B介子两体非轻衰变中的疑难问题和一种可能的理论解决方案

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Based on works:

[1] S. Lü and **M. Z. Yang**, NPB972, 115550 (2021).
[2] S. Lü and **M. Z. Yang**, PRD107, 013004 (2023).
[3] R.X. Wang, **M.Z. Yang**, PRD108,013003 (2023).
[4] S. Lü, R.X. Wang, **M. Z. Yang**, PRD110,056025 (2024)
[5] Y.H. Gui, **M. Z. Yang**, PRD111,036007 (2024)
[6] R.X. Wang, **M.Z. Yang**, EPJC85, 146(2025).
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Outline

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- II B decays in perturbative QCD
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- IV. Cotribution of color-octet quark-antiquark compoents
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I Introduction

- High Precision data are collected by B factories after years' runing.
- Serious discrepancies have been revealed between experimental data and theoretical predictions.
- $B \to \pi\pi,\ \pi K$ puzzles are remarkable problems existing in B decays

(1) $B \to \pi\pi$ puzzle:

Perturbative QCD (PQCD) and QCD factorization (QCDF) are two main theoretical methods for calculating B decays in these several decades.

Decay Mode	Exp. Data	PQCD and QCDF
$B^+ \to \pi^+\pi^0$	$(5.5 \pm 0.4) \times 10^{-6}$	$\sim 5.0 \times 10^{-6}$
$B^0 \to \pi^+\pi^-$	$(5.12 \pm 0.19) \times 10^{-6}$	$\sim 5.0 \times 10^{-6}$
$B^0 \to \pi^0\pi^0$	$(1.59 \pm 0.26) \times 10^{-6}$	$\sim 1.0 \times 10^{-7}$

Theoretical prediction gives a much small branching ratio for $B^0 \to \pi^0\pi^0$ decay mode.

Calculation of NLO can't solve this problem.

Generally, based on the factorization theroem, the amplitude for $B \to \pi\pi$ decays are generally like

$$A(B^0 \to \pi^+\pi^-) = -T\left(1 + \frac{P}{T}e^{i\phi_2}\right),$$

$$\sqrt{2}A(B^+ \to \pi^+\pi^0) = -T\left[1 + \frac{C}{T} + \frac{P_{\rm ew}}{T}e^{i\phi_2}\right],$$

$$\sqrt{2}A(B^0 \to \pi^0\pi^0) = T\left[\left(\frac{P}{T} - \frac{P_{\rm ew}}{T}\right)e^{i\phi_2} - \frac{C}{T}\right],$$

T ~tree, C~ color suppressed tree, p~penguin, P_{ew}~electroweak penguin

There are the counting rules for the decay amplitudes in the SM

$$\frac{P}{T} \sim \lambda, \qquad \frac{C}{T} \sim \lambda, \qquad \frac{P_{\rm ew}}{T} \sim \lambda^2.$$

where λ is Wolfenstein parameter $\lambda \sim 0.22$

H.N. Li, S. Mishima, A.I. Sanda, PRD72,114005(2005)

(2) $B \to \pi K$ puzzle:

Experimental data for $\mathrm{B} \to \pi\mathrm{K}$ decays

Decay Mode	Exp. Br	Exp. CP Violation
$B^+ \to K^0\pi^+$	$(2.37 \pm 0.08) \times 10^{-5}$	-0.017 ± 0.016
$B^+ \to K^+\pi^0$	$(1.29 \pm 0.05) \times 10^{-5}$	0.037 ± 0.021
$B^0 \to K^+\pi^-$	$(1.96 \pm 0.05) \times 10^{-5}$	-0.083 ± 0.004
$B^0 \to K^0\pi^0$	$(9.9 \pm 0.5) \times 10^{-6}$	0.00 ± 0.13

From theoretical side, based on the factorization theroem, the amplitude for $B \to \pi K$ decays are generally like

$$A(B^+ \to \pi^+ K^0) = P',$$

$$\sqrt{2}A(B^+ \to \pi^0 K^+) = -P'\left[1 + \frac{P'_{\rm ew}}{P'} + \left(\frac{T'}{P'} + \frac{C'}{P'}\right)e^{i\phi_3}\right]$$

$$A(B^0 \to \pi^- K^+) = -P'\left(1 + \frac{T'}{P'}e^{i\phi_3}\right),$$

$$\sqrt{2}A(B^0 \to \pi^0 K^0) = P'\left(1 - \frac{P'_{\rm ew}}{P'} - \frac{C'}{P'}e^{i\phi_3}\right),$$

T' ~tree, C'~ color suppressed tree, P'~penguin, P'_{ew}~electroweak penguin

The amplitudes obey the counting rule In the SM

$$\frac{T'}{P'} \sim \lambda, \qquad \frac{P'_{\rm ew}}{P'} \sim \lambda, \qquad \frac{C'}{P'} \sim \lambda^2. \qquad \lambda \sim 0.22$$

H.N. Li, S. Mishima, A.I. Sanda, PRD72,114005(2005)

The relation about CP violation is expected

$$A_{CP}(B^{+} \to \pi^{0}\mathrm{K}^{+}) \simeq A_{CP}(B^{0} \to \pi^{-}\mathrm{K}^{+})$$

Experimental data give:

$$A_{CP}(B^{+} \to \pi^{0}\mathrm{K}^{+}) - A_{CP}(B^{0} \to \pi^{-}\mathrm{K}^{+}) = 0.120 \pm 0.021$$

This is contradictory to the theoretical expectation in the SM

The amplitude is also puzzling when confronting the SM expectation to the experimental data

based on factorization method

A.J. Buras, R. Fleischer, S. Recksiegel, F. Schwab, EPJC 32, 45 (2003)

(3) The brief status for solving $B \to \pi\pi,\ \pi K$ puzzles

a) Calculation up to next-to-leading order in QCD (PQCD)

[1] H. N. Li, S. Mishima, and A. I. Sanda, PRD 72, 114005 (2005).
[2] Y. L. Zhang, X. Y. Liu, Y. Y. Fan, S. Cheng, and Z. J. Xiao, PRD 90, 014029 (2014).
[3] W. Bai, M. Liu, Y.Y. Fan, W.F. Wang, S. Cheng, and Z.J. Xiao, CPC 38, 033101 (2014).
[4] J. Chai, S. Cheng, Y. H. Ju, D. C. Yan, C. D. Lü, and Z. J. Xiao, CPC 46, 123103 (2022).
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b) Penguin annihilation and power correction (QCDF)

- Endpoint divergence in annihilation is modelled as

$$X_A \equiv \int_0^1 dx/\bar{x} \quad \rightarrow \quad X_A = \ln\left(\frac{m_B}{\Lambda_h}\right)(1+\rho_A e^{i\phi_A})$$

- Power corrections to the color-suppressed topology are parametrized as

$$a_2 \to a_2\,(1+\rho_C e^{i\phi_C})$$

[1] H. Y. Cheng and C. K. Chua, Phys. Rev. D 80, 074031 (2009).
[2] H. Y. Cheng and C. K. Chua, Phys. Rev. D 80, 114008 (2009).
[3] Q. Chang, J. Sun, Y. Yang, and X. Li, Phys. Rev. D 90, 054019 (2014)

c) A soft factor associated with pseudoscalar is introduced due to the residuel divergence in kT factorization in loop process (PQCD)

[1] H.N. Li and S. Mishima, Phys. Rev. D 83, 034023 (2011).
[2] H.N. Li and S. Mishima, Phys. Rev. D 90, 074018 (2014).
[3] X. Liu, H.N. Li, Z.J. Xiao, Phys. Rev. D 93, 014024 (2016).

d) Ressort to Final-state Rescattering effect

[1] H. Y. Cheng, C. K. Chua, and A. Soni,,Phys. Rev. D 71, 014030 (2005).
[2] C. K. Chua, Phys. Rev. D 97, 093004 (2018)

1. CPVs of $B \to \pi\pi$ are either too smal or wrong sign compare to exp. data, The cases of $B \to K\pi$ are similar
2. The gloabal fit is not completly satisfactory either. Br. of $B \to \pi^0\pi^0$ is still 1/3 smaller then exp. data, but CPV is larger.

d) New physics is invoked

[1] V. Barger, C.W. Chiang, P. Langacker, H.S. Lee, Phys.Lett. B 598, 218 (2004).
[2] S. Baek, P. Hamel, D. London, A. Datta, D.A. Suprun, Phys. Rev. D 71, 057502 (2005).
[3] R. Arnowitt, B. Dutta, B. Hu, S. Oh, Phys. Lett. B 633, 748 (2006).
[4] C. Kim, S. Oh, and Y.W. Yoon, Phys. Lett. B 665, 231 (2008).
[5] N.B. Beaudry, A. Datta, D. London, A. Rashed, J.S. Roux, JHEP 01, 074 (2018).
[6] A. Datta, J. Waite, and D. Sachdeva, Phys. Rev. D 100, 055015 (2019).
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etc.

II B decays in perturbative QCD based on k_T factorization (PQCD)

The decay amplitude of $B \to M_1 M_2$ is

$$M = \int d^3k_1 \int d^3k_2 \int d^3k_3 \;\Phi^B(k_1, \mu)$$

$$\cdot\, C(\mu) H(k_1, k_2, k_3, \mu) \Phi^{M_1}(k_2, \mu)\, \Phi^{M_2}(k_3, \mu)$$

for $\mu > \mu_c = 1$ GeV

The spinor wave function of B meson is taken from solving relativistic potential model

$$\Phi^B_{\alpha\beta}(\vec{k}) = \frac{-if_B m_B}{4}K(\vec{k})$$
$$\times \left\{(E_Q + m_Q)\frac{1+\not{v}}{2}\left[\left(\frac{k_+}{\sqrt{2}} + \frac{m_q}{2}\right)\not{n}_+\right.\right.$$
$$\left. + \left(\frac{k_-}{\sqrt{2}} + \frac{m_q}{2}\right)\not{n}_- - k_\perp^\mu\gamma_\mu\right]\gamma_5$$
$$- (E_q + m_q)\frac{1-\not{v}}{2}\left[\left(\frac{k_+}{\sqrt{2}} - \frac{m_q}{2}\right)\not{n}_+\right.$$
$$\left.\left. + \left(\frac{k_-}{\sqrt{2}} - \frac{m_q}{2}\right)\not{n}_- - k_\perp^\mu\gamma_\mu\right]\gamma_5\right\}_{\alpha\beta},$$

$$K(\vec{k}) = \frac{2N_B\Psi_0(\vec{k})}{\sqrt{E_q E_Q(E_q + m_q)(E_Q + m_Q)}},$$

$$\langle 0|\bar{q}(z)_\beta[z,0]b(0)_\alpha|\bar{B}\rangle = \int d^3k\Phi^B_{\alpha\beta}(\vec{k})e^{-ik\cdot z}$$

[1] M.Z. Yang, EPJC 72, 1880 (2012).
[2] J.B. Liu and M.Z. Yang, JHEP 2014, 106 (2014).
[3] J.B. Liu and M.Z. Yang, PRD 91, 094004 (2015).
[4] H.K. Sun and M.Z. Yang, PRD 95, 113001 (2017).
[5] H.K. Sun and M.Z. Yang, PRD 99, 093002 (2019).

	Meson	J^P	Mass (GeV)	GI (GeV)	Exp. (MeV) [7]
	B	0^-	5271	5310	5279.25 ± 0.17
	B^*	1^-	5317	5320	5325.2 ± 0.4
			6087		
		0^+	5696		
	$B_1(5721)$	1^+	5732		5723.5 ± 2.0
			5759		
	$B_2^*(5747)$	2^+	5775	5800	5743 ± 5
			6374		
		2^-	6051		
			6105		
$(b\bar{q})$		3^-	6061		
			6614		
		3^+	6296		
			6383		
		4^+	6302		
			6826		
		4^-	6511		
			6619		

n=1

	B_s	0^-	5360	5390	5366.77 ± 0.24
	B_s^*	1^-	5408	5450	$5415.4^{+2.4}_{-2.1}$
			6120		
		0^+	5731		
		1^+	5765		
	$B_{s1}(5830)$		5850		5829.4 ± 0.7
	$B_{s2}^*(5840)$	2^+	5866	5880	5839.7 ± 0.6
			6405		
		2^-	6134		
			6142		
$(b\bar{s})$		3^-	6144		
			6641		
		3^+	6374		
			6416		
		4^+	6380		
			6849		
		4^-	6585		
			6647		

n=1

	J^P	Mass (GeV)	GI (GeV)
	0^-	5836	5900
	1^-	5878	5930
		6422	
	0^+	6070	
	1^+	6137	
		6196	
	2^+	6220	
		6676	
$(b\bar{q})$	2^-	6417	
		6446	
	3^-	6430	
		6891	
	3^+	6618	
		6689	
	4^+	6626	
		7085	

n=2

	0^-	5928	5980
	1^-	5970	6010
		6460	
	0^+	6109	
	1^+	6187	
		6289	
	2^+	6308	
		6709	
$(b\bar{s})$	2^-	6483	
		6499	
	3^-	6512	
		6919	
	3^+	6695	
		6722	
	4^+	6704	
		7108	

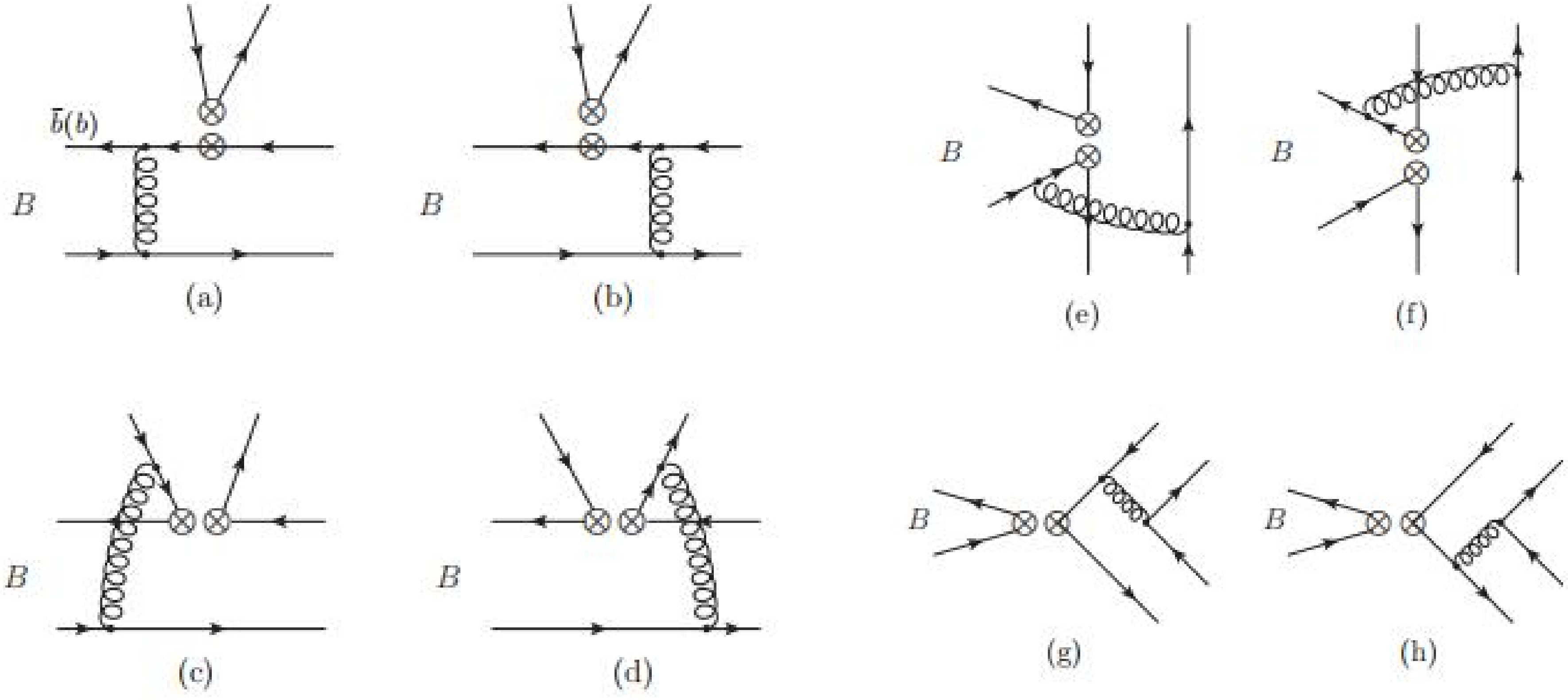
n=2

The spinor wave function for light meson

$$\Phi^{\pi}_{\gamma\delta} = \frac{i f_\pi}{4}\left[\not{p}\gamma_5\phi_\pi(u, k_{q\perp}) - \mu_\pi\gamma_5\phi^\pi_P(u, k_{q\perp}) + \mu_\pi\gamma_5\sigma^{\mu\nu}p_\mu z_\nu \frac{\phi^\pi_\sigma(u, k_{q\perp})}{6}\right]_{\gamma\delta},$$

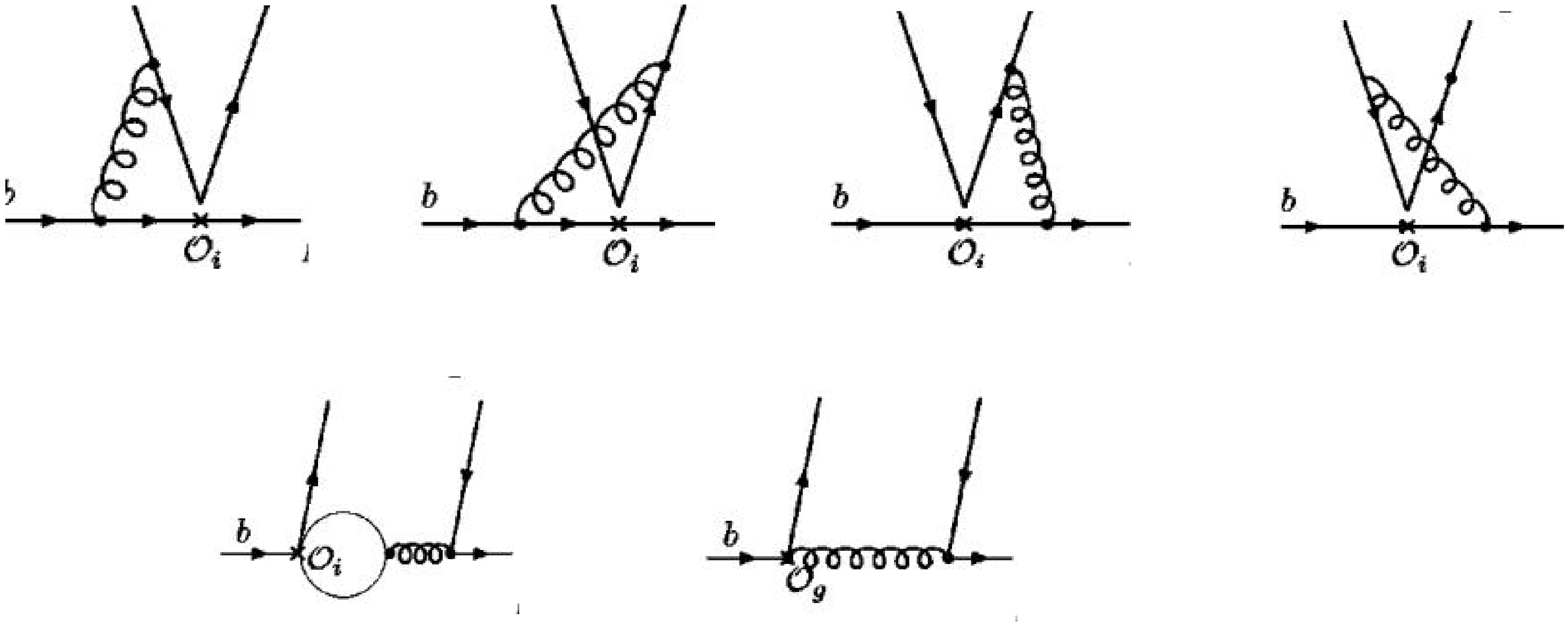
[1] V.M. Braun and I. Filyanov, Z Phys. C 48, 239 (1990).
[2] P. Ball, JHEP 01, 010 (1999).
[3] P. Ball, V.M. Braun, and A. Lenz, JHEP 05, 004 (2006).

Leading order diagrams in QCD



- Transverse momenta are kept to remove endpoint singlarity
- Sudakov factor is introduced to suppress long-distance contribution

Most important next-to-leading order diagrams in QCD



Naive calculation of the hard diagrams in PQCD

●With the B meson wave function taken from Relativistic potential model, the suppression of Sudakov factor to soft contribution becomes weak.

●Soft contributions are still large

(1)For dirgrams (a) (b) (g) (h):

more than 40% in the range $\alpha_s/\pi > 0.2$

(2) For dirgrams (c) (d) :

more than 93% in the range $\alpha_s/\pi < 0.2$

(3) For dirgrams (e) (f) :

contribution only a few percent, very small

[1] S. Lü and M. Z. Yang, NPB972, 115550 (2021).
[2] S. Lü and M. Z. Yang, PRD107, 013004 (2023).
[3] R.X. Wang, M.Z. Yang, Eprint, arxiv:2212.09054, to appear in PRD.

III. Introduction of soft form factors

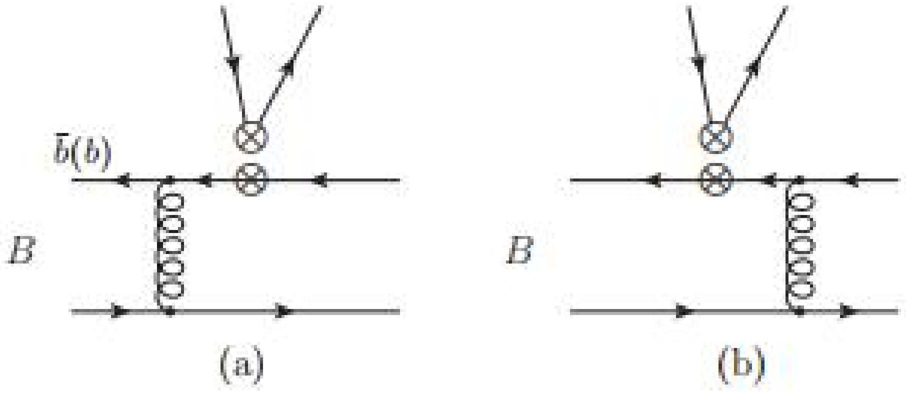
A momentum cutoff is introduced for perturbative calculation:

Taking μ_c as a critical scale that separating soft and hard interaction

(1) $\mu > \mu_c$ for hard scale

(2) $\mu < \mu_c$ for soft scale

$\mu_c = 1$ GeV



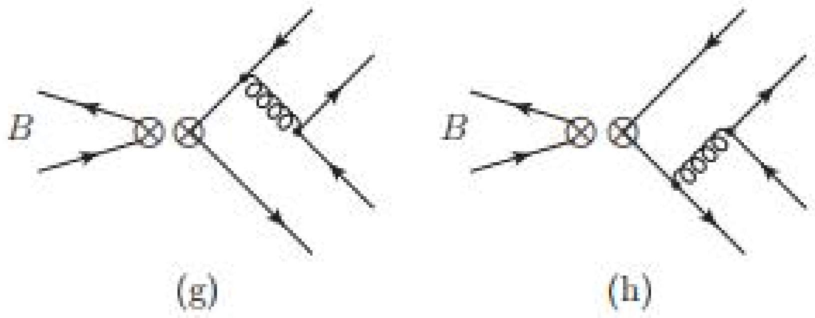
Transition form factor

$$F_0^{BM} = h_0^{BM} + \xi_0^{BM}$$

soft form factor

$$h_0^{BM} = \Phi^B \otimes H \otimes \Phi^\pi$$

hard form factor



M_1

M_2

Production form factor

$$\langle M_1 M_2 | S | 0 \rangle \equiv -\frac{1}{2}\sqrt{\mu_{M1}\mu_{M2}} F_+^{M_1 M_2}$$

$$\boxed{\mu_M = m_M^2/(m_{q1} + m_{q2})}$$

Separation of hard and soft form factor:

$$F_+^{M_1 M_2} = h_+^{M_1 M_2} + \xi_+^{M_1 M_2}$$

hard

soft

The $B\pi$ and BK transition form factors calculated perturbatively with $\mu > \mu_c$ are：

$$h_0^{B\pi} = 0.23 \pm 0.01 \qquad h_0^{BK} = 0.29 \pm 0.02$$

The total $B\pi$ and BK transition form factors from LQCD and experimental constraint are

$$F_0^{B\pi} = 0.27 \pm 0.02 \qquad F_0^{BK} = 0.33 \pm 0.04$$

which result in

$$\xi_0^{B\pi} = 0.04 \pm 0.01 \qquad \xi_0^{BK} = 0.04 \pm 0.02$$

IV. Cotribution of color-octet quark-antiquark compoents

There is a relation for the generators of the color SU(3) group

$$T^a_{ik}T^a_{jl} = -\frac{1}{2N_c}\delta_{ik}\delta_{jl} + \frac{1}{2}\delta_{il}\delta_{jk}$$

It can change the color non-singlet quark current into color singlet and octet operators

$$(\bar{q}_{1i}q_{2j})(\bar{q}_{3j}q_{4i}) = \frac{1}{N}(\bar{q}_{1i}q_{2i})(\bar{q}_{3j}q_{4j}) + 2(\bar{q}_1 T^a q_2)(\bar{q}_3 T^a q_4).$$

The first term is the color suppressed trem, and the second color-octet one

Hadronic matrix element of color-octet operators

For example, the contribution of the operator O_1 to the decay of $\overline{B}^0 \to K^-\pi^+$

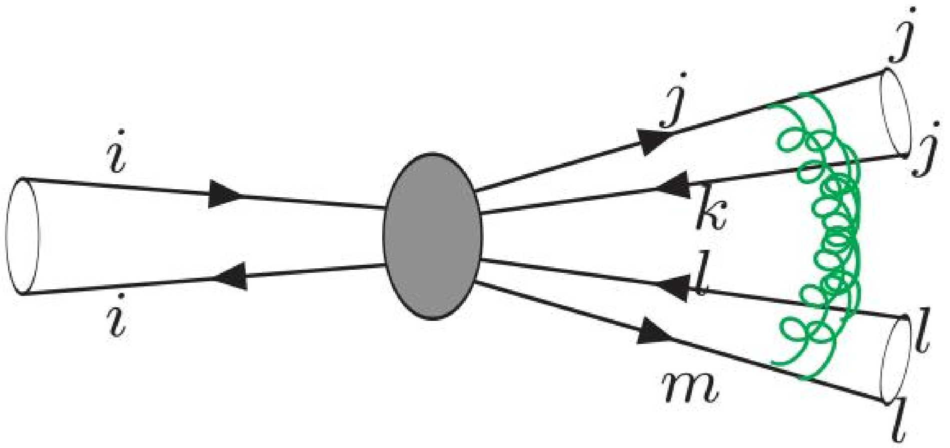
$$O_1 = \bar{s}_i\gamma^\mu(1-\gamma_5)u_j\bar{u}_j\gamma_\mu(1-\gamma_5)b_i$$

$$A = \langle K^-\pi^+|C_1O_1|\overline{B}^0\rangle$$

$$= \frac{C_1}{N_c}\left\langle K^-\pi^+\middle|\bar{s}_i\gamma^\mu(1-\gamma_5)u_i\bar{u}_j\gamma_\mu(1-\gamma_5)b_j\middle|\overline{B}^0\right\rangle$$
$$+2C_1\left\langle K^-\pi^+\middle|\bar{s}\gamma^\mu(1-\gamma_5)T^a u\ \bar{u}\gamma_\mu(1-\gamma_5)T^a b\middle|\overline{B}^0\right\rangle$$

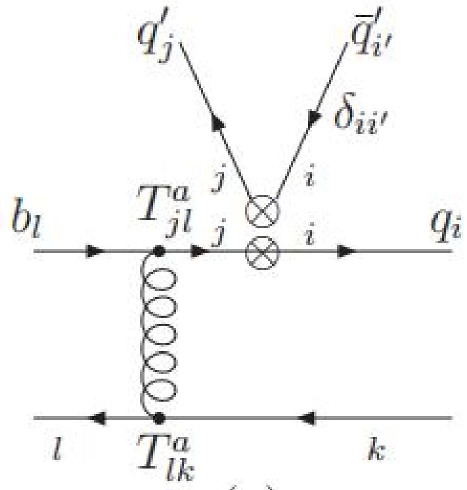
color-octet matrix element

Such matrix element may have nonzero value!

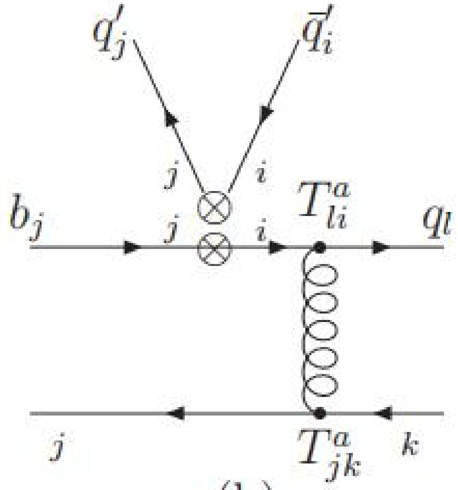


The color-octet quark pairs can be changed into color singlet by exchanging soft gluons at hadronic scale

The contributions of the final quark pair in color-octet state are considered by treating the final quark-antiquark pairs in the hard decay diagrams in non-singlet states



(a)



(b)

$$\sum_{ijkl} T^a_{jl} T^a_{lk} = \sum_{ijk} C_F \delta_{jk} = \sum_{ijki'} C_F \delta_{jk} \delta_{ii'}$$
$$= \sum_{ijki'} C_F \left(\frac{1}{N_c} \delta_{ji'} \delta_{ik} + 2 T^a_{ji'} T^a_{ik} \right),$$

$$\sum_{ijkl} T^a_{li} T^a_{jk} = \sum_{ijkl} \left[-\frac{1}{2N_c} \delta_{li} \delta_{jk} + \frac{1}{2} \delta_{lk} \delta_{ji} \right]$$
$$= \sum_{ijkl} \left[-\frac{1}{2N_c} \left(\frac{1}{N_c} \delta_{lk} \delta_{ji} + 2 T^b_{lk} T^b_{ji} \right) + \frac{1}{2} \delta_{lk} \delta_{ji} \right]$$
$$= \sum_{ijkl} \left(\frac{C_F}{N_c} \delta_{lk} \delta_{ji} - \frac{1}{N_c} T^b_{lk} T^b_{ji} \right),$$

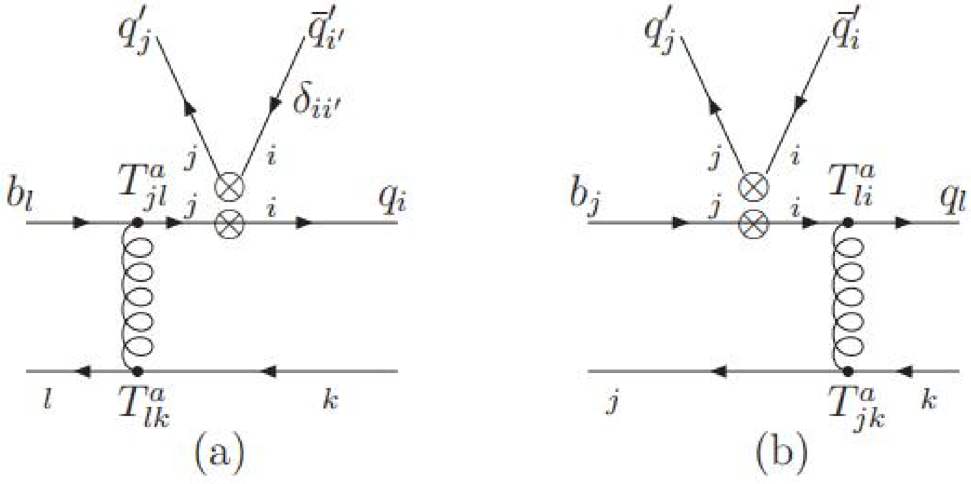
●The contributions of the other diagrams can be analyzed similarly.

●Two parameters Y_F^8 and Y_M^8 need to be introduced, which descibe the effect of color-octet quark pair changing to color singlet states by exchanging soft gluons.

Y_F^8 is for the factorizable diagrams

Y_M^8 is for the non-factorizable diagrams

For (V-A)(V-A) and (S+P)(S-P) operators, these two diagrams’ controbutions are:



$$Y^8_F F^{(P)8}_e$$

$$F^{(P)8}_e \equiv 2N_c^2 F^{(P)a}_e - \frac{N_c}{C_F} F^{(P)b}_e$$

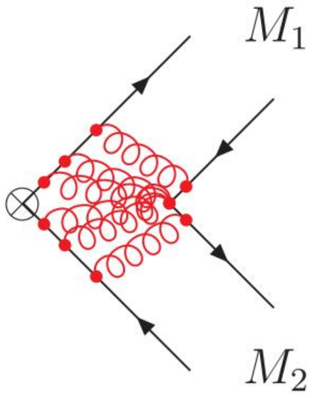
The color-octet contributions of the other diagrams and operator insertions can be also obtained

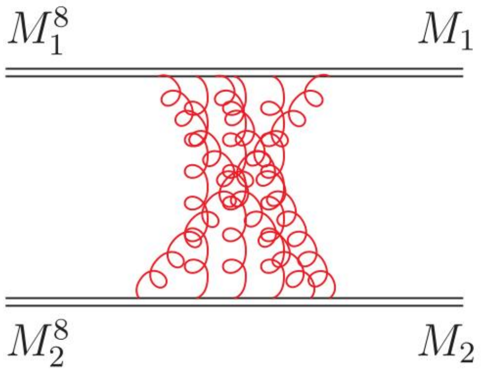
V. The nonperturbative parameters and SU(3) symmetry

The residual free parameters are

$$\xi^{M_1 M_2} \qquad Y_F^8(M_1 M_2) \qquad Y_M^8(M_1 M_2)$$

which are in principle nonperturbative and final-state-dependent





These nonperturbative parameters are final-state-dependent

The pseudoscalar mesons compose a nonet under SU(3) flavor symmetry:

$$M = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} \end{pmatrix}$$

$$= \begin{pmatrix} \frac{\pi^0+\eta_q}{\sqrt{2}} & \pi^+ & K^+ \\ \pi^- & \frac{-\pi^0+\eta_q}{\sqrt{2}} & K^0 \\ K^- & \bar{K}^0 & \eta_s \end{pmatrix}$$

$$\eta_q = \frac{u\bar{u} + d\bar{d}}{\sqrt{2}}$$

$$\eta_s = s\bar{s}$$

The mixing of η–η' is

$$\begin{pmatrix} \eta \\ \eta' \end{pmatrix} = U(\phi) \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix} = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \begin{pmatrix} \eta_q \\ \eta_s \end{pmatrix}$$

where ϕ is the mixing angle, which is taken numerically as

$$\phi = 39.3^\circ \pm 1.0^\circ$$

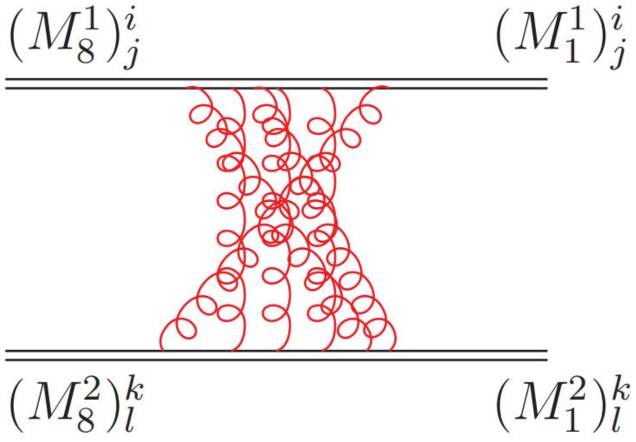
[1] Th. Feldmann, P. Kroll, B. Stech, Phys. Rev. D 58, 114006 (1998).

[2] Th. Feldmann, P. Kroll, B. Stech, Phys. Lett. B 449, 339 (1999).

A: The SU(3) symmetry and symmetry breaking for color-octet parameters:

The Hamiltonian of the scattering of $M_8^1\ M_8^2 \to M_1^1\ M_1^2$ is

$$H_0 = c_0\ (M_8^1)^i_j\ (M_8^2)^k_l \cdot (M_1^{1+})^j_i\ (M_1^{2+})^l_k$$



Under SU(3) symmetry the parameters Y_F^8 and Y_M^8 should be c_0 : $\to c_0^a, c_0^b$

The consideratopn of SU(3) symmetry breaking

The flavor SU(3) symmetry is broken by the mass difference for the quarks u, d and s

$$\begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} = \frac{1}{3}(m_u + m_d + m_s)I + \frac{1}{2}(m_u - m_d)X + \frac{1}{6}(m_u + m_d - 2m_s)W,$$

$$X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \qquad W = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

[1] X.G. HE, G.N. Li, D. Xu, Phys. Rev. D91,014029 (2015).

$$\to W = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

Put W everywhere to consider SU(3) symmetry

$$H_0 = c_0 \ (M_8^1)^i_j \ (M_8^2)^k_l \cdot (M_1^{1+})^j_i \ (M_1^{2+})^l_k$$

$$H_1^1 = c_1^1 \left(W^i_j (M_8^1)^j_m \ (M_1^{1+})^m_i \right) \left((M_8^2)^k_l \ (M_1^{2+})^l_k \right)$$

$$H_2^1 = c_1^2 \left((M_8^1)^i_j \, W^j_m \, (M_1^{1+})^m_i \right) \left((M_8^2)^k_l \ (M_1^{2+})^l_k \right)$$

Put W in the second term is the finally the same thing

$$\left(W^i_j (M_8^1)^j_m (M_1^{1\dagger})^m_i\right) = (\bar{K}_1^0)^+ (\bar{K}_8^0) + (K_1^-)^+ K_8^- + \eta_{s1}^+ \eta_{s8},$$

$$\left((M_8^1)^i_j W^j_m (M_1^{1\dagger})^m_i\right) = (K_1^0)^+ K_8^0 + (K_1^+)^+ K_8^+ + \eta_{s1}^+ \eta_{s8}.$$

CP symmetry of strong interaction $\Rightarrow c_1^1 = c_1^2$

SU(3) symmetry breaking at second order is

$$H_2^1 = c_2^1 \left(W_j^i (M_8^1)_m^j (M_1^{1\dagger})_i^m \right) \left(W_l^k (M_8^2)_n^l (M_1^{2\dagger})_k^n \right)$$
$$H_2^2 = c_2^2 \left(W_j^i (M_8^1)_m^j (M_1^{1\dagger})_i^m \right) \left((M_8^2)_l^k W_n^l (M_1^{2\dagger})_k^n \right)$$
$$H_2^3 = c_2^3 \left((M_8^1)_j^i W_m^j (M_1^{1\dagger})_i^m \right) \left((M_8^2)_l^k W_n^l (M_1^{2\dagger})_k^n \right)$$
$$H_2^4 = c_2^4 \left(W_j^i (M_8^1)_m^j W_n^m (M_1^{1\dagger})_i^n \right) \left((M_8^2)_l^k (M_1^{2\dagger})_k^l \right)$$

Similarly, CP symmetry requires $c_2^1 = c_2^3$.

$c_0,\ c_1^1,\ c_2^1,\ c_2^2,\ c_2^4$

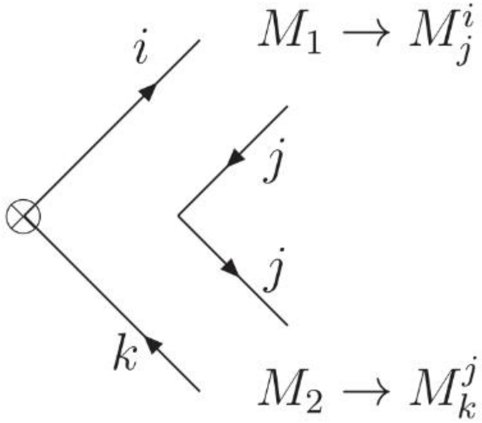
B: The SU(3) symmetry and symmetry breaking for time-like production form factors:

The production form factor is defined by the matrix

$$\langle M_1 M_2 | S | 0 \rangle$$

$$S_i^k \equiv \bar{q}_i \, q_k$$

$$i, k = 1,2,3$$



The Hamitonian with SU(3) symmetry is

$$H_0 = c_0^c \; M_j^i \;\; M_k^j \, S_i^k$$

Under SU(3) symmetry c_0^c is relevant to the time-like production form factor

- The leading-order symmetry breaking term is

$$H_1^1 = c_1^{1c} W_l^i M_j^l M_k^j S_i^k,$$
$$H_1^2 = c_1^{2c} M_j^i W_l^j M_k^l S_i^k,$$
$$H_1^3 = c_1^{3c} M_j^i M_k^j W_l^k S_i^l.$$

CP symmetry requires $c_1^{1c} = c_1^{3c}$

- The secpnd-order symmetry breaking term is

$$H_2^1 = c_2^{1c} W_j^i M_l^j W_m^l M_k^m S_i^k,$$
$$H_2^2 = c_2^{2c} W_j^i M_l^j M_m^l W_k^m S_i^k,$$
$$H_2^3 = c_2^{3c} M_j^i W_l^j M_m^l W_k^m S_i^k.$$

CP symmetry requires $c_2^{1c} = c_2^{3c}$

TABLE I. The coefficients of LO and NLO SU(3) symmetry breaking parameters for different final states, where $R_j^{ia(b)} = c_j^{ia(b)}/c_0^{a(b)}$, $i\ j = 1, 2, 4$.

	Y_F^8	Y_M^8
$\pi\pi(\eta_q)$	c_0^a	c_0^b
$K\pi(\eta_q)$	$c_0^a(1+\frac{1}{2}R_1^{1a})$	$c_0^b(1+\frac{1}{2}R_1^{1b})$
$\pi\eta_s$	$c_0^a(1+R_1^{1a}+\frac{1}{2}R_2^{4a})$	$c_0^a(1+R_1^{1a}+\frac{1}{2}R_2^{4a})$
$K\eta_s$	$c_0^a(1+\frac{3}{2}R_1^{1a}+R_2^{1a}+\frac{1}{2}R_2^{2a}+\frac{1}{2}R_2^{4a})$	$c_0^b(1+\frac{3}{2}R_1^{1b}+R_2^{1b}+\frac{1}{2}R_2^{2b}+\frac{1}{2}R_2^{4b})$
$K\bar{K}$	$c_0^a(1+R_1^{1a}+\frac{1}{2}R_2^{2a})$	$c_0^b(1+R_1^{1b}+\frac{1}{2}R_2^{2b})$

TABLE II. The coefficients of LO and NLO SU(3) symmetry breaking parameters for production form factors with different final states, where $R_j^{ic} = c_j^{ic}/c_0^c$, $i\ j = 1, 2, 3, 4$.

	$\sqrt{\mu_{M_1}\mu_{M_2}}F_+^{M_1M_2}$
$\pi\pi(\eta_q)$	c_0^c
$K\pi(\eta_q)$	$c_0^c(1+R_1^{1c})$
$\pi\eta_s$	0
$K\eta_s$	$c_0^c(1+R_1^{1c}+R_1^{2c}+R_2^{1c})$
$K\bar{K}$	$c_0^c(1+R_1^{2c})$

VI. Confronting the theoretical framework to experimental data

A. The soft transition form factors

The hard form factor with $\mu > \mu_c$ and $\mu_c = 1$ GeV:

$$h_+^{B\pi} = 0.23 \pm 0.01,$$
$$h_+^{BK} = 0.29 \pm 0.02,$$
$$h_+^{B\eta_q} = 0.17 \pm 0.01,$$

The total form factors from exp. data and LQCD:

$$F_+^{B\pi} = 0.27 \pm 0.02,$$
$$F_+^{BK} = 0.33 \pm 0.04,$$
$$F_+^{B\eta_q} = 0.23 \pm 0.03.$$

The difference of them is the soft form factor

$$\xi^{B\pi} = 0.04 \pm 0.01,$$
$$\xi^{BK} = 0.04 \pm 0.02,$$
$$\xi^{B\eta_q} = 0.06 \pm 0.02.$$

●The color-octet parameters and the meson-pair production form factors can not be calculated with PQCD because of their nonperturbative property.

●These parameters can be fitted from the experimental data.

$$
\begin{aligned}
c_0^a &= (0.179 \pm 0.015)\mathrm{Exp}[(-0.61 \pm 0.02)\pi i],\\
R_1^{1a} &= (0.89 \pm 0.03)\mathrm{Exp}[(-0.79 \pm 0.02)\pi i],\\
R_2^{1a} &= (0.20 \pm 0.03)\mathrm{Exp}[(0.60 \pm 0.04)\pi i],\\
R_2^{2a} &= (0.27 \pm 0.04)\mathrm{Exp}[(0.60 \pm 0.08)\pi i],\\
R_2^{4a} &= (0.23 \pm 0.03)\mathrm{Exp}[(0.08 \pm 0.08)\pi i].
\end{aligned}
$$

$$
\begin{aligned}
c_0^b &= (0.078 \pm 0.04)\mathrm{Exp}[(-0.58 \pm 0.02)\pi i],\\
R_1^{1b} &= (0.84 \pm 0.02)\mathrm{Exp}[(0.32 \pm 0.02)\pi i],\\
R_2^{1b} &= (0.31 \pm 0.04)\mathrm{Exp}[(-0.60 \pm 0.07)\pi i],\\
R_2^{2b} &= (0.22 \pm 0.03)\mathrm{Exp}[(-0.30 \pm 0.08)\pi i],\\
R_2^{4b} &= (0.23 \pm 0.03)\mathrm{Exp}[(0.50 \pm 0.10)\pi i].
\end{aligned}
$$

$$
\begin{aligned}
c_0^c &= (0.58 \pm 0.04)\mathrm{Exp}[(-0.66 \pm 0.02)\pi i],\\
R_1^{1c} &= (0.31 \pm 0.02)\mathrm{Exp}[(0.72 \pm 0.07)\pi i],\\
R_1^{2c} &= (0.63 \pm 0.07)\mathrm{Exp}[(0.78 \pm 0.08)\pi i],\\
R_2^{1c} &= (0.41 \pm 0.02)\mathrm{Exp}[(-0.52 \pm 0.04)\pi i].
\end{aligned}
$$

TABLE III. Branching ratio ($\times 10^{-6}$) and direct CP violation with NLO contributions for decay modes of $\pi\pi$, $K\pi$ and $K\bar{K}$ final states.

	$\mathrm{LO_{NLOWC}}$	NLO	NLO+soft	Data
$\mathrm{Br}(B^0 \to \pi^+\pi^-)$	3.90	4.82	$5.07 \pm 0.71^{+0.17+0.28}_{-0.25-0.24}$	5.12 ± 0.19
$\mathrm{Br}(B^+ \to \pi^+\pi^0)$	3.59	3.24	$5.42 \pm 0.53^{+0.19+0.18}_{-0.22-0.20}$	5.5 ± 0.4
$\mathrm{Br}(B^0 \to \pi^0\pi^0)$	0.36	0.12	$1.34 \pm 0.34^{+0.04+0.16}_{-0.06-0.18}$	1.59 ± 0.26
$\mathrm{Br}(B^+ \to K^0\pi^+)$	13.4	13.8	$23.5 \pm 3.6^{+0.6+0.8}_{-1.0-0.8}$	23.7 ± 0.8
$\mathrm{Br}(B^+ \to K^+\pi^0)$	9.0	8.4	$12.4 \pm 1.8^{+0.4+0.3}_{-0.5-0.3}$	12.9 ± 0.5
$\mathrm{Br}(B^0 \to K^+\pi^-)$	13.7	13.2	$21.0 \pm 3.4^{+0.5+0.7}_{-0.6-0.7}$	19.6 ± 0.5
$\mathrm{Br}(B^0 \to K^0\pi^0)$	4.9	5.2	$9.7 \pm 1.8^{+0.3+0.4}_{-0.4-0.3}$	9.9 ± 0.5
$\mathrm{Br}(B^+ \to K^+\bar{K}^0)$	0.92	0.66	$1.22 \pm 0.37^{+0.03+0.04}_{-0.05-0.02}$	1.31 ± 0.17
$\mathrm{Br}(B^0 \to K^0\bar{K}^0)$	0.98	0.68	$1.35 \pm 0.38^{+0.04+0.06}_{-0.05-0.04}$	1.21 ± 0.16
$\mathrm{Br}(B^0 \to K^+K^-)$	0.034	0.034	$0.051 \pm 0.012^{+0.002+0.005}_{-0.002-0.004}$	0.078 ± 0.015
$A_{CP}(B^0 \to \pi^+\pi^-)$	0.27	0.16	$0.33 \pm 0.03^{+0.01+0.04}_{-0.00-0.04}$	0.32 ± 0.04
$A_{CP}(B^+ \to \pi^+\pi^0)$	0.00	0.00	$0.0022 \pm 0.0007^{+0.0001+0.0001}_{-0.0000-0.0001}$	0.03 ± 0.04
$A_{CP}(B^0 \to \pi^0\pi^0)$	-0.60	0.30	$0.47 \pm 0.06^{+0.01+0.07}_{-0.01-0.06}$	0.33 ± 0.22
$A_{CP}(B^+ \to K^0\pi^+)$	-0.004	0.010	$0.0108 \pm 0.0012^{+0.0002+0.0010}_{-0.0001-0.0010}$	-0.017 ± 0.016
$A_{CP}(B^+ \to K^+\pi^0)$	-0.15	-0.039	$0.060 \pm 0.027^{+0.001+0.011}_{-0.001-0.012}$	0.037 ± 0.021
$A_{CP}(B^0 \to K^+\pi^-)$	-0.175	-0.107	$-0.085 \pm 0.028^{+0.002+0.016}_{-0.003-0.017}$	-0.083 ± 0.004
$A_{CP}(B^0 \to K^0\pi^0)$	0.018	-0.036	$-0.13 \pm 0.04^{+0.00+0.01}_{-0.00-0.01}$	0.00 ± 0.13
$A_{CP}(B^+ \to K^+\bar{K}^0)$	0.07	0.12	$0.00 \pm 0.04^{+0.00+0.03}_{-0.00-0.02}$	0.04 ± 0.014
$A_{CP}(B^0 \to K^0\bar{K}^0)$	0.00	0.05	$-0.02 \pm 0.04^{+0.00+0.01}_{-0.00-0.02}$	$-0.58^{+0.73}_{-0.66}$
$A_{CP}(B^0 \to K^+K^-)$	0.001	0.26	$-0.28 \pm 0.12^{+0.03+0.10}_{-0.03-0.12}$	$-$

TABLE IV. Branching ratio ($\times 10^{-6}$) and direct CP violation with NLO contributions with decay modes involving η and η' mesons.

	$\mathrm{LO}_{\mathrm{NLOWC}}$	NLO	NLO+gg	NLO+soft	Data
$\mathrm{Br}(B^0 \to \pi^0 \eta)$	0.09	0.18	0.18	$0.45 \pm 0.07^{+0.01+0.02}_{-0.01-0.02}$	0.41 ± 0.17
$\mathrm{Br}(B^+ \to \pi^+ \eta)$	0.97	1.49	1.48	$4.41 \pm 0.69^{+0.09+0.11}_{-0.10-0.11}$	4.02 ± 0.27
$\mathrm{Br}(B^0 \to \pi^0 \eta')$	0.04	0.14	0.12	$0.70 \pm 0.14^{+0.01+0.03}_{-0.01-0.03}$	1.2 ± 0.6
$\mathrm{Br}(B^+ \to \pi^+ \eta')$	0.50	0.60	0.60	$2.85 \pm 0.43^{+0.02+0.07}_{-0.05-0.08}$	2.7 ± 0.9
$\mathrm{Br}(B^0 \to K^0 \eta)$	3.29	3.76	3.69	$1.43 \pm 0.62^{+0.10+0.09}_{-0.09-0.09}$	$1.23^{+0.27}_{-0.24}$
$\mathrm{Br}(B^+ \to K^+ \eta)$	3.68	4.51	4.45	$2.25 \pm 0.97^{+0.10+0.07}_{-0.12-0.11}$	2.4 ± 0.4
$\mathrm{Br}(B^0 \to K^0 \eta')$	22.4	30.4	32.6	$62.5 \pm 12.5^{+1.5+3.9}_{-1.3-2.3}$	66 ± 4
$\mathrm{Br}(B^+ \to K^+ \eta')$	24.8	33.6	36.0	$68.5 \pm 13.5^{+1.2+4.4}_{-1.5-3.3}$	70.4 ± 2.5
$A_{CP}(B^0 \to \pi^0 \eta)$	0.42	-0.06	-0.06	$-0.94 \pm 0.03^{+0.01+0.02}_{-0.01-0.02}$	-
$A_{CP}(B^+ \to \pi^+ \eta)$	0.40	0.08	0.08	$-0.13 \pm 0.10^{+0.01+0.04}_{-0.01-0.03}$	-0.14 ± 0.07
$A_{CP}(B^0 \to \pi^0 \eta')$	0.43	0.02	0.04	$-0.37 \pm 0.08^{+0.01+0.05}_{-0.01-0.05}$	-
$A_{CP}(B^+ \to \pi^+ \eta')$	0.51	0.47	0.49	$0.10 \pm 0.09^{+0.02+0.04}_{-0.02-0.04}$	0.06 ± 0.16
$A_{CP}(B^0 \to K^0 \eta)$	-0.001	-0.05	-0.05	$-0.32 \pm 0.16^{+0.02+0.03}_{-0.02-0.03}$	-
$A_{CP}(B^+ \to K^+ \eta)$	0.05	-0.05	-0.06	$-0.29 \pm 0.20^{+0.01+0.04}_{-0.01-0.04}$	-0.37 ± 0.08
$A_{CP}(B^0 \to K^0 \eta')$	-0.005	0.02	0.02	$0.04 \pm 0.01^{+0.00+0.00}_{-0.00-0.00}$	0.06 ± 0.04
$A_{CP}(B^+ \to K^+ \eta')$	-0.06	-0.03	-0.02	$-0.008 \pm 0.010^{+0.001+0.002}_{-0.001-0.002}$	0.004 ± 0.011

TABLE I. Branching ratio ($\times 10^{-6}$) and direct CP violation with NLO contributions for decay modes of $\pi\rho$, $\pi\omega$ final states. Column "NLO*" incorporates additional contributions from soft form factors. NLO+soft includes all soft contributions. Column "Data" is for the averaged values given in PDG [1]. Experimental values of Data* are taken from the data of BABAR experimental collaboration [67–69].

	$\mathrm{LO_{NLOWC}}$	NLO	NLO*	NLO+soft	Data*	Data [1]
$\mathrm{Br}(B^0\to\pi^\pm\rho^\mp)$	15.7	17.7	33.5	$24.7\pm1.1^{+1.1+0.3}_{-1.3-0.3}$		23.0 ± 2.3
$\mathrm{Br}(B^+\to\pi^0\rho^+)$	6.70	7.08	13.1	$12.6\pm0.9^{+0.6+0.32}_{-0.7-0.33}$		$10.6^{+1.2}_{-1.3}$
$\mathrm{Br}(B^+\to\pi^+\rho^0)$	4.18	3.05	4.25	$6.04\pm0.47^{+0.19+0.03}_{-0.24-0.02}$	8.1 ± 1.7[67]	8.3 ± 1.2
$\mathrm{Br}(B^0\to\pi^0\rho^0)$	0.23	0.03	0.04	$1.95\pm0.34^{+0.06+0.05}_{-0.05-0.05}$		2.0 ± 0.5
$\mathrm{Br}(B^0\to\pi^0\omega)$	0.02	0.005	0.11	$0.30\pm0.04^{+0.06+0.01}_{-0.07-0.01}$		<0.5
$\mathrm{Br}(B^+\to\pi^+\omega)$	2.37	2.67	7.08	$7.17\pm0.57^{+0.32+0.09}_{-0.30-0.11}$		6.9 ± 0.5
$\mathcal{A}_{CP}(B^0\to\pi^-\rho^+)$	0.13	0.06	-0.19	$-0.142\pm0.032^{+0.002+0.004}_{-0.003-0.004}$	-0.18 ± 0.09[68]	0.13 ± 0.06
$\mathcal{A}_{CP}(B^0\to\pi^+\rho^-)$	-0.36	-0.37	0.03	$-0.04\pm0.04^{+0.00+0.01}_{-0.00-0.01}$		-0.08 ± 0.08
$\mathcal{A}_{CP}(B^+\to\pi^0\rho^+)$	0.26	0.20	-0.15	$0.017\pm0.051^{+0.004+0.003}_{-0.003-0.003}$		0.03 ± 0.10
$\mathcal{A}_{CP}(B^+\to\pi^+\rho^0)$	-0.45	-0.38	0.25	$-0.0025\pm0.075^{+0.0045+0.0071}_{-0.0051-0.0059}$		0.003 ± 0.014
$\mathcal{A}_{CP}(B^0\to\pi^0\rho^0)$	0.06	0.84	0.89	$0.23\pm0.05^{+0.01+0.01}_{-0.01-0.01}$	0.10 ± 0.66[69]	-0.27 ± 0.24
$\mathcal{A}_{CP}(B^0\to\pi^0\omega)$	0.80	0.87	0.46	$0.41\pm0.04^{+0.07+0.05}_{-0.07-0.04}$		...
$\mathcal{A}_{CP}(B^+\to\pi^+\omega)$	-0.02	-0.34	-0.26	$-0.031\pm0.044^{+0.015+0.017}_{-0.018-0.013}$		-0.04 ± 0.05

Table 1 Branching ratios ($\times 10^{-6}$) and CP violations of the $B \to K^*\pi$ and $B \to K\rho$ decays

Mode	$\mathrm{LO}_{\mathrm{NLOWC}}$	NLO	NLO+ξ^{BM}, $\xi^{M_1M_2}$	NLO+Soft	Data [1]
$\mathrm{Br}(B^+ \to K^{*0}\pi^+)$	5.0	5.0	16.8	$9.6^{+0.9+0.2+0.4}_{-0.9-0.4-0.4}$	10.1 ± 0.8
$\mathrm{Br}(B^+ \to K^{*+}\pi^0)$	4.1	3.4	9.0	$5.9^{+0.5+0.2+0.2}_{-0.6-0.3-0.2}$	6.8 ± 0.9
$\mathrm{Br}(B^0 \to K^{*+}\pi^-)$	6.4	5.4	13.4	$7.9^{+0.5+0.1+0.2}_{-0.5-0.2-0.2}$	7.5 ± 0.4
$\mathrm{Br}(B^0 \to K^{*0}\pi^0)$	1.8	1.9	6.1	$2.9^{+0.3+0.0+0.1}_{-0.3-0.0-0.0}$	3.3 ± 0.6
$\mathrm{Br}(B^+ \to K^0\rho^+)$	3.3	4.2	2.4	$8.1^{+0.8+0.1+0.1}_{-0.7-0.1-0.1}$	$7.3^{+1.0}_{-1.2}$
$\mathrm{Br}(B^+ \to K^+\rho^0)$	1.3	1.7	0.9	$3.2^{+0.6+0.1+0.1}_{-0.6-0.0-0.1}$	3.7 ± 0.5
$\mathrm{Br}(B^0 \to K^+\rho^-)$	1.8	2.2	0.4	$6.7^{+1.1+0.1+0.1}_{-1.0-0.1-0.1}$	7.0 ± 0.9
$\mathrm{Br}(B^0 \to K^0\rho^0)$	1.8	1.8	0.8	$4.3^{+0.5+0.0+0.1}_{-0.5-0.0-0.1}$	3.4 ± 1.1
$A_{CP}(B^+ \to K^{*0}\pi^+)$	-0.011	0.008	0.011	$0.001^{+0.002+0.000+0.002}_{-0.003-0.000-0.002}$	-0.021 ± 0.032
$A_{CP}(B^+ \to K^{*+}\pi^0)$	-0.39	-0.15	0.10	$-0.18^{+0.06+0.00+0.00}_{-0.06-0.00-0.01}$	-0.39 ± 0.21
$A_{CP}(B^0 \to K^{*+}\pi^-)$	-0.45	-0.26	0.03	$-0.27^{+0.05+0.01+0.03}_{-0.06-0.02-0.02}$	-0.27 ± 0.04
$A_{CP}(B^0 \to K^{*0}\pi^0)$	0.04	-0.04	-0.07	$-0.03^{+0.07+0.02+0.04}_{-0.06-0.03-0.04}$	-0.15 ± 0.13
$A_{CP}(B^+ \to K^0\rho^+)$	0.00	0.00	0.01	$0.00^{+0.01+0.00+0.01}_{-0.01-0.00-0.00}$	-0.03 ± 0.15
$A_{CP}(B^+ \to K^+\rho^0)$	0.459	0.423	0.616	$0.162^{+0.083+0.006+0.021}_{-0.078-0.009-0.021}$	0.160 ± 0.021
$A_{CP}(B^0 \to K^+\rho^-)$	0.47	0.44	0.84	$0.25^{+0.11+0.00+0.02}_{-0.10-0.01-0.02}$	0.20 ± 0.11
$A_{CP}(B^0 \to K^0\rho^0)$	-0.15	-0.05	-0.02	$0.23^{+0.07+0.01+0.01}_{-0.07-0.00-0.00}$	0.04 ± 0.20

TABLE II. $B \to \rho\eta^{(\prime)}$ branching ratios and CP violations.

	$\mathrm{LO_{NLOWC}}$	NLO	$\mathrm{NLO}+g^*g^*$	$\mathrm{NLO}+g^*g^*+\xi^{B\rho}+\xi^{B\eta_q}$	$\mathrm{NLO}+g^*g^*+\mathrm{soft}^a$	$\mathrm{NLO}+g^*g^*+\mathrm{soft}^b$	Data [8]
$\mathrm{Br}(B^0 \to \rho^0\eta)\times 10^{-6}$	0.002	0.01	0.01	0.05	$0.15 \pm 0.09^{+0.00+0.00}_{-0.00-0.00}$	$0.28 \pm 0.04^{+0.00+0.00}_{-0.00-0.00}$	<1.5
$\mathrm{Br}(B^+ \to \rho^+\eta)\times 10^{-6}$	3.45	3.66	3.68	6.49	$9.35 \pm 2.02^{+0.14+0.29}_{-0.23-0.29}$	$9.42 \pm 1.51^{+0.14+0.29}_{-0.23-0.29}$	7.0 ± 2.9
$\mathrm{Br}(B^0 \to \rho^0\eta')\times 10^{-6}$	0.01	0.01	0.02	0.07	$1.21 \pm 0.20^{+0.00+0.00}_{-0.00-0.00}$	$1.26 \pm 0.08^{+0.00+0.00}_{-0.00-0.00}$	<1.3
$\mathrm{Br}(B^+ \to \rho^+\eta')\times 10^{-6}$	2.08	1.84	1.94	2.58	$7.83 \pm 1.38^{+0.07+0.14}_{-0.11-0.14}$	$7.69 \pm 1.07^{+0.07+0.14}_{-0.11-0.14}$	9.7 ± 2.2
$A_{CP}(B^0 \to \rho^0\eta)$	0.94	0.84	0.82	0.22	$-0.72 \pm 0.35^{+0.01+0.07}_{-0.01-0.09}$	$-0.51 \pm 0.30^{+0.01+0.07}_{-0.01-0.09}$	$\cdots$
$A_{CP}(B^+ \to \rho^+\eta)$	0.00	−0.08	−0.09	−0.14	$0.14 \pm 0.08^{+0.00+0.00}_{-0.00-0.00}$	$0.15 \pm 0.07^{+0.00+0.00}_{-0.00-0.00}$	0.11 ± 0.11
$A_{CP}(B^0 \to \rho^0\eta')$	0.69	0.67	0.62	0.04	$-0.29 \pm 0.33^{+0.02+0.08}_{-0.03-0.09}$	$-0.35 \pm 0.33^{+0.02+0.08}_{-0.03-0.09}$	$\cdots$
$A_{CP}(B^+ \to \rho^+\eta')$	0.10	0.20	0.16	0.34	$0.14 \pm 0.11^{+0.00+0.03}_{-0.00-0.03}$	$0.17 \pm 0.11^{+0.00+0.03}_{-0.00-0.03}$	0.26 ± 0.17

TABLE I. The values of color-octet parameters.

	$Y_F^{8\rho\eta_q}$	$Y_F^{8\rho\eta_s}$	$Y_M^{8\rho\eta_q}$	$Y_M^{8\rho\eta_s}$
a	$(0.171^{+0.039}_{-0.078})e^{i\pi(1.394^{+0.106}_{-0.058})}$	$(0.172^{+0.038}_{-0.075})e^{i\pi(1.625^{+0.159}_{-0.284})}$	$(0.187^{+0.020}_{-0.038})e^{i\pi(1.159^{+0.054}_{-0.043})}$	$(0.196^{+0.013}_{-0.031})e^{i\pi(0.860^{+0.114}_{-0.057})}$
b	$(0.163^{+0.009}_{-0.008})e^{i\pi(0.174^{+0.032}_{-0.028})}$	$(0.017^{+0.019}_{-0.014})e^{i\pi(0.289^{+0.146}_{-0.204})}$	$(0.165^{+0.008}_{-0.004})e^{i\pi(0.203^{+0.029}_{-0.030})}$	$(0.205^{+0.002}_{-0.002})e^{i\pi(0.286^{+0.021}_{-0.019})}$

TABLE II. Branching fraction ($\times 10^{-6}$), CP asymmetry, and longitudinal polarization fraction of the $B \to \rho\rho$ decay.

Mode	$\mathrm{LO_{NLOWC}}$	NLO	NLO+$\xi^{B\rho}$	NLO+$\xi^{B\rho},\xi^{\rho\rho}$	NLO+Soft	Data [36, 61]
$\mathcal{B}(B^+ \to \rho^+\rho^0)$	7.4	6.7	12.0	12.0	$22.9^{+5.8+0.8+2.2}_{-5.4-1.7-2.3}$	24.0 ± 1.9
$\mathcal{B}(B^0 \to \rho^+\rho^-)$	10.7	12.0	24.4	22.0	$30.6^{+10.9+0.6+2.0}_{-9.6-1.3-1.8}$	27.7 ± 1.9
$\mathcal{B}(B^0 \to \rho^0\rho^0)$	0.30	0.06	0.06	0.39	$0.98^{+0.27+0.06+0.51}_{-0.25-0.11-0.36}$	0.96 ± 0.15
$A_{CP}(B^+ \to \rho^+\rho^0)$	0.00	0.00	0.00	0.00	$0.01^{+0.01+0.00+0.01}_{-0.01-0.00-0.00}$	-0.05 ± 0.05
$A_{CP}(B^0 \to \rho^+\rho^-)$	-0.03	-0.08	-0.08	0.05	$-0.01^{+0.04+0.00+0.01}_{-0.03-0.01-0.01}$	0.00 ± 0.09
$A_{CP}(B^0 \to \rho^0\rho^0)$	0.2	0.8	0.9	0.5	$0.3^{+0.2+0.0+0.1}_{-0.2-0.0-0.1}$	-0.2 ± 0.9
$f_L(B^+ \to \rho^+\rho^0)$	0.969	0.970	0.938	0.938	$0.972^{+0.009+0.000+0.003}_{-0.010-0.001-0.005}$	0.950 ± 0.016
$f_L(B^0 \to \rho^+\rho^-)$	0.921	0.920	0.888	0.879	$0.940^{+0.025+0.000+0.005}_{-0.031-0.002-0.007}$	0.990 ± 0.020
$f_L(B^0 \to \rho^0\rho^0)$	0.84	0.83	0.77	0.47	$0.76^{+0.04+0.02+0.07}_{-0.05-0.02-0.11}$	0.71 ± 0.06

Discussion of μ_c-dependence:

TABLE III. $B \to K\pi$ branching ratios and CP violations varying with the critical cutoff scale μ_c, where the total from factors are fixed with $F_0^{BK}(0) = 0.33$, $F_0^{B\pi}(0) = 0.27$ and$F_+^{K\pi} = 0.20\exp(-0.47i\pi)$ [a]

Mode	$\mu_c = 0.9$GeV	1.0GeV	1.1GeV	1.3GeV	1.5GeV	2.0GeV	Data [3]
$B(B^+ \to K^0\pi^+) \times 10^{-6}$	24.5	24.3	24.3	23.5	23.0	20.9	23.7 ± 0.8
$B(B^+ \to K^+\pi^0) \times 10^{-6}$	12.4	12.6	12.7	12.5	12.4	11.5	12.9 ± 0.5
$B(B^0 \to K^+\pi^-) \times 10^{-6}$	19.5	20.0	20.4	20.5	20.4	19.0	19.6 ± 0.5
$B(B^0 \to K^0\pi^0) \times 10^{-6}$	9.3	9.4	9.5	9.3	9.1	8.4	9.9 ± 0.5
$A_{CP}(B^+ \to K^0\pi^+)$	0.012	0.011	0.011	0.010	0.009	0.008	-0.017 ± 0.016
$A_{CP}(B^+ \to K^+\pi^0)$	0.055	0.041	0.031	0.012	0.001	-0.011	0.037 ± 0.021
$A_{CP}(B^0 \to K^+\pi^-)$	-0.055	-0.084	-0.102	-0.133	-0.150	-0.178	-0.083 ± 0.004
$A_{CP}(B^0 \to K^0\pi^0)$	-0.100	-0.112	-0.118	-0.128	-0.133	-0.148	0.00 ± 0.13

[a] The total form factors should not vary with the critical cutoff scale. So the value of them can be obtained by adding the hard and soft part at any value of μ_c. Here $F_+^{K\pi}$ is taken by adding the values of $h^{K\pi}$ and $\xi^{K\pi}$ at $\mu_c = 1$ GeV. The color-octet contributions are taken as μ_c-independent quantities.

- Br and CPV are not changed much around $\mu_c \sim 1$GeV
- The change becomes large when $\mu_c > 2$ GeV, where the scale of soft interaction is pushed too high

VII. Summary

1) We used the B meson wave function obtained from relativistic potential model. Then the suppression of Sudakov factor to LD contribution is no longer sufficient.

2) A critical cutoff scale μ_c is introduced to insure perturbation calculation applicable.

3) Soft form factors, transition and production form factors, have to be introduced.

4) Contribution of color-octet final quark-antiquark pair is considered, which is crucial to explain the experimental data.

谢谢大家！