重点研发项目“粲强子衰变和标准模型的精确检验”2025夏季年会

2025.8.12-16，贵州民族大学，贵阳

Selected topics on hadronic tau decays

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Current status on muon g-2

[2505.21476, White Paper 25]

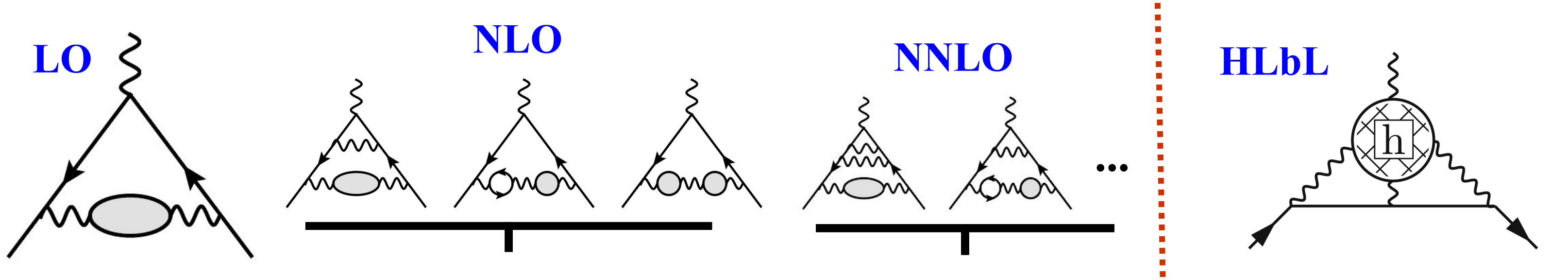
Contribution	Section	Equation	Value $\times 10^{11}$	References
Experiment (E989, E821)		Eq. (9.5)	116 592 071.5(14.5)	Refs. [5–8, 10–13]
HVP LO (lattice)	Sec. 3.6.1	Eq. (3.37)	7132(61)	Refs. [14–30]
HVP LO (e^+e^-, τ)	Sec. 2	Table 5	Estimates not provided at this point	
HVP NLO (e^+e^-)	Sec. 2.9	Eq. (2.47)	−99.6(1.3)	Refs. [31, 32]
HVP NNLO (e^+e^-)	Sec. 2.9	Eq. (2.48)	12.4(1)	Ref. [33]
HLbL (phenomenology)	Sec. 5.10	Eq. (5.69)	103.3(8.8)	Refs. [34–57]
HLbL NLO (phenomenology)	Sec. 5.10	Eq. (5.70)	2.6(6)	Ref. [58]
HLbL (lattice)	Sec. 6.2.8	Eq. (6.34)	122.5(9.0)	Refs. [59–63]
HLbL (phenomenology + lattice)	Sec. 9	Eq. (9.2)	112.6(9.6)	Refs. [34–57, 59–63]
QED	Sec. 7.5	Eq. (7.27)	116 584 718.8(2)	Refs. [64–70]
EW	Sec. 8	Eq. (8.12)	154.4(4)	Refs. [51, 71–73]
HVP LO (lattice) + HVP N(N)LO (e^+e^-)	Sec. 9	Eq. (9.1)	7045(61)	Refs. [14–33]
HLbL (phenomenology + lattice + NLO)	Sec. 9	Eq. (9.3)	115.5(9.9)	Refs. [34–63]
Total SM Value	Sec. 9	Eq. (9.4)	116 592 033(62)	Refs. [14–73]
Difference: $\Delta a_\mu \equiv a_\mu^{\rm exp} - a_\mu^{\rm SM}$	Sec. 9	Eq. (9.6)	38(63)	

Hadron

Error budget of $a_\mu^{\rm SM}$: 61(HVP-Lat), 10(HLbL), 0.4(EW), 0.2(QED)

HVP:

(dominated by e^+e^--> hadrons below 1 GeV)



WP20: ~7000(~40)

$\pi\pi$: (>70%) (>60%)

$\pi\pi\pi$, KK, ...

−99.6(1.3)

12.4(1)

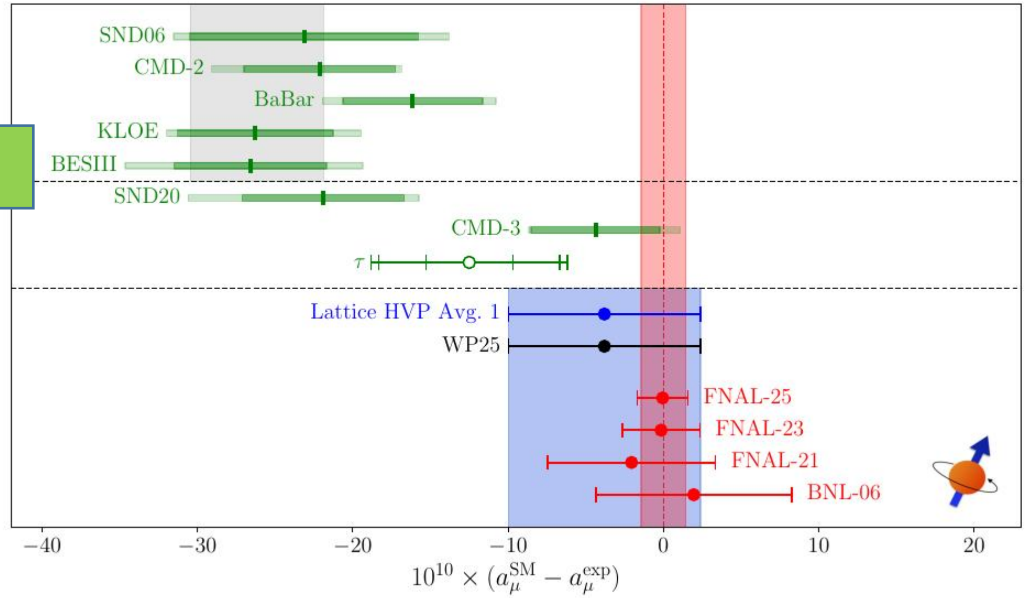
115.5(9.9)

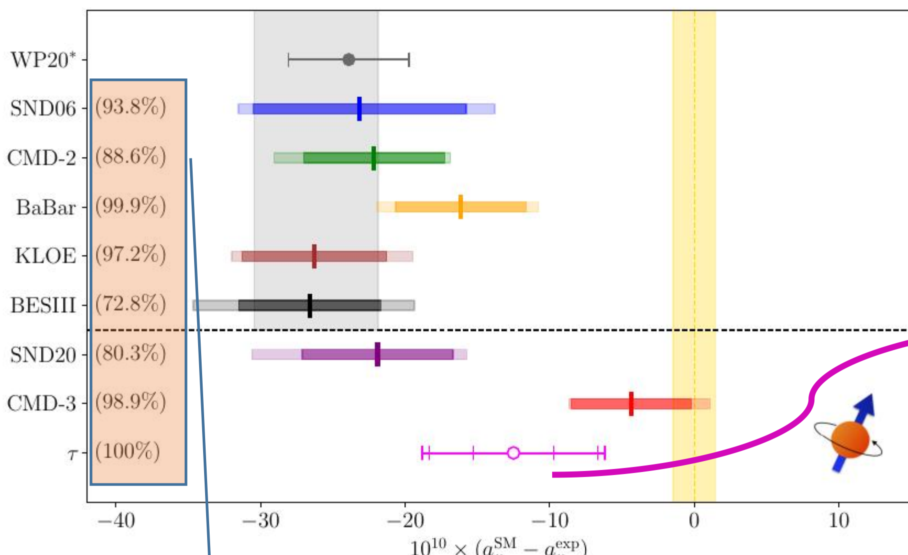
Greatly improved since WP20

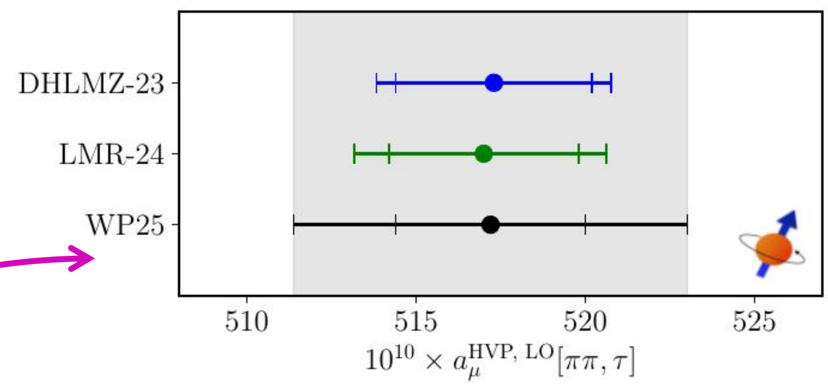
Current status on muon g-2

[2505.21476, White Paper 25]

- $e^+e^- \to \pi^+\pi^-$ from each indicated Exp
- HVP-LO beyond $\pi\pi$ from WP20
- Others, such as HLbL, EW, QED, from WP25



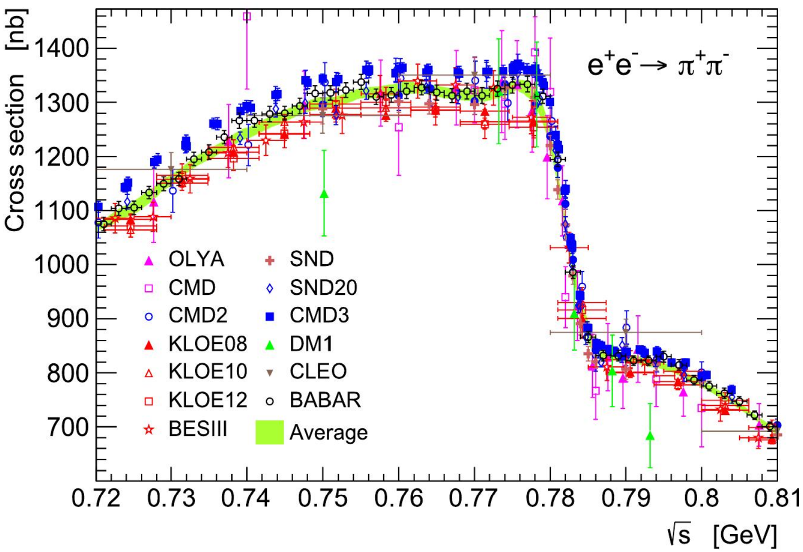


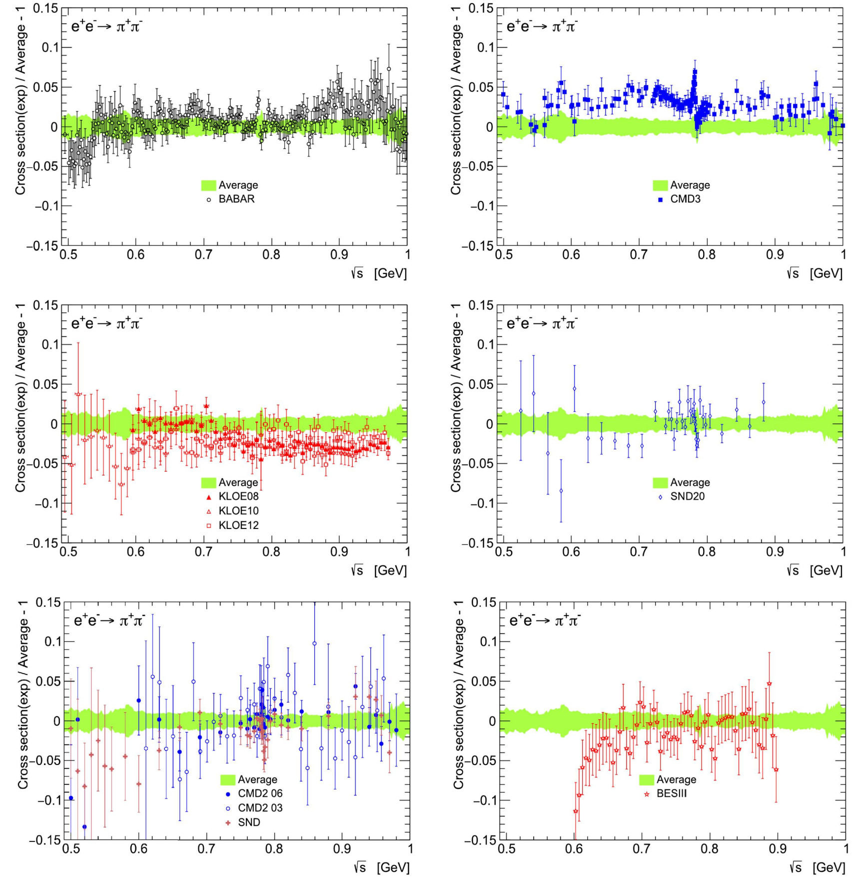


$\pi\pi$ contribution from each Exp to HVP integral

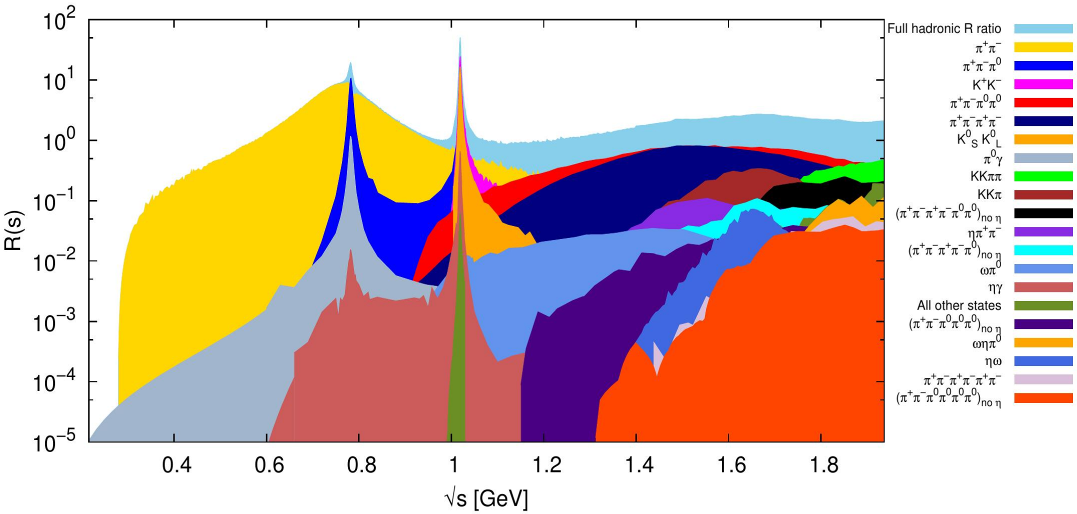
Two common methods to combine various data for $e^+e^- \to$ hadrons

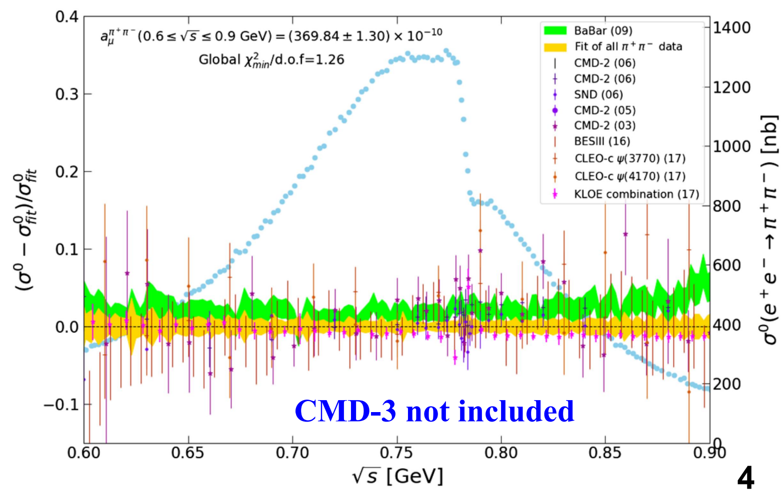
[Davier, et al., (DHLMZ average), EPJC'24]





[Keshavarzi, et al., (KNTW average), PRD'20]





Alternative way to address HVP from $\pi\pi$

$$a_\mu^{\mathrm{HVP,LO}} = \frac{1}{4\pi^3}\int_{4M_2^2}^{t_{max}} dt K(t)\sigma^0_{e^+e^-\to\mathrm{hadrons}}(t)$$

Known kernel function (enhanced contribution from energy below 1GeV)

$$\sigma^0_{e^+e^-\to\pi^+\pi^-} = \frac{\pi\alpha^2}{3t}\beta_{\pi^+\pi^-}\left|F^{(0)}_{\pi\pi}(t)\right|^2$$

$$\frac{d\Gamma(\tau_{2\pi})}{dt} = \frac{G_F^2\left|V_{ud}\right|^2 m_\tau^3 S_{\mathrm{EW}}}{384\pi^3}\left(1-\frac{t}{m_\tau^2}\right)^2\left(1+\frac{2t}{m_\tau^2}\right)\beta_{\pi^-\pi^0}\left|F^{(-)}_{\pi\pi}(t)\right|^2$$

$\tau^- \to \pi^-\pi^0\nu_\tau$: $\langle\pi^-\pi^0|\bar{d}\gamma_\mu u|0\rangle \sim F^{(-)}_{\pi\pi}(t)$ [I=1, I_3=-1]

$e^+e^- \to \pi^+\pi^-$: $\langle\pi^+\pi^-|\bar{u}\gamma_\mu u - \bar{d}\gamma_\mu d|0\rangle \sim F^{(0)}_{\pi\pi}(t)$ [I=1, I_3=0]

Isospin limit → **Conserved vector current (CVC)**

$$F^{(0)}_{\pi\pi}(t) = F^{(-)}_{\pi\pi}(t)$$

$$\sigma^0_{e^+e^-\to\pi^+\pi^-} = \frac{K_\sigma(t)}{K_\Gamma(t)}\frac{\beta_{\pi^+\pi^-}}{S_{\mathrm{EW}}\beta_{\pi^-\pi^0}}\frac{d\Gamma(\tau_{2\pi})}{dt}$$

$$K_\sigma(t) = \frac{\pi\alpha^2}{3t}\,,\ K_\Gamma(t) = \frac{G_F^2\left|V_{ud}\right|^2 m_\tau^3}{384\pi^3}\left(1-\frac{t}{m_\tau^2}\right)^2\left(1+2\frac{t}{m_\tau^2}\right)$$

- **Isospin breaking (IB) effects become CRUCIAL at the sub-percent level.**
- **Full control of all the IB terms is yet to be reached.**

Results on the estimation of a_μ based on the tau data in WP25 are based on:

[Davier, et al., (DHLMZ), EPJC'24] [Lopez Castro, et al., (LMR) PRD'25]

Isospin breaking corrections to a_μ

$$\sigma^0_{e^+e^-\to\pi^+\pi^-} = \left[\frac{K_\sigma(t)}{K_\Gamma(t)}\frac{d\Gamma(\tau_{2\pi[\gamma]})}{dt}\right]\times\left(\frac{R_{\rm IB}(t)}{S_{\rm EW}}\right)$$

Exp tau data (photon inclusive)

$$R_{\rm IB}(t) = \frac{{\rm FSR}(t)}{G_{\rm EM}(t)}\frac{\beta^3_{\pi^+\pi^-}}{\beta^3_{\pi^-\pi^0}}\frac{\left|F^{(0)}_{\pi\pi}(t)\right|^2}{\left|F^{(-)}_{\pi\pi}(t)\right|^2}$$

$$\Delta a_\mu^{\rm HVP,LO} = \frac{1}{4\pi^3}\int_{4m_\pi^2}^{t_{max}} dt K(t)\left[\frac{K_\sigma(t)}{K_\Gamma(t)}\frac{d\Gamma(\tau_{2\pi[\gamma]})}{dt}\right]\times\left(\frac{R_{\rm IB}(t)}{S_{\rm EW}}-1\right)$$

- **Final-state radiation (FSR) corrections to $\pi^+\pi^-$**
- **$\beta_{\pi\pi}=2q_{\rm CM}(t)/\sqrt{t}$: kinematical factor caused by the π^{+-} - π^0 mass difference [important near thresh.]**
- **Raito of form factors: $F^{(0)}_{\pi\pi}(t)/F^{(0)}_{\pi\pi}(t)$ [carrying the largest uncertainty]**

  $F^{(0)}_{\pi\pi}(t)$ [$e^+e^-\to\pi^+\pi^-$]: **$M_{\rho0}$, $\Gamma_{\rho0}$, ρ^0-ω mixing**

  $F^{(-)}_{\pi\pi}(t)$ [$\tau^-\to\nu_\tau\pi^0\pi^-$]: **$M_{\rho-}$, $\Gamma_{\rho-}$**

Not only depend on $\Delta M_\rho = M_{\rho-} - M_{\rho0}$, $\Delta\Gamma_\rho=\Gamma_{\rho-}-\Gamma_{\rho0}$, ρ^0-ω mixing , but also on the FF parameterization.

[2505.21476, WP25] $\Delta a_\mu^{\rm HVP,\,LO}[\pi\pi,\tau]$ (in units of 10^{-10})

	ΔM_ρ	$0.20(^{+27}_{-19})(9)$	$1.95^{+1.56}_{-1.55}$
	$\Delta\Gamma_\rho(\Delta M_\pi)$	$4.09(0)(7)$	3.37
$\frac{F_\pi^V}{f_+}$ (w/o ρ–ω)	$\Delta\Gamma_\rho(\pi\pi\gamma)$	$-5.91(59)(48)$	$-6.66(73)$
	$\Delta\Gamma_\rho(g_{\rho\pi\pi})$	–	–
	Total	$-1.62(65)(63)$	$(-1.34)^{+1.72}_{-1.71}$
		(DHLMZ)	**(LMR)**

ρ–ω mixing	$4.0(4)$	$2.87(8)$
	(DHLMZ)	**(LMR)**

- **$G_{\rm EM}(t)$: long-distance radiative corrections to $\tau^-\to\nu_\tau\pi^0\pi^-$**

➢ $G_{EM}(t)$: long-distance EM corrections to $\tau^- \to \nu_\tau \pi^0 \pi^-$

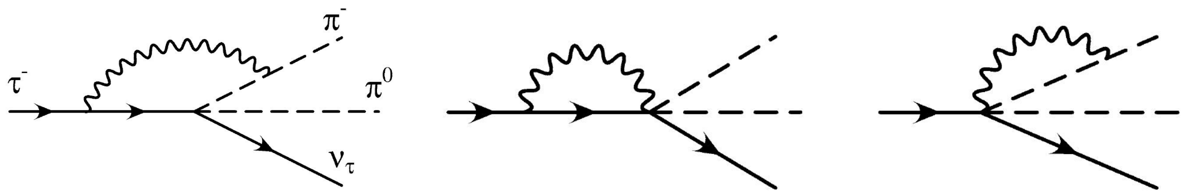
$$\frac{d\Gamma(\tau_{2\pi[\gamma]})}{dt} = \frac{G_F^2 |V_{ud}|^2 m_\tau^3 S_{EW}}{384\pi^3}\left(1-\frac{4m_\pi^2}{t}\right)\left(1-\frac{t}{m_\tau^2}\right)^2\left(1+\frac{2t}{m_\tau^2}\right)\left|F_{\pi\pi}^{(-)}(t)\right|^2 G_{EM}(t)$$

$d\Gamma_{\tau\to\pi\pi\nu}/dt$

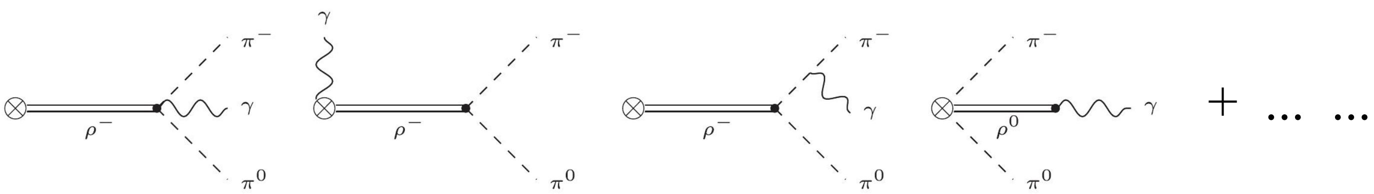
$G_{EM}(t) \sim$ virtual photon + real photon

virtual photon (loops)

[Cirigliano, et al.,PLB'01]



real photon (radiative decay, hadronic modeling needed)



➢ **G_{EM} is infrared finite: cancellation between photon loop and bremsstrahlung of the real photon.**

➢ **Experimental measurement of $\tau \to \pi\pi\gamma\nu_\tau$ is absent: theoretical estimation needed.**

. [Cirigliano et al, JHEP'02]: Minimal Resonance Chiral Theory interactions

. [Flores-Baez et al., PRD'06]: VMD with anomalous vector interactions

$$a_\mu^\tau[2\pi] = (517.3 \pm 1.9 \pm 2.2 \pm 1.9) \times 10^{-10}$$ [Davier et al., EPJC'24]

. [Miranda, Roig., PRD'20]: extended RChT with many free parameters

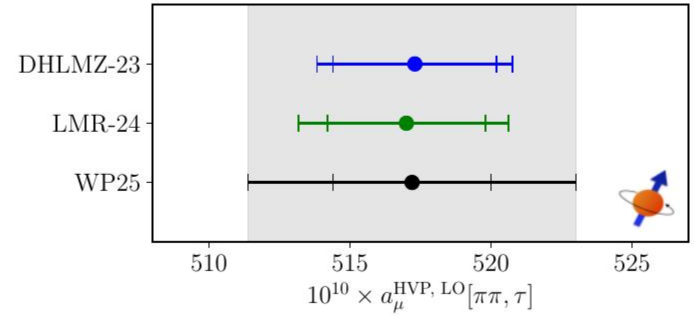
$$a_\mu^\tau[2\pi] = (519.6 \pm 2.8[\mathrm{exp}]^{+1.9}_{-2.1}[\mathrm{IB}]) \times 10^{-10}$$ [O(p^4)]

Muon g-2 based on the $\tau \to \nu_\tau \pi\pi$ data

$\tau \to \nu_\tau \pi\pi$ data (Belle, ALEPH, CLEO, OPAL)

+

Isospin breaking factors ([Davier, et al., (DHLMZ), EPJC'24][Lopez Castro, et al., (LMR) PRD'25])



$$a_\mu^{\text{HVP, LO}}[\pi\pi, \tau] = 517.2(2.8)_{\text{exp}}(5.1)_{\text{th}} \times 10^{-10}$$

Adding other contributions to HVP-LO ($\pi\pi\pi$, KK, $\pi\gamma$ ...) from WP20

$$a_\mu^{\text{HVP, LO}}[(\pi\pi, \tau) + \text{WP20}] = 704.5(6.2) \times 10^{-10}$$

Caveat in WP25: "The above offset from WP20 is not updated in this work, we instead focus on the major tensions in the 2π channel. ... ... As described in Secs. 2.2.6 and 2.6.2, tensions between the Belle-II 3π data and previous measurements are now visible, other tensions are present in the K+ K− channel and in the comparison of the BESIII inclusive R-ratio measurement with pQCD. ... ... "

- **To futher take HLbL, HVP-N(N)LO, EW, QED from WP25, one would obtain**

$$a_\mu^{\text{SM}}\,[(\pi\pi,\tau)+\text{WP25}] = 116\,591\,946\,(63) \times 10^{-11}$$

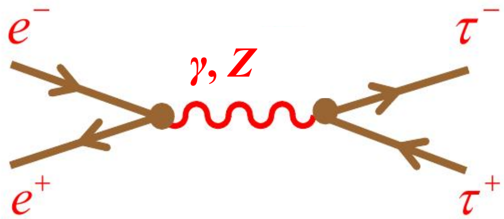
$$a_\mu^{\text{Exp}} = 116\,592\,071.5\,(14.5) \times 10^{-11}$$

$$\Delta a_\mu\,[(\pi\pi,\tau)] = a_\mu^{\text{Exp}} - a_\mu^{\text{SM}} = 126\,(65) \times 10^{-11} \quad (1.9\sigma)$$

to compare with: $\Delta a_\mu\,[(\pi\pi,\text{lattice})] = a_\mu^{\text{Exp}} - a_\mu^{\text{SM}} = 38\,(63) \times 10^{-11}$ (reference value in WP25)

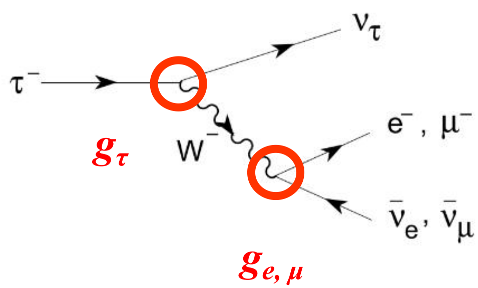
tau轻子概览

- tau产生机制



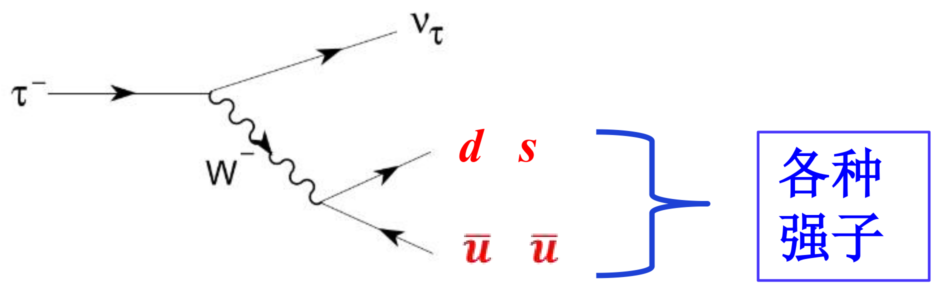
对于BES/STCF/Belle实验，Z 交换被 q^2/M_z^2 因子压低，γ交换机制起主导作用。

- 轻子衰变(标准模型)



- 半轻衰变/强衰变(标准模型)

tau跟胶子没有直接作用，
也不能直接通过QED进行衰变！



- ➢ 标准模型中tau轻子的性质几乎跟 electron、muon是相同的
- ➢ 三代轻子的规范相互作用(W、Z、光子、胶子)是完全一样的，包括形式和大小
- ➢ 三代轻子与Higgs相互作用形式一样，但是 Yukawa耦合常数大小不同： $y_\tau \sim 3500\ y_e$，$y_\tau \sim 17\ y_\mu$
- ➢ 正是因为上述Yukawa常数（轻子质量）的不同，使得三代轻子的现象学讨论皆然不同！

分支比概览

- ➢ **Br($\tau\rightarrow e\nu_\tau\overline{\nu_e}$) : 17.8%**

  Br($\tau\rightarrow \mu\nu_\tau\overline{\nu_\mu}$) : 17.4%

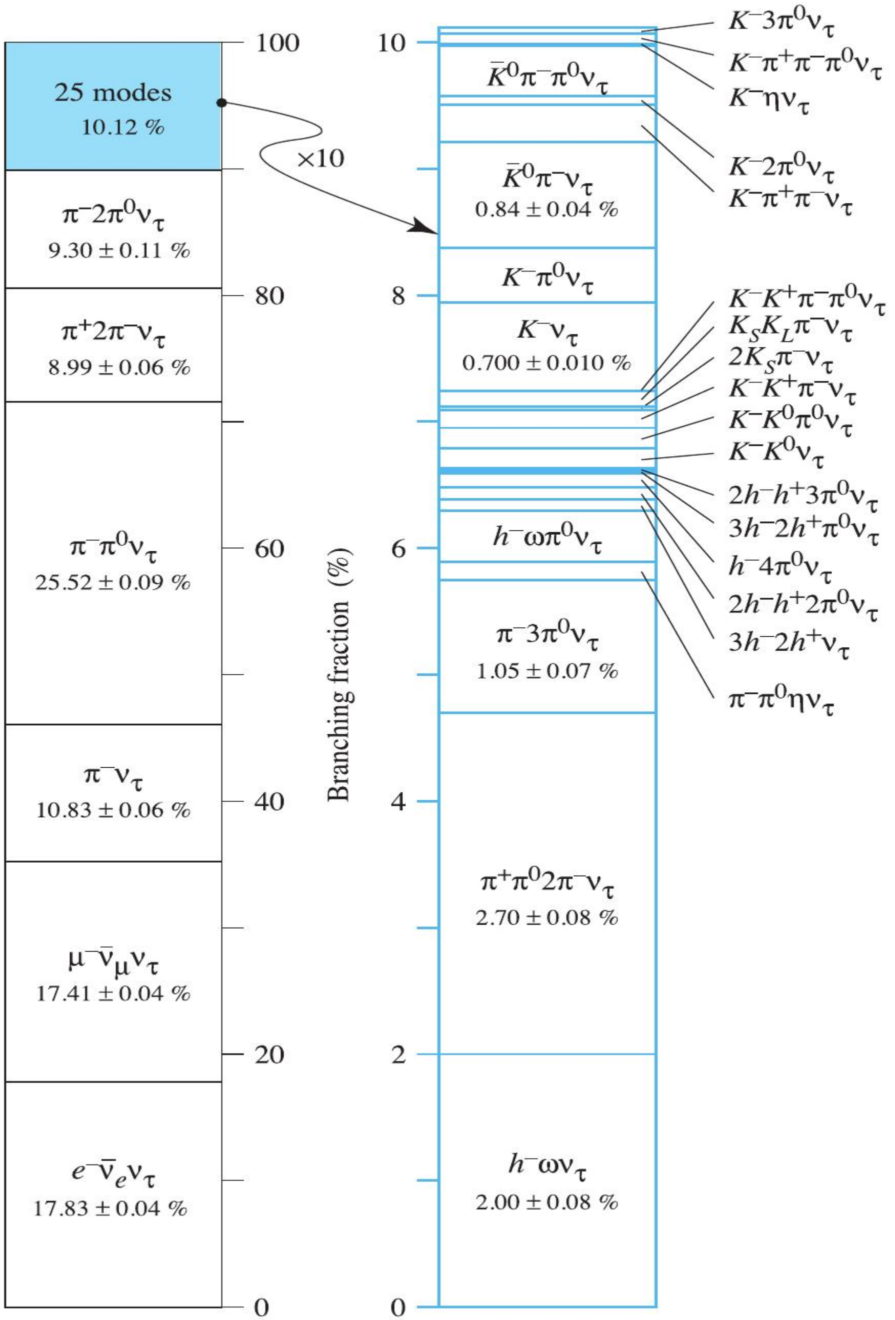
- ➢ **Br($\tau\rightarrow\nu$+Cabbibo allowed hadrons) ~ 62%**
- ➢ **Br($\tau\rightarrow\nu$+Cabbibo suppressed hadrons) : ~3%**

- □ **Br($\tau\rightarrow\nu\pi\pi$) ~ 25%, 单举衰变中分支比最大**
- □ **tau的衰变末态只有轻味强子，不涉及重味粒子 ($m_\tau < m_D$)**
- □ **在重子数守恒的假设下，tau不能衰变至含有重子的末态($m_\tau < 2m_N$)**

名词澄清：

- **单举(exclusive): 只包含某一个具体物理过程**
- **遍举(inclusive): 包含所有可能的单举过程或者包含某一类单举过程**

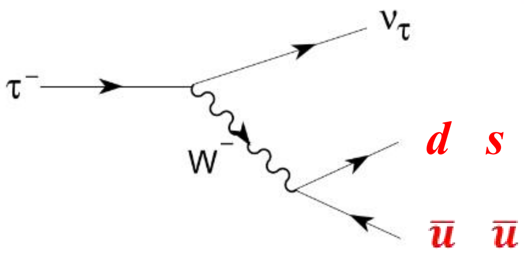
例如，**Cabbibo允许的inclusive过程是指末态不含奇数个K介子的所有exclusive过程**



基础理论回顾

标准模型的带电流

$$\mathcal{L}_{CC} = -\frac{g}{\sqrt{2}} W_\mu^\dagger \left(V_{uD}\, \bar{u}_L \gamma^\mu D_L + \bar{\nu}_L \gamma^\mu \tau_L \right) + h.c.$$



- $$M_{\tau\to H\nu_\tau} = \frac{g^2 V_{uD}^*}{2} \frac{L_\mu H^\mu}{s-M_W^2} \approx -\frac{g^2 V_{uD}^*}{2M_W^2} L_\mu H^\mu$$

$$H_\mu = \langle n | \bar{d}_L \gamma_\mu u | 0 \rangle, L_\mu = \bar{u}_{L,\nu} \gamma_\mu u_{L,\tau}$$

因为$m_\tau \ll m_W$，另外如果在电弱能标下没有质量轻的新粒子， 四费米子等价相互作用可以用来很好地描述tau衰变（标准模型或者新物理模型均可）。

- tau强衰变相关的标准模型下的四费米子作用可总结为：

$$\mathcal{L}_{CC}^{\rm SM} = -\frac{G_F}{\sqrt{2}} V_{uD}\, \underline{\bar{l}_{\nu_\tau} \gamma^\mu (1-\gamma_5) l_\tau}\, \underline{\bar{u}\gamma_\mu(1-\gamma_5) D}\,, \quad [D = d, s]$$

V_{uD} → **CKM: V_{ud}/V_{us}**

$\bar{l}_{\nu_\tau} \gamma^\mu (1-\gamma_5) l_\tau$ → 轻子流

$\bar{u}\gamma_\mu(1-\gamma_5) D$ → 夸克流

$$G_F = \frac{g^2}{4\sqrt{2}M_W^2}$$

- **SMEFT → LEFT** [Cirigliano et al, '10] [Y.Liao et al., '21] [J.H.Yu et al., '21][F.Z.Chen et al, '22] ...

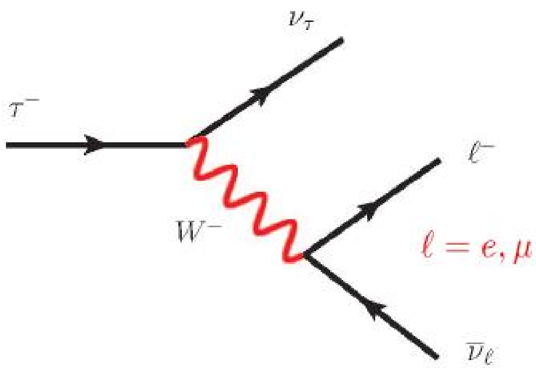
$$\mathcal{L}_{\rm eff} = -\frac{G_\mu V_{uD}}{\sqrt{2}} \Big[\left(1+\epsilon_L^{D\ell}\right) \bar{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell \cdot \bar{u}\gamma^\mu(1-\gamma_5)D + \epsilon_R^{D\ell} \bar{\ell}\gamma_\mu(1-\gamma_5)\nu_\ell \cdot \bar{u}\gamma^\mu(1+\gamma_5)D$$
$$+ \bar{\ell}(1-\gamma_5)\nu_\ell \cdot \bar{u}\left[\epsilon_S^{D\ell} - \epsilon_P^{D\ell}\gamma_5\right] D + \frac{1}{4}\hat{\epsilon}_T^{D\ell} \bar{\ell}\sigma_{\mu\nu}(1-\gamma_5)\nu_\ell \cdot \bar{u}\sigma^{\mu\nu}(1-\gamma_5)D \Big] + \text{h.c.},$$

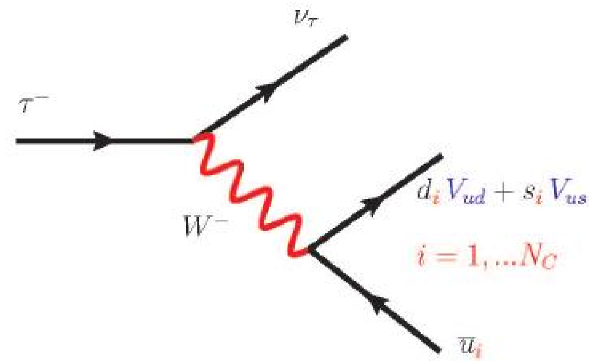
ε_X parameterize various new physics at high energy scale

基于SM的粗略分支比估计

universal coupling *g* of W and fermions

V_{ud}=0.974，V_{us}=0.224





	粗略理论估计	实验值
➢ **Br($\tau \to e \nu_\tau \overline{\nu_e}$)**	$\frac{1}{2 + N_C\left(\|V_{ud}\|^2 + \|V_{us}\|^2\right)} \simeq 20\,\%$	**17.8%**
Br($\tau \to \mu \nu_\tau \overline{\nu_\mu}$)		**17.4%**
➢ **Br(τ→ν+Cabbibo allowed channels)**	$\frac{N_C \|V_{ud}\|^2}{2 + N_C\left(\|V_{ud}\|^2 + \|V_{us}\|^2\right)} \simeq 57\%$	**(62±4)%**
➢ **Br(τ→ν+Cabbibo suppressed channels)**	$\frac{N_C \|V_{us}\|^2}{2 + N_C\left(\|V_{ud}\|^2 + \|V_{us}\|^2\right)} \simeq 3\%$	**（2.6±0.7）%**

❖ **对于单举强衰变过程，因为QCD非微扰效应，其理论预言要复杂的多！**

强衰变过程

tau的强衰变可以给我们提供什么信息?

- **inclusive衰变:（某类）所有的强子末态**

$$\tau^- \to \nu_\tau (\bar{u}d, \bar{u}s)$$

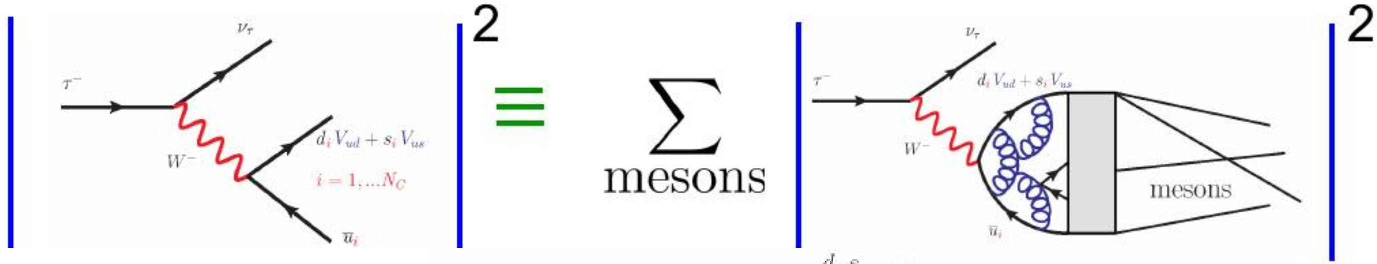
➡ 可以用来研究标准模型的基本参数：α_S, V_{us}, ...

- **exclusive衰变：衰变至特定的强子末态**

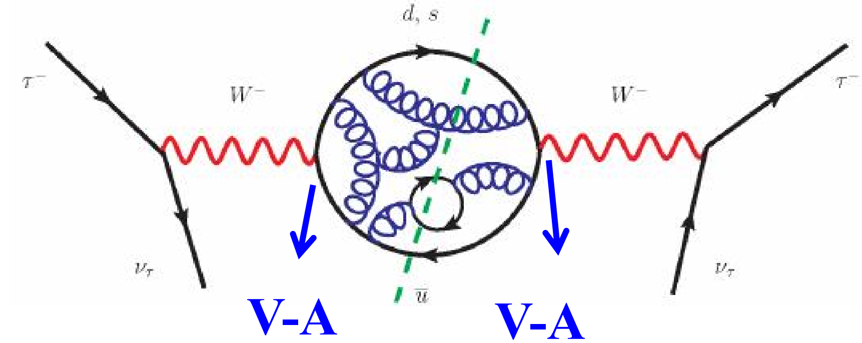
$$\tau^- \to \nu_\tau (P,\ PP',\ P_1P_2P_3,\ \ldots)$$

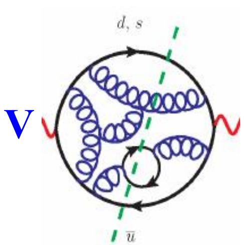
➡
- 强作用形状因子，强子共振态，手征对称性，...
- CPV、轻子味道破坏、... ...

tau的 inclusive衰变



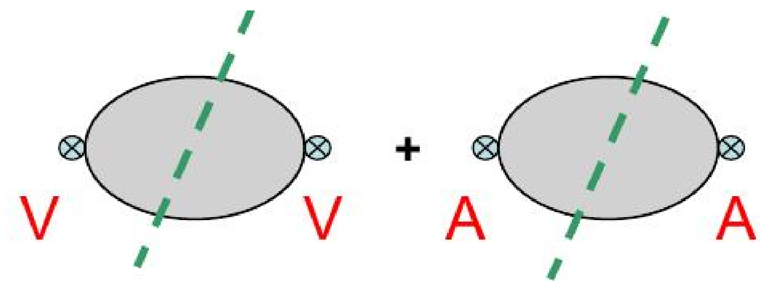
$$\Gamma\left(\tau \longrightarrow \nu_\tau \,\text{mesons}\right) \quad \propto$$





=0 （强作用下宇称守恒）

$$R_\tau \equiv \frac{\Gamma\left(\tau^- \rightarrow \nu_\tau \,\text{mesons}\right)}{\Gamma\left(\tau^- \rightarrow e^- \overline{\nu}_e \nu_\tau\right)} \quad \propto$$



$$R_\tau = \underbrace{R_{\tau,V} + R_{\tau,A}}_{S=0} + \underbrace{R_{\tau,S}}_{S=1} \simeq \frac{N_C}{2}|V_{ud}|^2 + \frac{N_C}{2}|V_{ud}|^2 + N_C|V_{us}|^2$$

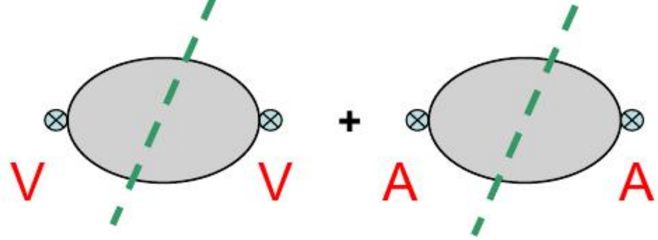
$$R_\tau \simeq N_C$$

$$R_\tau^{\text{exp}} = \frac{\sum_i \Gamma(\tau \to \nu_\tau h_i)}{\Gamma(\tau^- \to \nu_\tau e^- \overline{\nu}_e)} = 3.6355 \pm 0.0081$$

$$R_\tau^{\text{exp}} = \frac{1 - \mathrm{B}_e - \mathrm{B}_\mu}{\mathrm{B}_e} = 3.6370 \pm 0.0075$$

QCD corrections amount to 20%!

$$R_\tau \equiv \frac{\Gamma(\tau^- \to \nu_\tau \,\mathrm{mesons})}{\Gamma(\tau^- \to e^- \overline{\nu}_e \nu_\tau)} \propto$$



两点关联函数

$$V_{ij}^\mu = \bar{\psi}_j \gamma^\mu \psi_i \qquad A_{ij}^\mu = \bar{\psi}_j \gamma^\mu \gamma_5 \psi_i$$

$$\Pi_{ij,J}^{\mu\nu}(q) \equiv i \int d^4x\; e^{iqx} \langle 0 | T[J_{ij}^\mu(x) J_{ij}^\nu(0)^\dagger] | 0 \rangle$$

$$= \left(-g^{\mu\nu} q^2 + q^\mu q^\nu\right) \Pi_{ij,J}^{(1)}(q^2) + q^\mu q^\nu \; \Pi_{ij,J}^{(0)}(q^2)$$

于是有：

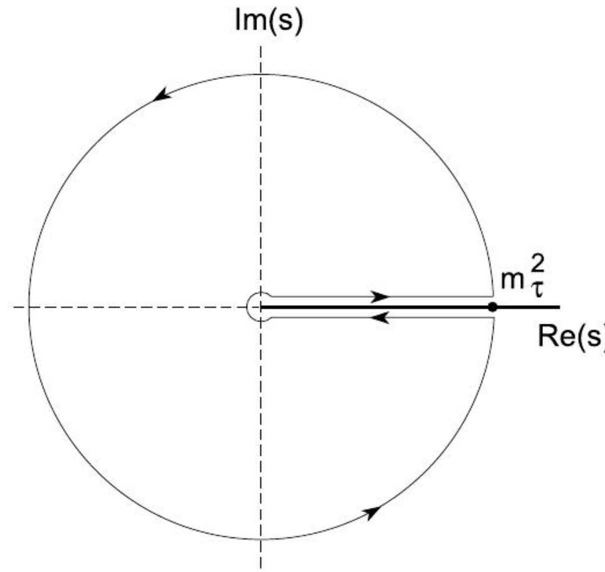
$$R_\tau = 12\pi \int_0^{M_\tau^2} \frac{ds}{M_\tau^2} \left(1 - \frac{s}{M_\tau^2}\right)^2 \left[\left(1 + 2\frac{s}{M_\tau^2}\right) \mathrm{Im}\Pi^{(1)}(s) + \mathrm{Im}\Pi^{(0)}(s)\right]$$

谱函数 （两点关联函数的虚部）

说明：

- 谱函数ImΠ(s)实验可测：tau的inclusive衰变过程
- 谱函数ImΠ(s) 在 s~(0,m_τ^2) 区间内的理论计算完全涉及非微扰QCD，很难有可靠的计算
- 理论出路？

利用函数Π(s)的解析性质



- 柯西定理
- Π(s)在除去正实轴以外的其他地方解析
- f(s)为任一解析函数

$$\frac{1}{\pi}\int_0^{s_0} ds\, f(s)\, \mathrm{Im}\Pi(s) = -\frac{1}{2\pi i}\oint_{|s|=s_0} ds\, f(s)\, \Pi(s)$$

$$R_\tau = 12\pi \int_0^{M_\tau^2} \frac{ds}{M_\tau^2}\left(1-\frac{s}{M_\tau^2}\right)^2 \left[\left(1+2\frac{s}{M_\tau^2}\right)\mathrm{Im}\Pi^{(1)}(s) + \mathrm{Im}\Pi^{(0)}(s)\right]$$

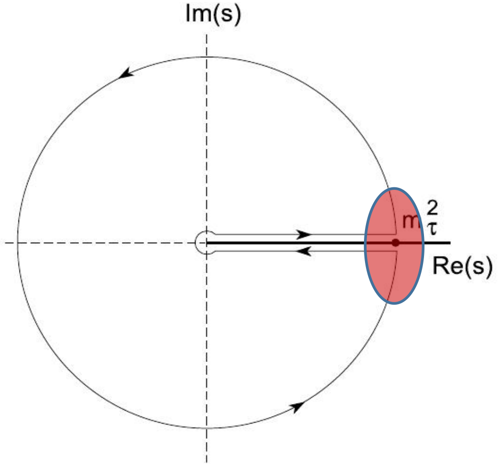
$$= 6\pi i \oint_{|s|=M_\tau^2} \frac{ds}{M_\tau^2}\left(1-\frac{s}{M_\tau^2}\right)^2 \left[\left(1+2\frac{s}{M_\tau^2}\right)\Pi^{(0+1)}(s) - \frac{2s}{M_\tau^2}\Pi^{(0)}(s)\right]$$

✓ 在|s|=m_τ^2的圆周上，利用算符乘积展开(operator product expansion, OPE)，可对Π(s)进行可靠的理论计算。

算符乘积展开(OPE)

$$\Pi^{(J)}(s) = \sum_{D=0,2,4,\ldots} \frac{1}{(-s)^{D/2}} \sum_{\dim \mathcal{O}=D} C_D^{(J)}(s,\mu)\, \langle \mathcal{O}_D(\mu) \rangle$$

- **D=0，QCD微扰部分（以α_s为参数进行展开）**
- **D>0, QCD非微扰部分（以各种凝聚量为展开）**
- **可能的Quark-hadron duality violation (DV) 效应**



$$R_\tau = N_C\, S_{\mathrm{EW}} \left(1 + \delta_{\mathrm{P}} + \delta_{\mathrm{NP}}\right)$$

$S_{\mathrm{EW}} = 1.0201\,(3)$; $\delta_{\mathrm{NP}} = -0.0064 \pm 0.0013$

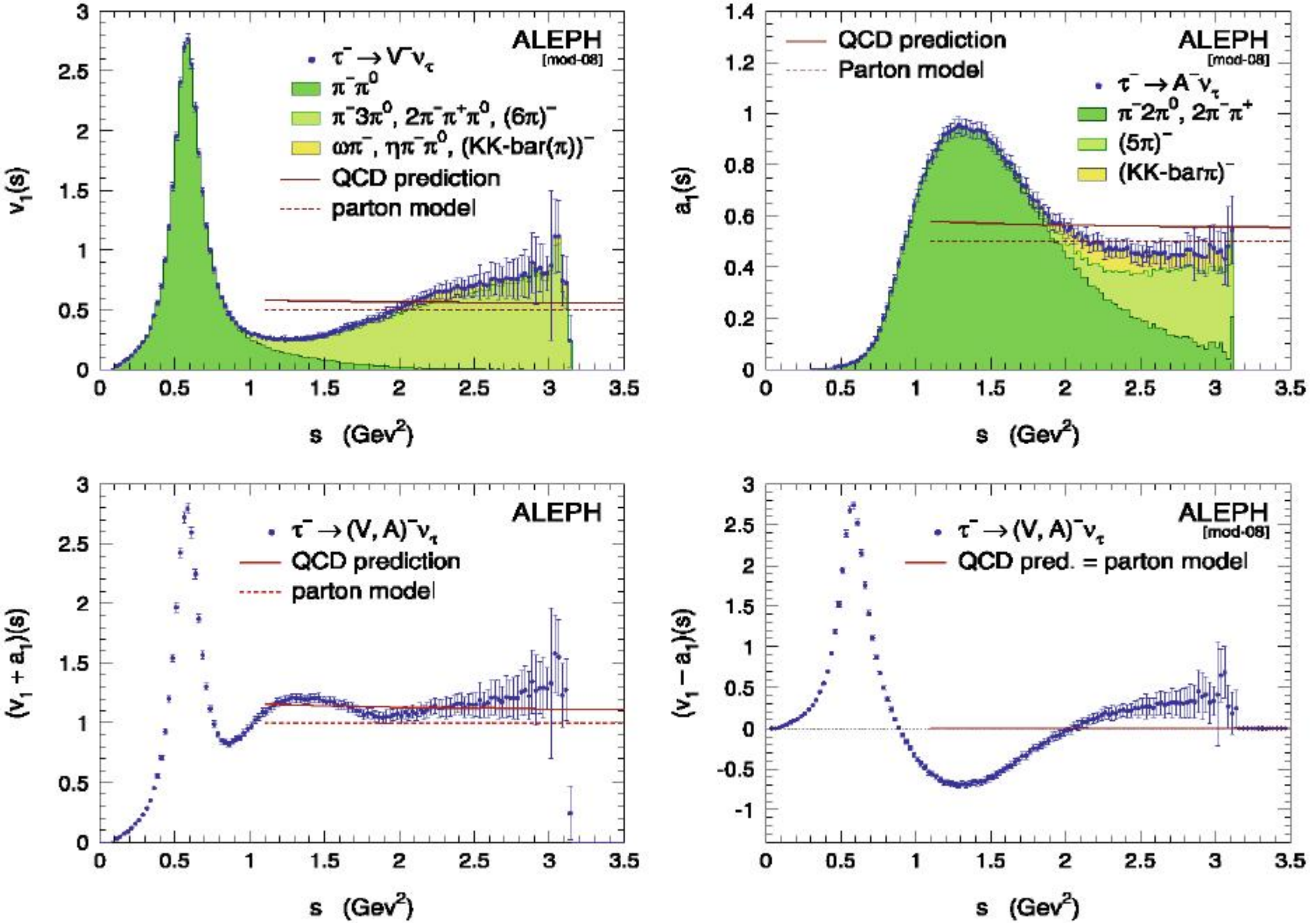
Marciano-Sirlin, Braaten-Li, Erler **Fitted from data (Davier et al)**

$$\delta_{\mathrm{P}} = a_\tau + 5.20\, a_\tau^2 + 26\, a_\tau^3 + 127\, a_\tau^4 + \ldots \approx 20\% \quad ; \quad a_\tau \equiv \alpha_s(m_\tau)/\pi$$

Baikov-Chetyrkin-Kühn

- **tau遍举衰变中的微扰修正非常重要，其对α_s依赖敏感，因此可以有效地确定α_s数值。**

$$R_\tau = 12\pi \int_0^{M_\tau^2} \frac{ds}{M_\tau^2}\left(1-\frac{s}{M_\tau^2}\right)^2 \left[\left(1+2\frac{s}{M_\tau^2}\right)\mathrm{Im}\Pi^{(1)}(s) + \mathrm{Im}\Pi^{(0)}(s)\right] = N_C\, S_{EW}\,(1+\delta_P+\delta_{NP})$$



- **矢量谱函数(V)：**

$I^G J^P = 1^+ 1^-$

末态为偶数个π

- **轴矢谱函数(A)：**

$I^G J^P = 1^- 1^+$

末态为奇数个π

- **末态含K介子时：**

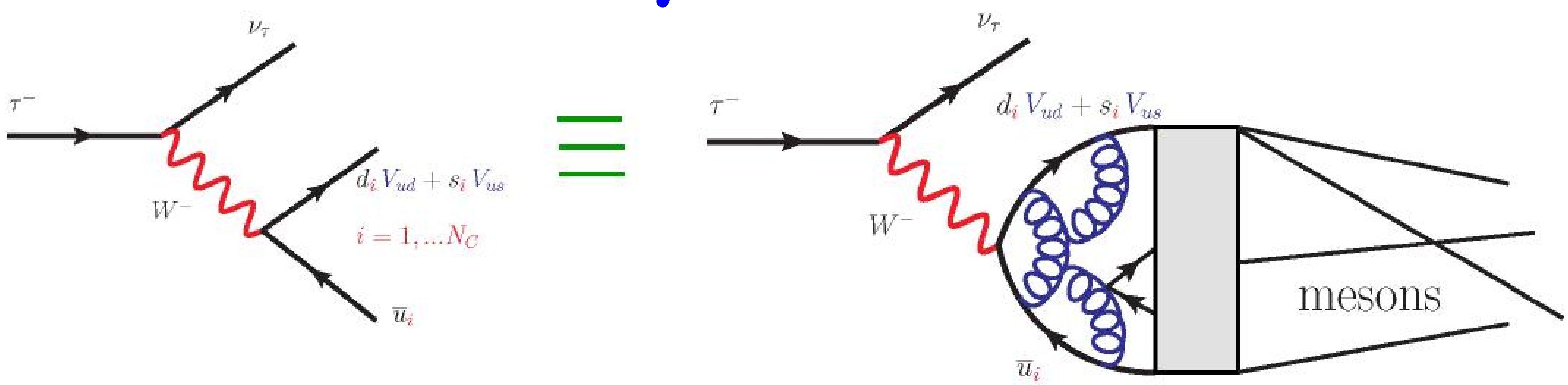
需要通过理论模型分别抽取V、A谱函数

$$(V-A)\big|_\chi \quad \propto \quad \text{non-perturbative} \qquad (m_u = m_d = m_s = 0)$$

$$(V+A) \quad \propto \quad \left(\text{perturbative} + \frac{1}{M_\tau^6}\,\text{non-perturbative}\right)$$

$\alpha_S(M_\tau)$ **(微扰贡献大致20%，非微扰贡献大致在 0.5%)**

Tau exclusive decays



$$\mathcal{M}\left(\tau \to \nu_\tau \mathrm{H}\right) = \frac{G_F}{\sqrt{2}} V_{\mathrm{CKM}}\, \overline{u}_{\nu_\tau} \gamma^\mu \left(1 - \gamma_5\right) u_\tau \left\langle H \right| \left(V_\mu - A_\mu\right) e^{i\,\mathrm{L_{QCD}}} \left|\Omega_{\mathrm{H}}\right\rangle$$

Hadronic V-A currents

Chiral EFT is the low energy realization of QCD:

$$e^{i\mathcal{Z}(v_\mu, a_\mu, s, p)} = \int \mathcal{D}q\mathcal{D}\bar{q}\mathcal{D}G_\mu\; e^{i\int d^4x \mathcal{L}_{QCD}(v_\mu, a_\mu, s, p)} = \int \mathcal{D}u\; e^{i\int d^4x \mathcal{L}_{EFT}(v_\mu, a_\mu, s, p)}$$

$$\mathcal{L}^{QCD} = \mathcal{L}_0^{QCD} + \bar{q}\gamma^\mu(v_\mu + a_\mu\gamma_5)q - \bar{q}(s - i\gamma_5 p)q$$

v_μ, a_μ, s, p are the external source fields .

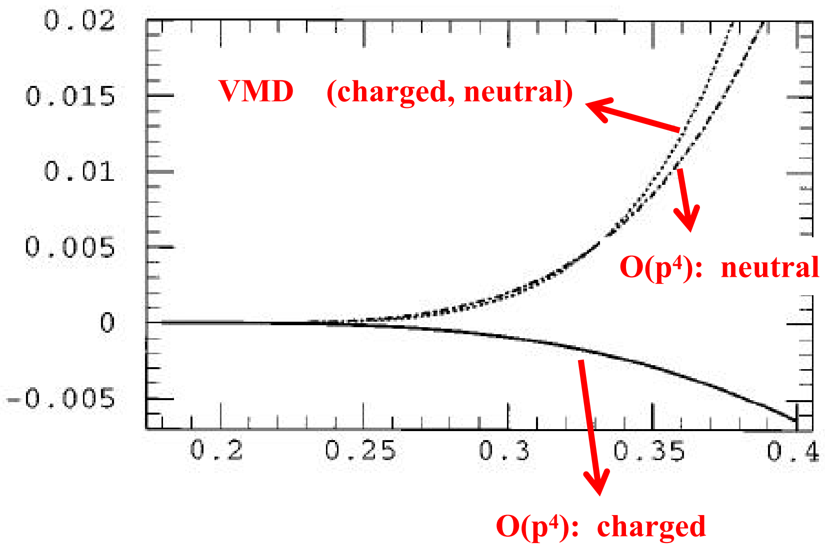
Chiral symmetry is RELEVANT to tau decays

Example: $\tau \rightarrow \nu_\tau \pi\pi\pi$ transition amplitudes in the low energy region
VMD models do not automatically respect chiral symmetry.

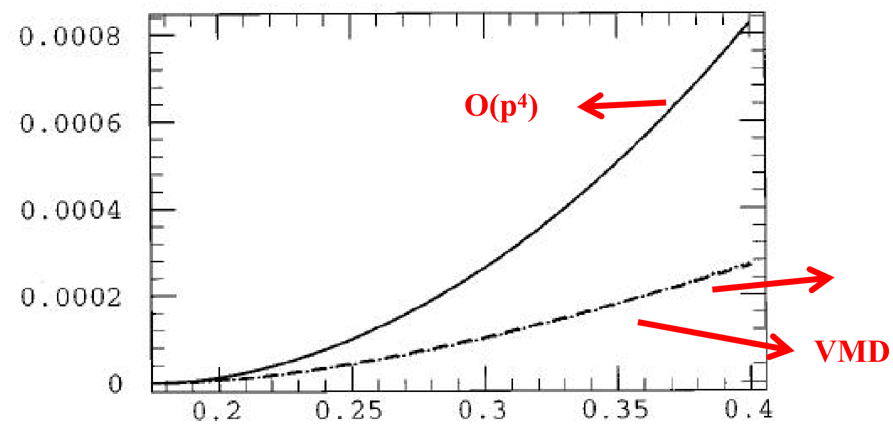
$$J_\alpha = -i\frac{2\sqrt{2}}{3f_\pi}\mathrm{BW}_a(Q^2)(B_\rho(s_2)V_{1\alpha} + B_\rho(s_1)V_{2\alpha})$$

[Kuhn,Santamaria, ZPC'90]

W_D structure function



W_{SA} structure function (neutral channel)



[Colangelo, et al., PRD'96]

➢ **Resonance chiral theory** **implements the constraint of chiral symmetry from the very beginning in the construction of the Lagrangians.**

Resonance chiral theory (RχT)

Chiral group: $G = SU(3)_L \times SU(3)_R$, $H = SU(3)_V$, $u(\phi) = G/H$

Resonances : $R \stackrel{G}{\Longrightarrow} h\, R\, h^\dagger , \quad h \in H$

pNGB and external sources : $X = u_\mu,\ \chi_\pm,\ f_\pm^{\mu\nu},\ h_{\mu\nu}$

Operators	P	C	h.c.	chiral order
u	$u^\dagger$	u^T	$u^\dagger$	1
Γ_μ	Γ^μ	$-{\Gamma_\mu}^T$	$-\Gamma_\mu$	p
u_μ	$-u^\mu$	u_μ^T	u_μ	p
$\chi_\pm$	$\pm\chi_\pm$	$\chi_\pm^T$	$\pm\chi_\pm$	p^2
$f_{\mu\nu\,\pm}$	$\pm f_\pm^{\mu\nu}$	$\mp f_{\mu\nu\,\pm}^T$	$f_{\mu\nu\,\pm}$	p^2
$h_{\mu\nu}$	$-h^{\mu\nu}$	$h_{\mu\nu}^T$	$h_{\mu\nu}$	p^2

Operators	P	C	h.c.
$V_{\mu\nu}$	$V^{\mu\nu}$	$-V_{\mu\nu}^T$	$V_{\mu\nu}$
$A_{\mu\nu}$	$-A^{\mu\nu}$	$A_{\mu\nu}^T$	$A_{\mu\nu}$
S	S	S^T	S
P	$-P$	P^T	P

Minimal RχT Lagrangian [Ecker, *et al.*, '89]

$$
\begin{aligned}
\mathcal{L}_{2V} &= \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + \frac{iG_V}{2\sqrt{2}}\langle V_{\mu\nu}[u^\mu, u^\nu]\rangle \\
\mathcal{L}_{2A} &= \frac{F_A}{2\sqrt{2}}\langle A_{\mu\nu} f_-^{\mu\nu}\rangle , \\
\mathcal{L}_{2S} &= c_d\langle\, S u_\mu u^\mu\,\rangle + c_m\langle\, S\chi_+\,\rangle , \\
\mathcal{L}_{2P} &= i d_m\langle\, P\chi_-\,\rangle .
\end{aligned}
$$

Operators beyond minimal [Cirigliano, *et al.*, '04]:

$$\mathcal{L}_{VAP} = \lambda_1^{VA}\langle [V^{\mu\nu}, A_{\mu\nu}]\chi_-\rangle + \ldots ,$$

[Ruiz-Femenia, Pich and Portolés, '03]

$$
\begin{aligned}
\mathcal{L}_{VVP} &= d_1 \varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\nu}, V^{\rho\alpha}\}\nabla_\alpha u^\sigma\rangle + \ldots , \\
\mathcal{L}_{VJP} &= \frac{c_1}{M_V}\varepsilon_{\mu\nu\rho\sigma}\langle\{V^{\mu\nu}, f_+^{\rho\alpha}\}\nabla_\alpha u^\sigma\rangle + \ldots ,
\end{aligned}
$$

$$d_4 \varepsilon_{\mu\nu\rho\sigma}\langle\{\nabla^\sigma V^{\mu\nu}, V^{\rho\alpha}\}u_\alpha\rangle$$

QCD dynamics in RχT

- Low energy QCD: implemented from the construction of RχT

- Intermediate energy: explicit resonance states

- High energy information: to match the same physical objects in RχT and QCD, $\langle\, J(x_n)\cdot\cdot J(0)\,\rangle^{\mathrm{R}\chi\mathrm{T}} = \langle\, J(x_n)\cdot\cdot J(0)\,\rangle^{\mathrm{QCD}}$.

  For example: $\pi\pi$ vector form factor

$$
\begin{aligned}
\left[\mathcal{F}^{v}_{\pi\pi}(q^2)\right]^{\mathrm{R}\chi\mathrm{T}} &= 1 + \frac{F_V G_V}{F^2}\frac{q^2}{M_V^2 - q^2}\,, \\
\left[\mathcal{F}^{v}_{\pi\pi}(q^2)\right]^{\mathrm{QCD}} &\to 0, \qquad \text{for } q^2 \to \infty
\end{aligned}
$$

  This leads to

$$
[\mathcal{F}^{v}_{\pi\pi}(q^2)]^{\mathrm{R}\chi\mathrm{T}} = [\mathcal{F}^{v}_{\pi\pi}(q^2)]^{\mathrm{QCD}} \Longrightarrow F_V G_V = F^2
$$

例1： $\tau \rightarrow P + \nu_\tau$

辐射修正项

$$\Gamma(\tau \to H\nu_\tau) = \frac{m_\tau^3 f_H^2 G_F^2 |V_{uD}|^2}{16\pi}\left(1 - \frac{m_H^2}{m_\tau^2}\right)^2 (1 + \delta_{RC}^{(H)})$$

$\delta_{\tau\pi} = (-0.24 \pm 0.56)\%$
$\delta_{\tau K} = (-0.15 \pm 0.57)\%$

[Arroyo-Urena, et al., PRD'21]

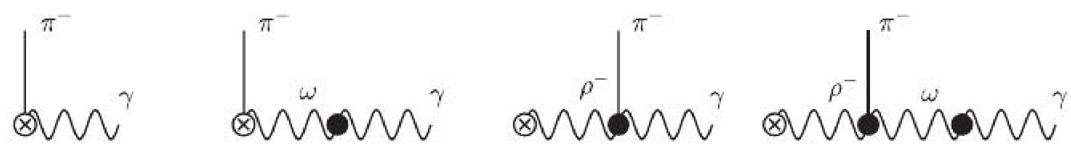
轻子普适性检验：

$$R_{\tau/P} \equiv \frac{\Gamma(\tau \to P\nu_\tau[\gamma])}{\Gamma(P \to \mu\nu_\mu[\gamma])} = \left|\frac{g_\tau}{g_\mu}\right|_P^2 \frac{1}{2}\frac{M_\tau^3}{m_\mu^2 m_P}\frac{(1 - m_P^2/M_\tau^2)^2}{(1 - m_\mu^2/m_P^2)^2}(1 + \delta R_{\tau/P})$$

$\delta R_{\tau/\pi} = (0.18 \pm 0.57)\%$
$\delta R_{\tau/K} = (0.97 \pm 0.58)\%$

Predictions to $\tau \to \pi/K\ \gamma\nu_\tau$ [ZHG,Roig,PRD'10]

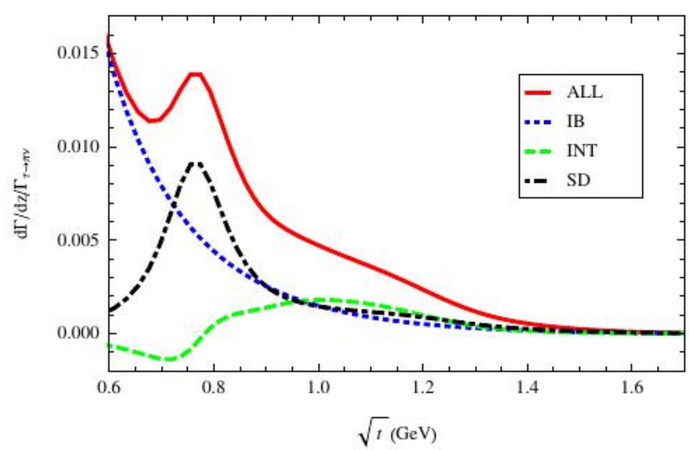
vector currents:



axial-vector currents:

	E_γ=50 MeV	E_γ=400MeV
IB	13.09×10^{-3}	1.48×10^{-3}
$IB - V$	0.02×10^{-3}	0.04×10^{-3}
$IB - A$	0.34×10^{-3}	0.29×10^{-3}
VV	0.99×10^{-3}	0.73×10^{-3}
VA	~ 0	0.02×10^{-3}
AA	0.15×10^{-3}	0.14×10^{-3}
ALL	14.59×10^{-3}	2.70×10^{-3}

in units of $\mathrm{Br}(\Gamma_{\tau\to\pi\nu_\tau}) \sim 11\%$



例2： $\tau \to PP' + \nu_\tau$

$$\langle P_1 P_2 | \bar{D}\gamma^\mu u | 0 \rangle = \left[(p_2 - p_1)^\mu - \frac{\Delta_{P_2 P_1}}{s} q^\mu \right] F_+^{P_1 P_2}(s) + \frac{\Delta_{Du}}{s} q^\mu \widehat{F}_0^{P_1 P_2}(s)$$

(vector FF) (scalar FF)

$$\Delta_{P_2 P_1} = m_{P_2}^2 - m_{P_1}^2 \,, \qquad \Delta_{Du} = B_0(m_D - m_u)\,, \qquad q_\mu = (p_1 + p_2)_\mu, \qquad s = q^2.$$

- **Invariant-mass distribution of P_1P_2**

$$\frac{d\Gamma_{\tau\to P_1P_2\nu_\tau}}{d\sqrt{s}} = \frac{G_F^2 M_\tau^3}{48\pi^3 s} S_{\rm EW} \, |V_{uD}|^2 \left(1 - \frac{s}{M_\tau^2}\right) \left\{ \left(1 + \frac{2s}{M_\tau^2}\right) q_{P_1P_2}^3(s) \left| F_+^{P_1P_2}(s) \right|^2 + \frac{3\Delta_{Du}^2}{4s} q_{P_1P_2}(s) \left| \widehat{F}_0^{P_1P_2}(s) \right|^2 \right\}$$

- **Forward-Backward (FB) asymmetry distribution**

$$A_{FB}(s) = \frac{\int_0^1 d\cos\alpha \frac{d^2\Gamma_{\tau\to P_1P_2\nu_\tau}}{d\sqrt{s}d\cos\alpha} - \int_{-1}^0 d\cos\alpha \frac{d^2\Gamma_{\tau\to P_1P_2\nu_\tau}}{d\sqrt{s}d\cos\alpha}}{\int_0^1 d\cos\alpha \frac{d^2\Gamma_{\tau\to P_1P_2\nu_\tau}}{d\sqrt{s}d\cos\alpha} + \int_{-1}^0 d\cos\alpha \frac{d^2\Gamma_{\tau\to P_1P_2\nu_\tau}}{d\sqrt{s}d\cos\alpha}} = \frac{\Delta_{Du}\, q_{P_1P_2}(s)\, \Re\left[F_+^{P_1P_2}(s)\, \widehat{F}_0^{P_1P_2 *}(s) \right]}{\frac{2\sqrt{s}}{3}\left(1 + \frac{2s}{M_\tau^2}\right) q_{P_1P_2}^2(s) \left| F_+^{P_1P_2}(s) \right|^2 + \frac{\Delta_{Du}^2}{2\sqrt{s}} \left| \widehat{F}_0^{P_1P_2}(s) \right|^2}$$

α： angle between the momenta of P_1 and τ in the P_1P_2 rest frame

$\tau \to \pi^-\pi^0\nu_\tau$： $\Delta_{PP'} \to 0$ (同位旋破坏项)，标量F_0项可忽略，矢量F_+绝对主导!

$\tau \to K\pi\nu_\tau$： $\Delta_{PP'} \neq 0$，标量形状因子F_0项以及矢量形状因子F_+项都有贡献!

Calculation of P_1P_2 form factors

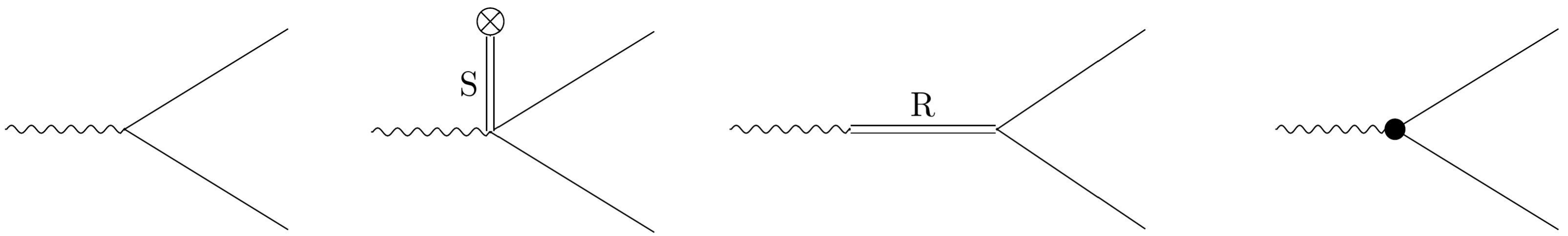
[Hao, Duan, ZHG, 2507.00383]

$$\mathcal{L}^{\rm LO} = \frac{F^2}{4}\langle u_\mu u^\mu\rangle + \frac{F^2}{4}\langle \chi_+\rangle + \frac{F^2}{12}M_0^2 X^2$$

$$\mathcal{L}_V = \frac{F_V}{2\sqrt{2}}\langle V_{\mu\nu} f_+^{\mu\nu}\rangle + \frac{iG_V}{\sqrt{2}}\langle V_{\mu\nu} u^\mu u^\nu\rangle \,,$$

$$\mathcal{L}_\Lambda^{\rm NLO,U(3)} = -\frac{F^2\Lambda_1}{12} D^\mu X D_\mu X - \frac{F^2\Lambda_2}{12} X\langle\chi_-\rangle$$

$$\mathcal{L}_S = c_d\langle S u_\mu u^\mu\rangle + c_m\langle S\chi_+\rangle \,,$$



(a) (b) (c) (d)

(a) (b)

Mixing relations of π^0-η-η'-a (axion)

[Gao,ZHG,Oller,Zhou, JHEP'23]
[Gao,Hao,ZHG,Oller,Zhou, EPJC'25]

$$\begin{pmatrix} \hat\pi^0 \\ \hat\eta \\ \hat\eta' \\ \hat a \end{pmatrix} = \begin{pmatrix} 1+z_{11} & c_\theta(-v_{12}+z_{12}) + s_\theta(-v_{13}+z_{13}) & -s_\theta(-v_{12}+z_{12}) + c_\theta(-v_{13}+z_{13}) & -v_{14}+z_{14} \\ v_{12}+z_{21} & c_\theta(1+z_{22}) + s_\theta(z_{23}-v_{23}) & -s_\theta(1+z_{22}) + c_\theta(z_{23}-v_{23}) & -v_{24}+z_{24} \\ v_{13}+z_{31} & c_\theta(z_{32}+v_{23}) + s_\theta(1+z_{33}) & -s_\theta(z_{32}+v_{23}) + c_\theta(1+z_{33}) & -v_{34}+z_{34} \\ v_{41}+z_{41} & c_\theta(v_{42}+z_{42}) + s_\theta(v_{43}+z_{43}) & -s_\theta(v_{42}+z_{42}) + c_\theta(v_{43}+z_{43}) & 1+v_{44}+z_{44} \end{pmatrix} \begin{pmatrix} \pi^0 \\ \eta_8 \\ \eta_0 \\ a \end{pmatrix}$$

v_{ij}: LO terms

z_{ij}: NLO terms (L_5, L_8, Λ_1, Λ_2) or [(L_5, L_8)~ ($c_d c_m$, $c_m c_m$)/M^2_S , Λ_1, Λ_2]

Some explicit expressions for Form Factors

- **VFF-ππ**

$$F_+^{\pi^-\pi^0}(s) = -\frac{\sqrt{2}}{F^2} G_{\mathrm{LO}+\rho\,\mathrm{Ex}}(s)$$

$$G_{\mathrm{LO}+\rho\,\mathrm{Ex}}(s) = \frac{G_V F_V s + F^2(M_\rho^2 - s)}{M_\rho^2 - s - iM_\rho \Gamma_\rho(s)} - \frac{G_V' F_V' s}{M_{\rho'}^2 - s - iM_{\rho'}\Gamma_{\rho'}(s)} - \frac{G_V'' F_V'' s}{M_{\rho''}^2 - s - iM_{\rho''}\Gamma_{\rho''}(s)} ,$$

$$\Gamma_\rho(s) = \frac{M_\rho s}{96\pi F_\pi^2}\left[\sigma_{\pi\pi}^3(s) + \frac{1}{2}\sigma_{KK}^3(s)\right] , \quad \Gamma_{\rho',\rho''}(s) = \Gamma_{\rho',\rho''} \frac{s}{M_{\rho',\rho''}^2} \frac{\sigma_{\pi\pi}^3(s)}{\sigma_{\pi\pi}^3(M_{\rho',\rho''}^2)} , \quad \sigma_{P_1P_2}(s) = \frac{2q_{P_1P_2}(s)}{\sqrt{s}}\theta[s - (m_{P_1} + m_{P_2})^2]$$

- **VFF-πη/πη'/πa**

$$F_+^{\pi^-\eta}(s) = -\frac{\sqrt{2}v_{12}}{F^2} G_{\mathrm{LO}+\rho\,\mathrm{Ex}}(s) - \sqrt{2}\left(y_{12} - v_{13} y_{23}^{(0)}\right) ,$$

$$F_+^{\pi^-\eta'}(s) = -\frac{\sqrt{2}v_{13}}{F^2} G_{\mathrm{LO}+\rho\,\mathrm{Ex}}(s) - \sqrt{2}\left(y_{13} + v_{12} y_{23}^{(0)}\right) ,$$

$$F_+^{\pi^- a}(s) = -\frac{\sqrt{2}v_{41}}{F^2} G_{\mathrm{LO}+\rho\,\mathrm{Ex}}(s) - \sqrt{2}\left(y_{14} + v_{12}\, y_{24}^{(0)} + v_{13}\, y_{34}^{(0)}\right)$$

- **SFF-πη**

$$F_0^{\pi^-\eta}(s) = \sqrt{\frac{2}{3}}(c_\theta - \sqrt{2}s_\theta) + \frac{1}{\sqrt{3}}(\Lambda_1 - 2\Lambda_2)s_\theta - \frac{1}{\sqrt{3}} y_{23}(2c_\theta + \sqrt{2}s_\theta) + 4\sqrt{\frac{2}{3}}\frac{c_\theta - \sqrt{2}s_\theta}{F^2}\Bigg\{ \left[\frac{c_m(c_m - c_d)2m_\pi^2 + c_m c_d(s + m_\pi^2 - m_\eta^2)}{M_{a_0}^2 - s - iM_{a_0}\Gamma_{a_0}(s)} - \frac{2c_m(c_m - c_d)(2m_K^2 - m_\pi^2)}{M_S^2}\right] + \left[c_{m,d}, M_{a_0}, \Gamma_{a_0}, M_S \to c'_{m,d}, M_{a_0'}, \Gamma_{a_0'}, M_{S'}\right]\Bigg\} ,$$

Fits to experimental spectra and BRs

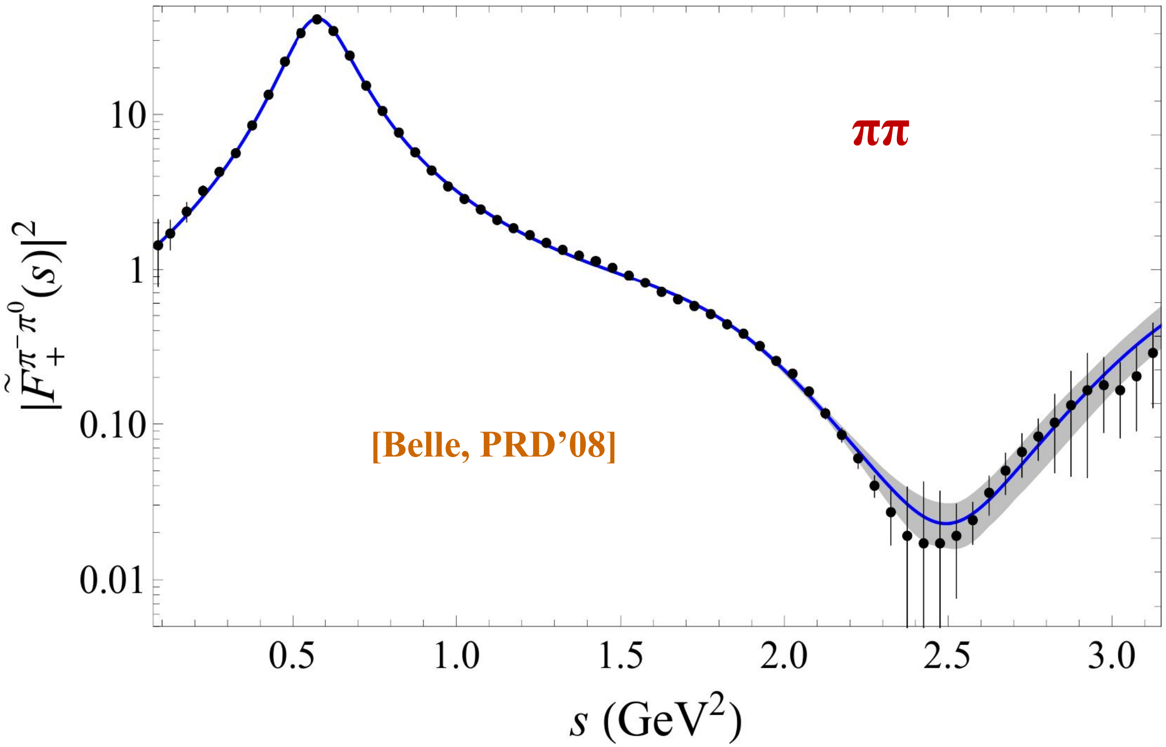
$G_V\, F_V(\mathrm{GeV}^2) \times 10^3$	$10.26^{+0.01}_{-0.01}$	$G'_V\, F'_V(\mathrm{GeV}^2) \times 10^3$	$0.64^{+0.03}_{-0.02}$
$G''_V\, F''_V(\mathrm{GeV}^2) \times 10^3$	$-0.94^{+0.05}_{-0.05}$	$M_\rho(\mathrm{GeV})$	$0.7738^{+0.0003}_{-0.0003}$
$M_{\rho'}(\mathrm{GeV})$	$1.409^{+0.004}_{-0.004}$	$\Gamma_{\rho'}(\mathrm{GeV})$	$0.338^{+0.012}_{-0.010}$
$M_{\rho''}(\mathrm{GeV})$	$1.842^{+0.012}_{-0.013}$	$\Gamma_{\rho''}(\mathrm{GeV})$	$0.268^{+0.025}_{-0.026}$
$c'_m(\mathrm{GeV})$	$0.053^{+0.007}_{-0.009}$	$M_{K^*}(\mathrm{GeV})$	$0.8956^{+0.0002}_{-0.0002}$
$\Gamma_{K^*}(\mathrm{GeV})$	$0.0477^{+0.0005}_{-0.0005}$	$M_{K^{*\prime}}(\mathrm{GeV})$	$1.339^{+0.009}_{-0.009}$
$\overline{B}_{K_S\pi^-} \times 10^3$	$3.98^{+0.04}_{-0.04}$	$\overline{B}_{K^-\eta} \times 10^4$	$1.34^{+0.04}_{-0.04}$
$\chi^2/\mathrm{d.o.f}$	$271.5/(182-14) = 1.61$		

$$c_m c_d + c'_m c'_d = \frac{F^2}{4}$$

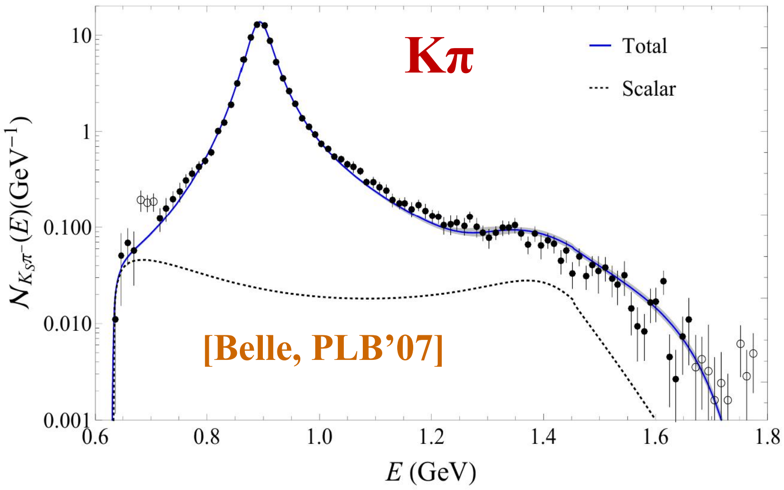
c_m= 27 MeV, c_d=15 MeV

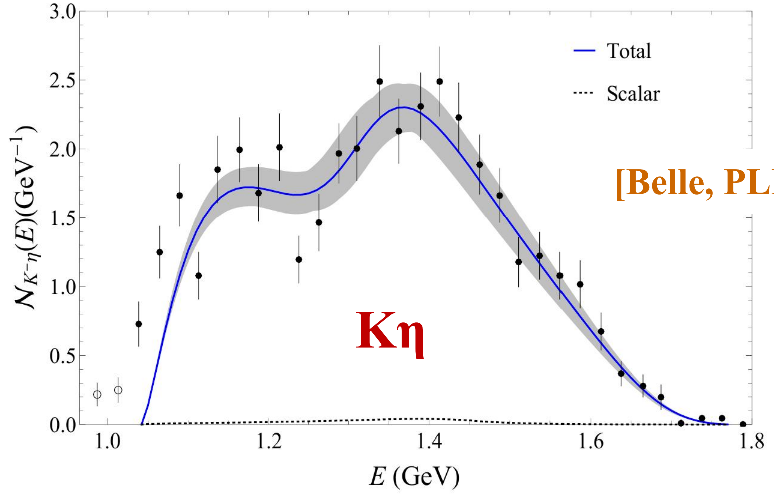
[ZHG,Oller, PRD'11]

Parameters for a_0(980)/a_0(1450)/K_0*(700)//K_0*(1430) are fixed to their pole positions .



- **Crucial inputs to address muon g-2**
- **Most precise spectra is from Belle;**
  but most precise BR is from ALEPH: 25.47(13)%
- **Coherent precise measurements of both spectra and BR from one Exp would be invaluable!**

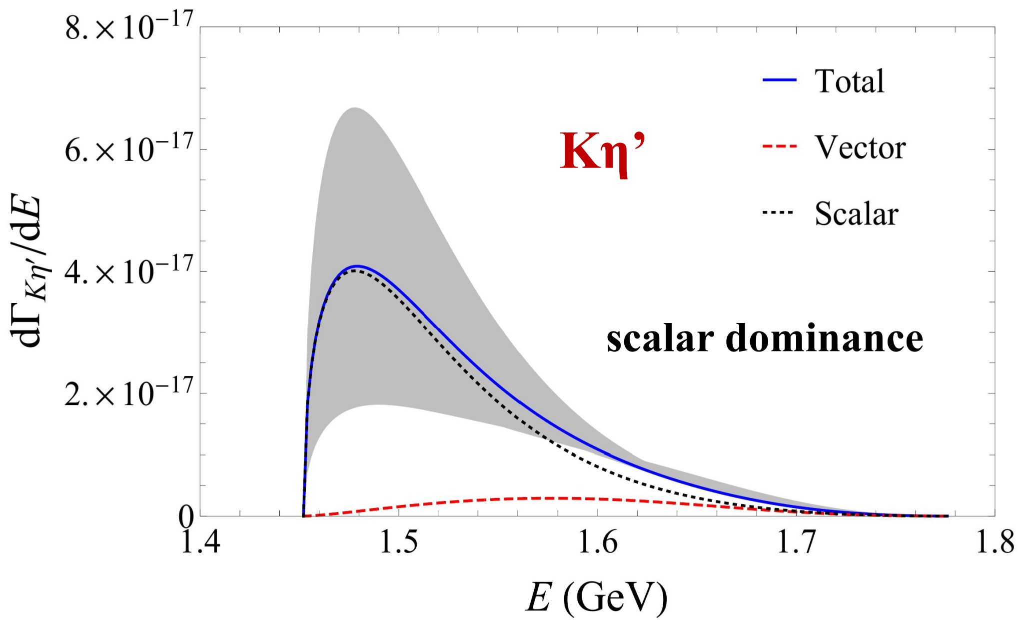




F_+^{KP}: K*, K*(1410)

F_0^{KP}: κ, K^*_0(1430)

prediction to Kη'



$BR(K^- \eta')^{Theo} = (2.0 \pm 1.0) \times 10^{-6}$

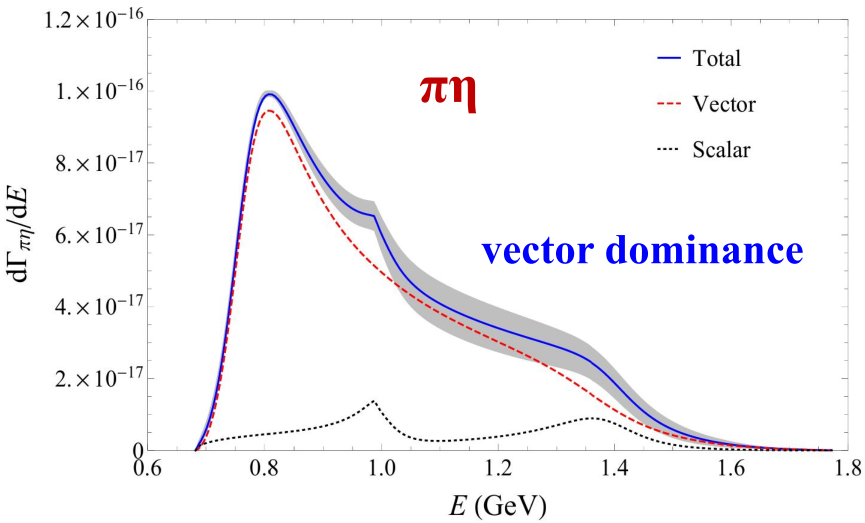
$BR(K^- \eta')^{Exp,\ BaBar} < 2.4 \times 10^{-6}$

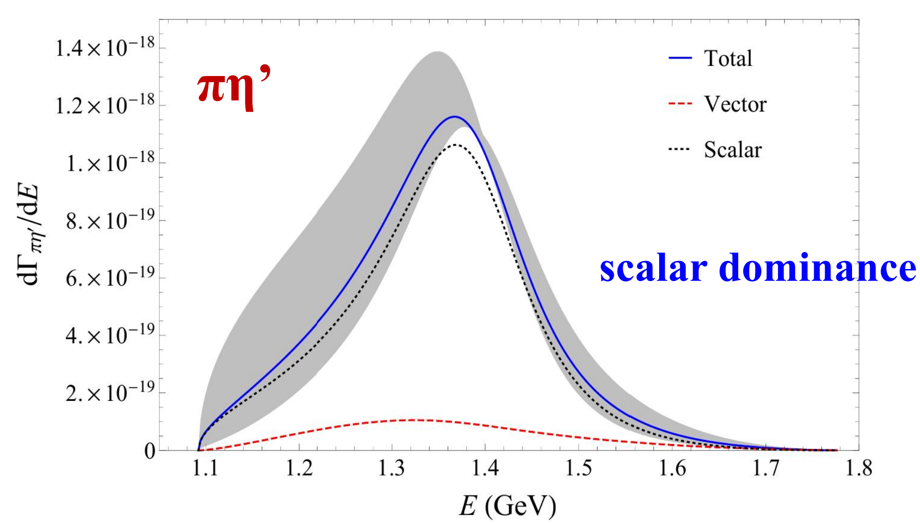
Predictions to $\tau \to \pi^- \eta^{(\prime)} \nu_\tau$ (Cabibbo allowed)：second-class currents

若$\pi^-\eta$ 的J=0，则其P=+1 （V-A型的流不允许）； 若J=1，则P=-1 （矢量流）

但是对于SM来讲，1-矢量流对应的G宇称为正，而$\pi^-\eta$的G宇称为负，表明这是一个破坏G宇称的过程（second-class current），因此可能是寻找新物理的一个有效途径。

$$\langle \pi^- P | \bar{d} \gamma^\mu u | 0 \rangle = \left[(p_P - p_\pi)^\mu - \frac{\Delta_{P\pi}}{s} q^\mu \right] F_+^{\pi^- P}(s) + \frac{\Delta_{du}^{\rm Phy}}{s} q^\mu F_0^{\pi^- P}(s)$$



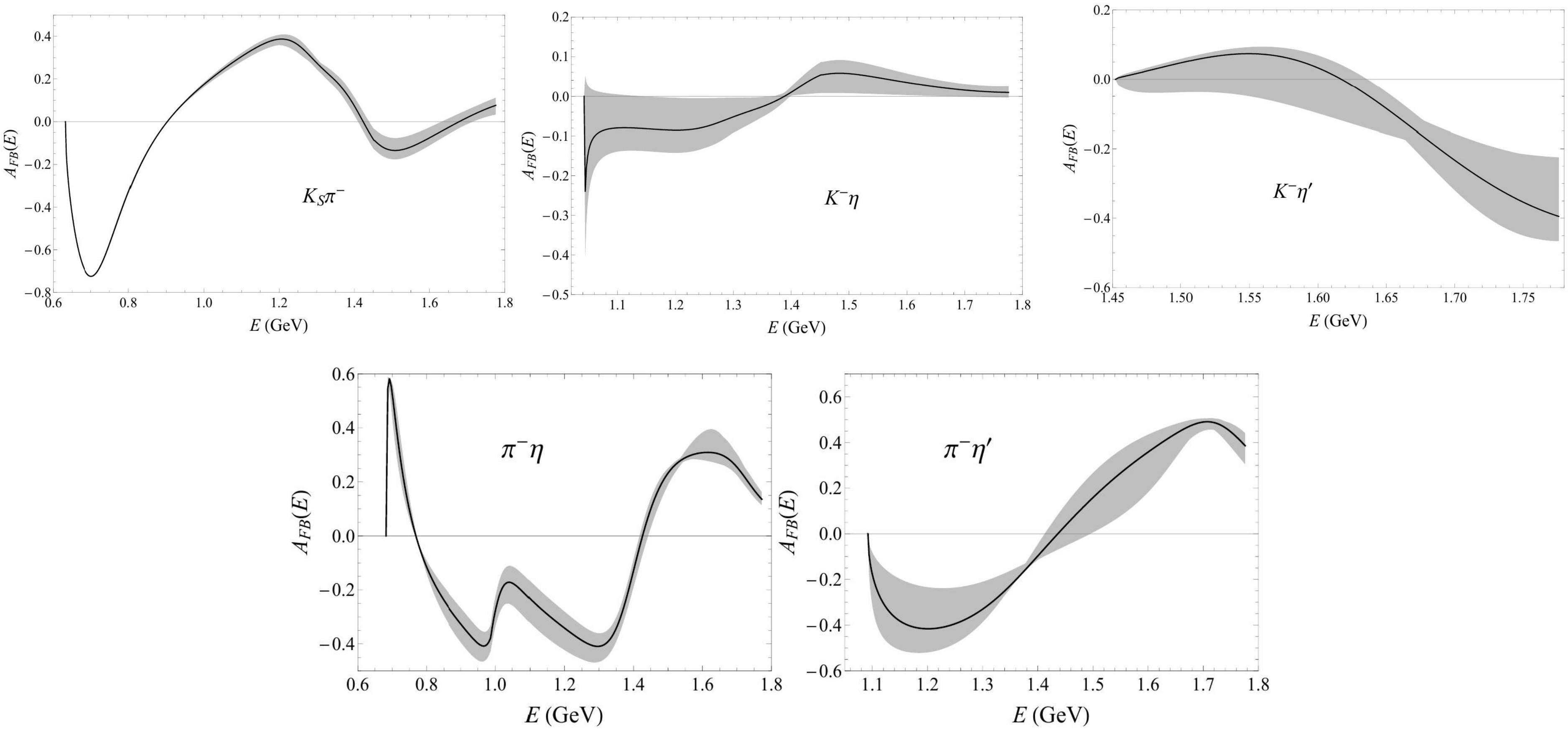


Channel	Total	Vector	Scalar	Exp Limits
$\tau^- \to \pi^- \eta \nu_\tau$ $(\times 10^5)$	$1.63^{+0.14}_{-0.14}$	$1.43^{+0.18}_{-0.21}$	$0.20^{+0.07}_{-0.04}$	< 9.9 (BaBar) [69] < 7.3 (Belle) [70]
$\tau^- \to \pi^- \eta' \nu_\tau$ $(\times 10^7)$	$1.17^{+0.36}_{-0.07}$	$0.14^{+0.09}_{-0.08}$	$1.03^{+0.44}_{-0.16}$	< 40 (BaBar) [71]

[Hao, Duan, ZHG, 2507.00383]

Predictions to Forward-Backward asymmetries

$$\frac{\Delta_{Du}\, q_{P_1P_2}(s)\, \Re\left[F_+^{P_1P_2}(s)\, \widehat{F}_0^{P_1P_2*}(s)\right]}{\frac{2\sqrt{s}}{3}\left(1+\frac{2s}{M_\tau^2}\right) q_{P_1P_2}^2(s)\left|F_+^{P_1P_2}(s)\right|^2 + \frac{\Delta_{Du}^2}{2\sqrt{s}}\left|\widehat{F}_0^{P_1P_2}(s)\right|^2}$$

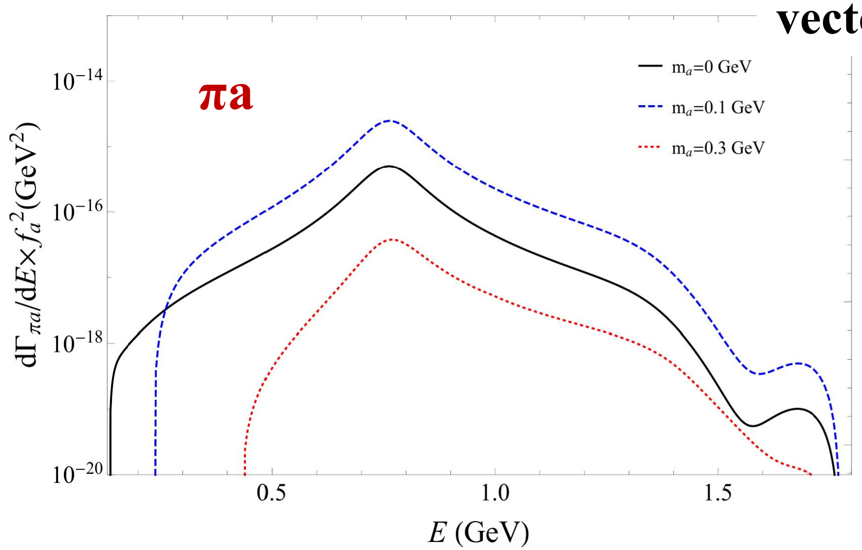


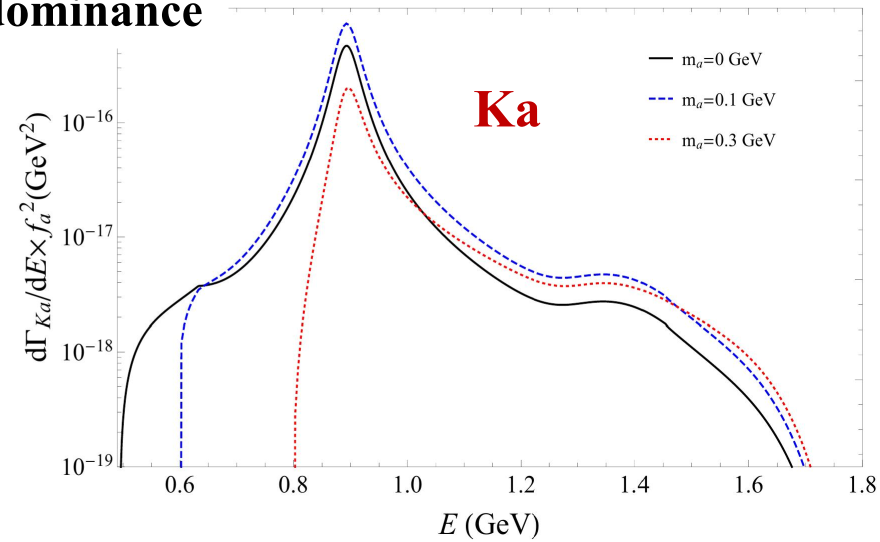
- ➢ **Measurement on A_{FB} can determine the curcial inputs for CPV**

Predictions to axion-meson production in tau decays

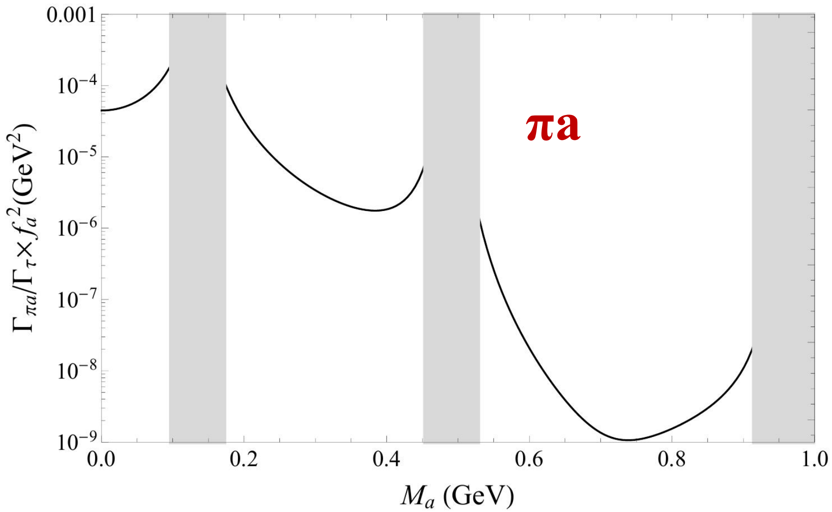
Spectra

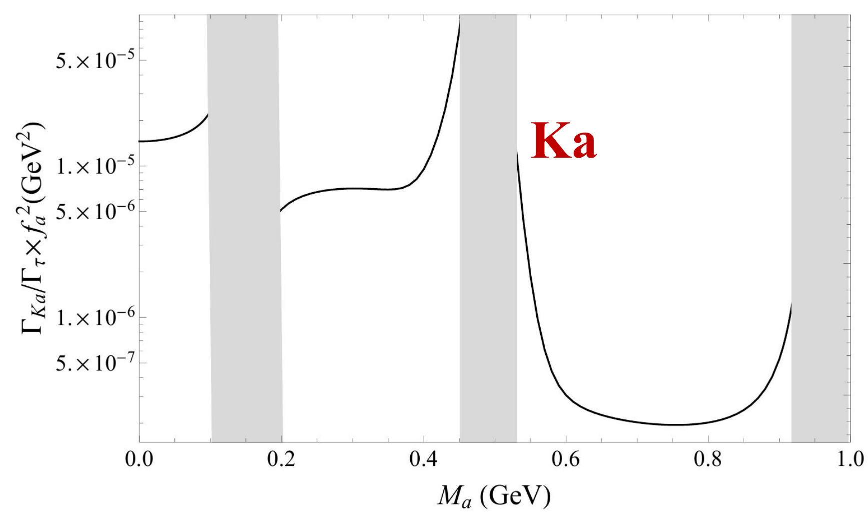
vector dominance





BRs





例3： $\tau \to P_1P_2P_3 + \nu_\tau$

$$\langle P_1^- P_2^- P_3^+ | (V_\mu - A_\mu)\, e^{i\mathrm{L}_{\mathrm{QCD}}}) | \Omega_h \rangle =$$

$$\begin{aligned} Q &= p_1 + p_2 + p_3 \\ s &= (p_2 + p_3)^2 \\ t &= (p_1 + p_3)^2 \end{aligned}$$

$$\left(g_{\mu\nu} - \frac{Q_\mu Q_\nu}{Q^2}\right)\left[F_1(Q^2,s,t)\,(p_1 - p_3)^\nu + F_2(Q^2,s,t)\,(p_2 - p_3)^\nu\right]$$
$$+F_3(Q^2,s,t)\,Q_\mu + iF_4(Q^2,s,t)\,\varepsilon_{\mu\alpha\beta\gamma}\,p_3^\alpha p_2^\beta p_1^\gamma$$

$\tau \to \pi\pi\pi\nu_\tau$

$F_2(Q^2,s,t) = F_1(Q^2,t,s)$

Bose symmetry, Axial-Vector only

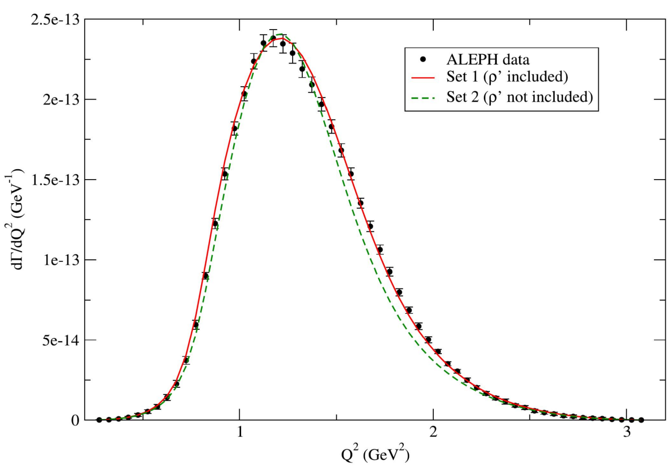
$\tau \to KK\pi\nu_\tau$

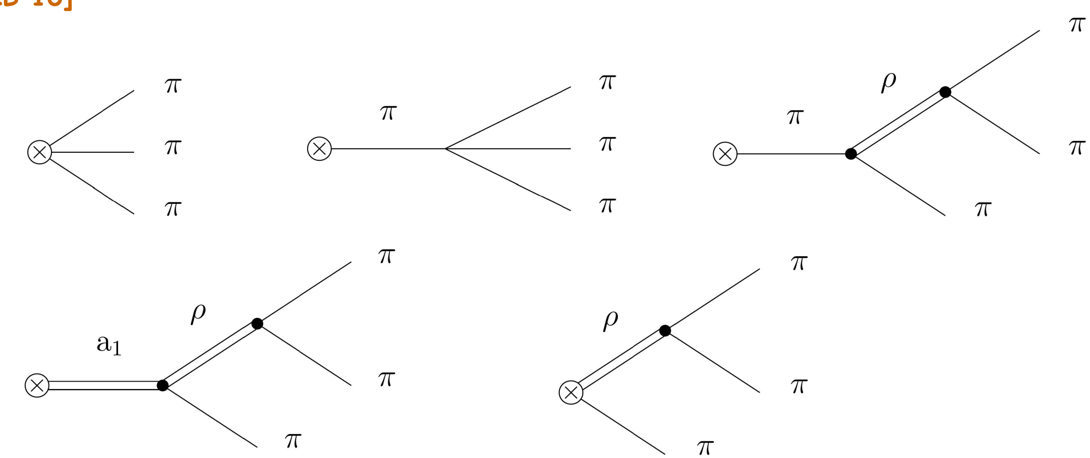
Vector and Axial-Vector

F_3 is suppressed by m_π/Q; F_4 is suppressed by IB.

$\tau^- \to \pi^-\pi^-\pi^+\nu_\tau$

[Gomez Dumm, et al., PLB'10]

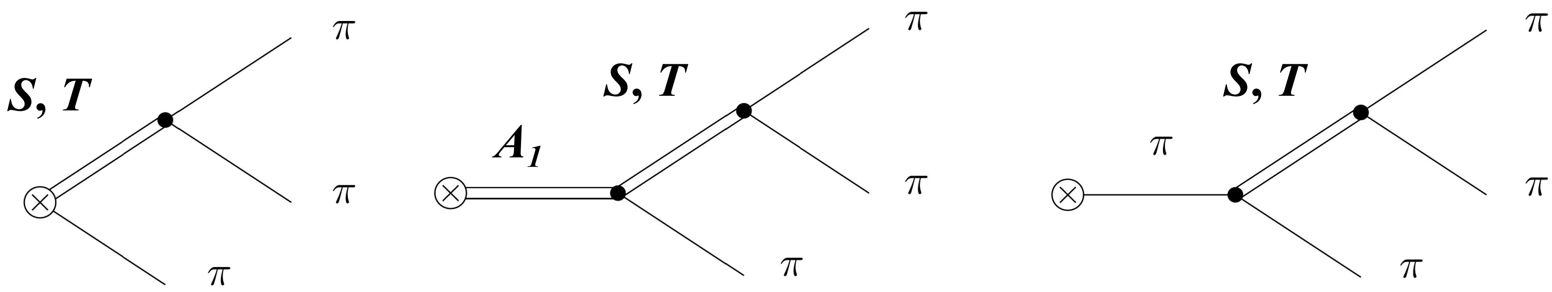




- To describe the ALEPH data, it seems good enough to include vector and axial-vector resonances in chiral EFT.

➢ Additional contributions from scalar and tensor resonances and extra axial-vector resonance ?

[CLEO, PRD'99] [Nugent (BaBar), NPBSP'13] [Nugent, et al., PRD'13] [Sanz-Cillero, et al., JHEP'17] [Rabusov, et al., 2405.19264] ......

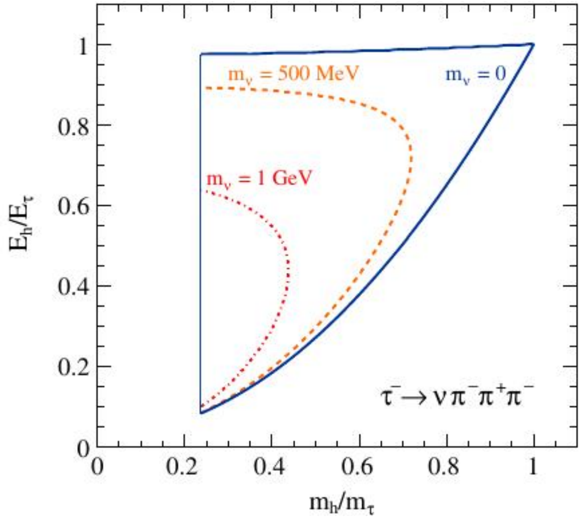


S: $f_0(500)$、 $f_0(980)$、 $f_0(1370)$; ***T:*** $f_2(1270)$; A_1: $a_1(1420)$

- **It is still under debate about the existence of scalar and tensor resonances.**
- **Measurements of the $\pi\pi$ line shapes could be very helpful to address this issue.**
- **Possible to have partial-wave analyses ?**

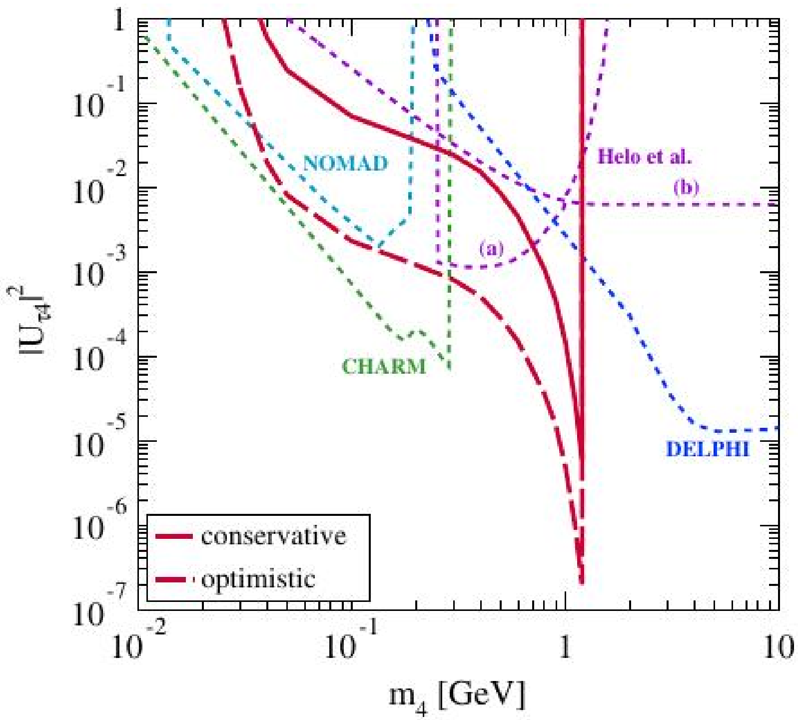
Proposal to search for massive neutrino in tau decay

$\tau \to \pi\pi\pi\nu_4$ [Kobach, Dobbs, PRD'15]

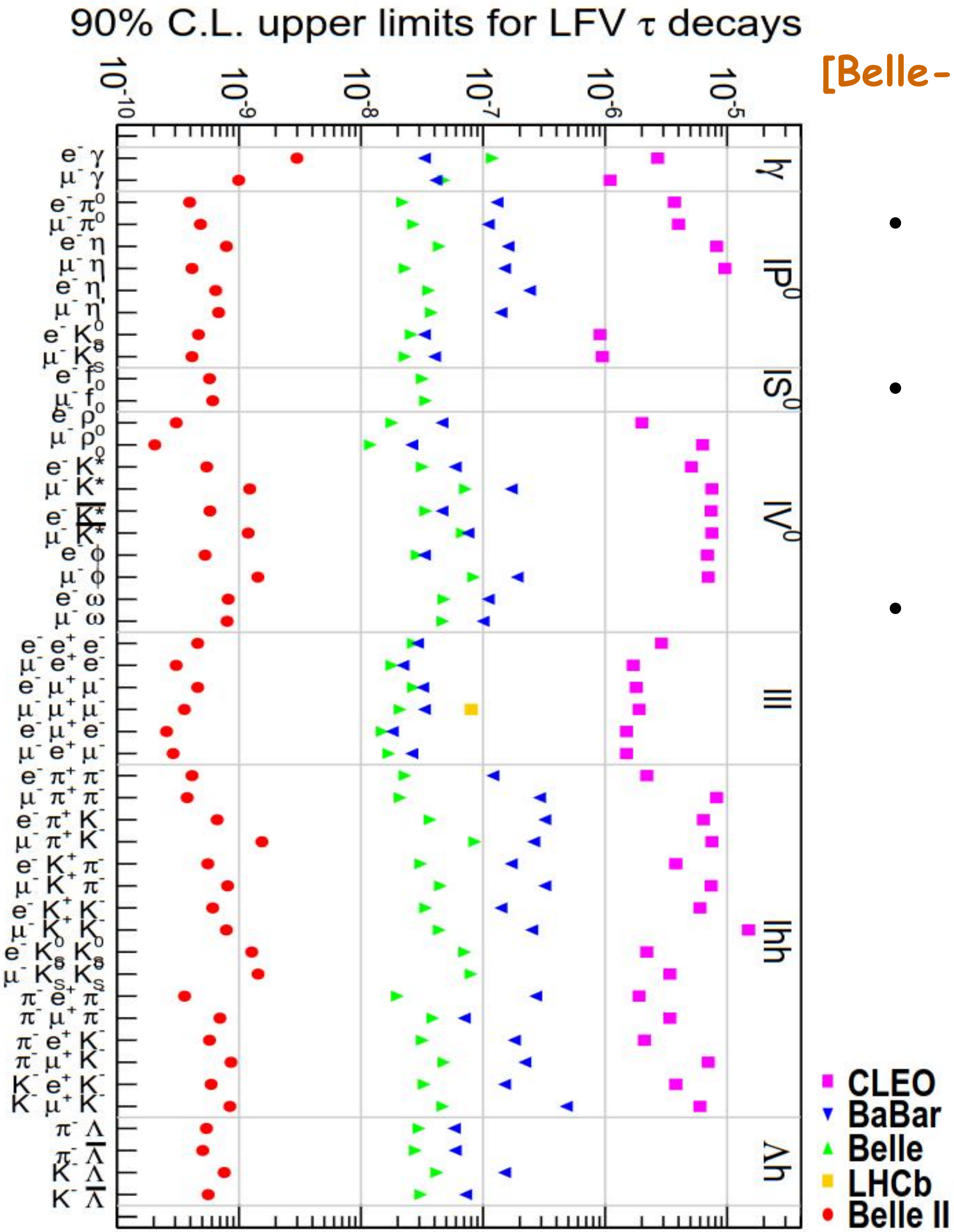


$$\frac{d\Gamma_{\rm tot}(\tau^- \to \nu h^-)}{dm_h dE_h} = (1 - |U_{\tau 4}|^2) \frac{d\Gamma(\tau^- \to \nu h^-)}{dm_h dE_h}\bigg|_{m_\nu=0} + |U_{\tau 4}|^2 \frac{d\Gamma(\tau^- \to \nu h^-)}{dm_h dE_h}\bigg|_{m_\nu=m_4}$$

Strong interaction of the $\pi\pi\pi$ [dominated by $a_1(1260)$] system will greatly affect the final results!



➢ Charged lepton flavor violation in tau decays



[Belle-II, '22]

- Not only statistic but also systematic uncertainties are important in $\tau \to l\,\gamma$
- Clean backgroud makes $\tau \to l\, l'\, l''$ one of the best channels to search for LFV signals.
- $\tau \to l + hadrons$ provides a different laboratory to probe different LFV origins, comparing with the pure leptonic processes.

总结

Tau衰变为粒子物理提供了一个非常广阔的平台:

- ✓ **电弱物理的精确检验:**

  V_{CKM}, 轻子普适性(lepton universality), (g-2)$_{\mu}$, ...

- ✓ **强相互作用相关的物理:**

  强作用基本耦合常数α_s, 强子共振态, 手征对称性，形状因子, ...

- ✓ **可能的新物理现象:**

  轻子味道破坏(LFV), CPV, ...

谢谢大家！