Phase between strong and EM amplitudes in charmonium decays

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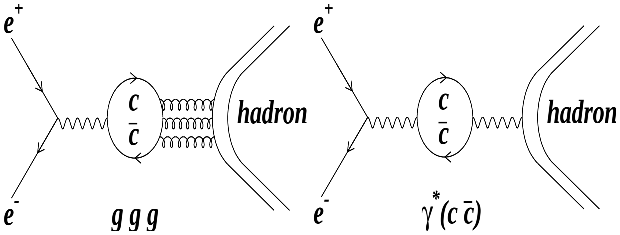
Physics: The phase between strong and EM amplitudes

As we understand today, the charmonium 1^{--} resonances decay into light hadrons through both strong and electromagnetic (EM) interactions. The diagrams are



The phase by SU(3) symmetry

In standard model, the phase between these two ampltudes comes from higher order perturbation. This phase has drawn interest from both theorists as well as experiment people ever since the early days of J/ψ study. It had first been obtained by comparing decay rates of different final states of the same category. For example, the decay amplitudes of charmonium into vector-pseudoscalar final states $(1^{--}0^{-+})$ are decompsed as

$$\begin{aligned} A_{\omega\pi^0} &= 3a_\gamma \;, \\ A_{\rho\pi} &= a_{3g} + a_\gamma \;, \\ A_{K^{*+}K^-} &= a_{3g} + \varepsilon + a_\gamma, \\ A_{K^{*0}\bar{K}^0} &= a_{3g} + \varepsilon - 2a_\gamma, \end{aligned}$$

where a_{3g} is the strong decay amplitude (via gluon), a_γ is the electromagnetic decay amplitude (via photon), ε is introduced as a SU(3) symetry breaking parameter. (Howard E. Haber and Jacques Perrier, Phys.Rev.D32(1985)2961.)

The phase by SU(3) symmetry

And for charmonium decaying into a pair of pseudoscalar mesons

$$
\begin{aligned}
A_{\pi^+\pi^-} &= E, \\
A_{K^+K^-} &= E + S, \\
A_{K_S^0 K_L^0} &= S.
\end{aligned}
$$

where S is the strong decay amplitude (via gluon), E is the electromagnetic decay amplitude (via photon).
Similar for vector-vector final state

The phase obtained by SU(3)

Such analysis has been done for J/ψ in the 1990s when data were accumulated at J/ψ by BES. It reaches the conclustion

- 1^+0^- 90° M. Suzuki, Phys. Rev. **D63**, 054021 (2001)
- 1^-0^- $(106 \pm 10)^\circ$ J. Jousset *et al.*, Phys. Rev. **D41**, 1389 (1990); D. Coffman *et al.*, Phys. Rev. **D38**, 2695 (1988); J. Jousset *et al.*, *Phys. Rev.* D **41**, 1389 (1990); A. Bramon, R. Escribano and M. D. Scadron, *Phys. Lett.* B **403**, 339 (1997); M. Suzuki, *Phys. Rev.* D **58**, 111504 (1998); N.N.Achasov, Talk at Hadron2001; G. López Castro *et al.*, in CAM-94, Cancum, Mexico.
- 1^-1^- $(138 \pm 37)^\circ$ L. Köpke and N. Wermes, Phys. Rep. **174**, 67 (1989).
- 0^-0^- $(89.6 \pm 9.9)^\circ$ M. Suzuki, Phys. Rev. **D60**, 051501(1999); G. López Castro *et al.*, ibid; L. Köpke and N. Wermes, ibid.
- $N\bar{N}$ $(89 \pm 15)^\circ$ R. Baldini, *et al.* Phys. Lett.**B444**, 111 (1998); G. López Castro *et al.*, ibid.

The phase obtained by SU(3)

In 2001, Mahiko Suzuki summarized the experimental situation of the two-body J/ψ decays. (Phys. Rev. **D63**, 054021(2001))

He stated:

The existing data strongly favor large relative phase close to 90° between the gluon and the photon decay amplitudes for 1^-0^-, 0^-0^-, 1^-1^- and $N\bar{N}$, and are consistent with a large phase for 1^+0^-.

He then reached the conclusion:

The relative phase between the gluon and the photon decay amplitudes are universally large for all two-body decays of J/ψ.

The phase obtained by SU(3)

But we know that SU(3) symmetry is far from exact. For example, CLEOc reported the measured cross section at 3.67GeV and $\psi(3770)$ peak CLEO collaboration, G.S.adams et al, Phys.Rev.D73:012002,2006

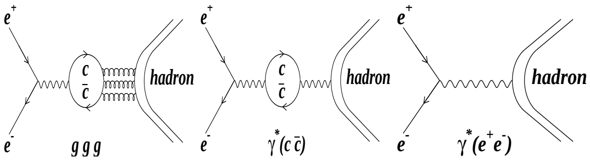
Channel	σ(3.67 GeV) [pb]	σ(3.77 GeV) [pb]
	VP	
$\rho^+\pi^-,\ \rho^-\pi^0,\ \rho^-\pi^+$	$8.0^{+1.7}_{-1.4}\pm 0.9$	$4.4\pm 0.3\pm 0.5$
$\omega\pi^0$	$15.2^{+2.8}_{-2.4}\pm 1.5$	$14.6\pm 0.6\pm 1.5$
$K^{*0}\overline{K^0},\ \overline{K}^{*0}K^0$	$23.5^{+4.6}_{-3.9}\pm 3.1$	$23.5\pm 1.1\pm 3.1$
$K^{*+}K^-,\ K^{*-}K^+$	$1.0^{+1.1}_{-0.7}\pm 1.8$	< 0.6

We see that the ratios of the cross sections between $K^{*0}\overline{K^0},\ \overline{K}^{*0}K^0$ and $K^{*+}K^-,\ K^{*-}K^+$ as well as between $\omega\pi^0$ and $\rho^+\pi^-,\ \rho^-\pi^0,\ \rho^-\pi^+$ deviate from SU(3) symmetry (factor 4 and 3 respectively) significantly.

So the analysis of this phase by SU(3) symmetry is questionable.

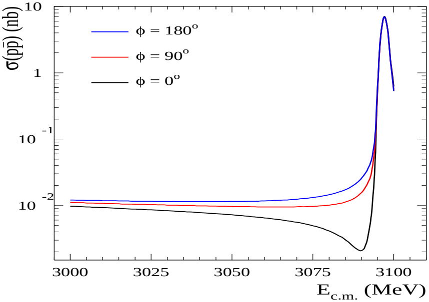
The amplitude in e^+e^- collision

BESIII measures this phase by scanning the charmonium resonances which is model independent. Here we have three diagrams



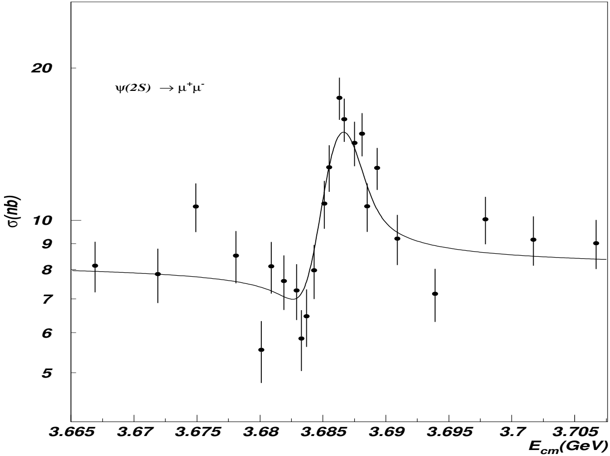
The amplitude in e^+e^- collision

With different phase between strong and electromagnetic amplitudes, the interference pattern would make the cross section across the resonance different. For example, $p\bar{p}$



The amplitude in e^+e^- collision

A pure electromagnetic process is $e^+e^- \to \mu^+\mu^-$ which is shown in the vicinity of $\psi(2S)$ (BES)



The amplitude in e^+e^- collision

For hadronic final state which go only through electromagnetic interactions, such as $\omega\pi^0$ and $\pi^+\pi^-$, the cross section is proportinal to $\mu^+\mu^-$, by a form factor. The form factor varies slowly on s relative to the width of the narrow resoance.

The amplitude in e^+e^- collision

In QED, the $e^+e^- \to \mu^+\mu^-$, near a quarkonium $J^{PC} = 1^{--}$ state is described by replacing the photon propagator by

$$\frac{1}{s} \to \frac{1}{s}\left(1 + \frac{3s\Gamma_{ee}/(M\alpha)}{s - M^2 + iM\Gamma_t}\right)$$

F.A.Berends and R. Gastmans in “Radiative Corrections in e^+e^- Collisions” in “Electromagnetic Interactions of Hadrons”, editted by A.Donnachie and G.Shaw published by Springer Science+Bussiness Media 1978 page 505.

This is the amplitude of $e^+e^- \to \psi$ through electromagnetic interaction by QED.

with

$$\frac{3\Gamma_{ee}s/M\alpha}{s - M^2 + iM\Gamma_\psi}$$

due to the EM decay of the resonance.

The amplitude in e^+e^- collision

Next, parametrize the strong decay amplitude. It is beyond QED. Let us assume that it has a phase relative to the electromagnetic decay amplitude, as well as a magnitude which we may conveniently define relative to the electromagnetic amplitude too.
i.e. we parametrize the strong decay amplitude in the way

$$a_{3g}(s) = \mathscr{C} e^{i\phi} a_\gamma(s).$$

This parametrization is transparent. Put together, the contributions of all the three diagrams to the amplitude are written as

$$\mathscr{A} = \frac{1}{s}[1 + \frac{3\Gamma_{ee} M/\alpha}{s - M^2 + iM\Gamma_\psi}(1 + \mathscr{C} e^{i\phi})]$$

Here the first term is due to the virtual photon, in the last bracket, the first term is due to electromagnetic decay, the second is due to strong decay.

The amplitude in e^+e^- collision

Notice that here in the amplitude

$$\frac{1}{s}[1+\frac{3s\Gamma_{ee}/(M\alpha)}{s-M^2+iM\Gamma_t}(1+\mathscr{C}e^{i\phi})]$$

On top of the resonance, $s=M^2$, so the electromagnetic decay amplitude

$$\frac{3\Gamma_{ee}M/\alpha}{s-M^2+iM\Gamma_t}$$

is pure imaginary. If the strong decay ampltude

$$a_{3g}(s)=\mathscr{C}e^{i\phi}a_\gamma(s)$$

has a phase of $\phi=\pm\pi/2$ relative to electromagnetic, then the phase between strong decay amplitude and virtual photon amplitude is either 0 or 180 degree, which leads to constructive or destructive interference between strong decay amplitude and virtual photon amplitude.

The amplitude in e^+e^- collision

The quark have positive or negative charge; On the other hand, the strong interaction is flavor-blind. So the interference between strong amplitude and continuum may flip sign for different final states, depending on their quark content. For example, among $J^{PC} = 1^{--} \to 1^{--}0^{-+}$ decays,

$$\begin{aligned}
A_{\omega\pi^0} &= 3(a_\gamma + a_c) \ , \\
A_{\rho\pi} &= a_{3g} + a_\gamma + a_c \ , \\
A_{K^{*+}K^-} &= a_{3g} + \varepsilon + a_\gamma + a_c \ , \\
A_{K^{*0}\bar{K}^0} &= a_{3g} + \varepsilon - 2(a_\gamma + a_c) \ , \\
A_{\omega\eta} &= X_\eta(a_{3g} + a_\gamma + a_c) \ , \\
A_{\phi\eta} &= Y_\eta[a_{3g} - 2(a_\gamma + a_c)].
\end{aligned}$$

here a_{3g} is strong decay amplitude; a_γ is the electromagnetic decay amplitude of the resonance; a_c is the non-resonant continuum amplitude; ε is a SU(3) breaking parameter; X_η and Y_η are the mixing angle between η_1 and η_8.

If in J/ψ decay the interference is destructive for $\rho\pi$, then it should also be so for $K^{*+}K^-$ and $\omega\eta$, and it must be constructive for $K^{*0}\bar{K}^0$ and $\phi\eta$. Such interference pattern has been observed at $\psi(3770)$.

The amplitude in e^+e^- collision

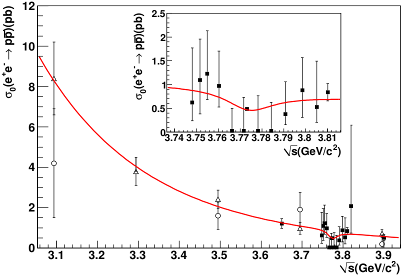
CLEO measured the cross section on and off $\psi(3770)$ states for some exclusive light hadron states.

TABLE I. The number of events N in the mass signal windows ("sw") and sidebands ("sb") in data taken at $\sqrt{s} = 3.671$ GeV and $\sqrt{s} = 3.773$ GeV data; the efficiency ϵ in percent; the level of consistency or significance, expressed in units of standard deviations, between continuum background and observed yield, for the two methods of determining the continuum background described in the text, S^{I} and S^{II}; the cross sections at $\sqrt{s} = 3.671$ GeV and $\sqrt{s} = 3.773$ GeV; the cross section $\psi(3770) \to h_1 h_2$, computed as the excess over the continuum prediction as established using Method I or Method II (see text).

Channel	$N_{\mathrm{sw}}^{3.67}$	$N_{\mathrm{sb}}^{3.67}$	$N_{\mathrm{sw}}^{3.77}$	$N_{\mathrm{sb}}^{3.77}$	ϵ	S^{I}	S^{II}	$\sigma^{3.67\ \mathrm{GeV}}$ [pb]	$\sigma^{3.77\ \mathrm{GeV}}$ [pb]	$\sigma^{\mathrm{I}}_{\psi(3770)}$ [pb]	$\sigma^{\mathrm{II}}_{\psi(3770)}$ [pb]
$\pi^+\pi^-\pi^0$	74	6.8	576	72.3	29.0	-2.7		$13.1^{+1.9}_{-1.7} \pm 2.1$	$7.4 \pm 0.4 \pm 1.2$	<0.04	
$\rho\pi$	43	5.4	314	44.8	26.3	-2.2	-1.7	$8.0^{+1.7}_{-1.4} \pm 0.9$	$4.4 \pm 0.3 \pm 0.5$	<0.04	<0.04
$\rho^0\pi^0$	21	3.4	130	33.0	32.5	-2.2	-2.1	$3.1^{+1.0}_{-0.8} \pm 0.4$	$1.3 \pm 0.2 \pm 0.2$	<0.03	<0.03
$\rho^+\pi^-$	22	2.0	184	11.8	23.1	-0.9	-0.5	$4.8^{+1.5}_{-1.2} \pm 0.5$	$3.2 \pm 0.3 \pm 0.3$	<0.05	<0.05
$\omega\pi^0$	54	6.2	696	39.2	19.0	0.9	-0.2	$15.2^{+2.8}_{-2.4} \pm 1.5$	$14.6 \pm 0.6 \pm 1.5$	<4.5	<0.06
$\phi\pi^0$	1	1.6	2	4.0	16.5	0.0	-0.0	<2.2	<0.2	<0.2	<0.2
$\rho\eta$	36	3.1	508	31.0	19.6	1.1	0.7	$10.0^{+2.2}_{-1.9} \pm 1.0$	$10.3 \pm 0.5 \pm 1.0$	<4.0	<1.3
$\omega\eta$	4	0.0	15	6.0	9.9	-1.7	-2.9	$2.3^{+1.8}_{-1.0} \pm 0.5$	$0.4 \pm 0.2 \pm 0.1$	<0.1	<0.1
$\phi\eta$	5	1.0	132	15.9	11.0	2.5	≥ 5	$2.1^{+1.9}_{-1.2} \pm 0.2$	$4.5 \pm 0.5 \pm 0.5$	<4.5	<3.3
$\rho\eta'$	1	0.0	27	0.9	2.9	1.0	-1.3	$2.1^{+4.7}_{-1.6} \pm 0.2$	$3.8^{+0.9}_{-0.8} \pm 0.4$	<4.7	<0.4
$\omega\eta'$	0	0.0	2	0.0	1.5	≥ 5	0.0	<17.1	$0.6^{+0.8}_{-0.3} \pm 0.6$	<3.0	<1.9
$\phi\eta'$	0	0.0	9	2.0	1.2	2.4	1.2	<12.6	$2.5^{+1.5}_{-1.1} \pm 0.4$	<5.2	<3.8
$K^{*0}\overline{K^0}$	38	0.4	501	18.1	8.8	1.1	≥ 5	$23.5^{+4.6}_{-3.9} \pm 3.1$	$23.5 \pm 1.1 \pm 3.1$	<9.0	<20.8
$K^{*+}K^-$	4	1.0	36	32.4	16.0	-1.4	-4.1	$1.0^{+1.1}_{-0.7} \pm 0.5$	<0.6	<0.1	<0.1
$b_1\pi$	20	4.5	268	100.3	11.3	-0.1		$7.9^{+3.1}_{-2.5} \pm 1.8$	$6.3 \pm 0.7 \pm 1.5$	<0.1	
$b_1^0\pi^0$	5	3.0	49	82.5	4.2	-1.2		<17.1	<2.5	<0.4	
$b_1^+\pi^-$	15	1.5	219	17.8	18.4	1.0		$4.2^{+1.6}_{-1.2} \pm 0.6$	$4.7 \pm 0.4 \pm 0.6$	<2.7	

The amplitude in e^+e^- collision

A beautiful example is the $p\bar{p}$ cross section across the $\psi(3770)$ resonance measured by BESIII.



The fitting yields two solutions, with the phase $\phi = (255.8 \pm 37.9 \pm 4.8)^\circ$ and $\phi = (266.9 \pm 6.1 \pm 0.9)^\circ$. $B_{\psi(3770)\to p\bar{p}}$ is determined to be $(7.1 \pm 4.2) \times 10^{-6}$ or $(3.1 \pm 0.3) \times 10^{-4}$.
(Phys.Lett. B735 (2014) 101-107)

The phase and BR by e^+e^- experiment

These interference pattern tells us, in e^+e^- colliding experiments, the branching ratios can not be measured by solely the data collected on top of the resonance. The branching ratios must be determened simultaneously together with the phase between strong and EM interactions.

For such measurements, we need at least 3 energy points across the resonance peak, in order to fit three parameters : the form factor, the strong decay amplitude and the phase between strong and electromagnetic.
(P.Wang, X.H.Mo and C.Z.Yuan, Int. J. Mod. Phys.A 21(2006)5163.)

In general, two solutions are obtained from the fitting.

The phase and BR by e^+e^- experiment

For exmaple, for VP final state

$$\sigma_{Born}(s) = \frac{4\pi\alpha^2}{3s^{3/2}}|\mathscr{F}(s)|^2 p_{VP}^3 |1 + \frac{3s\Gamma_{ee}/(M\alpha)}{s - M^2 + iM\Gamma_t}(1 + \mathscr{C}e^{i\phi})|^2$$

where $\mathscr{F}(s)$ is the VP form factor which carries dimension $1/M$ and p_{VP} is the modulus of the three momentum of the hadron V (or P). In the total cross section of $e^+e^- \to VP$

$$\frac{4\pi\alpha^2}{3s^{3/2}}|\mathscr{F}(s)|^2 p_{VP}^3 |\frac{3s\Gamma_{ee}/(M\alpha)}{s - M^2 + iM\Gamma_t}(1 + \mathscr{C}e^{i\phi})|^2$$

is the resonant part.

The phase and BR by e^+e^- experiment

The resonant part at $s = M^2$ divided by the total cross section of $e^+e^- \rightarrow resonance$

$$\sigma_{tot} = \frac{12\pi\Gamma_{ee}\Gamma_{tot}}{(s-M^2)^2 + M^2\Gamma_{tot}^2}$$

at $s = M^2$ gives the branching ratio

$$(\frac{p_{VP}^3}{M})|\mathscr{F}(s)|^2 B_{ee} M^2 |1 + \mathscr{C} e^{i\phi}|^2$$

The form factors are enegy dependent.
Fortunately our scan usually covers a limited energy range, in which the form factor varies slowly as s varies. We may approximate it by some power of s, like C/s for mesons, and C/s^2 for baryons (please consult Rinaldo Baldini), and then add in systematic uncertainties.

The phase and BR by e^+e^- experiment

Notice that for different final state, the dimension of the form factor and the expression of the cross section could be somewhat different. (see N.N.Achasov and A.A. Kozhevnikov, hep-ph/9801308)
For example, the cross section of K^+K^- would be

$$\sigma_{Born}(s) = \frac{8\pi\alpha^2}{3s^{5/2}}|\mathscr{F}(s)|^2 p_{K^+}^3 |1 + \frac{3s\Gamma_{ee}/(M\alpha)}{s - M^2 + iM\Gamma_t}(1 + |\mathscr{C}|e^{i\phi})|^2$$

Here the form factor $\mathscr{F}(s)$ is dimensionless and the branching ratio is

$$2(\frac{p_{K^+}}{M})^3 |\mathscr{F}(s)|^2 B_{ee} |1 + |\mathscr{C}|e^{i\phi}|^2$$

The phase and BR by e^+e^- experiment

In case of J/ψ, the branching ratio can be obtained independently using $\psi(2S)$ data. If such branching ratio is input into the fitting, then a unique solution of the phase can be obtained. This is what Francesca has done in her analysis of $J/\psi \to K^+K^-$.

So in our measurement of the phase through scan of J/ψ resonance, the J/ψ branching ratio obtained from $\psi(2S)$ data can serve both as a check to our scan data analysis, as well as selection from the two solutions. One of the two solutions must coincide with the measured branching ratio. If not so, we must check where we go wrong in the analysis of the scan data.

I suggest that charmonium group redo all the J/ψ branching ratios by $\psi(2S)$ data, and the results are used in our phase measurement by scan data.

Further question: OZI allowed process

So far, the study of the phase between strong and elecromagnetic interactions has been only on the strong decays which are OZI suppressed processes. In QCD these are described by three gluon annihilation.

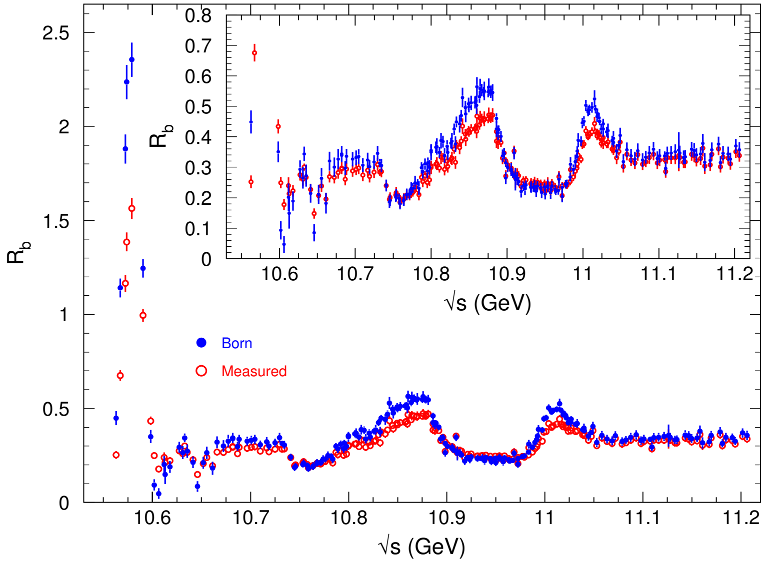
What happens to those strong decays which are not OZI suppressed?

A few years ago, C.Z.Yuan supervised a UCAS undergraduate student, who reanalyzed the BaBar and belle data of $e^+e^- \to b\bar{b}$, with proper treatment of radiative correction and vacuum polarization and obtained Born cross section as well as dressed cross section.

Xiang-Kun Dong, Xiaohu Mo, Ping Wang and Changzhneg Yuan, Chinese Physics C Vol 44, No 8, (2020) 083001

Among the structure of Born order cross section of $e^+e^- \to b\bar{b}$ they found a dip at 10.75GeV;

Further question: OZI allowed process



Further question: OZI allowed process

On the other hand, by the study of $e^+e^- \to \pi^+\pi^-\Upsilon$ process, belle had revealed a state $\Upsilon(10750)$ at exactly the same position of this dip in $e^+e^- \to b\bar{b}$ cross section.

This indicates that the dip may probably due to the destructive interference between Υ resonance and the continuum amplitude, as what we have seen in $\psi(3770)$, e.g. $\psi(3770) \to p\bar{p}$.

While previously the phase (between strong and EM interactions) has been found to be large and negative in OZI suppressed strong decays, but $\Upsilon \to b\bar{b}$ is not suppresed by OZI rule. That is to say, the large negative phase close to -90° between strong and electromagnetic decay ampliutdes may hold even for the strong processes which does not break OZI rule. Or this phase does not depend on the three gluon picture of strong interaction. It could be more general.

If it was the case, it could be more important.

Discussions

In 2001, Mahiko Suzuki summarized the experimental situation of the two-body J/ψ decays. (Phys. Rev. **D63**, 054021(2001))

He stated:

The relative phase between the gluon and the photon decay amplitudes are universally large for all two-body decays of J/ψ.

He made the commnents

Despite lack of a good theoretical argument at present, we suspect nevertheless that the universal large phase so far found are not an accident.

These were written more than 25 years ago.

Discussions

A quarter of century has passed since Suzuki wrote these words. No progress on the theory; on the other hand, now we have known that this large phase is also true in some of $\psi(3770)$ decays (BESIII on $p\bar{p}$); and we are measuring this phase in $\psi(2S)$ decays. Also there is some (although vague) indication that it could hold for the proceses which are not OZI suppressed. With these new evidence now we take Suzuki's comments more seriously that the universal large (and negative) phase so far found are not an accident.

The meaning of our study of this phase is that there is lack of a good theoretical argument.

This is a phenomena we found in experimental data which we do not expect and we do not understand. The discovery of such phenomena is the role of experment in physics. Only at next step, can we base our knowledge on mathematical form. Such interplay between experiments and theory is how physics has been developed.

Is the large phase really universal? And is this phase a contant? If it is a constant, is this constant -90°? This is to be answered by us in experiments. Our results will eventually lead to a theory in which this large phase comes in a natural way.