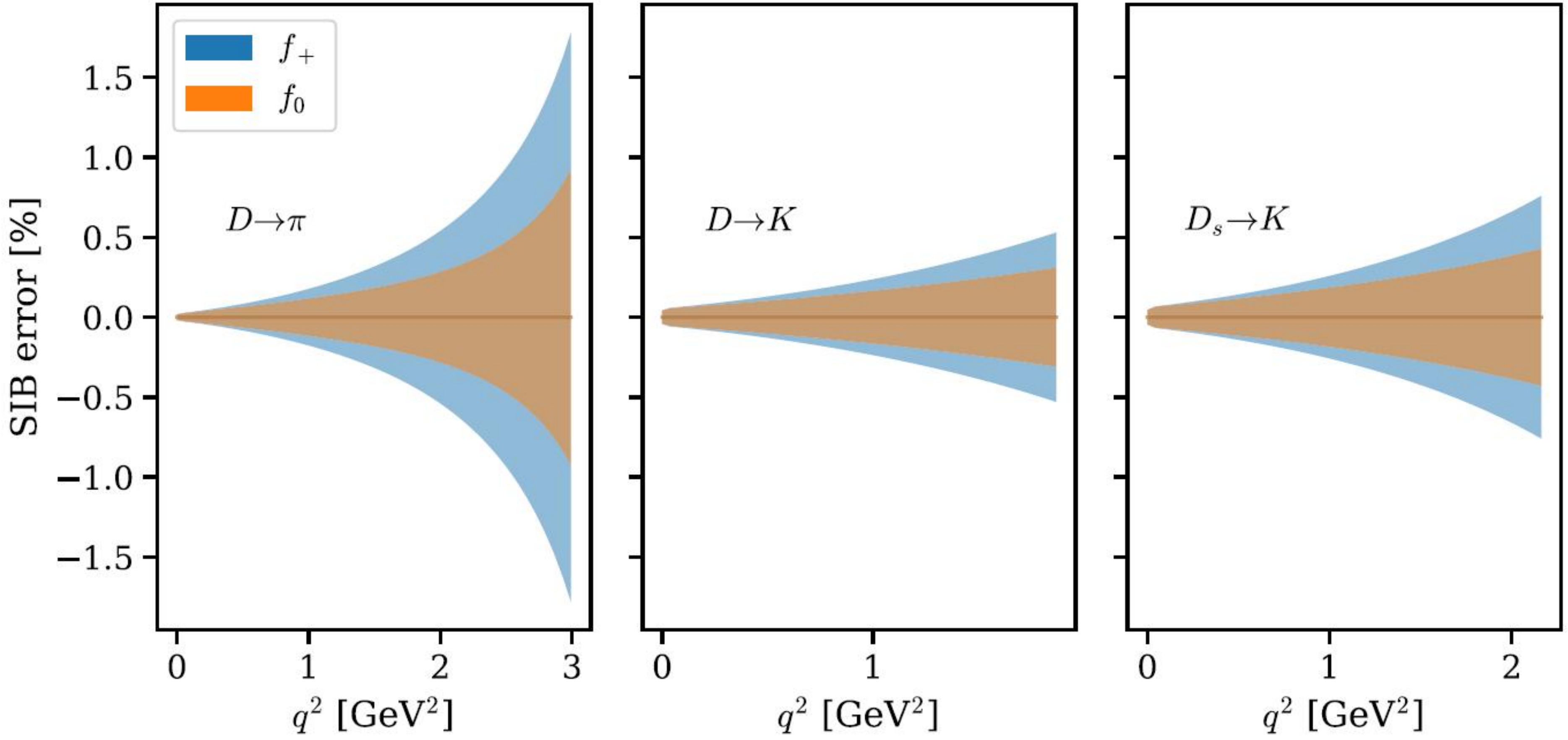
$$f_0^{\text{alt}}(q^2) = \frac{\sqrt{2M_H}}{M_H^2 - M_L^2} \times [(M_H - E_L)f_\parallel(q^2) + (E_L^2 - M_L^2)f_\perp(q^2)]$$

A. Bazavov et al., 2212.12648, PRD107.094516

系统误差：强同位旋破缺效应 $m_u \neq m_d$

A. Bazavov et al., 2212.12648, PRD107.094516

- 格点QCD模拟：上、下夸克简并$m_l = (m_u + m_d)/2$，不带电荷
- 现实世界：$m_u \neq m_d$，带电
- 误差估计：使用中性介子质量 vs 带电介子质量，结果差异 $(1 - f_{+,0}^{\mathrm{neutral}}/f_{+,0}^{\mathrm{charged}})$



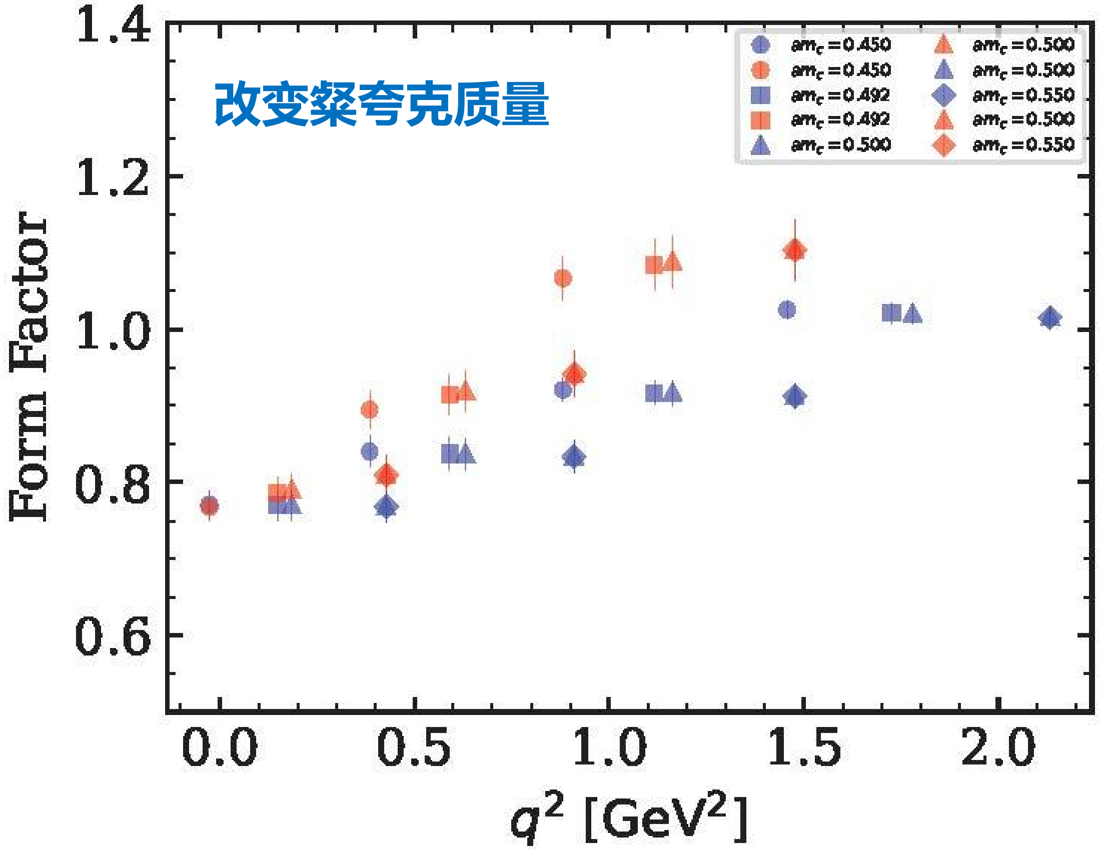
大q^2区域受相空间压低，同位旋破缺效应对唯象结果的影响较小

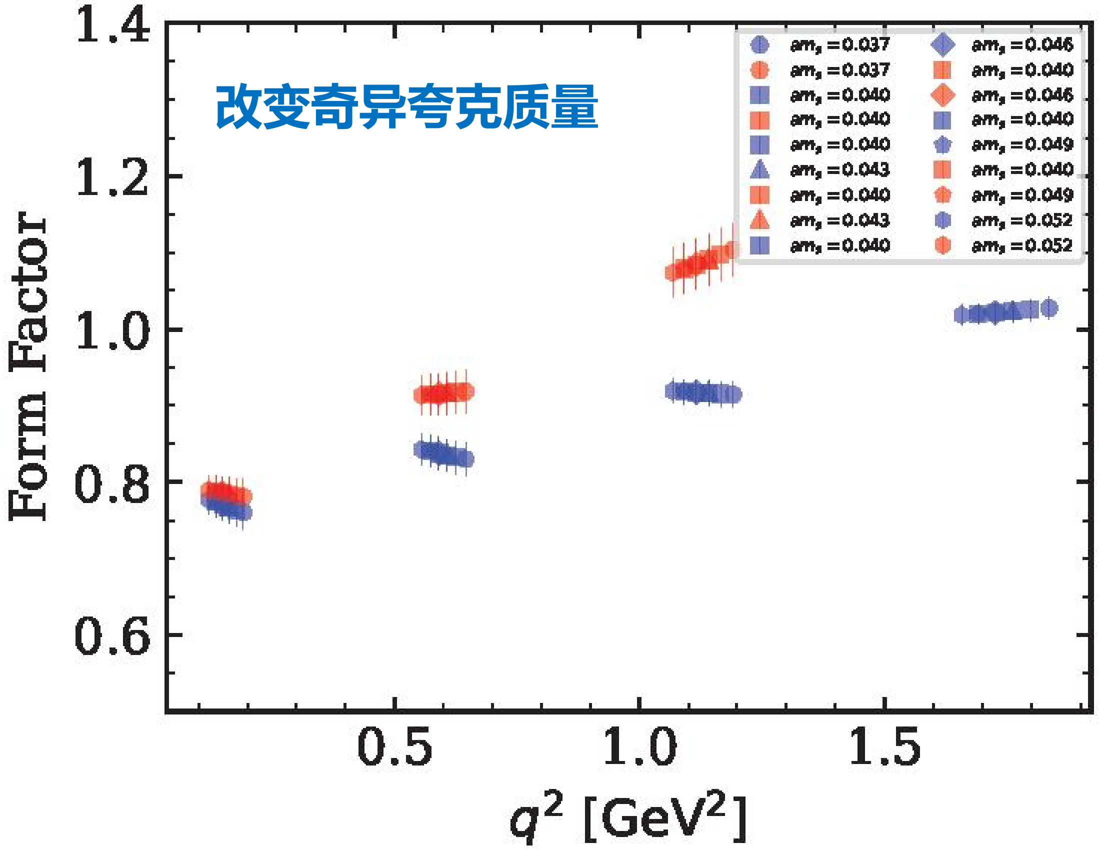
$D \rightarrow Kl\nu$形状因子$f_+(q^2)$和$f_0(q^2)$

- Overlap价夸克质量点分别有 4(u/d)、6(s)、4(c)个
- 海夸克质量点3个($m_\pi^{sea} \approx$ 302, 360, 412 MeV)
  - f004和f008组态上的分析正在进行
- 内插/外推到物理点

$1/a$ (GeV)	Label	am_l/am_s	Volume	$N_{conf} \times N_{src}$
2.383(9)	f004	0.004/0.03	$32^3 \times 64$	628×1
($m_\pi^{sea} \approx$ 360 MeV)	f006	0.006/0.03	$32^3 \times 64$	42×16
	f008	0.008/0.03	$32^3 \times 64$	49×16

RBC-UKQCD，2+1味domain-wall fermion组态





T. Shen, ..., ZL, Z. Zhang et al., arXiv:2503.01219, PoS LATTICE2024 (2025) 294

$f_+(q^2)$和$f_0(q^2)$的参数化及夸克质量依赖

定义： $z(q^2;t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$

对于给定的夸克质量，$t_+ \equiv (m_D + m_K)^2$用对应的强子质量计算

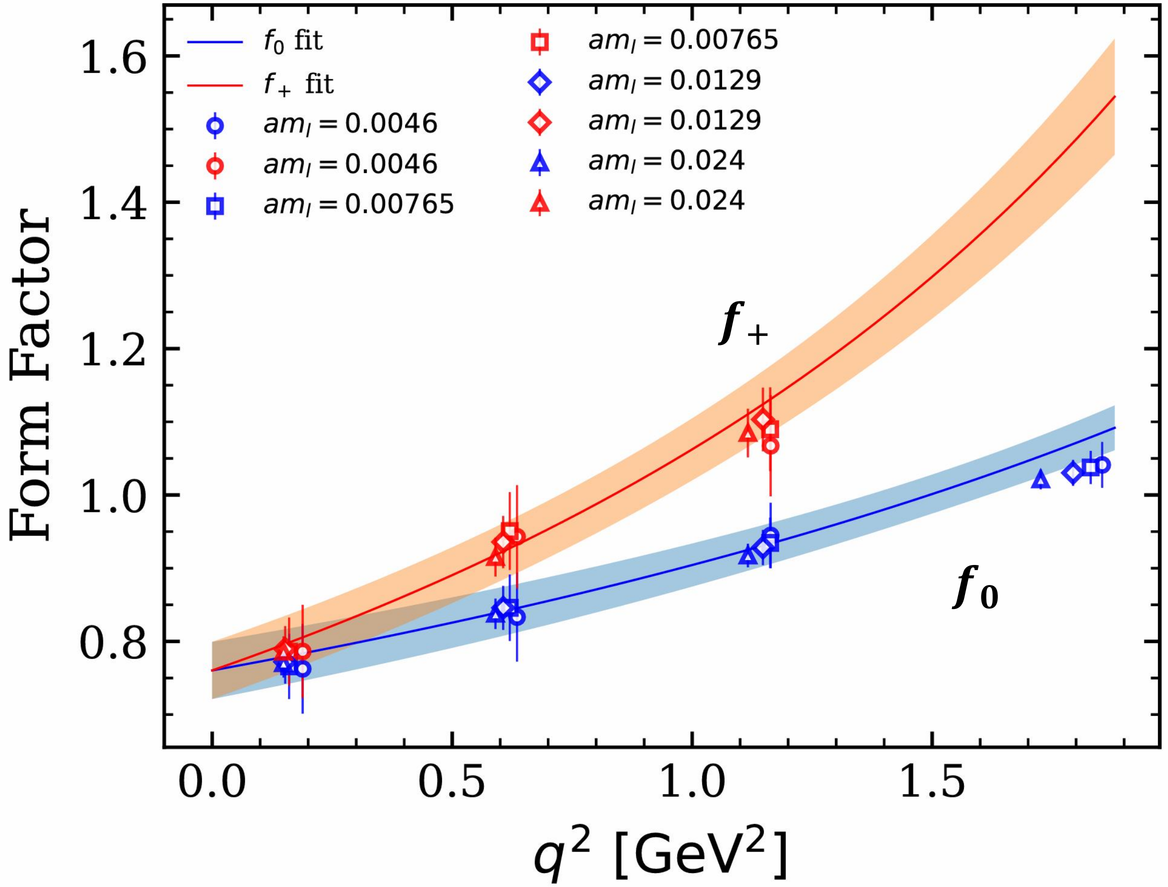
- **取 $t_0 = 0$，此时$q^2 = 0$与$z = 0$对应**

$$f_+(q^2; m_c, m_s, m_l) = \frac{1}{1 - q^2/m_{D_s^*}^2} \sum_{i=0}^{n} a_i D_i z^i$$

$$f_0(q^2; m_c, m_s, m_l) = \frac{1}{1 - q^2/m_{D_{s0}^*}^2} \sum_{i=0}^{n} b_i D_i z^i$$

$$D_i = 1 + c_{i1}\left(m_\pi^2 - (m_\pi^{\rm phys})^2\right) + c_{i2}(m_{\eta_s}^2 - (m_{\eta_s}^{\rm phys})^2) + c_{i3}(m_{J/\psi} - m_{J/\psi}^{\rm phys})$$

- **拟合参数：$a_i, b_i, c_{i1}, c_{i2}, c_{i3}$**
- $f_+(q^2 = 0) = f_0(0)$
- **$m_{D_s^*}$、$m_{D_{s0}^*}$取对应的格点计算结果或实验值**
- $m_{D_{s0}^*} = 2.3178$ GeV (PDG)
- **截断到 z 的一次、二次或三次项**



- **At the physical point (valence quark)**

$f_+(0) = f_0(0) = 0.760(39)$ $(n = 1)$

Preliminary results

$D \to K$张量流形状因子$f_T(q^2)$

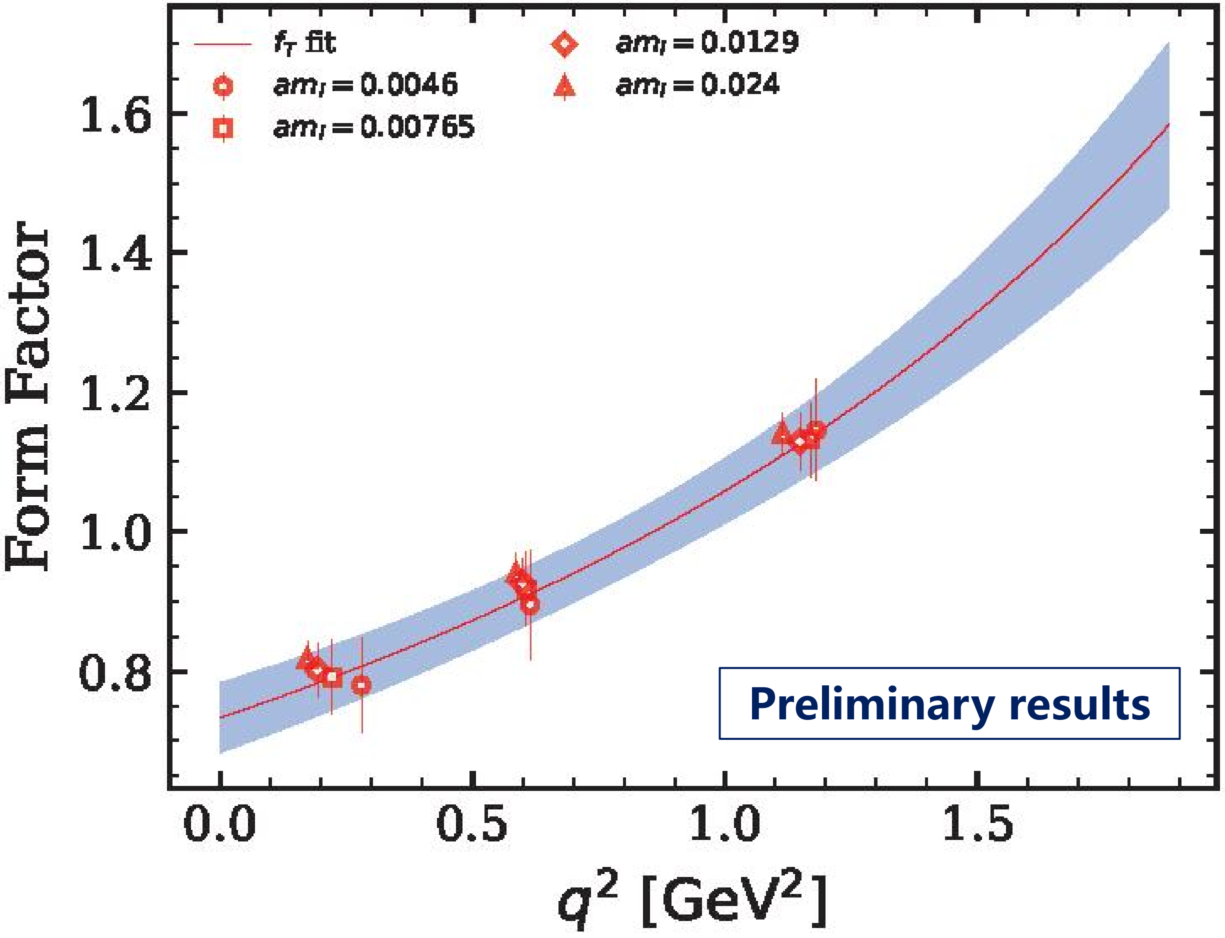
"Form factors in semileptonic decay of D mesons" ,T. Shen, Y. Chen, M. Gong, D. Li., K.-F. Liu, **ZL**, Z. Zhang, arXiv:2503.01219, PoS LATTICE2024 (2025) 294

- **张量流形状因子** $$\langle P|T_{\mu\nu}|D\rangle = \frac{2}{m_D + m_P}\left[p_{P\mu}p_{D\nu} - p_{P\nu}p_{D\mu}\right] f_T(q^2)$$ **助力新物理寻找**

- **以新物理方式影响$D \to K/\pi l\nu$**
- **影响FCNC稀有过程$D \to K/\pi ll$**

- **At the physical point (valence quark)**

$f_T(0) = 0.733(50)$ **($n = 1$)**

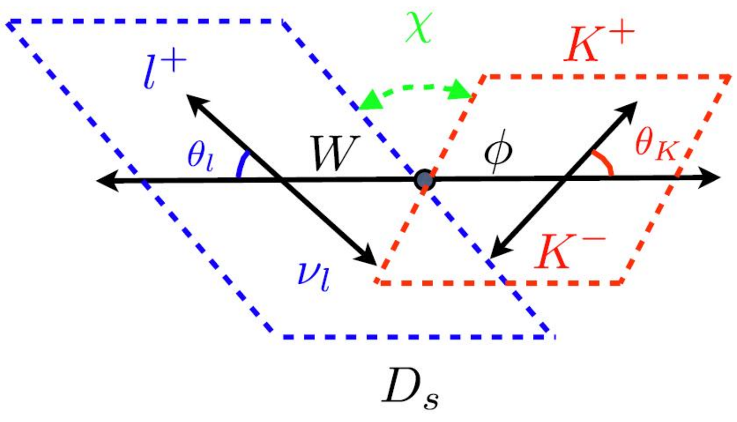


重整化常数： $Z_T^{\overline{\rm MS}}$**(2 GeV)**$= 1.157(11)$

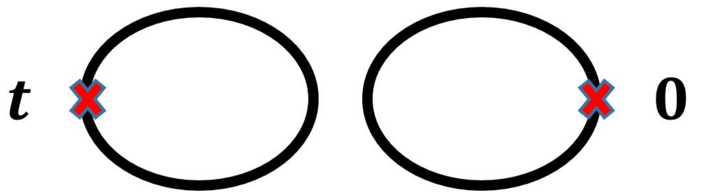
Y. Bi, ..., ZL et al., PRD108.054506 (2023)

$D_s \to \phi l \nu$

- HPQCD之前的计算 [PRD90, 1311.6669]
  - 除了在q^2-bin中比较截面，还有更多观测量，$\phi \to K\bar{K}$
  - 忽略非连通图贡献(OZI压低)
  - 模拟中ϕ不衰变；ϕ宽度较窄，预期阈效应小
  - 形状因子V，A_2，A_1，A_0； z-expansion
  - 2+1味；两个格距；HISQ价夸克作用量
  - 和*BABAR*（$D_s \to \phi e^+ \nu_e$），CLEO（$D \to K^*$）的结果做比较（假设spectator quark的影响很小）



$$
\begin{aligned}
&\langle \phi(p',\varepsilon)|V^\mu - A^\mu|D_s(p)\rangle \\
&= \frac{2i\epsilon^{\mu\nu\alpha\beta}}{M_{D_s}+M_\phi}\varepsilon^*_\nu p'_\alpha p_\beta V(q^2) - (M_{D_s}+M_\phi)\varepsilon^{*\mu}A_1(q^2) \\
&\quad + \frac{\varepsilon^*\cdot q}{M_{D_s}+M_\phi}(p+p')^\mu A_2(q^2) + 2M_\phi\frac{\varepsilon^*\cdot q}{q^2}q^\mu A_3(q^2) \\
&\quad - 2M_\phi\frac{\varepsilon^*\cdot q}{q^2}q^\mu A_0(q^2).
\end{aligned}
$$



两点函数中的非连通图

$$A_3(q^2) = \frac{M_{D_s}+M_\phi}{2M_\phi}A_1(q^2) - \frac{M_{D_s}-M_\phi}{2M_\phi}A_2(q^2)$$

$$A_0(0) = A_3(0)$$

$D_s \to \phi l \nu$

2+1味组态，clover fermions [CLQCD]

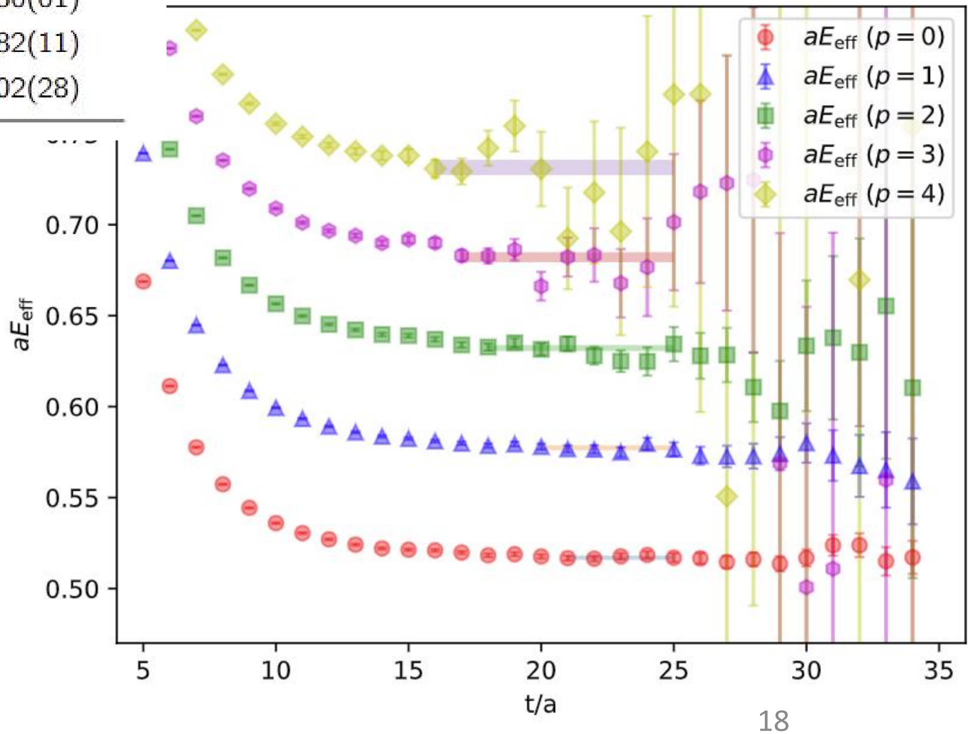
G. Fan, Y. Meng, ZL et al., in progress

Ensemble	C24P29	C32P23	F32P30	F48P21	G36P29	H48P32
a (fm)	0.10524(05)(62)	0.10524(05)(62)	0.07753(03)(45)	0.07753(03)(45)	0.06887(12)(41)	0.05199(08)(31)
$\tilde{m}_s^b$	−0.2400	−0.2400	−0.2050	−0.2050	−0.1926	−0.1700
$\tilde{m}_l^b$	−0.2770	−0.2790	−0.2295	−0.2320	−0.2150	−0.1850
$\tilde{m}_c^b$	0.4159(07)	0.4190(07)	0.1974(05)	0.1997(04)	0.1433(12)	0.0551(07)
$L^3 \times T$	$24^3 \times 72$	$32^3 \times 64$	$32^3 \times 96$	$48^3 \times 96$	$36^3 \times 108$	$48^3 \times 144$
$N_{cfg} \times N_{src}$	450×72	200×64	180×96	150×96	200×108	150×144
m_π (MeV)	292.3(1.0)	227.9(1.2)	300.4(1.2)	207.5(1.1)	297.2(0.9)	316.6(1.0)
t	2 − 17	2 − 20	4 − 22	4 − 26	2 − 32	8 − 30
Z_V^s	0.85184(06)	0.85350(04)	0.86900(03)	0.86880(02)	0.87473(05)	0.88780(01)
Z_V^c	1.57353(18)	1.57644(12)	1.30566(07)	1.30673(04)	1.23990(13)	1.12882(11)
Z_A/Z_V	1.07244(70)	1.07375(40)	1.05549(54)	1.05434(88)	1.04500(22)	1.03802(28)

- 四个格距，~ 4个pion 质量
- D_s静止，改变 ϕ 的三动量 $\vec{p} = \frac{2\pi}{La}(k_x, k_y, k_z)$
- $(k_x, k_y, k_z) = (0,0,0), (1,0,0), (1,1,0), (1,1,1), (2,0,0)$
- （轴）矢量流重正化常数 [CLQCD, arXiv:2408.03548]
- 另加一套组态 C32P29 进一步检查有限体积效应

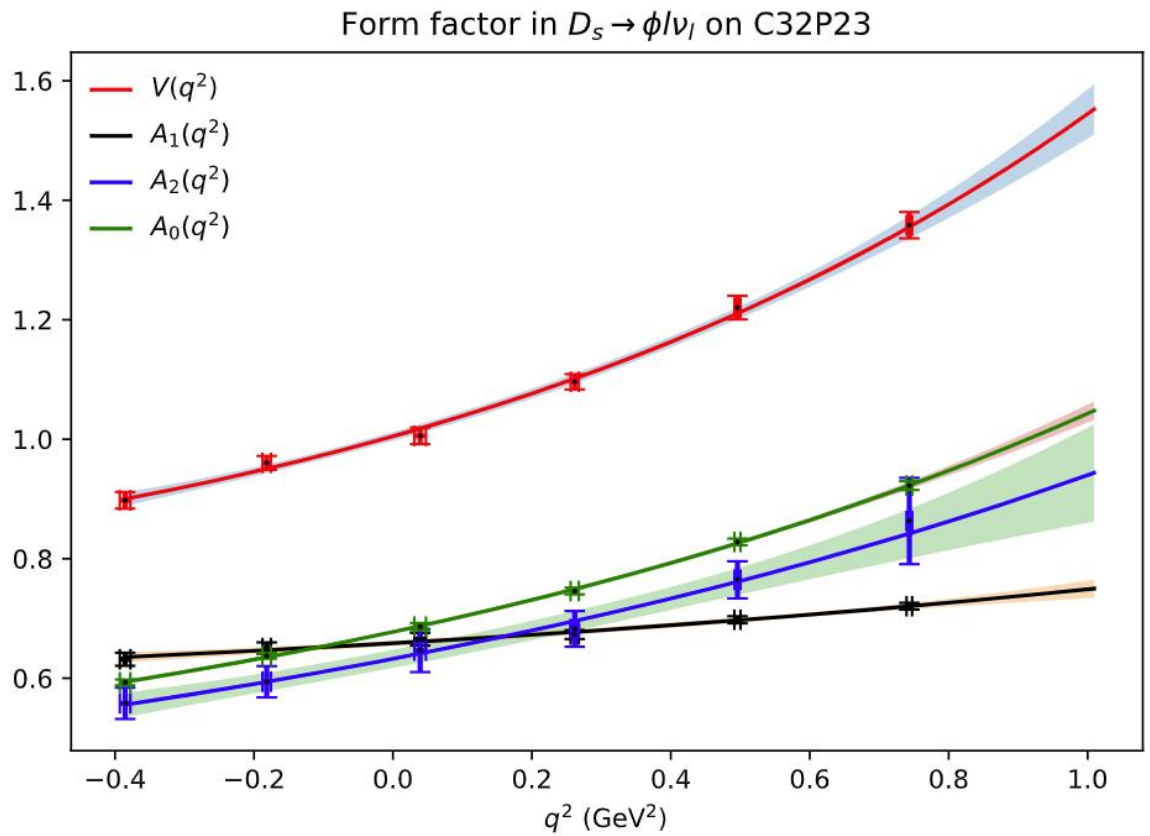
ϕ 两点函数能量平台

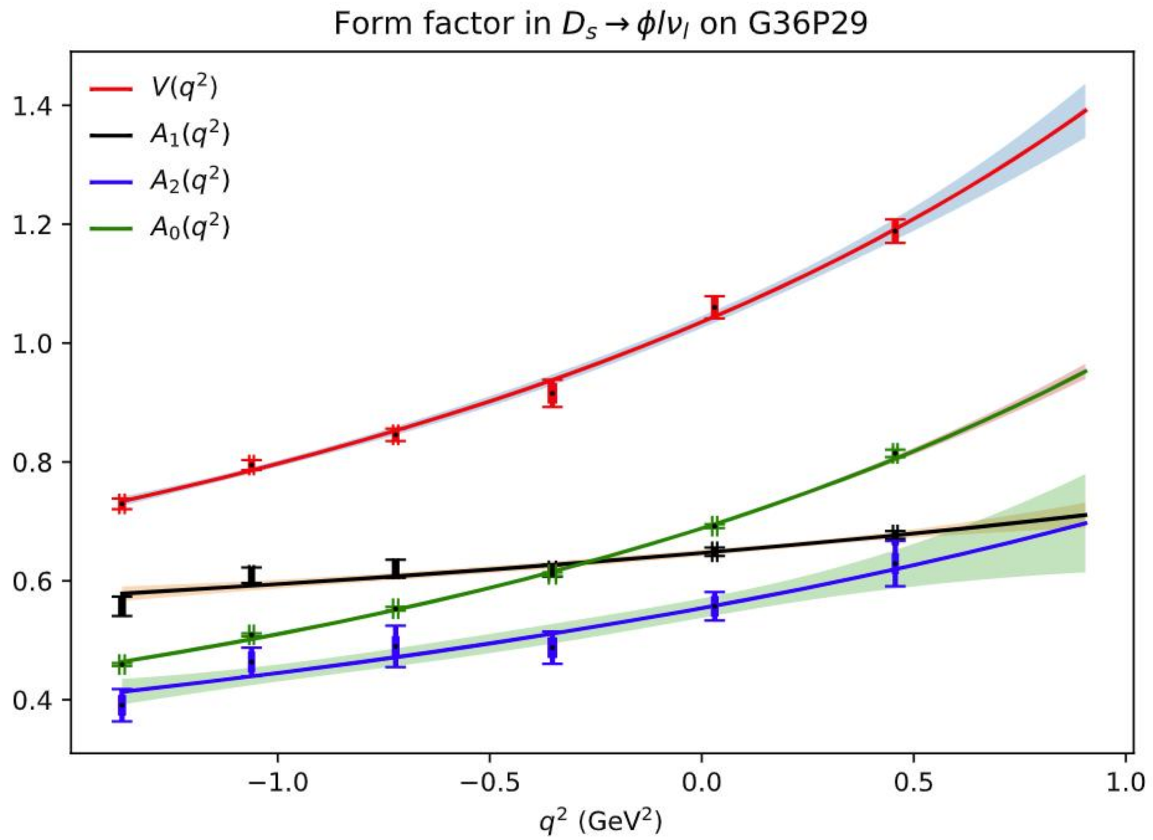
(组态 C24P29)



$D_s \to \phi l\nu$ 形状因子q^2依赖

Preliminary results





$$z\left(q^2, t_0\right) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}}$$

where $t_+ = (m_{D_s} + m_\phi)^2,\ t_0 = 0$

$$V = \frac{1}{1 - q^2/m_{D_s^*}^2}\left(a_0 + a_1 z + a_2 z^2\right)$$

$$A_{0,1,2} = \frac{1}{1 - q^2/m_{D_{s1}}^2}\left(a_0 + a_1 z + a_2 z^2\right)$$

$$A_3(q^2) = \frac{M_{D_s} + M_\phi}{2M_\phi} A_1(q^2) - \frac{M_{D_s} - M_\phi}{2M_\phi} A_2(q^2)$$

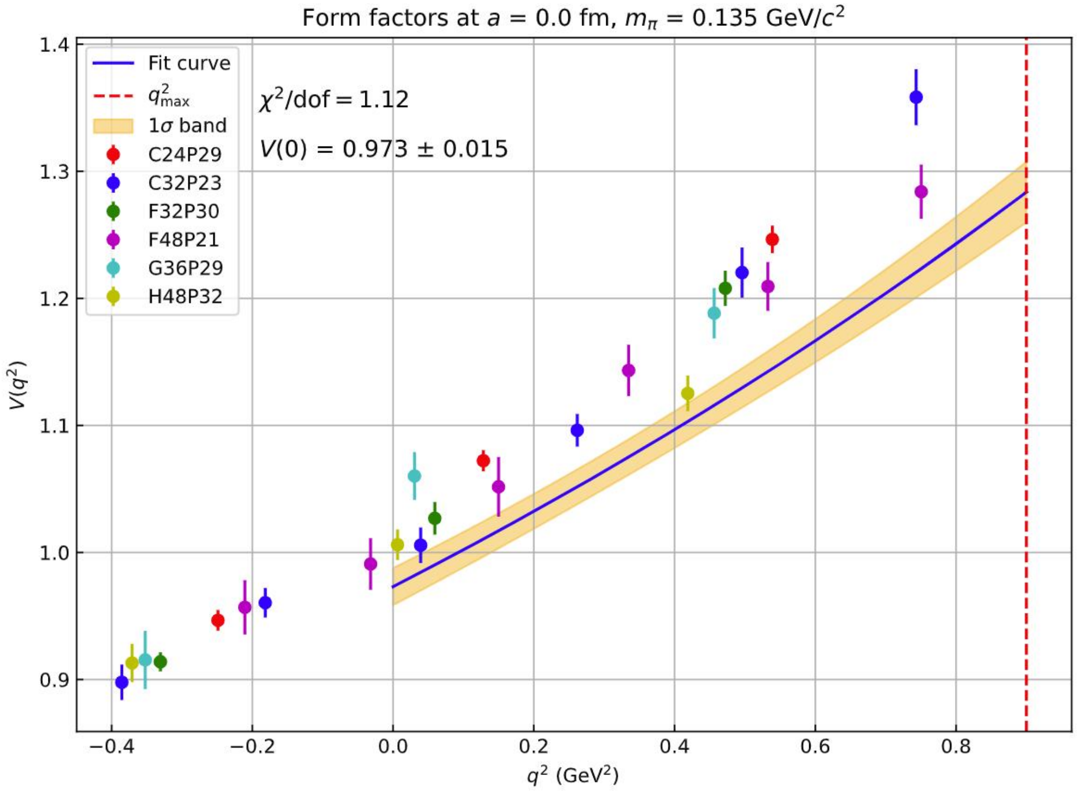
物理质量点及连续极限下的形状因子 $D_s \to \phi l \nu$

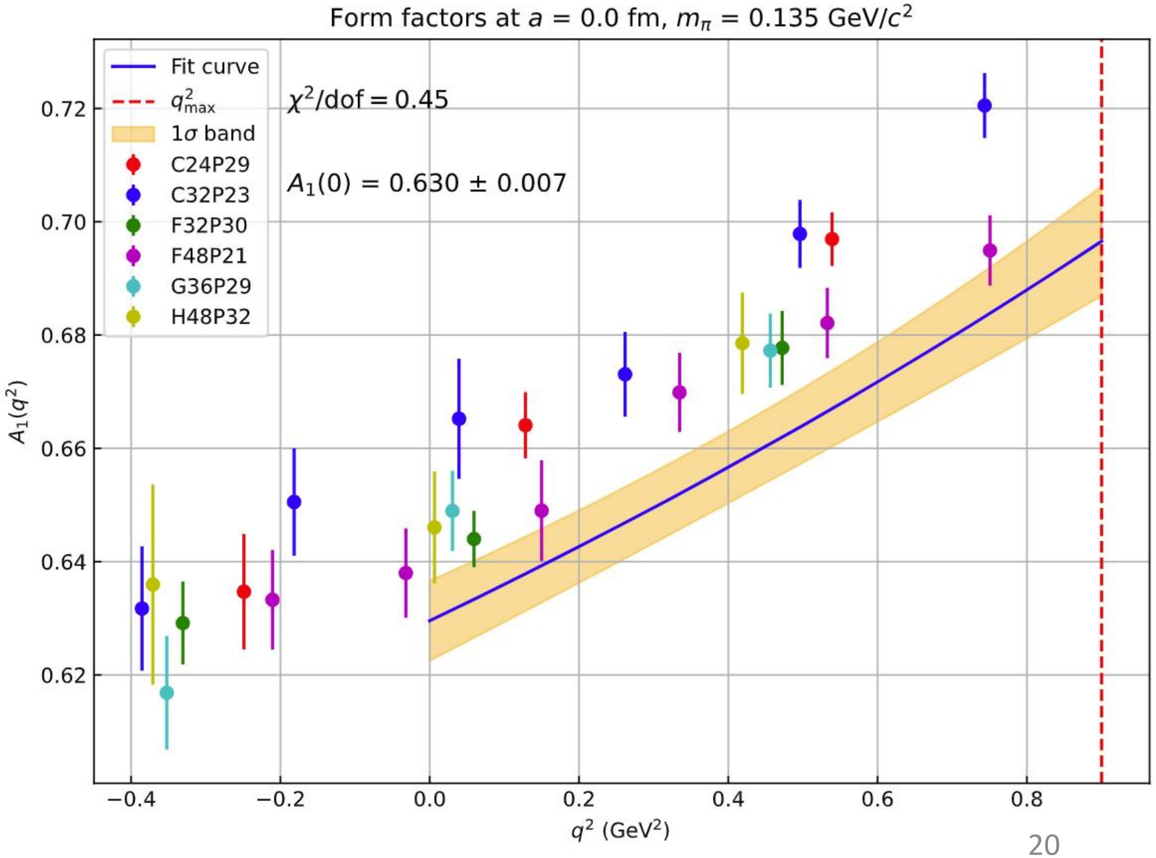
$$V\left(q^2, a, m_\pi\right) = \frac{1}{1 - q^2/m_{D_s^*}^2} \sum_{i=0}^{2} \left(c_i + d_i a^2\right) \left[1 + f_i\left(m_\pi^2 - m_{\pi,\mathrm{phys}}^2\right) + g_i\left(m_\pi^2 - m_{\pi,\mathrm{phys}}^2\right)^2\right] z^i$$

$$A_{0,1,2}\left(q^2, a, m_\pi\right) = \frac{1}{1 - q^2/m_{D_{s1}}^2} \sum_{i=0}^{2} \left(c_i + d_i a^2\right) \left[1 + f_i\left(m_\pi^2 - m_{\pi,\mathrm{phys}}^2\right) + g_i\left(m_\pi^2 - m_{\pi,\mathrm{phys}}^2\right)^2\right] z^i$$

单极点参数化形式给出误差范围内一致结果

$$m_{\pi,\mathrm{phys}}^2 = 135.0\ \mathrm{MeV}/c^2,\ m_{D_s^*} = 2112.2\ \mathrm{MeV}/c^2,\ m_{D_{s1}} = 2459.5\ \mathrm{MeV}/c^2$$

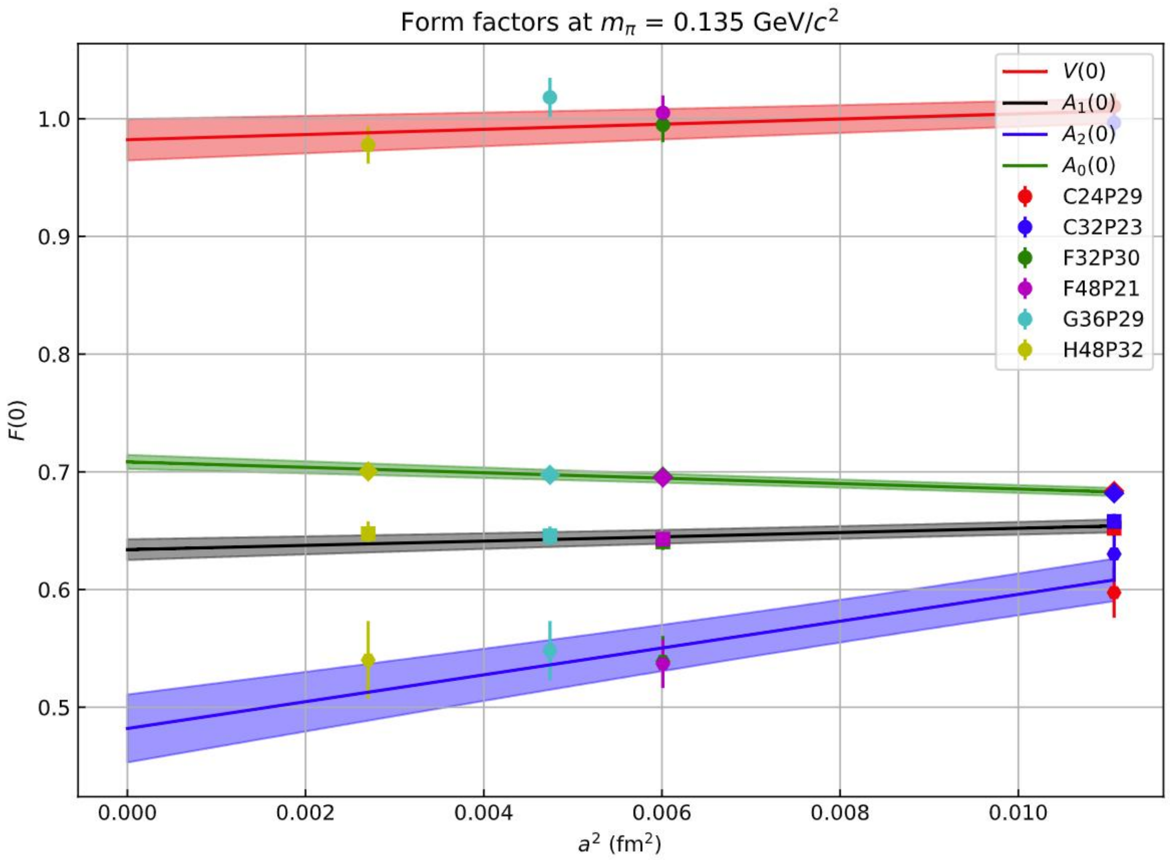




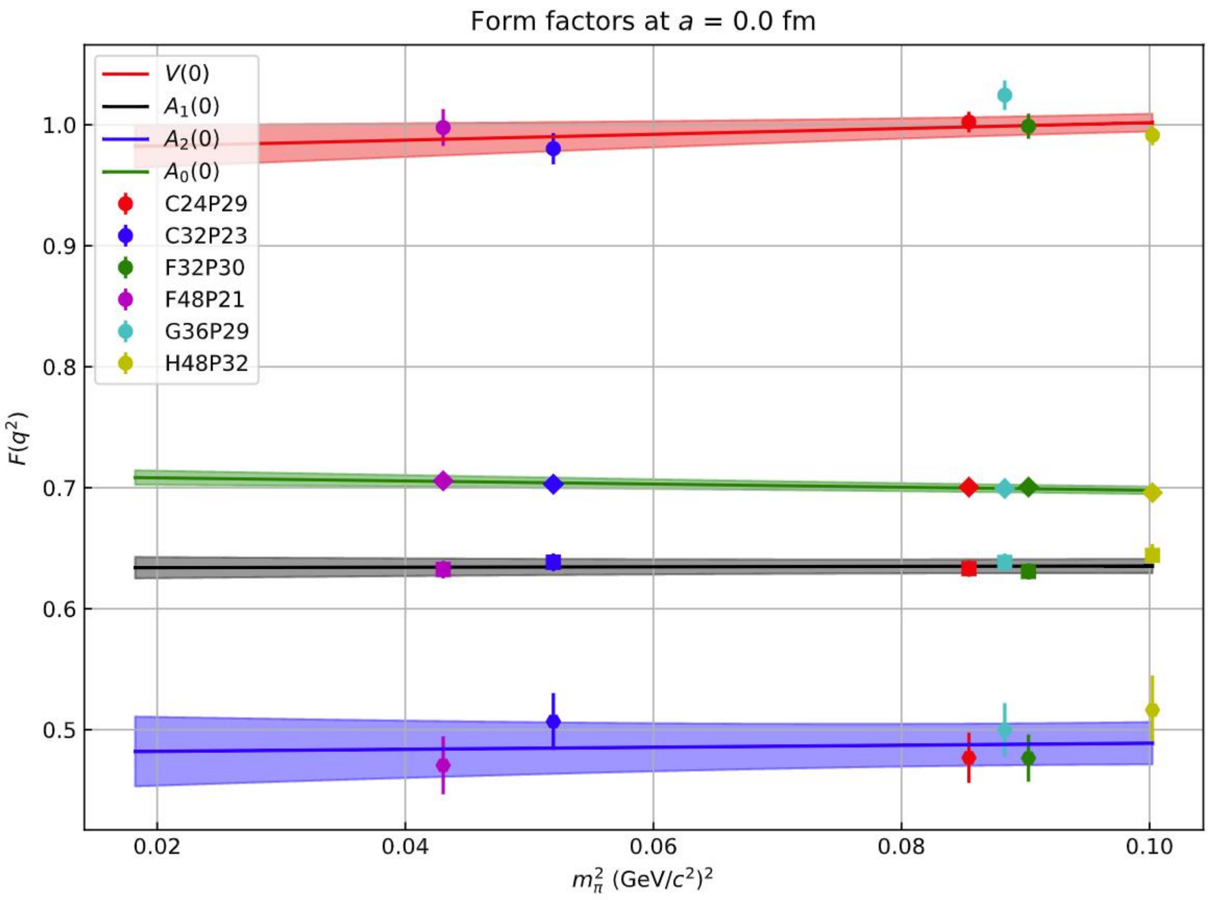
$$A_0(0) - A_3(0) = -0.004(16)$$

和零一致，符合运动学限制

$D_s \to \phi l\nu$



形状因子的连续极限外推

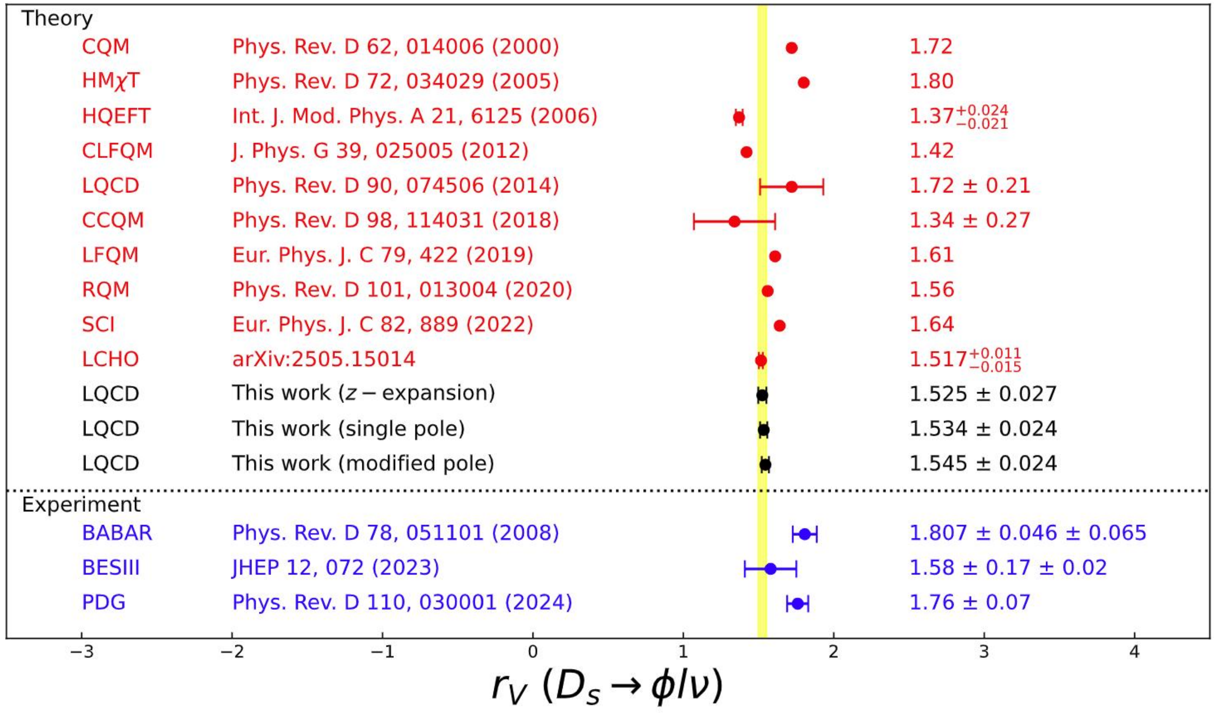


形状因子的轻夸克质量依赖

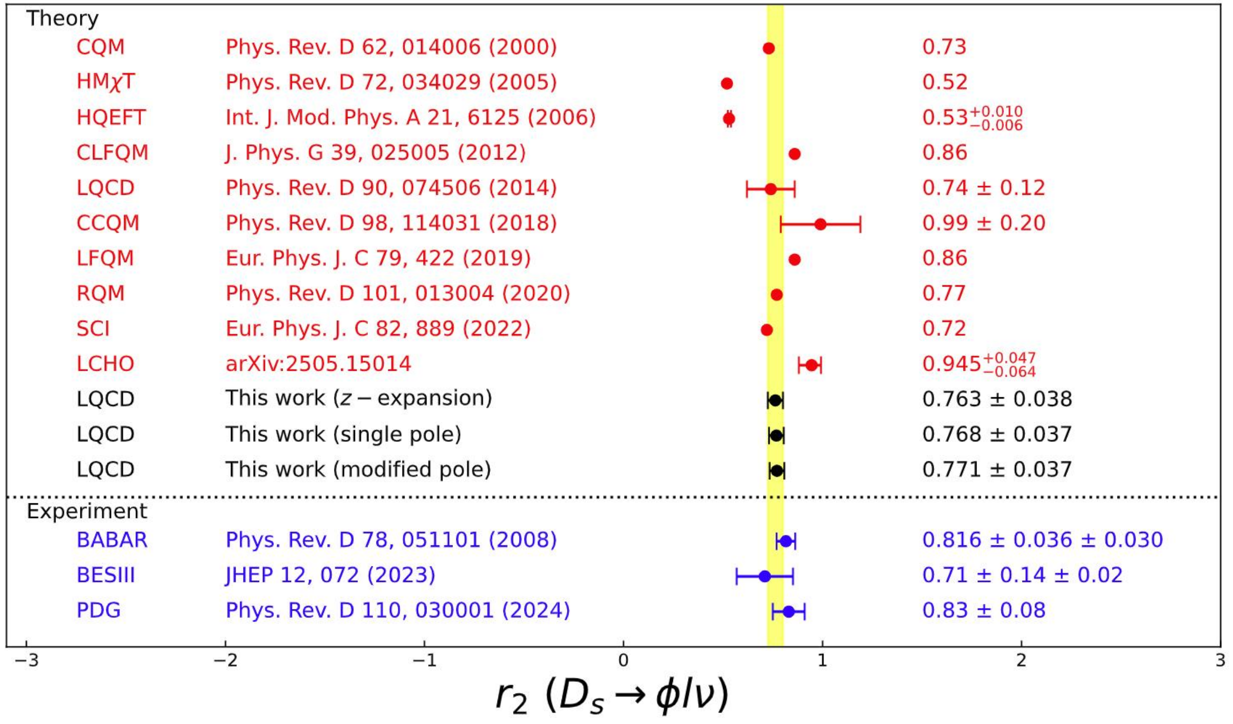
$D_s \to \phi l\nu$

Preliminary results

$r_V = V(0)/A_1(0)$



$r_2 = A_2(0)/A_1(0)$



分支比及比值:

	$\mathcal{B}_\mu(\%)$	$\mathcal{B}_e(\%)$	$\mathcal{R}_{\mu/e}$
z-expansion	$2.382(82)$	$2.524(89)$	$0.9437(18)$
single pole	$2.396(62)$	$2.540(68)$	$0.9434(14)$
modified pole	$2.407(57)$	$2.550(63)$	$0.9438(13)$
PDG	$2.24(11)$	$2.34(12)$	$0.957(68)$

BABAR, PRD78.051101(R) (2008)
BESIII, JHEP12 (2023) 072

小结

- 粲介子半轻衰变：标准模型精确检验和新物理寻找
- 强子矩阵元（形状因子）的格点QCD研究
  - $D \to Kl\nu$，overlap fermion on domain-wall fermion configurations
  - $D_s \to \phi l\nu$，Wilson clover fermion

谢谢！

复合算符重整化

粲介子衰变常数

形状因子

ZL et al. (χQCD), 1312.7628, PRD90.034505

Y. Bi, ..., ZL et al., 1710.08678, PRD97.094501

F. He, ..., ZL et al., 2204.09246, PRD106.114506

Y. Bi, ..., ZL et al., 2302.01659, PRD108.054506

χQCD, arXiv:1410.3343, PRD92.034517 (2015)

χQCD, arXiv:2008.05208, CPC45.023109 (2021)

D. Li, ..., ZL et al., 2407.03697, CPC48.123104 (2024)

Y. Meng, ..., ZL et al., 2401.13475, PRD109.074511

T. Shen, ..., ZL et al., 2503.01219, PoS LAT2024 (2025) 294

G. Fan, Y. Meng, ZL et al., in progress