Two-body Hadronic B-meson Decays in QCD Factorization Approach

李 新 强

華中師範大學
HUAZHONG UNIVERSITY

IOPP
Institute of Particle Physics
粒子物理研究所

In collaboration with G. Bell, M. Beneke, T. Huber, and S. Krankl

Based on JHEP 04 (2020), JHEP 09 (2016) 112, PLB 750 (2015) 348, NPB 832 (2010) 109

河南师范大学“八角楼”讲坛，2025/05/19，新乡

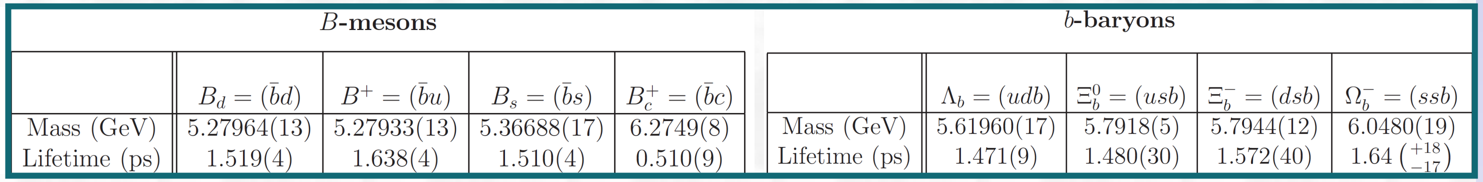
Outline

- Introduction & Motivation
- Theoretical framework & QCDF approach for hadronic B decays
- NNLO perturbative QCD corrections to hadronic matrix elements
- Possible higher-order power corrections motivated by data
- Summary

Introduction & Motivation

B physics and B decays

- **B physics: productions & decays of various b hadrons**



B-mesons				
	$B_d = (\bar{b}d)$	$B^+ = (\bar{b}u)$	$B_s = (\bar{b}s)$	$B_c^+ = (\bar{b}c)$
Mass (GeV)	5.27964(13)	5.27933(13)	5.36688(17)	6.2749(8)
Lifetime (ps)	1.519(4)	1.638(4)	1.510(4)	0.510(9)

b-baryons				
	$\Lambda_b = (udb)$	$\Xi_b^0 = (usb)$	$\Xi_b^- = (dsb)$	$\Omega_b^- = (ssb)$
Mass (GeV)	5.61960(17)	5.7918(5)	5.7944(12)	6.0480(19)
Lifetime (ps)	1.471(9)	1.480(30)	1.572(40)	$1.64\,(^{+18}_{-17})$

- **b-hadron weak decays:** at the quark level, all governed by flavor-changing charged-currents mediated by W-boson

$$\mathcal{L}_{\rm CC} = -\frac{g}{\sqrt{2}} J^{\mu}_{\rm CC}\, W^{\dagger}_{\mu} + \text{h.c.}$$

$$J^{\mu}_{\rm CC} = (\bar{\nu}_e, \bar{\nu}_\mu, \bar{\nu}_\tau)\, \gamma^\mu \begin{pmatrix} e_{\rm L} \\ \mu_{\rm L} \\ \tau_{\rm L} \end{pmatrix} + (\bar{u}_{\rm L}, \bar{c}_{\rm L}, \bar{t}_{\rm L})\, \gamma^\mu\, V_{\rm CKM} \begin{pmatrix} d_{\rm L} \\ s_{\rm L} \\ b_{\rm L} \end{pmatrix}$$

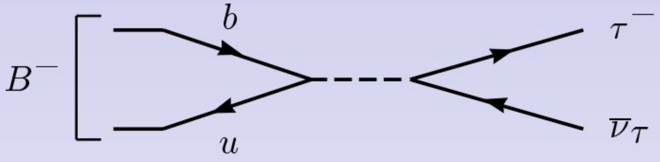
g: $SU(2)_L$ gauge coupling

$V_{\rm CKM}$: CKM matrix for quark mixing

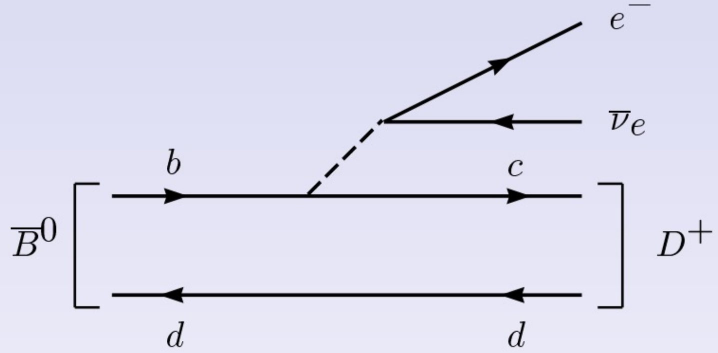
$$V_{\rm CKM} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix}$$

- **Classification of b-hadron weak decays:** three classes

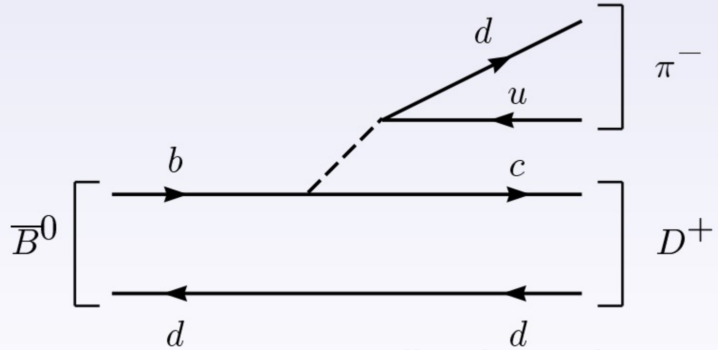
purely leptonic, semi-leptonic, non-leptonic (hadronic)



purely leptonic



semi-leptonic



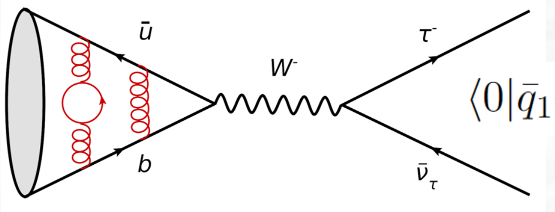
non-leptonic (hadronic)

Interplay between weak & strong forces

☐ **QCD effect always matters:** in real world, quarks confined inside hadrons and no free quarks;

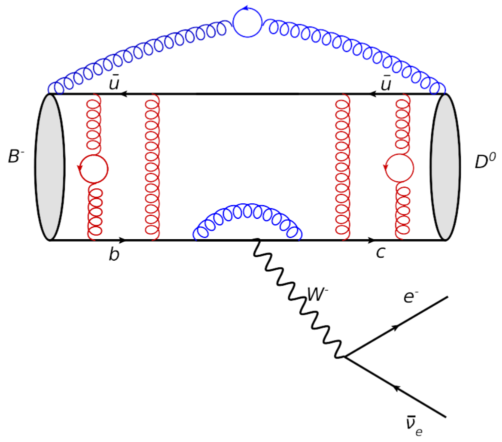
↳ the simplicity of weak interactions overshadowed by the complexity of strong interactions

- **Purely leptonic decays:** decay constant



$$\langle 0|\bar{q}_1\gamma_\mu\gamma_5 q_2|P(p)\rangle = i p_\mu f_P$$

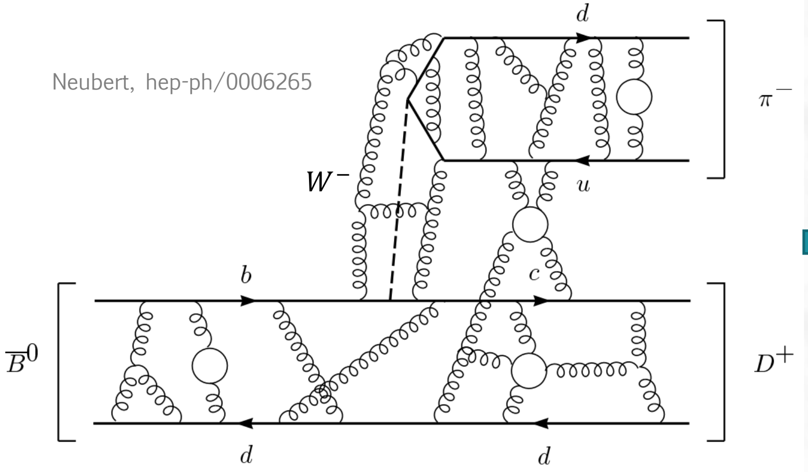
- **Semi-leptonic decays:** transition form factors



$$\langle D|\,\bar{c}\gamma^\mu b\,|\bar{B}\rangle \equiv f_+(q^2)(p_B+p_D)^\mu + [f_0(q^2)-f_+(q^2)]\,\frac{m_B^2-m_D^2}{q^2}\,q^\mu$$

Lattice QCD or LCSR etc.

- **Hadronic decays:** hadronic matrix elements


Neubert, hep-ph/0006265

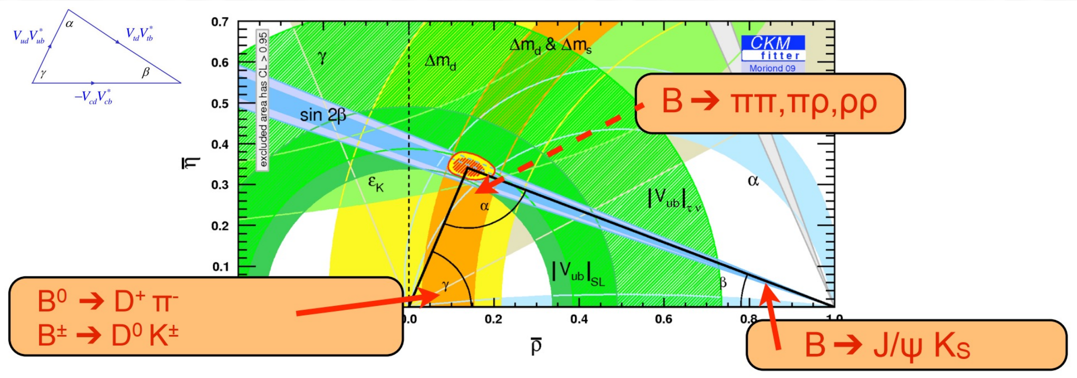
multi-scale problem with highly hierarchical scales!

EW interaction scale	$\gg$	ext. mom'a in B rest frame	$\gg$	QCD-bound state effects
$m_W \sim 80$ GeV $m_Z \sim 91$ GeV	$\gg$	$m_b \sim 5$ GeV	$\gg$	$\Lambda_{\rm QCD} \sim 1$ GeV

the most complicated, but also most interesting!

Why hadronic B decays

- ☐ **direct access to the CKM parameters, especially to the three angles of UT**



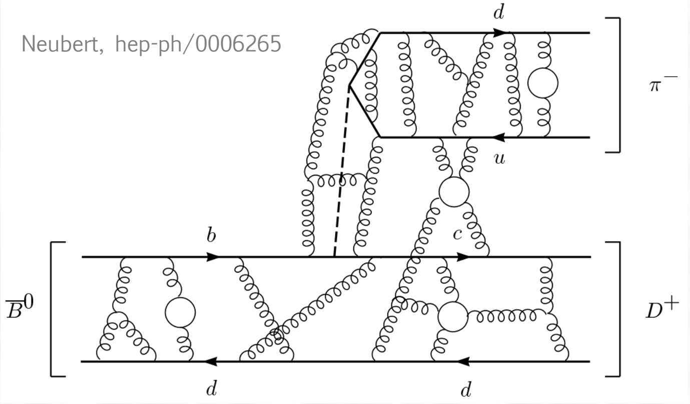
B → ππ,πρ,ρρ

$B^0 \to D^+\pi^-$
$B^\pm \to D^0 K^\pm$

B → J/ψ Ks

- ☐ **further insight into the strong-interaction effects involved in hadronic weak decays**

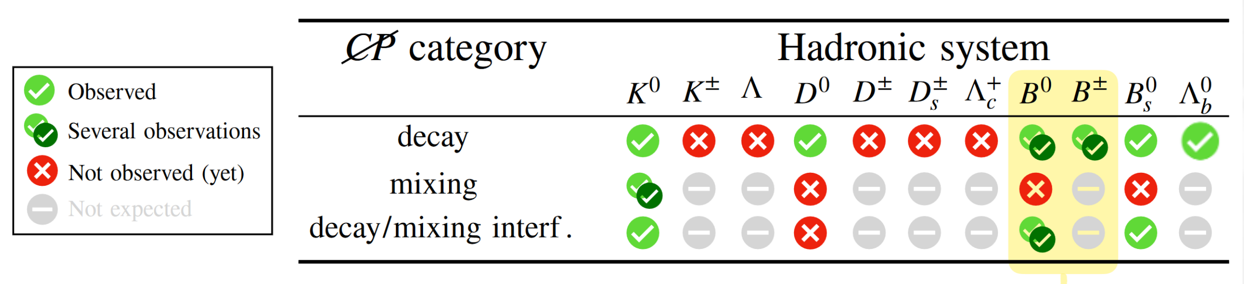
factorization? strong phase origin?...



Neubert, hep-ph/0006265

- ☐ **deep insight into the hadron structures: especially exotic hadronic states**

- ☐ **deepen our understanding of the origin & mechanism of CPV**



✓ Observed
✓✓ Several observations
✗ Not observed (yet)
⊖ Not expected

$\not{C}\not{P}$ category	Hadronic system										
	K^0	$K^\pm$	Λ	D^0	$D^\pm$	$D_s^\pm$	Λ_c^+	B^0	$B^\pm$	B_s^0	Λ_b^0
decay	✓	✗	✗	✓	✗	✗	✗	✓✓	✓✓	✓	✓
mixing	✓✓	⊖	⊖	✗	⊖	⊖	⊖	✗	⊖	✗	⊖
decay/mixing interf .	✓	⊖	⊖	✗	⊖	⊖	⊖	✓✓	⊖	✓	⊖

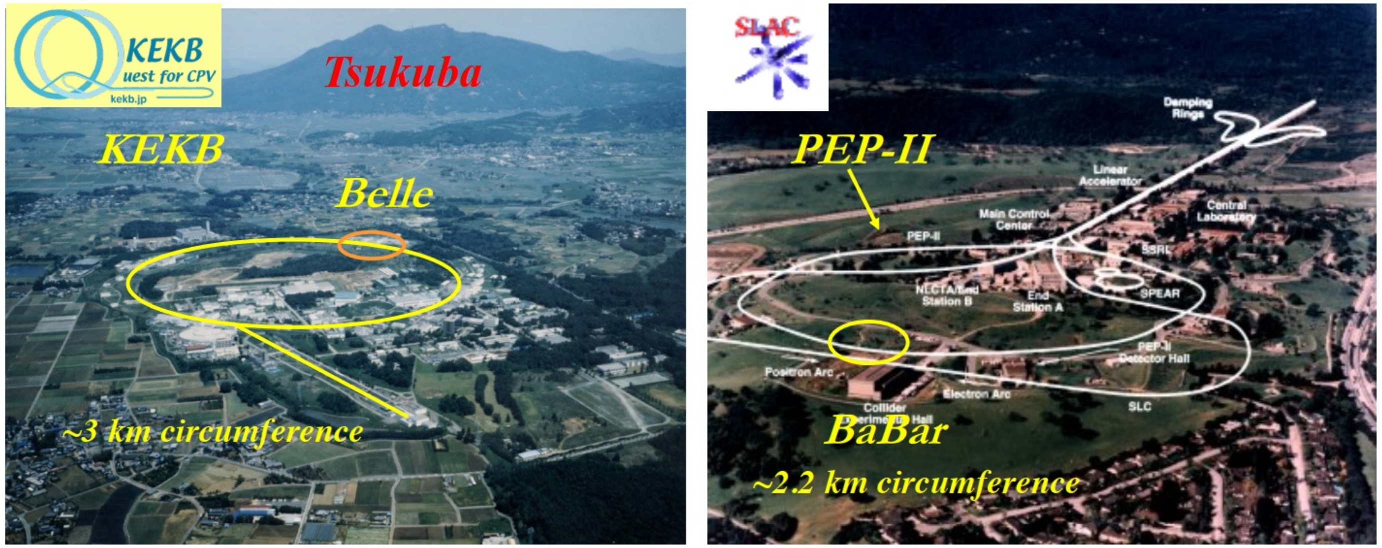
➡ ***Hadronic B decays always play key roles in testing SM & probing NP beyond it***

Exp. facilities of B physics

- □ **B-factories (e^+e^-): Belle & BaBar**
- □ **Hadron colliders ($p\bar{p}$): CDF & D0 @ Tevatron**

https://www-d0.fnal;
https://www-cdf.fnal.gov/gov/

observation of B_s mixing



3.5 GeV e^+ 8 GeV e^-

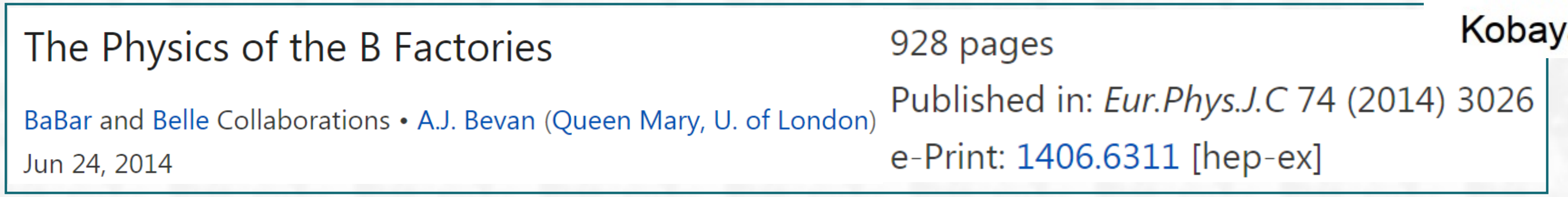
3.1 GeV e^+ 9 GeV e^-

BaBar & Belle confirmed the KM mechanism of CPV in the SM!

Nobel Prize 2008 for

Makoto Kobayashi

Toshihide Maskawa



The Physics of the B Factories

BaBar and Belle Collaborations • A.J. Bevan (Queen Mary, U. of London)

Jun 24, 2014

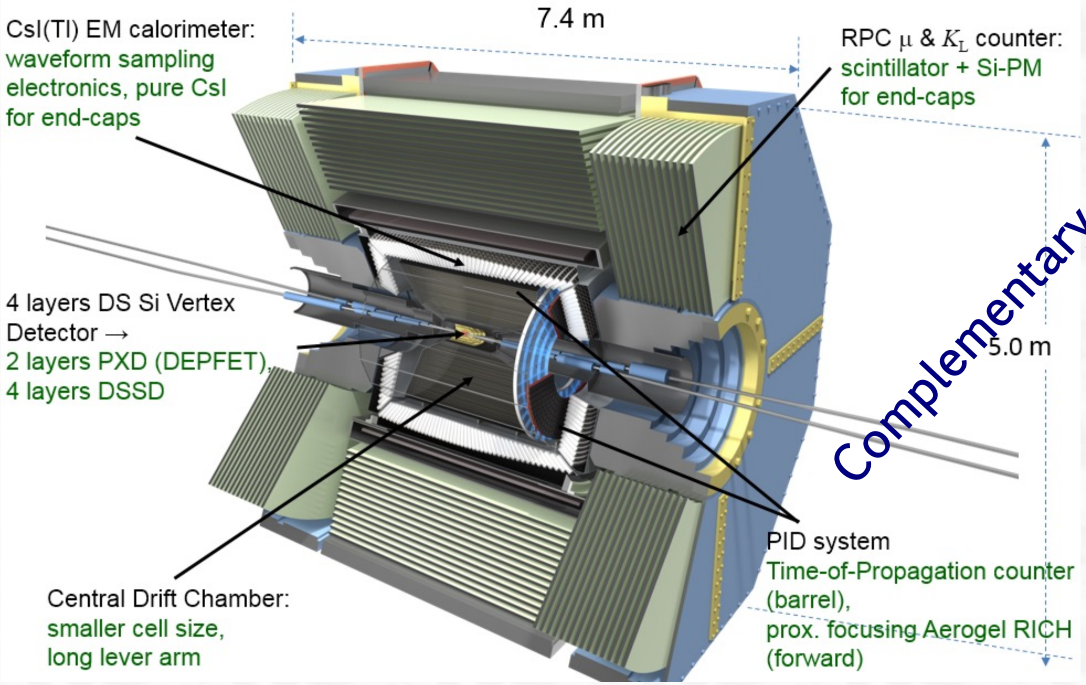
928 pages

Published in: *Eur.Phys.J.C* 74 (2014) 3026

e-Print: 1406.6311 [hep-ex]

Exp. facilities of B physics

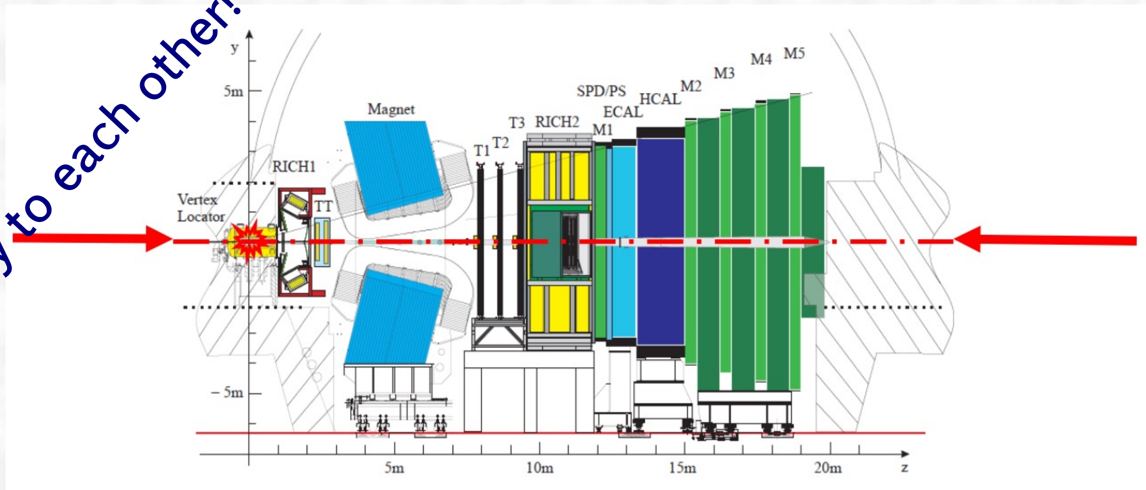
☐ Super B-factories (e^+e^-): **Belle II**



[E. Kou *et al.* [Belle II], PTEP 2019 (2019) 123C01]

☐ Hadron colliders (pp): **LHCb @LHC**

Complementary to each other!



[R. Aaij *et al.* [LHCb Collaboration], arXiv:1808.08865]

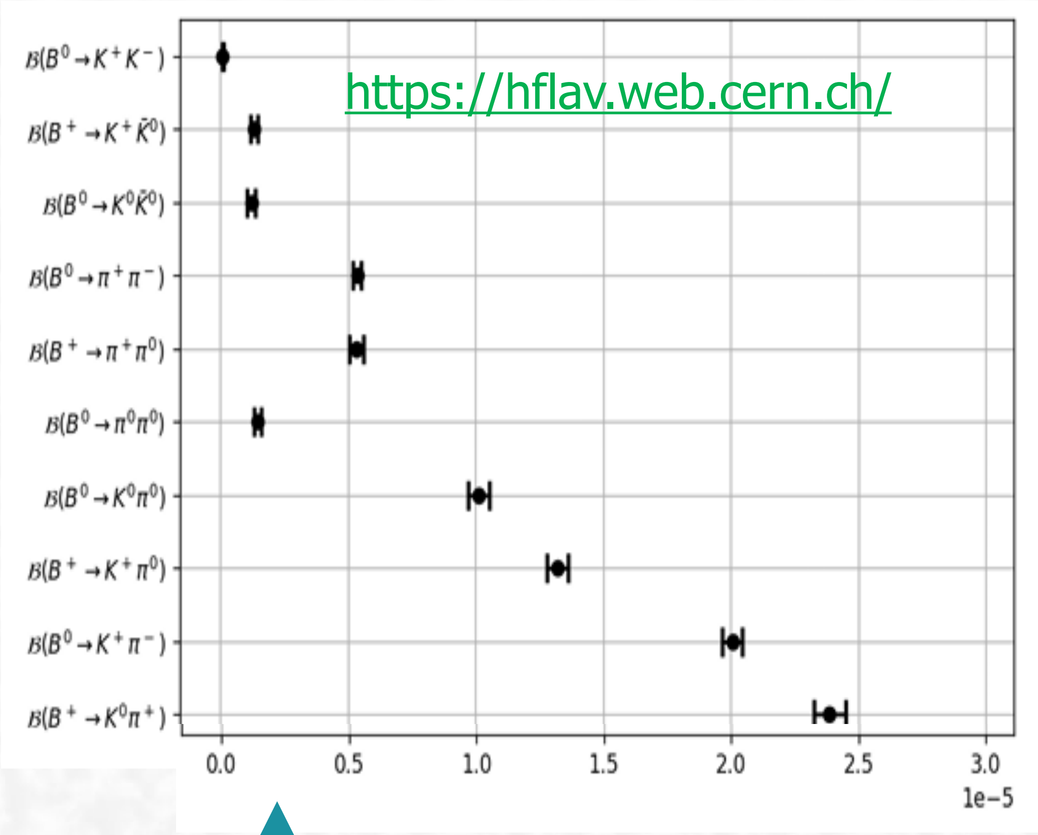
LHCb & Belle II: the two currently running experiments aimed at heavy flavor physics!

☐ Two main goals among others:

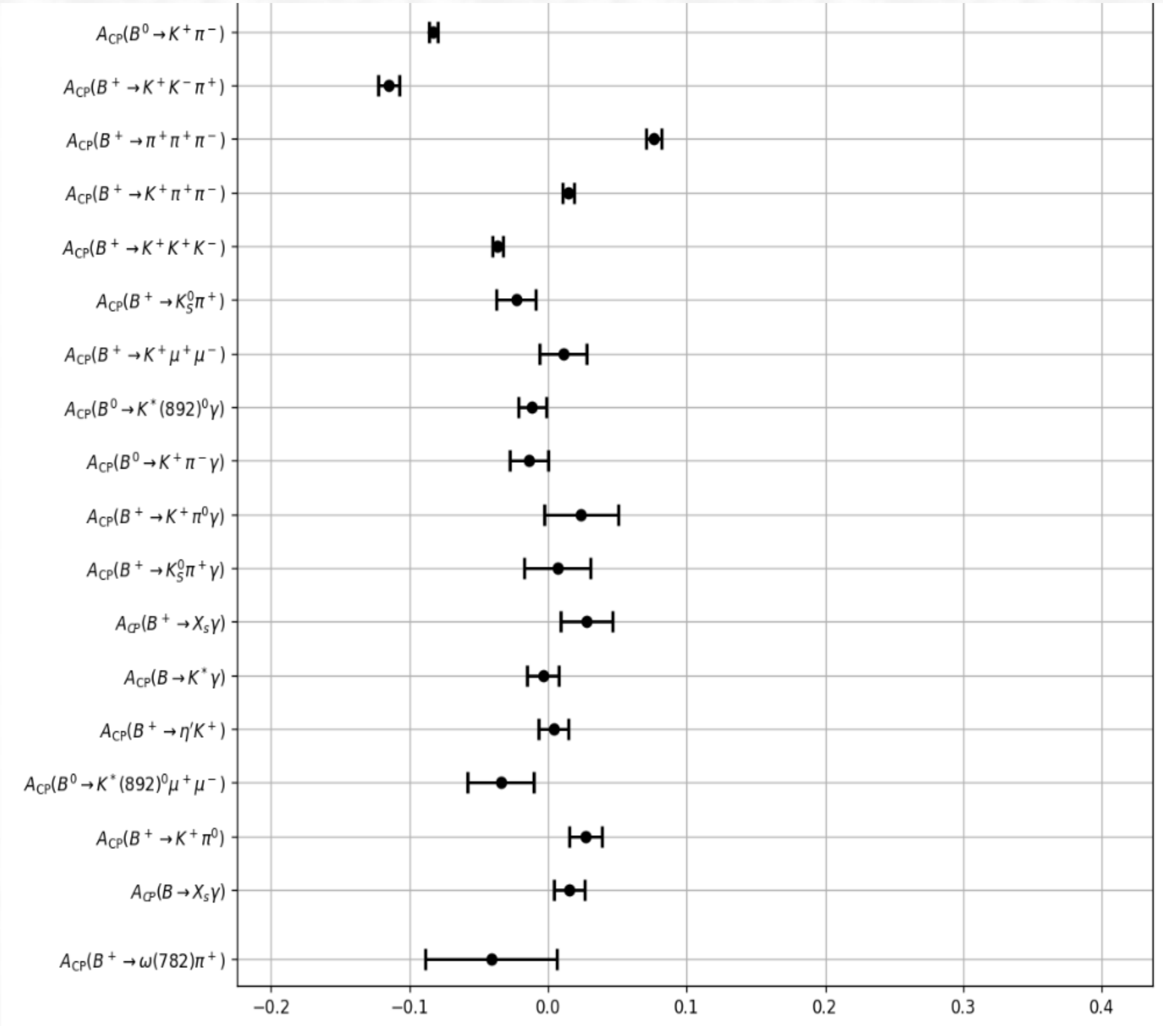
- **Check if there are any extra new CP-violation mechanisms beyond the KM?**
- **Check if there are new particles/interactions that are sensitive to flavor structures?**

Precision era of B physics

□ With LHCb & Belle II running, we are now having more precise data:



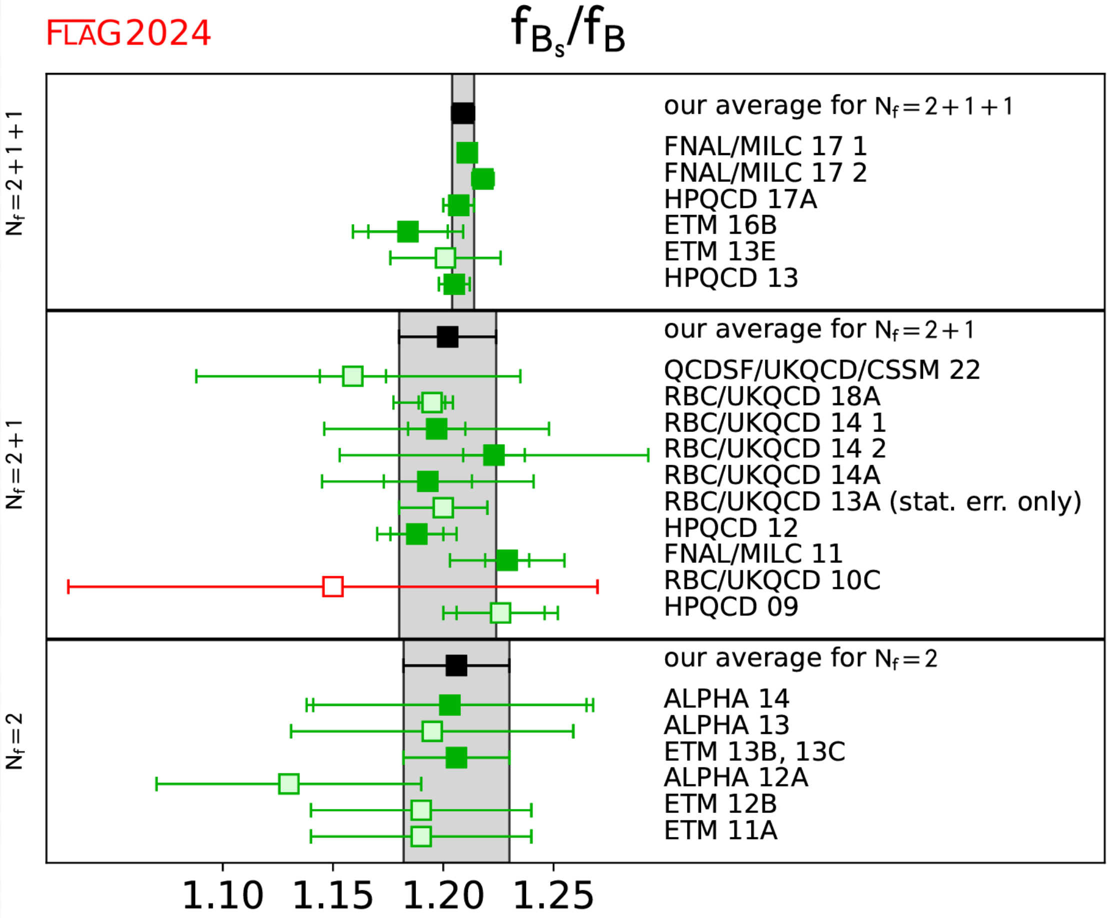
https://hflav.web.cern.ch/

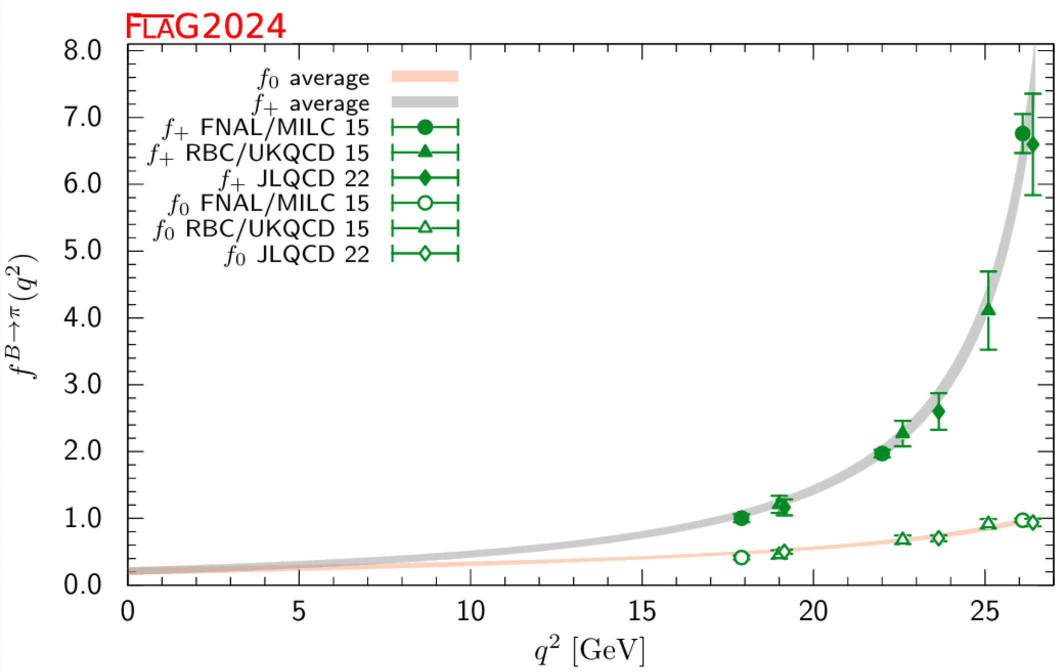


Branching ratio **Direct CPV**

Precision era of B physics

□ **Lattice QCD & LCSR provide more precise results for the non-pert. hadronic parameters**





http://flag.itp.unibe.ch/2024/

With both exp. and theo. progress, we are now entering an era of *precision flavor physics*

Theoretical framework & QCDF approach for hadronic B-meson decays

Effective Hamiltonian for hadronic B decays

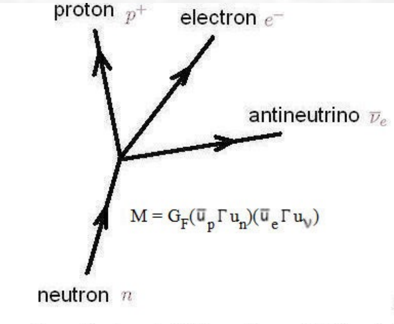
- **For hadronic B decays: typical multi-scale problem**

multi-scale problem with highly hierarchical scales!

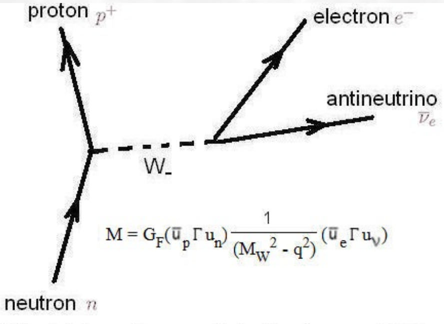
EW interaction scale	$\gg$	ext. mom'a in B rest frame	$\gg$	QCD-bound state effects
$m_W \sim 80$ GeV $m_Z \sim 91$ GeV	$\gg$	$m_b \sim 5$ GeV	$\gg$	$\Lambda_{\rm QCD} \sim 1$ GeV

EFT formalism more suitable!

- **Example: 4-Fermi theory of beta decay**



a. Fermi's 4-point Interaction, 1934

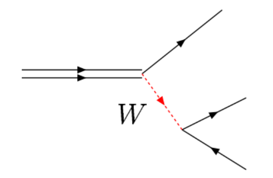


b. Weak Interaction mediated by boson, 1938

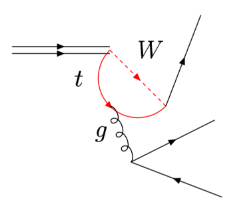
- **Starting point $\mathcal{H}_{\rm eff} = -\mathcal{L}_{\rm eff}$: obtained after integrating out heavy d.o.f. ($m_{W,Z,t} \gg m_b$)**

[Buras, Buchalla, Lautenbacher '96; Chetyrkin, Misiak, Munz '98]

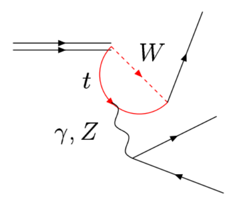
$$\mathcal{L}_{\rm eff} = -\frac{G_F}{\sqrt{2}} \sum_{p=u,c} V_{pb} V_{pD}^* \left(C_1 \mathcal{O}_1 + C_2 \mathcal{O}_2 + \sum_{i={\rm pen}} C_i \mathcal{O}_{i,{\rm pen}} \right)$$



tree



QCD penguin



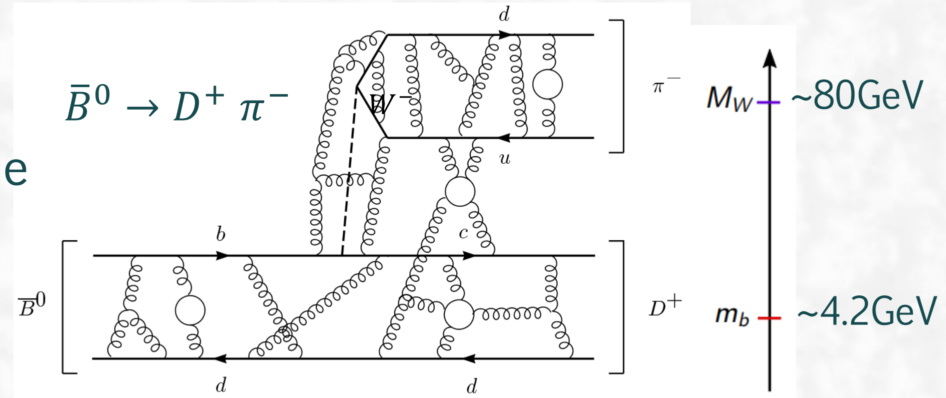
EW penguin

- **Wilson coefficients C_i: all physics above m_b; perturbatively calculable & NNLL program now complete!** [Gorbahn, Haisch '04; Misiak, Steinhauser '04]

Calculation of $C_i(\mu_b)$

- ☐ **Problem:** well-separated multiple scales would spoil the perturbative convergence due to large logs

$$\boxed{P(M_W, m_b) = 1 + \alpha_s\left(\# \ln\frac{M_W}{m_b} + *\right) + \alpha_s^2\left(\# \ln^2\frac{M_W}{m_b} + *\right) + \ldots}$$



$\bar{B}^0 \to D^+\,\pi^-$

- ☐ **Solution:** the perturbative series needs to be re-organized, and all $(\alpha_s \ln\frac{m_W}{m_b})^n$ re-summed!

  - ➢ **Point 1:** through matching to achieve a separation of scales, sometimes also called "factorization";

$$\left[1 + \alpha_s\left(\# \ln\frac{M_W}{\mu} + *\right) + \ldots\right] \cdot \left[1 + \alpha_s\left(\# \ln\frac{\mu}{m_b} + *\right) + \ldots\right]$$

$$P(M_W, m_b) = C(M_W, \mu) D(m_b, \mu)$$

$\boxed{\mu \text{ arbitrary}}$

at the cost of introducing a "factorization scale" μ.

  - ➢ **Point 2:** solve RGE and evolve

$$\text{RGEs:} \left\{\begin{array}{rcl} \mu\frac{d}{d\mu}C(M_W,\mu) & = & \gamma(\mu)\,C(M_W,\mu) \\ \mu\frac{d}{d\mu}D(M_W,\mu) & = & -\gamma(\mu)\,D(M_W,\mu) \end{array}\right\} \Rightarrow \mu\frac{d}{d\mu}(CD) = 0$$

["C and D run with μ."]

$\boxed{\mu_{\text{high}} \sim M_W}$

$$\begin{array}{rcl} C(M_W,\mu) & = & C(M_W,\mu_{\text{high}})\,U(\mu_{\text{high}},\mu) \\ D(m_b,\mu) & = & D(m_b,\mu_{\text{low}})\,U(\mu,\mu_{\text{low}}) \end{array}$$

$\boxed{\mu_{\text{low}} \sim m_b}$

- ☐ **Final result:**

RG-improved P.T.

$$\boxed{P(M_W, m_b) = \underbrace{C(M_W,\mu_{\text{high}})\,U(\mu_{\text{high}},\mu_{\text{low}})}_{C_{\text{RGimproved}}(M_W,\mu_{\text{low}})}\,D(m_b,\mu_{\text{low}})}$$

$U(\mu_{\text{high}}, \mu_{\text{low}})$ is generally an exponential, and hence re-sums large logs $(\alpha_s \ln\frac{\mu_{\text{high}}}{\mu_{\text{low}}})^n$!

Calculation of $C_i(\mu_b)$

☐ **Three steps to get $C_i(\mu_b)$:**

- Matching calculation of $C_i(M_W)$ in fixed-order perturbation theory:

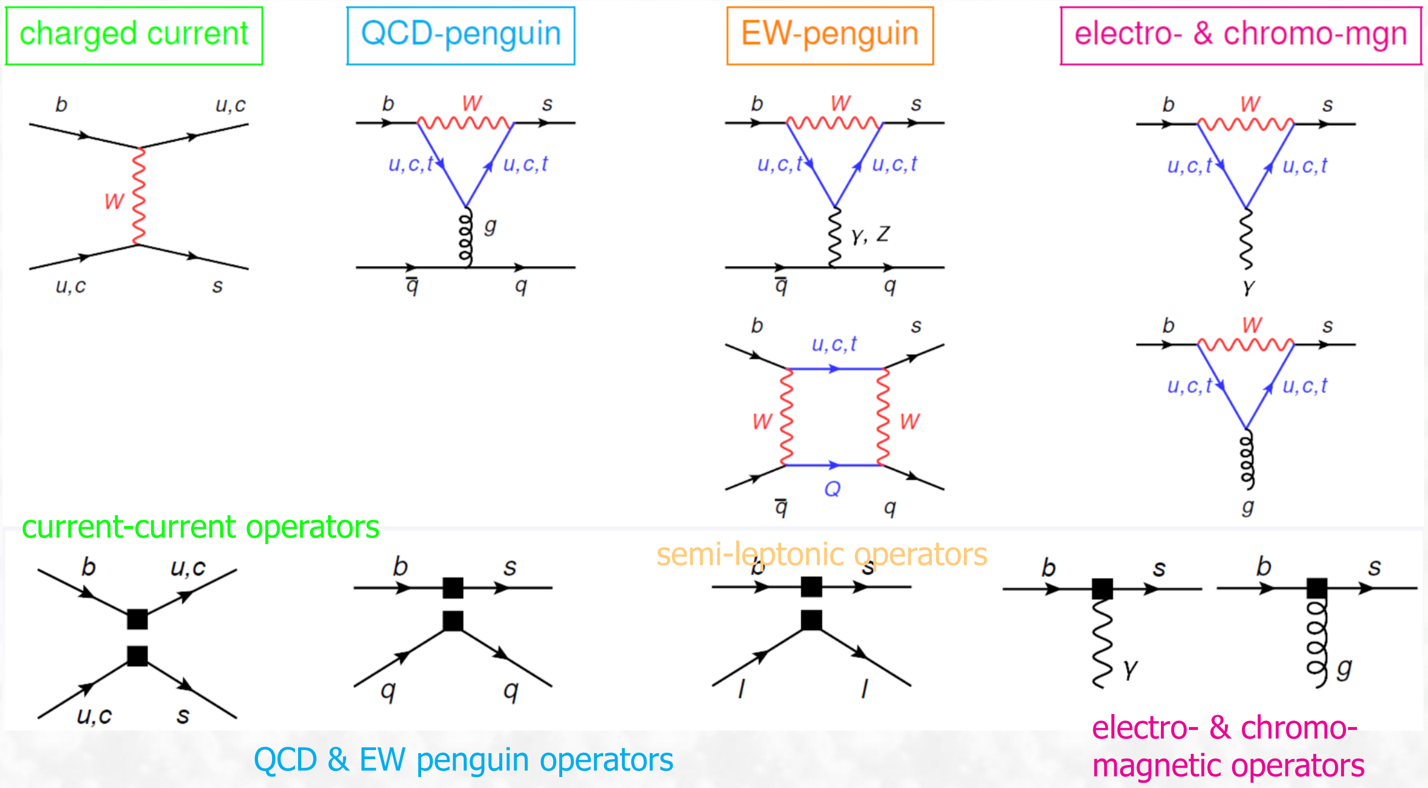
$$C_i(M_W) = C_i^{(0)}(M_W) + \frac{\alpha_s}{4\pi} C_i^{(1)}(M_W) + \cdots$$

- Calculation of anomalous dimensions γ_{ij} of local operators in $\mathcal{H}_{\text{eff}}$:

$$\gamma_{ij} = \gamma_{ij}^{(0)} + \frac{\alpha_s}{4\pi}\gamma_{ij}^{(1)} + \cdots$$

- Use renormalization group to evolve the Wilson coefficients from the high to the low scale:

$$C_i(M_W) \to C_i(m_b) = \left(\frac{\alpha_s(m_b)}{\alpha_s(M_W)}\right)^{-\gamma_{ij}^{(0)}/2\beta_0} C_j(M_W) + \cdots$$



$$O_i = \begin{cases} (\bar s\Gamma_i c)(\bar c\Gamma_i' b), & i = 1,2, \quad |C_i(m_b)| \sim 1 \\ (\bar s\Gamma_i b)\Sigma_q(\bar q\Gamma_i' q), & i = 3,4,5,6, \quad |C_i(m_b)| < 0.07 \\ \frac{em_b}{16\pi^2}\bar s_L\sigma^{\mu\nu}b_R F_{\mu\nu}, & i = 7, \quad C_7(m_b) \sim -0.3 \\ \frac{gm_b}{16\pi^2}\bar s_L\sigma^{\mu\nu}T^a b_R G^a_{\mu\nu}, & i = 8, \quad C_8(m_b) \sim -0.15 \\ \frac{e^2}{16\pi^2}(\bar s_L\gamma_\mu b_L)(\bar l\gamma^\mu\gamma_5 l), & i = 9, 10 \quad |C_i(m_b)| \sim 4 \end{cases}$$

Hadronic matrix elements

- ☐ **For a typical two-body decay $\bar{B} \to M_1 M_2$:**

$$\boxed{\mathcal{A}(\bar{B} \to M_1 M_2) = \sum_i [\lambda_{\text{CKM}} \times C_i \times \langle M_1 M_2 | \mathcal{O}_i | \bar{B} \rangle]}$$

- ☐ $\langle M_1 M_2 | \mathcal{O}_i | \bar{B} \rangle$: depending on spin & parity of $M_{1,2}$; final-state re-scattering introduces strong phases, and hence non-zero direct CPV; ➡ *A quite difficult, multi-scale, strong-interaction problem!*

- ☐ **Different methods proposed for $\langle M_1 M_2 | \mathcal{O}_i | \bar{B} \rangle$:** naïve fact., generalized fact., ……

- Dynamical approaches based on factorization theorems: PQCD, QCDF, SCET, · · ·

[Keum, Li, Sanda, Lü, Yang '00;
Beneke, Buchalla, Neubert, Sachrajda, '00;
Bauer, Flemming, Pirjol, Stewart, '01; Beneke, Chapovsky, Diehl, Feldmann, '02]

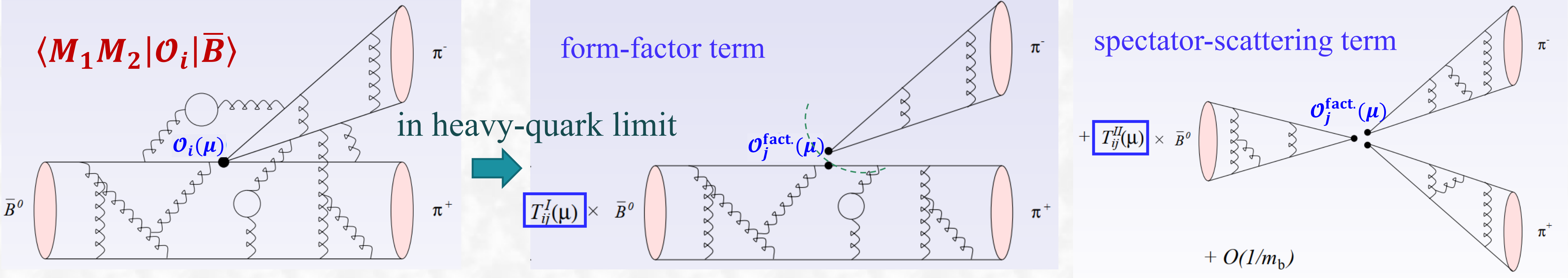
how to include higher-order perturbative & power corrections?

- Symmetries of QCD: Isospin, U-Spin, V-Spin, and flavour SU(3) symmetries, · · ·

[Zeppenfeld, '81;
London, Gronau, Rosner, He, Chiang, Cheng *et al.*]

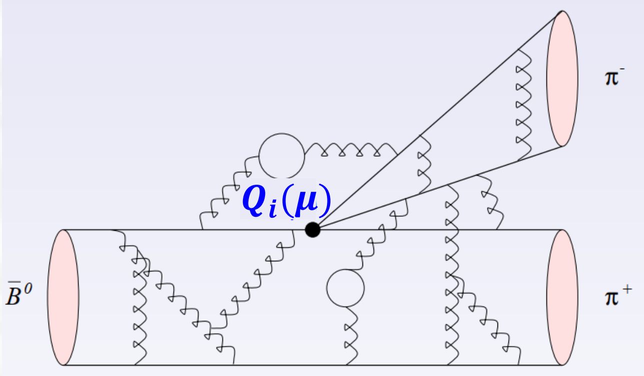
how to systematically estimate symmetry-breaking effects?

- ☐ **QCDF/SCET:** systematic framework from QCD, valid to all orders in α_s, limited by $\frac{\Lambda_{\text{QCD}}}{m_b}$ corrections



QCDF formula for charmless B decays

☐ QCDF formula: [BBNS '99-'03]



$$\langle M_1 M_2|Q_i|\bar B\rangle \sim F^{B\to M_1}(q^2=0)\int_0^1 \mathrm{d}x\, \mathbf{T}_i^{\mathrm{I}}(x)\,\phi_{M_2}(x)$$

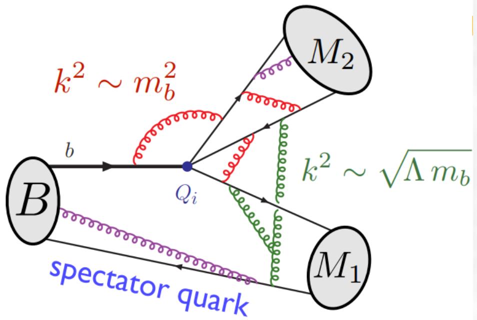
form-factor term

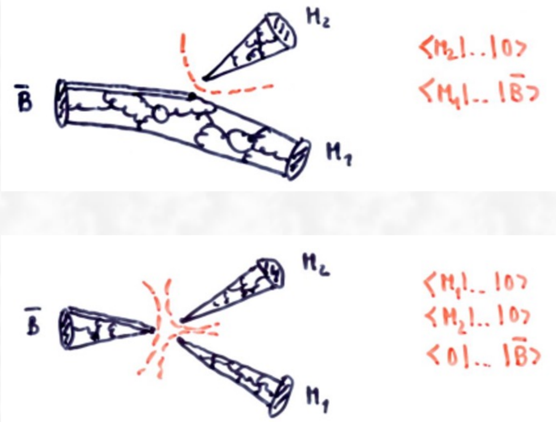
$$+\int_0^\infty \frac{\mathrm{d}\omega}{\omega}\int_0^1 \mathrm{d}x\,\mathrm{d}y\;\mathbf{T}_i^{\mathrm{II}}(x,y,\omega)\,\phi_{M_1}(y)\,\phi_{M_2}(x)\,\phi_B^+(\omega)$$

spectator-scattering term

☐ How to proof QCDF formula:

- based on diagrammatic factorization [BBNS '99-'03]
- method of expansion by regions [Beneke, Smirnov '97]
- combining heavy-quark & collinear expansion for hard exclusive processes [Lepage, Brodsky '80]



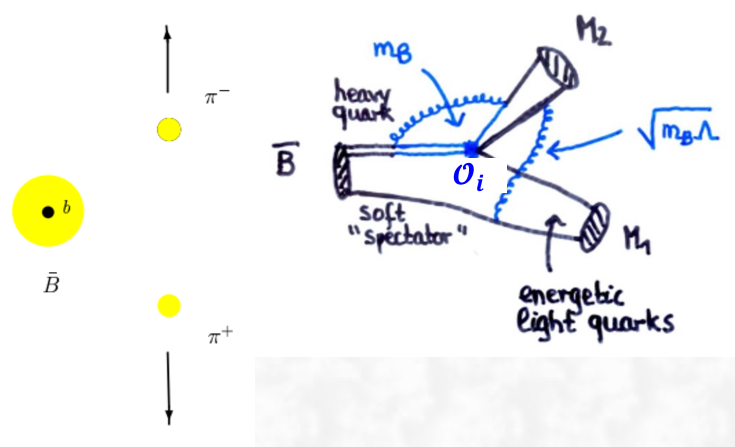


non-perturbative but universal hadronic parameters

$\langle M_1 M_2|Q_i|\bar B\rangle$ factorized into $\langle M|j_\mu|\bar B\rangle$ (transition form factors), $\langle M|j_\mu|0\rangle$, $\langle 0|j_\mu|\bar B\rangle$ (decay constants & LCDAs)

Soft-collinear factorization from SCET

☐ **For a two-body decay:** simple kinematics, but complicated dynamics with several typical modes



- low-virtuality modes:
  - ⋆ HQET fields: $p - m_b v \sim \mathcal{O}(\Lambda)$
  - ⋆ soft spectators in B meson: $p_s^\mu \sim \Lambda \ll m_b, \quad p_s^2 \sim \mathcal{O}(\Lambda^2)$
  - ⋆ collinear quarks and gluons in pion: $E_c \sim m_b, \quad p_c^2 \sim \mathcal{O}(\Lambda^2)$
- high-virtuality modes:
  - ⋆ hard modes: (heavy quark + collinear)$^2 \sim \mathcal{O}(m_b^2)$
  - ⋆ hard-collinear modes: (soft + collinear)$^2 \sim \mathcal{O}(m_b\Lambda)$

☐ **SCET:** a very suitable framework for studying factorization and re-summation for processes involving energetic & light particles/jets [Bauer *et al.* '00; Beneke *et al.* '02]

☐ **From SCET point of view:** introduce different fields/modes for different momentum regions, and SCET diagrams must reproduce QCD diagrams in collinear & soft momentum region!

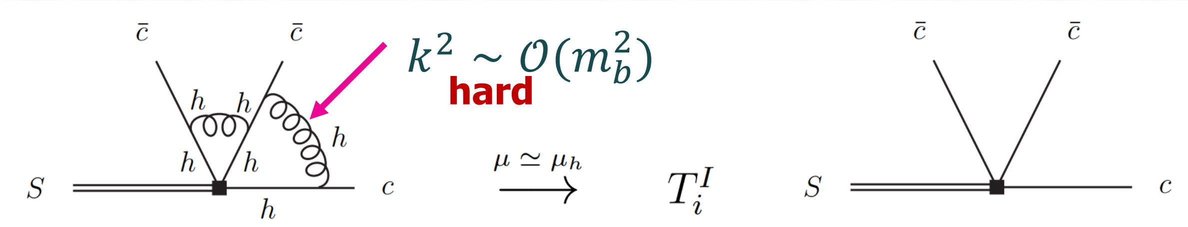
↳ **achieve *soft-collinear factorization* & hence QCDF formula via strict QFT machinery** [Beneke, 1501.07374]

Soft-collinear factorization from SCET

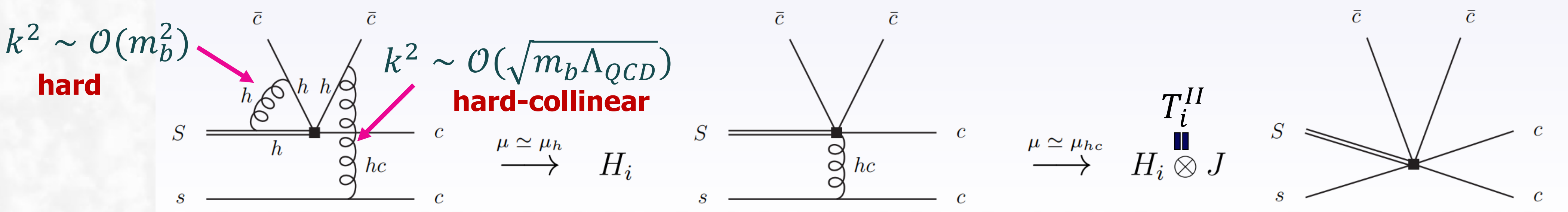
- **QCDF formula from SCET:** hard kernels $T_i^{I,II}$ = matching coefficients from QCD to SCET

$$\langle M_1 M_2|\mathcal{Q}_i|\bar{B}\rangle \simeq F^{B\to M_1}\, T_i^I \otimes \phi_{M_2} + T_i^{II} \otimes \phi_B \otimes \phi_{M_1} \otimes \phi_{M_2} \Longrightarrow \boxed{\text{QCD - SCET} = T^I \;\&\; T^{II}}$$

- **For T_i^I:** only hard scale involved, one-step matching from QCD → $\mathrm{SCET_I(hc, c, s)}$!



- **For T_i^{II}:** two scales involved, two-step matching from QCD → $\mathrm{SCET_I(hc, c, s)}$ → $\mathrm{SCET_{II}(c, s)}$!



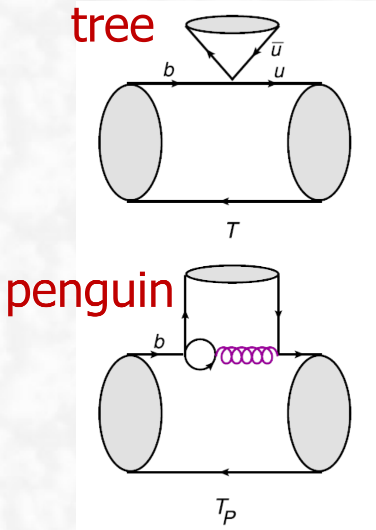
- **SCET formalism reproduces exact QCDF formula, but more apparent & efficient;** [Beneke, 1501.07374]

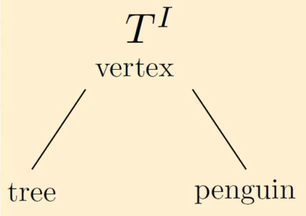
$$\boxed{\langle M_1 M_2|\mathcal{Q}_i|\bar{B}\rangle = T^I(\mu_h) * \phi_{M_2}(\mu_h) f_+^{BM_1}(0) + H_i(\mu_h) * U_\parallel(\mu_h, \mu_{hc}) * J(\mu_{hc}) * \phi_{M_2}(\mu_h) * \phi_{M_1}(\mu_{hc}) * \phi_B(\mu_{hc})}$$

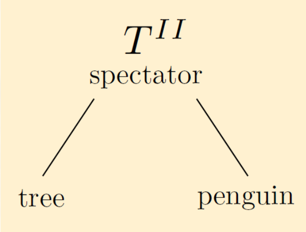
Status of NNLO calculation of T^I & T^{II}

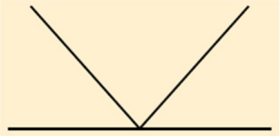
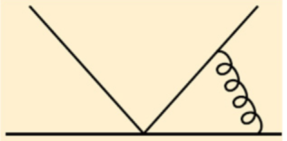
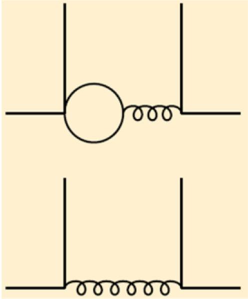
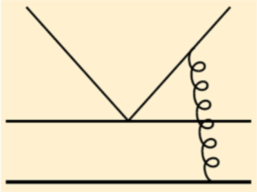
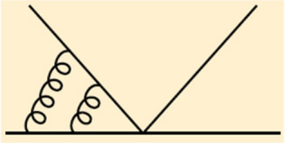
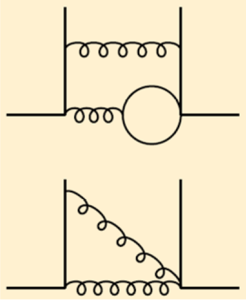
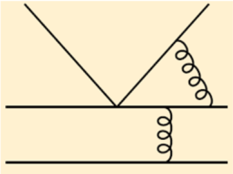
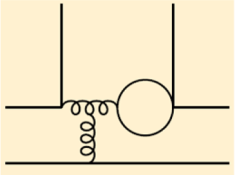
- **For each Q_i insertion, both tree & penguin topologies relevant for charmless decays**

$$\boxed{\begin{aligned}\langle M_1 M_2|Q_i|\bar{B}\rangle \simeq F^{B\to M_1}\, T_i^I \otimes \phi_{M_2}\\ +T_i^{II} \otimes \phi_B \otimes \phi_{M_1} \otimes \phi_{M_2}\end{aligned}}$$







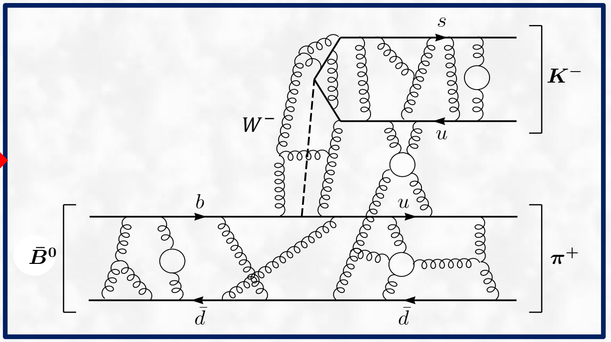
	T_i^I, tree	T_i^I, penguin	T_i^{II}, tree	T_i^{II}, penguin
LO: $\mathcal{O}(1)$		$T^I = 1 + \mathcal{O}(\alpha_s) + \cdots$		
NLO: $\mathcal{O}(\alpha_s)$ BBNS '99-'03				$T^{II} = \mathcal{O}(\alpha_s) + \cdots$
NNLO: $\mathcal{O}(\alpha_s^2)$	 Bell '07, '09 Beneke, Huber, Li '09 Huber, Krankl, Li '16	 Kim, Yoon '11 Bell, Beneke, Huber, Li '15, '20	 Beneke, Jager '05 Kivel '06, Pilipp '07	 Beneke, Jager '06 Jain, Rothstein, Stewart '07

- **For tree & penguin topologies, both contribute to T_i^I & T_i^{II}**

Example

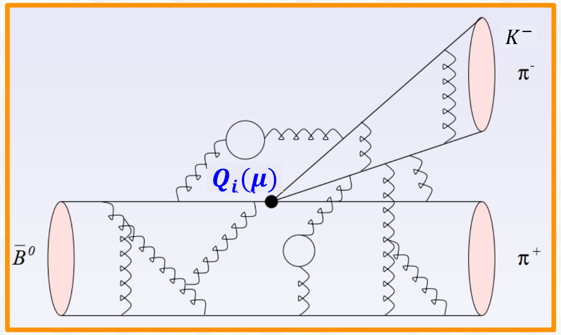
- With $\langle M_1 M_2|Q_i|\bar B\rangle_{\rm QCD,QED}$ at hand, we can then do what we want to do:

$\bar B^0 \to \pi^+ K^-$



EW interaction scale	≫	ext. mom'a in B rest frame	≫	QCD-bound state effects
$m_W \sim 80$ GeV $m_Z \sim 91$ GeV	≫	$m_b \sim 5$ GeV	≫	$\Lambda_{\rm QCD} \sim 1$ GeV

$$\mathcal{H}_{\rm eff} = -\frac{4G_F}{\sqrt{2}} \sum_{p=u,c} V_{pD}^* V_{pb} \left(C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3}^{10} C_i Q_i + C_{7\gamma} Q_{7\gamma} + C_{8g} Q_{8g} \right)$$



electroweak parameters

WCs due to NP

non-perp. parameters

$$\mathcal{A}(\bar B^0 \to \pi^+ K^-) = \frac{G_F}{\sqrt{2}} \sum_{ij} V_{\rm CKM} \left(\mathcal{C}_i^{\rm SM} + \mathcal{C}_i^{\rm NP}\right) \Big[F_j^{B\to\pi}(m_K^2) \int_0^1 du\, T_{ij}^{\rm I}(u) \Phi_K(u) + (\pi \leftrightarrow K) + \int_0^1 d\xi\, du\, dv\, T_i^{\rm II}(\xi,u,v) \Phi_B(\xi) \Phi_\pi(v) \Phi_K(u) \Big]$$

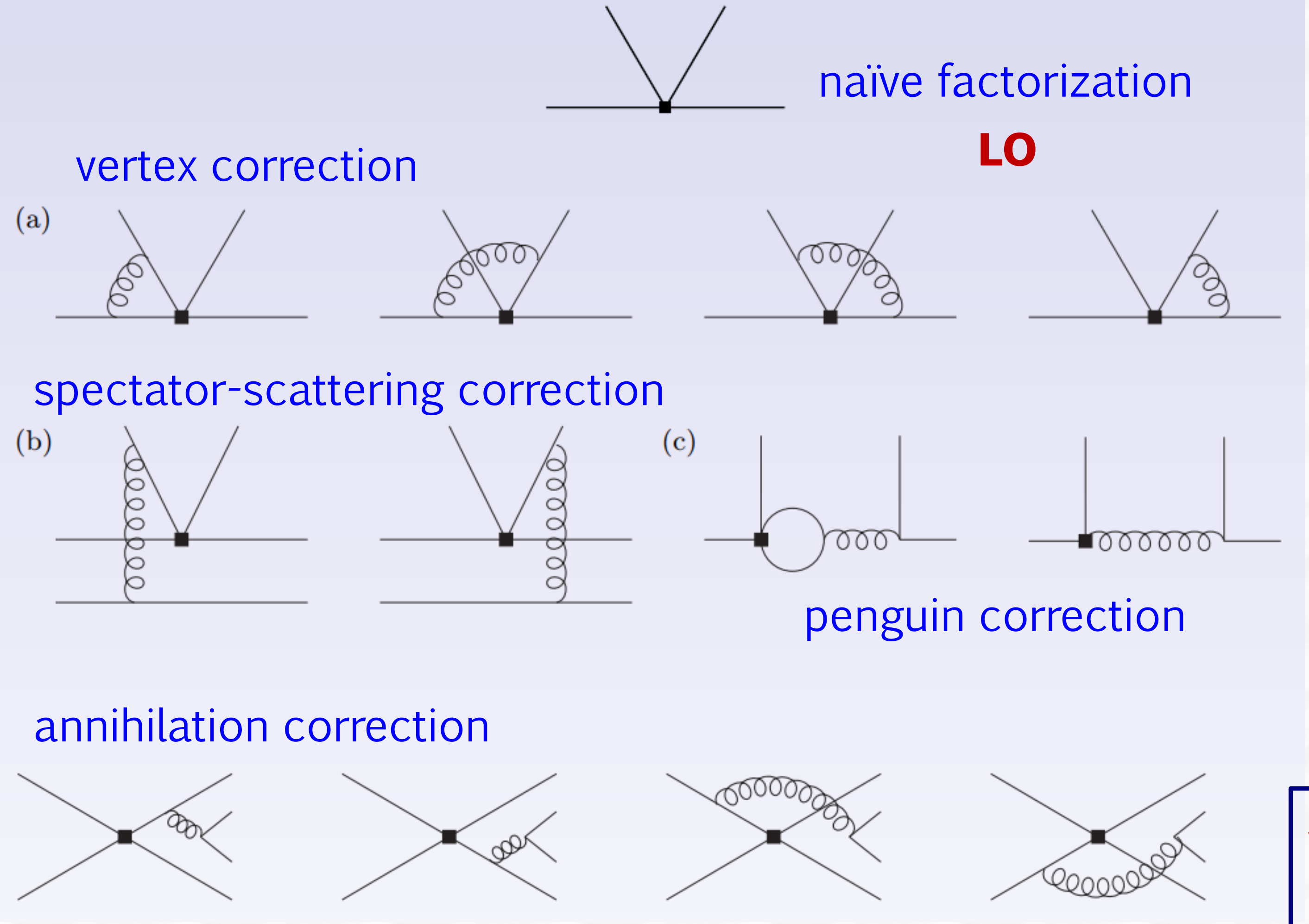
related to exp. Br & CPV

WCs from SM, also perp. calculable

perp. calculable in QCD & QED

Phenomenological analyses based on **NLO**

- **Various analyses based on NLO hard kernels**



naïve factorization

LO

vertex correction

spectator-scattering correction

penguin correction

annihilation correction

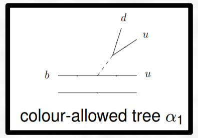
- **complete sets of final states:**

  - $B \to PP, PV$: [Beneke, Neubert, hep-ph/0308039; Cheng, Chua, 0909.5229, 0910.5237;]
  - $B \to VV$: [Beneke, Rohrer, Yang, hep-ph/0612290; Cheng, Yang, 0805.0329; Cheng, Chua, 0909.5229, 0910.5237;]
  - $B \to AP, AV, AA$: [Cheng, Yang, 0709.0137, 0805.0329;]
  - $B \to SP, SV$: [Cheng, Chua, Yang, hep-ph/0508104, 0705.3079; Cheng, Chua, Yang, Zhang, 1303.4403;]
  - $B \to TP, TV$: [Cheng, Yang, 1010.3309;]

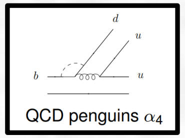
very successful but also with some problems phenomenologically. !

Phenomenological successes based on NLO

☐ Successes at NLO:



colour-allowed tree α_1



QCD penguins α_4

- ➢ For color-allowed tree- & penguin-dominated decay modes, branching ratios usually quantitatively OK
- ➢ Dynamical explanation of intricate patterns of penguin interference seen in *PP*, *PV*, *VP* and *VV* modes

$$PP \sim a_4 + r_\chi a_6, \quad PV \sim a_4 \approx \frac{PP}{3}$$
$$VP \sim a_4 - r_\chi a_6 \sim -PV$$
$$VV \sim a_4 \sim PV$$

$$\boxed{r_\chi = \frac{2m_L^2}{m_b\,(m_q + m_s)}}$$

➡ $\mathrm{Br}\left(B^{\pm,0} \to \eta^{(\prime)} K^{(*)\pm,0}\right)$

- ➢ Qualitative explanation of polarization puzzle in $B \to VV$ decays, due to the large weak annihilation
- ➢ Strong phases start at $\mathcal{O}(\alpha_s)$, dynamical explanation of smallness of direct CP asymmetries

☐ Some problems encountered at NLO:

- ➢ Factorization of power corrections generally broken, due to endpoint divergence
- ➢ Could not account for some data, such as $\mathrm{Br}(B^0 \to \pi^0\pi^0)$ and $\Delta A_{CP}(\pi K)$
- ➢ How important the higher-order pert. corr.? Fact. theorem is still established for them?
- ➢ As strong phases start at $\mathcal{O}(\alpha_s)$, NNLO is only NLO to them; quite relevant for A_{CP}?

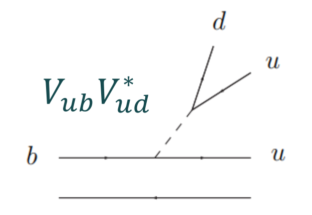
↳ **we need go beyond the LO in pert. and power corrections!**

NNLO perturbative QCD corrections to hadronic matrix elements

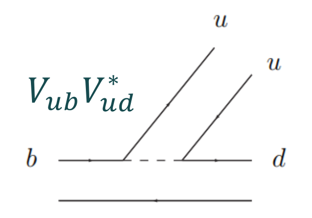
Tree-dominated B decays

☐ **$B \to \pi\pi$ decay amplitudes in QCDF:**

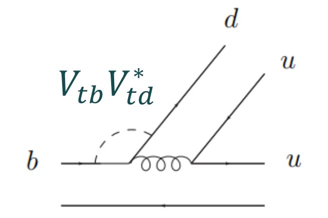
$$\begin{aligned}\sqrt{2}\,\langle \pi^-\pi^0|\,\mathcal{H}_{eff}\,|B^-\rangle &= \lambda_u\big[\alpha_1(\pi\pi)+\alpha_2(\pi\pi)\big]\;A_{\pi\pi}\\ \langle \pi^+\pi^-|\,\mathcal{H}_{eff}\,|\bar{B}^0\rangle &= \big\{\lambda_u\big[\alpha_1(\pi\pi)+\alpha_4^u(\pi\pi)\big]+\lambda_c\,\alpha_4^c(\pi\pi)\big\}\;A_{\pi\pi}\\ -\,\langle \pi^0\pi^0|\,\mathcal{H}_{eff}\,|\bar{B}^0\rangle &= \big\{\lambda_u\big[\alpha_2(\pi\pi)-\alpha_4^u(\pi\pi)\big]-\lambda_c\,\alpha_4^c(\pi\pi)\big\}\;A_{\pi\pi}\end{aligned}$$



colour-allowed tree α_1



colour-suppressed tree α_2



QCD penguins α_4

$b \to u\bar{u}d$: $\lambda_u = V_{ub}V_{ud}^* \sim \mathcal{O}(\lambda^3) \sim \lambda_c = V_{cb}V_{cd}^* \sim \mathcal{O}(\lambda^3)$ → α_4 loop-suppressed vs $\alpha_{1,2}$ → Tree-dominated!

☐ **α_2 at NLO:**

$$\boldsymbol{\alpha_2(\pi\pi) = 0.220 - [0.179 + 0.077i]_{\text{NLO}} + \left[\frac{r_{\text{sp}}}{0.485}\right]\{[0.123]_{\text{LOsp}} + [0.072]_{\text{tw3}}\}}$$

large cancellation between 1-loop vertex correction & LO result,

hence it is dominated by spectator-scattering contributions

$$r_{\text{sp}} = \frac{9f_{M_1}\hat{f}_B}{m_b f_+^{B\pi}(0)\lambda_B}$$

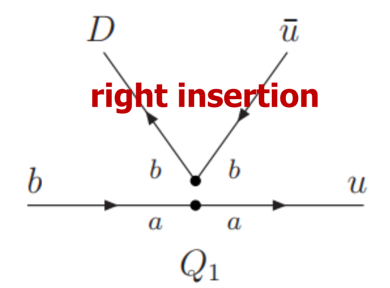
making α_2 sensitive to NNLO corrections, and large effect possible?

Hard kernel T_i^I at NNLO

☐ QCD → SCETI matching calculation:

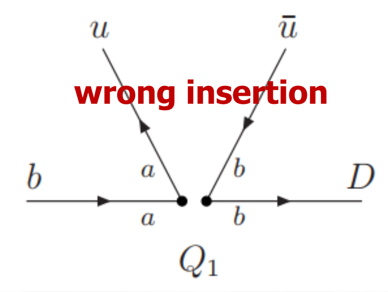
- For "right insertion":

$$\langle Q_i \rangle = T_i \, \langle O_{\rm QCD} \rangle + \sum_{a>1} H_{ia} \langle O_a \rangle$$



- For "wrong insertion":

$$\langle Q_i \rangle = \tilde{T}_i \, \langle O_{\rm QCD} \rangle + \tilde{H}_{i1} \langle \tilde{O}_1 - O_1 \rangle + \sum_{a>1} \tilde{H}_{ia} \langle \tilde{O}_a \rangle$$



☐ On-shell matrix elements at NNLO: full QCD side

$$\begin{aligned}\langle Q_i \rangle = \Big\{ & A_{ia}^{(0)} + \frac{\alpha_s}{4\pi} \left[A_{ia}^{(1)} + Z_{ext}^{(1)} \, A_{ia}^{(0)} + Z_{ij}^{(1)} A_{ja}^{(0)} \right] \\ & + \left(\frac{\alpha_s}{4\pi} \right)^2 \Big[A_{ia}^{(2)} + Z_{ij}^{(1)} A_{ja}^{(1)} + Z_{ij}^{(2)} A_{ja}^{(0)} + Z_{ext}^{(1)} \, A_{ia}^{(1)} + Z_{ext}^{(2)} \, A_{ia}^{(0)} \\ & + Z_{ext}^{(1)} \, Z_{ij}^{(1)} A_{ja}^{(0)} + Z_{\alpha}^{(1)} A_{ia}^{(1)} + (-i) \, \delta m^{(1)} \, A_{ia}^{\prime(1)} \Big] + \mathcal{O}(\alpha_s^3) \Big\} \, \langle O_a \rangle^{(0)}\end{aligned}$$

☐ On-shell matrix elements at NNLO: SCET side

$$\begin{aligned}\langle O_a \rangle = \Big\{ & \delta_{ab} + \frac{\hat{\alpha}_s}{4\pi} \left[M_{ab}^{(1)} + Y_{ext}^{(1)} \, \delta_{ab} + Y_{ab}^{(1)} \right] + \left(\frac{\hat{\alpha}_s}{4\pi} \right)^2 \Big[M_{ab}^{(2)} + Y_{ac}^{(1)} M_{cb}^{(1)} \\ & + Y_{ab}^{(2)} + Y_{ext}^{(1)} \, M_{ab}^{(1)} + Y_{ext}^{(2)} \, \delta_{ab} + Y_{ext}^{(1)} \, Y_{ab}^{(1)} + \hat{Z}_{\alpha}^{(1)} M_{ab}^{(1)} \Big] + \mathcal{O}(\alpha_s^3) \Big\} \, \langle O_b \rangle^{(0)}\end{aligned}$$

☐ Master formula for T_i^I: right insertion

$$\begin{aligned} T_i^{(0)} &= A_{i1}^{(0)} \, , \\ T_i^{(1)} &= A_{i1}^{(1)\rm nf} + Z_{ij}^{(1)} \, A_{j1}^{(0)} \, , \\ T_i^{(2)} &= A_{i1}^{(2)\rm nf} + Z_{ij}^{(1)} \, A_{j1}^{(1)} + Z_{ij}^{(2)} \, A_{j1}^{(0)} + Z_{\alpha}^{(1)} \, A_{i1}^{(1)\rm nf} + (-i) \, \delta m^{(1)} \, A_{i1}^{\prime(1)\rm nf} \\ & \quad - T_i^{(1)} \big[C_{FF}^{(1)} + Y_{11}^{(1)} - Z_{ext}^{(1)} \big] - \sum_{b>1} H_{ib}^{(1)} \, Y_{b1}^{(1)} \, . \end{aligned}$$

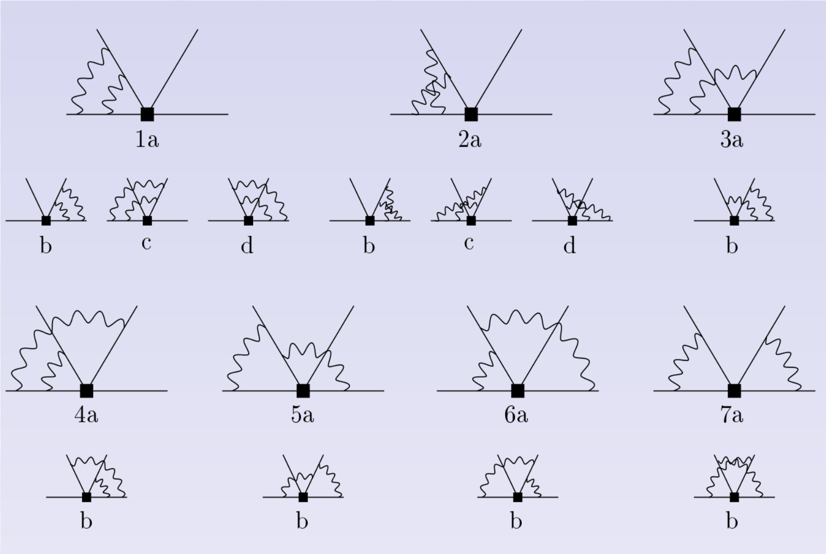
☐ Master formula for T_i^I: wrong insertion

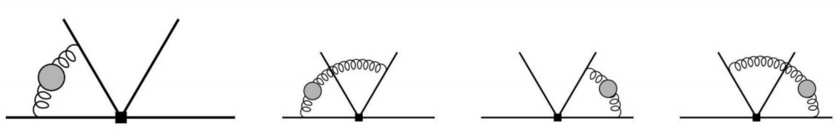
$$\begin{aligned} \widetilde{T}_i^{(0)} &= \widetilde{A}_{i1}^{(0)} \, , \\ \widetilde{T}_i^{(1)} &= \widetilde{A}_{i1}^{(1)\rm nf} + Z_{ij}^{(1)} \, \widetilde{A}_{j1}^{(0)} + \underbrace{\widetilde{A}_{i1}^{(1)\rm f} - A_{21}^{(1)\rm f} \, \widetilde{A}_{i1}^{(0)}}_{\mathcal{O}(\epsilon)} - \underbrace{[\widetilde{Y}_{11}^{(1)} - Y_{11}^{(1)}] \, \widetilde{A}_{i1}^{(0)}}_{\mathcal{O}(\epsilon)} , \\ \widetilde{T}_i^{(2)} &= \widetilde{A}_{i1}^{(2)\rm nf} + Z_{ij}^{(1)} \, \widetilde{A}_{j1}^{(1)} + Z_{ij}^{(2)} \, \widetilde{A}_{j1}^{(0)} + Z_{\alpha}^{(1)} \, \widetilde{A}_{i1}^{(1)\rm nf} \\ & \quad + (-i) \, \delta m^{(1)} \, \widetilde{A}_{i1}^{\prime(1)\rm nf} + Z_{ext}^{(1)} \, [\widetilde{A}_{i1}^{(1)\rm nf} + Z_{ij}^{(1)} \, \widetilde{A}_{j1}^{(0)}] \\ & \quad - \widetilde{T}_i^{(1)} \big[C_{FF}^{(1)} + \widetilde{Y}_{11}^{(1)} \big] - \sum_{b>1} \widetilde{H}_{ib}^{(1)} \, \widetilde{Y}_{b1}^{(1)} \\ & \quad + [\widetilde{A}_{i1}^{(2)\rm f} - A_{21}^{(2)\rm f} \, \widetilde{A}_{i1}^{(0)}] + (-i) \, \delta m^{(1)} \, [\widetilde{A}_{i1}^{\prime(1)\rm f} - A_{21}^{\prime(1)\rm f} \, \widetilde{A}_{i1}^{(0)}] \\ & \quad + (Z_{\alpha}^{(1)} + Z_{ext}^{(1)}) \, [\widetilde{A}_{i1}^{(1)\rm f} - A_{21}^{(1)\rm f} \, \widetilde{A}_{i1}^{(0)}] \\ & \quad - [\widetilde{M}_{11}^{(2)} - M_{11}^{(2)}] \, \widetilde{A}_{i1}^{(0)} \\ & \quad - (C_{FF}^{(1)} - \xi_{45}^{(1)}) \, [\widetilde{Y}_{11}^{(1)} - Y_{11}^{(1)}] \, \widetilde{A}_{i1}^{(0)} - [\widetilde{Y}_{11}^{(2)} - Y_{11}^{(2)}] \, \widetilde{A}_{i1}^{(0)} \, . \end{aligned}$$

Two-loop QCD diagrams

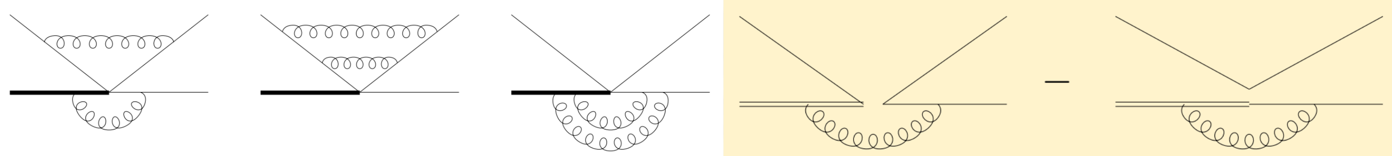
□ $\widetilde{A}_{i1}^{(2)\mathrm{nf}}$: **relevant two-loop non-factorizable Feynman diagrams in full QCD:**

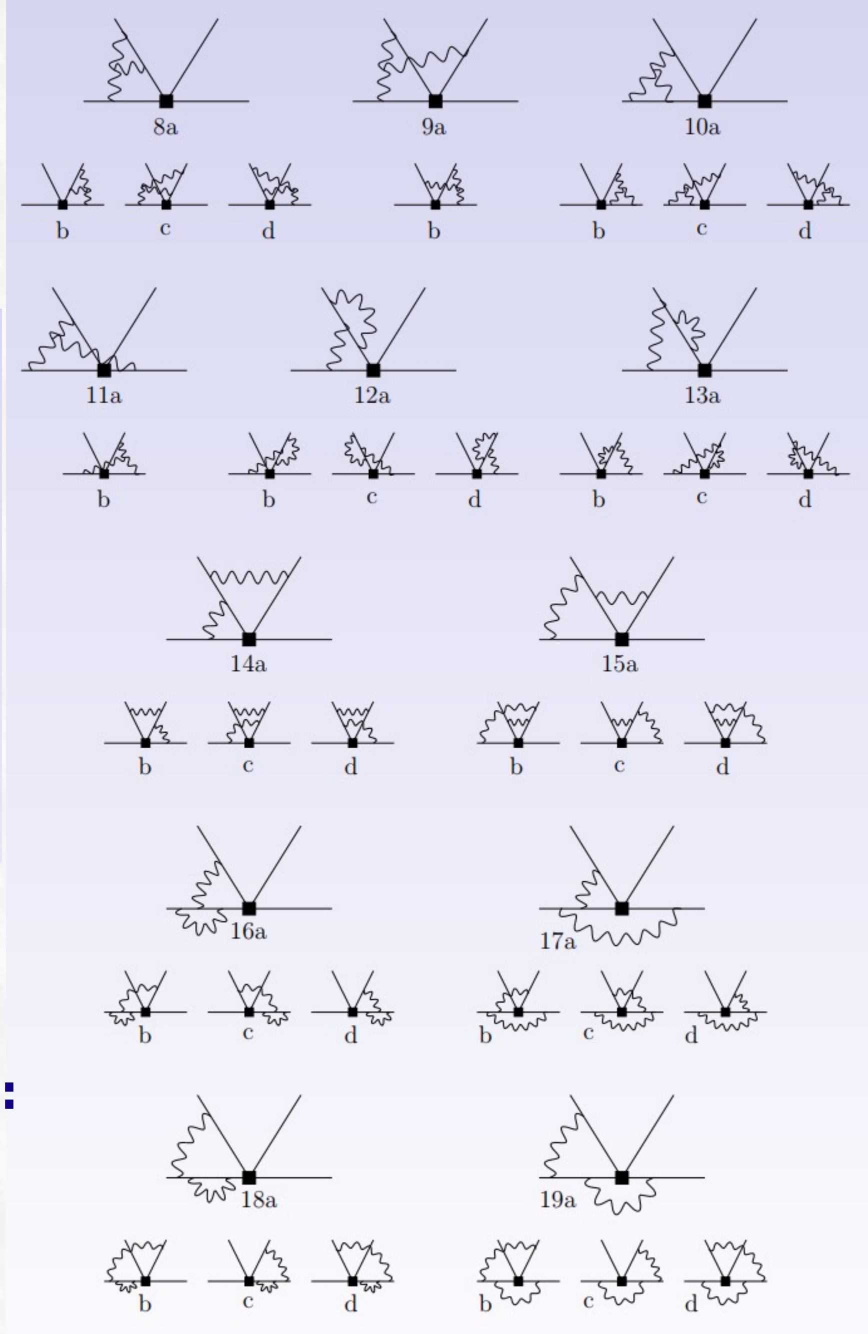
- ✓ totally ~ 70 diagrams
- ✓ needs the modern multi-loop Feynman integral techniques: IBP reduction, Mellin-Barnes representation, Differential equations, …





□ **Complicated counter-terms from QCD & SCET operators:**





Final results for $\alpha_{1,2}$

☐ Tree amplitudes $\alpha_{1,2}$, after convolution with LCDAs:

$$\alpha_i(M_1M_2) = \sum_j C_j V_{ij}^{(0)} + \sum_{l\geqslant 1}\left(\frac{\alpha_s}{4\pi}\right)^l \left[\frac{C_F}{2N_c}\sum_j C_j V_{ij}^{(l)} + P_i^{(l)}\right] + \cdots$$

no endpoint divergence

collinear factorization

established @ NNLO

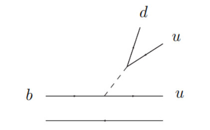
$$V_{1j}^{(0)} = \int_0^1 du\, T_j^{(0)} \phi_M(u), \qquad \frac{C_F}{2N_c} V_{1j}^{(l)} = \int_0^1 du\, T_j^{(l)}(u)\phi_M(u),$$

$$V_{2j}^{(0)} = \int_0^1 du\, \widetilde{T}_j^{(0)} \phi_M(u), \qquad \frac{C_F}{2N_c} V_{2j}^{(l)} = \int_0^1 du\, \widetilde{T}_j^{(l)}(u)\phi_M(u).$$

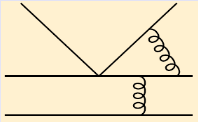
☐ Numerical results

$$\langle M_1M_2|\mathcal{O}_i|\bar{B}\rangle \simeq F^{B\to M_1}\, T_i^I \otimes \phi_{M_2} + T_i^{II} \otimes \phi_B \otimes \phi_{M_1} \otimes \phi_{M_2}$$

$$\begin{aligned}
\alpha_1(\pi\pi) \quad &= \quad 1.009 + [0.023 + 0.010\,i]_{\text{NLO}} + [0.026 + 0.028\,i]_{\text{NNLO}} \\
&\quad - \left[\frac{r_{\text{sp}}}{0.445}\right]\Big\{[0.014]_{\text{LOsp}} + [0.034 + 0.027i\,]_{\text{NLOsp}} + [0.008]_{\text{tw3}}\Big\} \\
&\quad 1.000^{+0.029}_{-0.069} + (0.011^{+0.023}_{-0.050})i
\end{aligned}$$

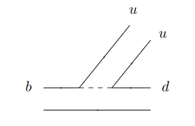


colour-allowed tree α_1

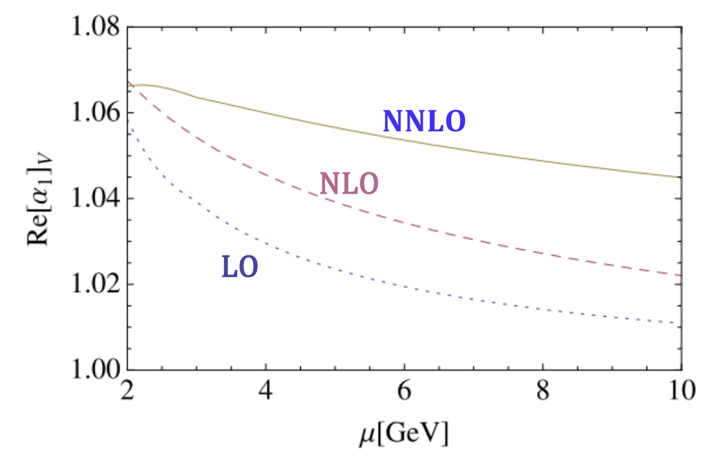


Beneke, Jager '05
Kivel '06, Pilipp '07

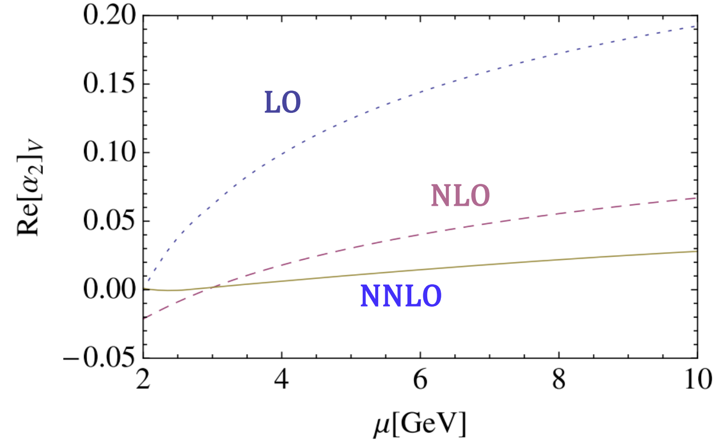
$$\begin{aligned}
\alpha_2(\pi\pi) \quad &= \quad 0.220 - [0.179 + 0.077\,i]_{\text{NLO}} - [0.031 + 0.050\,i]_{\text{NNLO}} \\
&\quad + \left[\frac{r_{\text{sp}}}{0.445}\right]\Big\{[0.114]_{\text{LOsp}} + [0.049 + 0.051i\,]_{\text{NLOsp}} + [0.067]_{\text{tw3}}\Big\} \\
&= \quad 0.240^{+0.217}_{-0.125} + (-0.077^{+0.115}_{-0.078})i
\end{aligned}$$



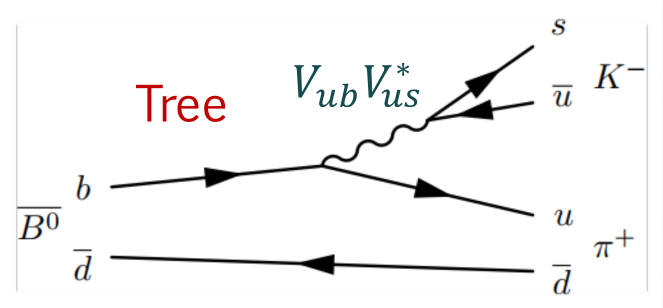
colour-suppressed tree α_2

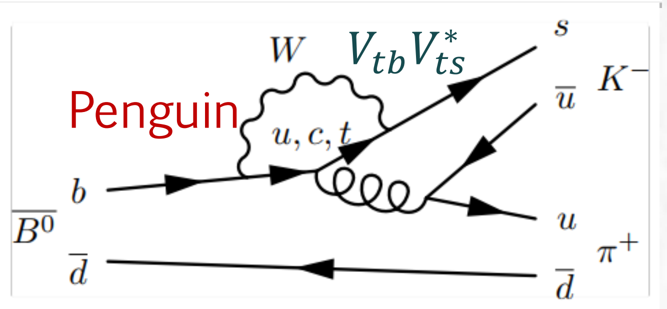


☐ Scale-dependence much reduced!



Penguin-dominated B decays





□ **$B\to\pi K$ decay amplitudes:** mediated by $b\to sq\bar q$ transitions

$$\sqrt{2}\,\mathcal{A}_{B^-\to\pi^0K^-} = A_{\pi\bar K}\big[\delta_{pu}\,\alpha_1+\hat\alpha_4^p\big]+A_{\bar K\pi}\big[\underline{\delta_{pu}\alpha_2+\delta_{pc}\tfrac{3}{2}\alpha^c_{3,\rm EW}}\big],$$

$$\mathcal{A}_{\bar B^0\to\pi^+K^-} = A_{\pi\bar K}\big[\delta_{pu}\,\alpha_1+\hat\alpha_4^p\big],$$

$\lambda_u = V_{ub}V_{us}^* \sim \mathcal{O}(\lambda^4) \ll \lambda_c = V_{cb}V_{cs}^* \sim \mathcal{O}(\lambda^2)$ ➡ Penguin-dominated!

□ **In QCDF, strong phases generated firstly at NLO in α_s**

$$A_{\rm CP} = [c\times\alpha_s]_{\rm NLO} + \mathcal{O}(\alpha_s^2,\Lambda/m_b)$$

➡ ***NNLO is only NLO for A_{CP}, large effects still possible?***

⬇

□ **To predict accurately direct CPV, we must calculate both tree & penguin up to NNLO!**

□ **Driven by the current exp. data on $B\to\pi K$:**

$$\Delta A_{CP}(\pi K) = A_{CP}(B^-\to\pi^0K^-) - A_{CP}(\bar B^0\to\pi^+K^-)$$

$$= (11.0\pm1.2)\%$$ **differs from 0 by ~9σ**

ΔA_{CP} puzzle

➡ ***How about the situation @ NNLO?***

Penguin topologies with various insertions

☐ Effective Hamiltonian including penguin operators: [BBL ’96; CMM ’98]

$$\mathcal{H}_{\rm eff} = \frac{4G_F}{\sqrt{2}} \sum_{p=u,c} V_{pD}^* V_{pb} \left(C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3}^{10} C_i Q_i + C_{7\gamma} Q_{7\gamma} + C_{8g} Q_{8g} \right) + \text{h.c.}$$

CMM operator basis

$$Q_1^p = (\bar{p}_L \gamma^\mu T^A b_L)\,(\bar{D}_L \gamma_\mu T^A p_L),$$
$$Q_2^p = (\bar{p}_L \gamma^\mu b_L)\,(\bar{D}_L \gamma_\mu p_L),$$

current-current operators

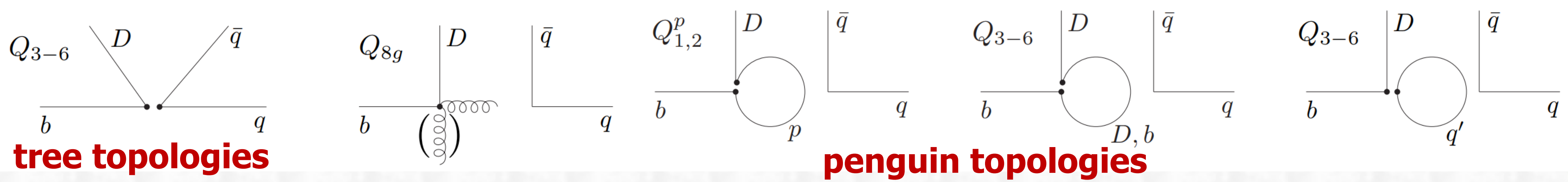
$$Q_3 = (\bar{D}_L \gamma^\mu b_L) \textstyle\sum_q (\bar{q} \gamma_\mu q),$$
$$Q_4 = (\bar{D}_L \gamma^\mu T^A b_L) \textstyle\sum_q (\bar{q} \gamma_\mu T^A q),$$
$$Q_5 = (\bar{D}_L \gamma^\mu \gamma^\nu \gamma^\rho b_L) \textstyle\sum_q (\bar{q} \gamma_\mu \gamma_\nu \gamma_\rho q),$$
$$Q_6 = (\bar{D}_L \gamma^\mu \gamma^\nu \gamma^\rho T^A b_L) \textstyle\sum_q (\bar{q} \gamma_\mu \gamma_\nu \gamma_\rho T^A q)\,.$$

QCD penguin operators

$$Q_{8g} = \frac{-g_s}{32\pi^2}\, \overline{m}_b\, \bar{D} \sigma_{\mu\nu} (1+\gamma_5) G^{\mu\nu} b,$$

chromo-magnetic dipole operators

☐ Various operator insertions:



tree topologies

penguin topologies

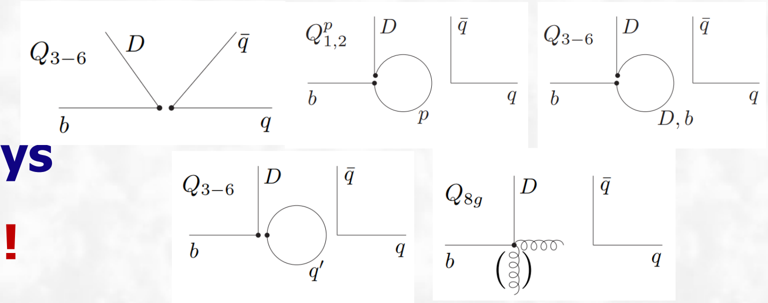
(i) Dirac structure of Q_i, (ii) color structure of Q_i, (iii) types of contraction, (iv) quark masses in fermion loop

Hard kernel T^I at NNLO

☐ **QCD → SCETI matching calculation:**

$$\boxed{\langle Q_i\rangle = \sum_a \widetilde{H}_{ia}\,\langle\widetilde{O}_a\rangle}$$

☐ **Reminder: always wrong insertion!**



☐ **Complete SCET operator basis:**

$$Q_3 = (\bar D_L\gamma^\mu b_L)\sum_q(\bar q\gamma_\mu q),$$
$$Q_4 = (\bar D_L\gamma^\mu T^A b_L)\sum_q(\bar q\gamma_\mu T^A q),$$
$$Q_5 = (\bar D_L\gamma^\mu\gamma^\nu\gamma^\rho b_L)\sum_q(\bar q\gamma_\mu\gamma_\nu\gamma_\rho q),$$
$$Q_6 = (\bar D_L\gamma^\mu\gamma^\nu\gamma^\rho T^A b_L)\sum_q(\bar q\gamma_\mu\gamma_\nu\gamma_\rho T^A q)\,.$$

+ evanescent operators

$$O_1 = \sum_{q=u,d,s}\left[\bar\chi_D\frac{\not{n}_-}{2}(1-\gamma_5)\chi_q\right]\,\left[\bar\xi_q\,\not{n}_+(1-\gamma_5)h_v\right]\,,$$

the only physical operator and factorizes into FF*LCDA

$$\tilde O_n = \sum_{q=u,d,s}\left[\bar\xi_q\,\gamma_\perp^\alpha\gamma_\perp^{\mu_1}\gamma_\perp^{\mu_2}\ldots\gamma_\perp^{\mu_{2n-2}}\chi_q\right]\,\left[\bar\chi_q(1+\gamma_5)\gamma_{\perp\alpha}\gamma_{\perp\mu_{2n-2}}\gamma_{\perp\mu_{2n-3}}\ldots\gamma_{\perp\mu_1}h_v\right],$$

$\tilde O_1 - O_1/2$ is another evanescent operator

n now up to 4, with 7 gamma matrices

☐ **On-shell matrix elements at NNLO: on the full QCD side**

$$\langle Q_i\rangle = \Big\{\widetilde{A}^{(0)}_{ia} + \frac{\alpha_s}{4\pi}\Big[\widetilde{A}^{(1)}_{ia} + Z^{(1)}_{\rm ext}\,\widetilde{A}^{(0)}_{ia} + Z^{(1)}_{ij}\,\widetilde{A}^{(0)}_{ja}\Big]$$
$$+\left(\frac{\alpha_s}{4\pi}\right)^2\Big[\widetilde{A}^{(2)}_{ia} + Z^{(1)}_{ij}\,\widetilde{A}^{(1)}_{ja} + Z^{(2)}_{ij}\,\widetilde{A}^{(0)}_{ja} + Z^{(1)}_{\rm ext}\,\widetilde{A}^{(1)}_{ia} + Z^{(2)}_{\rm ext}\,\widetilde{A}^{(0)}_{ia}$$
$$+Z^{(1)}_{\rm ext}\,Z^{(1)}_{ij}\,\widetilde{A}^{(0)}_{ja} + Z^{(1)}_{\alpha}\,\widetilde{A}^{(1)}_{ia} + (-i)\,\delta m^{(1)}\,\widetilde{A}'^{(1)}_{ia}\Big] + \mathcal{O}(\alpha_s^3)\Big\}\,\langle\widetilde{O}_a\rangle^{(0)}$$

☐ **On-shell matrix elements at NNLO: SCET side**

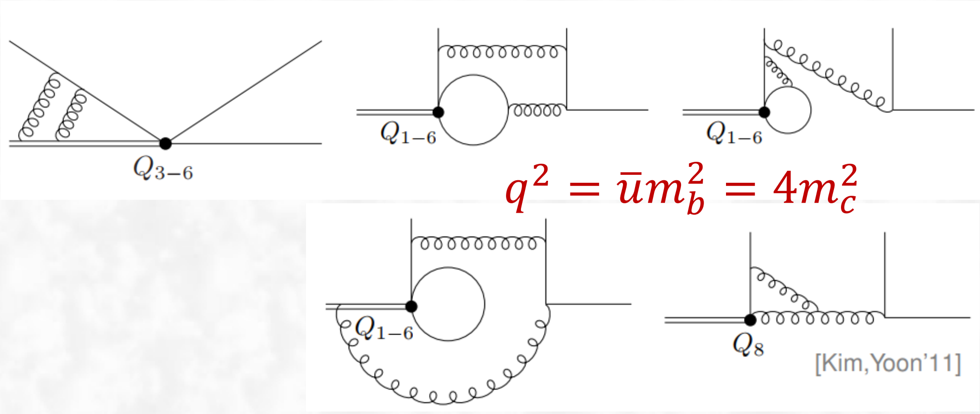
$$\langle O_a\rangle = \Big\{\delta_{ab} + \frac{\hat\alpha_s}{4\pi}\Big[M^{(1)}_{ab} + Y^{(1)}_{ext}\,\delta_{ab} + Y^{(1)}_{ab}\Big] + \left(\frac{\hat\alpha_s}{4\pi}\right)^2\Big[M^{(2)}_{ab} + Y^{(1)}_{ac}M^{(1)}_{cb}$$
$$+Y^{(2)}_{ab} + Y^{(1)}_{ext}\,M^{(1)}_{ab} + Y^{(2)}_{ext}\,\delta_{ab} + Y^{(1)}_{ext}\,Y^{(1)}_{ab} + \hat Z^{(1)}_{\alpha}M^{(1)}_{ab}\Big] + \mathcal{O}(\alpha_s^3)\Big\}\,\langle O_b\rangle^{(0)}$$

T_i^I up to NNLO

☐ Master formulae for T_i^I:

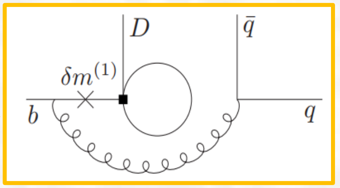
$$
\begin{aligned}
\frac{1}{2}\,\widetilde{T}_i^{(2)} = {} & \boxed{\widetilde{A}_{i1}^{(2)\mathrm{nf}}} + Z_{ij}^{(1)}\,\widetilde{A}_{j1}^{(1)} + Z_{ij}^{(2)}\,\widetilde{A}_{j1}^{(0)} + Z_{\alpha}^{(1)}\,\widetilde{A}_{i1}^{(1)\mathrm{nf}} \\
& + \boxed{(-i)\,\delta m^{(1)}\,\widetilde{A}_{i1}^{\prime(1)\mathrm{nf}}} - Z_{\mathrm{ext}}^{(1)}\,[\widetilde{A}_{i1}^{(1)\mathrm{nf}} + Z_{ij}^{(1)}\,\widetilde{A}_{j1}^{(0)}] \\
& - \frac{1}{2}\,\widetilde{T}_i^{(1)}\,[C_{FF}^{(1)} + \widetilde{Y}_{11}^{(1)}] - \sum_{b>1}\widetilde{H}_{ib}^{(1)}\,\widetilde{Y}_{b1}^{(1)} \\
& + [\widetilde{A}_{i1}^{(2)\mathrm{f}} - A_{31}^{(2)\mathrm{f}}\,\widetilde{A}_{i1}^{(0)}] + (-i)\,\delta m^{(1)}\,[\widetilde{A}_{i1}^{\prime(1)\mathrm{f}} - A_{31}^{\prime(1)\mathrm{f}}\,\widetilde{A}_{i1}^{(0)} \\
& + (Z_{\alpha}^{(1)} + Z_{\mathrm{ext}}^{(1)})\,[\widetilde{A}_{i1}^{(1)\mathrm{f}} - A_{31}^{(1)\mathrm{f}}\,\widetilde{A}_{i1}^{(0)}] \\
& - [\widetilde{M}_{11}^{(2)} - M_{11}^{(2)}]\,\widetilde{A}_{i1}^{(0)} \\
& - (C_{FF}^{(1)} - \xi_{45}^{(1)})\,[\widetilde{Y}_{11}^{(1)} - Y_{11}^{(1)}]\,\widetilde{A}_{i1}^{(0)} - [\widetilde{Y}_{11}^{(2)} - Y_{11}^{(2)}]\,\widetilde{A}_{i1}^{(0)} \\
& - \boxed{\sum_{b>1}\widetilde{A}_{ib}^{(0)}\,\widetilde{M}_{b1}^{(2)} - \sum_{b>1}\widetilde{A}_{ib}^{(0)}\,\widetilde{Y}_{b1}^{(2)}\,.}
\end{aligned}
$$

~ 100 two-loop Feynman diagrams

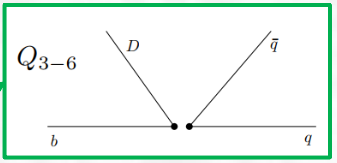


$q^2 = \bar{u}m_b^2 = 4m_c^2$

[Kim,Yoon'11]



non-vanishing fermion-tadpole contraction of QCD penguin operators



tree-level matching of Q_i involves already evanescent SCET operators

☐ Complication during calculations:

(i) fermion loop with either $m = 0, m = m_c$ or $m = m_b$

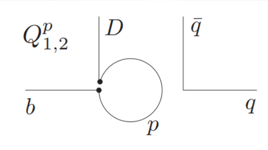
➡ genuine 2-loop two-scale problem: $\bar{u},\ z_c = m_c^2/m_b^2$

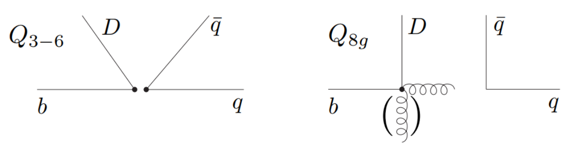
(ii) nontrivial threshold at $\bar{u} = 4z_c$ introduces strong phase

Final results for a_4^p

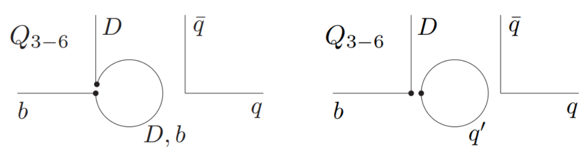
☐ Final numerical results:

$$\langle M_1 M_2 | \mathcal{O}_i | \bar{B} \rangle \simeq F^{B\to M_1}\, T_i^I \otimes \phi_{M_2} + T_i^{II} \otimes \phi_B \otimes \phi_{M_1} \otimes \phi_{M_2}$$





$$\begin{aligned} a_4^u(\pi\bar{K})/10^{-2} &= -2.87 - [0.09 + 0.09i]_{\mathrm{V}_1} + [0.49 - 1.32i]_{\mathrm{P}_1} - [0.32 + 0.71i]_{\mathrm{P}_2,\,\mathrm{Q}_{1,2}} + [0.33 + 0.38i]_{\mathrm{P}_2,\,\mathrm{Q}_{3-6,8}} \\ &\quad + \left[\frac{r_{\mathrm{sp}}}{0.434}\right] \Big\{ [0.13]_{\mathrm{LO}} + [0.14 + 0.12i]_{\mathrm{HV}} - [0.01 - 0.05i]_{\mathrm{HP}} + [0.07]_{\mathrm{tw3}} \Big\} \\ &= (-2.12^{+0.48}_{-0.29}) + (-1.56^{+0.29}_{-0.15})i\,, \end{aligned}$$

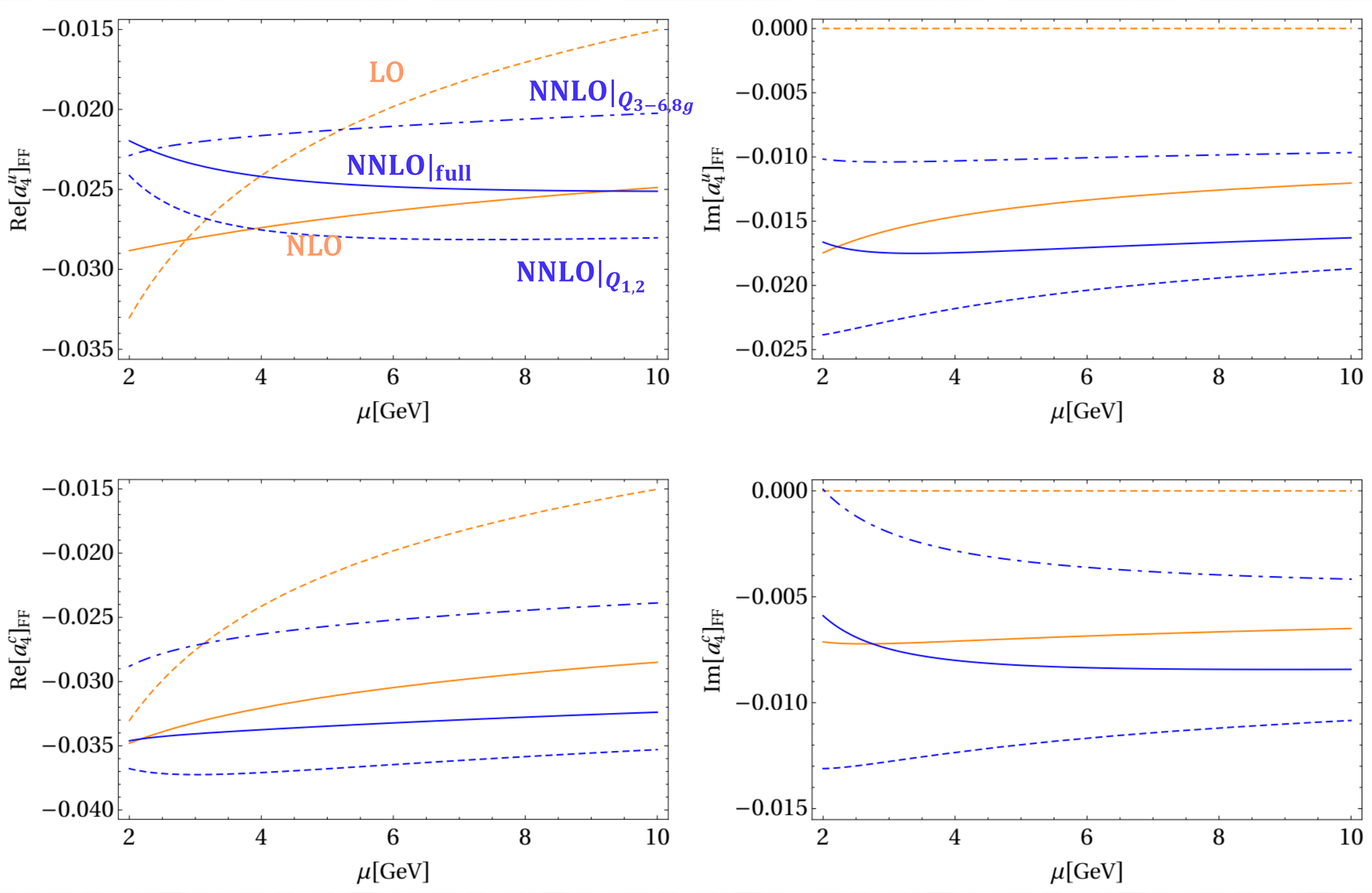


$$\begin{aligned} a_4^c(\pi\bar{K})/10^{-2} &= -2.87 - [0.09 + 0.09i]_{\mathrm{V}_1} + [0.05 - 0.62i]_{\mathrm{P}_1} - [0.77 + 0.50i]_{\mathrm{P}_2,\,\mathrm{Q}_{1,2}} + [0.33 + 0.38i]_{\mathrm{P}_2,\,\mathrm{Q}_{3-6,8}} \\ &\quad + \left[\frac{r_{\mathrm{sp}}}{0.434}\right] \Big\{ [0.13]_{\mathrm{LO}} + [0.14 + 0.12i]_{\mathrm{HV}} + [0.01 + 0.03i]_{\mathrm{HP}} + [0.07]_{\mathrm{tw3}} \Big\} \\ &= (-3.00^{+0.45}_{-0.32}) + (-0.67^{+0.50}_{-0.39})i\,. \end{aligned}$$

- **individual NNLO contributions from $Q^p_{1,2}$ and $Q_{3-6,8g}$ significant**
- **strong cancellation between NNLO corrections from $Q^p_{1,2}$ and $Q_{3-6,8g}$**

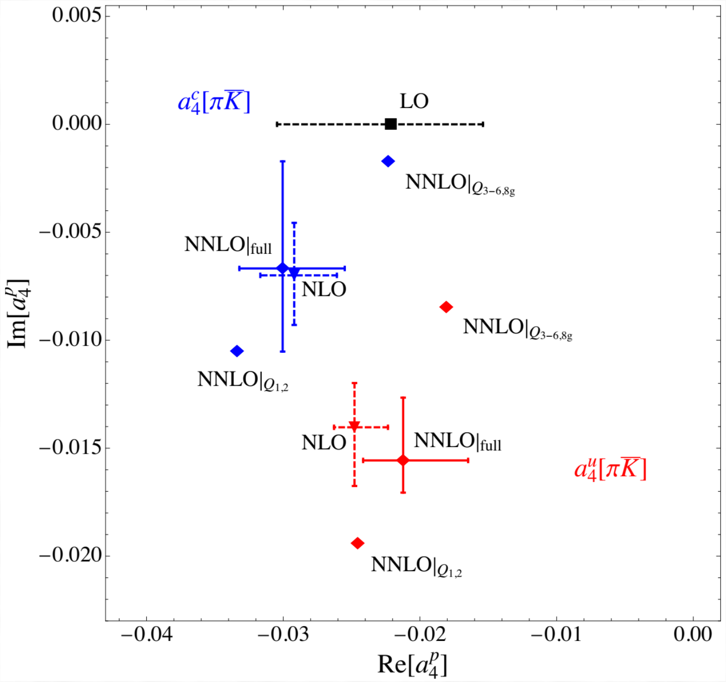
Scale dependence of a_4^p

□ Scale dependence of a_4^p: only form-factor term



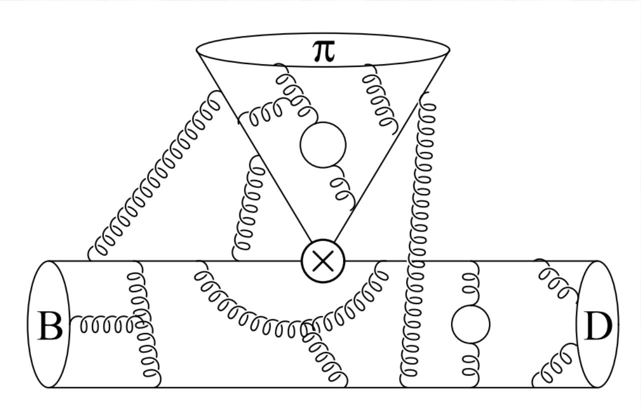
- ✓ scale dependence negligible, especially for μ > 4 GeV.

□ Results at different orders:



- ✓ total NNLO effects small
- ✓ uncertainty at NNLO larger than at NLO, due to non-trivial charm mass dependence

$B_q^0 \to D_q^{(*)-} L^+$ class-I decays



- **At quark-level, these decays mediated by $b \to c\bar{u}d(s)$**

all four flavors different from each other,

no penguin operators & no penguin topologies!

$$\mathcal{Q}_2 = \bar{d}\gamma_\mu(1-\gamma_5)u \;\; \bar{c}\gamma^\mu(1-\gamma_5)b$$
$$\mathcal{Q}_1 = \bar{d}\gamma_\mu(1-\gamma_5)T^A u \;\; \bar{c}\gamma^\mu(1-\gamma_5)T^A b$$

- **For class-I decays:** QCDF formula much simpler;

only the form-factor term at leading power

[Beneke, Buchalla, Neubert, Sachrajda '99-'03; Bauer, Pirjol, Stewart '01]

$$\langle D_q^{(*)+} L^- |\, \mathcal{Q}_i \,| \bar{B}_q^0 \rangle = \sum_j F_j^{\bar{B}_q \to D_q^{(*)}}(M_L^2) \times \int_0^1 du\, T_{ij}(u)\phi_L(u) + \mathcal{O}\left(\frac{\Lambda_{\rm QCD}}{m_b}\right)$$

i) only color-allowed tree topology a_1

ii) spectator & annihilation power-suppressed

iii) annihilation absent in $B^0_{d(s)} \to D^-_{d(s)} K(\pi)^+$ etc.

iv) they are theoretically simpler and cleaner

these decays used to test factorization theorems

- **Hard kernel T:** both NLO and NNLO results known;

[Beneke, Buchalla, Neubert, Sachrajda '01; Huber, Kränkl, Li '16]

$$T = T^{(0)} + \alpha_s T^{(1)} + \alpha_s^2 T^{(2)} + O(\alpha_s^3)$$

Calculation of T^I

☐ Matching QCD onto SCET$_\mathrm{I}$: [Huber, Kränkl, Li '16]

m_c also heavy, must keep m_c/m_b fixed as $m_b \to \infty$,

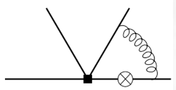
thus needing two sets of SCET operator basis.

$$\boxed{\langle \mathcal{Q}_i \rangle = \hat{T}_i \langle \mathcal{Q}^{\mathrm{QCD}} \rangle + \hat{T}'_i \langle \mathcal{Q}'^{\mathrm{QCD}} \rangle + \sum_{a>1} \left[H_{ia} \langle \mathcal{O}_a \rangle + H'_{ia} \langle \mathcal{O}'_a \rangle \right]}$$

☐ Renormalized on-shell QCD amplitudes:

$$\begin{aligned}\langle \mathcal{Q}_i \rangle = \Big\{ & A_{ia}^{(0)} + \frac{\alpha_s}{4\pi} \left[A_{ia}^{(1)} + Z_{ext}^{(1)} A_{ia}^{(0)} + Z_{ij}^{(1)} A_{ja}^{(0)} \right] \\ & + \left(\frac{\alpha_s}{4\pi} \right)^2 \left[A_{ia}^{(2)} + Z_{ij}^{(1)} A_{ja}^{(1)} + Z_{ij}^{(2)} A_{ja}^{(0)} + Z_{ext}^{(1)} A_{ia}^{(1)} + Z_{ext}^{(2)} A_{ia}^{(0)} + Z_{ext}^{(1)} Z_{ij}^{(1)} A_{ja}^{(0)} \right. \\ & \left. + (-i)\delta m_b^{(1)} A_{ia}^{*(1)} + \boxed{(-i)\delta m_c^{(1)} A_{ia}^{**(1)}} + Z_\alpha^{(1)} A_{ia}^{(1)} \right] + \mathcal{O}(\alpha_s^3) \Big\} \langle \mathcal{O}_a \rangle^{(0)} \\ & + (A \leftrightarrow A') \langle \mathcal{O}'_a \rangle^{(0)} .\end{aligned}$$

on QCD side



☐ Renormalized on-shell SCET amplitudes:

$$\begin{aligned}\langle \mathcal{O}_a \rangle = \Big\{ & \delta_{ab} + \frac{\hat{\alpha}_s}{4\pi} \left[M_{ab}^{(1)} + Y_{ext}^{(1)} \delta_{ab} + Y_{ab}^{(1)} \right] \\ & + \left(\frac{\hat{\alpha}_s}{4\pi} \right)^2 \left[M_{ab}^{(2)} + Y_{ext}^{(1)} M_{ab}^{(1)} + Y_{ac}^{(1)} M_{cb}^{(1)} + \hat{Z}_\alpha^{(1)} M_{ab}^{(1)} + Y_{ext}^{(2)} \delta_{ab} \right. \\ & \left. + Y_{ext}^{(1)} Y_{ab}^{(1)} + Y_{ab}^{(2)} \right] + \mathcal{O}(\hat{\alpha}_s^3) \Big\} \langle \mathcal{O}_b \rangle^{(0)} ,\end{aligned}$$

on SCET side

physical operators and factorizes into FF*LCDA.

$$\begin{aligned}\mathcal{O}_1 &= \bar{\chi} \frac{\not{n}_-}{2} (1-\gamma_5) \chi \;\; \bar{h}_{v'} \not{n}_+ (1-\gamma_5) h_v , \\ \mathcal{O}_2 &= \bar{\chi} \frac{\not{n}_-}{2} (1-\gamma_5) \gamma_\perp^\alpha \gamma_\perp^\beta \chi \;\; \bar{h}_{v'} \not{n}_+ (1-\gamma_5) \gamma_{\perp,\beta} \gamma_{\perp,\alpha} h_v , \\ \mathcal{O}_3 &= \bar{\chi} \frac{\not{n}_-}{2} (1-\gamma_5) \gamma_\perp^\alpha \gamma_\perp^\beta \gamma_\perp^\gamma \gamma_\perp^\delta \chi \;\; \bar{h}_{v'} \not{n}_+ (1-\gamma_5) \gamma_{\perp,\delta} \gamma_{\perp,\gamma} \gamma_{\perp,\beta} \gamma_{\perp,\alpha} h_v\end{aligned}$$

$$\begin{aligned}\mathcal{O}'_1 &= \bar{\chi} \frac{\not{n}_-}{2} (1-\gamma_5) \chi \;\; \bar{h}_{v'} \not{n}_+ (1+\gamma_5) h_v , \\ \mathcal{O}'_2 &= \bar{\chi} \frac{\not{n}_-}{2} (1-\gamma_5) \gamma_\perp^\alpha \gamma_\perp^\beta \chi \;\; \bar{h}_{v'} \not{n}_+ (1+\gamma_5) \gamma_{\perp,\alpha} \gamma_{\perp,\beta} h_v , \\ \mathcal{O}'_3 &= \bar{\chi} \frac{\not{n}_-}{2} (1-\gamma_5) \gamma_\perp^\alpha \gamma_\perp^\beta \gamma_\perp^\gamma \gamma_\perp^\delta \chi \;\; \bar{h}_{v'} \not{n}_+ (1+\gamma_5) \gamma_{\perp,\alpha} \gamma_{\perp,\beta} \gamma_{\perp,\gamma} \gamma_{\perp,\delta} h_v\end{aligned}$$

evanescent operators and must be renormalized to zero.

☐ Master formulas for hard kernels:

$$T = T^{(0)} + \alpha_s T^{(1)} + \alpha_s^2 T^{(2)} + O(\alpha_s^3)$$

$$\boxed{\begin{aligned}\hat{T}_i^{(0)} &= A_{i1}^{(0)} \\ \hat{T}_i^{(1)} &= A_{i1}^{(1)nf} + Z_{ij}^{(1)} A_{j1}^{(0)} \\ \hat{T}_i^{(2)} &= A_{i1}^{(2)nf} + Z_{ij}^{(1)} A_{j1}^{(1)} + Z_{ij}^{(2)} A_{j1}^{(0)} + Z_\alpha^{(1)} A_{i1}^{(1)nf} - \hat{T}_i^{(1)} \left[C_{FF}^{\mathrm{D}(1)} + Y_{11}^{(1)} - Z_{\mathrm{ext}}^{(1)} \right] \\ & \quad - C_{FF}^{\mathrm{ND}(1)} \hat{T}_i'^{(1)} + (-i)\delta m_b^{(1)} A_{i1}^{*(1)nf} + (-i)\delta m_c^{(1)} A_{i1}^{**(1)nf} - \sum_{b\neq 1} H_{ib}^{(1)} Y_{b1}^{(1)} .\end{aligned}}$$

Decay amplitudes for $B_q^0 \to D_q^- L^+$

- ☐ **Color-allowed tree amplitude a_1: collinear factorization established @ NNLO!**

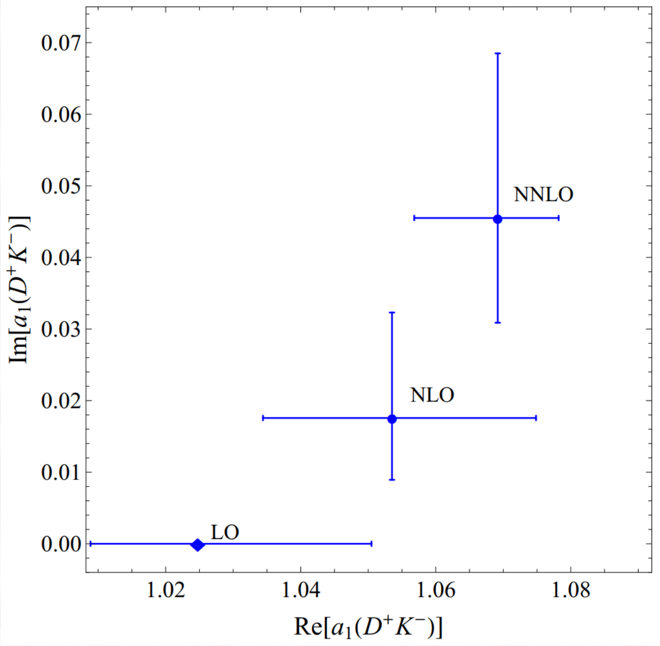
$$\boxed{\begin{aligned} a_1(D^+L^-) &= \sum_{i=1}^{2} C_i(\mu) \int_0^1 du \, \left[\hat{T}_i(u,\mu) + \hat{T}_i'(u,\mu)\right] \, \Phi_L(u,\mu)\,, \\ a_1(D^{*+}L^-) &= \sum_{i=1}^{2} C_i(\mu) \int_0^1 du \, \left[\hat{T}_i(u,\mu) - \hat{T}_i'(u,\mu)\right] \, \Phi_L(u,\mu)\,, \end{aligned}}$$

free from the

endpoint divergence

⬇

collinear factorization established



- ☐ **Numerical result:**

$$\boxed{\begin{aligned} a_1(D^+K^-) &= 1.025 + [0.029 + 0.018i]_{\rm NLO} + [0.016 + 0.028i]_{\rm NNLO} \\ &= (1.069^{+0.009}_{-0.012}) + (0.046^{+0.023}_{-0.015})i\,, \end{aligned}}$$

- ✓ both NLO and NNLO add always constructively to LO result!
- ✓ NNLO corrections to real part quite small (2%), but rather large to imaginary part (60%).

- ☐ **For different decay modes:** *quasi-universal*, with small process dependence from *different LCDA of light mesons*.

$$\boxed{\begin{aligned} a_1(D^+K^-) &= (1.069^{+0.009}_{-0.012}) + (0.046^{+0.023}_{-0.015})i\,, \\ a_1(D^+\pi^-) &= (1.072^{+0.011}_{-0.013}) + (0.043^{+0.022}_{-0.014})i\,, \\ a_1(D^{*+}K^-) &= (1.068^{+0.010}_{-0.012}) + (0.034^{+0.017}_{-0.011})i\,, \\ a_1(D^{*+}\pi^-) &= (1.071^{+0.012}_{-0.013}) + (0.032^{+0.016}_{-0.010})i\,. \end{aligned}}$$

Possible higher-order power corrections motivated by current data

Non-leptonic/semi-leptonic ratios

☐ Non-leptonic/semi-leptonic ratios : [Bjorken '89; Neubert, Stech '97; Beneke, Buchalla, Neubert, Sachrajda '01]

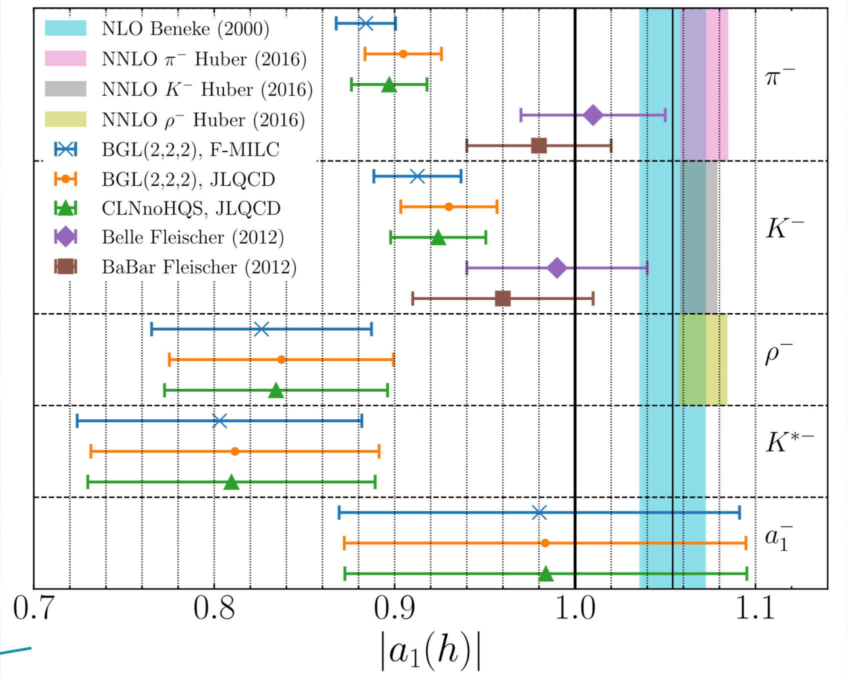
$$R^{(*)}_{(s)L} \equiv \frac{\Gamma(\bar{B}^0_{(s)} \to D^{(*)+}_{(s)} L^-)}{d\Gamma(\bar{B}^0_{(s)} \to D^{(*)+}_{(s)} \ell^- \bar{\nu}_\ell)/dq^2 \,|_{q^2=m_L^2}} = 6\pi^2 \,|V_{uq}|^2 \, f_L^2 \,|a_1(D^{(*)+}_{(s)} L^-)|^2 \, X^{(*)}_L$$

free from uncertainties from V_{cb} & $B_{d,s} \to D^{(*)}_{d,s}$ form factors

☐ Updated predictions vs data: [Huber, Kränkl, Li '16; Cai, Deng, Li, Yang '21]

$R^{(*)}_{(s)L}$	LO	NLO	NNLO	Exp.	Deviation (σ)
R_π	1.01	$1.07^{+0.04}_{-0.04}$	$1.10^{+0.03}_{-0.03}$	0.74 ± 0.06	5.4
R^*_π	1.00	$1.06^{+0.04}_{-0.04}$	$1.10^{+0.03}_{-0.03}$	0.80 ± 0.06	4.5
R_ρ	2.77	$2.94^{+0.19}_{-0.19}$	$3.02^{+0.17}_{-0.18}$	2.23 ± 0.37	1.9
R_K	0.78	$0.83^{+0.03}_{-0.03}$	$0.85^{+0.01}_{-0.02}$	0.62 ± 0.05	4.4
R^*_K	0.72	$0.76^{+0.03}_{-0.03}$	$0.79^{+0.01}_{-0.02}$	0.60 ± 0.14	1.3
R_{K^*}	1.41	$1.50^{+0.11}_{-0.11}$	$1.53^{+0.10}_{-0.10}$	1.38 ± 0.25	0.6
$R_{s\pi}$	1.01	$1.07^{+0.04}_{-0.04}$	$1.10^{+0.03}_{-0.03}$	0.72 ± 0.08	4.4
R_{sK}	0.78	$0.83^{+0.03}_{-0.03}$	$0.85^{+0.01}_{-0.02}$	0.46 ± 0.06	6.3

☐ Latest Belle data: 2207.00134



$|a_1(\bar{B} \to D^{*+}\pi^-)| = 0.884 \pm 0.004 \pm 0.003 \pm 0.016 \; [1.071^{+0.020}_{-0.016}]$;

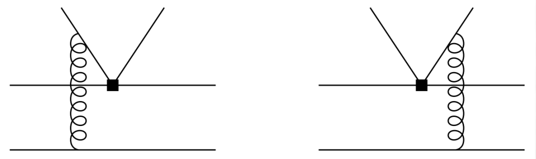
15% lower than SM

$|a_1(\bar{B} \to D^{*+}K^-)| = 0.913 \pm 0.019 \pm 0.008 \pm 0.013 \; [1.069^{+0.020}_{-0.016}]$;

Power corrections

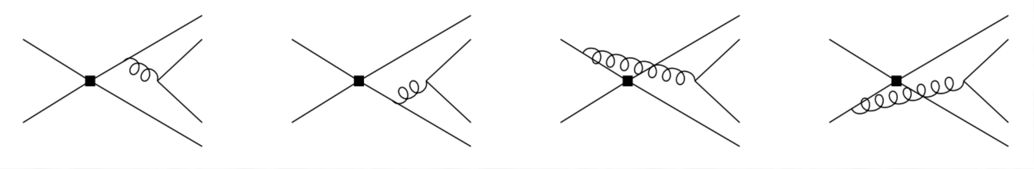
☐ **Sources of sub-leading power corrections:** [Beneke, Buchalla, Neubert, Sachrajda '01; Bordone, Gubernari, Huber, Jung, van Dyk '20]

- non-factorizable spectator-spectatorings

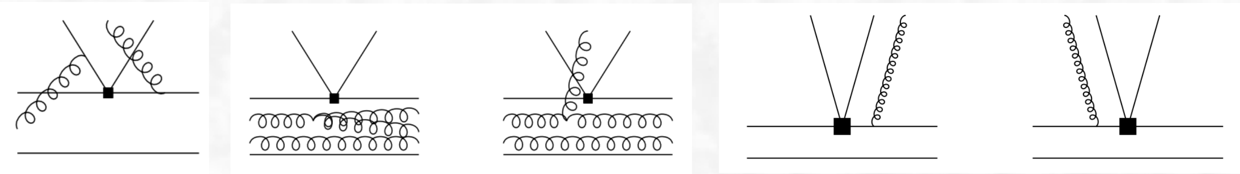


$$\frac{\Lambda_{\text{QCD}}}{m_b}$$

- annihilation topologies



- non-leading higher Fock-state contributions



$$\left(\frac{\Lambda_{\text{QCD}}}{m_b}\right)^2$$

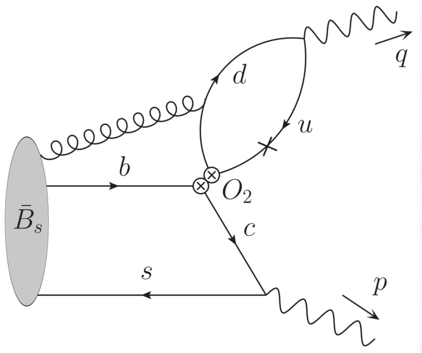
- non-factorizable soft-gluon contributions in LCSR: [Maria Laura Piscopo, Aleksey V. Rusov '23]

$$\langle D_q^{(*)+} L^- |\, \mathcal{Q}_i \,| \bar{B}_q^0 \rangle = \sum_j F_j^{\bar{B}_q \to D_q^{(*)}}(M_L^2) \times \int_0^1 du\, T_{ij}(u)\phi_L(u) + \boxed{\mathcal{O}\left(\frac{\Lambda_{\text{QCD}}}{m_b}\right)}$$

☐ **Scaling of the leading-power contribution:** [BBNS '01]

$$\mathcal{A}(\bar{B}_d \to D^+\pi^-) \sim G_F m_b^2\, F^{B\to D}(0)\, f_\pi \sim G_F m_b^2\, \Lambda_{\text{QCD}}$$

- all these corrections ESTIMATED to be power-suppressed
- difficult to explain why measured $|a_1(h)|$ smaller than SM?
- *must consider sub-leading power corrections carefully!*



$$\text{Br}(\bar{B}_s^0 \to D_s^+\pi^-) = \left(2.15^{+2.14}_{-1.35}\right)[2.98 \pm 0.14] \times 10^{-3}$$

$$\text{Br}(\bar{B}^0 \to D^+K^-) = \left(2.04^{+2.39}_{-1.20}\right)[2.05 \pm 0.08] \times 10^{-4}$$

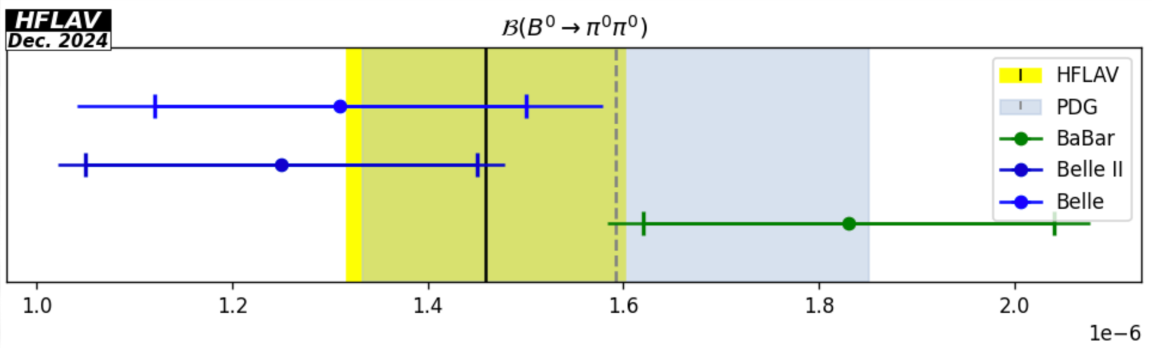
Charmless two-body hadronic B decays

- ☐ **Long-standing puzzle in $\mathrm{Br}(\overline{B}^0 \to \pi^0\pi^0)$:** [HFLAV '24]

$$\mathrm{Br}_{\mathrm{SM}}(B^0 \to \pi^0\pi^0) = (0.3-0.9)\times 10^{-6}$$

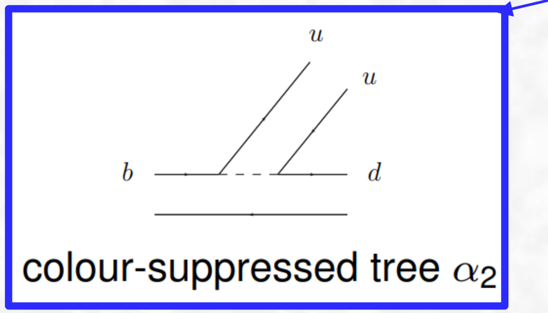
based on QCDF & PQCD

$$\mathrm{Br}_{\mathrm{exp}}(B^0 \to \pi^0\pi^0) = (1.46 \pm 0.14)\times 10^{-6}$$



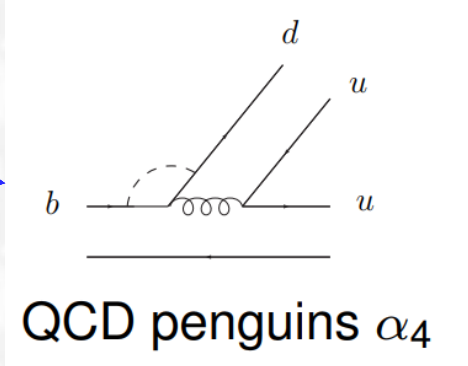
- ☐ **Decay amplitudes in QCDF: dominated by colour-suppressed tree amplitude α_2**

$$-\mathcal{A}_{\overline{B}^0\to\pi^0\pi^0} = A_{\pi\pi}\left[\delta_{pu}(\alpha_2-\beta_1) - \hat{\alpha}_4^p - 2\beta_4^p\right]$$



colour-suppressed tree α_2

necessary to consider sub-leading power corrections within the SM!



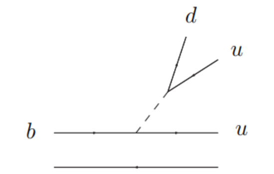
QCD penguins α_4

- ☐ **Find some other mechanism to enhance α_2, and hence explain the puzzle!**

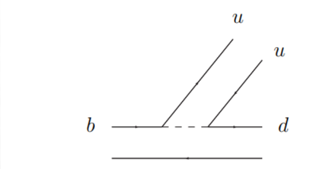
Charmless two-body hadronic B decays

- **Long-standing puzzles in $\Delta A_{CP}(\pi K) = A_{CP}(\pi^0 K^-) - A_{CP}(\pi^+ K^-)$:** [HFLAV '24]
- **Decay amplitudes in QCDF: dominated by $\hat{\alpha}_4$, but having strong interference with α_1**

$$\sqrt{2}\,\mathcal{A}_{B^-\to\pi^0K^-} = A_{\pi\bar{K}}\big[\delta_{pu}\,\alpha_1 + \hat{\alpha}_4^p\big] + A_{\bar{K}\pi}\big[\delta_{pu}\alpha_2 + \delta_{pc}\tfrac{3}{2}\alpha_{3,\mathrm{EW}}^c\big],$$
$$\mathcal{A}_{\bar{B}^0\to\pi^+K^-} = A_{\pi\bar{K}}\big[\delta_{pu}\,\alpha_1 + \hat{\alpha}_4^p\big],$$

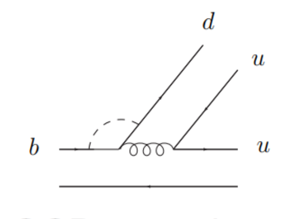


colour-allowed tree α_1

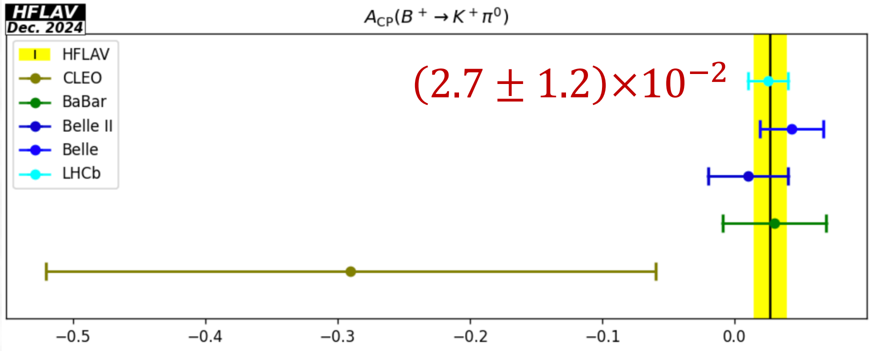


colour-suppressed tree α_2

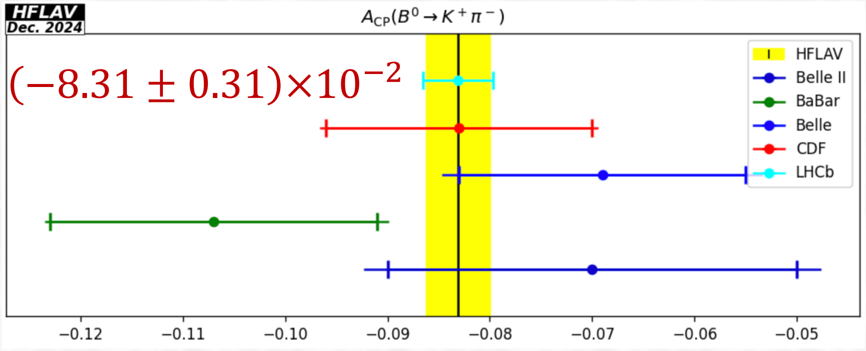
$$\Delta A_{CP}(\pi K)_{\mathrm{SM}} = A_{CP}(\pi^0 K^-) - A_{CP}(\pi^+ K^-)$$
$$= -2\sin\gamma\big(\mathrm{Im}(r_C) - \mathrm{Im}(r_T\, r_{EW})\big) + \cdots$$



QCD penguins α_4



$(2.7 \pm 1.2)\times 10^{-2}$



$(-8.31 \pm 0.31)\times 10^{-2}$

$\Delta A_{CP}(\pi K)_{\mathrm{exp}} = (11.0 \pm 1.2)\%$ **differs from 0 by ~9σ**

- **Find some mechanism to enhance α_2 or $\alpha_{3,\mathrm{EW}}^c$, and hence explain the observed puzzles!**

Power-suppressed color-octet contribution

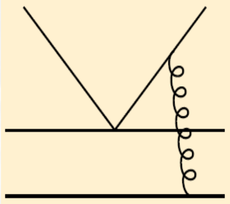
- ☐ **Sub-leading power corrections to a_2: spectator scattering or final-state re-scatterings**

- ☐ **Every four-quark operator in H_{eff} has a color-octet piece in QCD:**

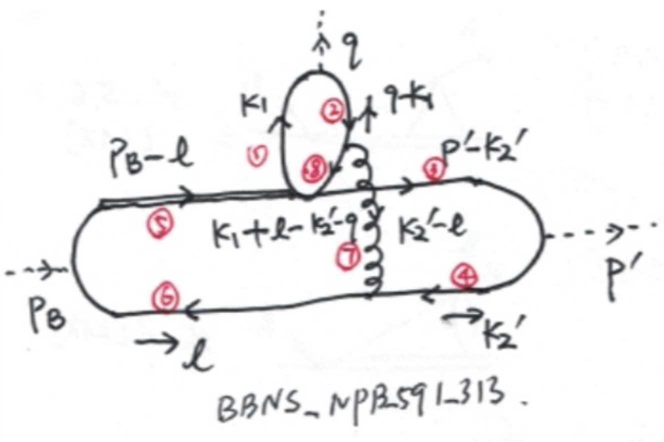
$$\boxed{t^a_{ik}t^a_{jl} = \frac{1}{2}\delta_{il}\delta_{jk} - \frac{1}{2N_c}\delta_{ik}\delta_{jl},}$$

$$Q_1 = (\bar{u}_i b_i)_{V-A} \otimes (\bar{s}_j u_j)_{V-A} = \frac{1}{N_c}(\bar{s}_i b_i)_{V-A} \otimes (\bar{u}_j u_j)_{V-A} + 2(\bar{s}T^A b)_{V-A} \otimes (\bar{u}T^A u)_{V-A}$$

$$Q_2 = (\bar{u}_i b_j)_{V-A} \otimes (\bar{s}_j u_i)_{V-A} = \frac{1}{N_c}(\bar{u}_i b_i)_{V-A} \otimes (\bar{s}_j u_j)_{V-A} + 2(\bar{u}T^A b)_{V-A} \otimes (\bar{s}T^A u)_{V-A}$$



- ☐ **Soft-gluon contributions with color-octet operator insertions:**



method of regions: 6 regions

- ➢ the gluon propagator can be in the hard-collinear region
  - ➡ hard-spectator scattering contribution
- ➢ can also be in the soft region; expected to be $\mathcal{O}(1/m_b)$
  - ➡ can be non-zero at sub-leading power, numerically relevant
- ➢ other four regions suppressed by more powers of $1/m_b$

Soft-exchange effects from emission topology

- ☐ **Real realization of the mechanism requires three-loop three-point correlators** [w.i.p.]

- ☐ **Matching from QCD to SCET$_{\mathbf{I}}$:**

$$Q_1 \;\to\; H_1(u)\otimes[\bar u_c h_v]_{\Gamma_1}\,[\bar s_{\bar c} u_{\bar c}]_{\Gamma_2}(u) + H_2(u)\otimes\frac{1}{N_c}\,[\bar s_c h_v]_{\tilde\Gamma_1}\,[\bar u_{\bar c} u_{\bar c}]_{\tilde\Gamma_2}(u)$$
$$+\,H_3(u)\otimes 2\,\underline{[\bar s_c T^A h_v]_{\tilde\Gamma_1}\,[\bar u_{\bar c} T^A u_{\bar c}]_{\tilde\Gamma_2}(u)}$$

$$Q_2 \;=\; [\bar u_i b_j]_{\Gamma_1}\,[\bar s_j u_i]_{\Gamma_2} = [\bar s b]_{\tilde\Gamma_1}\,[\bar u u]_{\tilde\Gamma_2}$$
$$\to\; H_1(u)\otimes[\bar s_c h_v]_{\tilde\Gamma_1}\,[\bar u_{\bar c} u_{\bar c}]_{\tilde\Gamma_2}(u) + H_2(u)\otimes\frac{1}{N_c}\,[\bar u_c h_v]_{\Gamma_1}\,[\bar s_{\bar c} u_{\bar c}]_{\Gamma_2}(u)$$
$$+\,H_3(u)\otimes 2\,\underline{[\bar u_c T^A h_v]_{\Gamma_1}\,[\bar s_{\bar c} T^A u_{\bar c}]_{\Gamma_2}}(u)\,,$$

colour-octet SCET$_\mathrm{I}$ operators

  - ➢ $H_i(u)$: hard matching coefficients; at tree-level, $H_i(u) = 1$;

- ☐ **How to implement $\langle M_1M_2\left|[\bar u_c T^A h_v]_{\Gamma_1}[\bar s_{\bar c} T^A u_{\bar c}]_{\Gamma_2}\right|\bar B\rangle$: function of u, depending on $M_{1,2}$ & $\bar B$**

  - ➢ for color-singlet SCET$_\mathrm{I}$ operators: factorization well established

$$\left\langle M_1M_2\middle|[\bar u_c h_v]_{\Gamma_1}[\bar s_{\bar c} u_{\bar c}]_{\Gamma_2}(u)\middle|\bar B\right\rangle = c\,\hat A_{M_1M_2}\phi_{M_2}(u),\ \text{with}\ \ \hat A_{M_1M_2} = i\,m_B^2 F^{B\to M_1}(0) f_{M_2}$$

  - ➢ for color-octet SCET$_\mathrm{I}$ operators: normalized to the naïve factorizable amplitude

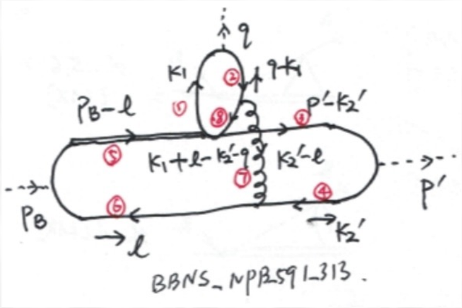
$$\left\langle M_1M_2\middle|[\bar u_c T^A h_v]_{\Gamma_1}[\bar s_{\bar c} T^A u_{\bar c}]_{\Gamma_2}(u)\middle|\bar B\right\rangle = \hat A_{M_1M_2}\mathfrak{F}_{M_2}^{BM_1}(u),\ \text{with}\ \mathfrak{F}_{M_2}^{BM_1}(u)\ \text{an arbitrary function}$$

Soft-exchange effects from emission topology

- ☐ **To have predictive power, make the following two approximations:**

  - ➢ working to lowest order in the hard QCD → SCET$_{\text{I}}$ matching, then $H_i(u) = 1$

$$\Rightarrow \quad \mathfrak{F}_{M_2}^{BM_1} = \int_0^1 du\, \mathfrak{F}_{M_2}^{BM_1}(u)$$



BBNS_NPB591_313.

  - ➢ When the gluon propagator is soft, the propagator 8 is anti-hard-collinear;

    ➡ the SCET$_{\text{I}}$ operator naively factorizes after matching to SCET$_{\text{II}}$:

$$\mathfrak{F}_{M_2}^{BM_1}(u) = \frac{1}{\hat{A}_{M_1M_2}} \frac{f_{M_2}\phi_{M_2}(u)}{8N_c u\bar{u}} \times (-1)\int_0^\infty ds \left\langle M_1 \middle| \left[\bar{u}_c T^A h_v\right]_{\Gamma_1} \text{Ú}_{\mu\nu\alpha\beta} n_+^\nu g_s G^{A,\alpha\beta}(-sn_+) \middle| \bar{B} \right\rangle$$

$$= \frac{1}{\hat{A}_{M_1M_2}} \frac{f_{M_2}\phi_{M_2}(u)}{8N_c u\bar{u}} \times (-i) F^{B\to M_1}(0) g_{\Gamma_1}^{BM_1} = \frac{\phi_{M_2}(u)}{8N_c u\bar{u}} g_{\Gamma_1}^{BM_1}$$

independent of M_2

  - ➢ with the asymptotic $\phi_{M_2}(u) = 6u\bar{u}$, we have:

$$\boxed{\mathfrak{F}_{M_2}^{BM_1} = \int_0^1 du\, \mathfrak{F}_{M_2}^{BM_1}(u) = \frac{1}{4} g_{\Gamma_1}^{BM_1}}$$

- ☐ **Pheno. impacts on two-body hadronic B decays:** [Bell, Beneke, Huber, Li, w.i.p.]

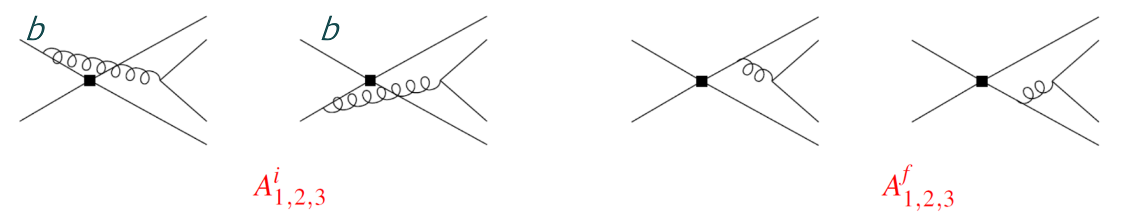
Pure annihilation B decays

☐ **Two typical pure annihilation decay modes:** $\bar{B}_s^0 \to \pi^+\pi^-$ vs $\bar{B}_d^0 \to K^+K^-$ related by SU(3)

$$\mathcal{A}(\bar{B}_s \to \pi^+\pi^-) = B_{\pi\pi}\left[\delta_{pu} b_1 + 2b_4^p + \frac{1}{2} b_{4,\mathrm{EW}}^p\right]$$

$$\mathcal{A}(\bar{B}_d \to K^+K^-) = A_{\bar{K}K}\left[\delta_{pu}\beta_1 + \beta_4^p + b_{4,\mathrm{EW}}^p\right] + B_{K\bar{K}}\left[b_4^p - \frac{1}{2} b_{4,\mathrm{EW}}^p\right]$$

$$= A_{\bar{K}K}\left[\delta_{pu}\beta_1 + \beta_4^p\right] + B_{K\bar{K}}\left[b_4^p\right]$$



☐ **Both involve $b_1 = \frac{C_F}{N_c^2} C_1 A_1^i$ & $b_4^p = \frac{C_F}{N_c^2}[C_4 A_1^i + C_6 A_2^i]$ and kernels A_1^i & A_2^i :**

$A_1^i: (\mathrm{V}-\mathrm{A}) \otimes (V-A)$

$A_2^i: (\mathrm{V}-\mathrm{A}) \otimes (V+A)$

$$A_1^i(M_1M_2) = \pi\alpha_s \int_0^1 dxdy \left\{\Phi_{M_2}(x)\,\Phi_{M_1}(y)\left[\frac{1}{y(1-x\bar{y})} + \frac{1}{\bar{x}^2 y}\right] + r_\chi^{M_1} r_\chi^{M_2}\,\Phi_{m_2}(x)\,\Phi_{m_1}(y)\,\frac{2}{\bar{x}y}\right\},$$

$$A_2^i(M_1M_2) = \pi\alpha_s \int_0^1 dxdy \left\{\Phi_{M_2}(x)\,\Phi_{M_1}(y)\left[\frac{1}{\bar{x}(1-x\bar{y})} + \frac{1}{\bar{x}y^2}\right] + r_\chi^{M_1} r_\chi^{M_2}\,\Phi_{m_2}(x)\,\Phi_{m_1}(y)\,\frac{2}{\bar{x}y}\right\},$$

☐ **With the asymptotic LCDAs $\Phi_M(x) = 6x\bar{x}$, we have $A_1^i = A_2^i$:**

[BBNS '99-'03]

$$A_1^i(M_1M_2) = \pi\alpha_s \left\{18X_A - 18 - 6(9-\pi^2) + r_\chi^{M_1} r_\chi^{M_2}\left(2X_A^2\right)\right\},$$

$$A_2^i(M_1M_2) = \pi\alpha_s \left\{18X_A - 18 - 6(9-\pi^2) + r_\chi^{M_1} r_\chi^{M_2}\left(2X_A^2\right)\right\},$$

$$X_A = (1 + \rho_A e^{i\varphi_A})\ln\left(\frac{m_B}{\Lambda_h}\right)$$

Ways to improve the modelling of annihilations

☐ With universal X_A and different scenarios, we have:

[BBNS '03]

Mode	Theory	S1 (large γ)	S2 (large a_2)	S3 ($\varphi_A = -45^\circ$)	S4 ($\varphi_A = -55^\circ$)	Exp.
$\bar{B}_s^0 \to \pi^+\pi^-$	$0.024^{+0.003+0.025+0.000+0.163}_{-0.003-0.012-0.000-0.021}$	0.027	0.032	0.149	0.155	0.72 ± 0.11
$\bar{B}^0 \to K^-K^+$	$0.013^{+0.005+0.008+0.000+0.087}_{-0.005-0.005-0.000-0.011}$	0.007	0.014	0.079	0.070	0.080 ± 0.015

Large SU(3)-flavor symmetry breaking or flavor-dependent $A^i_{1,2}$?

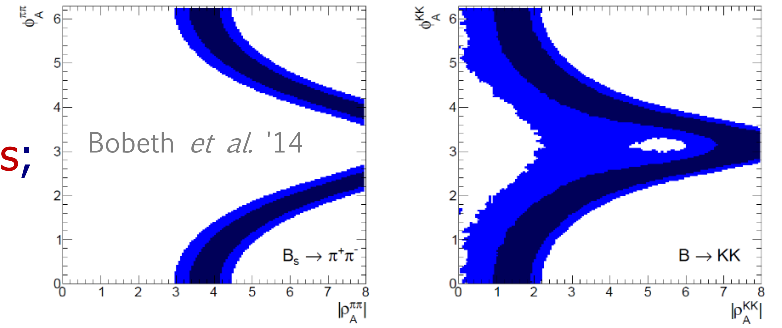
[Wang, Zhu '03; Bobeth *et al.* '14; Chang, Sun *et al.* '14-15]

☐ How to improve the situation:

- including higher Gegenbauer moments to include SU(3)-breaking effects;

$$\Phi_M(x,\mu) = 6x\bar{x}\left[1 + \sum_{n=1}^{\infty} a_n^M(\mu)\, C_n^{(3/2)}(2x-1)\right]$$

due to G-parity, $a^{\pi}_{odd} = 0$, but $a^{K}_{odd} \neq 0$



Bobeth *et al.* '14

FIGURE 5.8: 68% and 95% CRs for the complex parameter $\rho_A^{\pi^+\pi^-}$ and $\rho_A^{K^+K^-}$ obtained from a branching-ratio fit assuming the SM.

$$X_A = (1 + \rho_A e^{i\varphi_A})\ln\left(\frac{m_B}{\Lambda_h}\right)$$

- including the difference between the chirality factors to include SU(3)-breaking effects;

$$r_\chi^\pi(1.5\text{GeV}) = \frac{2m_\pi^2}{m_b(\mu)\big(m_u(\mu) + m_d(\mu)\big)} \simeq 0.86, \qquad r_\chi^K(1.5\text{GeV}) = \frac{2m_K^2}{m_b(\mu)\big(m_u(\mu) + m_s(\mu)\big)} \simeq 0.91$$

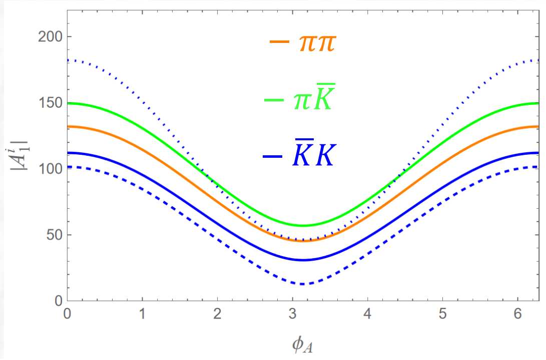
Ways to improve the modelling of annihilations

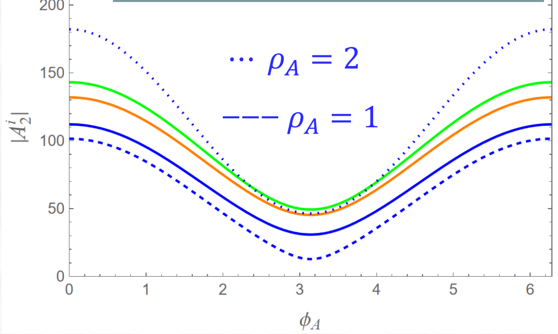
☐ SU(3)-breaking effects in $A_{1,2}^i$: due to higher Gengengauber moments and quark masses

$$A_1^i(M_1M_2) = \pi\alpha_s \Big\{ 18(1 - a_1^{M_1} + a_2^{M_1})\Big[(1 + 3a_1^{M_2} + 6a_2^{M_2})X_A - (1 + 6a_1^{M_2} + 16a_2^{M_2})\Big] \quad +54(69 - 7\pi^2)a_1^{M_1}a_1^{M_2} - 36(385 - 39\pi^2)(a_1^{M_1}a_2^{M_2} - 2a_2^{M_1}a_1^{M_2})$$
$$-6(9 - \pi^2) - 18(10 - \pi^2)(3a_1^{M_1} - a_1^{M_2}) - 6(59 - 6\pi^2)(6a_2^{M_1} + a_2^{M_2}) \quad -18(9593 - 972\pi^2)a_2^{M_1}a_2^{M_2} + r_\chi^{M_1}r_\chi^{M_2}\left(2X_A^2\right)\Big\},$$

$$X_A = \ln\left(\frac{m_B}{\Lambda_h}\right)(1 + \rho_A e^{i\phi_A})$$

$$A_2^i(M_1M_2) = \pi\alpha_s \Big\{ 18(1 + a_1^{M_2} + a_2^{M_2})\Big[(1 - 3a_1^{M_1} + 6a_2^{M_1})X_A - (1 - 6a_1^{M_1} + 16a_2^{M_1})\Big]$$
$$-6(9 - \pi^2) - 18(10 - \pi^2)(a_1^{M_1} - 3a_1^{M_2}) - 6(59 - 6\pi^2)(a_2^{M_1} + 6a_2^{M_2})$$
$$+54(69 - 7\pi^2)a_1^{M_1}a_1^{M_2} - 36(385 - 39\pi^2)(2a_1^{M_1}a_2^{M_2} - a_2^{M_1}a_1^{M_2})$$
$$-18(9593 - 972\pi^2)a_2^{M_1}a_2^{M_2} + r_\chi^{M_1}r_\chi^{M_2}\left(2X_A^2\right)\Big\},$$





	$\pi\pi$	$\pi\bar{K}$	$\bar{K}K$
A_1^i	$31.7X_A - 51.5+6.2 + 1.5X_A^2$ $[18X_A - 18+5.2 + 1.5X_A^2]$	$37.6X_A - 63.4+6.5 + 1.6X_A^2$ $[18X_A - 18+5.2 + 1.6X_A^2]$	$23.4X_A - 36.0+5.2 + 1.7X_A^2$ $[18X_A - 18+5.2 + 1.7X_A^2]$
A_2^i	$31.7X_A - 51.5+6.2 + 1.5X_A^2$ $[18X_A - 18+5.2 + 1.5X_A^2]$	$34.6X_A - 56.2+6.9 + 1.6X_A^2$ $[18X_A - 18+5.2 + 1.6X_A^2]$	$23.4X_A - 36.0+5.2 + 1.7X_A^2$ $[18X_A - 18+5.2 + 1.7X_A^2]$

$Br(\bar{B}_s^0 \to \pi^+\pi^-)$: $(0.72 \pm 0.11)\times 10^{-6}$
$Br(\bar{B}^0 \to K^-K^+)$: $(0.080 \pm 0.015)\times 10^{-6}$

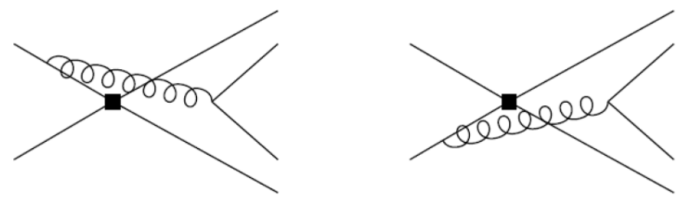
- ➢ $|A_{1,2}^i|$ can differ by more than **20%** in the **BBNS+ model**!
- ➢ The amplitude ratios $A_{1,2}^i(\pi\pi)/A_{1,2}^i(KK)$ get **enhanced in the BBNS+ model**! ➡what we need!

Endpoint divergence in annihilation decays

☐ Finally, endpoint divergence in QCDF may be solved in SCET:

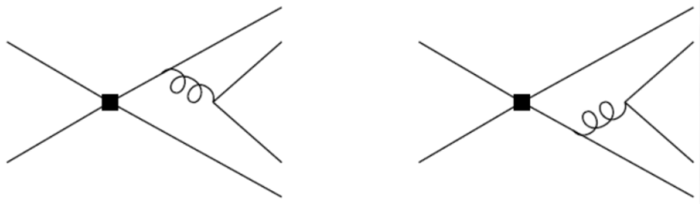
See M. Stillger talk @ SCET 2025: https://indico.physics.lbl.gov/event/3051/contributions/9656/attachments/4895/6797/SCET_2025_Stillger.pdf

$$A_1^i(M_1M_2) = \pi\alpha_s \int_0^1 dxdy \left\{\Phi_{M_2}(x)\,\Phi_{M_1}(y)\left[\frac{1}{y(1-x\bar{y})}+\frac{1}{\bar{x}^2 y}\right] + r_\chi^{M_1} r_\chi^{M_2}\,\Phi_{m_2}(x)\,\Phi_{m_1}(y)\,\frac{2}{\bar{x}y}\right\}$$



$A^i_{1,2,3}$

- ➢ with asymptotic LCDA $\Phi(x) \sim 6x\bar{x}$, divergent integral at $x \to 1$
- ➢ physical picture: one constituent becomes soft



$A^f_{1,2,3}$

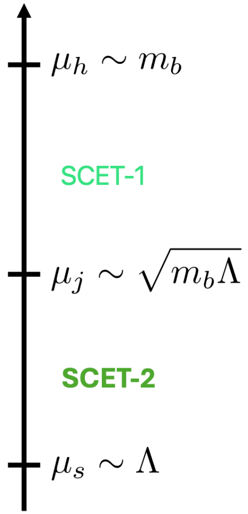
☐ Successive two-step matching: QCD → $\mathrm{SCET_I}$ → $\mathrm{SCET_{II}}$

(1) QCD → SCET-1

- at scale $\mu_h \sim m_b$ **integrate out hard** modes
- obtain EFT with (anti-)hard-collinear and soft modes
- Wilson coefficients: hard functions H

(2) SCET-1 → SCET-2

- at scale $\mu_j \sim \sqrt{m_b\Lambda}$ **integrate out (anti-)hard-collinear** modes
- obtain EFT with (anti-)collinear and soft modes
- Wilson coefficients: jet functions J



$\mu_h \sim m_b$

SCET-1

$\mu_j \sim \sqrt{m_b\Lambda}$

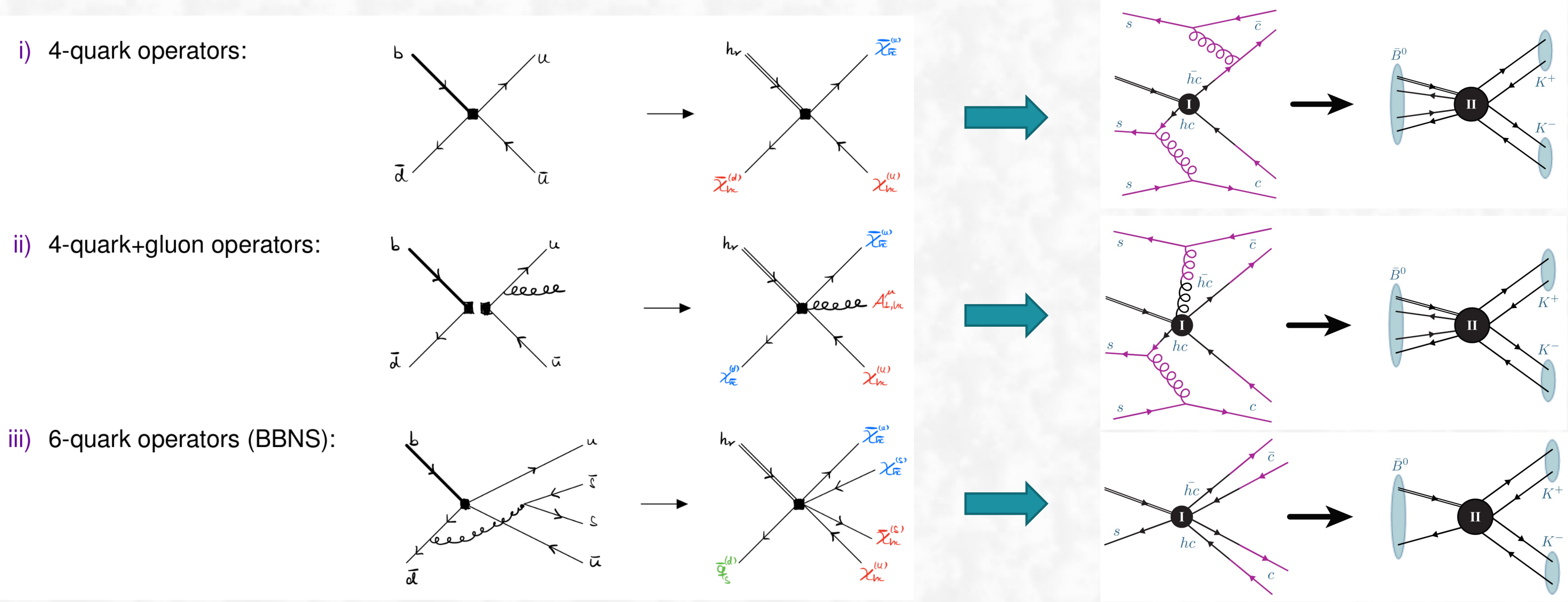
SCET-2

$\mu_s \sim \Lambda$

See also P. Boer talk @ SCET 2023

Endpoint divergence in annihilation decays

- ☐ **Cancellation of endpoint divergence:** see also P. Boer talk @ SCET 2023



i) 4-quark operators:

ii) 4-quark+gluon operators:

iii) 6-quark operators (BBNS):

8-quark operators

6-quark operators

- **4-quark** operators give new contribution
  - ➢ endpoint **divergent** for $\bar{x} \to 0$ or $y \to 0$
- **6-quark** operators reproduce BBNS contributions
  - ➢ endpoint **divergent** for $\bar{x} \to 0$ or $y \to 0$

- 4-quark + gluon operators yield endpoint finite contributions

Summary

- **With exp. and theor. progress, we are now entering a precision era for flavour physics**
- **Within QCDF/SCET framework, NNLO QCD corrections to color-allowed, color-suppressed tree & leading-power penguin amplitudes complete, factorization at 2-loop established**
- **Due to delicate cancellation, NNLO corrections found small; some puzzles still remain:**
  - long-standing $\mathrm{Br}(\bar{B}^0 \to \pi^0\pi^0)$ and $\Delta A_{CP}(\pi K) = A_{CP}(B^- \to \pi^0 K^-) - A_{CP}(\bar{B}^0 \to \pi^+ K^-)$;
  - for class-I $B_q^0 \to D_q^{(*)-} L^+$ decays, $\mathcal{O}(4\text{–}5\sigma)$ discrepancies observed in branching ratios;

➡ **sub-leading power corrections in QCDF/SCET must be considered!**

  - sub-leading color-octet matrix elements $\langle M_1 M_2 | [\bar{u}_c T^A h_v]_{\Gamma_1} [\bar{s}_{\bar{c}} T^A u_{\bar{c}}]_{\Gamma_2}(u) | \bar{B} \rangle$ [w.i.p]
  - improved treatments of annihilation amplitudes: SU(3)-breaking effects & flavor-dependence of the building blocks $A_{1,2}^i$ [w.i.p]

Thank You for your attention!