QCD Sum Rule predictions for some charmed meson semileptonic decays

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Outline

I. Motivation

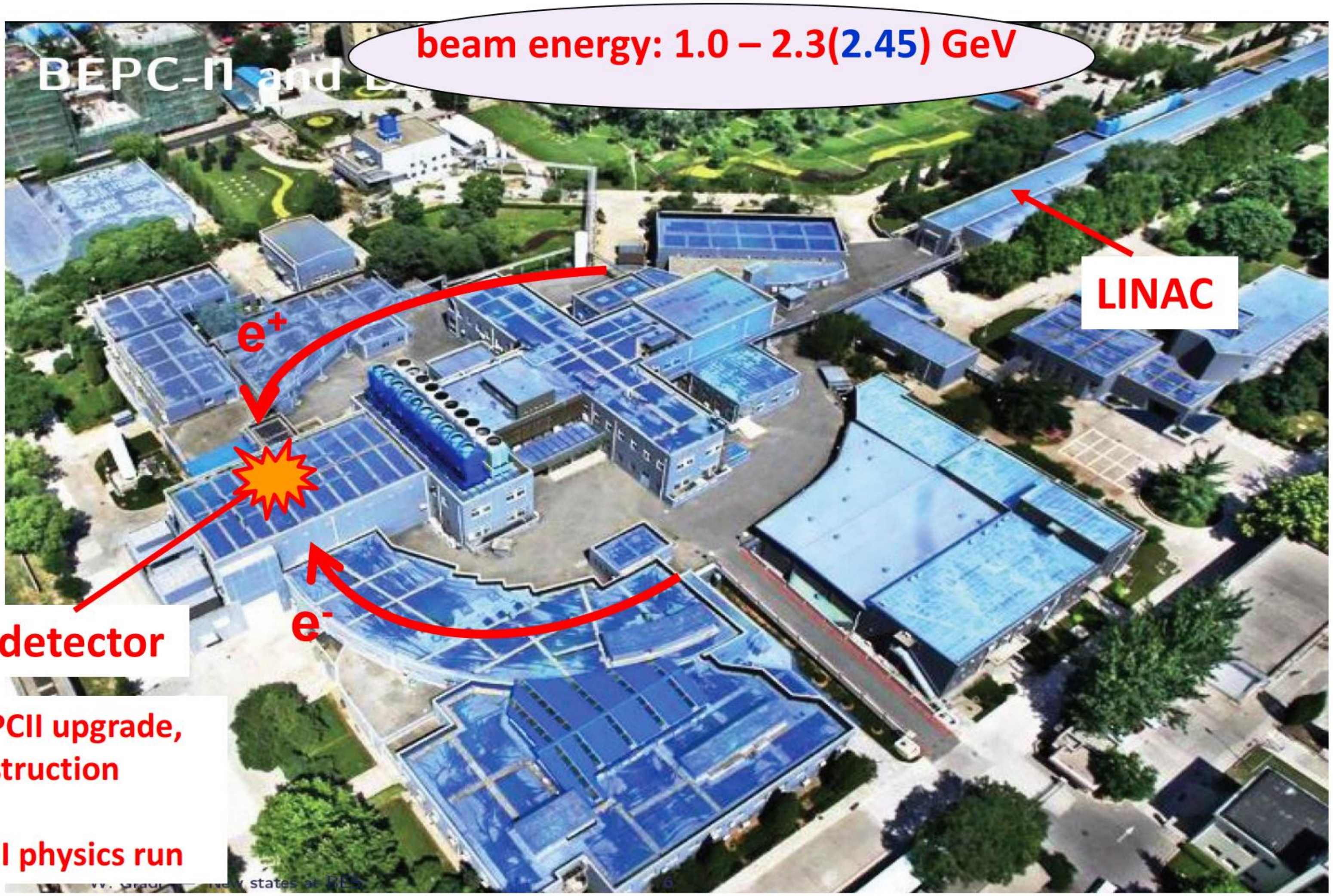
II. $D_s \to f_0(980)(\to \pi^+\pi^-)e^+\nu_e$decay process

III. $D_s \to \phi l^+\nu_l$ decay process

IV. Summary

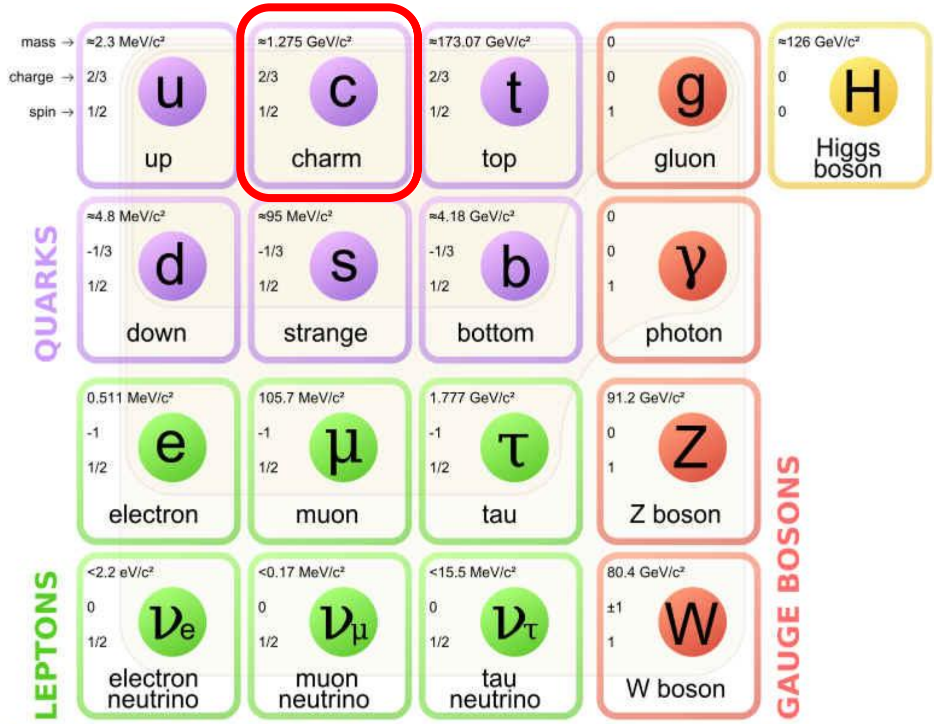
I. Motivation

Beijing Electron Positron Collider (BEPCII)



2004: started BEPCII upgrade,
BESIII construction
2008: test run
2009 - now: BESIII physics run

I. Motivation



Standard_Model_of_Elementary_Particles.svg (SVG file, nominally 774 × 581 pixels, file size: 419 KB)

$$U = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{tb} & V_{ts} & V_{tb} \end{pmatrix}$$

Semileptonic D decay

- Transition form factors
- CKM matrix element
- Decay width and anomalous
- Forward-backward asymmetry
- Polarization
- Lepton Flavor Universality

Non-perturbative method

- Lattice QCD
- QCD sum rules
- QCD factorization
- Model: Quark Model
- AdS/QCD

2. 实验测量：$f_0(980)$作为中间态

CLEO(2019): $\mathcal{B}(D_s^+ \to f_0(980) e^+ \nu_e) \times \mathcal{B}(f_0(980) \to \pi^+ \pi^-) = (0.13 \pm 0.04 \pm 0.01)\%$ [PRD80, 052007 (2019)]

CLEO(2009): $\mathcal{B}(D_s^+ \to f_0(980) e^+ \nu_e) \times \mathcal{B}(f_0(980) \to \pi^+ \pi^-) = (0.20 \pm 0.04 \pm 0.01)\%$ [PRD80, 052009 (2019)]

BESIII(2019): $\mathcal{B}(D_s^+ \to f_0(980) e^+ \nu_e) \times \mathcal{B}(f_0(980) \to \pi^+ \pi^-) < 2.8 \times 10^{-5}$ [PRL122, 062001 (2019)]

BESIII(2022): $\mathcal{B}(D_s^+ \to f_0(980) e^+ \nu_e) \times \mathcal{B}(f_0(980) \to \pi^0 \pi^0) = (0.079 \pm 0.014 \pm 0.004)\%$ [PRD105, L031101 (2022)]

BESIII(2024): $\mathcal{B}(D_s \to f_0(980) e^+ \nu_e) \times \mathcal{B}(f_0(980) \to \pi^+ \pi^-) = (0.172 \pm 0.013 \pm 0.01)\%$ [PRL132, 141901 (2024)]

I. Motivation

1. 不同的夸克组分

$f_0(980) = \bar{s}s$ ×

$$f_0(980) = \frac{1}{\sqrt{2}}\left(\bar{u}u + \bar{d}d\right)\bar{s}s$$

$f_0(980) \to K^*\bar{K}^*$ (分子态)

$$f_0(980) = \sin\theta\left[\frac{1}{\sqrt{2}}\left(\bar{u}u + \bar{d}d\right)\right] + \cos\theta\left(\bar{s}s\right)$$ $f_0(500) - f_0(980)$ mixing ⟹ 考虑混合态效应

2. 理论计算

Light-front quark model (LFQM) PRD 80, 074030 (2009)

Covariant light-front dynamics (CLFD) PRD 79, 076004(2009)

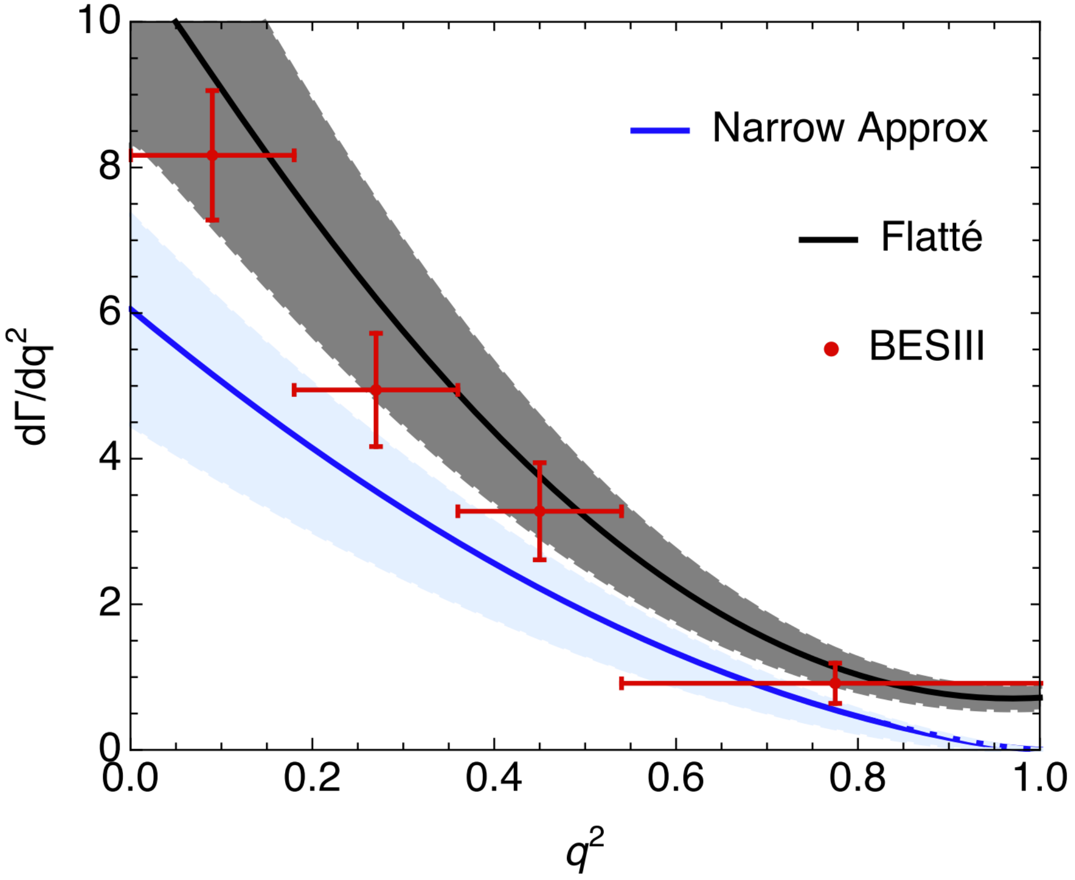
Three point sum rule (3PSR) PLB 579, 59-66 (2004) ($\bar{s}s$) PRD 68 (2003) 036001 (混合态)

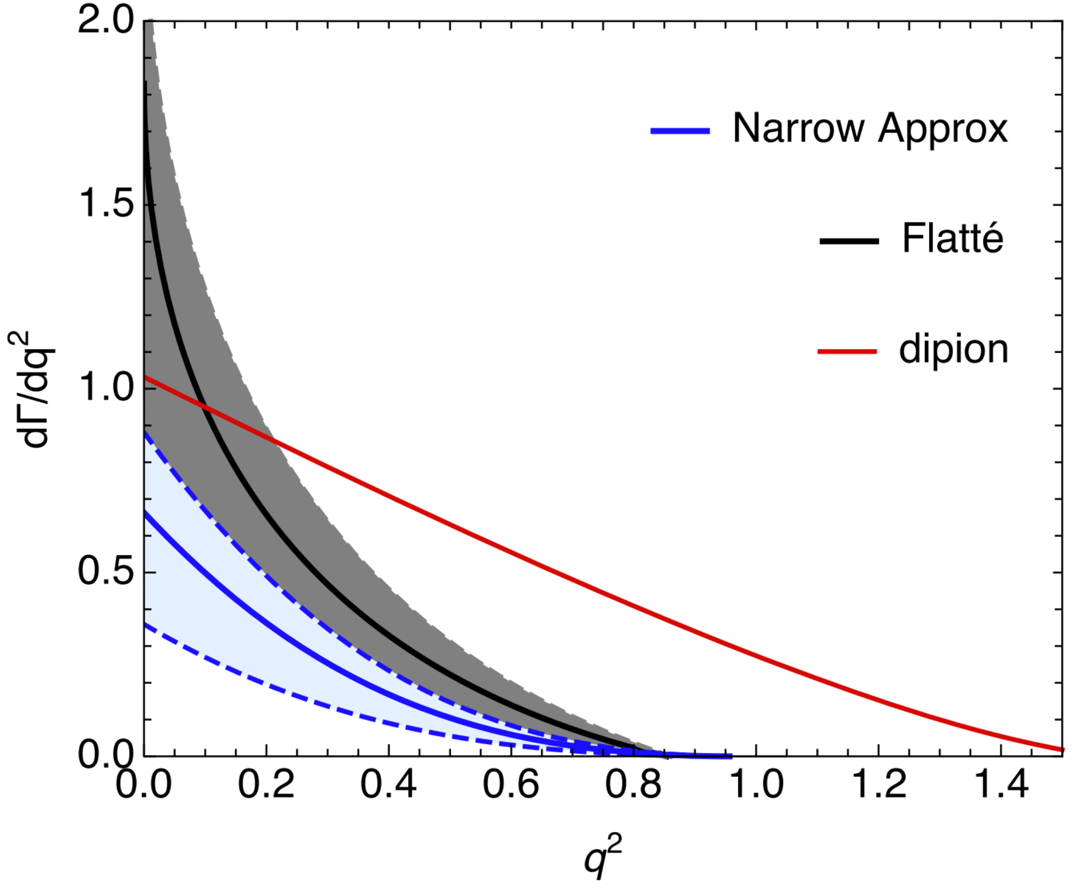
Light-cone sum rules (LCSR) PRD 81.074001 (2010)

I. Motivation

2. 理论计算

程山 EPJC 84 (2024) 379





I. Motivation

Decay Width, Ratio, Observables

↓

Transition form factors (TFFs)

↓

Light cone Sum Rule (LCSR) → 手征流光锥求和规则方法

↓

Light meson distribution amplitude (DAs) →
Gegenbauer展开
光锥谐振子模型
QCD sum rules

I. Motivation

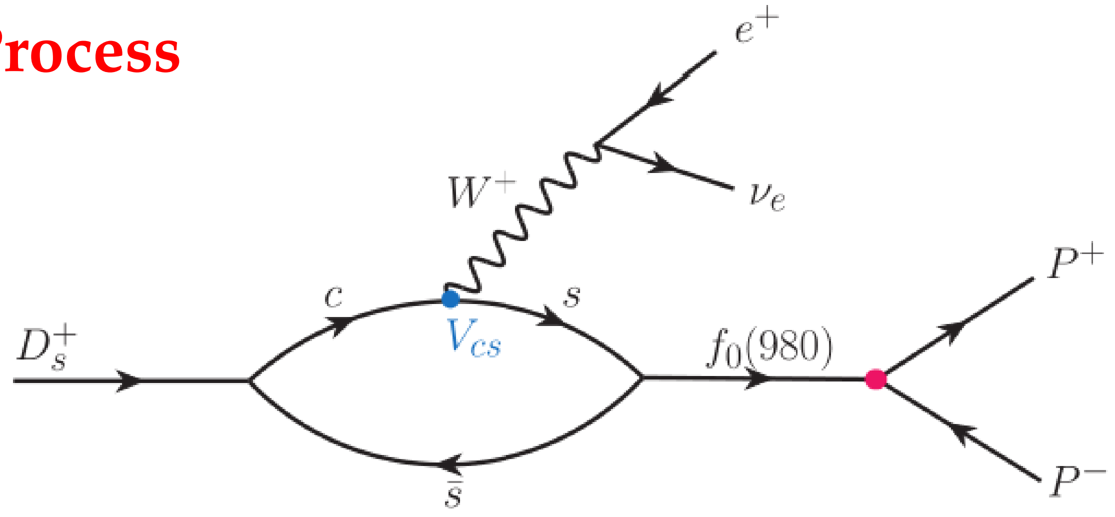
QCD求和规则 →
- ◆ SVZ求和规则
- ◆ 光锥求和规则

→
- ➢ 构造关联函数
- ➢ 算符乘积展开
- ➢ 强子表示
- ➢ 色散关系

SVZ求和规则(真空-真空) ⟶ 轻介子twist-2,3分布振幅

光锥求和规则(真空-强子态) ⟶ 衰变过程中的跃迁形状因子 ⟶ 衰变宽度、衰变分支比或CKM矩阵元等

I. Decay Process



PRL122, 062001 (2019) -- BESIII

$$\frac{d\Gamma(D_s^+ \to f_0(980)(\to \pi^+\pi^-)e^+\nu_e)}{ds\ dq^2} = \frac{G_F^2|V_{cs}|^2}{192\pi^3 m_{D_s^+}^3}\lambda^{3/2}(m_{D_s^+}^2, m_{f_0}^2, q^2)|f_+(q^2)|^2 \frac{P(s)}{\pi},$$

$$P(s) = \frac{g_1\rho_{\pi\pi}}{|m_{f_0}^2 - s - i[g_1\rho_{\pi\pi}(s) + g_2\rho_{K\bar{K}}(s)]|^2}.$$

$P(s)$为相对论的Flatte shape

常数g_1和g_2分别为$f_0(980)$耦合到$\pi^+\pi^-$ 和 K^+K^-的耦合常数

(1). 构建关联函数:

$$\Pi_\mu(p,q) = i\int d^4x e^{iq\cdot x}\langle f_0(980)(p)|T\{\bar{s}(x)\gamma_\mu\gamma_5 c(x), \bar{c}(0)i\gamma_5 s(0)\}|0\rangle$$
$$= F[q^2,(p+q)^2]p_\mu + \widetilde{F}[p^2,(p+q)^2]q_\mu.$$

(2). 算符乘积展开(OPE):

重夸克传播子
$$\langle 0|c^i_\alpha(x)\bar{c}^j_\beta(0)|0\rangle = i\int\frac{d^4k}{(2\pi^4)}e^{-ik\cdot x}\left(\delta^{ij}\frac{\not{k}+m_c}{k^2-m_c^2}+\cdots\right)_{\alpha\beta}.$$

真空--$f_0(980)$矩阵元
$$\langle f_0(980)(p)|\bar{s}^i_\alpha(x)s^j_\beta(0)|0\rangle = \frac{\delta^{ji}}{12}\bar{f}_{f_0}\int_0^1 du e^{iup\cdot x}\Big[\not{p}\phi_{2;f_0}(u) + m_{f_0}\phi^p_{3;f_0}(u)$$
$$-\frac{1}{6}m_{f_0}\sigma_{\mu\nu}p^\mu x^\nu\phi^\sigma_{3;f_0}(u)\Big]_{\beta\alpha} + \cdots$$

(3). 强子表示:

在关联函数中插入一组Ds介子基态和激发态的完备集得到

$$\Pi_\mu(p,q) = \frac{\langle f_0(980)(p)|\bar{s}\gamma_\mu\gamma_5 c|D_s^+(p+q)\rangle\langle D_s^+(p+q)|\bar{c}i\gamma_5 s|0\rangle}{m_{D_s^+}^2 - (p+q)^2} + \sum_H \frac{\langle f_0(980)(p)|\bar{s}\gamma_\mu\gamma_5 c|D_s^H(p+q)\rangle\langle D_s^H(p+q)|\bar{c}i\gamma_5 s|0\rangle}{m_{D_s^H}^2 - (p+q)^2}$$

对应的跃迁矩阵元、分布振幅矩阵元

$$\langle f_0(980)(p)|\bar{s}\gamma_\mu\gamma_5 c|D_s^+(p+q)\rangle = -2if_+(q^2)p_\mu - i[f_+(q^2)+f_-(q^2)]q_\mu,$$

$$\langle D_s^+(p+q)|\bar{c}i\gamma_5 s|0\rangle = m_{D_s^+}^2 f_{D_s^+}/(m_c+m_s)$$

$$F_{\rm had}[p^2,(p+q)^2] = \frac{-2im_{D_s^+}^2 f_{D_s^+} f_+(q^2)}{(m_c+m_s)[m_{D_s^+}^2 - (p+q)^2]} + \cdots$$

$$
\begin{aligned}
f_+(q^2) = & \frac{(m_c+m_s)\bar{f}_{f_0}}{2m_{D_s}^2 f_{D_s}} e^{m_{D_s^+}^2/M^2} \Bigg\{ \int_{u_0}^1 du e^{-(m_c^2-\bar{u}q^2+u\bar{u}m_{f_0}^2)/(uM^2)} \Bigg[-m_c \frac{\phi_{2;f_0}(u)}{u} \\
& + m_{f_0} \left(\frac{2}{u} + \frac{4um_c^2 m_{f_0}^2}{(m_c^2-q^2+u^2m_{f_0}^2)^2} - \frac{m_c^2+q^2-u^2m_{f_0}^2}{m_c^2-q^2+u^2m_{f_0}^2} \frac{d}{du} \right) \frac{\phi_{3;f_0}^\sigma(u)}{6} + m_{f_0} \\
& \times \phi_{3;f_0}^p(u) \Bigg] + e^{-(m_c^2-\bar{u}q^2+u\bar{u}m_{f_0}^2)/(uM^2)} \frac{m_{f_0}}{6m_c} \frac{m_c^2+q^2-u^2m_{f_0}^2}{m_c^2-q^2+u^2m_{f_0}^2} \phi_{3;f_0}^\sigma(u) \Big|_{u\to 1} \Bigg\}.
\end{aligned}
$$

被积函数中，不出现Borel参数，使得各项分布振幅前为一系列的常数和导数项。

II. Mixing Angle

$$f_0(980)=\left[\frac{1}{\sqrt{2}}(|\bar{u}u\rangle+|\bar{d}d\rangle)\right]\sin\theta+|\bar{s}s\rangle\cos\theta$$

$\theta=(19.7\pm12.8)^\circ$ BESIII Phys.Rev.Lett.132, 141901 (2024)

$|\theta|\leq30°$ LHCb Phys. Rev. D87, 052001 (2013)

$\theta=(20\pm10)^\circ$ ShanCheng EPJC 84, 379 (2024)

- 在计算twist-2分布振幅的时候，同时考虑$|n\bar{n}\rangle$和$|s\bar{s}\rangle$两种情况

III. LCDA

$$\phi_{2;a_0}(x,\mu) = 6x\bar{x}\left[a^0_{2;a_0}(\mu) + \sum_{n=1}^{\infty} a^n_{2;a_0}(\mu) C_n^{3/2}(\xi)\right]$$

⇓

$a^0_{2;f_0}, a^1_{2;f_0}, a^2_{2;f_0}, a^3_{2;f_0}, \dots$

⇓

$\xi^0_{2;f_0}, \xi^1_{2;f_0}, \xi^2_{2;f_0}, \xi^3_{2;f_0}, \dots$ — Anti-symmetry — $\xi^1_{2;f_0}, \xi^3_{2;f_0}, \dots$ ☆

⇓

QCD sum rule approach

⇓

Background field theory

1. 构建关联函数($s\bar{s}$):

Twist-2，3分布振幅矩阵元的定义：

$$\langle f_0(980)|\bar{q}_2(z_2)q_1(z_1)|0\rangle = m_{f_0}\bar{f}_{f_0}\int_0^1 dx e^{i(xp\cdot z_2+\bar{x}p\cdot z_1)}\phi^p_{3;f_0}(x)$$

$$\langle f_0(980)|\bar{q}_2(z_2)\gamma_\mu q_1(z_1)|0\rangle = p_\mu\bar{f}_{f_0}\int_0^1 dx e^{i(xp\cdot z_2+\bar{x}p\cdot z_1)}\phi_{2;f_0}(x)$$

$z^2 \to 0$，对两边进行泰勒展开：

$$\langle f_0(980)|\bar{q}_2(0)q_1(0)|0\rangle = m_{f_0}\bar{f}_{f_0}\langle\xi^{p,0}_{3;f_0}\rangle$$

$$\langle f_0(980)|\bar{q}_2(0)\not{z}(iz\cdot\overleftrightarrow{D})^n q_1(0)|0\rangle = \bar{f}_{f_0}(z\cdot p)^{n+1}\langle\xi^n_{2;f_0}\rangle$$

$$J_n^{(s\bar{s})}(x) = \bar{s}(x)\not{z}(iz\cdot\overleftrightarrow{D})^n s(x) \qquad \hat{J}_0^{(s\bar{s}),\dagger}(0) = \bar{s}(0)s(0)$$

2. 对关联函数作算符乘积展开(OPE):

$$\Pi_{2;f_0}^{(s\bar{s})}(z,q) = i\int d^4x e^{iq\cdot x}\langle 0|T\{\bar{s}(x)\not{z}(iz\cdot \overleftrightarrow{D})^n s(x), \bar{s}(0)s(0)\}|0\rangle$$
$$\Downarrow s \to s+\eta_s, \qquad \bar{s} \to \bar{s}+\bar{\eta}_s$$
$$= i\int d^4x e^{iq\cdot x}\langle 0|T\{[\bar{s}(x)+\bar{\eta}_s(x)]\not{z}(iz\cdot \overleftrightarrow{D})^n[s(x)+\eta_s(x)],$$
$$[\bar{s}(0)+\bar{\eta}_s(0)][s(0)+\eta_s(0)]\}|0\rangle$$

s是夸克场，η_s是对应的量子起伏。展开总共有16项，但只有三项有贡献

$$\Pi_{2;f_0}^{(s\bar{s})}(z,q) = i\int d^4x e^{iq\cdot x}\{-\mathrm{Tr}\langle 0|S_F^s(0,x)\not{z}(iz\cdot \overleftrightarrow{D})^n S_F^s(x,0)|0\rangle$$
$$+\mathrm{Tr}\langle 0|\bar{s}(x)s(0)\not{z}(iz\cdot \overleftrightarrow{D})^n S_F^s(x,0)|0\rangle$$
$$+\mathrm{Tr}\langle 0|S_F^s(0,x)\not{z}(iz\cdot \overleftrightarrow{D})^n \bar{s}(0)s(x)|0\rangle$$
$$+\cdots$$

3.强子表示

对关联函数插入强子态完备集可得：

$$\mathrm{Im}I_{2;f_0}^{\mathrm{had}}(s) = \pi m_{f_0}\delta(s-m_{f_0}^2)\bar{f}_{f_0}^{(s\bar{s})2}\langle\xi_{2;f_0}^{(s\bar{s}),n}\rangle|_\mu\langle\xi_{3;f_0}^{p(s\bar{s}),0}\rangle|_\mu + \mathrm{Im}I_{2;f_0}^{\mathrm{pert}}(s)\theta(s-s_{f_0})$$

4.色散关系

利用色散关系令两种表示相等 $\frac{1}{\pi}\frac{1}{M^2}\int_{4m_s^2}^{s_{f_0}} ds e^{-s/M^2}\mathrm{Im}I_{2;f_0}^{\mathrm{had}}(s) = L_{M^2}I_{2;f_0}^{\mathrm{QCD}}(q^2)$

再通过Borel变换压低高量纲与凝聚的贡献，可得最终的求和规则表达式，即：

Twist-2分布振幅的矩$\langle\xi_{3;f_0}^{p(s\bar{s}),0}\rangle\langle\xi_{2;f_0}^{(s\bar{s}),n}\rangle$

Twist-3分布振幅的零阶矩$\langle\xi_{3;f_0}^{p(s\bar{s}),0}\rangle$ ⟶ 利用其对称性，即归一的物理性质，来确定连续态阈值s_{f_0}

Twist-2分布振幅($s\bar{s}$)的矩表达式为：

$$\frac{m_{f_0}\tilde{f}_{f_0}^{2(s\bar{s})}}{M^2 e^{m_{f_0}^2/M^2}}\langle\xi_{3;f_0}^{p(s\bar{s}),0}\rangle\langle\xi_{2;f_0}^{(s\bar{s}),n}\rangle = [1-(-1)^n]\Big\{-\frac{1}{\pi M^2}\int_{4m_s^2}^{s_{f_0}} ds e^{-s/M^2}\frac{3m_s}{16\pi(n+1)(n+2)}$$
$$\times[2n+3+(-1)^n] + \left(\frac{1}{M^2}+\frac{nm_s^2}{M^4}\right)\langle\bar{s}s\rangle + \left(-\frac{n}{2M^4}+\frac{5nm_s^2}{18M^6}\right)\langle g_s\bar{s}\sigma TGs\rangle$$
$$+\frac{2m_s}{27M^6}\langle g_s\bar{s}s\rangle^2 + L_{M^2}I_{\langle G^2\rangle} + L_{M^2}I_{\langle G^3\rangle} + L_{M^2}I_{\langle q^4\rangle}\Big\},$$

$$L_{M^2}I_{\langle G^2\rangle} = -m_s\frac{\langle\alpha_s G^2\rangle}{48\pi M^4}\Big\{4\Big[\ln\Big(\frac{M^2}{\mu^2}\Big)+1-\gamma_E\Big]+2\,\frac{n+2}{n+1} - 4(\psi^{(0)}(n+1)+\gamma_E)$$
$$+\theta(n-2)\Big[(6n+4)\psi_3(n)+\frac{4n^3+10n^2-6n-8}{n(n+1)}\Big]+\theta(n-1)\Big[-\frac{2}{n+1}[3n^2$$
$$+5n+3+(n+1)\psi_2(n)]-8(n+1)\Big[\psi^{(0)}(n+1)+2\gamma_E-\ln\Big(\frac{M^2}{\mu^2}\Big)-1\Big]\Big]\Big\},$$

$$
\begin{aligned}
L_{M^2} I_{\langle G^3\rangle} = m_s \frac{\langle g_s^3 f G^3\rangle}{23040\pi^2 M^6} \Big\{ & (1584 - 7776n)\left[\ln\left(\frac{M^2}{\mu^2}\right) + \frac{3}{2} - \gamma_E\right] + \Big[72(108n-22) \\
& - 96(81n-5)\Big](\psi^{(0)}(n+1) + \gamma_E) - \frac{12(-1458n^2 - 1471n - 121)}{n+1} + \theta(n-2) \\
& \times \Big[72(n-1)\frac{-51n^3 - 35n^2 + 80n + 52}{n(n+1)} + 144(9n+13)\psi_3(n) + 864n(n-1) \\
& \times \left[\ln\left(\frac{M^2}{\mu^2}\right) + \frac{3}{2} - \gamma_E\right]\Big] + \theta(n-1)\Big[288(5n+8)\left[\ln\left(\frac{M^2}{\mu^2}\right) + \frac{3}{2} - \gamma_E\right] \\
& - (1440n + 1824)(\psi^{(0)}(n+1) + \gamma_E) - \frac{12}{n+1}(431n^2 - 1281n - 1030) \\
& - 96\psi_2(n)\Big] + \theta(n-3)\Big[\frac{54}{n(n^2-1)}(-77n^5 - 249n^4 + 67n^3 - 95n^2 - 294n \\
& - 72) - 216(20n^2 + 16n + 9)\psi_1(n) - 648(n^2 + 5n + 6)\Big[\ln\left(\frac{M^2}{\mu^2}\right) + \frac{3}{2} \\
& - \gamma_E\Big] + 648(n^2 + 5n + 6)(\psi^{(0)}(n+1) + \gamma_E)\Big]\Big\},
\end{aligned}
$$

$$L_{M^2} I_{\langle q^4\rangle} = m_s \frac{(2+\kappa^2)\langle g_s^2 q\bar{q}\rangle^2}{233280\pi^2 M^6}\Bigg\{648(66n-23)\left[\ln\left(\frac{M^2}{\mu^2}\right)+\frac{3}{2}-\gamma_E\right]+14904 \times\left(\psi^{(0)}(n+1)+\gamma_E\right)+\frac{216(27n^2+40n+82)}{n+1}+\theta(n-3)\Bigg[3564(n^2+5n +6)\left[\ln\left(\frac{M^2}{\mu^2}\right)+\frac{3}{2}-\gamma_E\right]+1188(20n^2+16n+9)\psi_1(n)-3564(n^2+5n +6)\left(\psi^{(0)}(n+1)+\gamma_E\right)+\frac{54}{n(n^2-1)}(581n^5+2355n^4+1013n^3-991n^2 +606n+396)\Bigg]+\theta(n-2)\Bigg[-4752n(n-1)\left[\ln\left(\frac{M^2}{\mu^2}\right)+\frac{3}{2}-\gamma_E\right]+72(46n -45)\psi_3(n)-108(n-1)\frac{-82n^3-35n^2+83n+60}{n^2(n+1)}\Bigg]+\theta(n-1)\Bigg\{414\,\psi_2(n) -648(3n-10)\left[\left(\psi^{(0)}(n+1)+\gamma_E\right)\right]+648(3n-10)\left[\ln\left(\frac{M^2}{\mu^2}\right)+\frac{3}{2}-\gamma_E\right] -\frac{18}{n+1}(1555n^2+2283n+368)\Bigg\}\Bigg\}.$$

Twist-3分布振幅($s\bar{s}$)的0阶矩表达式：

$$\frac{m_{f_0}^2 \bar{f}_{f_0}^{2(s\bar{s})}}{M^2 e^{m_{f_0}^2/M^2}} \langle \xi_{3;f_0}^{p(s\bar{s}),0} \rangle^2 = \frac{1}{\pi M^2} \int_{4m_s^2}^{s_{f_0}} ds e^{-s/M^2} \mathrm{Im} I_{3;f_0}^{\mathrm{pert}}(s) - \frac{3m_s}{M^2} \langle \bar{s}s \rangle + \frac{2}{27M^4} \left(4 + \frac{5m_s^2}{M^2}\right)$$
$$\times \langle g_s \bar{s}s \rangle^2 + \frac{1}{\pi M^2} \left\{ \frac{1}{8} + \frac{m_s^2}{M^2} \left[\frac{1}{3} + \frac{1}{2} \left(\ln\left(\frac{M^2}{\mu^2}\right) + 1 - \gamma_E \right) \right] \right\} \langle \alpha_s G^2 \rangle + \frac{m_s^2}{\pi^2 M^6}$$
$$\times \left[-\frac{97}{162} + \frac{4}{27} \left(\ln\left(\frac{M^2}{\mu^2}\right) + \frac{3}{2} - \gamma_E \right) \right] \langle g_s^3 f G^3 \rangle + \frac{(2+\kappa^2)}{M^4} \left\{ \frac{5}{54\pi^2} + \frac{m_s^2}{\pi^2 M^2} \right.$$
$$\left. \times \left[\frac{119}{729} - \frac{30}{253} \left(\ln\left(\frac{M^2}{\mu^2}\right) + \frac{3}{2} - \gamma_E \right) \right] \right\} \langle g_s^2 \bar{q}q \rangle^2 + \frac{m_s}{M^4} \langle g_s \bar{s} \sigma TGs \rangle,$$

$$\mathrm{Im} I_{3;f_0}^{\mathrm{pert}}(s) = \frac{3s}{8\pi} \left\{ \left(1 - \frac{m_s^2}{s}\right) \left[\frac{s^2 - (s - m_s^2)^2}{2s^2} + 1 \right] \right\} + \frac{3m_s^2}{4\pi} \left(\frac{2(1-s)^2 v^2}{s^2} \right),$$

Twist-2分布振幅($s\bar{s}$)矩的表达式为：

$$\langle \xi_{2;f_0}^{(s\bar{s}),n} \rangle|_\mu = \frac{(\langle \xi_{3;f_0}^{p(s\bar{s}),0} \rangle \langle \xi_{2;f_0}^{(s\bar{s}),n} \rangle)}{\sqrt{\langle \xi_{3;f_0}^{p(s\bar{s}),0} \rangle^2}}$$

夸克组分为轻夸克$(n\bar{n})$时的关联函数

$$\Pi_{2;f_0}^{(n\bar{n})}(z,q) = i\int d^4x e^{iq\cdot x}\langle 0|T\{J_n^{(n\bar{n})}(x), \hat{J}_0^{(n\bar{n}),\dagger}(0)\}|0\rangle$$
$$= (z\cdot q)^{n+1} I_{2;f_0}^{(n\bar{n})}(q^2)$$

对应的插入流

$$J_n^{(n\bar{n})}(x) = \frac{1}{\sqrt{2}}[\bar{u}(x)\not{z}(iz\cdot \overleftrightarrow{D})^n u(x) + \bar{d}(x)\not{z}(iz\cdot \overleftrightarrow{D})^n d(x)]$$
$$\hat{J}_0^{(n\bar{n}),\dagger}(0) = \frac{1}{\sqrt{2}}[\bar{u}(0)u(0) + \bar{d}(0)d(0)]$$

Twist-3分布振幅的0阶矩表达式：

$$\frac{m_{f_0}^2 f_{f_0}^{2(n\bar{n})}\langle\xi_{3;f_0}^{p(n\bar{n}),0}\rangle^2}{M^2 e^{m_{f_0}^2/M^2}} = \frac{1}{2}\left\{\frac{1}{\pi}\frac{1}{M^2}\int_{(m_u+m_d)^2}^{s_{f_0}}\left(\frac{3s}{4\pi}\right)ds e^{-s/M^2} - \frac{3(\langle\bar{u}u\rangle m_u + \langle\bar{d}d\rangle m_d)}{M^2} + \frac{\langle\alpha_s G^2\rangle}{4\pi M^2} + \frac{5(2+\kappa^2)\langle g_s^2\bar{q}q\rangle^2}{27\pi^2 M^4} + \frac{8(\langle g_s\bar{u}u\rangle^2 + \langle g_s\bar{d}d\rangle^2)}{27M^4} + \frac{(\langle g_s\bar{u}\sigma TGu\rangle m_u + \langle g_s\bar{d}\sigma TGd\rangle m_d)}{M^4}\right\}$$

Twist-2分布振幅的矩表达式为：

$$\frac{m_{f_0} f_{f_0}^{2(n\bar{n})}\langle\xi_{3;f_0}^{p(n\bar{n}),0}\rangle\langle\xi_{2;f_0}^{(n\bar{n}),n}\rangle}{M^2 e^{m_{f_0}^2/M^2}} = \frac{[1-(-1)^n]}{2}\left\{\frac{1}{\pi}\frac{1}{M^2}\int_{(m_u+m_d)^2}^{s_{f_0}} ds e^{-s/M^2}\left[-\frac{3(m_u+m_d)}{16\pi(n+1)(n+2)} \times [2n+(-1)^n+3]\right] + \frac{(\langle\bar{u}u\rangle + \langle\bar{d}d\rangle)}{M^2} - \frac{n(\langle g_s\bar{u}\sigma TGu\rangle + \langle g_s\bar{d}\sigma TGd\rangle)}{2M^4} + \frac{2m_u\langle g_s\bar{u}u\rangle^2}{27M^6} + \frac{2m_d\langle g_s\bar{d}d\rangle^2}{27M^6} + L_{M^2}I_{\langle G^2\rangle} + L_{M^2}I_{\langle G^3\rangle} + L_{M^2}I_{\langle q^4\rangle}\right\}$$

$$L_{M^2}I_{\langle G^2\rangle} = -\frac{(m_u+m_d)\langle\alpha_s G^2\rangle}{48\pi M^4}\left\{4\left[\ln\left(\frac{M^2}{\mu^2}\right)+1-\gamma_E\right]+2\,\frac{n+2}{n+1} - 4(\psi^{(0)}(n+1)+\gamma_E) + \theta(n-2)\left[(6n+4)\psi_3(n) + \frac{4n^3+10n^2-6n-8}{n(n+1)}\right] + \theta(n-1)\left[-\frac{2}{n+1}[3n^2+5n+3+(n+1)\psi_2(n)] - 8(n+1)\left[\psi^{(0)}(n+1)+2\gamma_E - \ln\left(\frac{M^2}{\mu^2}\right)-1\right]\right]\right\}$$

$$
\begin{aligned}
L_{M^2} I_{\langle G^3\rangle} = \frac{(m_u + m_d)\langle g_s^3 f G^3\rangle}{23040\pi^2 M^6} \Bigg\{ & (1584 - 7776n)\left[\ln\left(\frac{M^2}{\mu^2}\right) + \frac{3}{2} - \gamma_E\right] + \Bigg[72(108n - 22) \\
& - 96(81n - 5)\Bigg](\psi^{(0)}(n+1) + \gamma_E) - \frac{12(-1458n^2 - 1471n - 121)}{n+1} + \theta(n-2)\Bigg[72 \\
& \times (n-1)\frac{-51n^3 - 35n^2 + 80n + 52}{n(n+1)} + 144(9n+13)\psi_3(n) + 864n(n-1)\Bigg[\ln\left(\frac{M^2}{\mu^2}\right) \\
& + \frac{3}{2} - \gamma_E\Bigg]\Bigg] + \theta(n-1)\Bigg[288(5n+8)\left[\ln\left(\frac{M^2}{\mu^2}\right) + \frac{3}{2} - \gamma_E\right] - (1440n + 1824)(\psi^{(0)} \\
& \times (n+1) + \gamma_E) - \frac{12}{n+1}(431n^2 - 1281n - 1030) - 96\psi_2(n)\Bigg] + \theta(n-3)\Bigg[\frac{54}{n(n^2-1)} \\
& \times (-77n^5 - 249n^4 + 67n^3 - 95n^2 - 294n - 72) - 216(20n^2 + 16n + 9)\psi_1(n) - 324 \\
& \times (n^2 + 5n + 6)\left[\ln\left(\frac{M^2}{\mu^2}\right) + \frac{3}{2} - \gamma_E\right] + 648(n^2 + 5n + 6)(\psi^{(0)}(n+1) + \gamma_E)\Bigg]\Bigg\},
\end{aligned}
$$

$$L_{M^2} I_{\langle q^4\rangle} = \frac{(m_u+m_d)(2+\kappa^2)\langle g_s^2 q\bar{q}\rangle^2}{233280\pi^2 M^6} \Bigg\{ 648(66n-23)\left[\ln\left(\frac{M^2}{\mu^2}\right)+\frac{3}{2}-\gamma_E\right] + 14904$$
$$\times\left(\psi^{(0)}(n+1)+\gamma_E\right) + \frac{216(27n^2+40n+82)}{n+1} + \theta(n-3)\Bigg[3564(n^2+5n+6)$$
$$\times\left[\ln\left(\frac{M^2}{\mu^2}\right)+\frac{3}{2}-\gamma_E\right] + 1188(20n^2+16n+9)\psi_1(n) - 3564(n^2+5n+6)(\psi^{(0)}$$
$$\times(n+1)+\gamma_E) + \frac{54}{n(n^2-1)}(581n^5+2355n^4+1013n^3-991n^2+606n+396)\Bigg]$$
$$+\theta(n-2)\Bigg[-4752n(n-1)\left[\ln\left(\frac{M^2}{\mu^2}\right)+\frac{3}{2}-\gamma_E\right] + 72(46n-45)\psi_3(n) - 108$$
$$\times(n-1)\frac{-82n^3-35n^2+83n+60}{n^2(n+1)}\Bigg] + \theta(n-1)\Bigg\{414\psi_2(n) - 648(3n-10)$$
$$\times\left[\left(\psi^{(0)}(n+1)+\gamma_E\right)\right] + 648(3n-10)\left[\ln\left(\frac{M^2}{\mu^2}\right)+\frac{3}{2}-\gamma_E\right] - \frac{18}{n+1}(1555n^2$$
$$+2283n+368)\Bigg\}\Bigg\}.$$

$f_0(980)$的衰变常数

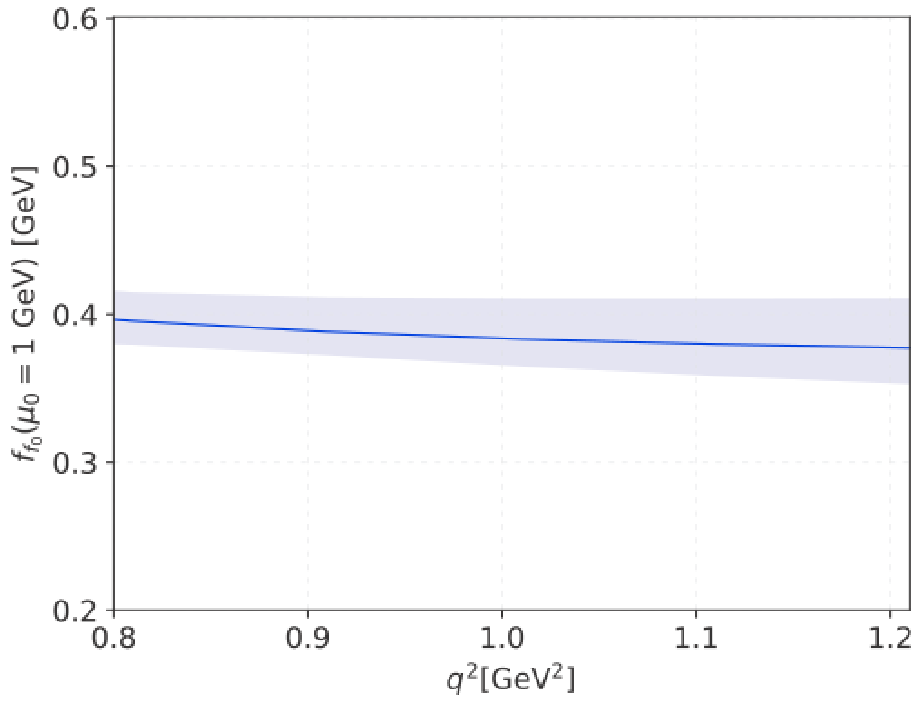
$$\bar{f}_{f_0} = \cos\theta \bar{f}_{f_0}^{(s\bar{s})} + \sin\theta \bar{f}_{f_0}^{(n\bar{n})}$$

Twist-3分布振幅是对称的，因此对应的0阶矩是归一的：

$$\frac{m_{f_0}^2 \bar{f}_{f_0}^{2(n\bar{n})} \langle \xi_{3;f_0}^{p(n\bar{n}),0} \rangle^2}{M^2 e^{m_{f_0}^2/M^2}} = \frac{1}{2}\left\{ \frac{1}{\pi}\frac{1}{M^2}\int_{(m_u+m_d)^2}^{s_{f_0}} \left(\frac{3s}{4\pi}\right) ds e^{-s/M^2} - \frac{3(\langle \bar{u}u\rangle m_u + \langle \bar{d}d\rangle m_d)}{M^2} + \frac{\langle \alpha_s G^2\rangle}{4\pi M^2} + \frac{5(2+\kappa^2)\langle g_s^2 \bar{q}q\rangle^2}{27\pi^2 M^4} + \frac{8(\langle g_s \bar{u}u\rangle^2 + \langle g_s \bar{d}d\rangle^2)}{27 M^4} + \frac{(\langle g_s \bar{u}\sigma TGu\rangle m_u + \langle g_s \bar{d}\sigma TGd\rangle m_d)}{M^4} \right\}$$

$$\frac{m_{f_0}^2 \bar{f}_{f_0}^{2(s\bar{s})}}{M^2 e^{m_{f_0}^2/M^2}} \langle \xi_{3;f_0}^{p(s\bar{s}),0} \rangle^2 = \frac{1}{\pi M^2}\int_{4m_s^2}^{s_{f_0}} ds e^{-s/M^2} \mathrm{Im} I_{3;f_0}^{\mathrm{pert}}(s) - \frac{3m_s}{M^2}\langle \bar{s}s\rangle + \frac{2}{27M^4}\left(4 + \frac{5m_s^2}{M^2}\right) \times \langle g_s \bar{s}s\rangle^2 + \frac{1}{\pi M^2}\left\{ \frac{1}{8} + \frac{m_s^2}{M^2}\left[\frac{1}{3} + \frac{1}{2}\left(\ln\left(\frac{M^2}{\mu^2}\right) + 1 - \gamma_E\right)\right]\right\} \langle \alpha_s G^2\rangle + \frac{m_s^2}{\pi^2 M^6} \times \left[-\frac{97}{162} + \frac{4}{27}\left(\ln\left(\frac{M^2}{\mu^2}\right) + \frac{3}{2} - \gamma_E\right)\right] \langle g_s^3 f G^3\rangle + \frac{(2+\kappa^2)}{M^4}\left\{ \frac{5}{54\pi^2} + \frac{m_s^2}{\pi^2 M^2} \times \left[\frac{119}{729} - \frac{30}{253}\left(\ln\left(\frac{M^2}{\mu^2}\right) + \frac{3}{2} - \gamma_E\right)\right]\right\} \langle g_s^2 \bar{q}q\rangle^2 + \frac{m_s}{M^4}\langle g_s \bar{s}\sigma TGs\rangle,$$

$$\mathrm{Im} I_{3;f_0}^{\mathrm{pert}}(s) = \frac{3s}{8\pi}\left\{ \left(1 - \frac{m_s^2}{s}\right)\left[\frac{s^2 - (s-m_s^2)^2}{2s^2} + 1\right]\right\} + \frac{3m_s^2}{4\pi}\left(\frac{2(1-s)^2 v^2}{s^2}\right),$$



确定Borel参数上下限。

1. 连续态贡献不超过30%,
2. 六维凝聚不超过1%,
3. 在Borel参数区间是平缓的

	$f_{f_0}(\mu_0 = 1\text{GeV})$
This work	0.386 ± 0.009
QCD SR'06 [40]	0.370 ± 0.020
3PSR [41]	0.180 ± 0.015

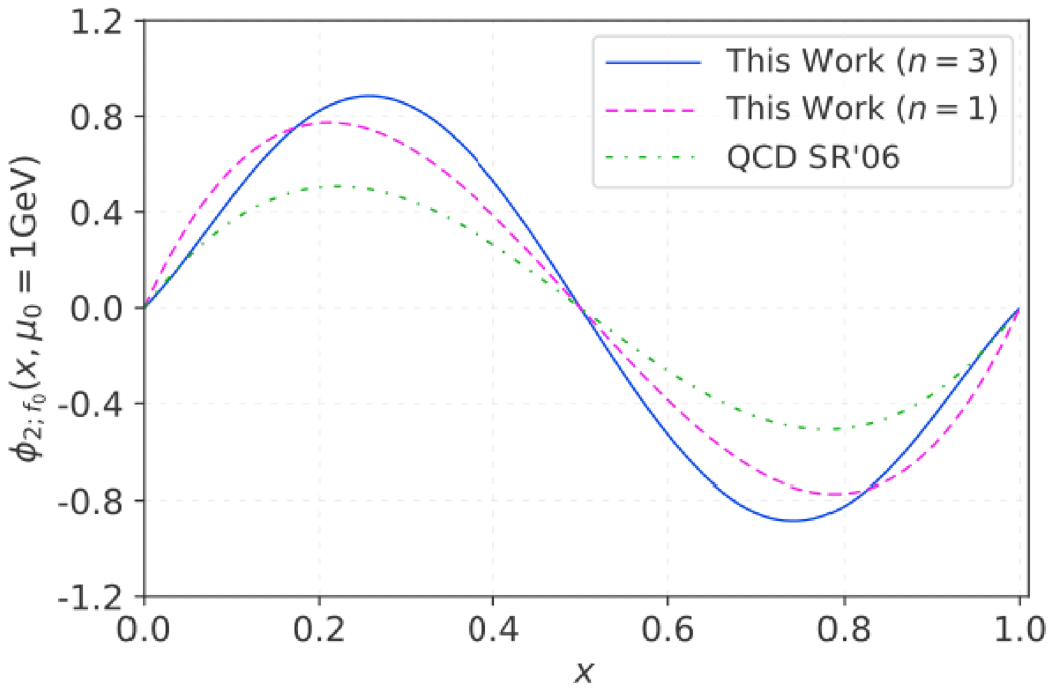
我们的预测结果在QCDSR的误差范围内，说明s夸克组分占据主导地位

1. $f_0(980)$twist-2分布振幅

$$\langle\xi^1_{2;f_0}\rangle|_{\mu_0} = -0.693^{+0.095}_{-0.081},$$
$$\langle\xi^1_{2;f_0}\rangle|_{\mu_k} = -0.542^{+0.074}_{-0.063},$$
$$\langle\xi^3_{2;f_0}\rangle|_{\mu_0} = -0.261^{+0.097}_{-0.095},$$
$$\langle\xi^3_{2;f_0}\rangle|_{\mu_k} = -0.217^{+0.079}_{-0.063}.$$

$$B_1(\mu) = \frac{5}{3}\langle\xi^1_{2;f_0}\rangle|_\mu,$$
$$B_3(\mu) = \frac{3}{4}(7\langle\xi^3_{2;f_0}\rangle|_\mu - 3\langle\xi^1_{2;f_0}\rangle|_\mu)$$

$$B_1(\mu_0) = -1.156^{+0.157}_{-0.133},$$
$$B_1(\mu_k) = -0.903^{+0.123}_{-0.104},$$
$$B_3(\mu_0) = 0.191^{+0.294}_{-0.317},$$
$$B_3(\mu_k) = 0.079^{+0.252}_{-0.269}.$$



$$\phi_{2;f_0}(x,\mu) = \bar{f}_{f_0}(\mu)6x\bar{x}\left[B_0(\mu) + \sum_{n=1}^{\infty} B_n(\mu)C_n^{3/2}(\xi)\right]$$

QCDSR 06: H. Y. Cheng PRD 73, 014017(2006)

预测的twist-2分布振幅是反对称行为，与QCDSR预测行为一致

2. $D_s \to f_0(980)$跃迁形状因子(TFF)

	$f_+(0)$
This work (LCSR)	$0.516^{+0.027}_{-0.024}$
BESIII'24 [28]	$0.518 \pm 0.018 \pm 0.036$
LCSR'10 [29]	0.300 ± 0.030
3PSR'04 [32]	0.50 ± 0.13
3PSR'10 [33]	0.48 ± 0.23
LCSR'24-s1 [35]	0.580 ± 0.070
LCSR'24-s2 [35]	0.400 ± 0.060
LCSR'24-s3 [35]	$0.780^{+0.130}_{-0.100}$

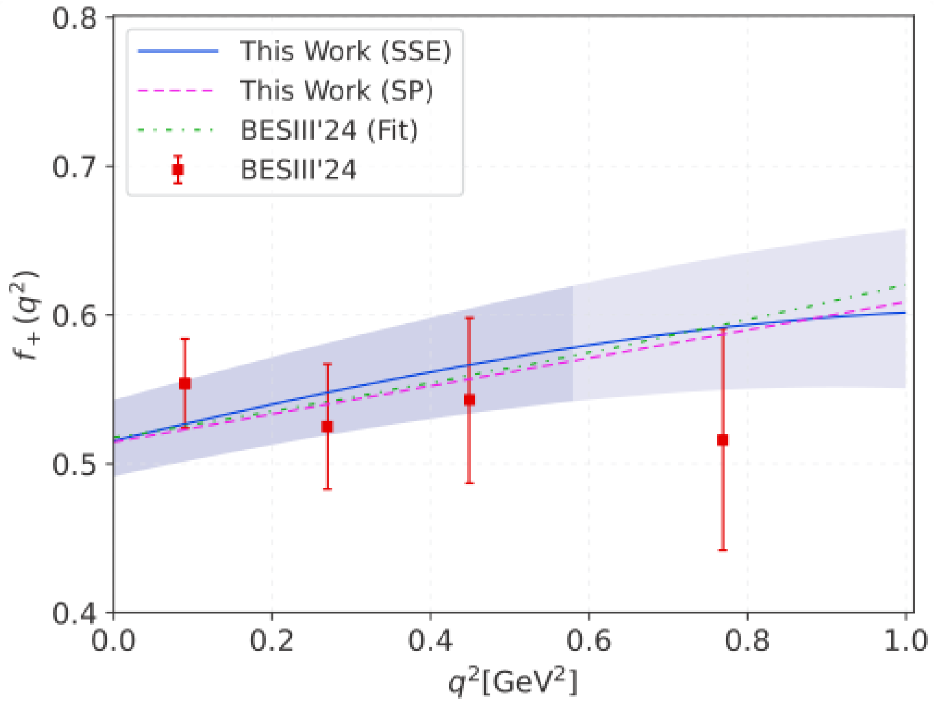
$$s_0 = 6.5 \pm 0.5\ GeV$$

$$M^2 = 7.0 \pm 1.0\ GeV^2$$

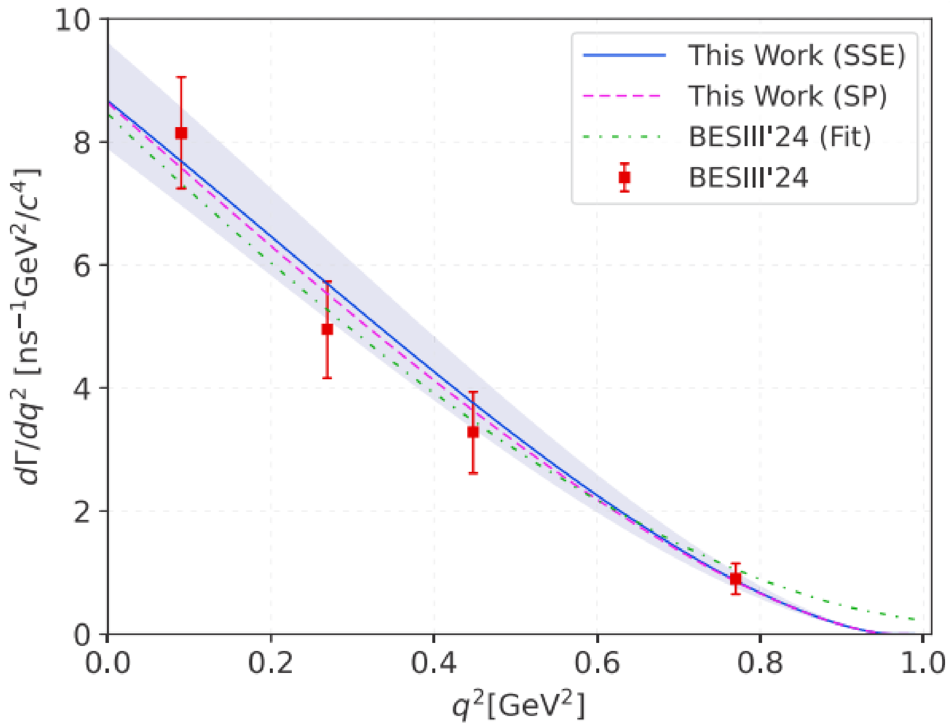
端点处的跃迁形状因子接近于最新的实验预测结果

简单级数展开参数化(SSE)

$$f_+(q^2) = \frac{1}{1 - q^2/m_{D_s^*}^2} \sum_{k=1}^{2} \alpha_k z^k(t, t_0)$$



3. $D_s \to f_0(980)e^+\nu_e$的衰变宽度与分支比



	$\mathcal{B}(D_s \to f_0(980)(\to \pi^+\pi^-)e^+\nu_e)$
This work(SSE)	$(1.783^{+0.227}_{-0.189}) \times 10^{-3}$
This work(SP)	$(1.743^{+0.190}_{-0.157}) \times 10^{-3}$
CLEO'09 [24]	$(1.3 \pm 0.4 \pm 0.1) \times 10^{-3}$
CLEO'09 [25]	$(2.0 \pm 0.3 \pm 0.1) \times 10^{-3}$
BESIII'24 [28]	$(1.72 \pm 0.13 \pm 0.10) \times 10^{-3}$

- 我们预测的微分衰变宽度中心值包含误差均在BESIII'24的误差范围之内；
- 可以进一步考虑twist-3分布振幅带来的影响。

arXiv: 2506.15967

III. $D_s \to \phi l^+ \nu_l$ decay process

See Liu Zhao-Feng's Report

1. 实验与理论组得到的分支比有一定差距

2018 BESIII 2015 CLEO 2008 BaBar 2022 PDG $\longrightarrow$ $D_s \to \phi l \nu_l \in [2.14, 2.61] \times 10^{-2}$

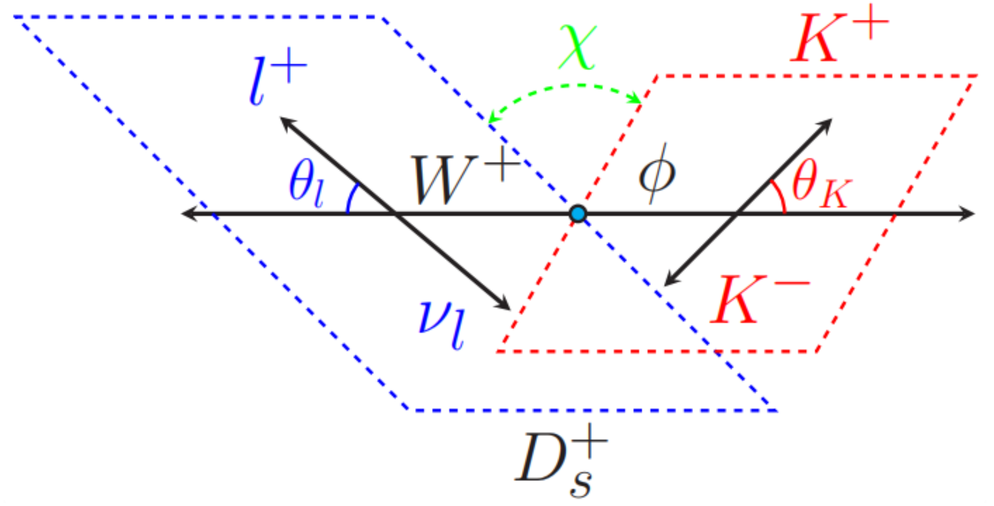
CLFQM CCQM 3PSR CQM HQEFT LCSR $\longrightarrow$ $D_s \to \phi l \nu_l \in [1.80, 3.10] \times 10^{-2}$

2. 目前只有一组理论组用LCSR方法计算$D_s \to \phi$的TFFs, γ_V, γ_2 与实验结果相差较大

3.对于ϕ介子分布振幅的研究比较少

P. Ball $a_{2;\phi}^{\parallel} = 0.18(8)$; $a_{2;\phi}^{\perp} = 0.14(7)$

III. $D_s \to \phi l^+ \nu_l$ decay process



$$\frac{d\Gamma(D_s^+ \to \phi\ell^+\nu_\ell,\ \phi \to K^+K^-)}{dq^2 d\cos\theta_K d\cos\theta_\ell d\chi} = \frac{3}{8(4\pi)^4} G_F^2 |V_{cs}|^2 \frac{|\mathbf{p_2}| m_\ell^2}{m_{D_s^+}^2} \mathcal{B}(\phi \to K^+K^-)$$
$$\times \{ \sin^2\theta_K \sin^2\theta_\ell |H_+(q^2)|^2 + \sin^2\theta_K \sin^2\theta_\ell |H_-(q^2)|^2$$
$$+ 4\cos^2\theta_K \cos^2\theta_\ell |H_0(q^2)|^2 + 4\cos^2\theta_K |H_t(q^2)|^2$$
$$+ \sin^2\theta_K \sin^2\theta_\ell \cos 2\chi H_+(q^2) H_-(q^2)$$
$$+ \sin 2\theta_K \sin 2\theta_\ell \cos 2\chi H_+(q^2) H_0(q^2)$$
$$+ \sin 2\theta_K \sin 2\theta_\ell \cos 2\chi H_-(q^2) H_0(q^2)$$
$$+ 2\sin 2\theta_K \sin\theta_\ell \cos\chi H_+(q^2) H_t(q^2)$$
$$+ 2\sin 2\theta_K \sin\theta_\ell \cos\chi H_-(q^2) H_t(q^2)$$
$$+ 8\cos^2\theta_K \cos\theta_\ell H_0(q^2) H_t(q^2) \},$$

III. $D_s \to \phi l^+ \nu_l$ decay process

角度积分以后，半轻衰变 $D_s \to \phi l^+ \nu_l$ 的宽度公式为：

$$\begin{aligned}\frac{\mathrm{d}\Gamma}{\mathrm{d}q^2} &= \frac{G_F^2|V_{cs}|^2}{192\pi^3 m_{D_s^+}^3}\lambda^{1/2}(m_{D_s^+}^2, m_\phi^2, q^2)q^2[|H_+|^2+|H_-|^2+|H_0|^2] \\ &= \frac{\mathrm{d}\Gamma_\mathrm{L}}{\mathrm{d}q^2}+\frac{\mathrm{d}\Gamma_\mathrm{T}^+}{\mathrm{d}q^2}+\frac{\mathrm{d}\Gamma_\mathrm{T}^-}{\mathrm{d}q^2}.\end{aligned}$$

$$\begin{aligned}\frac{\mathrm{d}\Gamma_\mathrm{L}}{\mathrm{d}q^2} = \frac{G_F^2|V_{cs}|^2}{192\pi^2 m_{D_s^+}^3}\lambda^{1/2}(m_{D_s^+}^2, m_\phi^2, q^2)q^2\left|\frac{1}{2m_\phi\sqrt{q^2}}\left[(m_{D_s^+}^2-m_\phi^2-q^2)(m_{D_s^+}\right.\right. \\ \left.\left. +m_\phi)A_1(q^2)-\frac{\lambda(m_{D_s^+}^2, m_\phi^2, q^2)}{m_{D_s^+}^2+m_\phi}A_2(q^2)\right]\right|^2\end{aligned}$$

$$\frac{\mathrm{d}\Gamma_\mathrm{T}^\pm}{\mathrm{d}q^2} = \frac{G_F^2|V_{cs}|^2}{192\pi^2 m_{D_s^+}^3}\lambda^{1/2}(m_{D_s^+}^2, m_\phi^2, q^2)q^2\left|(m_{D_s^+}+m_\phi)A_1(q^2)\mp\frac{\lambda^{1/2}(m_{D_s^+}^2, m_\phi^2, q^2)}{(m_{D_s^+}+m_\phi)}V(q^2)\right|^2.$$

III. $D_s \to \phi l^+ \nu_l$ decay process

关联函数

$$\Pi_\mu(p,q) = i\int d^4x e^{iq\cdot x}\langle \phi(p,e)| T\{\bar{s}(x)\gamma_\mu(1-\gamma_5)c(x), \bar{c}(0)i(1-\gamma_5)s(0)\}|0\rangle$$
$$= \Pi_1 e_\mu^{*(\lambda)} - \Pi_2(e^{*(\lambda)}\cdot q)(2p+q)_\mu - \Pi_3(e^{*(\lambda)}\cdot q)q_\mu - i\Pi_V \varepsilon_\mu^{\alpha\beta\gamma} e_\alpha^{*(\lambda)} q_\beta p_\gamma$$

$$A_1(q^2) = \frac{2m_c(m_c+m_s)m_\phi f_\phi^\parallel}{m_{D_s^+}^2(m_{D_s^+}+m_\phi)f_{D_s^+}}\left\{\int_{u_0}^1 du \exp\left(\frac{m_{D_s^+}^2 - s(u)}{M^2}\right)\left(\frac{1}{u}\phi_{3;\phi}^\perp(u) - \frac{m_\phi^2}{u^2M^2}\bar{G}_3(u)\right) + m_\phi^2 \right.$$
$$\left. \times \exp\left(\frac{m_{D_s^+}^2 - s(u)}{M^2}\right)\frac{\bar{G}_3(u)}{q^2 - m_c^2 - u^2m_\phi^2}\Bigg|_{u\to u_0}\right\},$$

$$A_2(q^2) = \frac{2m_c(m_c+m_s)(m_{D_s^+}+m_\phi)m_\phi f_\phi^\parallel}{m_{D_s^+}^2 f_{D_s^+}}\left\{\int_{u_0}^1 \frac{du}{M^2}\exp\left(\frac{m_{D_s^+}^2 - s(u)}{M^2}\right)\left(\frac{1}{u^2}\Phi_{2;\phi}^\parallel(u) + \frac{m_c^2m_\phi^2}{4u^4M^4}\Phi_{4;\phi}^\parallel(u)\right.\right.$$
$$\left. - \frac{\Phi_{3;\phi}^\perp(u)}{u^2} + \frac{m_\phi^2}{u^2M^2}\bar{G}_3(u)\right) + \exp\left(\frac{m_{D_s^+}^2 - s(u)}{M^2}\right)\left[\frac{\Phi_{2;\phi}^\parallel(u)}{q^2 - m_c^2 - u^2m_\phi^2}m_c^2m_\phi^2\right.$$
$$+ \left(\frac{m_c^2 - q^2}{(q^2 - m_c^2 - u^2m_\phi^2)^5}(m_c^2 - 2u^2m_\phi^2 - q^2)\Phi_{4;\phi}^\parallel(u) + \frac{u^3m_c^2m_\phi^4}{(q^2 - m_c^2 - u^2m_\phi^2)^4}\frac{d}{du}\Phi_{4;\phi}^\parallel(u)\right.$$
$$\left. - \frac{u^2m_c^2m_\phi^2}{4(q^2 - m_c^2 - u^2m_\phi^2)^3}\frac{d^2}{d^2u}\Phi_{4;\phi}^\parallel(u)\right) - \left(\frac{2u^3m_\phi^2}{(q^2 - m_c^2 - u^2m_\phi^2)^3}\bar{G}_3(u) + \frac{u^2}{(q^2 - m_c^2 - u^2m_\phi^2)^2}\frac{d}{du}\bar{G}_3(u)\right)$$
$$\left.\left. + \frac{\Phi_{3;\phi}^\perp(u)}{q^2 - m_c^2 - u^2m_\phi^2}\right]\Bigg|_{u\to u_0}\right\},$$

III. $D_s \to \phi l^+ \nu_l$ decay process

TFF具体表达式

$$A_0(q^2) = A_3(q^2) + \frac{q^2 m_c(m_c+m_s) m_\phi f_\phi^\parallel}{m_{D_s^+}^2 f_{D_s^+}} \Bigg\{ \int_{u_0}^1 \frac{du}{M^2} \exp\left(\frac{m_{D_s^+}^2 - s(u)}{M^2}\right) \left(\frac{1}{u^2} \Phi_{2;\phi}^\parallel(u) - \frac{m_c^2 m_\phi^2}{4u^4 M^4} \Phi_{4;\phi}^\parallel(u) - \frac{\Phi_{3;\phi}^\perp(u)}{u^2} - \frac{(2-u)m_\phi^2}{u^2 M^2} \bar{G}_3(u) \right) + \exp\left(\frac{m_{D_s^+}^2 - s(u)}{M^2}\right) \Bigg[\frac{1}{q^2 - m_c^2 - u^2 m_\phi^2} \Phi_{2;\phi}^\parallel(u) + \left(\frac{m_c^2 - q^2}{(q^2 - m_c^2 - u^2 m_\phi^2)^5} m_c^2 m_\phi^2 (m_c^2 - 2u^2 m_\phi^2 - q^2) \Phi_{4;\phi}^\parallel(u) + \frac{u^3 m_c^2 m_\phi^4}{(q^2 - m_c^2 - u^2 m_\phi^2)^4} \frac{d}{du} \Phi_{4;\phi}^\parallel(u) - \frac{u^2 m_c^2 m_\phi^2}{4(q^2 - m_c^2 - u^2 m_\phi^2)^3} \frac{d^2}{d^2u} \Phi_{4;\phi}^\parallel(u) \right) + \left(2 \frac{(m_c^2 + 3u^2 m_\phi^2 - q^2) - u^3 m_\phi^2}{(q^2 - m_c^2 - u^2 m_\phi^2)^3} \bar{G}_3(u) + u(2-u) \times \frac{1}{(q^2 - m_c^2 - u^2 m_\phi^2)^2} \frac{d}{du} \bar{G}_3(u) \right) + \frac{\Phi_{3;\phi}^\perp(u)}{q^2 - m_c^2 - u^2 m_\phi^2} \Bigg] \Bigg|_{u\to u_0} \Bigg\},$$

$$V(q^2) = \frac{m_c(m_c+m_s)(m_{D_s^+} + m_\phi) m_\phi f_\phi^\parallel}{2 m_{D_s^+}^2 f_{D_s^+}} \Bigg\{ \int_{u_0}^1 du \exp\left(\frac{m_{D_s^+}^2 - s(u)}{M^2}\right) \frac{\psi_{3;\phi}^\perp(u)}{u^2 M^2} - \exp\left(\frac{m_{D_s^+}^2 - s(u)}{M^2}\right) \times \frac{1}{q^2 - m_c^2 - u^2 m_\phi^2} \psi_{3;\phi}^\perp(u) \Bigg|_{u\to u_0} \Bigg\},$$

ϕ-介子的twist-2分布振幅(光锥谐振子模型LCHO)

$$\phi_{2;\phi}^{\|}(x,\mu) = \frac{2\sqrt{6}}{f_\phi^{\|}} \int_{|\mathbf{k}_\perp^2| \le \mu^2} \frac{d^2\mathbf{k}_\perp}{16\pi^3} A_{2;\phi}^{\|} \boxed{\varphi_{2;\phi}^{\|}(x)} \frac{m_s(\mathcal{M}+2m_s)+2\mathbf{k}_\perp^2}{(\mathcal{M}+2m_s)\sqrt{2(\mathbf{k}_\perp^2+m_s)}} \times \exp\left[-\frac{1}{8\beta_{2;\phi}^2}\left(\frac{\mathbf{k}_\perp^2+m_s^2}{x\bar{x}}\right)\right].$$

$$\varphi_{2;\phi}^{\|(\mathrm{I})}(x) = 1 + b_{2;\phi}^2 C_2^{3/2}(2x-1) + b_{2;\phi}^4 C_4^{3/2}(2x-1)$$

$$\varphi_{2;\phi}^{\|(\mathrm{II})}(x) = (x\bar{x})^{\alpha_{2;\phi}}[1 + B_{2;\phi}^2 C_2^{3/2}(2x-1)].$$

约束条件

$$\int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} \Psi_{2;\phi}^{\|}(x,\mathbf{k}_\perp) = \frac{f_\phi^{\|}}{2\sqrt{6}}.$$

$$P_\phi = \int_0^1 dx \int \frac{d^2\mathbf{k}_\perp}{16\pi^3} |\Psi_{2;\phi}^{\|}(x,\mathbf{k}_\perp)|^2$$

$A_{2;\phi}^{\|}$ $\beta_{2;\phi}$

III. $D_s \to \phi l^+ \nu_l$ decay process

模型I

$$a^n_{2;\phi}(\mu) = \frac{\int_0^1 dx \phi^{\|}_{2;\phi}(x,\mu) C_n^{3/2}(2x-1)}{\int_0^1 dx 6x(1-x)[C_n^{3/2}(2x-1)]^2}. \longrightarrow \quad b^2_{2;\phi} \quad b^4_{2;\phi}$$

$$\langle \xi^{\|;2}_{2;\phi} \rangle|_\mu = \frac{1}{5} + \frac{12}{35} a^2_{2;\phi}(\mu) \qquad \langle \xi^{\|;4}_{2;\phi} \rangle|_\mu = \frac{3}{35} + \frac{8}{35} a^2_{2;\phi}(\mu) + \frac{8}{77} a^4_{2;\phi}(\mu),$$

$$\langle \xi^{\|;6}_{2;\phi} \rangle|_\mu = \frac{1}{21} + \frac{12}{77} a^2_{2;\phi}(\mu) + \frac{120}{1001} a^4_{2;\phi}(\mu) + \frac{64}{2145} a^6_{2;\phi}(\mu)$$

模型II

$$\chi^2(\theta) = \sum_{i=1}^{5} \frac{(y_i - \mu(x_i,\theta))^2}{\sigma_i^2}$$

$$P_{\chi^2} = \int_{\chi^2}^{\infty} f(y; n_d) dy$$

$$f(y; n_d) = \frac{1}{\Gamma\left(\frac{n_d}{2}\right) 2^{\frac{n_d}{2}}} y^{\frac{n_d}{2}-1} e^{-\frac{y}{2}}$$

$\longrightarrow \quad \alpha_{2;\phi} \quad B^2_{2;\phi}$

III. $D_s \to \phi l^+ \nu_l$ decay process

ϕ介子是矢量介子，其夸克组分为： $\bar{s}\, s$

1. 构建关联函数：

$$\Pi(z\cdot q) = i\int d^4x e^{iq\cdot x}\langle 0|T\{J_n(x), J_0^?(0)\rangle|0\rangle$$

$$\Downarrow J_n(x) = \bar{s}(x)\not{z}(iz\cdot D)^n s(x) \quad J_0^\dagger(0) = \bar{s}(0)\not{z}s(0)$$

$$= i\int d^4x e^{iq\cdot x}\langle 0|T\{\bar{s}(x)\not{z}(iz\cdot D)^n s(x), \bar{s}(0)\not{z}s(0)\,\rangle|0\rangle$$

ϕ-介子的twist-2分布振幅矩阵元定义

$$\langle 0|\bar{s}(z)\gamma_\mu s(-z)|\phi(q,\lambda)\rangle = m_\phi f_\phi^{\parallel}\int_0^1 \mathrm{d}x\, e^{i(xz\cdot q - \bar{x}z\cdot q)} q_\mu \frac{e^{*(\lambda)}\cdot z}{q\cdot z}\phi_{2;\phi}^{\parallel}(x,\mu).$$

2. 流的定义

$$\langle 0|\bar{q}_2(0)\not{z}(iz\cdot \overleftrightarrow{D})^n q_1(0)|V(q,\lambda)\rangle = f_V(z\cdot q)^{n+1}\langle\xi_{2;\parallel}^n\rangle$$

$$\langle 0|\bar{q}_2(0)\not{z}q_1(0)|V(q,\lambda)\rangle = f_V(z\cdot q)\langle\xi_{2;\parallel}^0\rangle$$

ϕ-介子twist-2分布振幅矩的具体表达式

$$
\begin{aligned}
\frac{(f_\phi^{\parallel})^2 \langle\xi_{2;\phi}^{\parallel;n}\rangle|_\mu \langle\xi_{2;\phi}^{\parallel;0}\rangle|_\mu}{M^2 e^{m_\phi^2/M^2}} = & \frac{1}{\pi M^2}\int_{4m_s^2}^{s_\phi} ds\, e^{-s/M^2}\mathrm{Im} I_{2;\phi}^{\rm pert}(s) + \frac{2m_s\langle\bar{s}s\rangle}{M^4} + \frac{\langle\alpha_s G^2\rangle}{12\pi M^4}\frac{1+n\theta(n-2)}{n+1} \\
& - \frac{m_s\langle g_s\bar{s}\sigma TGs\rangle}{9M^6}(8n+1) + \frac{\langle g_s\bar{s}s\rangle}{81M^6}4(2n+1) - \frac{\langle g_s^3 fG^3\rangle}{48\pi^2 M^6}n\theta(n-2) + \frac{\sum\langle g_s^2\bar{q}q\rangle^2}{486\pi^2 M^6} \\
& \times\Big\{-2(51n+25)\Big(-\ln\frac{M^2}{\mu^2}\Big) + 3(17n+35) + \theta(n-2)\Big[2n\Big(-\ln\frac{M^2}{\mu^2}\Big) - 25 \\
& \times(2n+1)\tilde{\psi}(n) + \frac{1}{n}(49n^2+100n+56)\Big]\Big\} + m_s^2\Big\{-\frac{\langle\alpha_s G^2\rangle}{6\pi M^6}\Big[\theta(n-2)(n\tilde{\psi}(n)-2) \\
& -n-2+2n\Big(-\ln\frac{M^2}{\mu^2}\Big)\Big] + \frac{\langle g_s^3 fG^3\rangle}{288\pi^2 M^8}\Big\{-10\delta^{n0} + \theta(n-2)\Big[4n(2n-1)\Big(-\ln\frac{M^2}{\mu^2}\Big) \\
& -4n\tilde{\psi}(n) + 8(n^2-n+1)\Big] + \theta(n-4)\Big[2n(8n-1)\tilde{\psi}(n) - (19n^2+19n+6)\Big] + 8n \\
& \times(3n-1)\Big(-\ln\frac{M^2}{\mu^2}\Big) - (21n^2+53n-6)\Big\} - \frac{\sum\langle g_s^2 q\bar{q}\rangle^2}{972\pi^2 M^8}\Big\{6\delta^{n0}\Big[16\Big(-\ln\frac{M^2}{\mu^2}\Big) - 3\Big] \\
& +\theta(n-2)\Big[8(n^2+12n-12)\Big(-\ln\frac{M^2}{\mu^2}\Big) - 2(29n+22)\tilde{\psi}(n) + 4\Big(5n^2-2n-33 \\
& +\frac{46}{n}\Big)\Big] + \theta(n-4)\Big[2(56n^2-25n+24)\tilde{\psi}(n) - (139n^2+91n+54)\Big] + 8\Big(27n^2
\end{aligned}
$$

III. $D_s \to \phi l^+ \nu_l$ decay process

$$-15\,n-11\Big)\Big(-\ln\frac{M^2}{\mu^2}\Big)-3(63n^2+159n-50)\Big\}+\frac{4(n-1)}{3}\frac{m_s\langle\bar{s}s\rangle}{M^6}+\frac{8n-3}{9}$$
$$\times\frac{m_s\langle g_s\bar{s}\sigma TGs\rangle}{M^8}-\frac{4(2n+1)}{81}\frac{\langle g_s\bar{s}s\rangle^2}{M^8}\Big\}, \qquad (2.38)$$

$$\mathrm{Im}\,I^{\rm pert}_{2;\phi}(s)=\frac{3v^{n+1}}{8\pi(n+1)(n+3)}\Big\{[1+(-1)^n](n+1)\frac{1-v^2}{2}+[1+(-1)^n]\Big\},$$

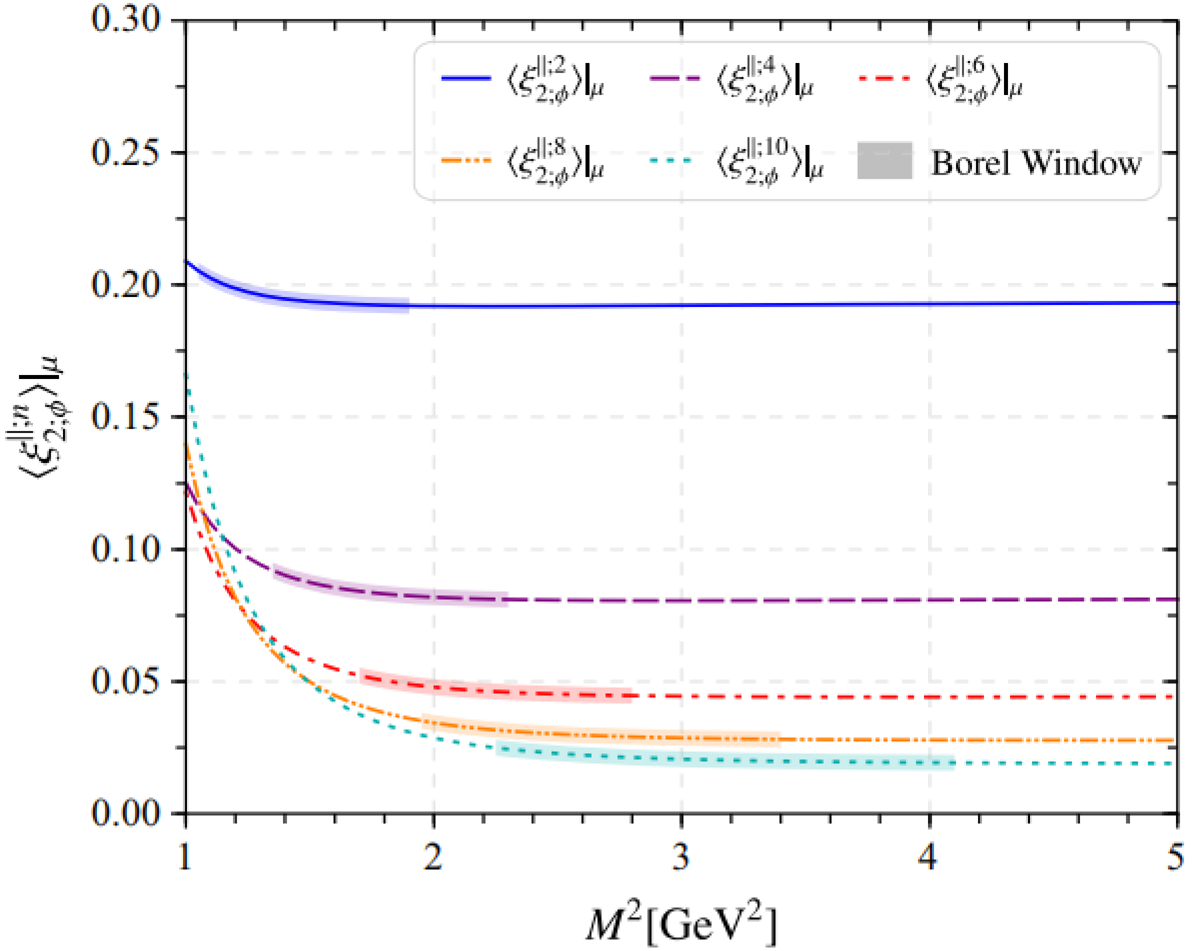
分布振幅0阶矩表达式

$$(\langle\xi^{\|;0}_{2;\phi}\rangle|_\mu)^2=\frac{e^{m_\phi^2/M^2}}{(f_\phi^{\|})^2}\int_{4m_s^2}^{s_\phi}\mathrm{d}s\,e^{-s/M^2}\frac{v}{8\pi^2}(3-v^2)+\frac{2m_s\langle\bar{s}s\rangle}{M^4}+\frac{\langle\alpha_sG^2\rangle}{12\pi M^4}-\frac{m_s\langle g_s\bar{s}\sigma TGs\rangle}{9M^6}$$
$$+\frac{4\langle g_s\bar{s}s\rangle}{81M^6}+\frac{\sum\langle g_s^2\bar{q}q\rangle^2}{486\pi^2M^6}\Big[-50\Big(-\ln\frac{M^2}{\mu^2}\Big)+105\Big]+m_s^2\Big\{\frac{\langle\alpha_sG^2\rangle}{3\pi M^6}-\frac{\langle g_s^3fG^3\rangle}{72\pi^2M^8}$$
$$-\frac{\sum\langle g_s^2q\bar{q}\rangle^2}{972\pi^2M^8}\Big[8\Big(-\ln\frac{M^2}{\mu^2}\Big)+132\Big]-\frac{4}{3}\frac{m_s\langle\bar{s}s\rangle}{M^6}-\frac{m_s\langle g_s\bar{s}\sigma TGs\rangle}{3M^8}-\frac{4}{81}\frac{\langle g_s\bar{s}s\rangle^2}{M^8}\Big\},$$

$$\langle\xi^{\|;n}_{2;\phi}\rangle|_\mu=\frac{\langle\xi^{\|;n}_{2;\phi}\rangle|_\mu\langle\xi^{\|;0}_{2;\phi}\rangle|_\mu|_{\text{From Eq. (2.38)}}}{\sqrt{(\langle\xi^{\|;0}_{2;\phi}\rangle|_\mu)^2}}.$$

III. $D_s \to \phi l^+ \nu_l$ decay process

ϕ-介子前五阶分布振幅矩的整体行为以及具体数值



$$\langle\xi^{\|;2}_{2;\phi}\rangle|_{\mu_0} = 0.199 \pm 0.010,$$

$$\langle\xi^{\|;2}_{4;\phi}\rangle|_{\mu_0} = 0.086 \pm 0.006,$$

$$\langle\xi^{\|;2}_{6;\phi}\rangle|_{\mu_0} = 0.049 \pm 0.004,$$

$$\langle\xi^{\|;2}_{8;\phi}\rangle|_{\mu_0} = 0.032 \pm 0.003,$$

$$\langle\xi^{\|;2}_{10;\phi}\rangle|_{\mu_0} = 0.022 \pm 0.003.$$

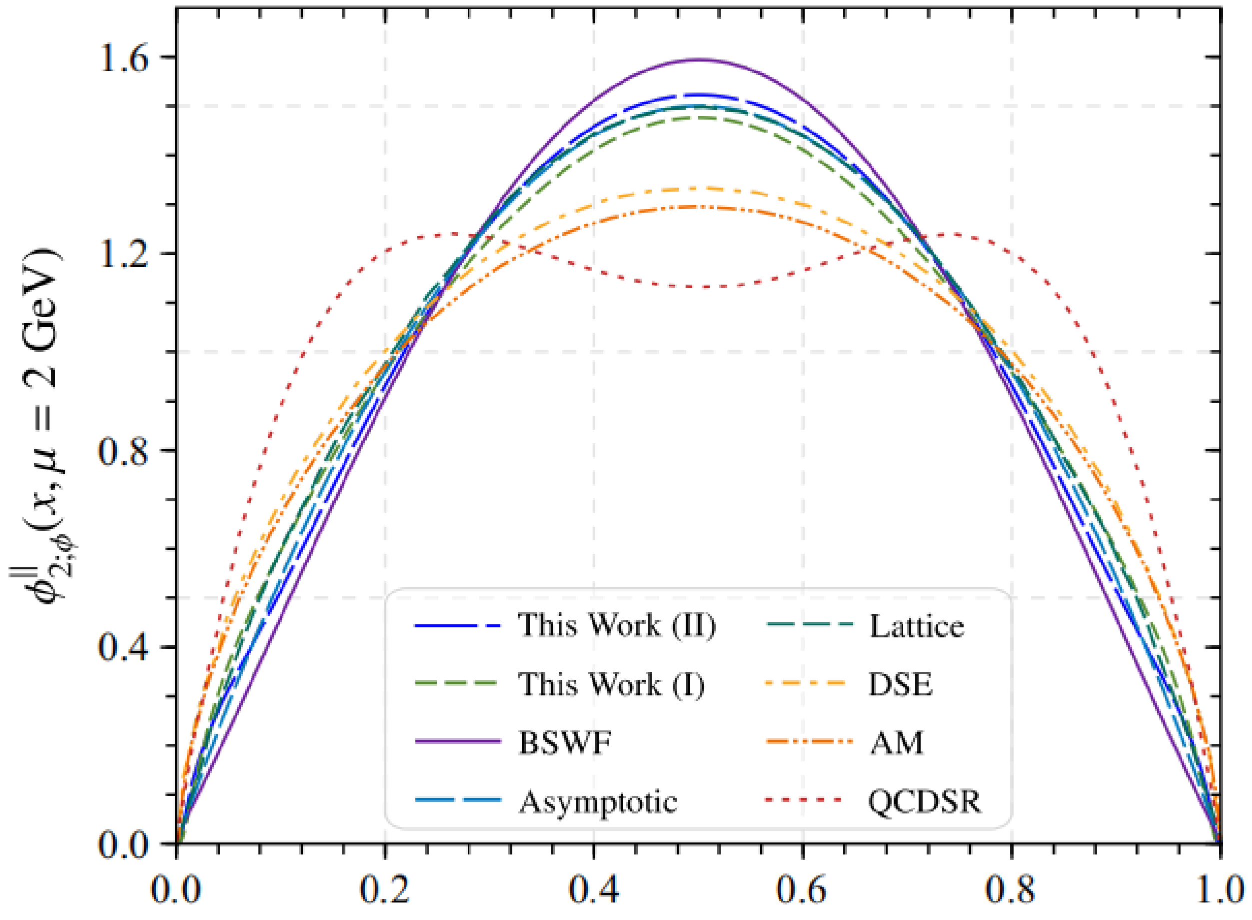
- 连续阈值$s_\phi = 2.1\ \mathrm{GeV}^2$
- 连续态贡献分别不超过20%，25%，30%，35%，40%，六维凝聚贡献不超过5%

III. $D_s \to \phi l^+ \nu_l$ decay process

ϕ-介子twist-2的分布振幅

模型I：$A_{2;\phi}^{\|(\mathrm{I})} = 11.508\ \mathrm{GeV}^{-1}$, $\beta_{2;\phi}^{(\mathrm{I})} = 1.212\ \mathrm{GeV}$, $b_{2;\phi}^2 = 0.061$, $b_{2;\phi}^4 = 0.020$

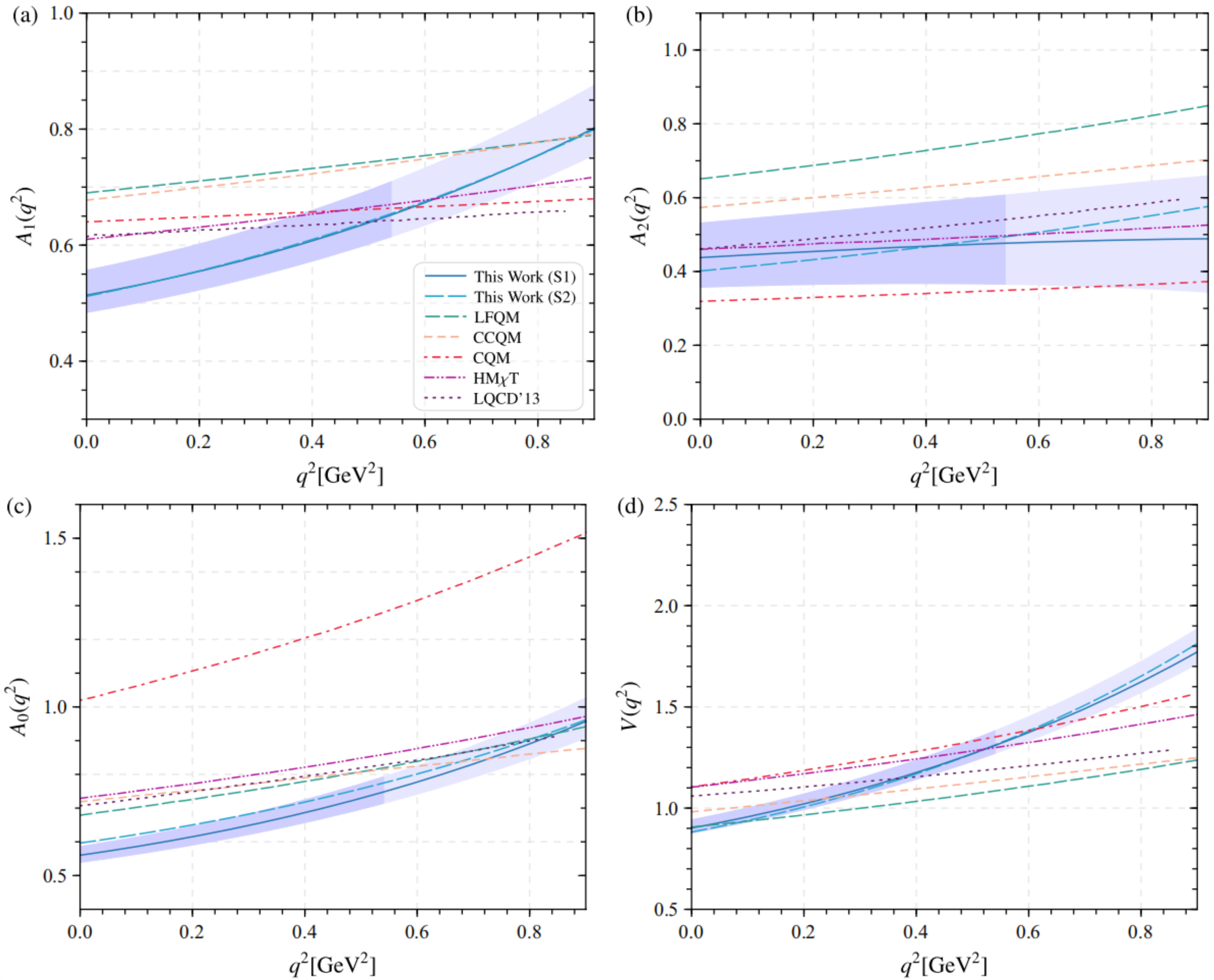
模型II $A_{2;\phi}^{\|(\mathrm{II})} = 2.515\ \mathrm{GeV}^{-1}$, $\alpha_{2;\phi} = -0.940$, $B_{2;\phi}^2 = -0.149$, $\beta_{2;\phi}^{(\mathrm{II})} = 1.207\ \mathrm{GeV}$



III. $D_s \to \phi l^+ \nu_l$ decay process

$D_s \to \phi$跃迁形状因子

	$V(0)$	$A_0(0)$	$A_1(0)$	$A_2(0)$
This work (S1)	$0.902^{+0.040}_{-0.024}$	$0.560^{+0.025}_{-0.021}$	$0.514^{+0.024}_{-0.016}$	$0.438^{+0.093}_{-0.080}$
This work (S2)	$0.882^{+0.040}_{-0.036}$	$0.596^{+0.025}_{-0.020}$	$0.512^{+0.030}_{-0.020}$	$0.402^{+0.078}_{-0.067}$
LQCD(2001) [22]	0.85(14)	0.63(2)	0.63(2)	0.62(78)
LQCD(2011) [23]	0.903(67)	0.686(17)	0.594(22)	0.401(80)
LQCD(2013) [24]	1.059(124)	0.706(37)	0.615(24)	0.457(78)
HQEFT [12]	$0.778^{+0.057}_{-0.062}$	$-0.757^{+0.029}_{-0.039}$	$0.569^{+0.046}_{-0.049}$	$0.304^{+0.021}_{-0.017}$
HMχT [14]	1.10	1.02	0.61	0.32
CQM [15]	1.10	0.73	0.64	0.47
3PSR [10]	1.21(33)	0.53(12)	0.55(15)	0.59(11)
CLFQM(2008) [18]	0.91	0.62	0.61	0.58
CLFQM(2011) [19]	0.98	0.72	0.69	0.57
LFQM [21]	1.24	0.71	0.77	0.66
LCSR [13]	0.70(10)	0.53(9)	0.54(9)	0.57(9)
CCQM [17]	0.91	0.68	0.68	0.67
RQM [27]	0.999	0.713	0.643	0.492
SCI [28]	1.00	0.66	0.61	0.44



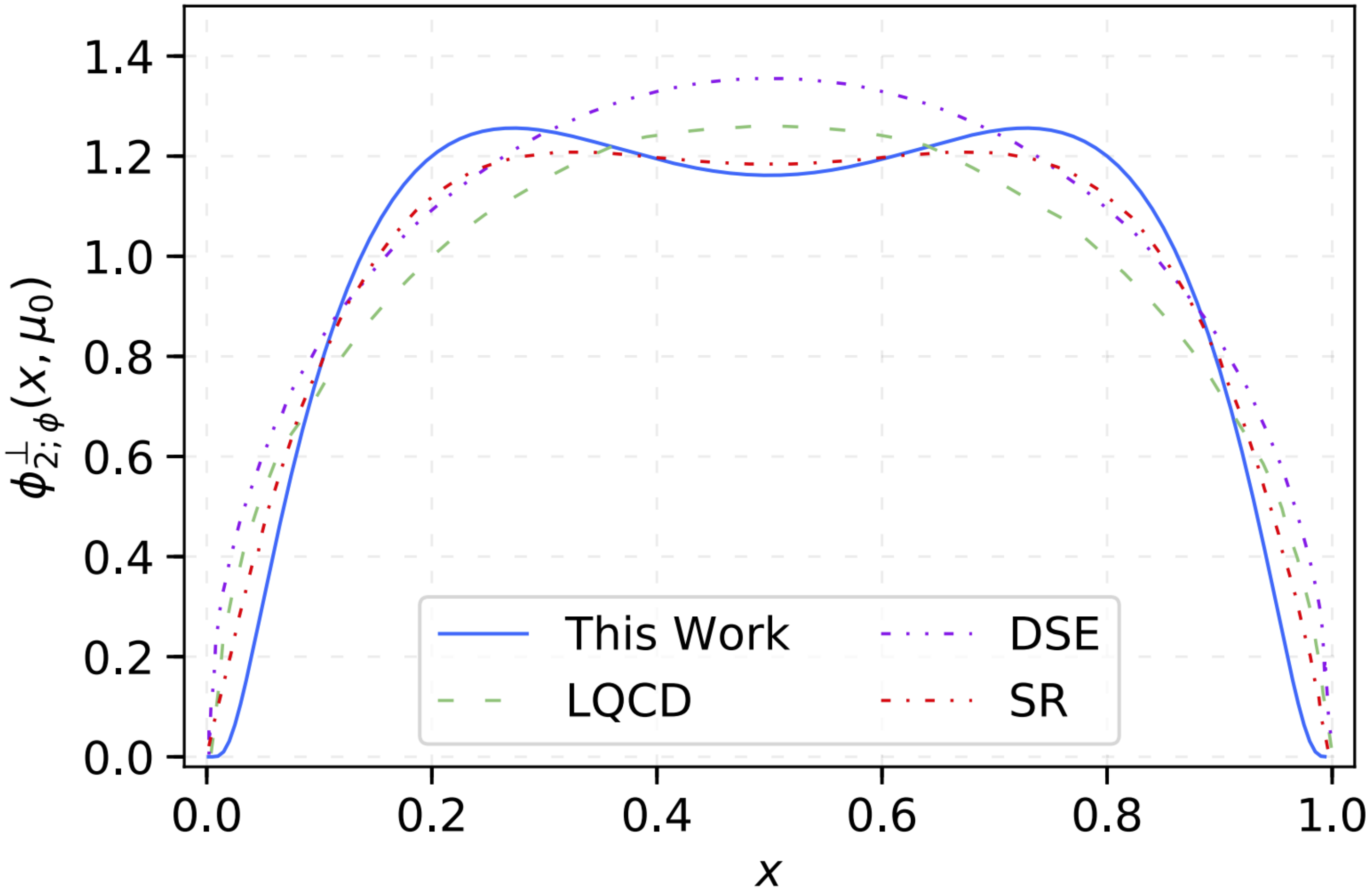
III. $D_s \to \phi l^+ \nu_l$ decay process

$D_s \to \phi l \nu_l$的分支比

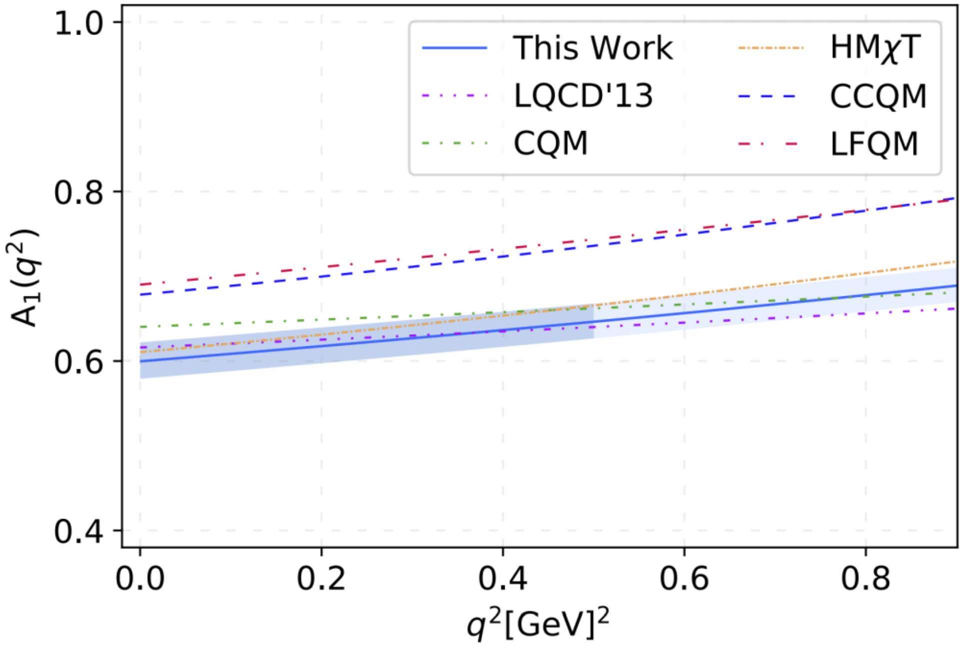
	$\mathcal{B}(D_s^+ \to \phi e^+ \nu_e)$	$\mathcal{B}(D_s^+ \to \phi \mu^+ \nu_\mu)$
This work (S1)	$2.347^{+0.342}_{-0.191}$	$2.330^{+0.341}_{-0.190}$
This work (S2)	$2.367^{+0.256}_{-0.132}$	$2.349^{+0.255}_{-0.132}$
BABAR [6]	$2.61 \pm 0.11 \pm 0.15$	$\cdots$
CLEO [7]	$2.14 \pm 0.17 \pm 0.08$	$\cdots$
BESIII(2017) [8]	$2.26 \pm 0.45 \pm 0.09$	1.94 ± 0.54
BESIII(2023) [9]	$\cdots$	$2.25 \pm 0.09 \pm 0.07$
PDG [79]	2.39 ± 0.16	1.90 ± 0.5
CLFQM(2017) [20]	3.1 ± 0.3	2.9 ± 0.3
3PSR(2004) [10]	1.80 ± 0.50	$\cdots$
CLFQM(2008) [18]	2.30	$\cdots$
LCSR [13]	$2.15^{+0.27}_{-0.31}$	$\cdots$
HQEFT [12]	$2.53^{+0.37}_{-0.40}$	$2.40^{+0.35}_{-0.40}$
CCQM [17]	3.01	2.85
CQM [15]	2.57	2.57
χUA [26]	2.12	1.94
RQM [27]	2.69	$\cdots$
SCI [28]	2.45	2.30

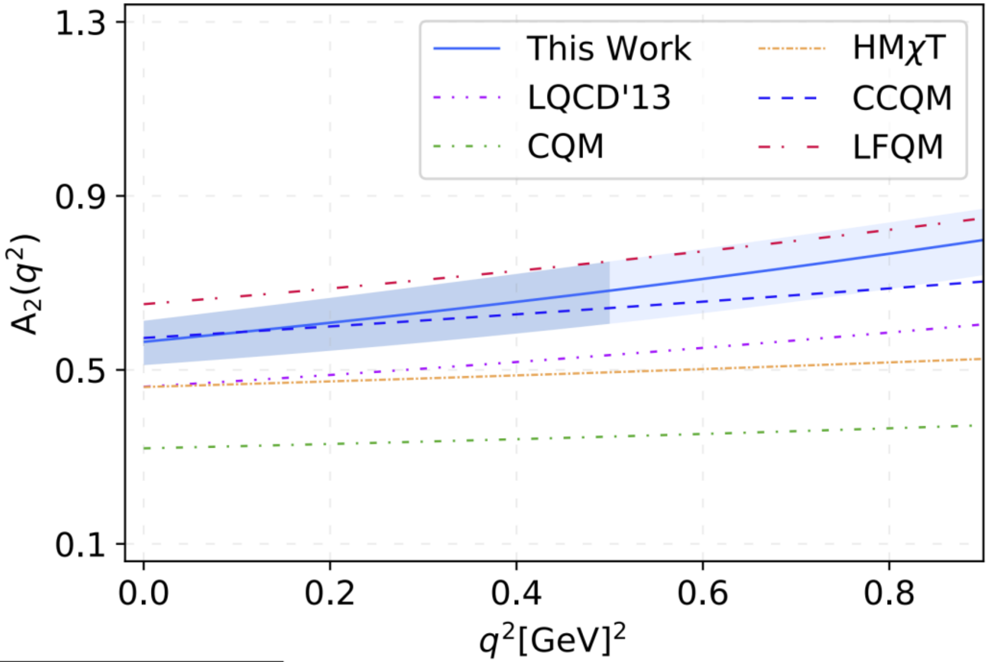
arXiv: 2403.10003

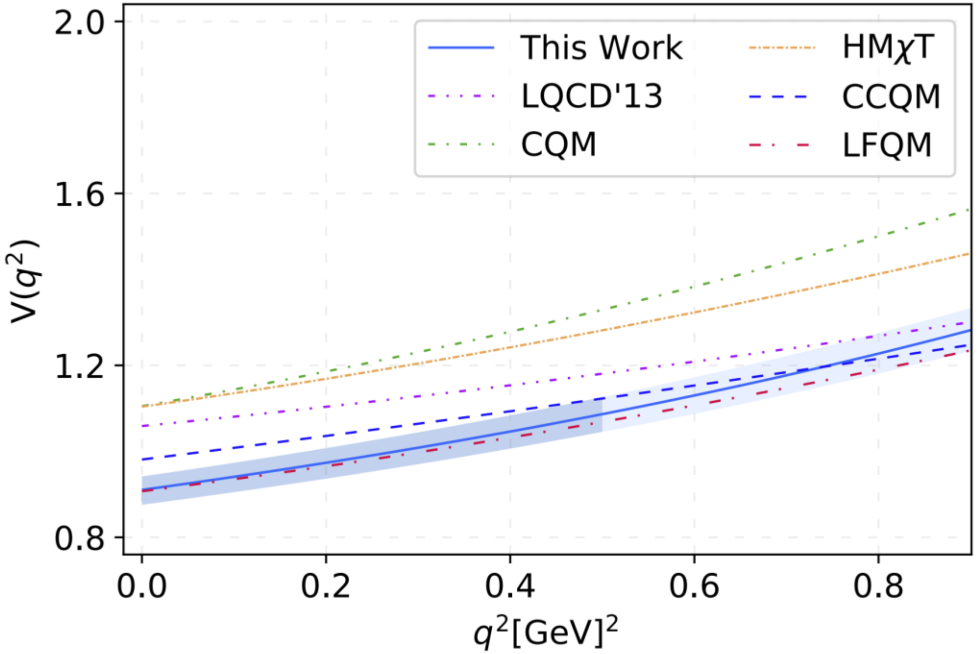
III. $D_s \to \phi l^+ \nu_l$ decay process



III. $D_s \to \phi l^+ \nu_l$ decay process







arXiv: 2505.15014

IV. Summary

- 背景场理论框架下采用SVZ求和规则计算$f_0(980)$介子的twist-2分布振幅的矩；
- 光锥求和规则计算$D_s \to f_0(980)$跃迁形状因子及半轻衰变过程的观测量
- ϕ介子的twist-2分布振幅，以及$D_s \to \phi$形状因子、衰变宽度、分支比以及极化和不对称性参数

Thanks for your attention!