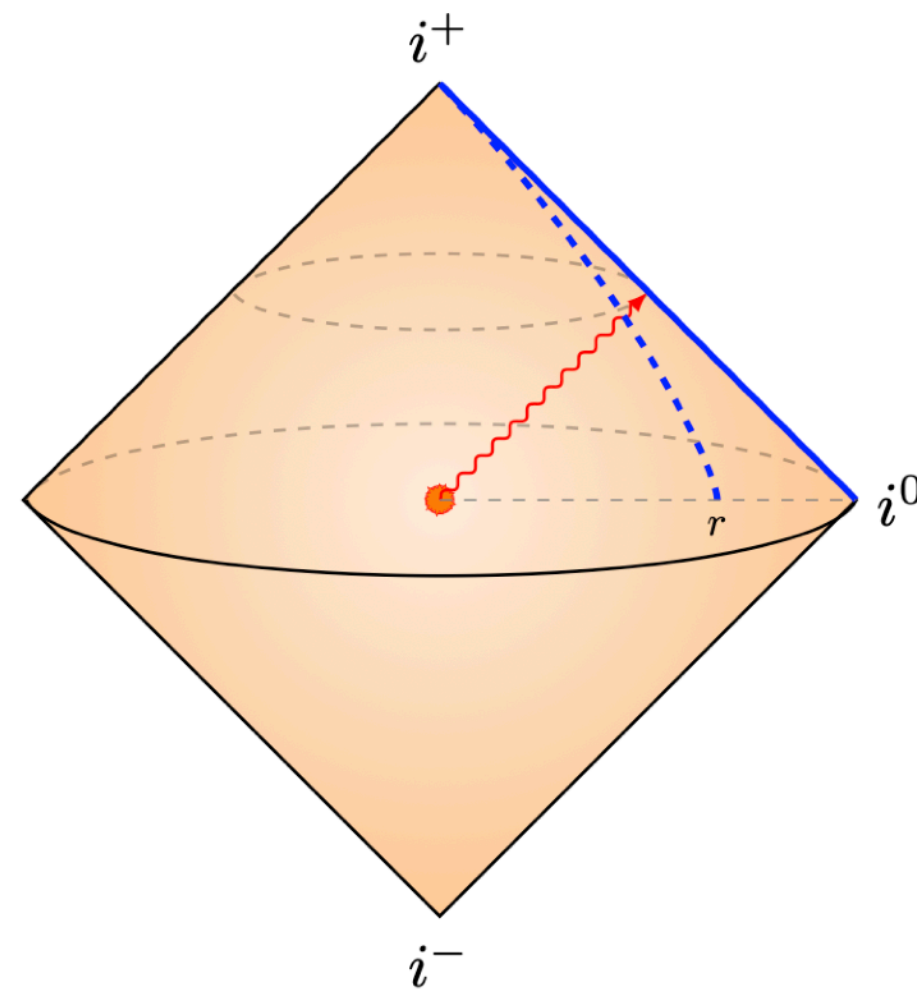
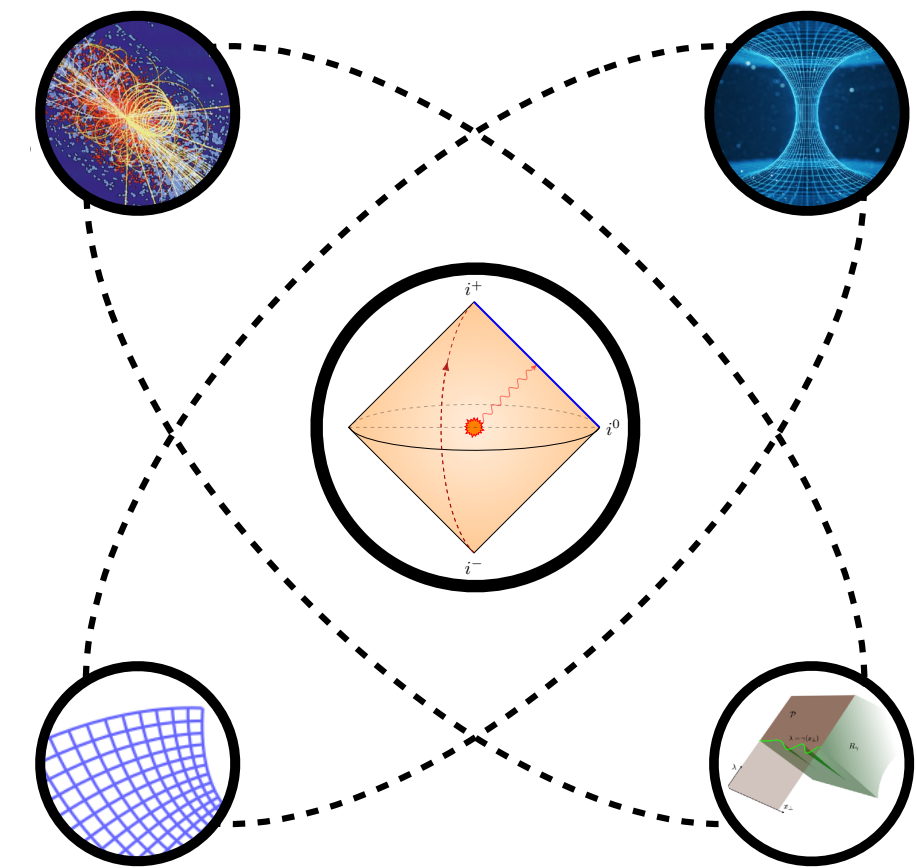


Energy Correlators: A Journey From Theory to Experiment



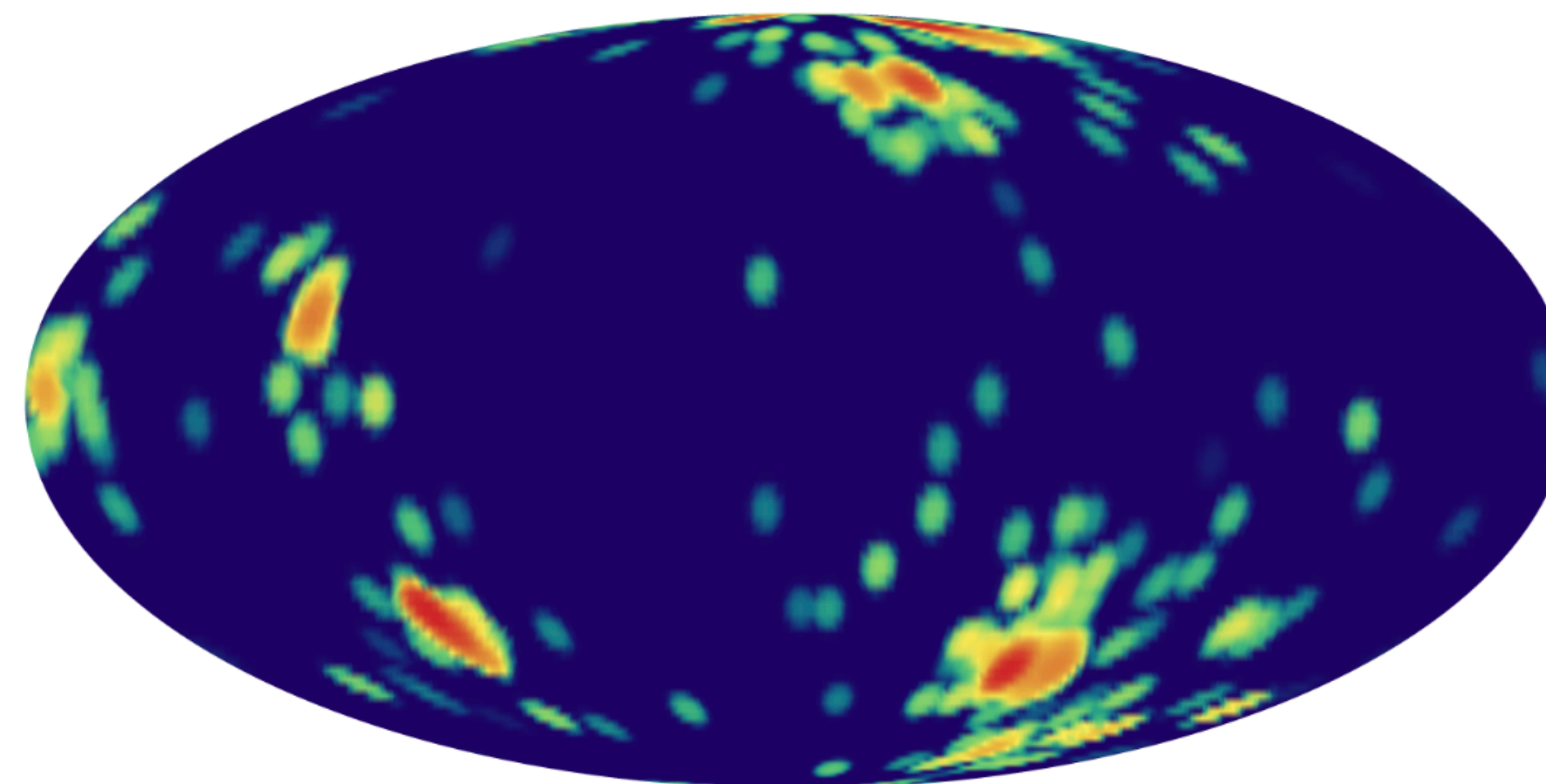
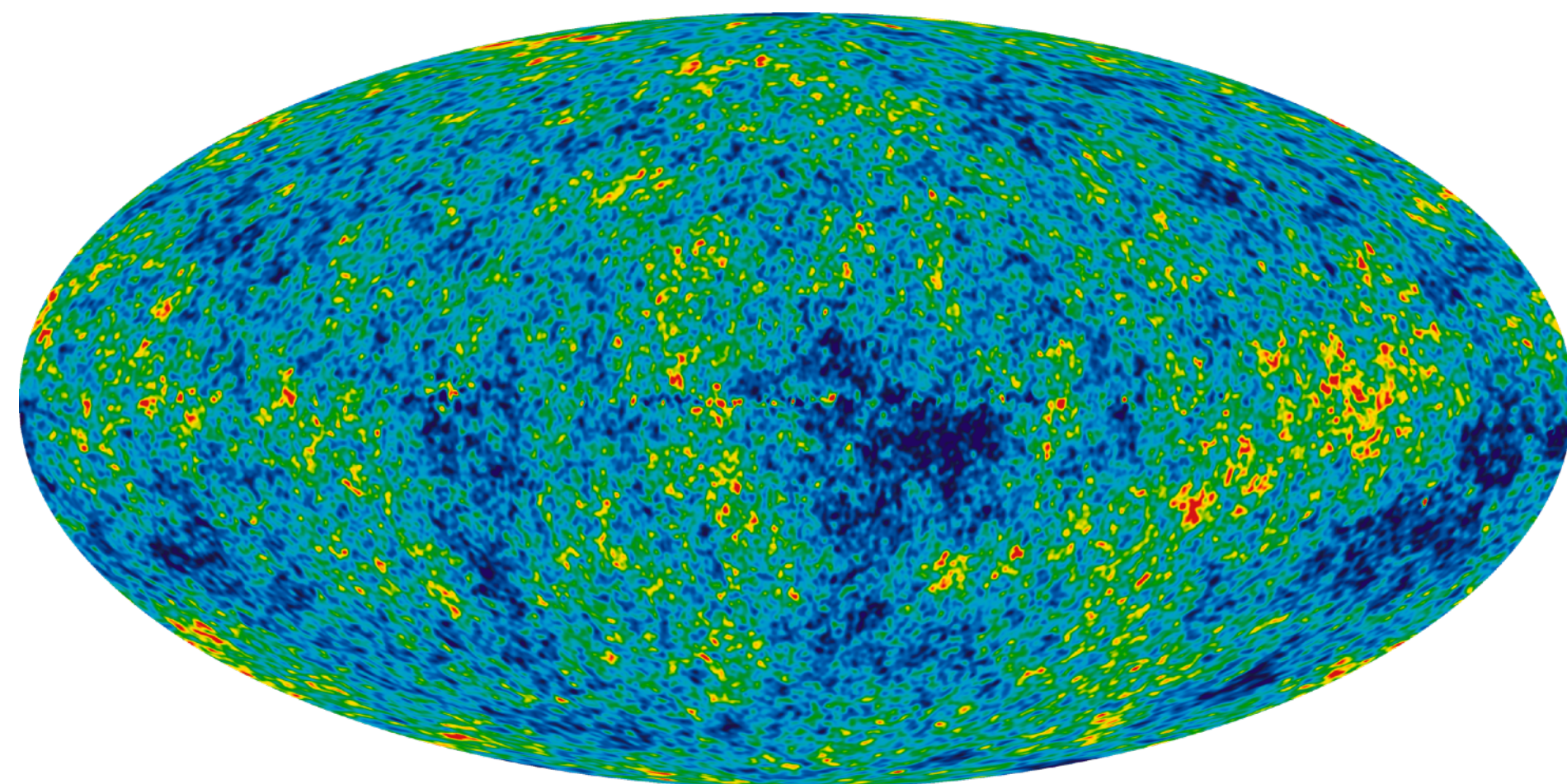
HuaXing Zhu
Peking University

Based on a review article with Ian Moutl



味物理讲座-第81期

April 24, 2025



After a particle collision, the underlying microscopic physics gets imprinted into detailed correlations in macroscopic fluxes, much in analogy to how our cosmic history is imprinted into correlations in the Cosmic Microwave Background.

Understanding how to map correlations in macroscopic fluxes to properties of the underlying quantum field theory (QFT) is therefore key to addressing a wide variety of questions in particle and nuclear physics.

Outline

- 50 years of energy flux theories of energy operators
 - QCD $\rightarrow$ CFT $\rightarrow$ QCD
- Energy operators in particle physics
- Energy operators in nuclear physics

The birth of energy operator

Jet Structure in e^+e^- Annihilation
with Massless Hadrons*

George Sterman

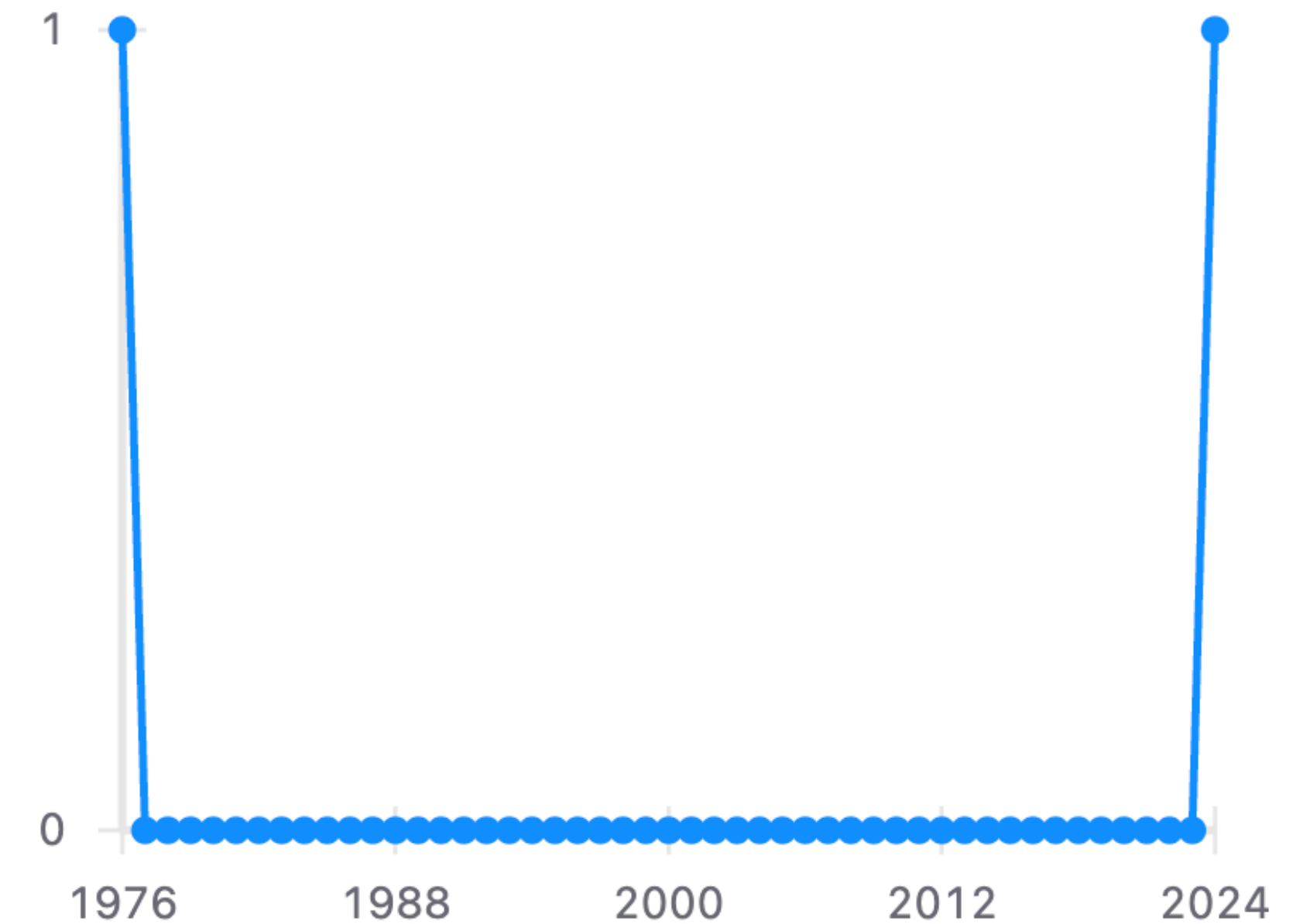
Department of Physics
University of Illinois at Urbana-Champaign
Urbana, Illinois 61801



Abstract

The cancellation of mass singularities in e^+e^- annihilation processes in the presence of directly coupled massless particles has been investigated. Ensembles of states have been identified, the summation over which produces transition probabilities which are free of mass singularities when the masses of one or more species of hadrons vanish. States included in these ensembles can be characterized as having a "jet-like" structure. Application to annihilation processes at high energies in massive theories is discussed.

Citations per year



$$\mathcal{E}(\hat{n})|X\rangle = \sum_{i \in X} k_i^0 \delta^2(\Omega_n - \Omega_k) |X\rangle$$

Correlation of energy operators

Energy Correlations in Electron-Positron Annihilation: Testing Quantum Chromodynamics

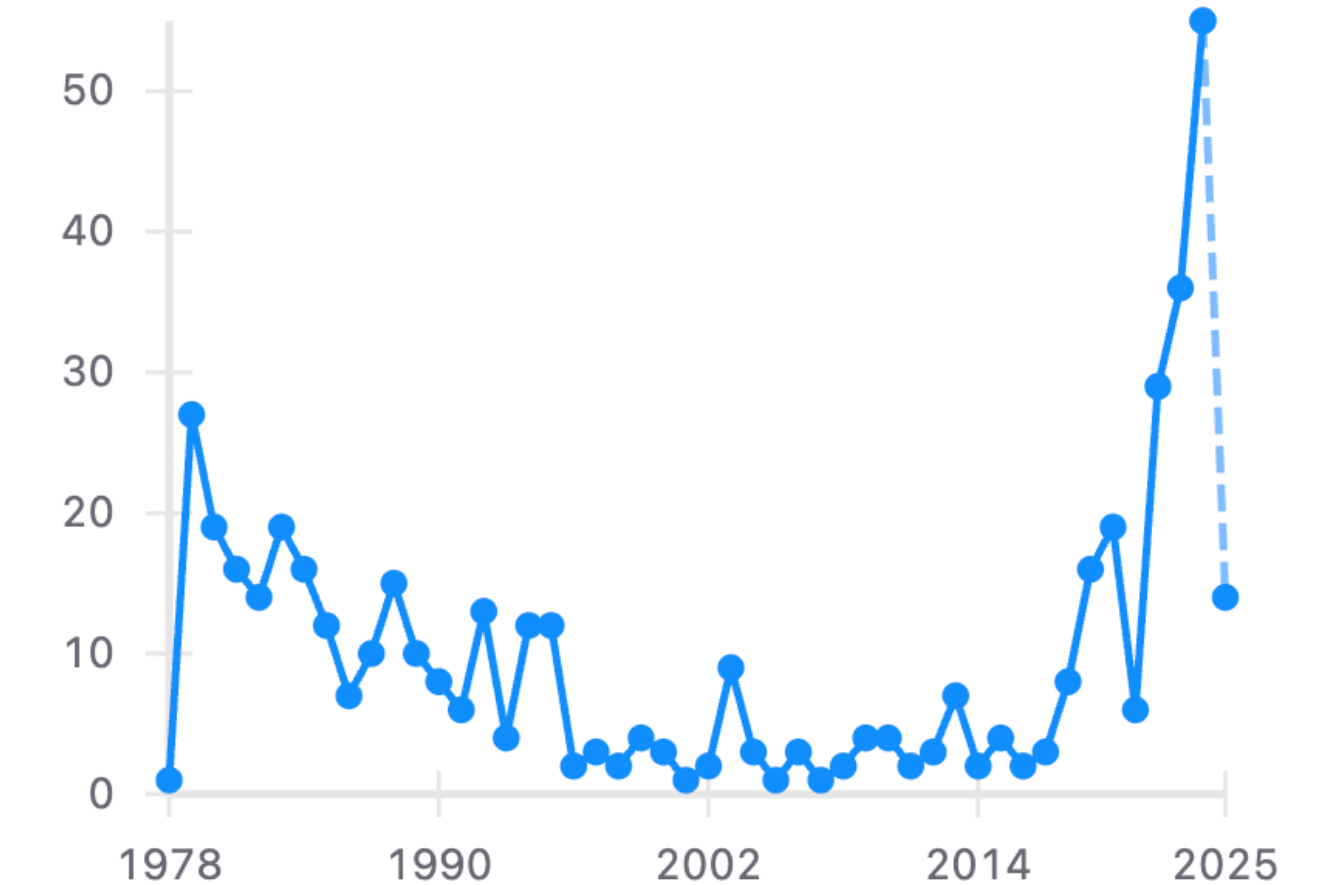
C. Louis Basham, Lowell S. Brown, Stephen D. Ellis, and Sherwin T. Love

Department of Physics, University of Washington, Seattle, Washington 98195

(Received 21 August 1978)

An experimental measure is presented for a precise test of quantum chromodynamics. This measure involves the asymmetry in the energy-weighted opening angles of the jets of hadrons produced in the process $e^+e^- \rightarrow \text{hadrons}$ at energy W . It is special for several reasons: It is reliably calculable in asymptotically free perturbation theory; it has rapidly vanishing (order $1/W^2$) corrections due to nonperturbative confinement effects; and it is straightforward to determine experimentally.

Citations per year



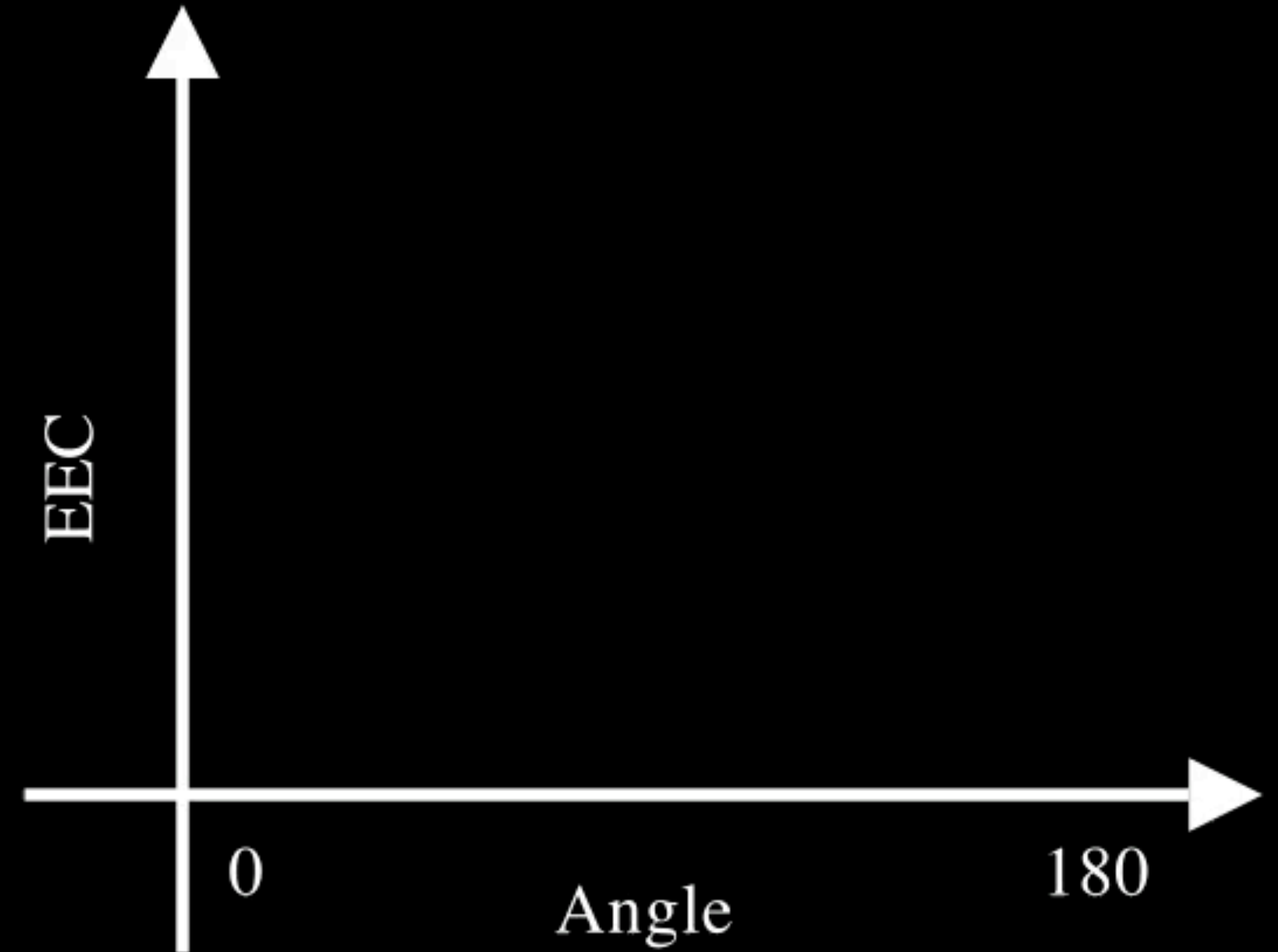
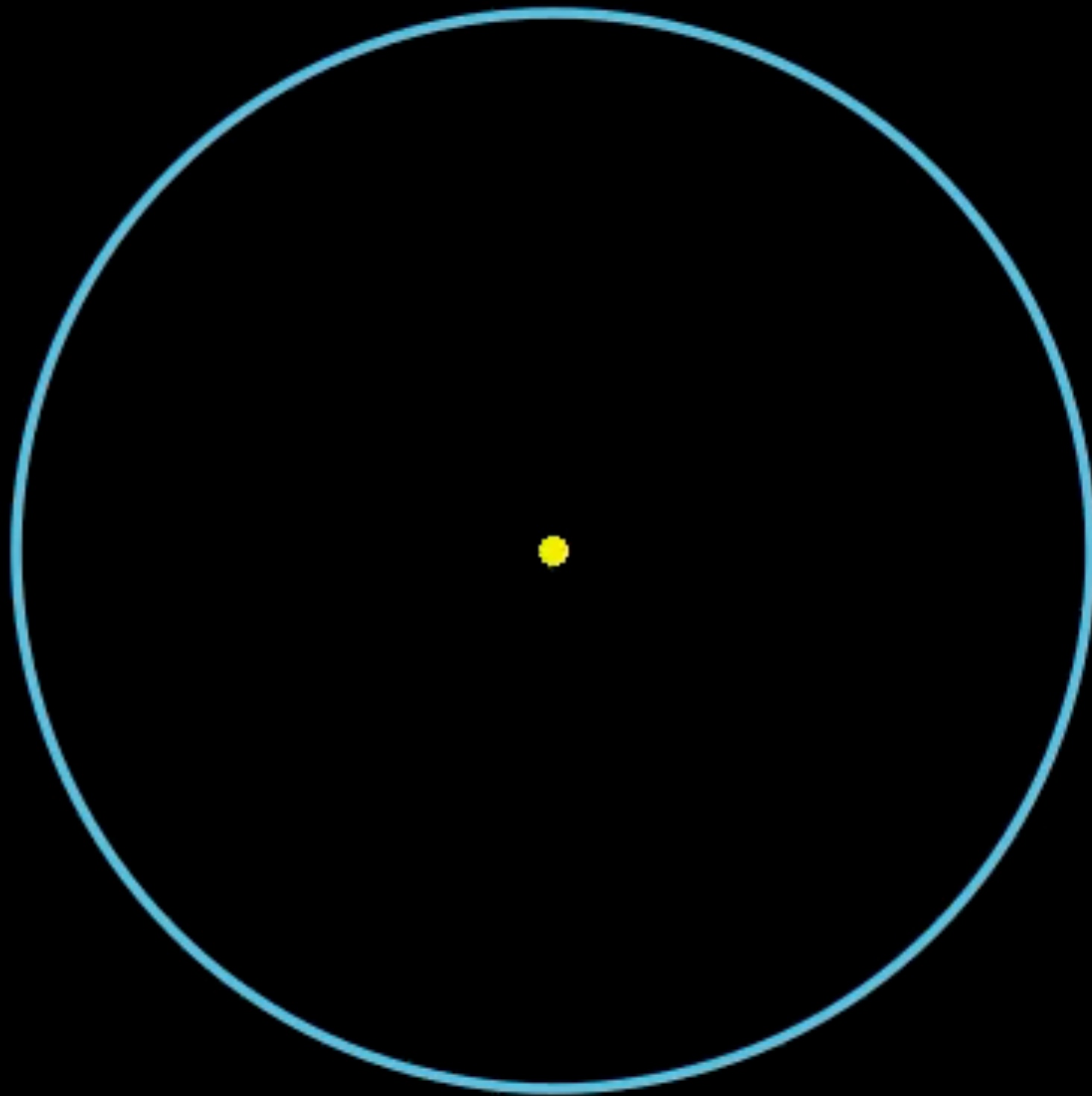
$$\sigma_0 = \int d^4x e^{iQ \cdot x} \langle 0 | J(x) J(0) | 0 \rangle \quad \text{Appelquist, Georgi, 1973}$$

$$\int d^4x e^{iQ \cdot x} \langle 0 | J(x) \mathcal{E}(\hat{n}_1) \mathcal{E}(\hat{n}_2) \cdots \mathcal{E}(\hat{n}_k) J(0) | 0 \rangle$$

Basham, Brown, Ellis, Love, 1978

What is Energy-Energy Correlator?

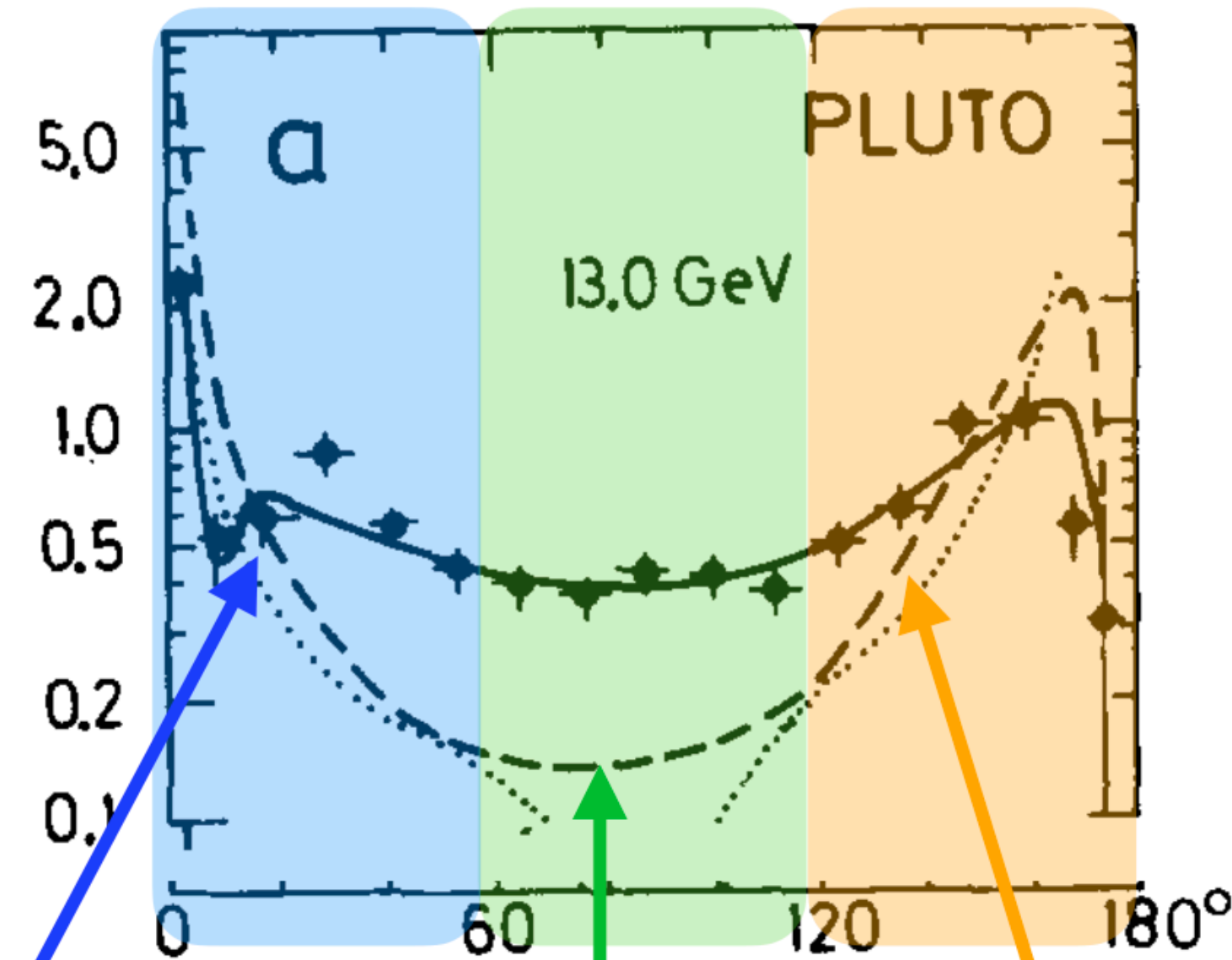
Basham, Brown, Ellis, Love, 1978



Credit: Hua Xing Zhu

$$\text{EEC}(\cos \theta) = \sum_{a,b} \frac{1}{\sigma_{\text{tot}}} \int d\sigma_{e^+e^- \rightarrow abX'} \frac{E_a E_b}{Q^2} \delta(\cos \theta - \hat{\mathbf{p}}_a \cdot \hat{\mathbf{p}}_b)$$

Experimentalists do not know what and how to extract from it



Collinear:

Konishi, Ukawa, Veneziano

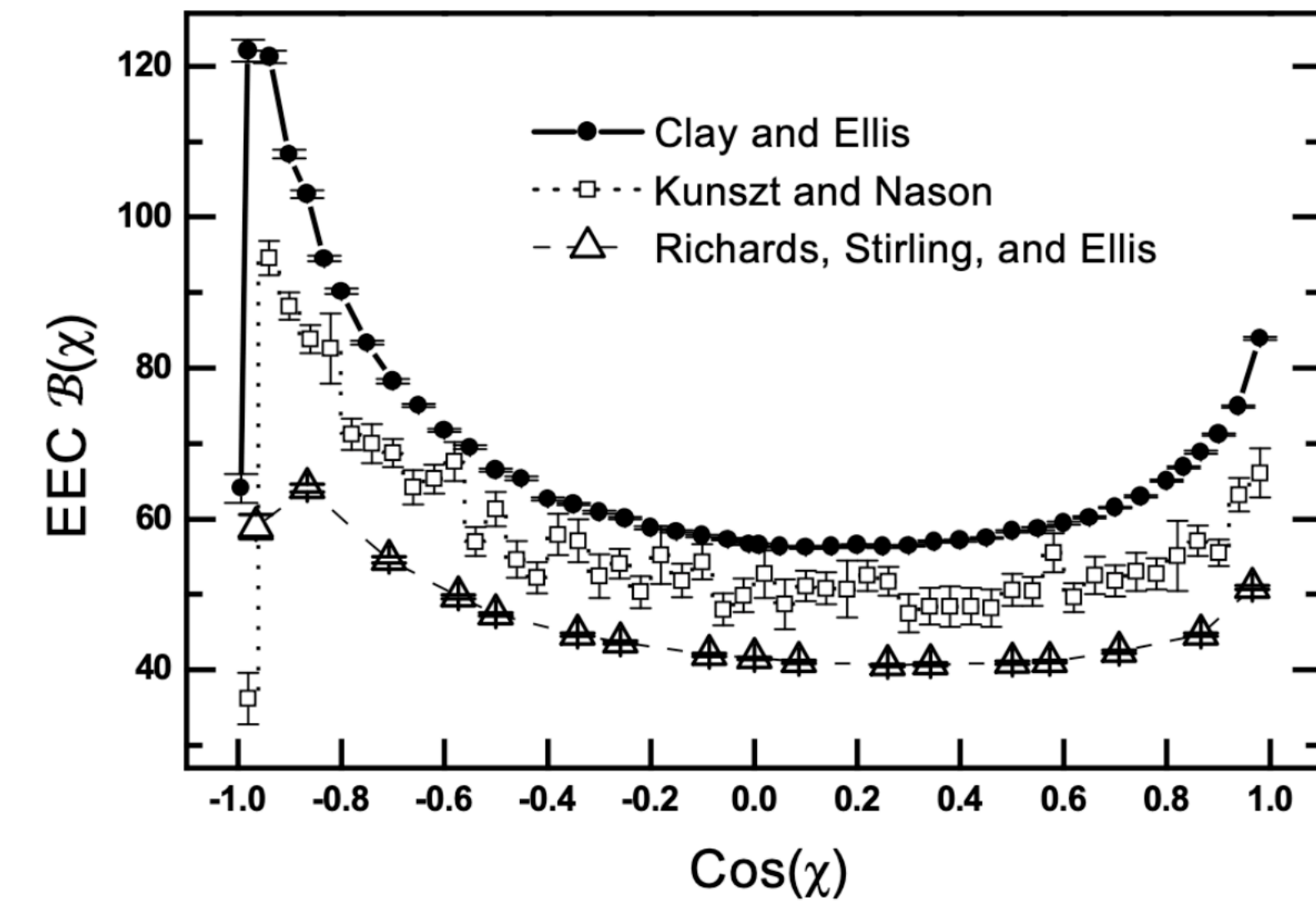
θ

Central:

Basham, Brown, Ellis, Love

b2b:

Parisi, Petronzio

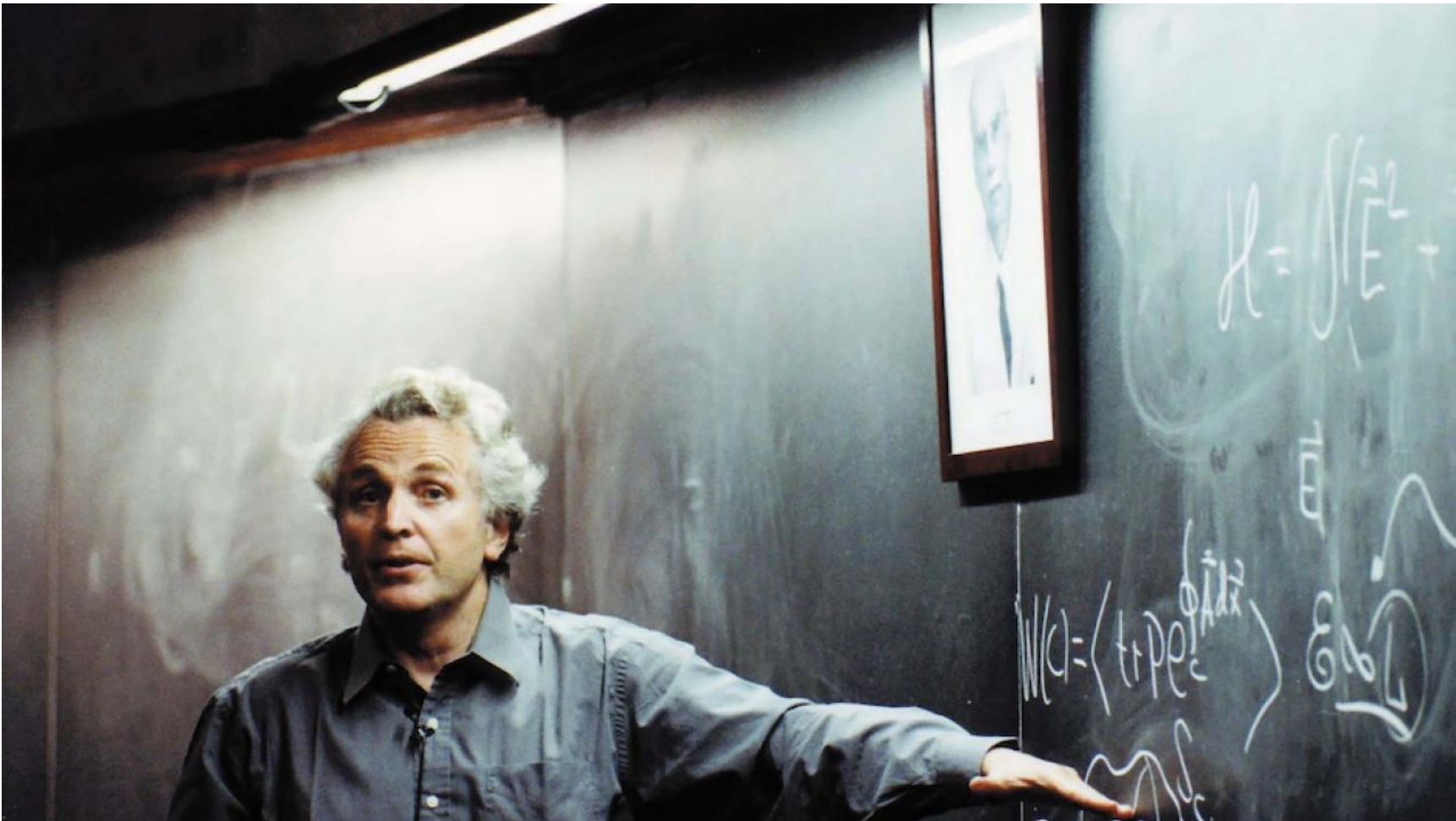


Clay, Ellis, 1995

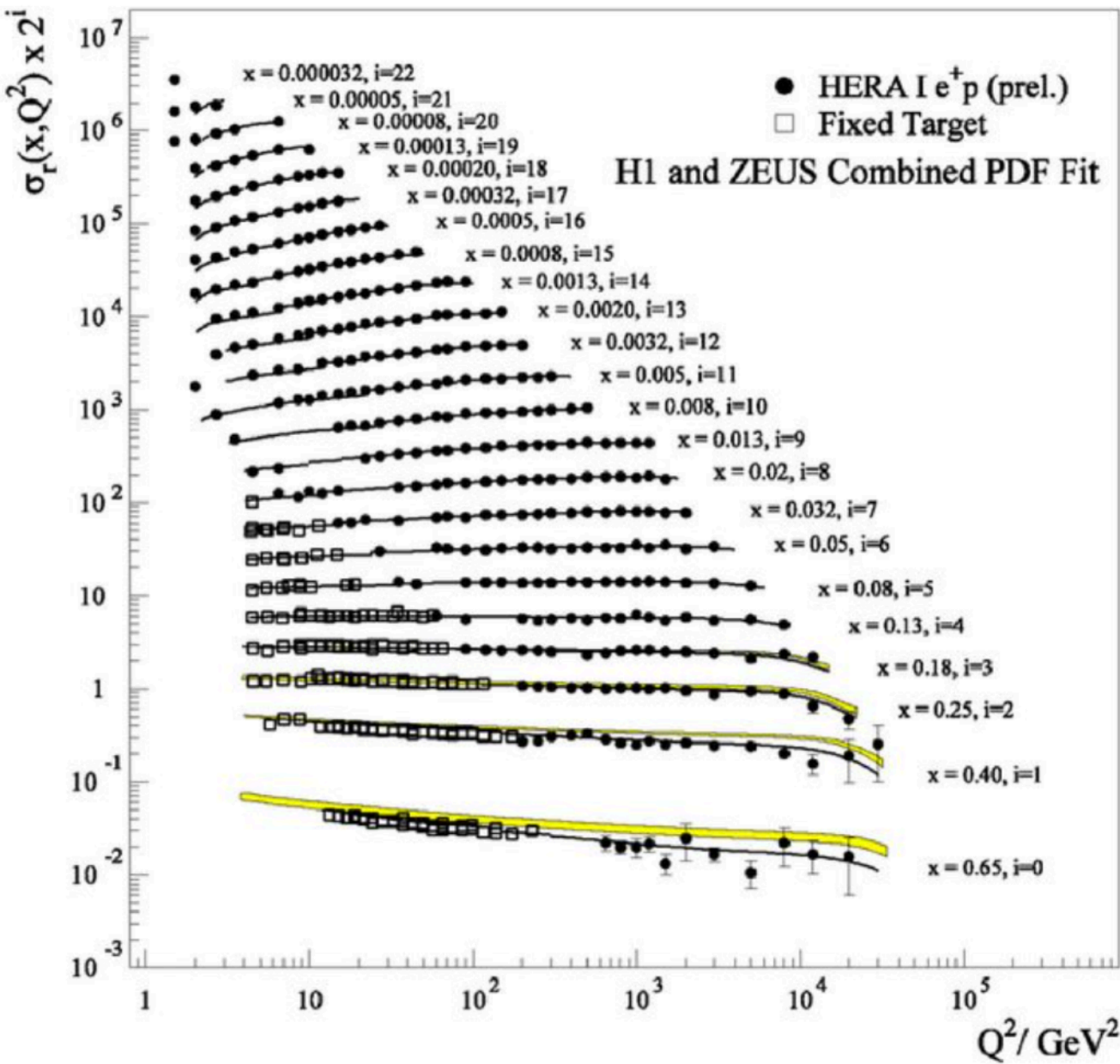
most precise and detailed experimental tests of QCD available [7, 8]. However, that potential has not been realized due to disagreement over the predicted value of the next-to-leading order correction in the strong coupling constant [9, 10, 11, 12, 13]. We report on a new calculation of the $O(\alpha_s^2)$

*I wanted to learn about elementary particles
by studying boiling water.*

— Polyakov, 1970



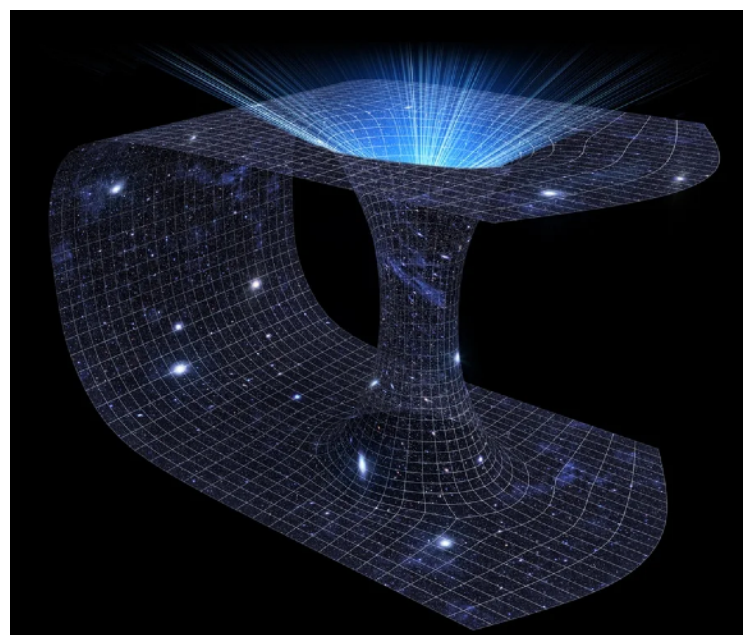
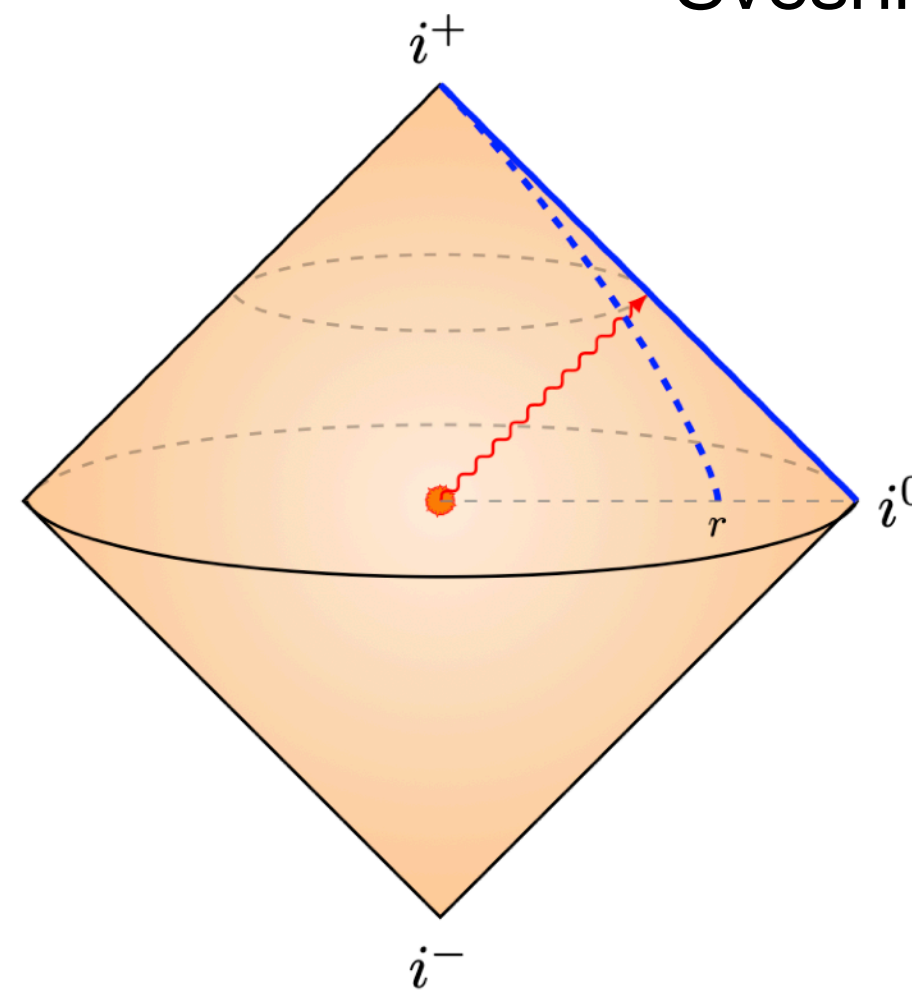
mass →	≈2.3 MeV/c ²	≈1.275 GeV/c ²	≈173.07 GeV/c ²	0	≈126 GeV/c ²
charge →	2/3	2/3	2/3	0	0
spin →	1/2	1/2	1/2	1	0
	u up	c charm	t top	g gluon	H Higgs boson
	≈4.8 MeV/c ²	≈95 MeV/c ²	≈4.18 GeV/c ²	0	
	-1/3	-1/3	-1/3	0	
	1/2	1/2	1/2	1	
	d down	s strange	b bottom	γ photon	
	0.511 MeV/c ²	105.7 MeV/c ²	1.777 GeV/c ²	91.2 GeV/c ²	
	-1	-1	-1	0	
	1/2	1/2	1/2	1	
	e electron	μ muon	τ tau	Z Z boson	
	<2.2 eV/c ²	<0.17 MeV/c ²	<15.5 MeV/c ²	80.4 GeV/c ²	
	0	0	0	±1	
	1/2	1/2	1/2	1	
	ν_e electron neutrino	ν_μ muon neutrino	ν_τ tau neutrino	W W boson	



Energy operators

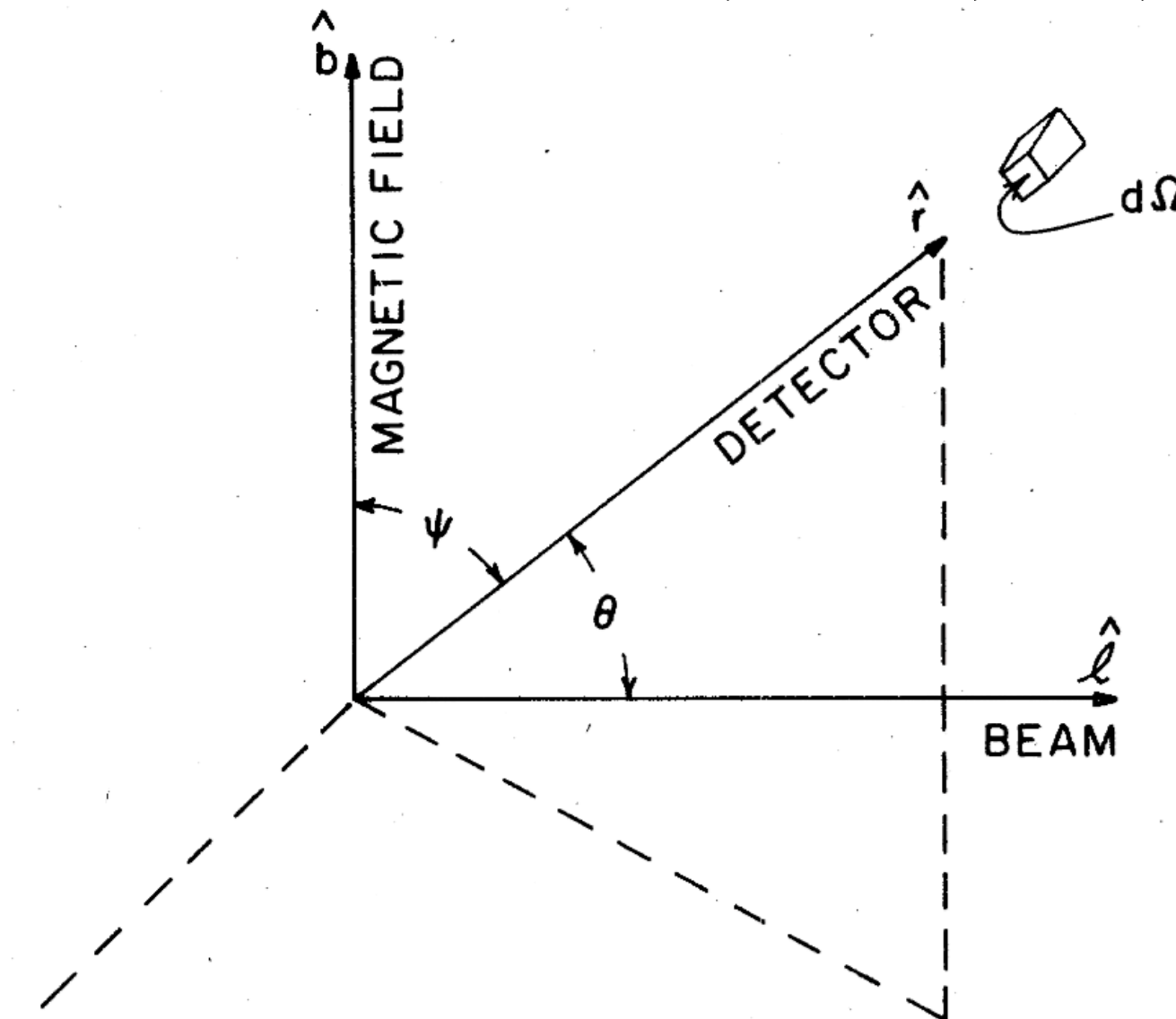
$$\mathcal{E}(\vec{n}) = \lim_{r \rightarrow \infty} r^2 \int_0^\infty dt \vec{n}_i T^{0i}(t, r\vec{n})$$

Sveshnikov, Tkachov, 1995



Morris, Thorne, 1989

Basham, Brown, Ellis, Love, 1978



**One-point detector function
is related to a three-point
correlation in QFT!**

$$\langle JTJ \rangle$$

Conformal collider bound

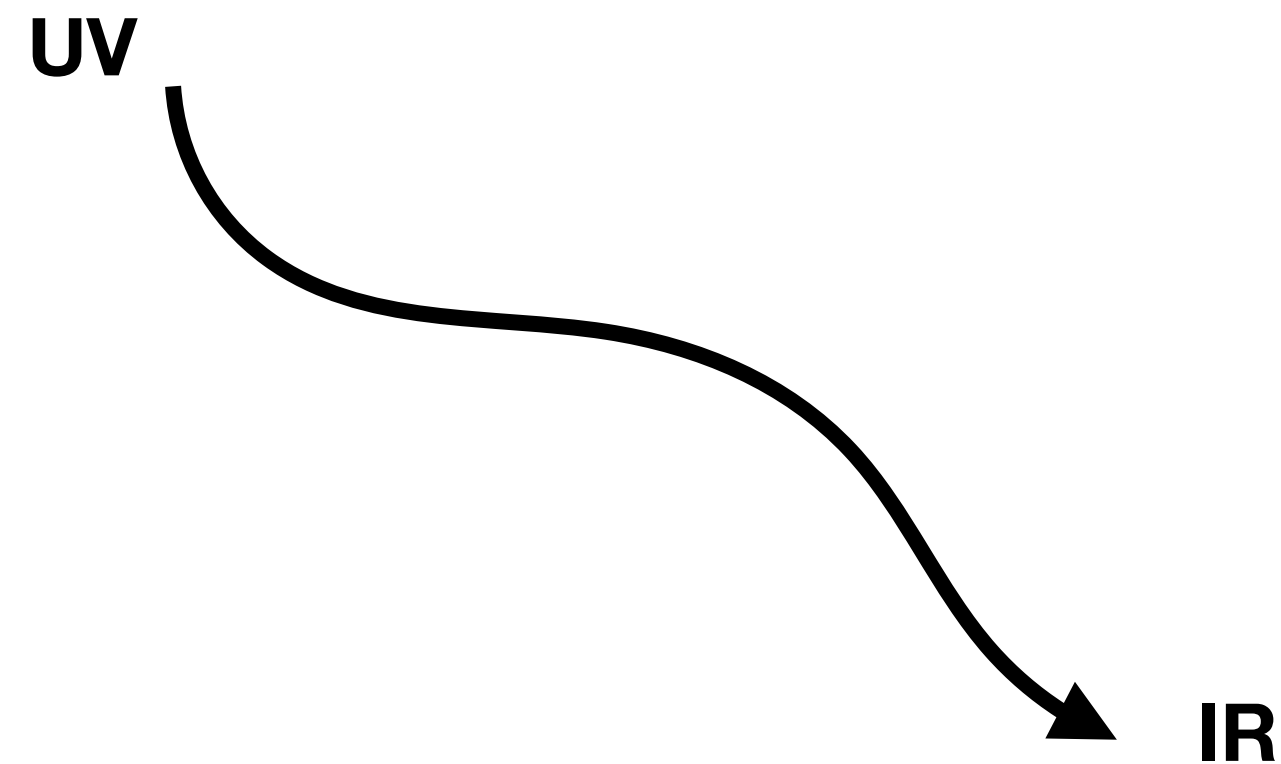


Zamolodchikov

Cardy

2024 fundamental physics breakthrough prize

$$T_{\mu}^{\mu} = \frac{c}{16\pi^2} W_{\mu\nu\delta\sigma} W^{\mu\nu\delta\sigma} - \frac{a}{16\pi^2} E$$



a and c monotonically decreases under RG flow

$$\langle \mathcal{E}(\theta) \rangle = 1 + 3 \frac{c-a}{c} \left(\cos^2 \theta - \frac{1}{3} \right)$$

$$\frac{3c}{2} \geq a \geq 0$$

Hofman, Maldacena, 2008

Lightray OPE

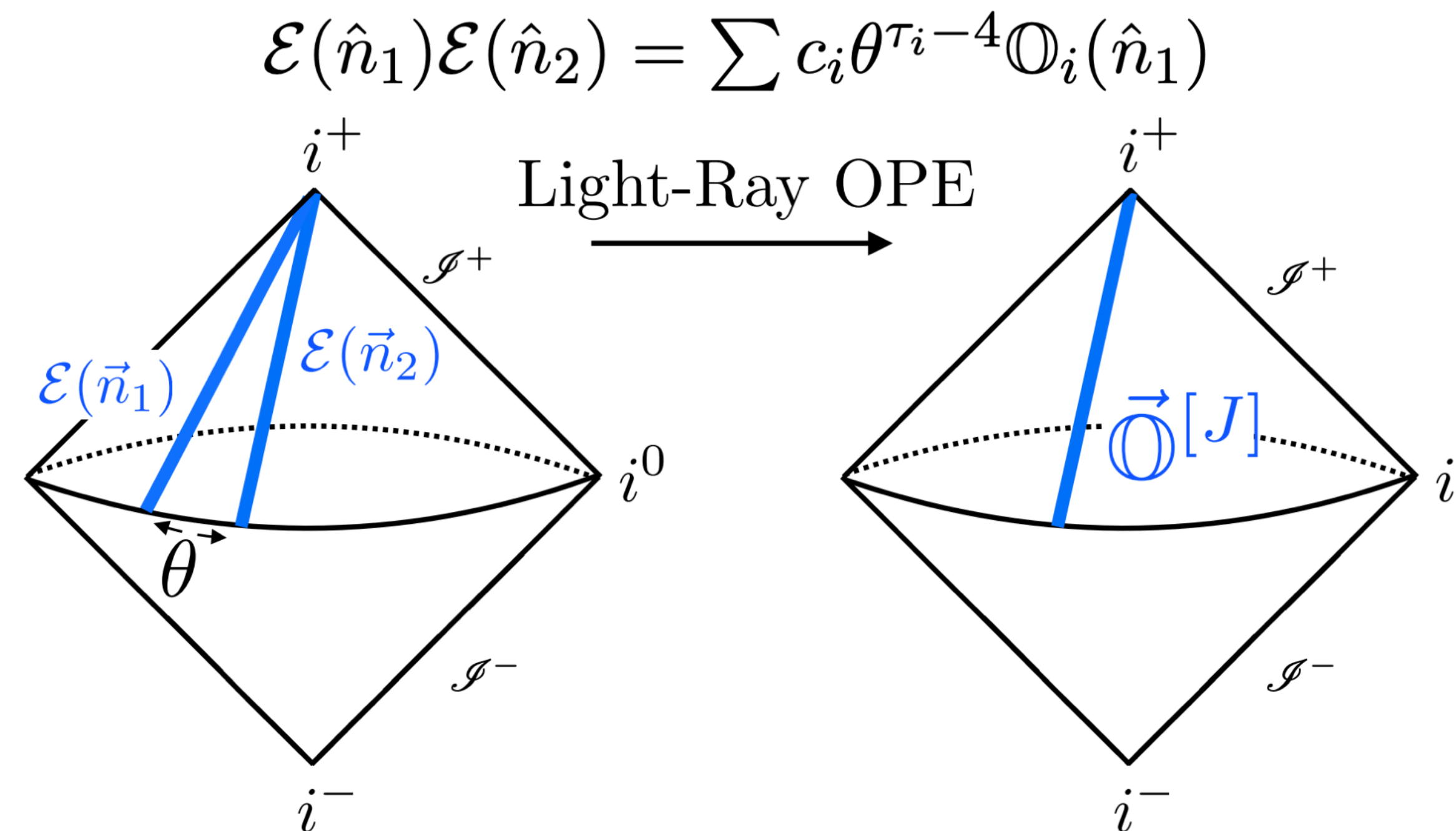
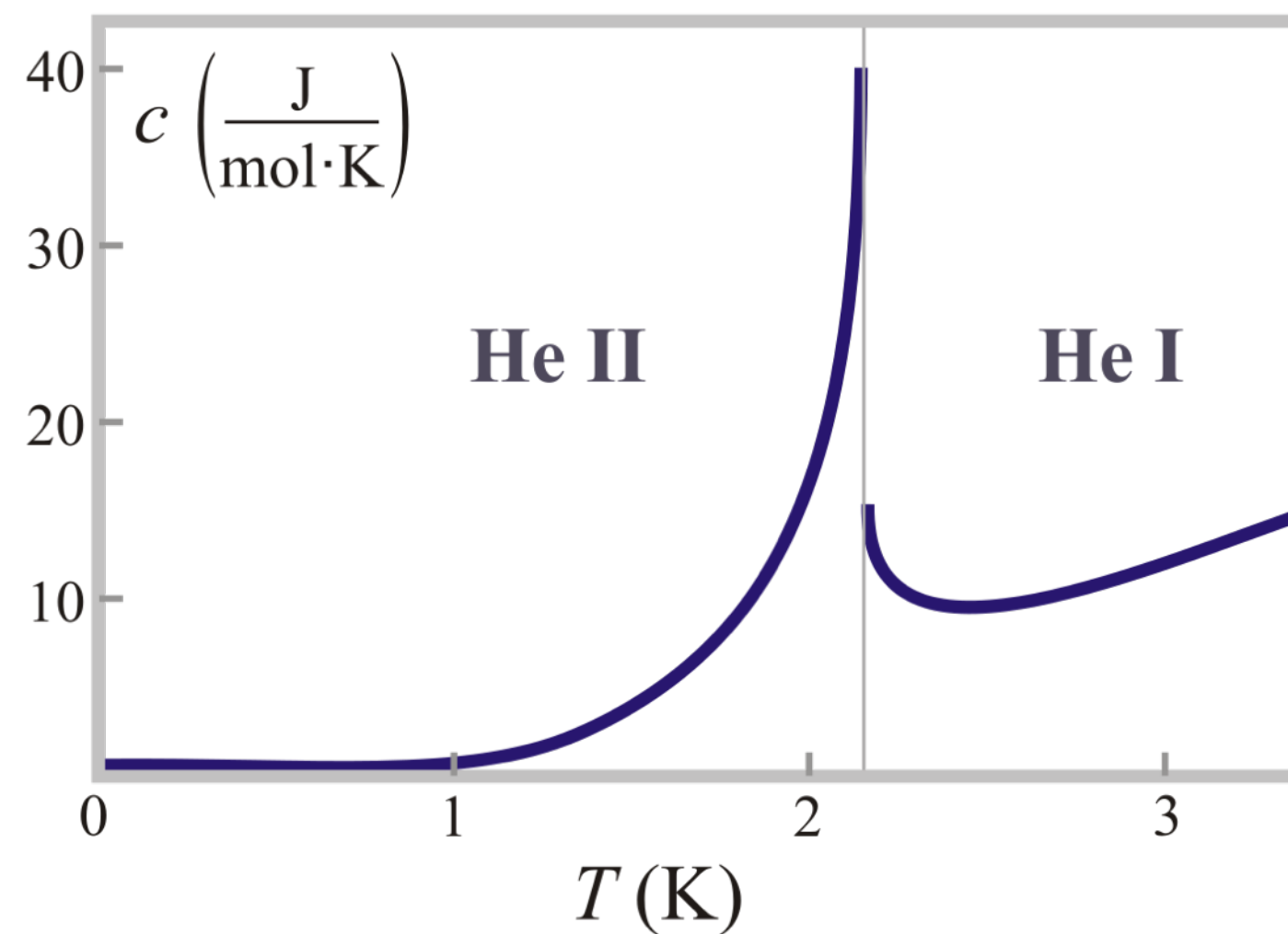
What happen if two detectors
are close to each other?

Hofman, Maldacena, 2008

Kravchuk, Simmons-Duffin, 2018

Kravchuk, Kologlu, Simmons-Duffin, Zhiboedov, 2019

$$O_1(x_1)O_2(x_2) = \sum_3 c_{123}(x_1 - x_2)O_3(x_2)$$



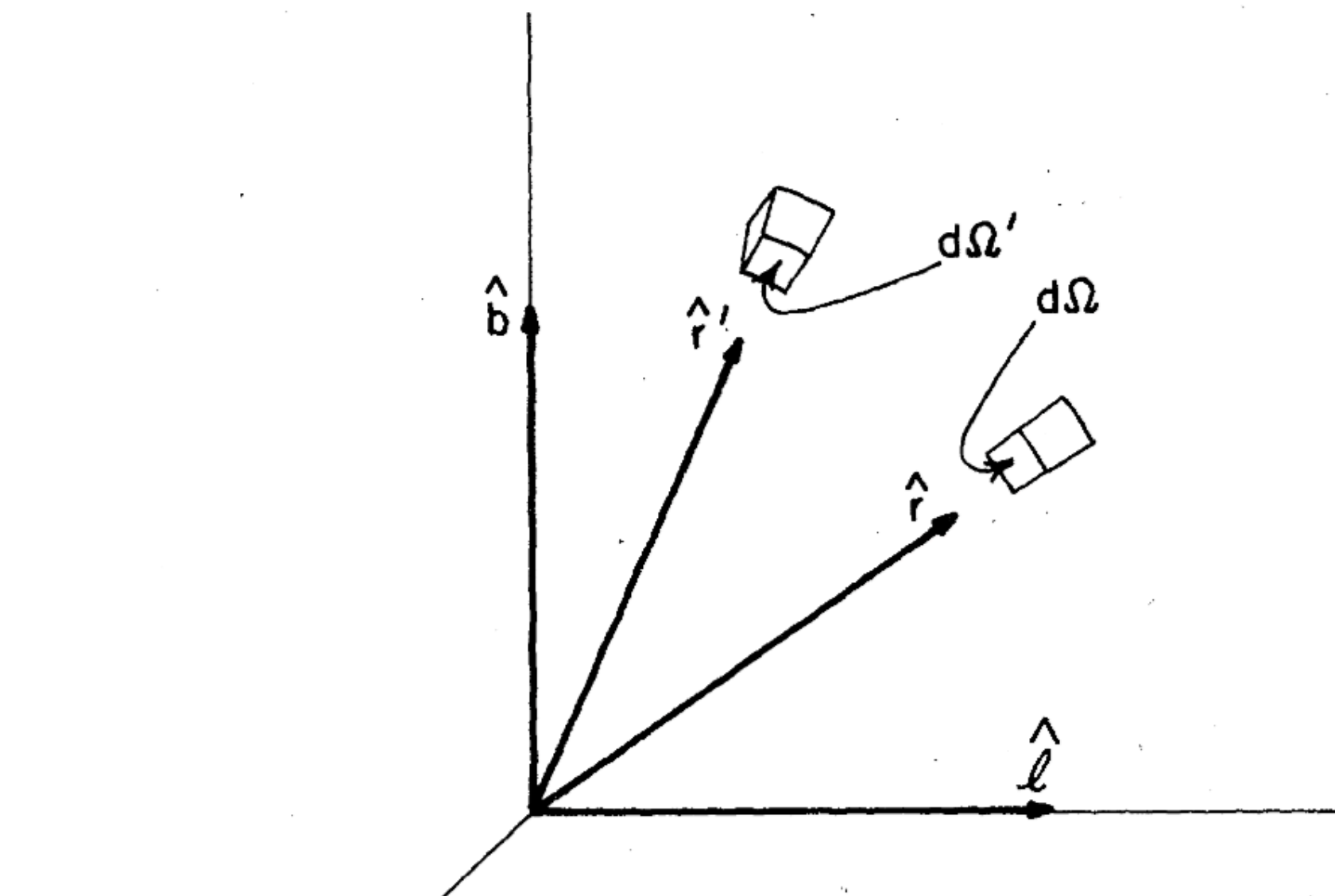
The analytic beauty of energy correlators

One-point detector function is related to a local four-point correlation in QFT!

$$\langle \mathcal{E}(\hat{n}_1) \mathcal{E}(\hat{n}_2) \rangle \sim \langle J T T J \rangle$$

Huge efforts in the past on computing 4-pt correlators and related integrals =>

Full analytic results of EEC up to NLO (N=4 SYM)



$$\frac{1}{\sigma_0} \frac{d\sigma^{(3)}}{dC} = \frac{\alpha_s}{2\pi} C_F \int_{x_2^-(C)}^{x_2^+(C)} dx \times \frac{6x \left[C(x^3 + (x-2)^2) - 6(1-x)(1+x^2) \right]}{C(C+6)^2(x-6/(C+6))\sqrt{(6/(C+6)-x)(x_2^+-x)(x-x_2^-)}x}$$

LO result for C parameter involves elliptic function
Difficult to make progress

Ellis, Ross, Terrano, 1981

$$F_2(z) = 4\sqrt{z} \left[\text{Li}_2(-\sqrt{z}) - \text{Li}_2(\sqrt{z}) + \frac{\ln z}{2} \ln \left(\frac{1+\sqrt{z}}{1-\sqrt{z}} \right) \right] + (1+z) [2\text{Li}_2(z) + \ln^2(1-z)] + 2\ln(1-z) \ln \left(\frac{z}{1-z} \right) + z \frac{\pi^2}{3}, \quad (46)$$

$$F_3(z) = \frac{1}{4} \left\{ (1-z)(1+2z) \left[\ln^2 \left(\frac{1+\sqrt{z}}{1-\sqrt{z}} \right) \ln \left(\frac{1-z}{z} \right) - 8\text{Li}_3 \left(\frac{\sqrt{z}}{\sqrt{z}-1} \right) - 8\text{Li}_3 \left(\frac{\sqrt{z}}{\sqrt{z}+1} \right) \right] - 4(z-4)\text{Li}_3(z) \right. \\ \left. + 6(3+3z-4z^2) \text{Li}_3 \left(\frac{z}{z-1} \right) - 2z(1+4z)\zeta_3 + 2[2(2z^2-z-2)\ln(1-z) + (3-4z)z\ln z] \text{Li}_2(z) \right. \\ \left. + \frac{1}{3} \ln^2(1-z) [4(3z^2-2z-1)\ln(1-z) + 3(3-4z)z\ln z] + \frac{\pi^2}{3} [2z^2\ln z - (2z^2+z-2)\ln(1-z)] \right\}. \quad (47)$$

Balitsky et al., 2013

N=4 SYM@NLO

Henn, Sokatchev, Yan, Zhiboedov, 2019

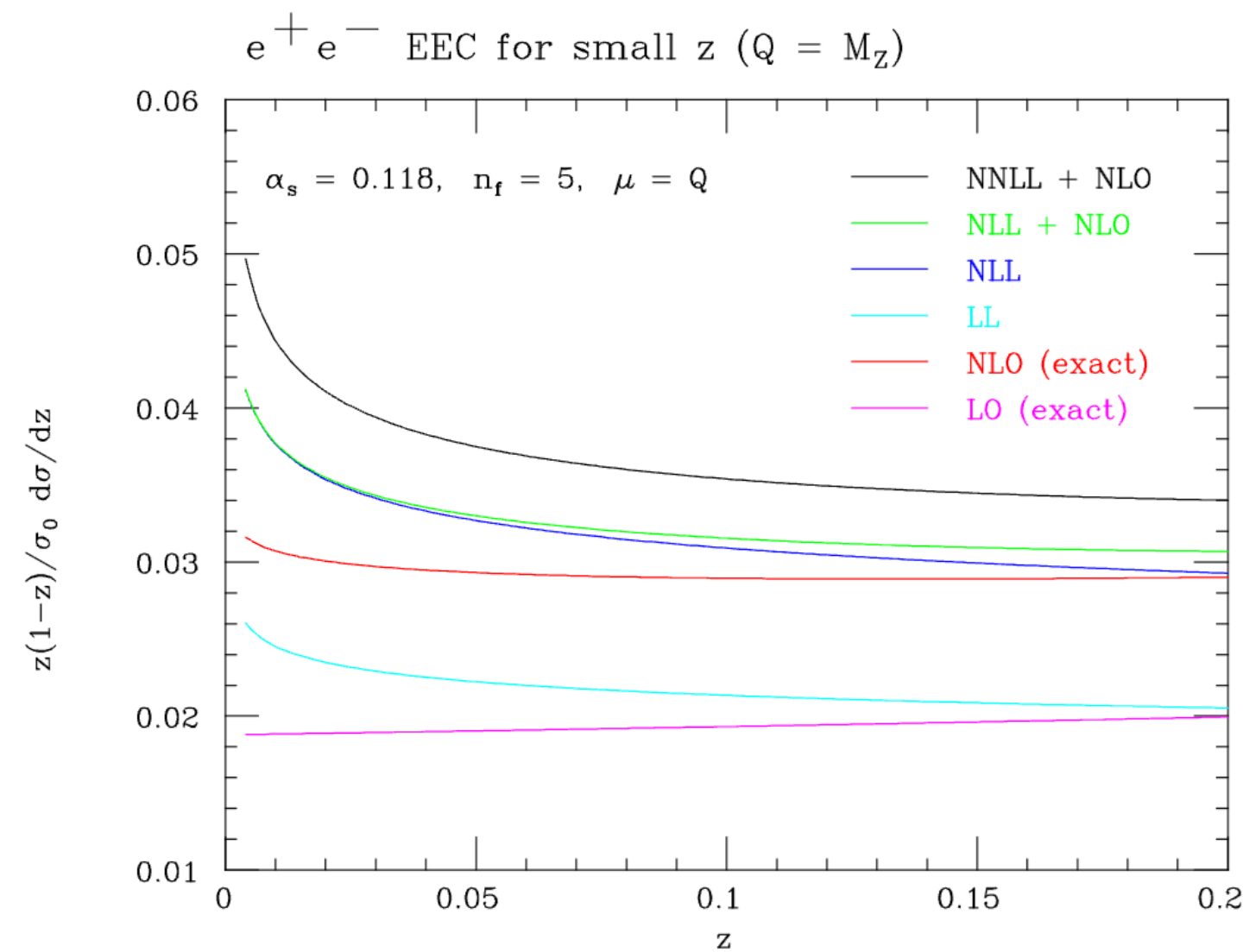
N=4 SYM@NNLO

Dixon, Luo, Shtabovenko, Yang, HXZ, 2018

QCD@NLO

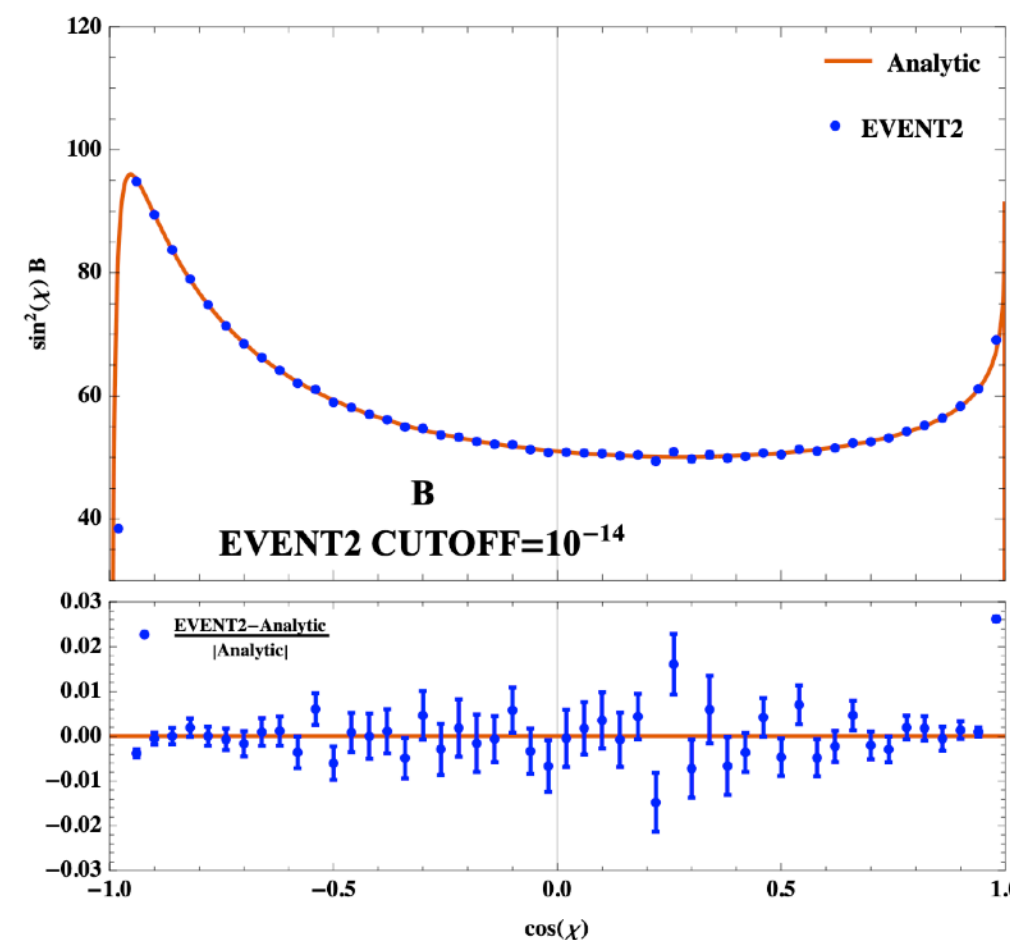
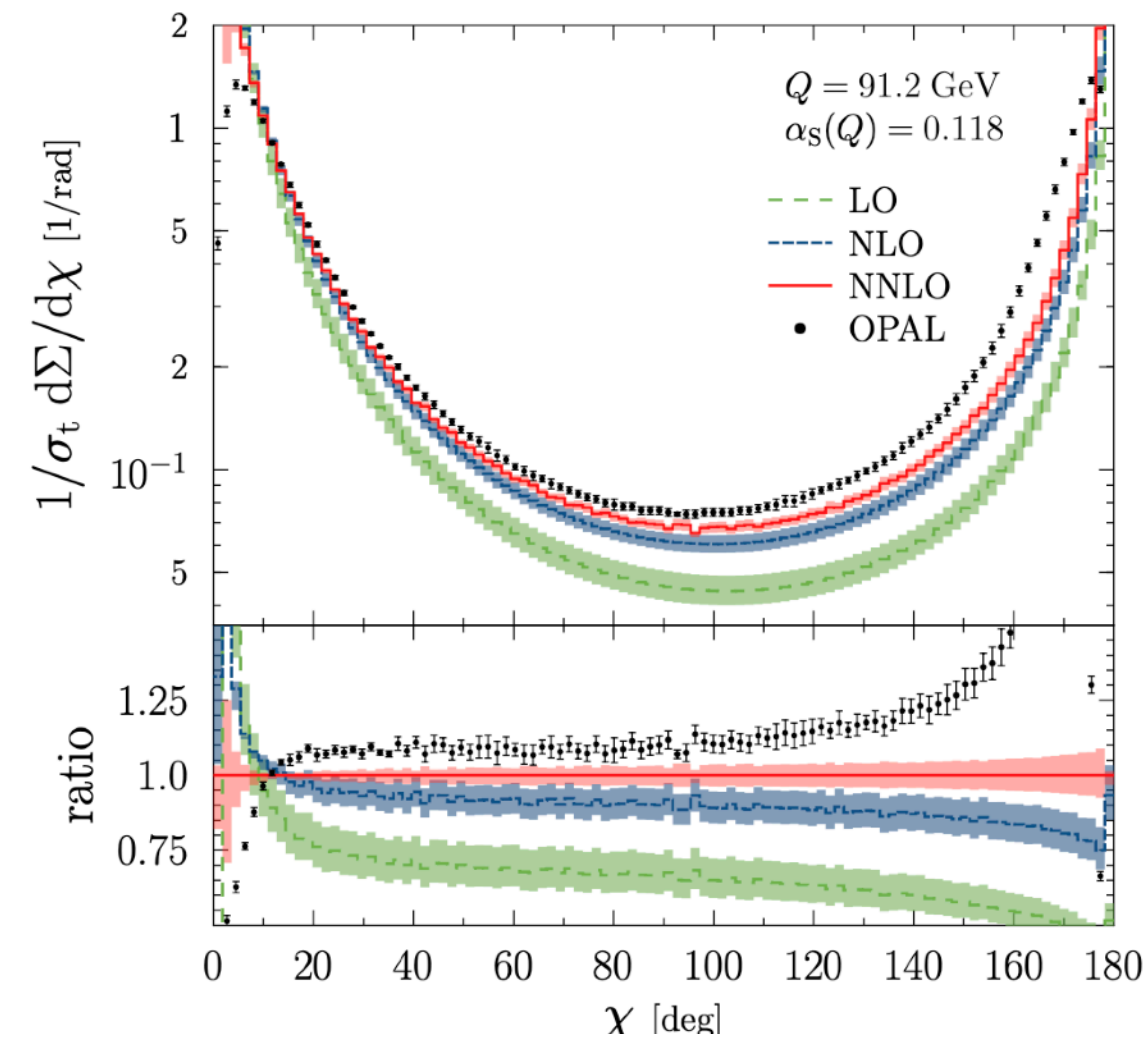
Back to QCD: pert. theory

Collinear resummation



Konishi, Ukawa, Veneziano, 1979
Ellis, Stirling, 1981
Dixon, Moul, HXZ, 1999

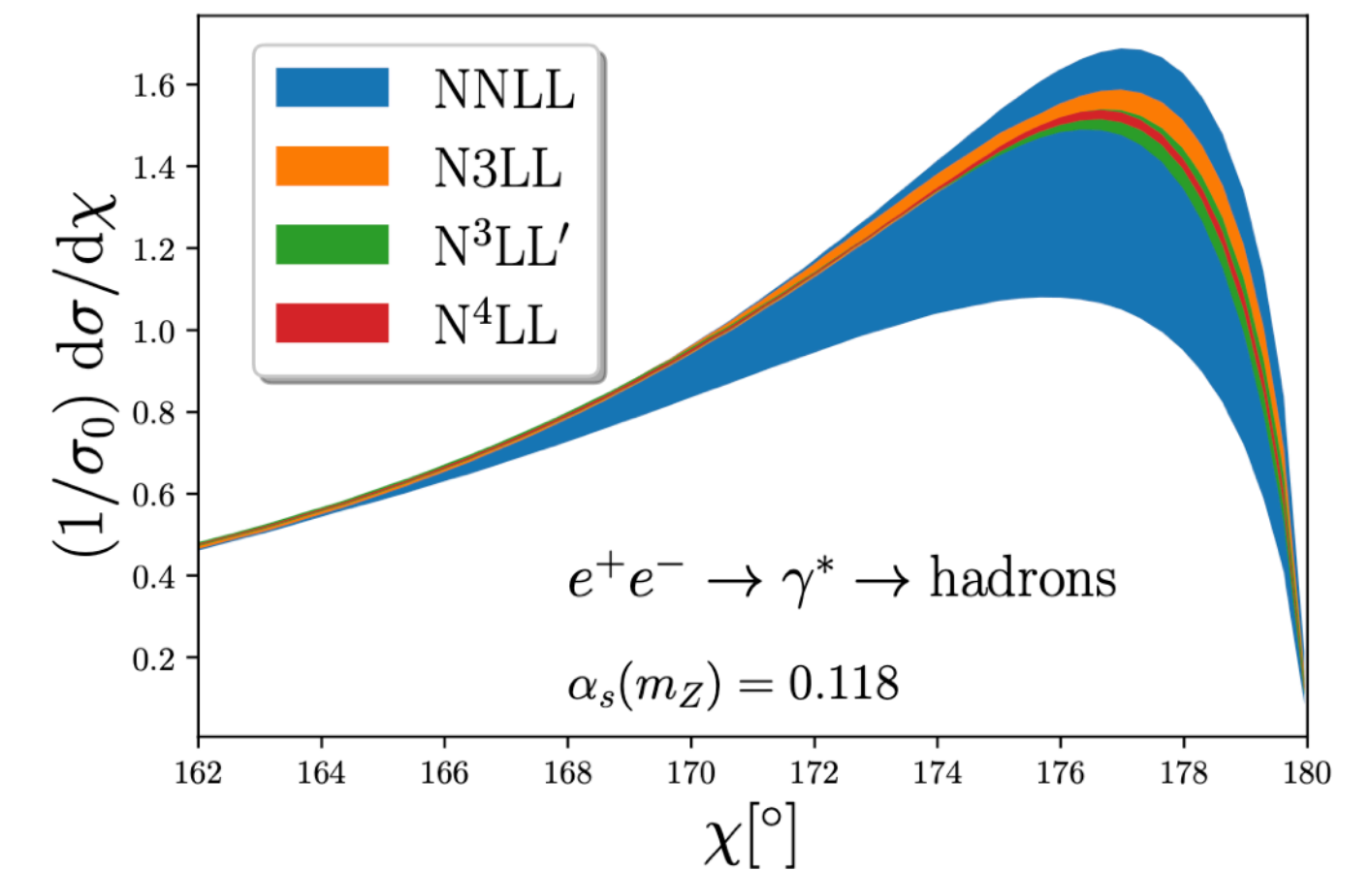
Fixed order



Del Duca, Duhr,
Kardos, Somogyi,
Trocsanyi, 2016

Dixon, Luo,
Shtabovenko, Yang,
HXZ, 2018

b2b resummation



Parasi, Petronzio, 1979
Chao, Soper, Collins, 1983
de Florian, Grazzini, 2004
Moul, HXZ, 2018
Mistlberger, Duhr, Vita, 2022

Back to QCD: non-pert. theory

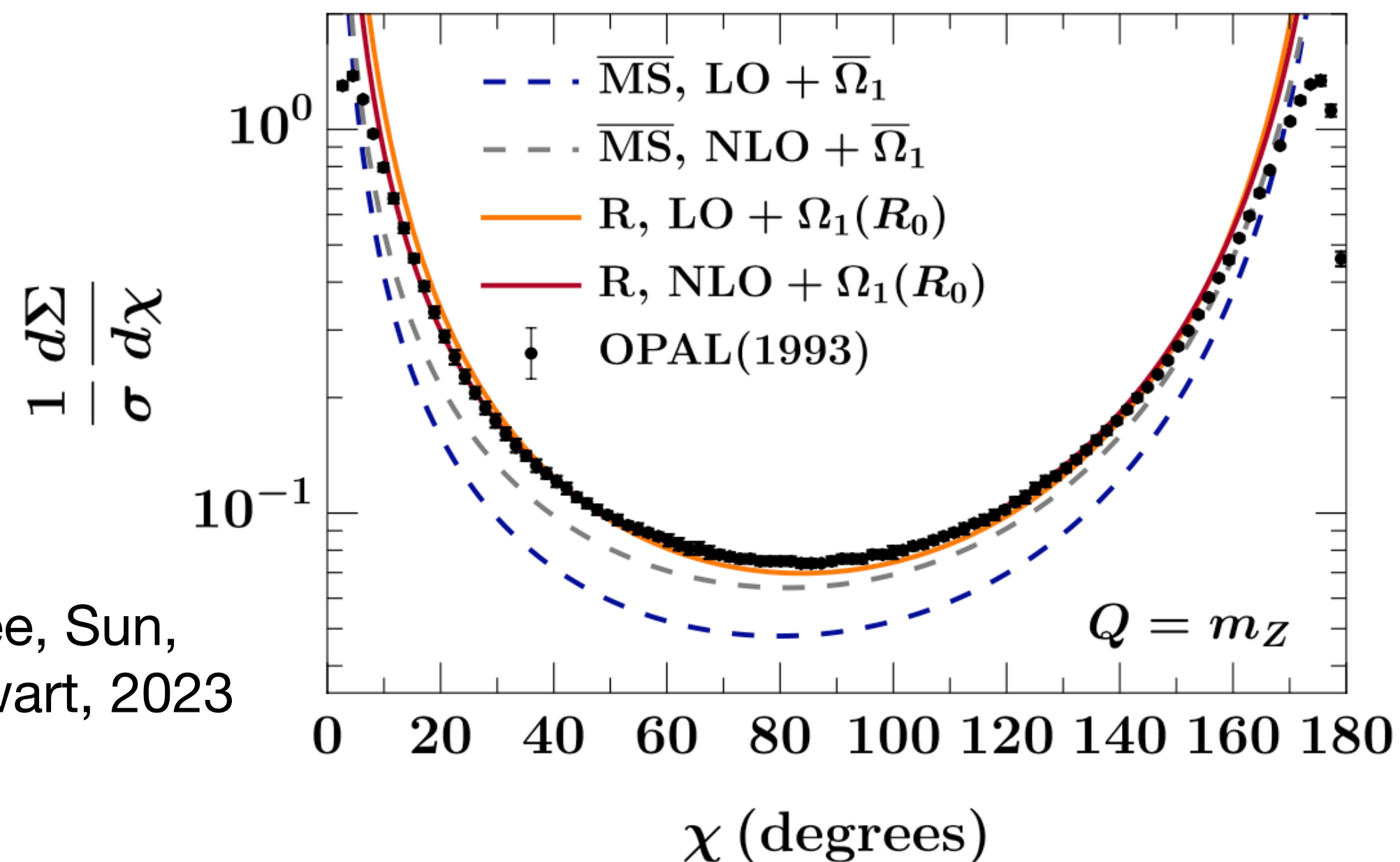
$$\langle \text{EEC}(\chi) \rangle_{\text{NP}} = \frac{1}{Q} \langle 0 | W^\dagger(0) \mathcal{E}(\vec{n}) W(0) | 0 \rangle = \frac{1}{Q} \mathcal{G}(\chi)$$

Korchinsky, Sterman, 1999

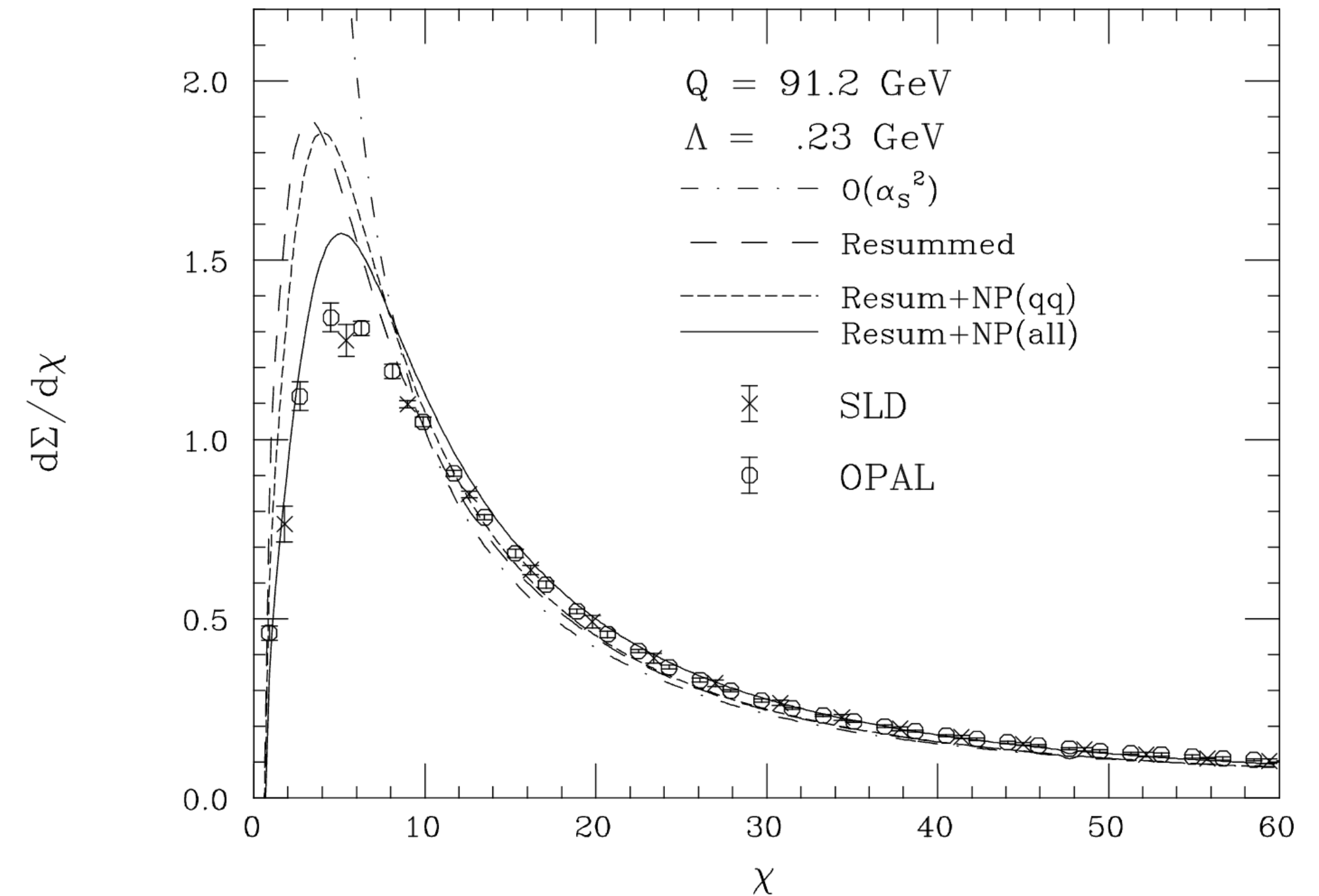
$$\mathcal{G}(\chi) = \frac{\lambda_1}{4\pi \sin^3 \chi}$$

enhanced power corrections
from symmetry analysis

Compare with perturbation: $\sim \frac{1}{\sin^2 \chi}$



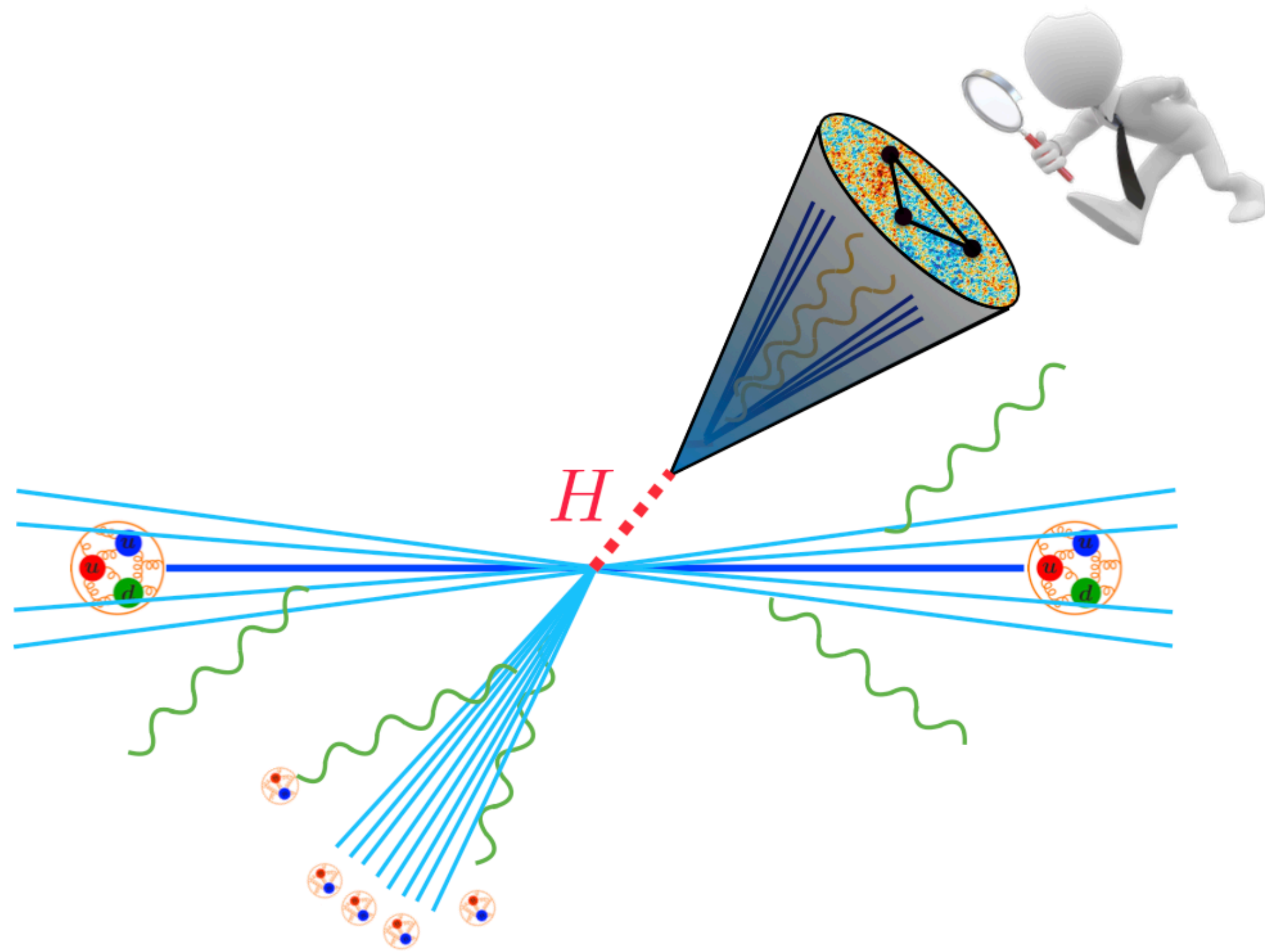
Lee, Sun,
Stewart, 2023



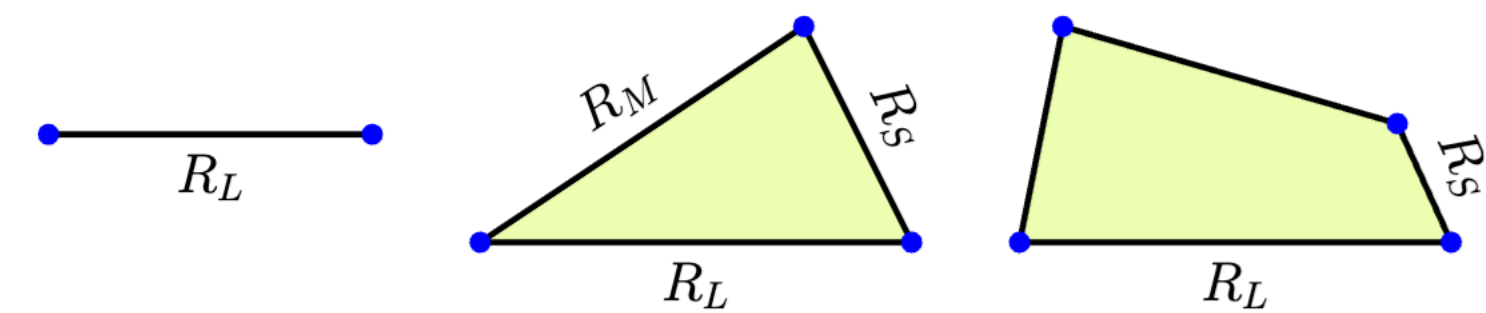
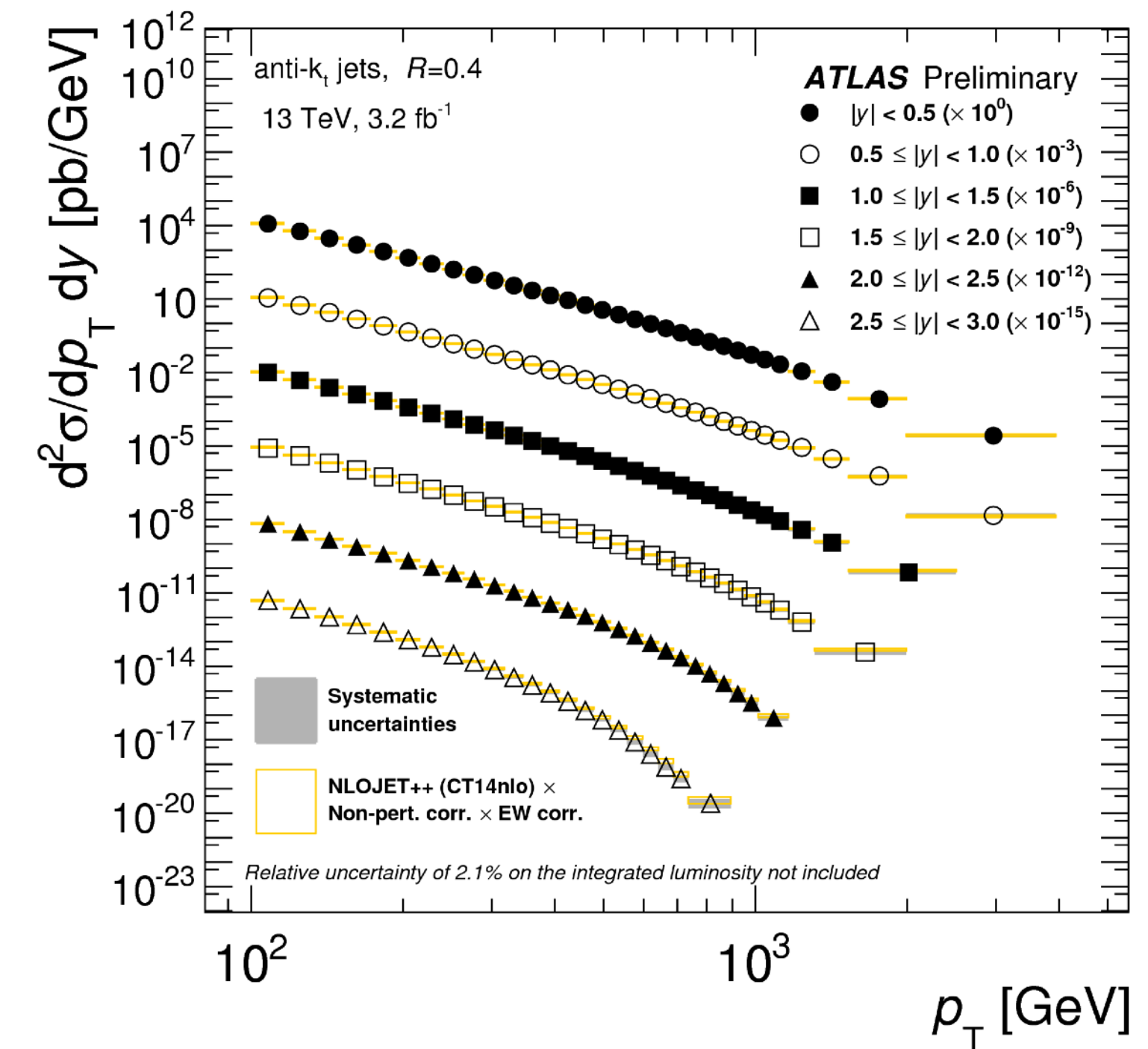
Dokshitzer, Marchesini, Webber, 1999

Importance of quark-gluon correlation

Looking inside a jet



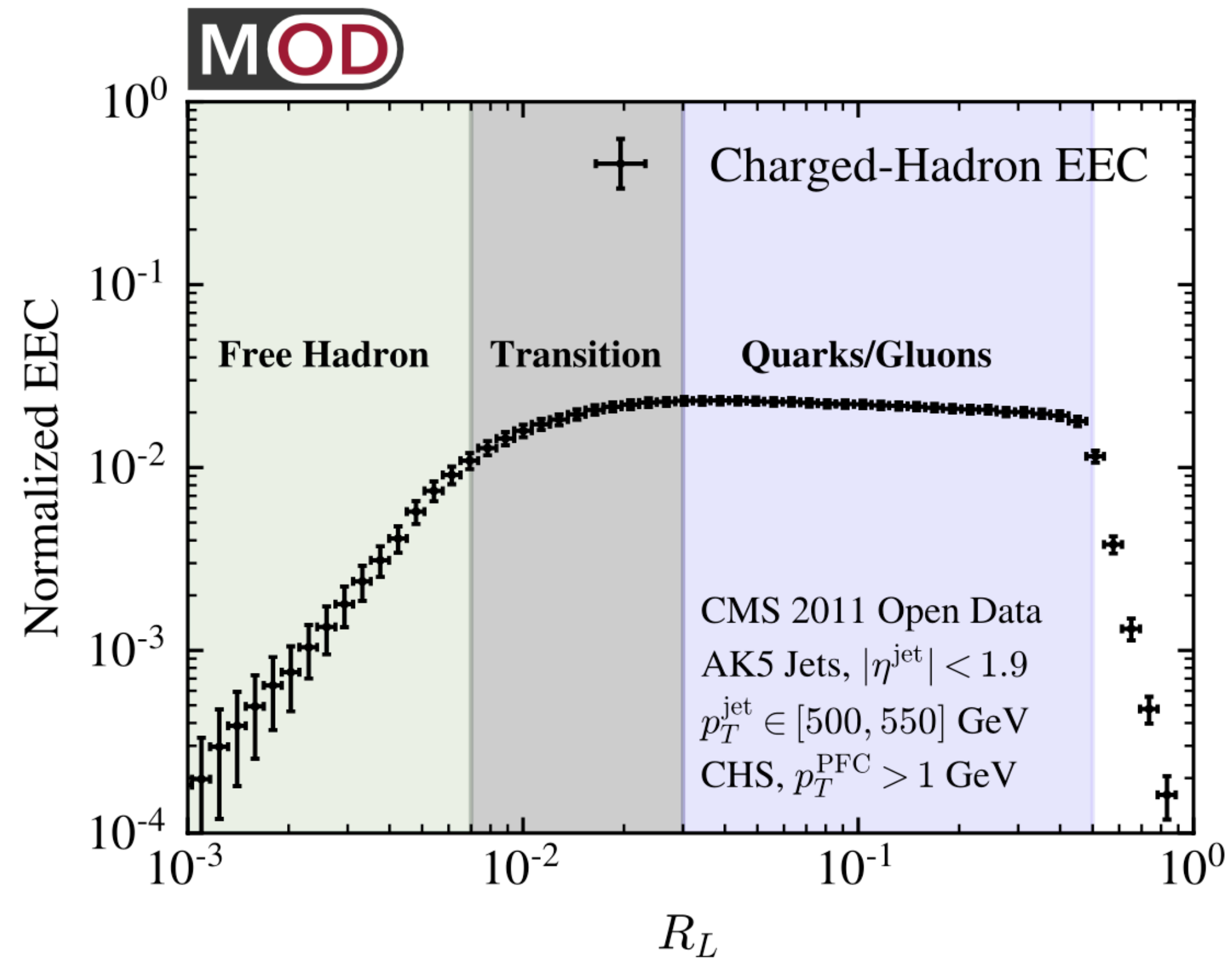
Rethinking jet substructure in terms of energy correlators



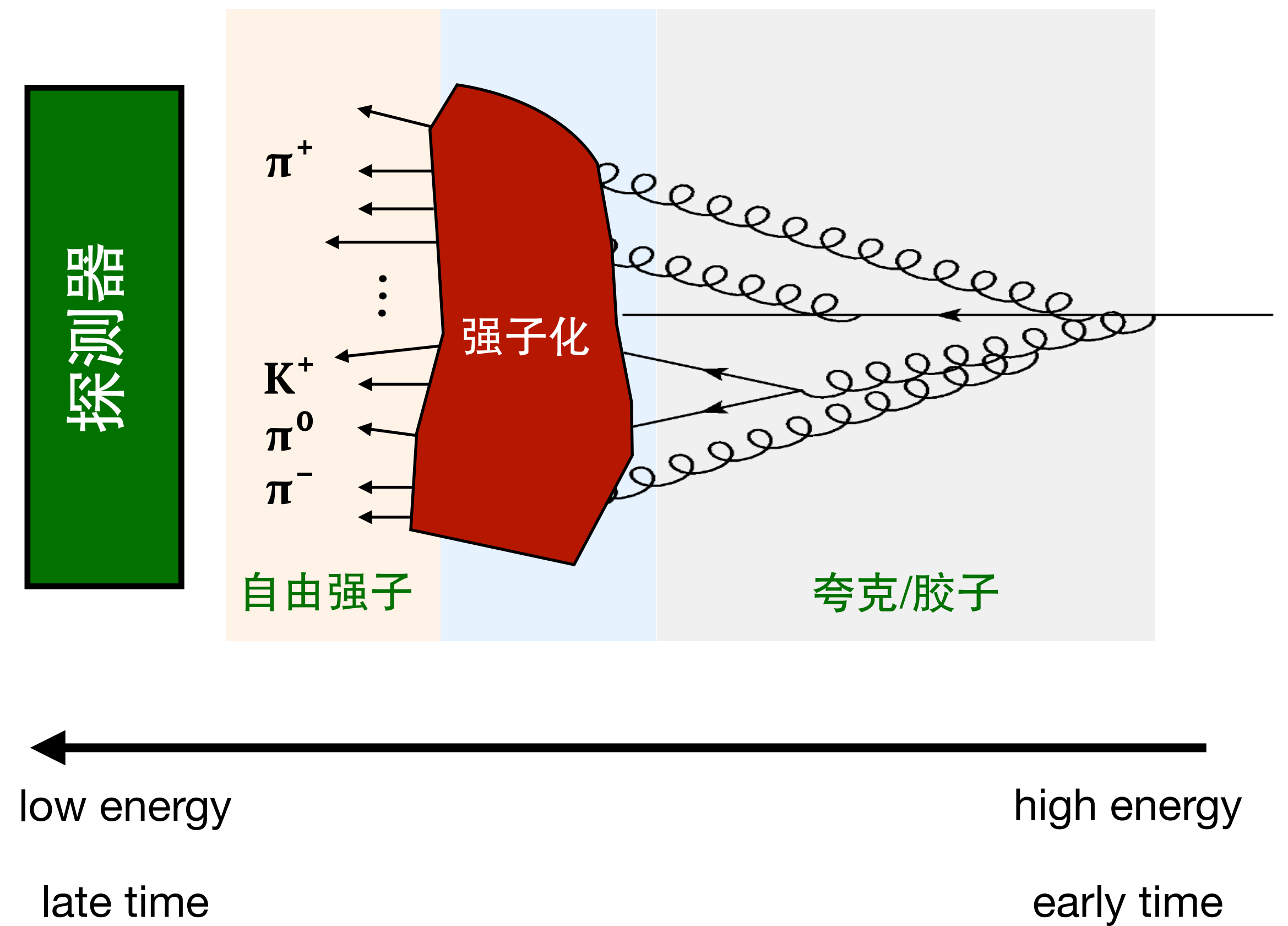
Chen, Moult, Zhang, HXZ, 2020

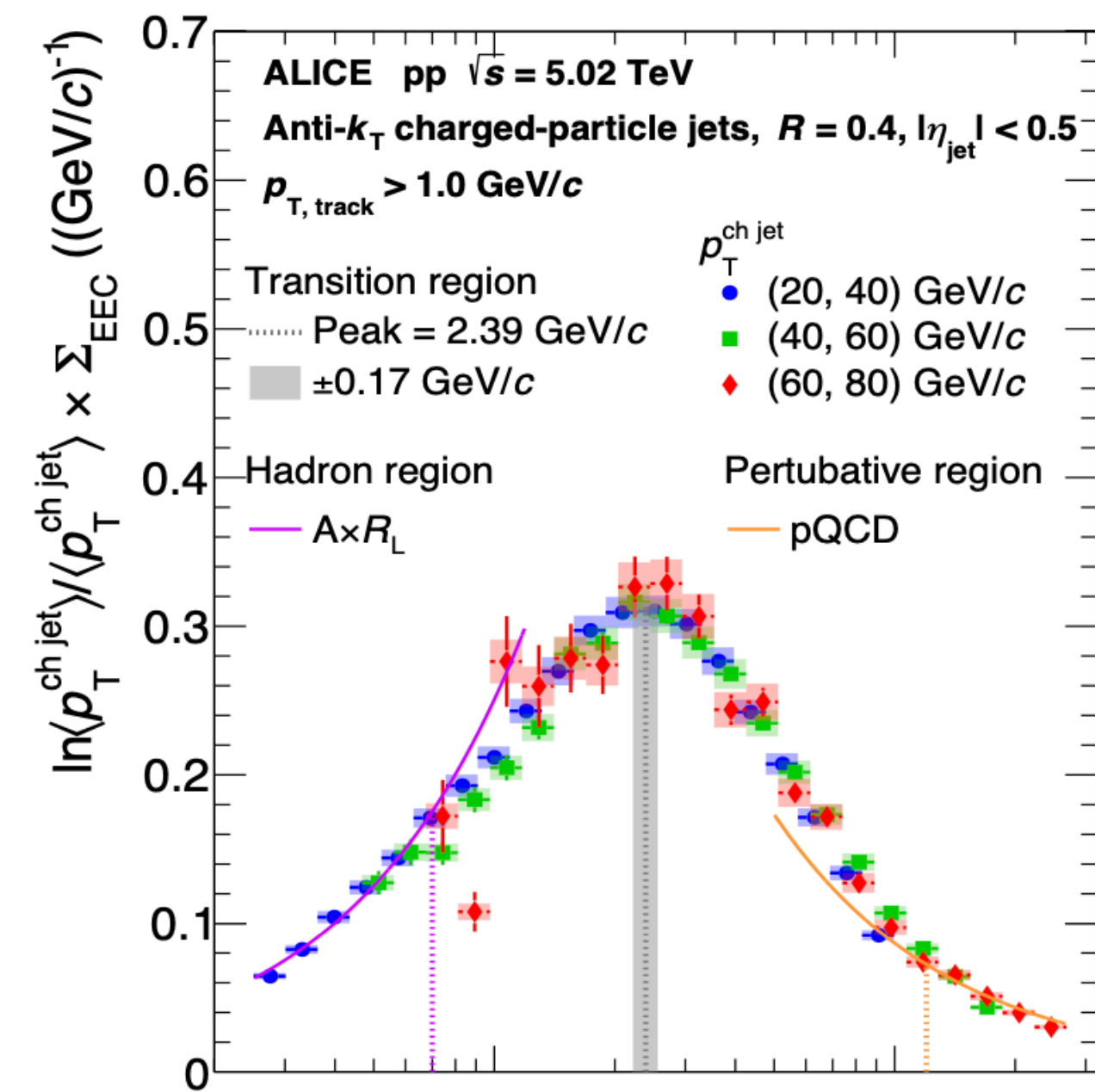
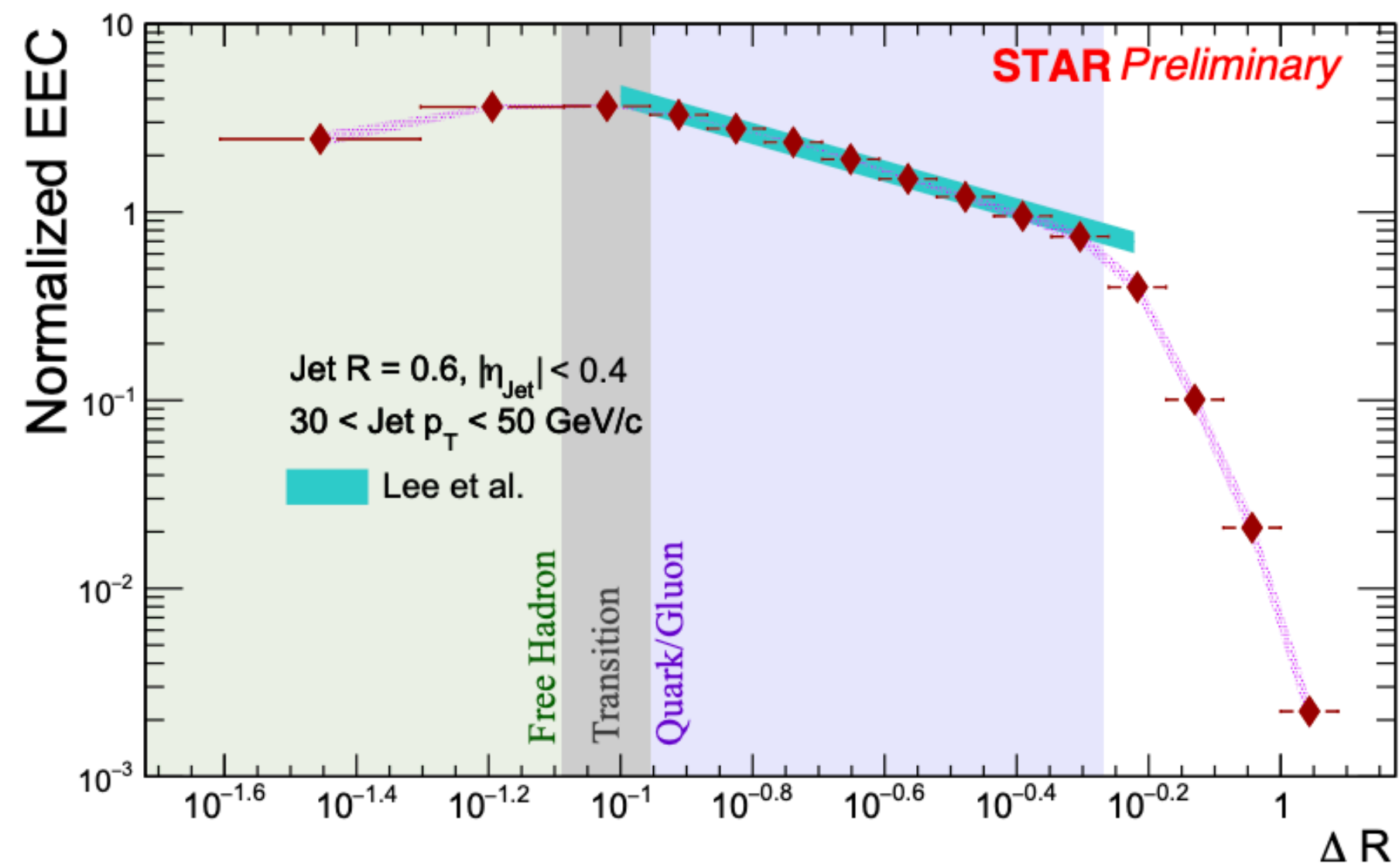
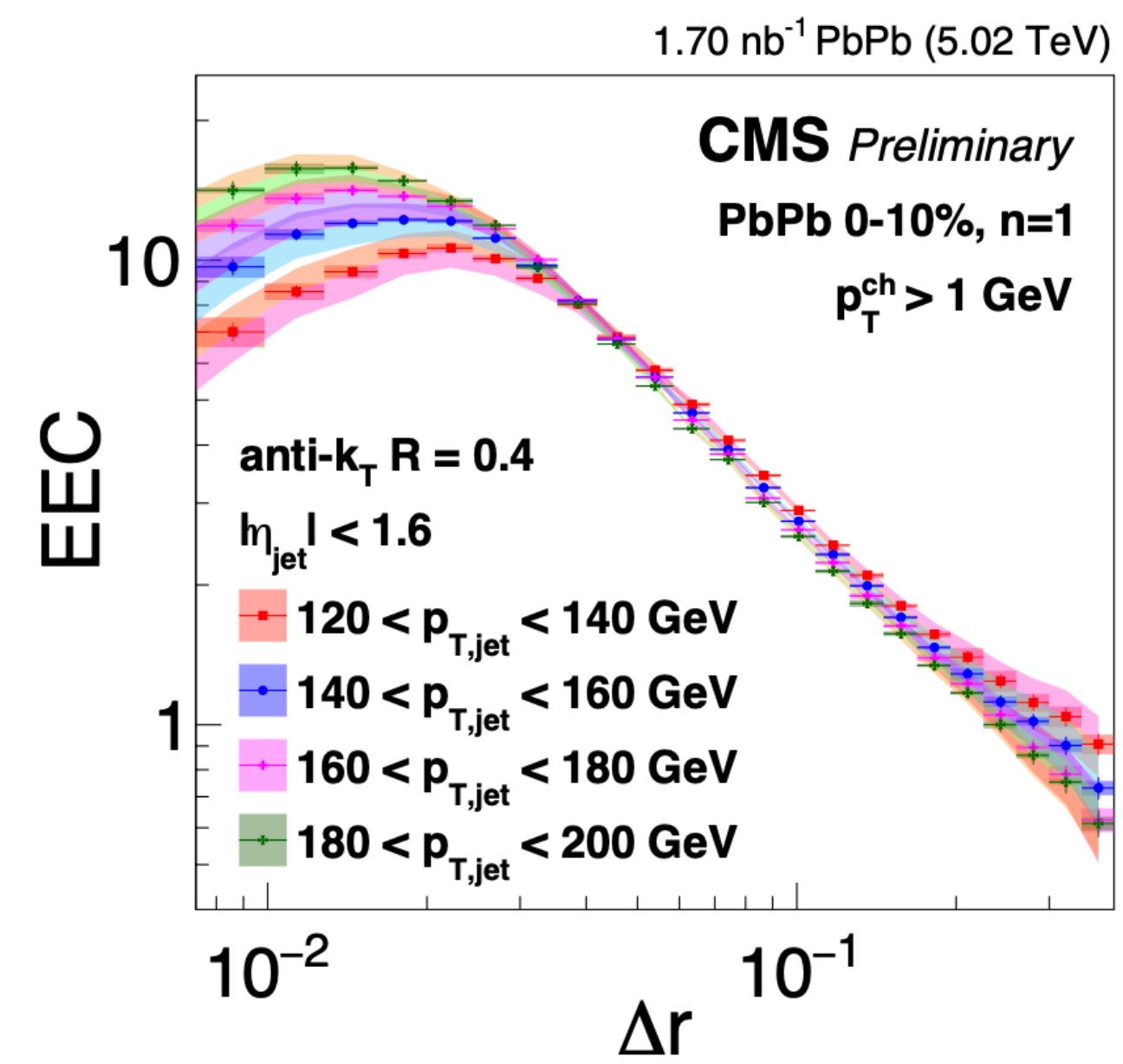
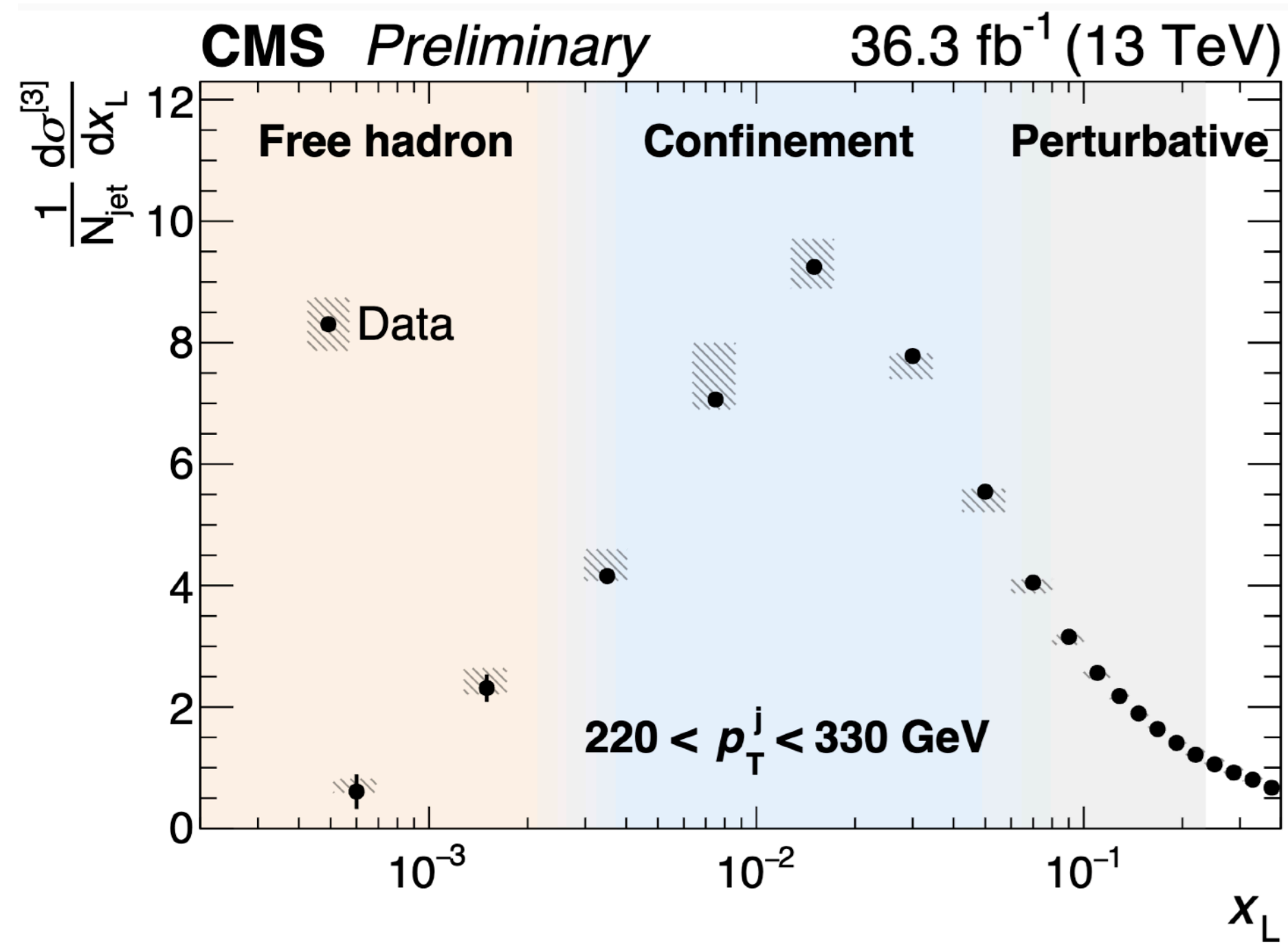
Chen, Luo, Moult, Yang, Zhang, HXZ, 2019

Discovery of scaling and imaging the hadronization transition



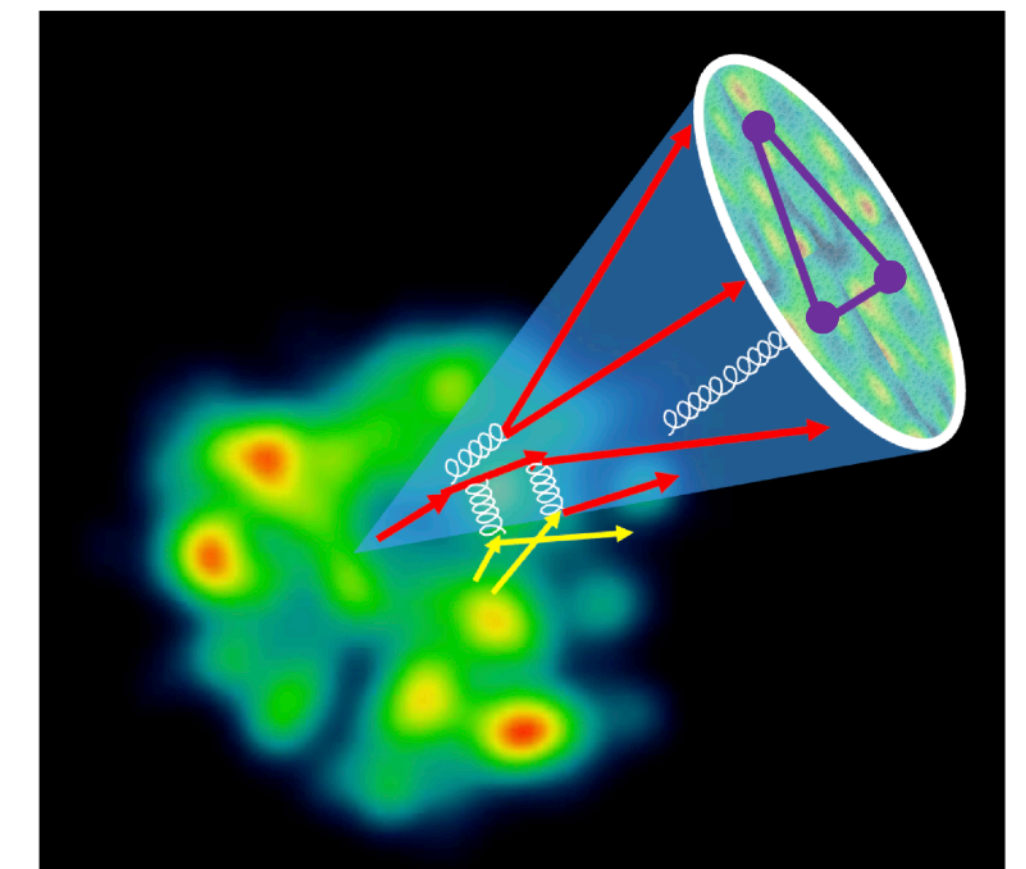
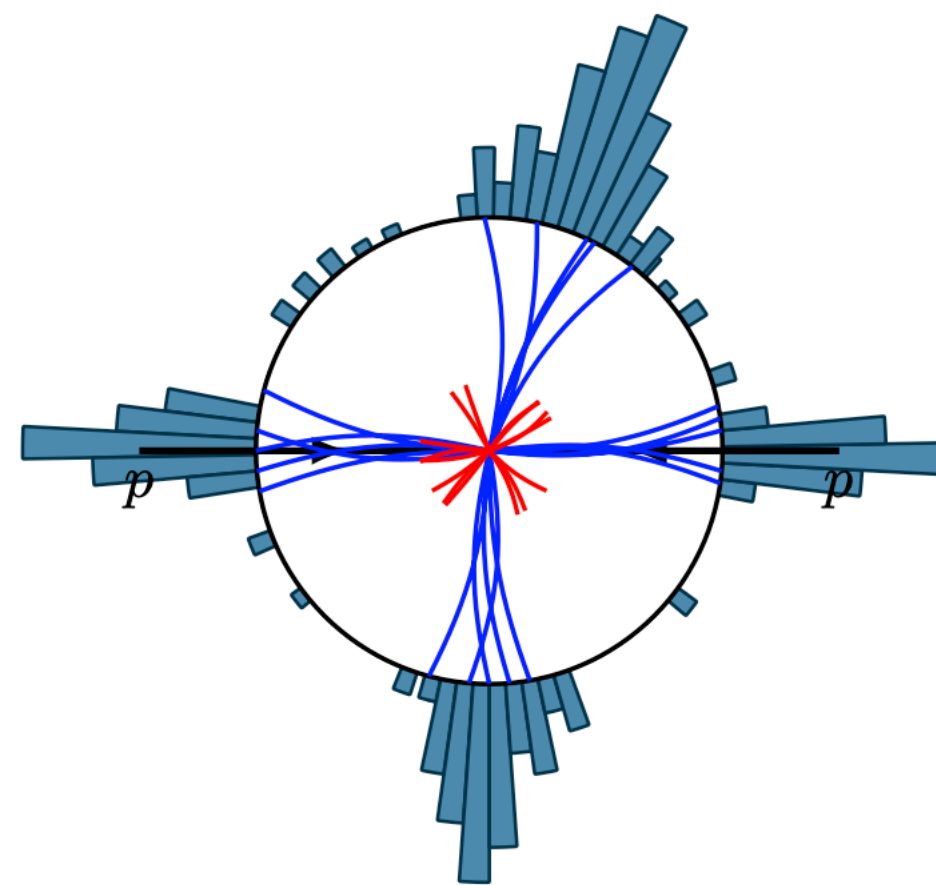
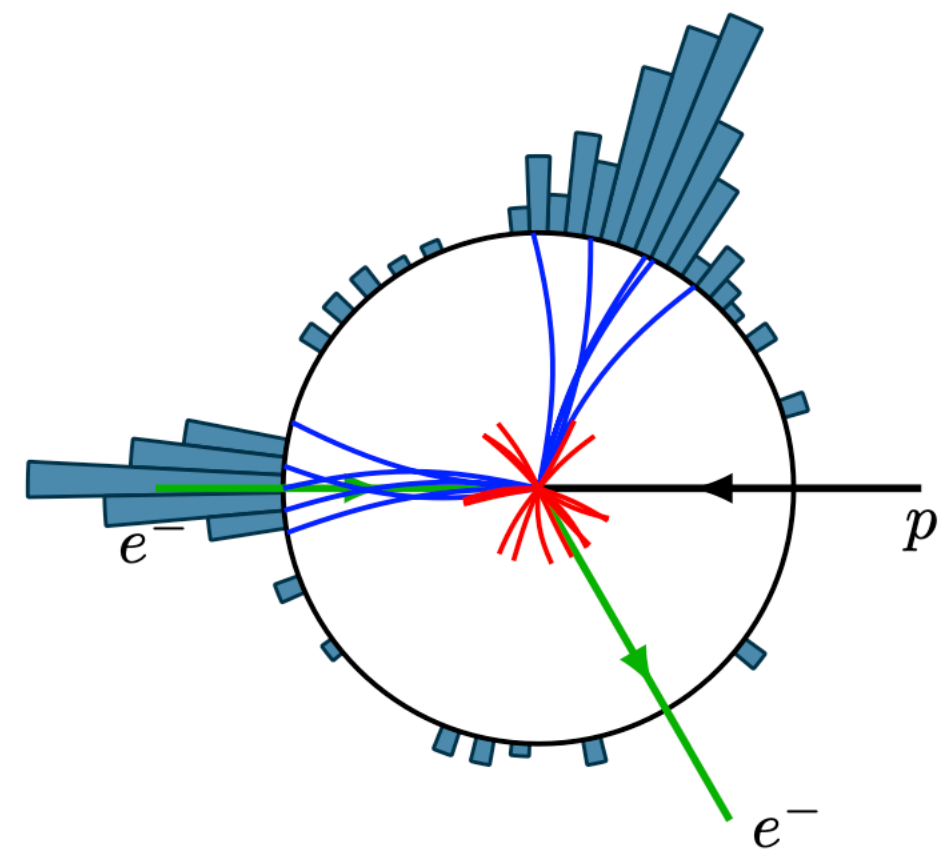
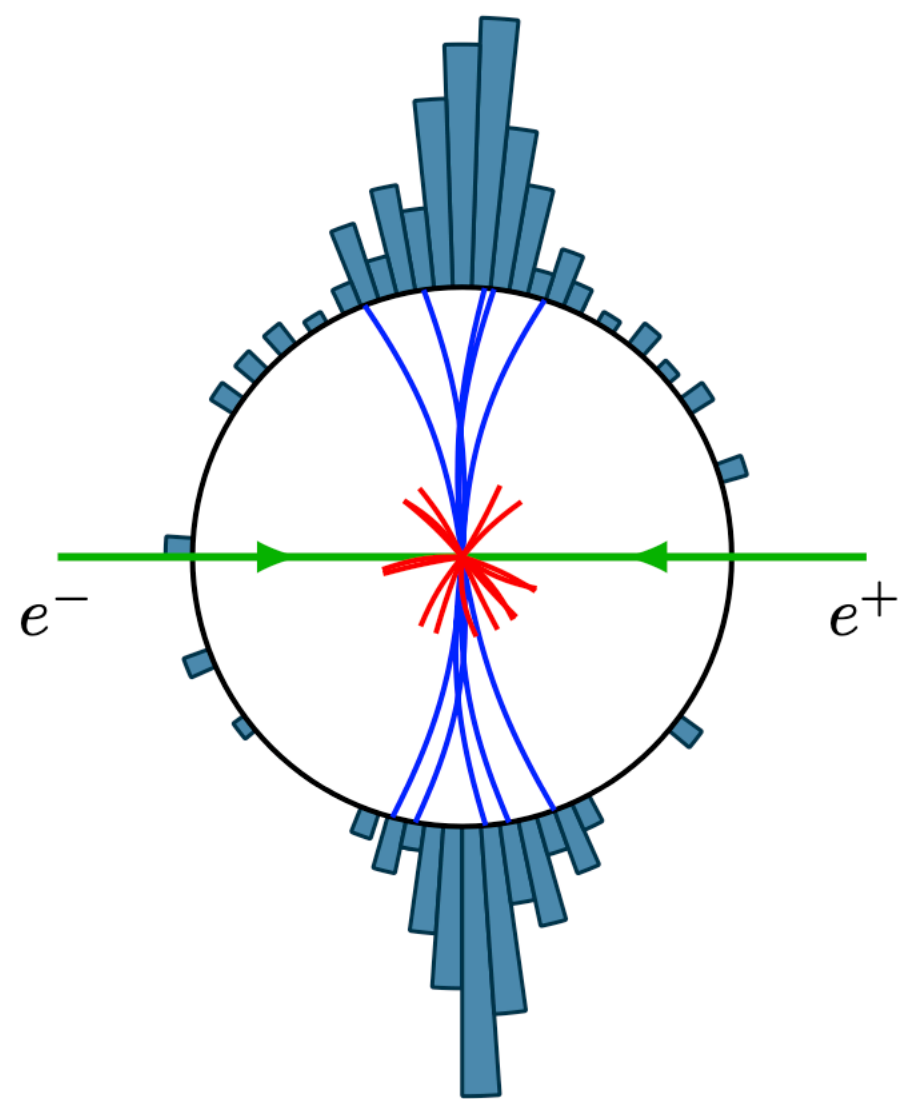
Komiske, Moulton, Thaler, HXZ, 2022





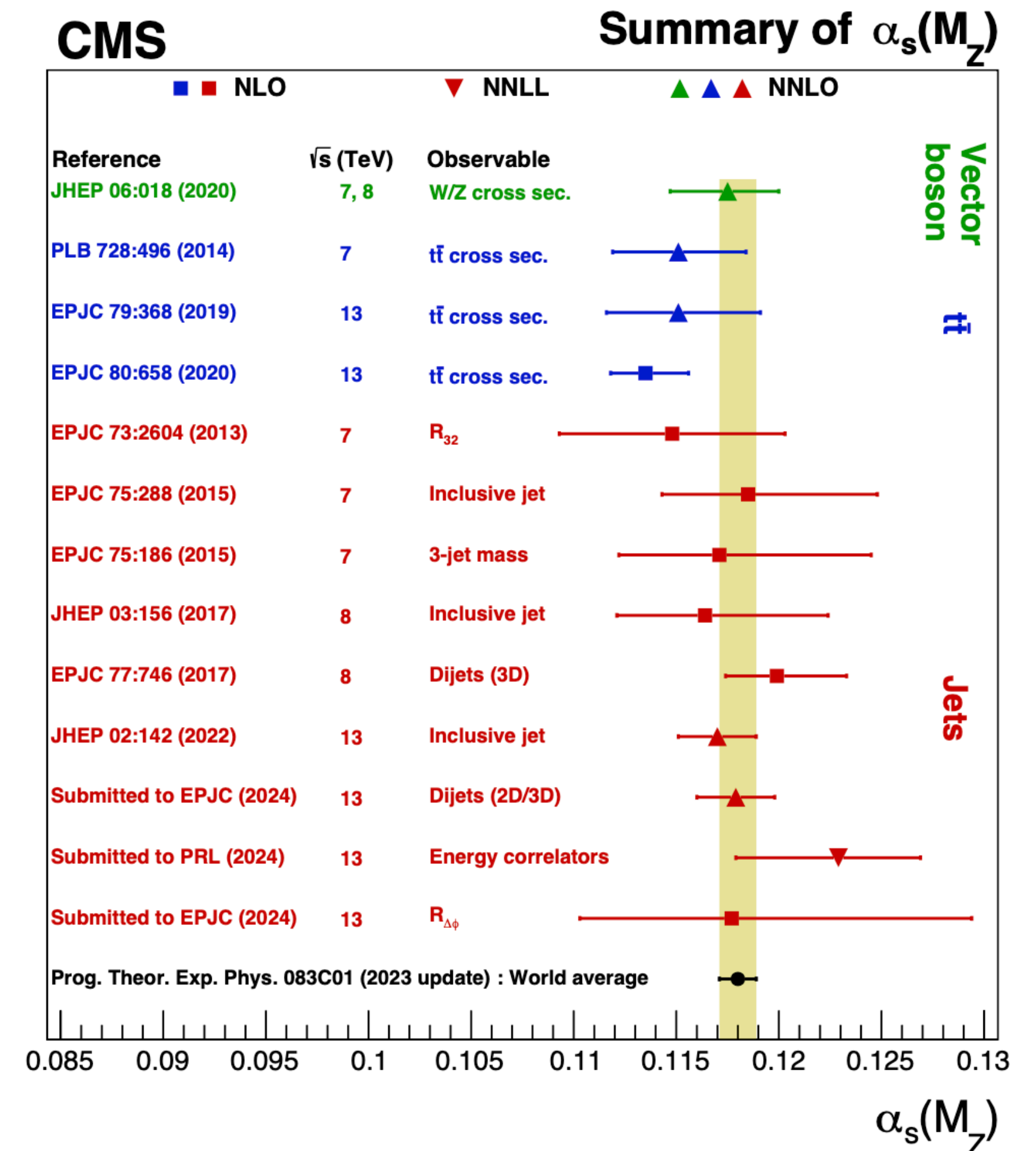
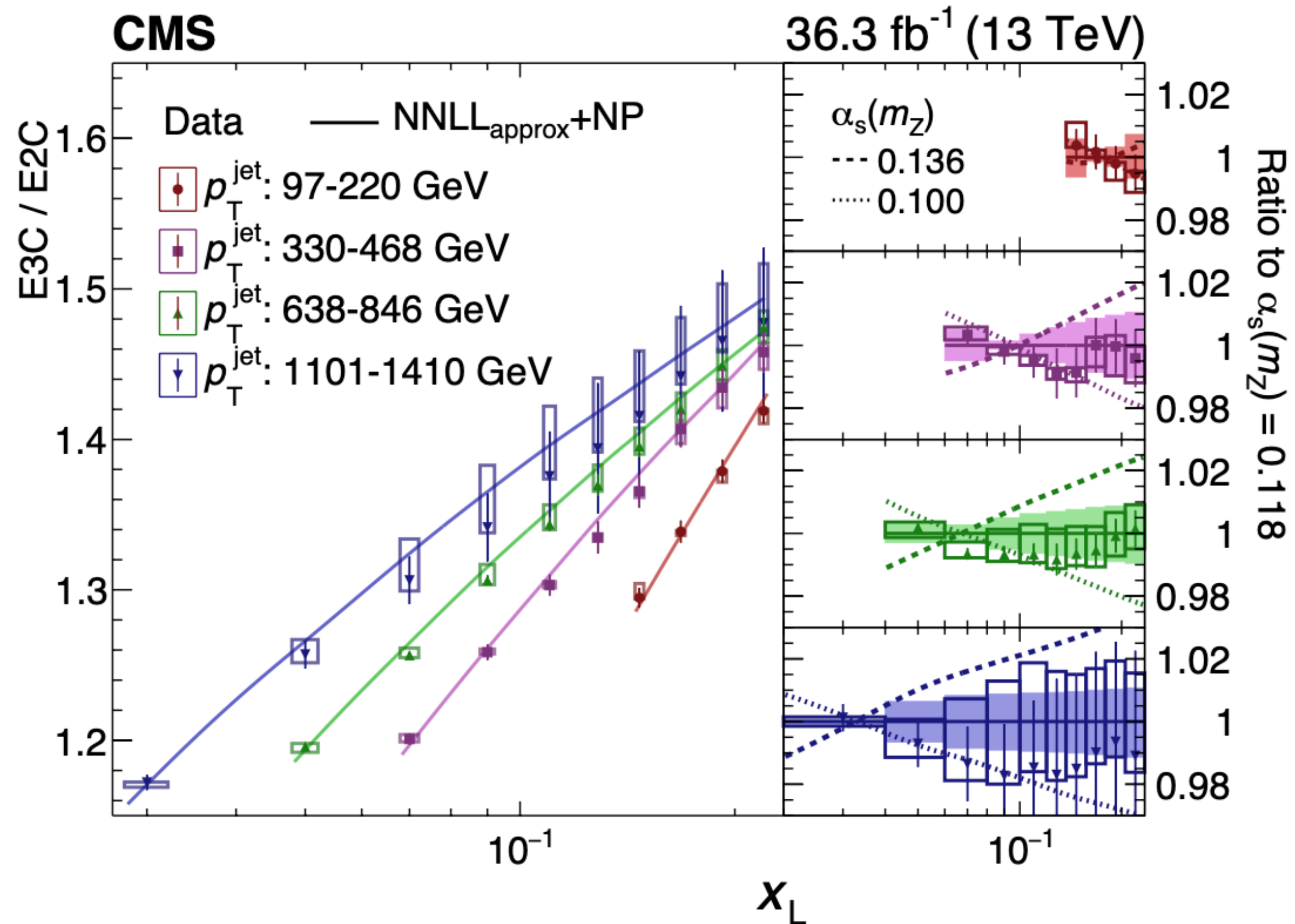
$$\langle \mathcal{E}(\hat{n}_1) \mathcal{E}(\hat{n}_2) \rangle_\psi$$

ψ : different states



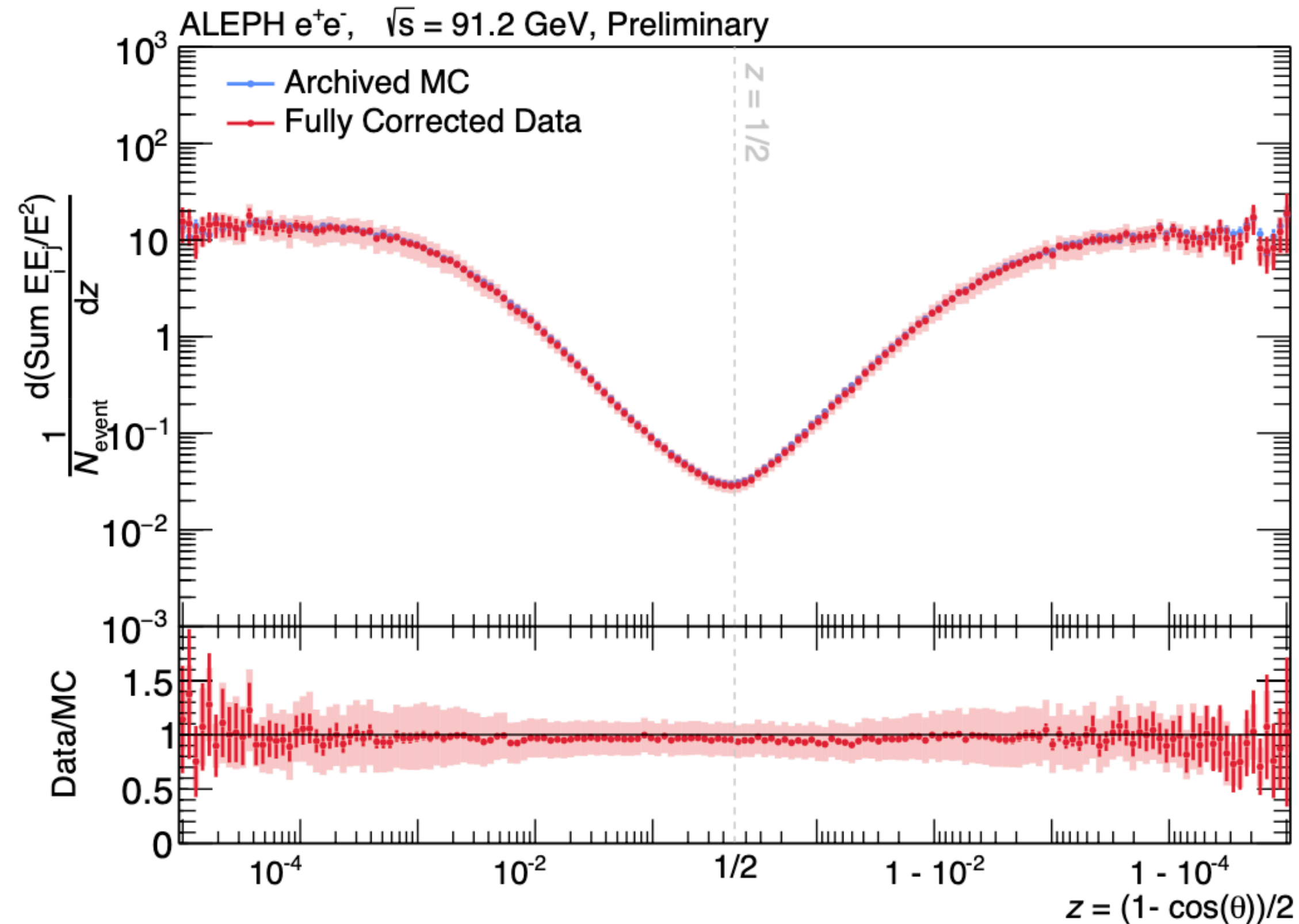
Energy operator in particle physics

Strong coupling constant

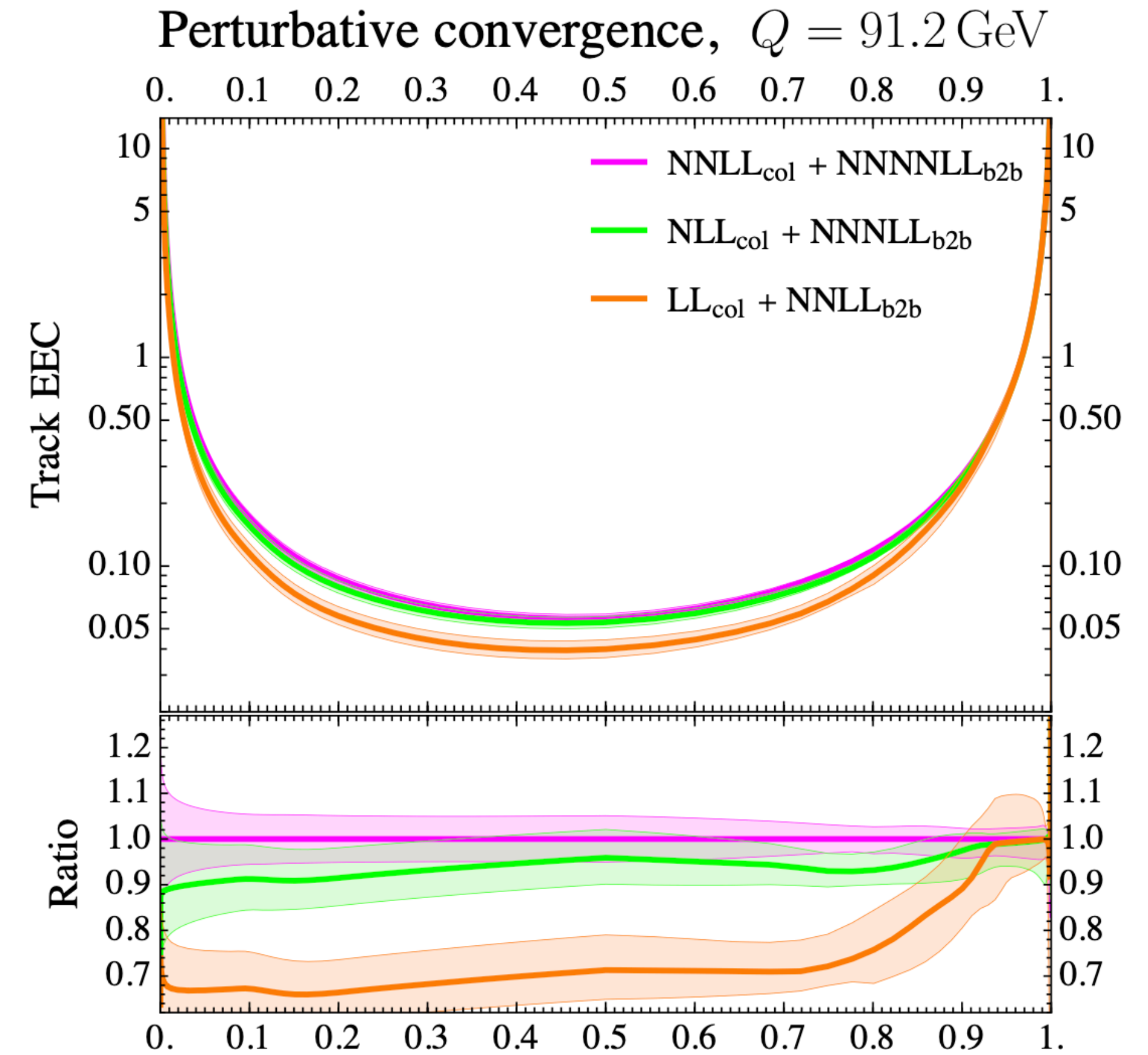


Most precise alphas from jet substructure!

LEP reloaded



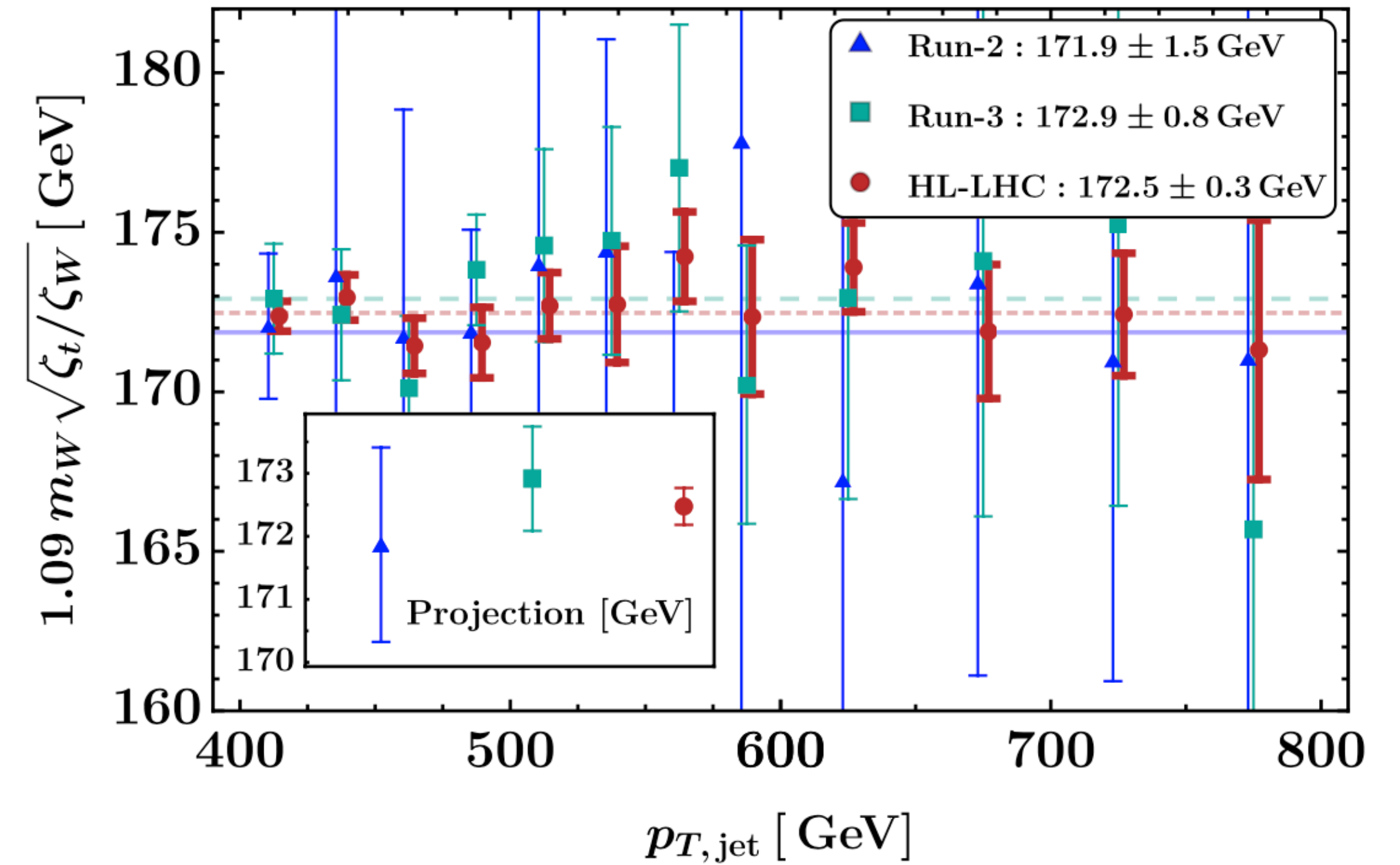
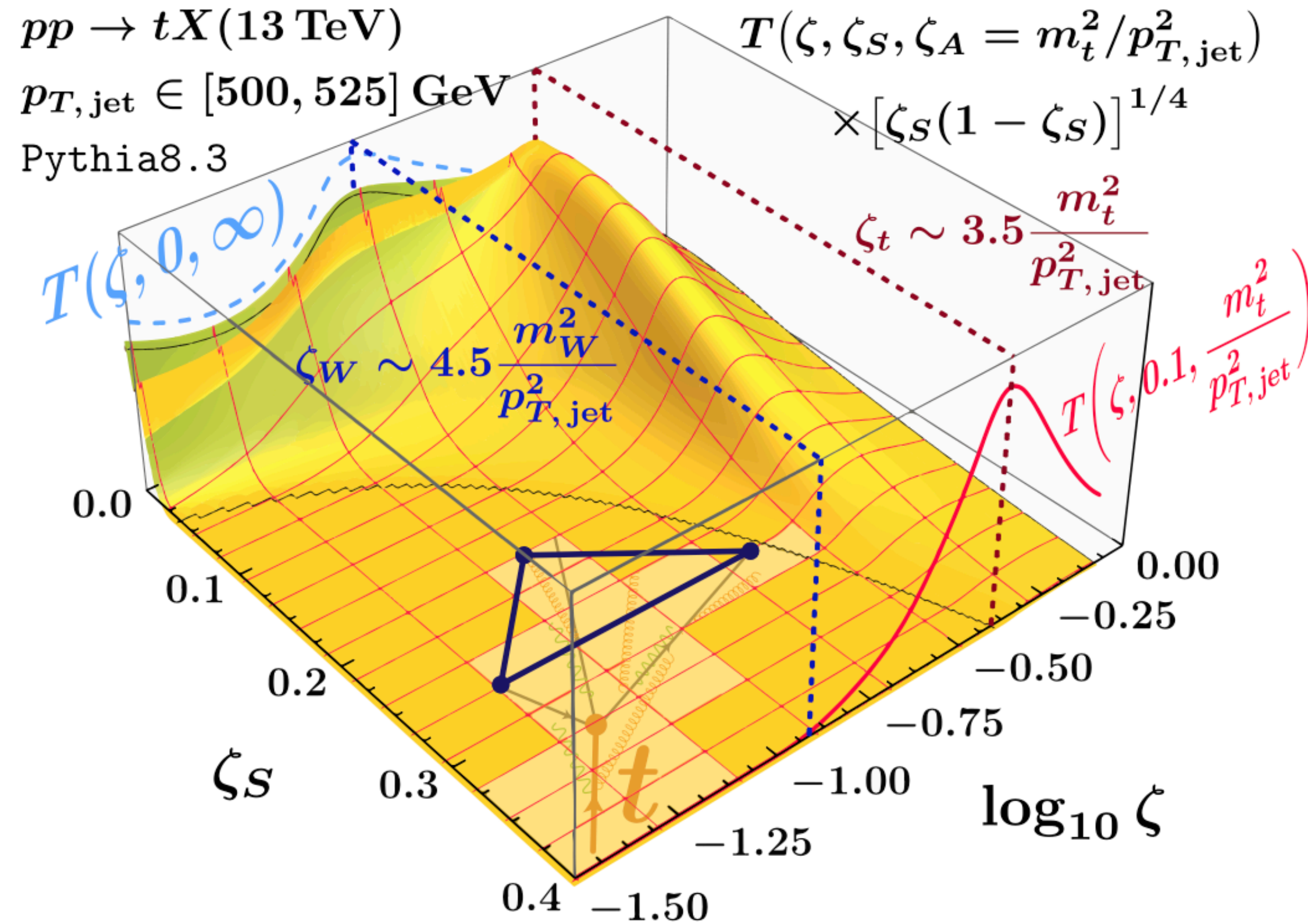
2024 Hard Probes Preliminary



$\approx$ Jaarsma et al., in progress

Complete analysis of EEC in the full phase space from experiment and theory

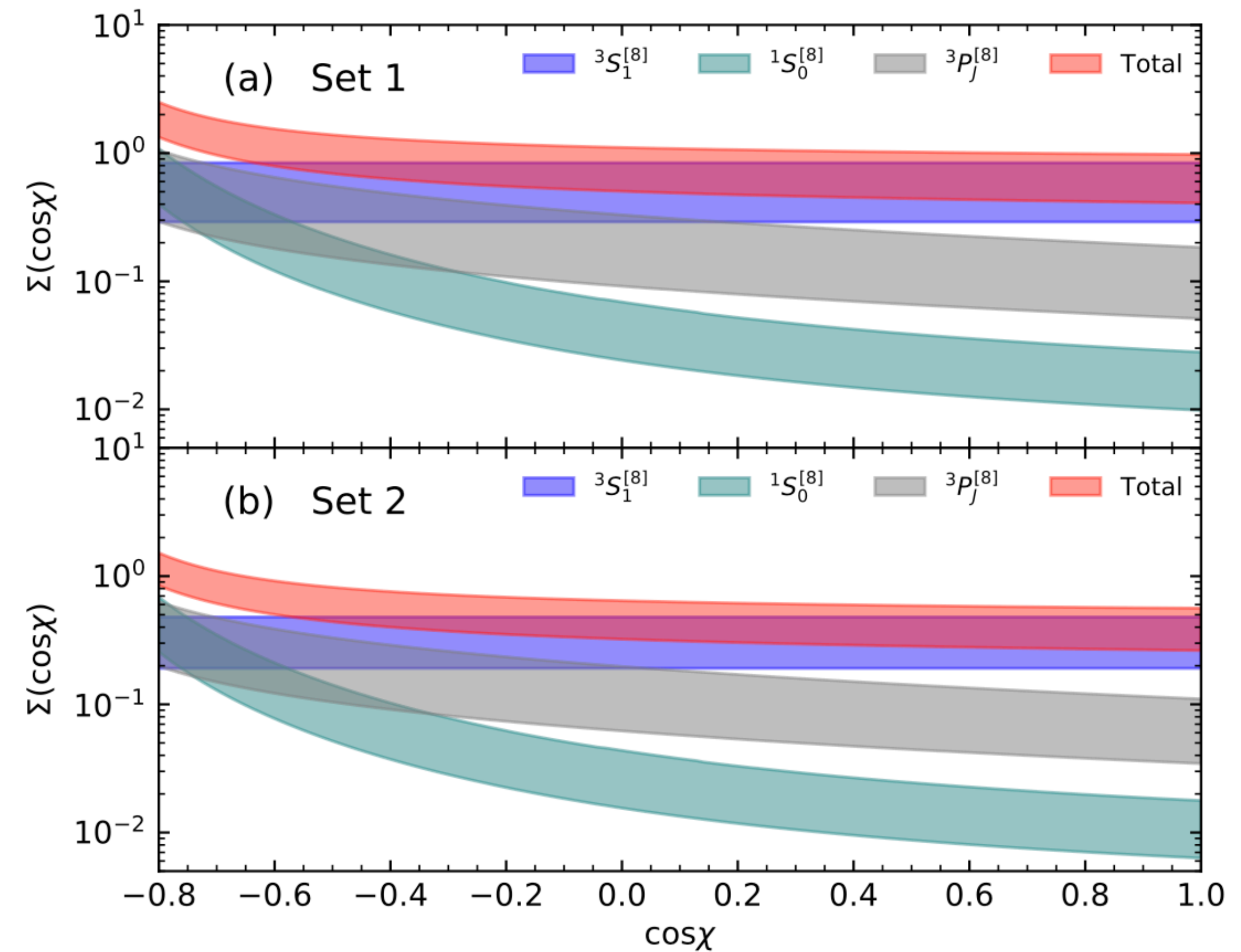
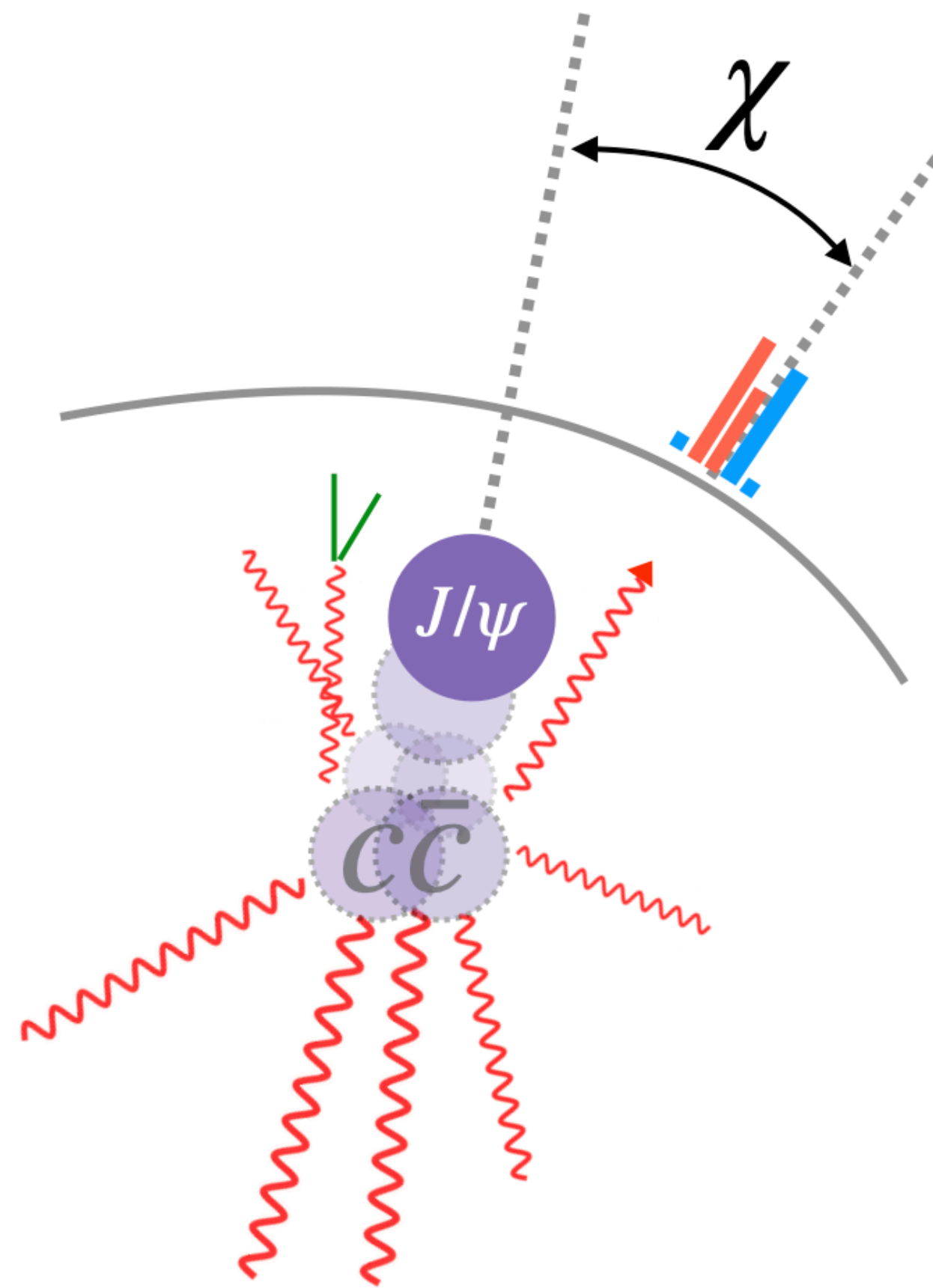
Top quark mass



Holguin, et al., 2024

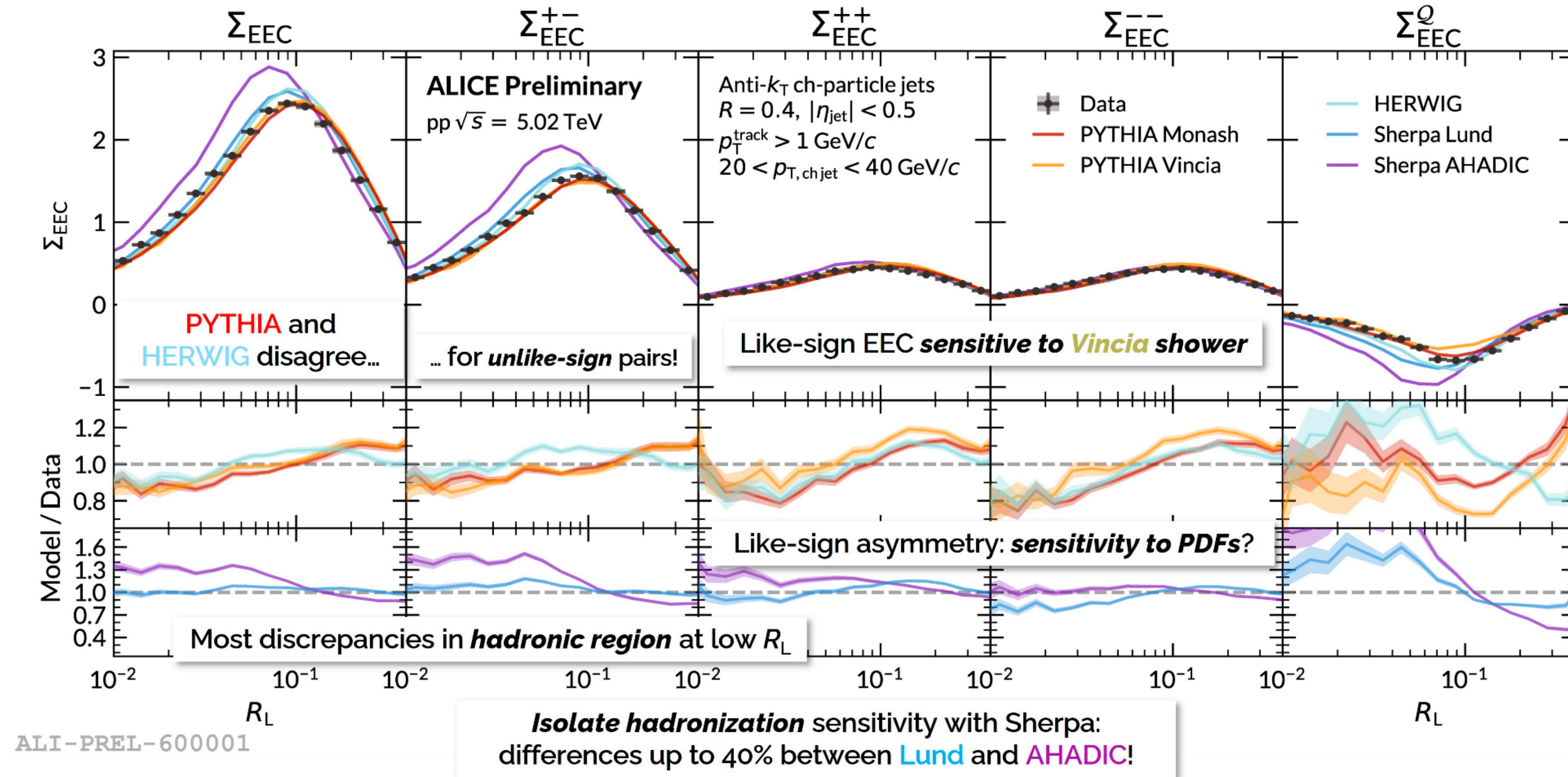
Three-point energy correlation can provide a precise measurement for top quark mass at high luminosity LHC

Quarkonium energy correlators



Chen, Liu, Ma, 2024

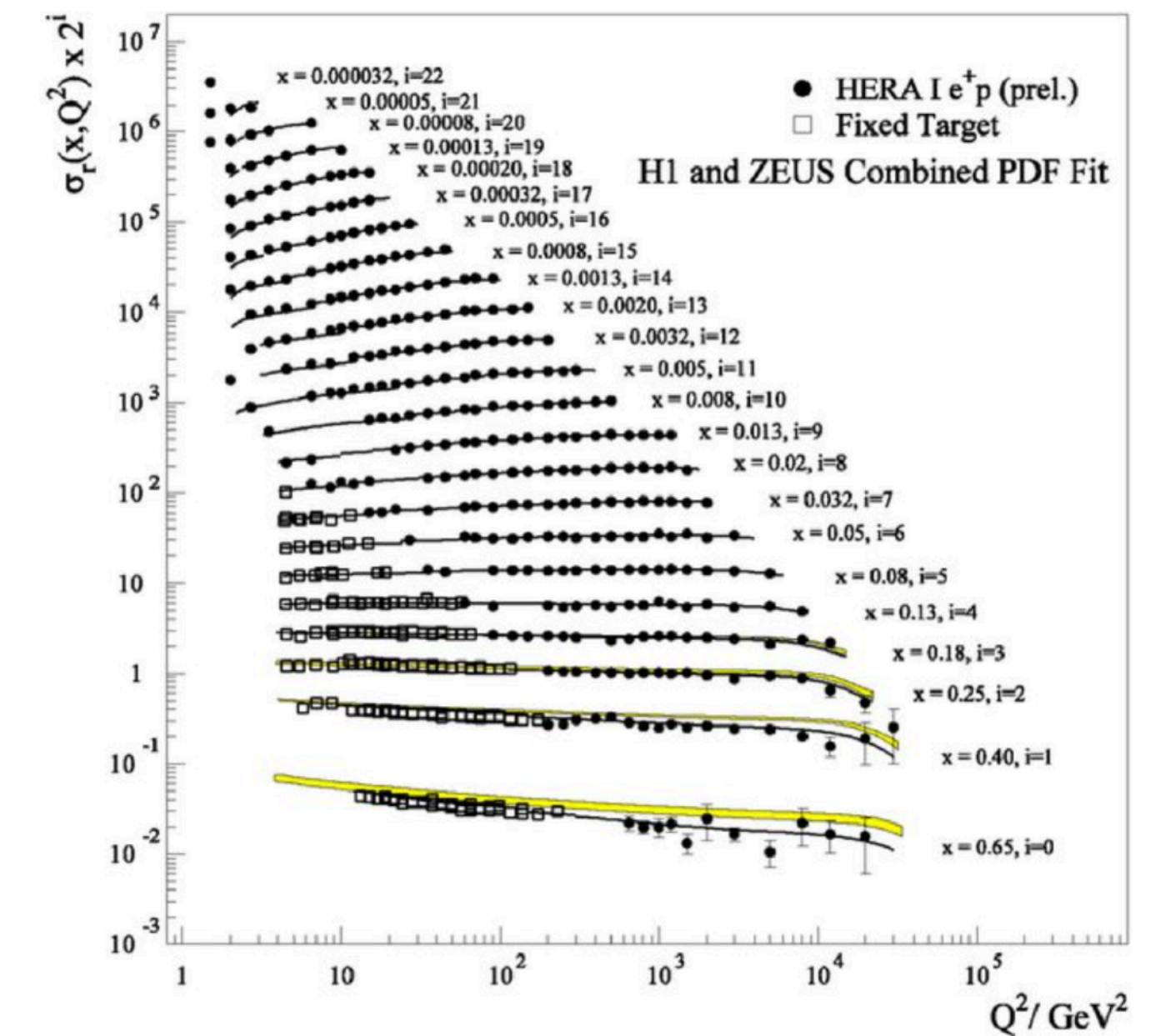
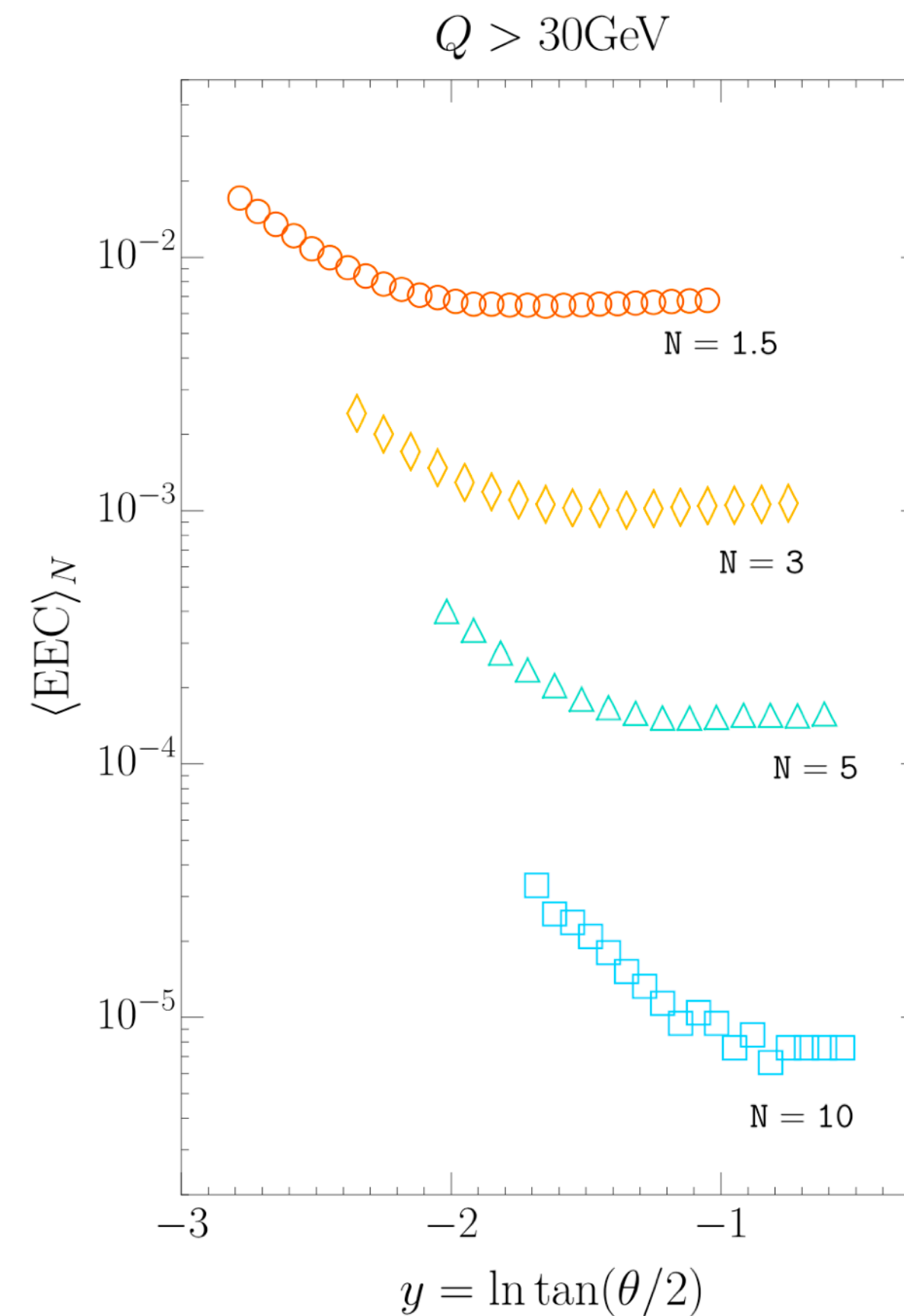
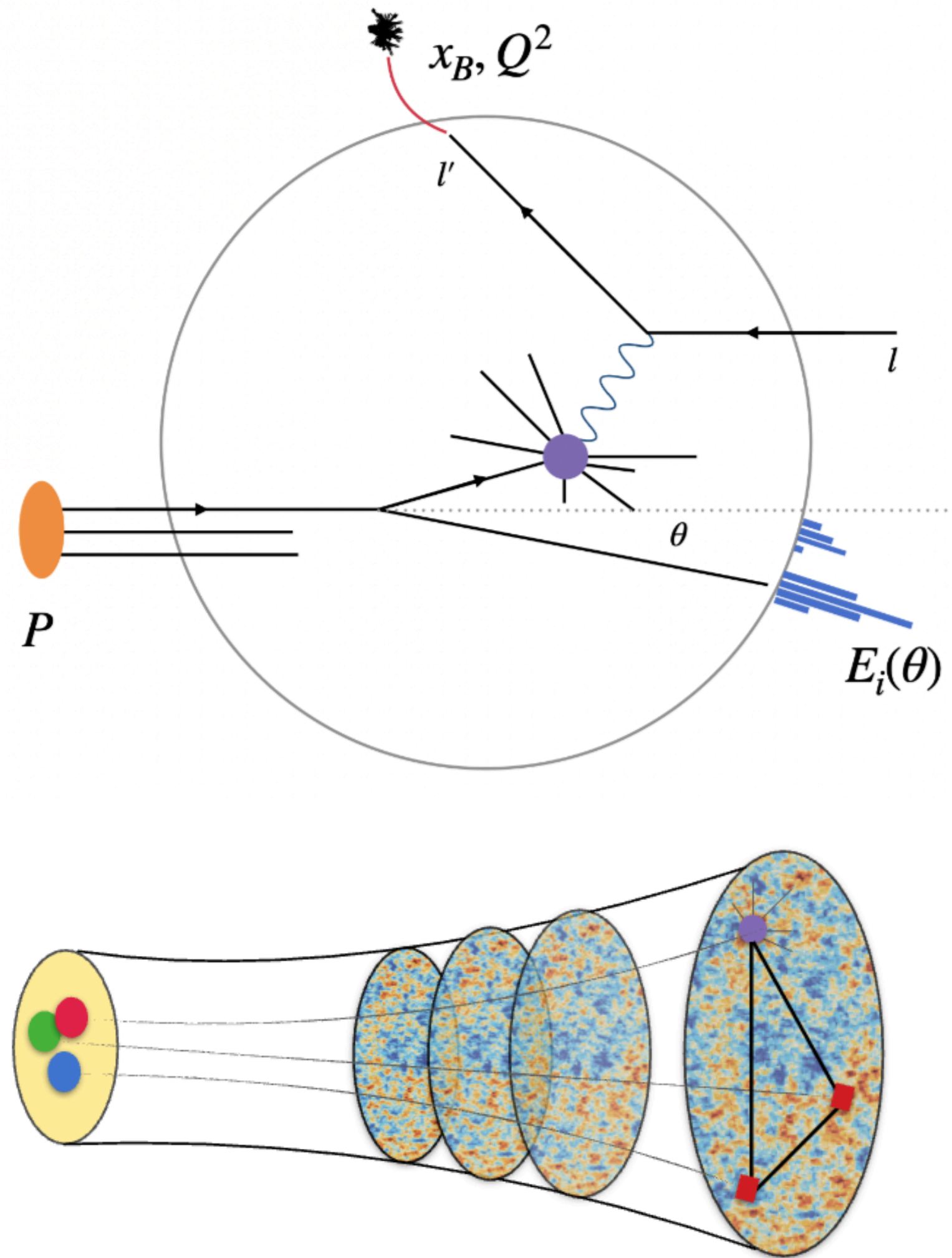
Charging the energy correlators



Poster by Hwang from Quark Matter 2025

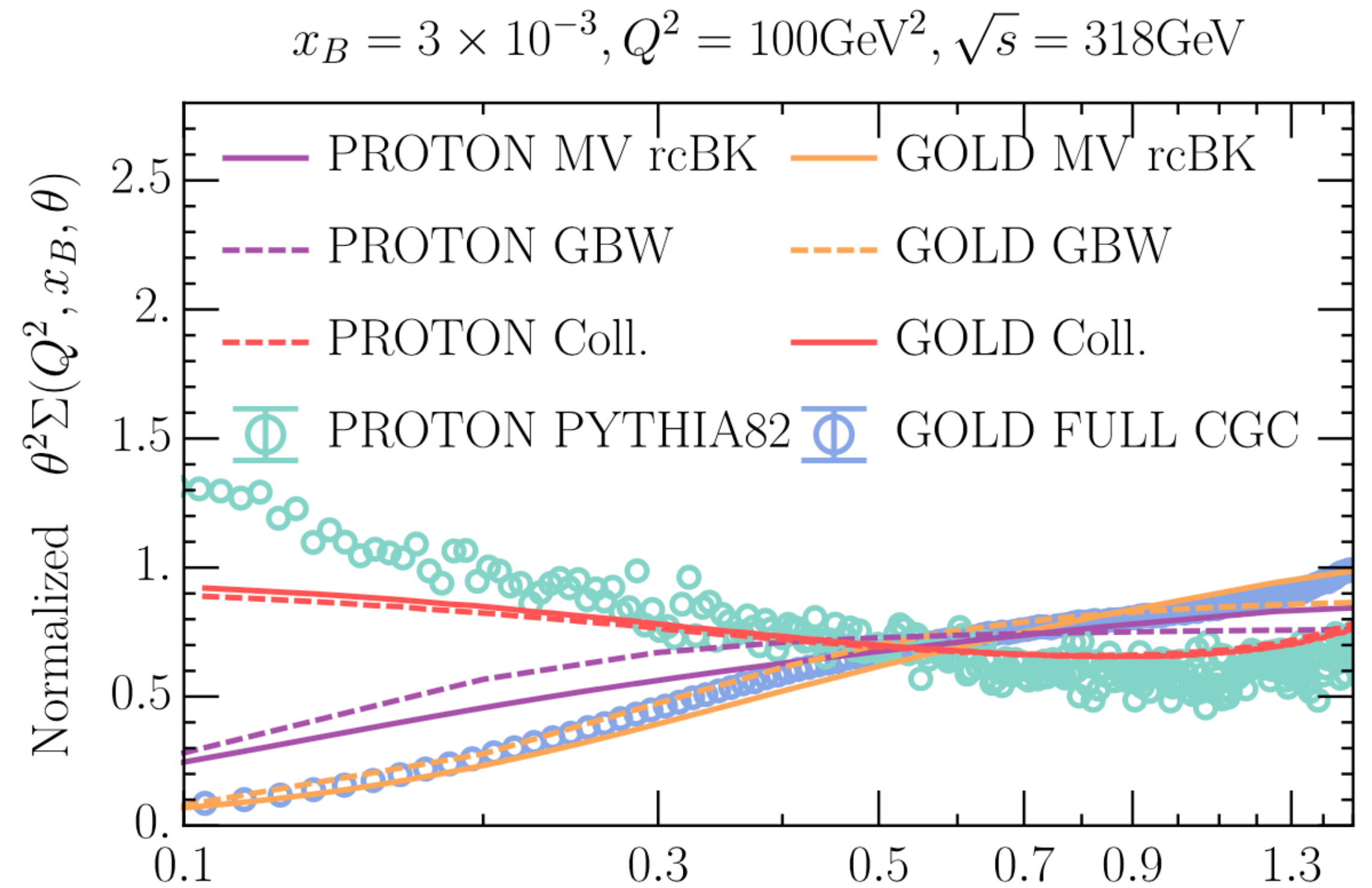
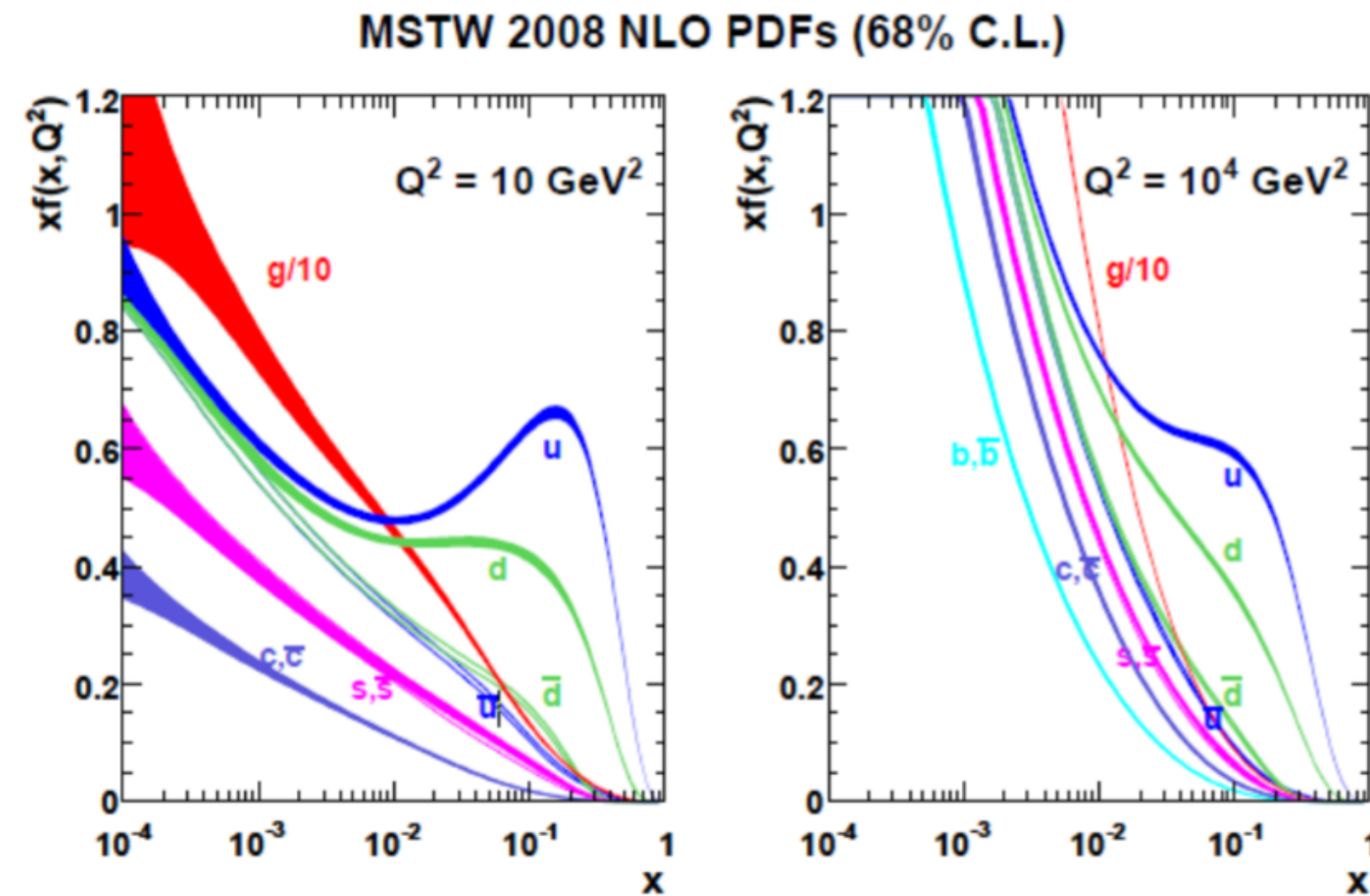
Energy operator in nuclear physics

Nucleon Energy Correlators

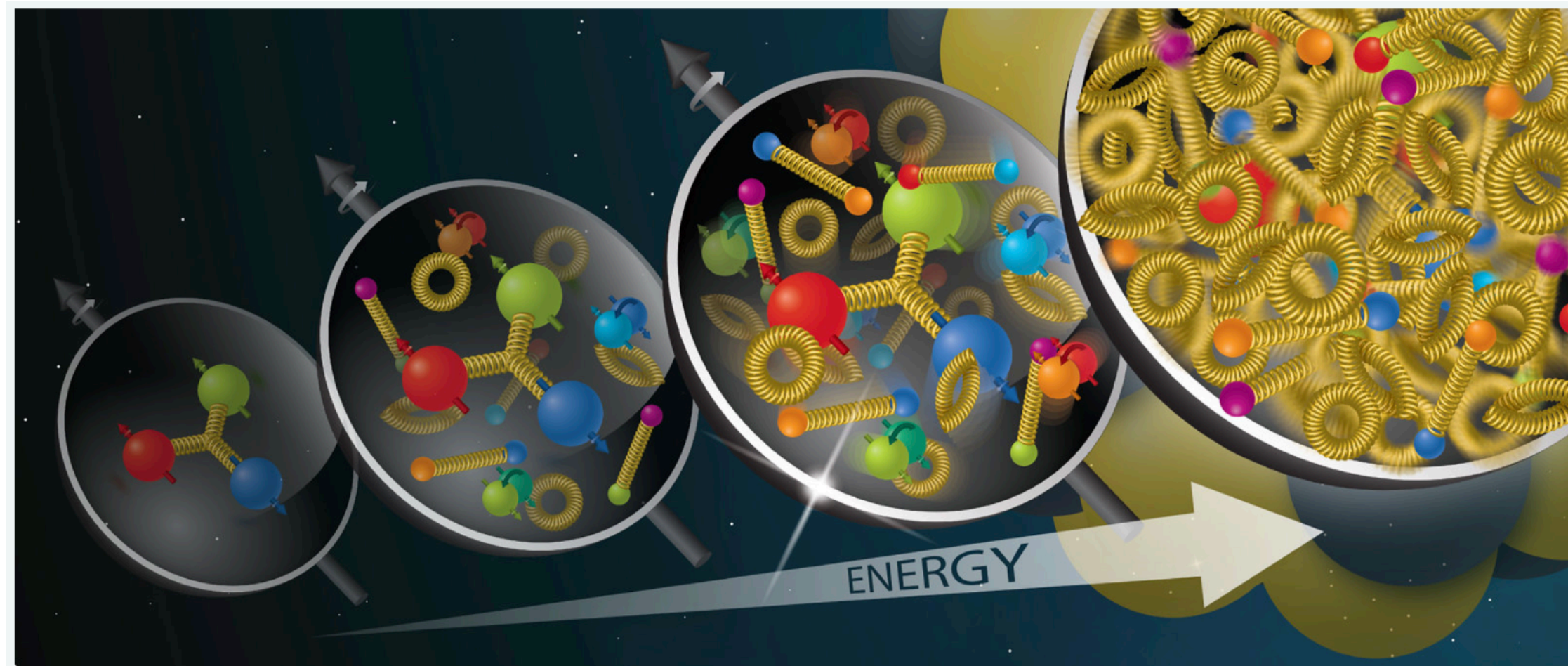


A new way to measure Bjorken scaling violation!

CGC and saturation scale

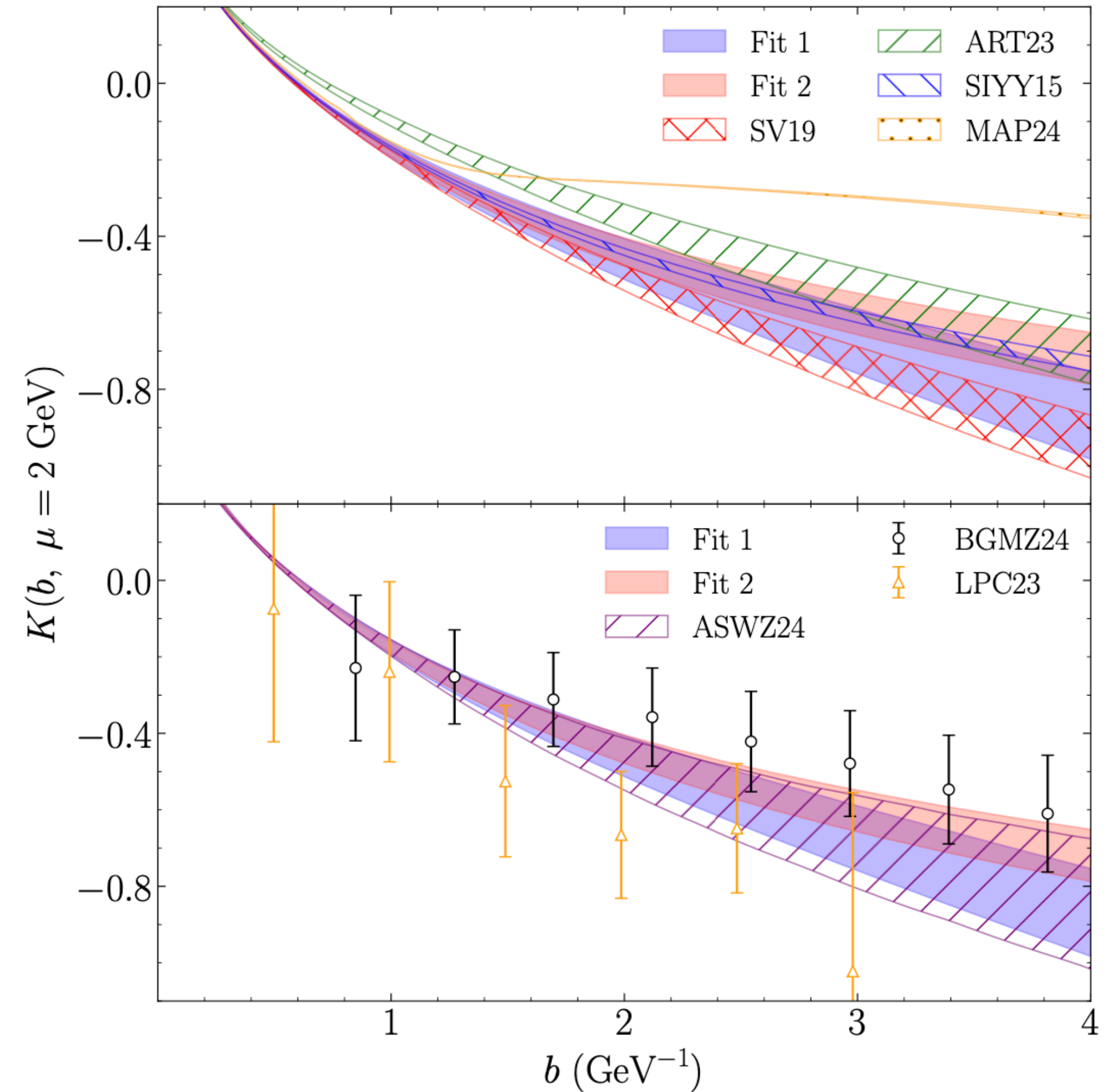
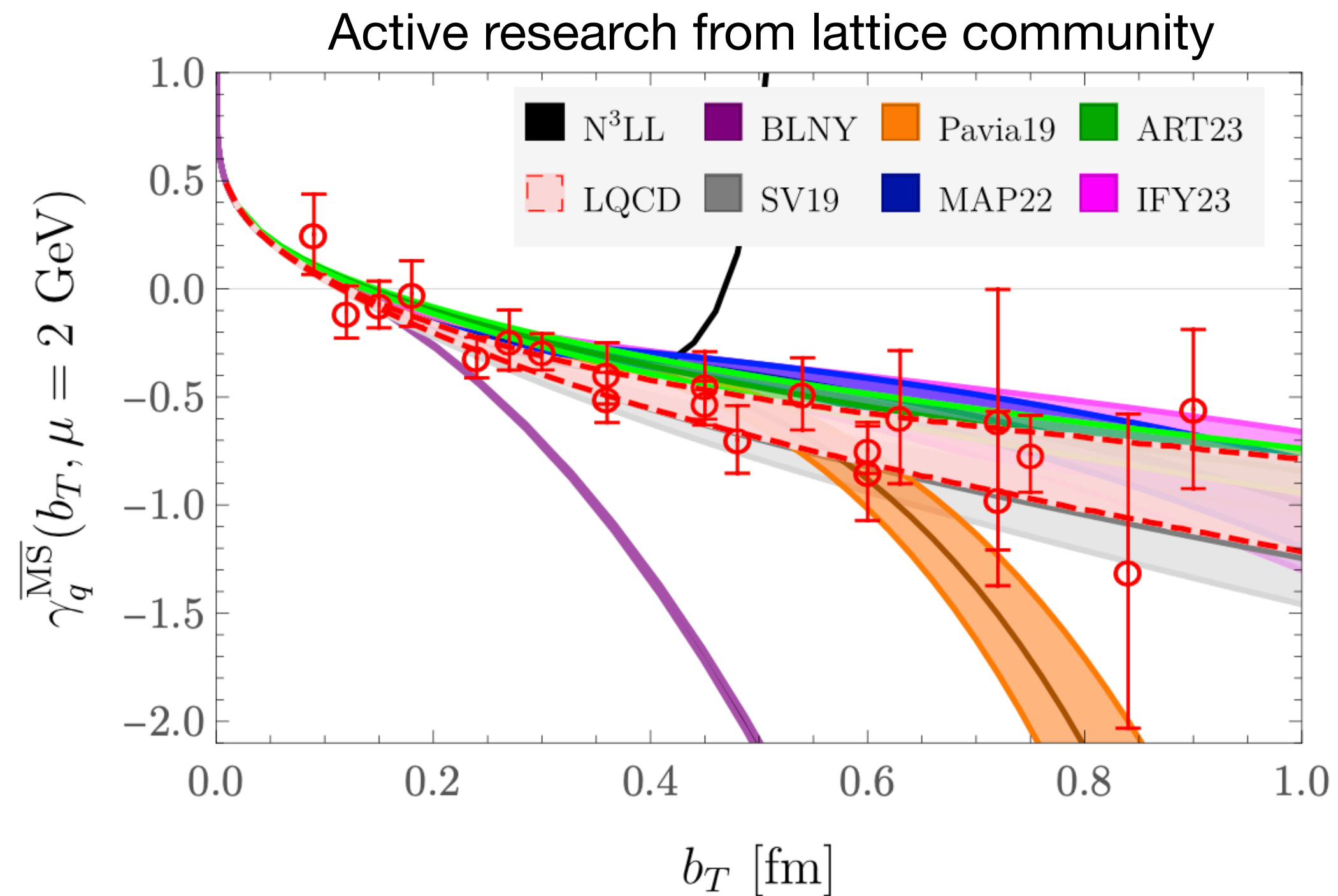
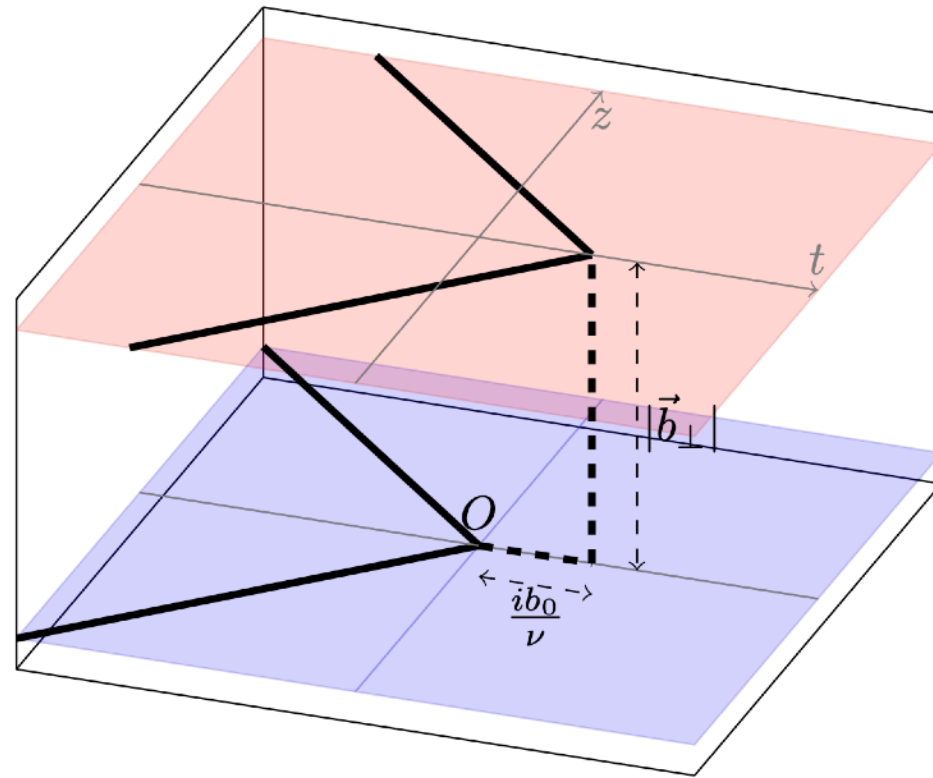


θ Liu, Liu, Pan, Yuan, HXZ, 2023



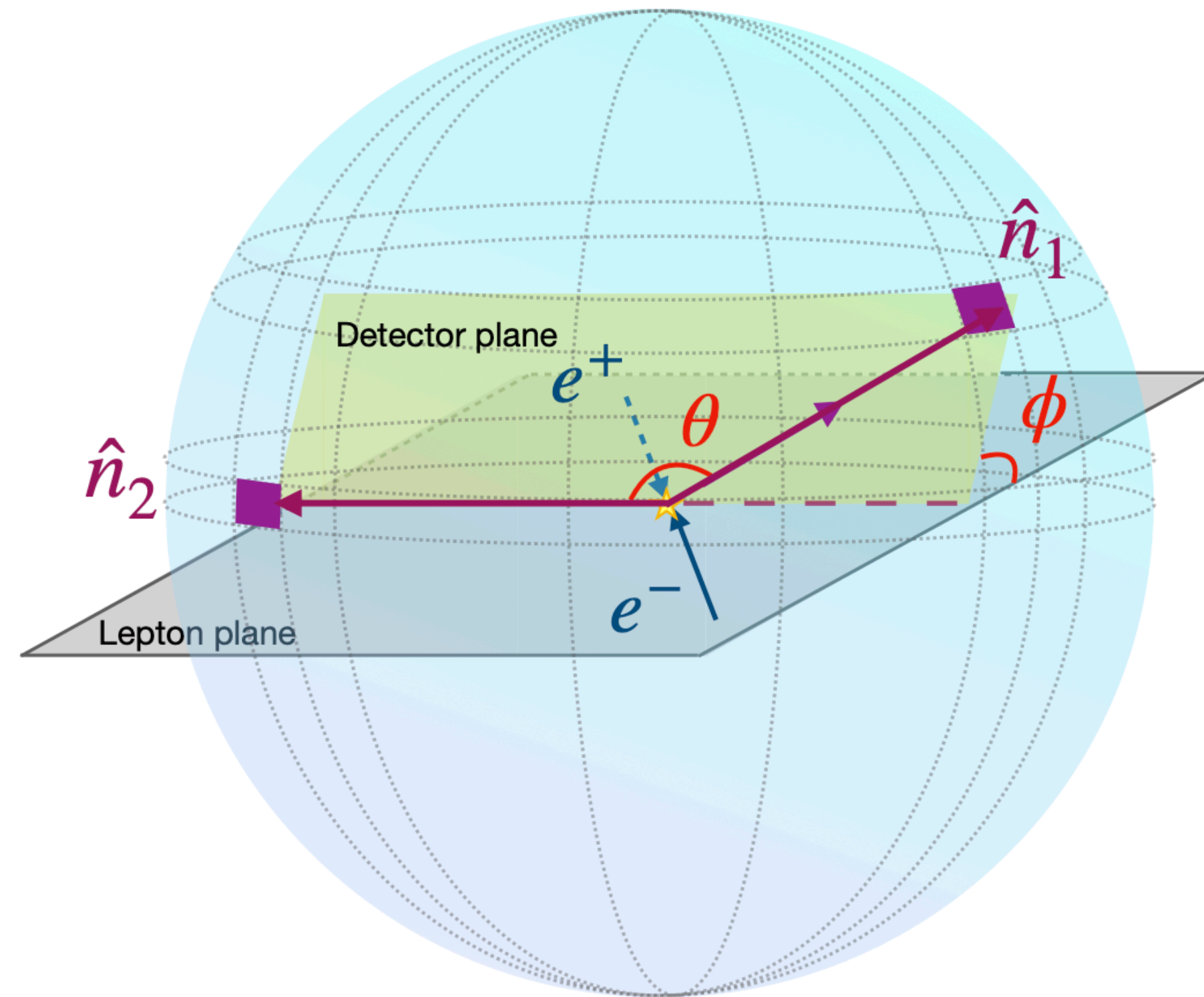
Search for the scale of gluon saturation
from angle scan

Collins-Soper Kernel



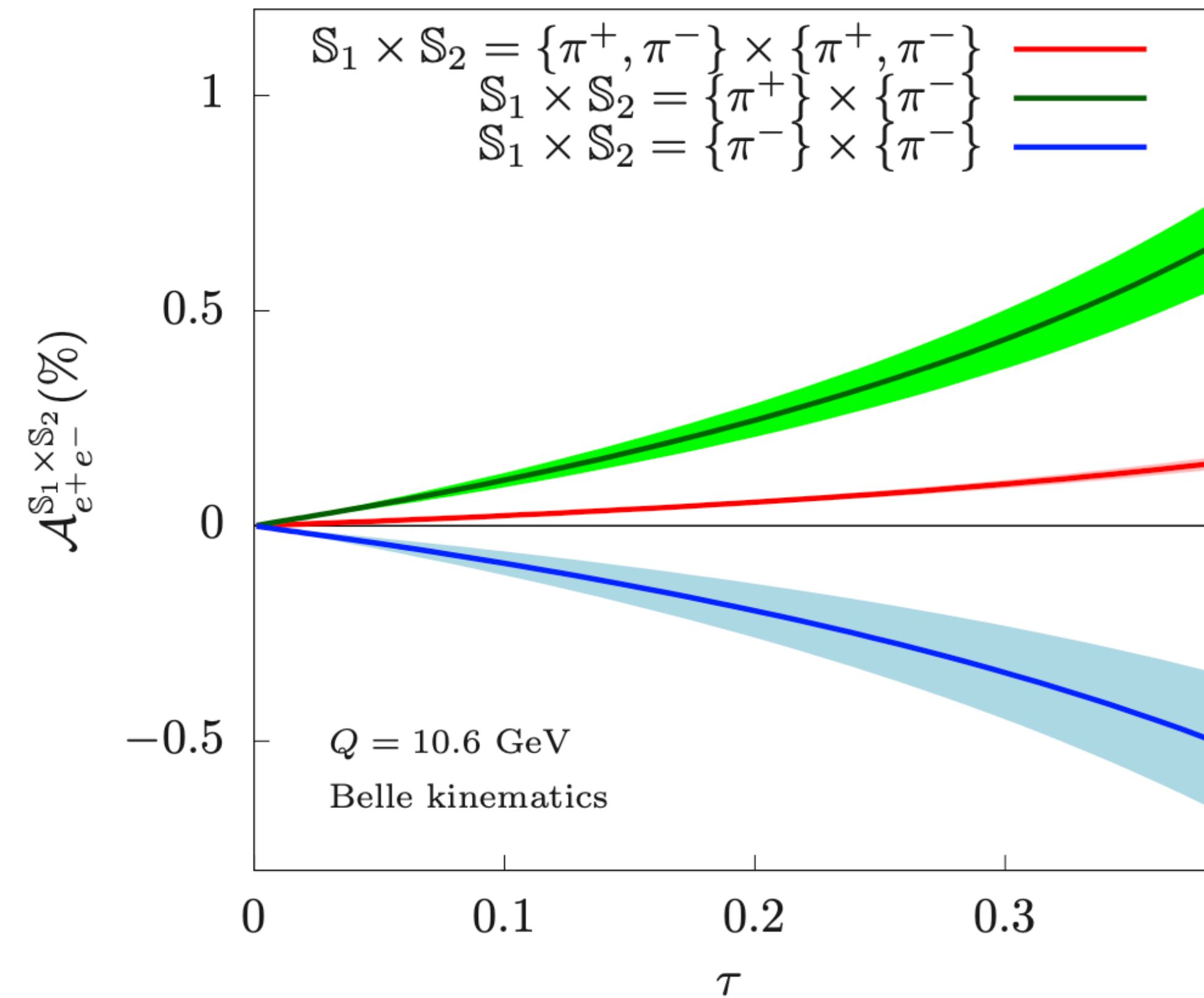
Kang, Penttala, Zhang, 2024

Non-perturbative Spin physics



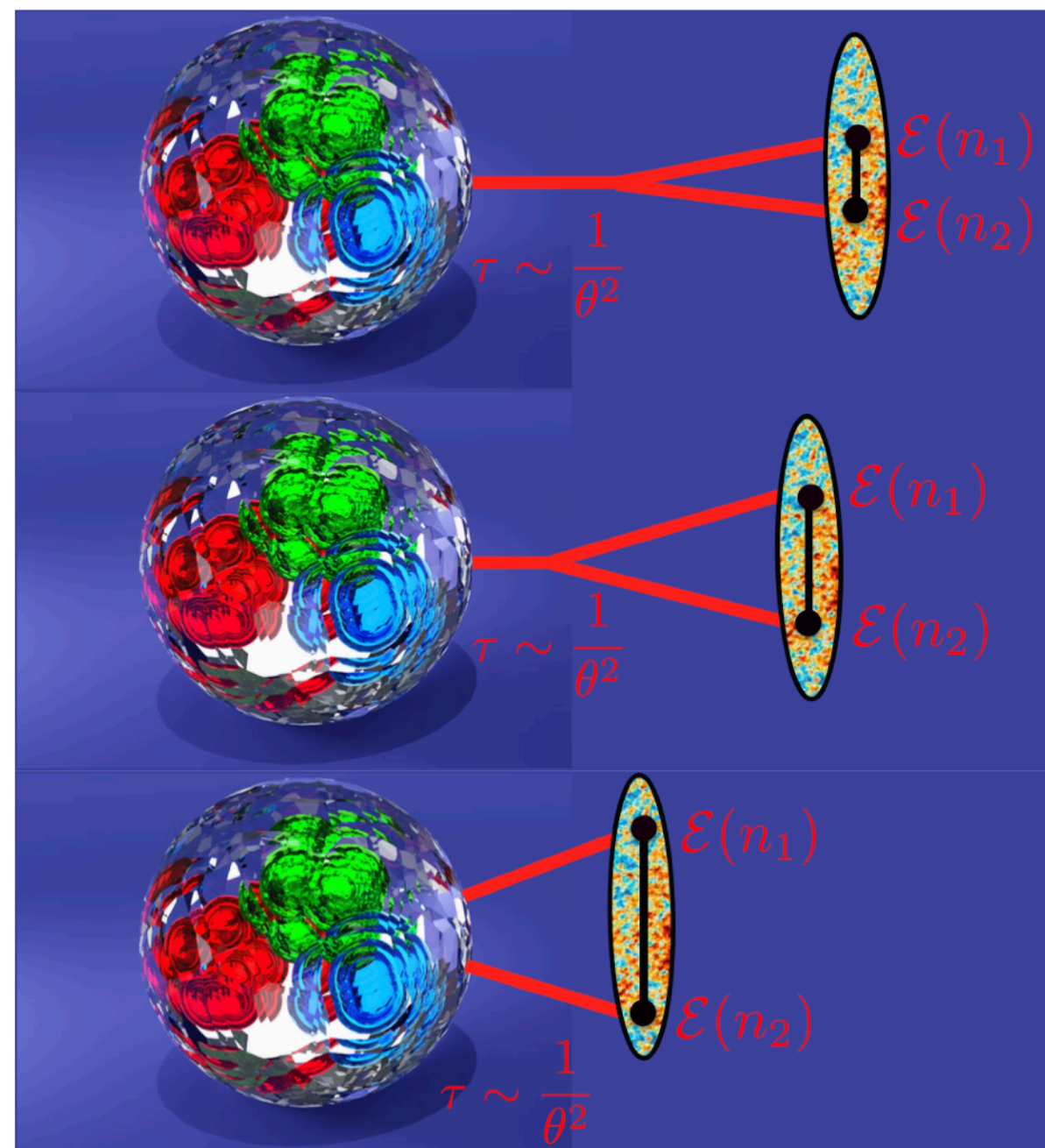
Probe Collins and Sivers function of QCD

Important for understanding QCD chiral symmetry breaking!

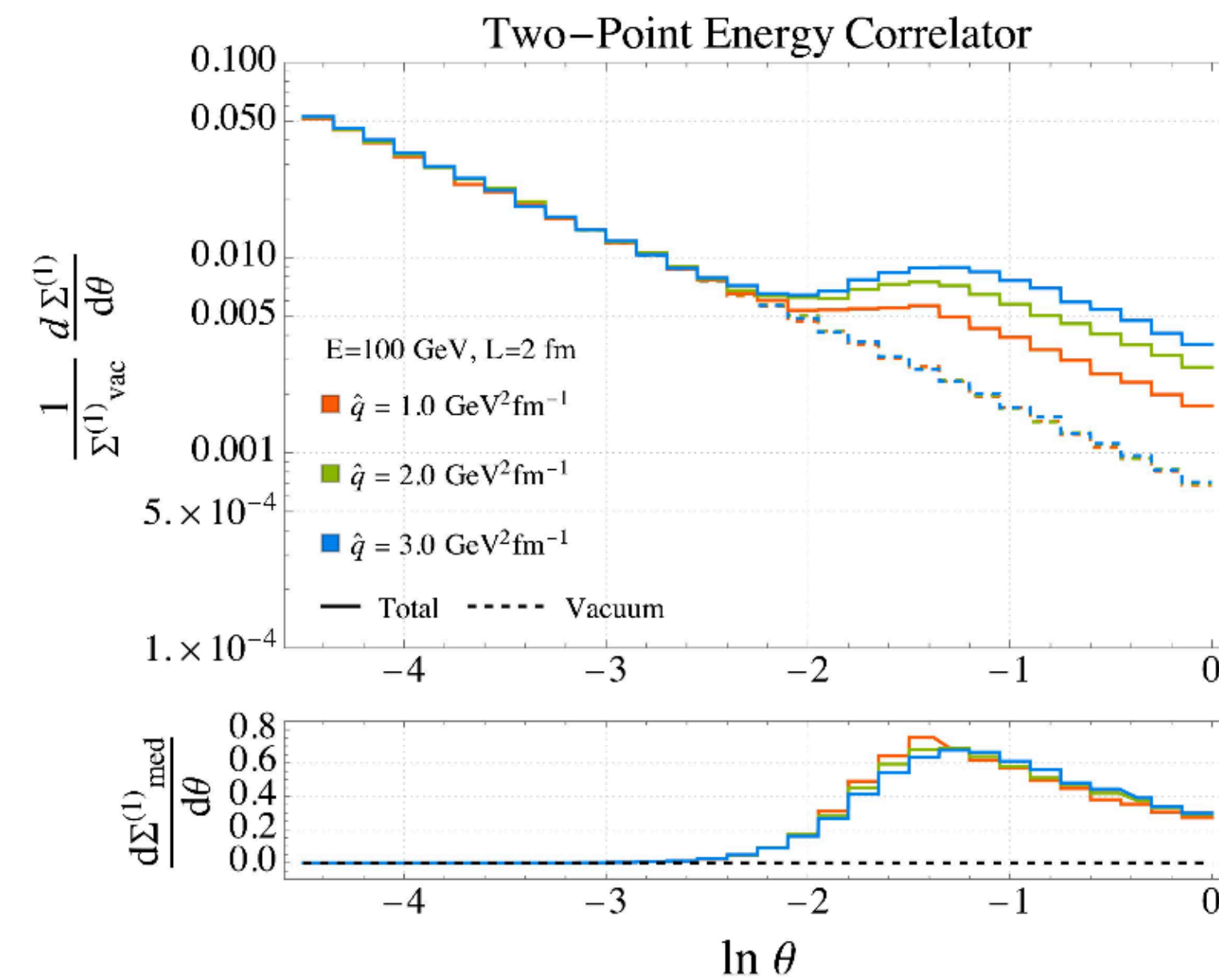


Kang, Lee, Fan, Shao, 2024

Probing properties of QGP



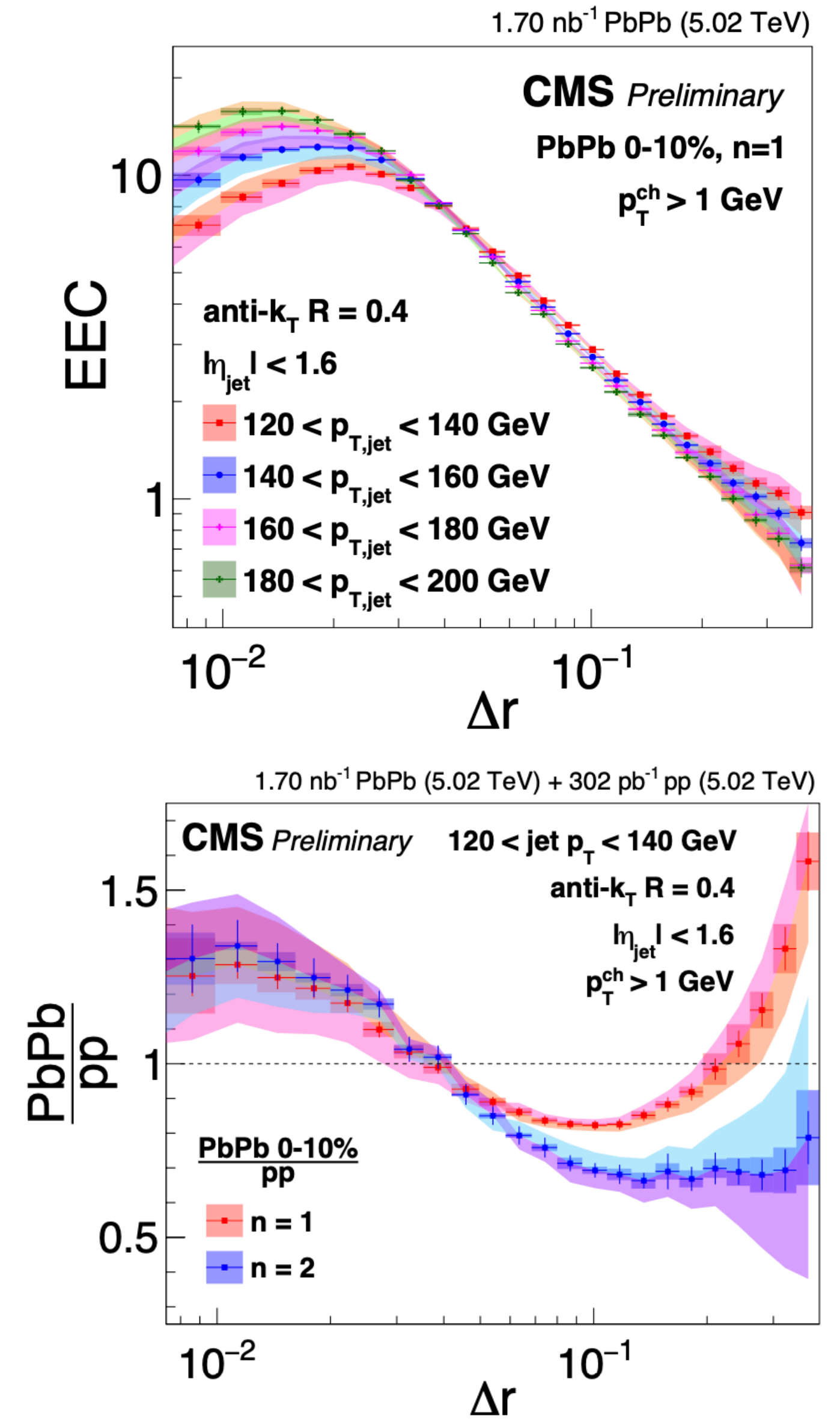
Increasing θ



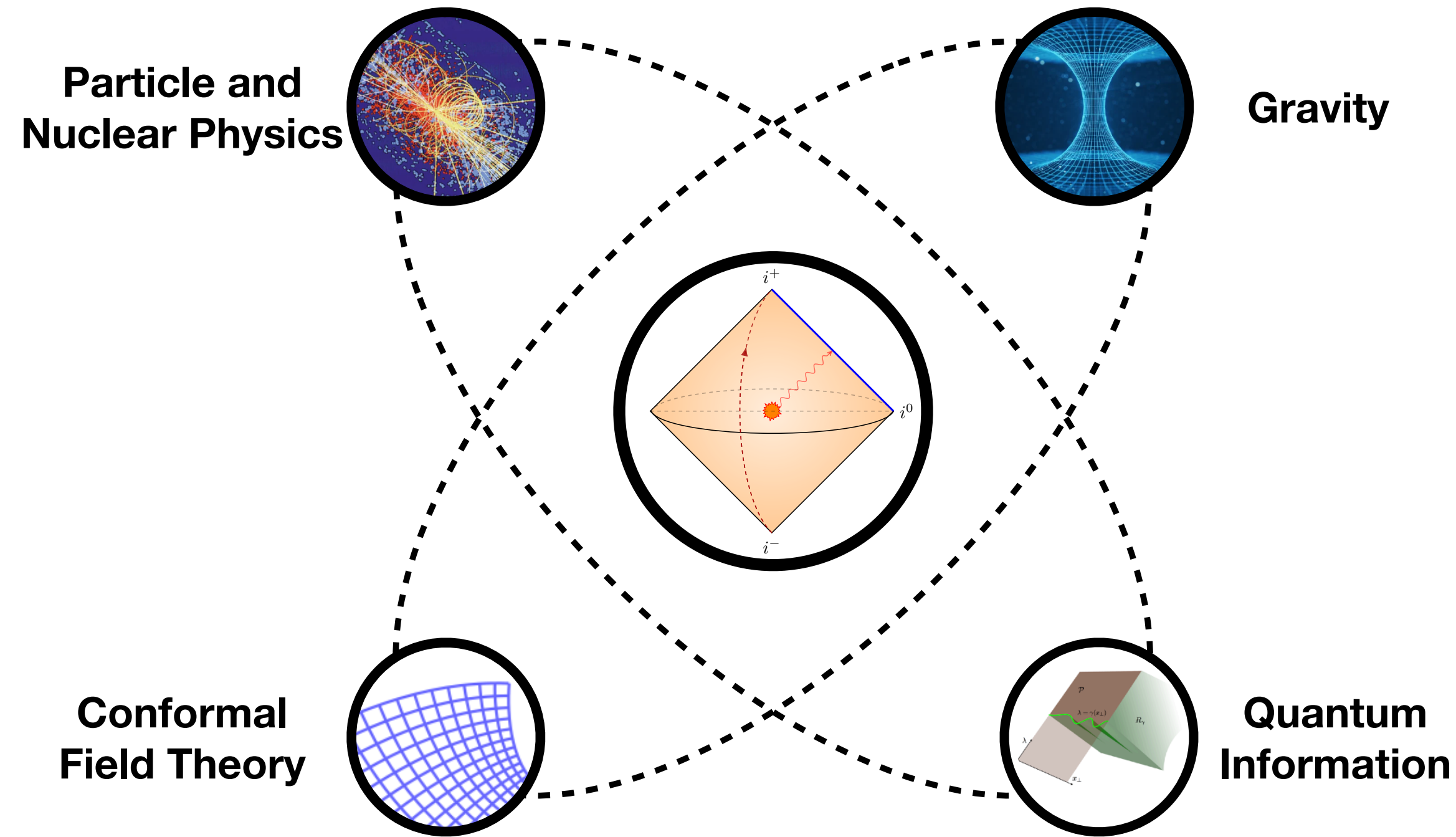
Andres et al., 2022

Yang et al., 2023

...

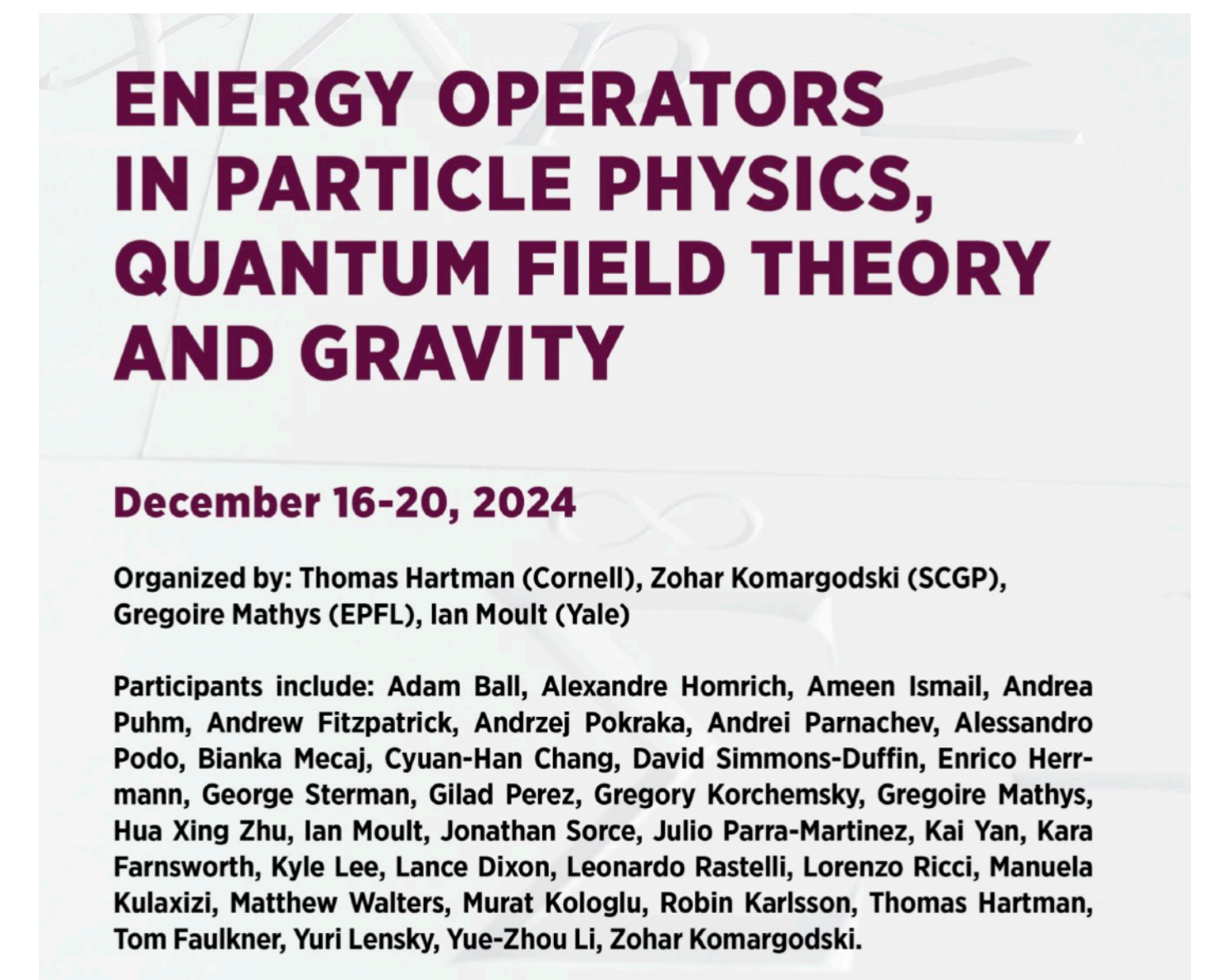
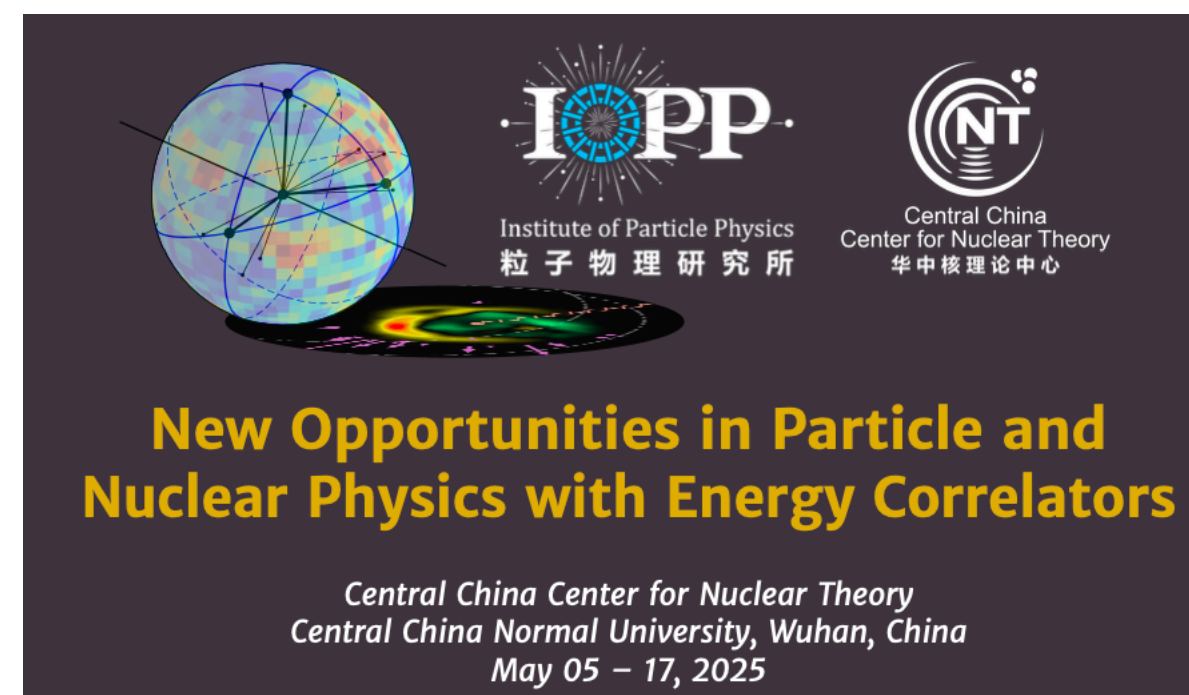
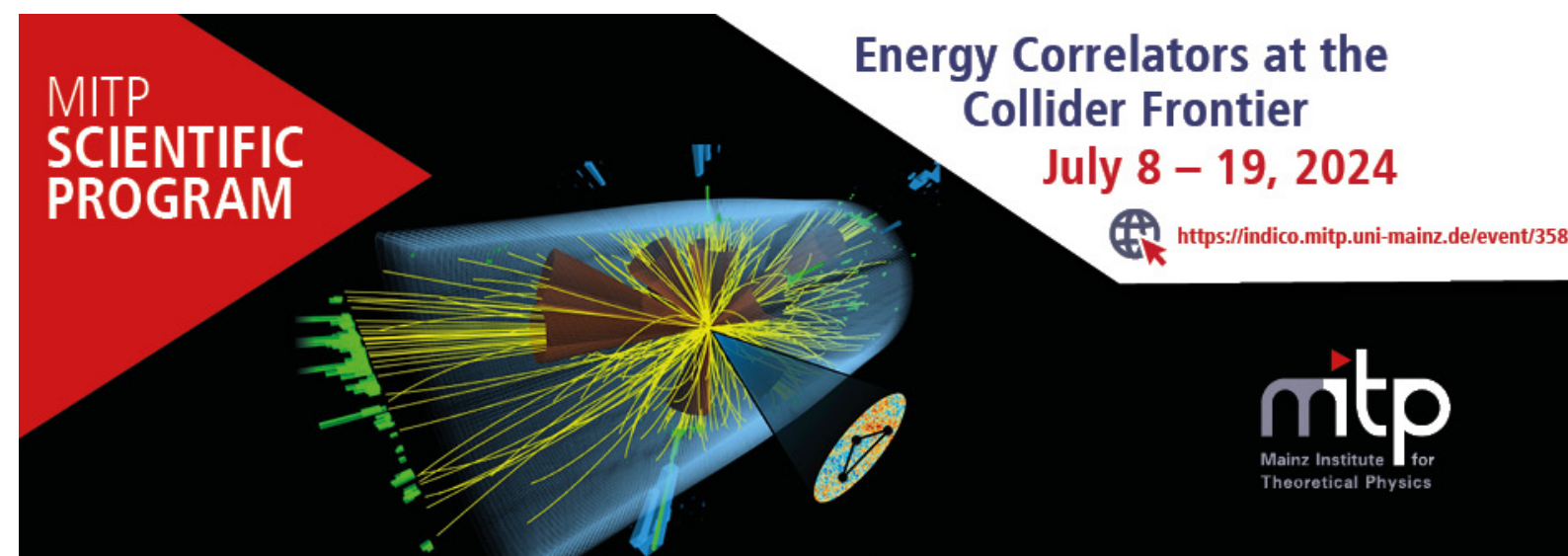


Summary



Energy correlators physics provides a window connecting particles and nuclear physics, conformal field theory, gravity and quantum information

1. Wuhan, Huazhong normal university EEC one-day workshop, 2023
2. Mainz MITP two-week program, 2024
3. Simons Center for Geometry and Physics, 2024
4. Wuhan C3NT inauguration two-week program, 2025
5. Lanzhou fragmentation function and EEC workshop, 2025
- 6. KITP 3 month program, 2027**



Expecting more surprises in the coming years. Stay tuned!