

粲介子四体半轻衰变的理论进展

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Overview

I Why semileptotic $D_{(s)}$ decays

II Opportunities in Semileptotic D decays

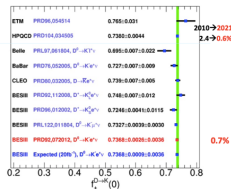
- i $D_s \rightarrow [f_0, \dots \rightarrow] \pi\pi e^+ \nu$
- ii $D \rightarrow \rho l^+ \nu$ and $D_{(s)} \rightarrow K^* l^+ \nu$

III Conclusions and Prospects

Semileptotic $D_{(s)}$ decays

play a crucial role in the precision era of particle physics

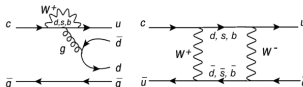
- **fundamental parameters**, like the CKM matrix element $|V_{cs}| = 0.975 \pm 0.006$ [PDG 2022]
- the result measured via $D \rightarrow K l \nu$ and $D_s \rightarrow \mu \nu \mu$ consist with each other ($\sim 1.5\sigma$ derivation)
- $\sim 3\sigma$ tension three years ago [PDG 2020, 2021]
- the improvement mainly due to the high precision of $D \rightarrow K$ form factor from lattice evaluation and the f_{D_s} from the BESIII



new physical mechanism via the FCNC

- anomalous measured in $B \rightarrow K^* \mu^+ \mu^-$, 3.6σ derivation of dB/dq^2 in $q^2 \in [1, 6] \text{ GeV}^2$, 1.9σ derivation of $p'_5 = S_5 / \sqrt{F_L(1 - F_L)}$ in $[4, 8] \text{ GeV}^2$
- a plausible effect in up-type FCNC process $c \rightarrow u l l$ [Bharucha 2011.12856] SM $\mathcal{B}(D \rightarrow \pi l^+ l^-) \sim \mathcal{O}(10^{-9})$, current best-world limit $\mathcal{O}(10^{-8})$
- LCSRs prediction $\mathcal{B}(D \rightarrow \pi \mu \mu) = 1.33^{+0.17}_{-0.24} \times 10^{-8}$, at the same order of LHCb limit 6.7×10^{-8} [A. Bansal, A. Khodjamirian and T. Mannel 2505.21369[hep-ph]],

- first measurement of $D^0 \rightarrow \pi^+ \pi^- e^+ e^-$ [LHCb 2412.09414] $(4.53 \pm 1.38) \times 10^{-7}$ in ρ/ω and $(3.84 \pm 0.96) \times 10^{-7}$ in ϕ
- LD contribution from ϕ , $D_s \rightarrow \pi, \rho(\phi \rightarrow) e^+ e^- \sim 10^{-5}$ [BESIII 2404.05973]



New physics hunter $D \rightarrow \pi\mu^+\mu^-$

- my talk in the " 超级陶粲装置研讨会 " at LZU, July 8th, 2024

- Experimentail potentials

Experiment	Measurement	Sensitivity	
LHCb [talk at Towards the Ultimate Precision in Flavour Physics, Durham U.K. (2019)]	Angular observables	$\sim 0.2\%$ with 50 fb^{-1} , $\sim 0.08\%$ with 300 fb^{-1}	Run 4 ~ 2030 Run 5 ~ 2038
LHCb	Branching ratio	$\sim 10^{-8}$ with 50 fb^{-1} , $\sim 3 \times 10^{-9}$ with 300 fb^{-1}	
[BABAR Collaboration 1107.4465] Belle-II	Branching ratio	$\sim 10^{-8}$ (rescaling BaBar)	

$N(D\bar{D}) \sim 10^9/\text{ab}^{-1}$ angular observables $\sim 0.2\%$

- BESIII Collaboration in the electron channel [BESIII Collaboration 1802.09752]
 $\mathcal{B}(D \rightarrow \pi^+\pi^-\text{e}^+\text{e}^-) < 0.7 \times 10^{-5}$ with $N(c\bar{c}) = 2 \times 10^7$ at 3.7 GeV

3.770	1	$D^0\bar{D}^0$	3.6	3.6×10^9	Single Tag Single Tag
		$D^+\bar{D}^-$	2.8	2.8×10^9	
		$D^0\bar{D}^0$		7.9×10^8	
		$D^+\bar{D}^-$		5.5×10^8	

STCF $N(D\bar{D}) \sim 8 \times 10^9$ Branching ratio $\sim 10^{-8}$

- STCF is still competitive in hunting the NP via $D \rightarrow \pi\mu^+\mu^-$, $\pi\pi\mu^+\mu^-$

Semileptotic $D_{(s)}$ decays

- a clean environment to study scalar mesons see *Chu-wen's talks*
 - $f_0(1370), f_0(1500), a_0(1450), K_0^*(1430)$ form a $SU(3)$ flavor nonet
 - $f_0(500)/\sigma, f_0(980), a_0(980), K_0^*(700)/\kappa$ form another flavor nonet
 - compact tetraquark and $K\bar{K}$ bound state are favored from spectral analyses, $\bar{s}s$ is dominated in B_s decay
- how about the energetic $q\bar{q}$ picture $f_0(980)$ in D_s decays ?
- depends on the precise perturbative QCD calculation, more important is **the accurate nonperturbative description of the structure**
 - the signal channel is $D_{(s)} \rightarrow [f_0 \rightarrow] \pi\pi, K\bar{K}l\nu$ invariant mass
 - dynamics of H_{I4} is governed by $H \rightarrow \pi\pi$ form factors, a big task of the QCD methods [S. Faller, et.al., 1310.6660, X.W. Kang, et.al., 1312.1193]
- **Dipion LCDAs** are introduced in the LCSR predictions of $B, D \rightarrow \pi\pi$ transitions
 - [SC, A. Khodjamirian and J. Virto, 1701.01633[hep-ph]] *B-meson LCSRs*
 - [C. Hambrock and A. Khodjamirian, 1511.02509[hep-ph]] *2 π DAs LCSRs of $F_{\parallel, \perp}$*
 - [SC, A. Khodjamirian and J. Virto, 1709.00173[hep-ph]] *timelike-helicity FF F_t and F_0*
 - [SC, 1901.06071[hep-ph]] *2 π DAs updates and $B \rightarrow [\pi\pi]_{S,P}$ FFs*
 - [SC and J.M Shen, 1907.08401[hep-ph]],
[SC and S.L Zhang, 2307.02309[hep-ph]] *$D_s \rightarrow [f_0 \rightarrow] \pi\pi l\nu$ leading twist*
 - [SC, 2502.07333[hep-ph]] *first study of twist-three 2 π DAs and $|V_{ub}|$ extraction*
 - [SC, L.Y Dai and S.L Zhang, to be appearing] *$D_s \rightarrow [f_0 \rightarrow] \pi\pi l\nu$ sub-leading twist*

Opportunities in SL $D_{(s)}$ decays

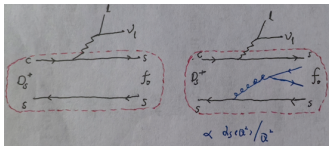
(private opinions)

i $D_s \rightarrow [f_0, \dots \rightarrow] \pi\pi e^+ \nu$

ii $D \rightarrow \rho l^+ \nu$ and $D_{(s)} \rightarrow K^* l^+ \nu$

$$D_s \rightarrow [f_0, \cdots \rightarrow] \pi \pi e^+ \nu$$

$$D_s \rightarrow [f_0, \cdots \rightarrow] \pi \pi e \nu$$



- ★ in SL B_s decays
- ★ tetraquark contribution is suppressed doubly by strong coupling and power
- ★ FSI is weak too

$$|f_0(980)\rangle = \psi_{q\bar{q}}|q\bar{q}\rangle + \psi_{q\bar{q}g}|q\bar{q}g\rangle + \psi_{q\bar{q}q\bar{q}}|q\bar{q}q\bar{q}\rangle + \cdots$$

$$|[\pi\pi]_S\rangle = \psi_{q\bar{q}}|q\bar{q}\rangle + \psi_{q\bar{q}g}|q\bar{q}g\rangle + \psi_{q\bar{q}q\bar{q}}|q\bar{q}q\bar{q}\rangle + \cdots$$

$$\psi_{f_0}^n(x_i, k_{\perp i}, \lambda_i) = \langle n, x_i, k_{\perp i}, \lambda_i | f_0 \rangle$$

- ★ physical observables are usually written in a QCD convolution

$$\frac{d\sigma}{d\Omega} = \sum_t \int_0^1 dx_i \mathcal{H}^t(x_i, Q) \psi^t(x_i, \mu)$$

- ★ ψ^t is universal, however $\mathcal{H}^t$ is process dependent, hence different observables might highlight the contributions from different components
- how about the energetic $q\bar{q}$ picture $f_0(980)$ in D_s decays ?
- ★ this talk focus on the contributions from lowest Fock state $|q\bar{q}\rangle$

$$D_s \rightarrow [f_0, \cdots \rightarrow] \pi\pi e\nu$$

- Semileptonic $D_{(s)}$ decays provide a clean environment to study scalar mesons

- $D_s \rightarrow f_0 e^+ \nu$ [CLEO '09], $D_{(s)} \rightarrow a_0 e^+ \nu$ [BESIII '18, '21], $D^+ \rightarrow f_0/\sigma e^+ \nu$ [BESIII '19]
- $D_s \rightarrow f_0 (\rightarrow \pi^0 \pi^0, K_s K_s) e^+ \nu$ [BESIII 22], $D_s \rightarrow f_0 (\rightarrow \pi^+ \pi^-) e^+ \nu$ [BESIII 23]

$$\mathcal{B}(D_s \rightarrow f_0 (\rightarrow \pi^0 \pi^0) e^+ \nu) = (7.9 \pm 1.4 \pm 0.3) \times 10^{-4}$$

$$\mathcal{B}(D_s \rightarrow f_0 (\rightarrow \pi^+ \pi^-) e^+ \nu) = (17.2 \pm 1.3 \pm 1.0) \times 10^{-4}$$

$$f_+^0(0) |V_{cs}| = 0.504 \pm 0.017 \pm 0.035$$

- single particle (narrow width limit) $D_s \rightarrow f_0 e^+ \nu$

$$\frac{d\Gamma(D_s^+ \rightarrow f_0 l^+ \nu)}{dq^2} = \frac{G_F^2 |V_{cs}|^2 \lambda^{3/2}(m_{D_s}^2, m_{f_0}^2, q^2)}{192\pi^3 m_{D_s}^3} |f_+(q^2)|^2, \quad D_s \rightarrow f_0 \text{ FF}$$

- improvement with the width effect by resonant model $D_s \rightarrow [f_0 \rightarrow] \pi\pi e^+ \nu$

$$\frac{d\Gamma(D_s^+ \rightarrow [\pi\pi]_S l^+ \nu)}{ds dq^2} = \frac{1}{\pi} \frac{G_F^2 |V_{cs}|^2}{192\pi^3 m_{D_s}^3} |f_+(q^2)|^2 \frac{\lambda^{3/2}(m_{D_s}^2, s, q^2) g_1 \beta_\pi(s)}{|m_S^2 - s + i(g_1 \beta_\pi(s) + g_2 \beta_K(s))|^2}, \quad \text{BESIII}$$

- calculate directly the signal channel $D_s \rightarrow [\pi\pi]_S e^+ \nu$

$$\frac{d^2\Gamma(D_s^+ \rightarrow [\pi\pi]_S l^+ \nu)}{dk^2 dq^2} = \frac{G_F^2 |V_{cs}|^2 \beta_{\pi\pi}(k^2) \sqrt{\lambda_{D_s} q^2}}{3(4\pi)^5 m_{D_s}^3} \sum_{\ell=0}^{\infty} |F_0^{(\ell)}(q^2, k^2)|^2, \quad D_s \rightarrow \pi\pi \text{ FF}$$

$D_s \rightarrow f_0$ form factor and $D_s^+ \rightarrow (f_0, [\pi\pi]_S) e^+ \nu_e$ decay

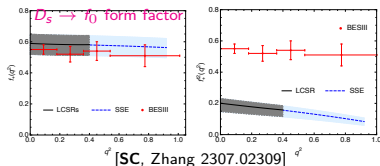
- $\{M^2, s_0\} = \{5.0 \pm 0.5, 6.0 \pm 0.5\} \text{GeV}^2$ see Hai-bing Fu's talk for the LCSRs

this work	3pSRs(07)	LFQM(09)	CLFD/DR(08)	LCSRs(10)
0.63 ± 0.04	0.96	0.87	0.86/0.90	0.30 ± 0.03

- the BESIII result in the $\pi^+ \pi^-$ system $f_+(0) = 0.518 \pm 0.018 \pm 0.036$ [BESIII 23]

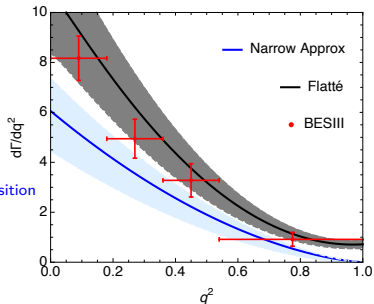
different input of the decay constant $\tilde{f}_{f_0} = 335$ MeV, much larger than 180 MeV in LCSR(10)
we add the first gegenbauer expansion terms in the LCDAs, up-to-date parameters

$\bar{s}s - n\bar{n}$ mixing scenario of f_0 with $\theta = 20^\circ \pm 10^\circ$



- Twist-3 LCDAs give dominate contribution in $D_s \rightarrow f_0$ transition

- the uncertainty estimation is conservative
- without NLO correction
- we need a model independent calculation
- not only for the QCD understanding
- but also for the future partial-wave measurement



Differential decay width of $D_s^+ \rightarrow (f_0, [\pi\pi]_S) e^+ \nu_e$

$D_s \rightarrow [\pi\pi]_S$ form factors

- Definition of $D_s \rightarrow [\pi\pi]_S$ form factor

$$\langle [\pi(k_1)\pi(k_2)]_S | \bar{s}\gamma_\mu(1 - \gamma_5)c | D_s^+(p) \rangle = -iF_t k_\mu^t - iF_0(q^2, s, \zeta) k_\mu^0 - iF_\parallel k_\mu^\parallel$$

$$k_\mu^t = \frac{q_\mu}{\sqrt{q^2}}, k_\mu^0 = \frac{2\sqrt{q^2}}{\sqrt{\lambda_{D_s}}} \left(k_\mu - \frac{k \cdot q}{q^2} q_\mu \right), k_\mu^\parallel = \frac{1}{\sqrt{k^2}} \left(\bar{k}_\mu - \frac{4(q \cdot k)(q \cdot \bar{k})}{\lambda_{D_s}} k_\mu + \frac{4k^2(q \cdot \bar{k})}{\lambda_{D_s}} q_\mu \right)$$

- LCSRs calculations start with the correlation functions

$$\Pi_\mu^{ab}(q, k_1, k_2) = i \int d^4x e^{iq \cdot x} \langle \pi^a(k_1) \pi^b(k_2) | T \{ j_{1,\mu}(x), j_2(0) \} | 0 \rangle$$

- The chiral even two quark isoscalar 2π DAs

leading twist, **twist-three LCDAs are available now** [SC, arXiv: 2502.07333]

$$\langle [\pi^a(k_1)\pi^b(k_2)]_S | \bar{s}(xn)\gamma_\mu s(0) | 0 \rangle = 2\delta^{ab} k_\mu \int du e^{iux(k \cdot n)} \Phi_{\parallel, [\pi\pi]_S}^{l=0}(u, \zeta, k^2)$$

$$\Phi_{\parallel, [\pi\pi]_S}^{l=0} = 6u(1-u) \sum_{n=1, \text{odd}}^{\infty} \sum_{l=0, \text{odd}}^{n+1} B_{\parallel, nl}^{l=0}(k^2, \mu) C_n^{3/2}(2u-1) C_l^{1/2}(2\zeta-1)$$

- Do the LCSRs QCD calculations and consider the partial-wave expansion

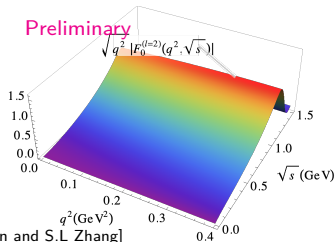
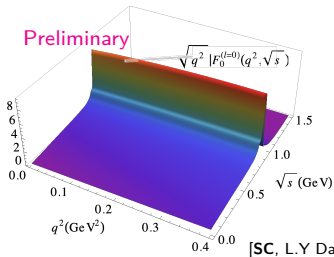
$$F_0(q^2, k^2, \zeta) = \sum_{\ell=0}^{\infty} \sqrt{2\ell+1} F_{0,t}^{(\ell)}(q^2, k^2) P_\ell^{(0)}(\cos \theta_\pi)$$

$D_s \rightarrow [\pi\pi]_S$ form factor and $D_s \rightarrow [\pi\pi]_S e^+ \nu$ decay

- The LCSR ℓ' -wave $D_s \rightarrow [\pi\pi]_S$ form factors ($\ell' = \text{even} \ \& \ \ell' \leq n+1$)

$$\sqrt{q^2} F_0^{(\ell')} (q^2, k^2) = \frac{m_c(m_c + m_s) \sqrt{q^2} \sqrt{\lambda_{D_s}}}{m_{D_s}^2 f_{D_s}} \sum_{n=1, \text{odd}}^{\infty} \frac{\beta_{\pi}(k^2)}{\sqrt{2\ell'+1}} J_n^0(q^2, k^2, M^2, s_0) B_{n\ell, \parallel}^{l=0}(k^2) I_{\ell\ell'}$$

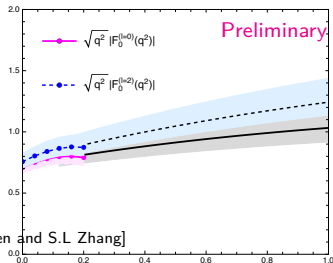
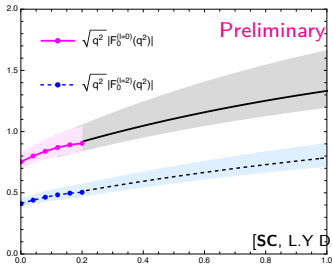
- Near threshold, $B_{n\ell}$ can be determined from [the low-energy effective theory](#) of pions interacting with massive "constituent" quarks, based on the instanton model of the QCD vacuum
- For the resonant regions, [Watson's theorem](#) yields the k^2 -dependence via Omnés solutions with the phase shifts implemented through subtracted dispersion relations



- S - and D -wave FFs: $\sqrt{q^2} |F_0^{(l=0)}(\sqrt{s}, q^2)|$ and $\sqrt{q^2} |F_0^{(l=2)}(\sqrt{s}, q^2)|$

$D_s \rightarrow [\pi\pi]_S$ form factor and $D_s \rightarrow [\pi\pi]_S e^+ \nu$ decay

- The q^2 -dependence of FFs obtained by adopting the phase shifts from $\pi^- p \rightarrow \pi^0 \pi^0 n$ reaction (peak) and $\pi\pi$ scattering (dip)



- Differential widths $d\Gamma/dq^2$ is two-order in magnitude smaller than the data
- the FFs at the two-particle level are one-order lower than the required
- the conventional $q\bar{q}$ is not the dominate component in the charm decays
- we have to go further to multi-particle DiPion LCDAs in CHARM ($q\bar{q}g, q\bar{q}q\bar{q}$)
- much different in B decays leading twist dominated [SC, arXiv: 2502.07333]

$$D \rightarrow \rho l^+ \nu \text{ and } D_{(s)} \rightarrow K^* l^+ \nu$$

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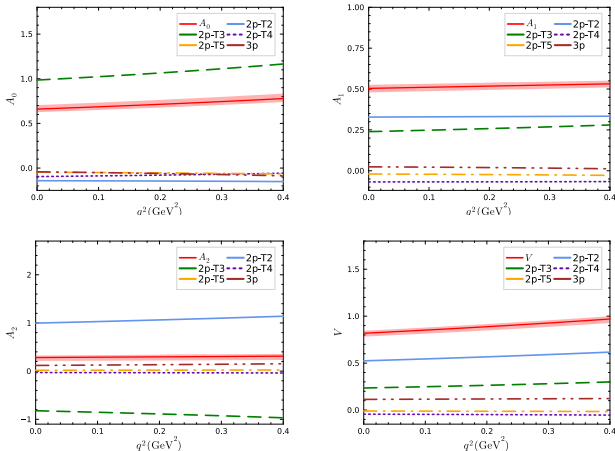
- motivated by the recent BESIII measurements

Decay Mode	Collaboration	Year	Reference
$D^0 \rightarrow K^{*-} \mu^+ \nu_\mu$	BESIII	2025	PRL 134(2025)1,011803
$D^0 \rightarrow \rho^- e^+ \nu_e$	BESIII	2024	PRD 110(2024)11,112018
$D_s^+ \rightarrow K^{*0} e^+ \nu_e$	BESIII	2019	PRL 122 (2019) 6, 061801
$D^+ \rightarrow \rho^0 e^+ \nu_e$	CLEO	2013	PRL 110(2013)13,131802
$D^+ \rightarrow K^{*0} \mu^+ \nu_\mu$	FOCUS	2006	PLB 637(2006)32-38

- state-of-the-art LCSR's calculation of $D \rightarrow V$ form factors with high twist LCDAs
- to examine the width effects and the non-resonant QCD backgrounds in the D_{l4} decays ($D_{(s)} \rightarrow V[\rightarrow P_1 P_2] l \nu$)

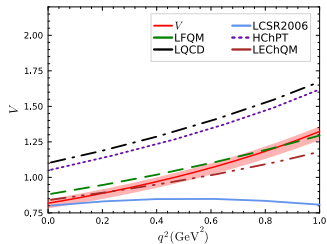
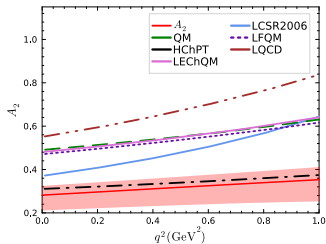
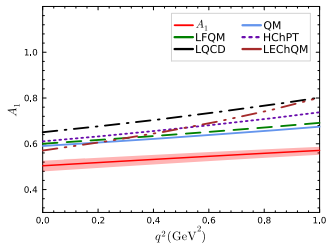
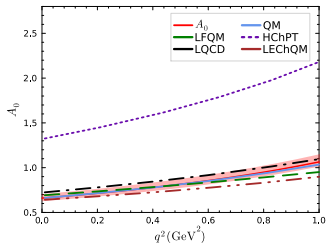
$D^0 \rightarrow \rho^-$ form factors from different twist (power) contributions

[W. Lin, X.E Huang, **SC** and D.L Yao, 2505.01329]



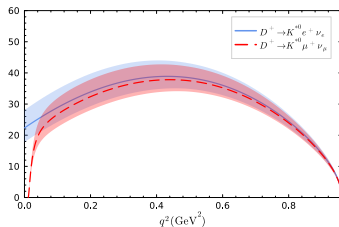
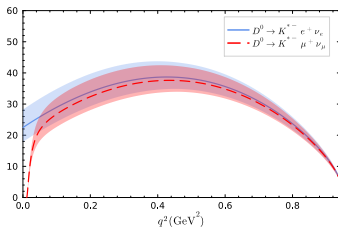
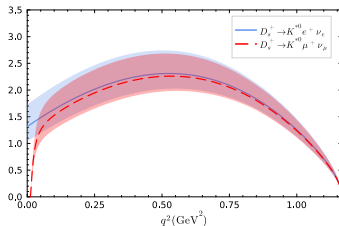
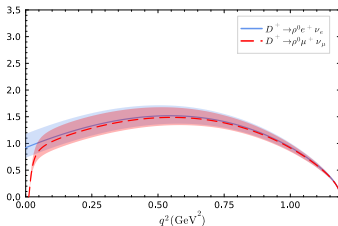
- both the three-particle LCDAs and the power correction from the heavy quark expansion at $\mathcal{O}(1/m_c)$ contribute significantly to the form factors A_1 and A_2 .
- The OPE expansion exhibits good convergence: while two-particle twist-three LCDAs contribute sizeably or even dominantly in the axial-vector transition, the higher-twist effects remain well under control

$D^0 \rightarrow \rho^-$ form factors



- LCSRs predictions in comparing to the results obtained from other approaches

$$D \rightarrow \rho l^+ \nu \text{ and } D_{(s)} \rightarrow K^* l^+ \nu$$



- Differential decay widths $d\Gamma/dq^2$ of $D \rightarrow \rho l^+ \nu$ and $D_{(s)} \rightarrow K^* l^+ \nu$ decays
- the lepton mass effect is mostly prominent in the large-recoil region of the semileptonic charm to vector meson decays

$$D \rightarrow \rho l^+ \nu \text{ and } D_{(s)} \rightarrow K^* l^+ \nu$$

- branching ratios of semileptonic charm decays (in unit of 10^{-6})

	$D^+ \rightarrow \rho^0 \ell^+ \nu_\ell$	$D_s^+ \rightarrow K^{*0} \ell^+ \nu_\ell$	$D^0 \rightarrow K^{*-} \ell^+ \nu_\ell$	$D^+ \rightarrow \bar{K}^{*0} \ell^+ \nu_\ell$
This work	$2.30^{+0.32}_{-0.25}$	$1.55^{+0.30}_{-0.20}$	$17.6^{+2.4}_{-1.9}$	$45.2^{+6.2}_{-5.0}$
	$2.20^{+0.30}_{-0.23}$	$1.48^{+0.29}_{-0.19}$	$16.6^{+2.2}_{-1.8}$	$42.7^{+5.7}_{-4.5}$
LCSR2006 [21]	$2.29^{+0.23}_{-0.16}$	$2.33^{+0.29}_{-0.30}$	21.2 ± 0.9	$53.7^{+2.4}_{-2.3}$
	$2.20^{+0.21}_{-0.16}$	$2.24^{+0.27}_{-0.29}$	20.1 ± 0.9	$51.0^{+2.3}_{-2.1}$
CLFQM [51]	2.32	1.90	—	73.2
see Xian-wei Kang's talk	2.22	1.82	—	69.3
HChPT [32]	2.50	2.20	22.0	56.0
PDG [2]	1.90 ± 0.10	2.15 ± 0.28	21.5 ± 1.60	54.0 ± 1.00
	2.40 ± 0.40	—	18.9 ± 2.40	52.7 ± 1.50

- 10%-20% discrepancy when confronting our LCSR predictions with experimental measurements
- a sizable $SU(3)$ flavor-breaking effect is observed in $D_{(s)} \rightarrow K^* l \nu$ decays
- the necessity to further implement the effects of vector meson widths and non-resonant QCD background

Conclusions and Prospects

- **The introduction of DiPion LCDAs provides an opportunity to study the width effects and the structures of nonstable mesons in H_{I4} processes**
 - a new booster on the accurate calculation in flavor physics
 - improvement study in the CKM determinations and the flavor anomalies
- **The studies of 2π DAs and H_{I4} decays are now at two-particle component**
 - universal phase shift in $\pi\pi$ scattering and heavy decay ?
- **Go further to high twist LCDAs associated to multi-particle components**
 - $B \rightarrow \pi\pi h\nu, B \rightarrow [\rho\rho \rightarrow] \rightarrow 4\pi, D_s \rightarrow \pi\pi h\nu, D \rightarrow K\pi\mu\nu, D \rightarrow \pi\pi e^+e^-$ et al.

Thank you for your patience.