Generalized symmetries in HEP and CMP

Low Entanglement Excitations in Invertible phases

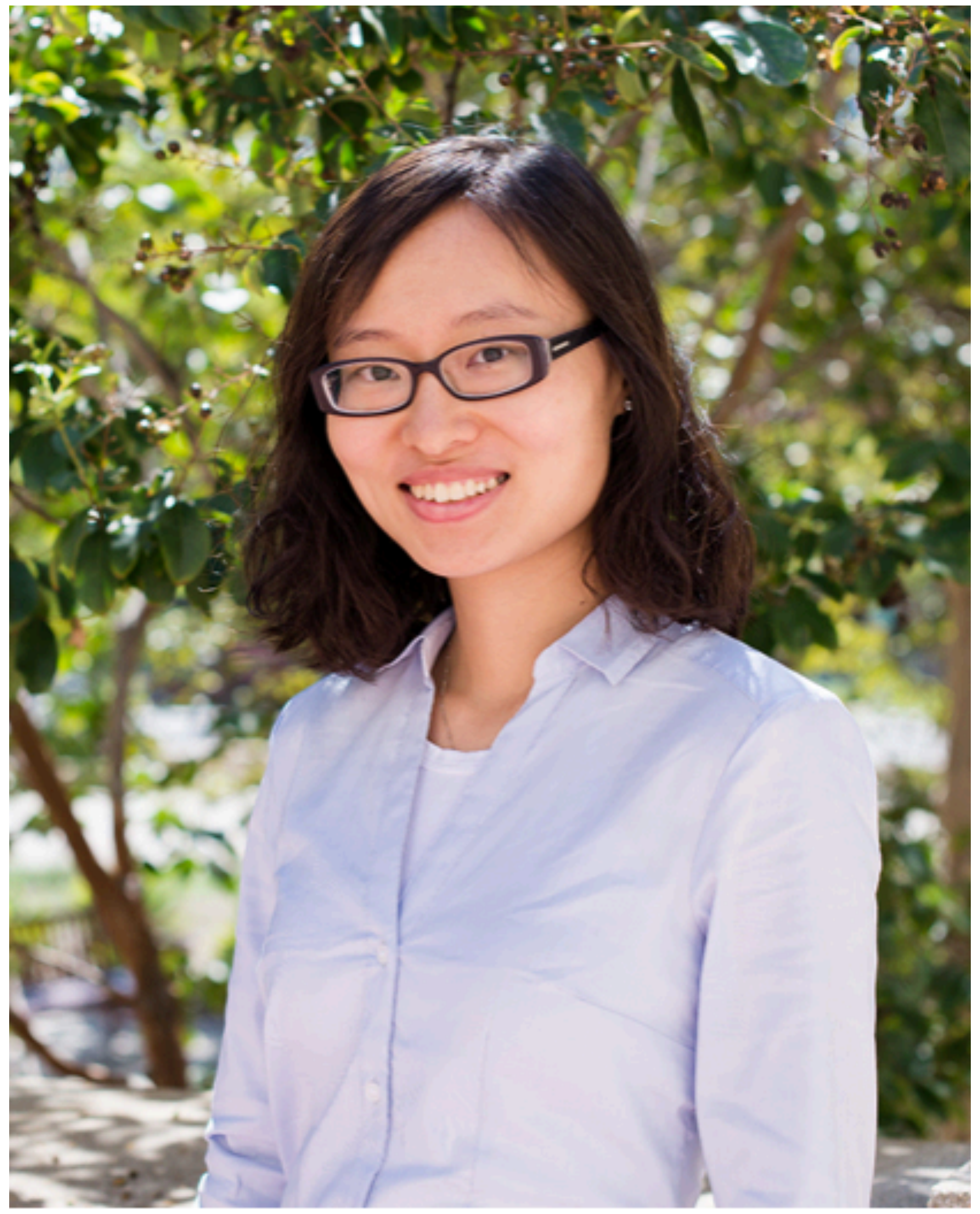
Wenjie Ji

July 2025

arXiv:2506.11288 with

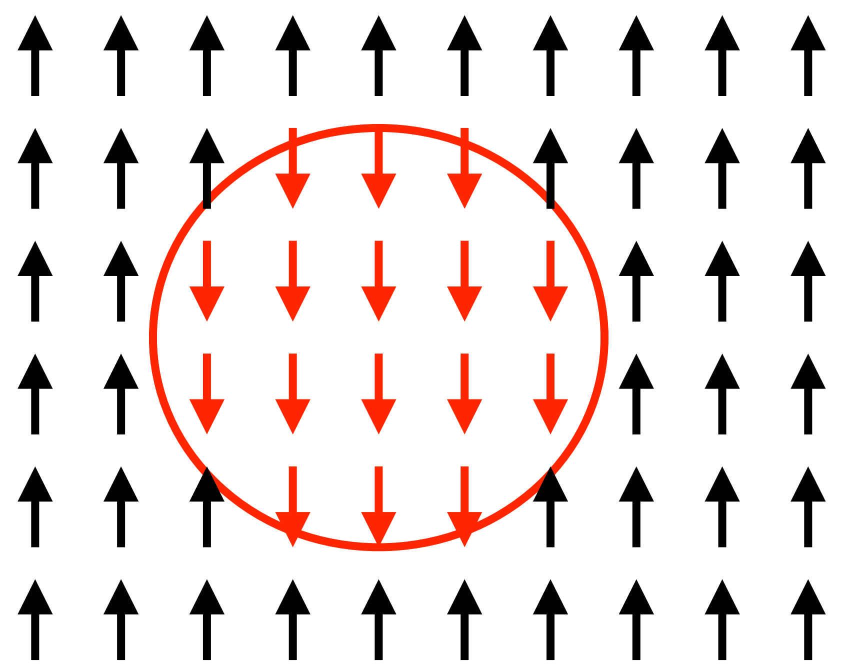
David Stephen
University of Colorado, Boulder
→ Quantinuum

Michael Levin
University of Chicago

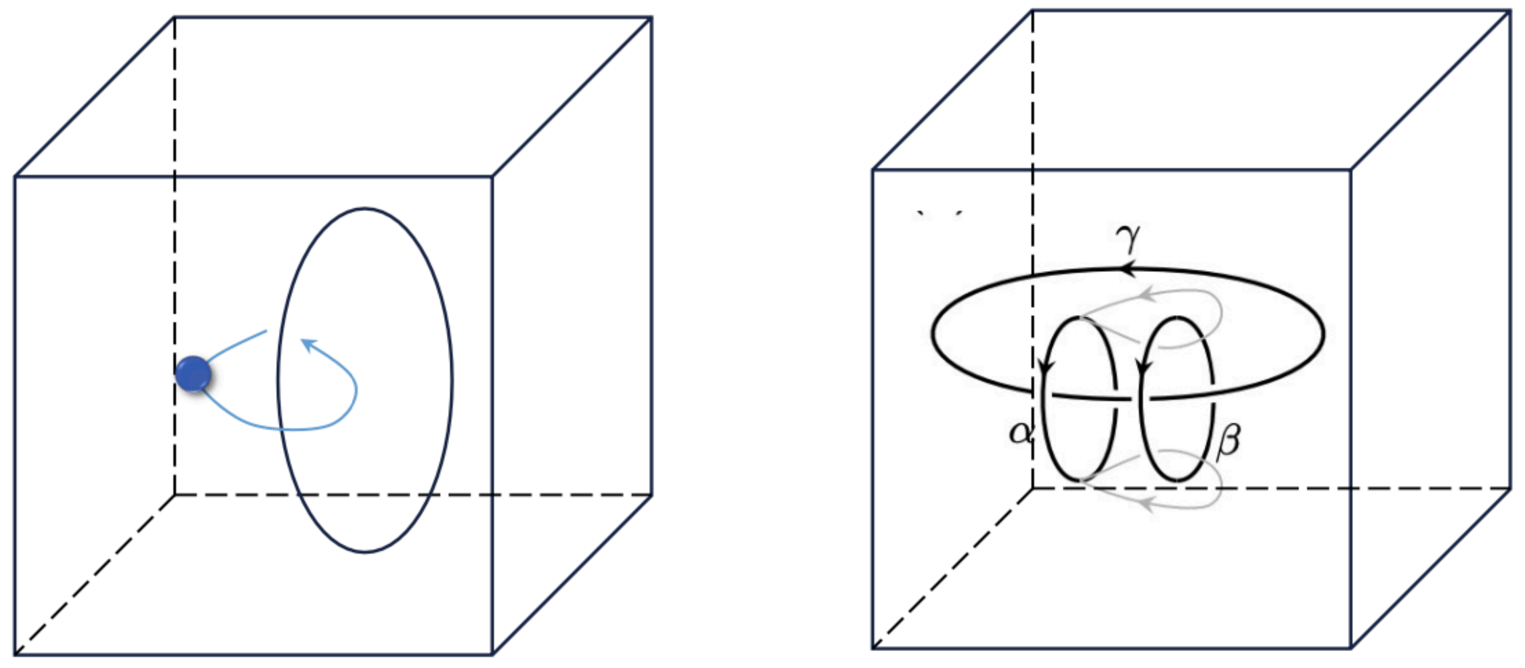


Xie Chen
Caltech

Higher dimensional excitations/defects

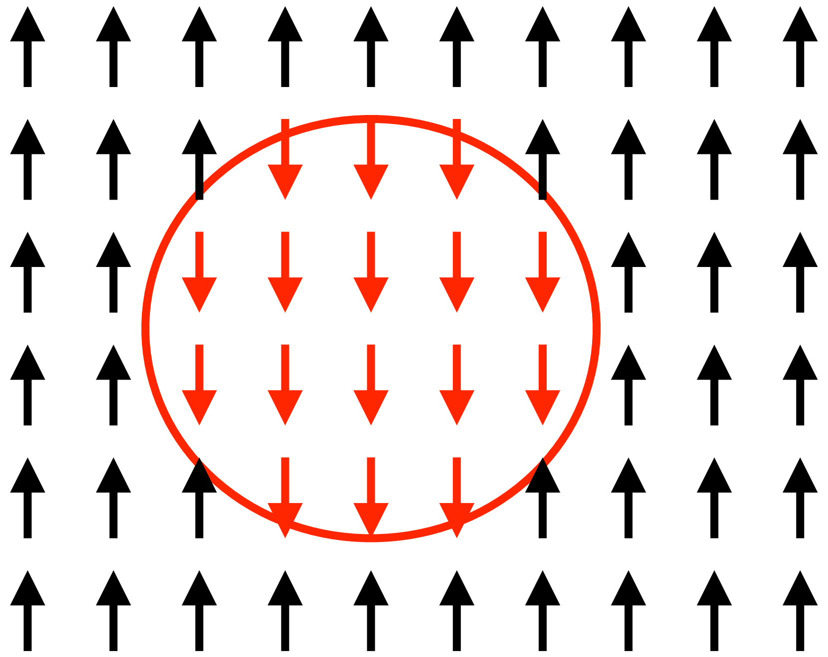


Domain wall
in Ising Ferromagetic phase

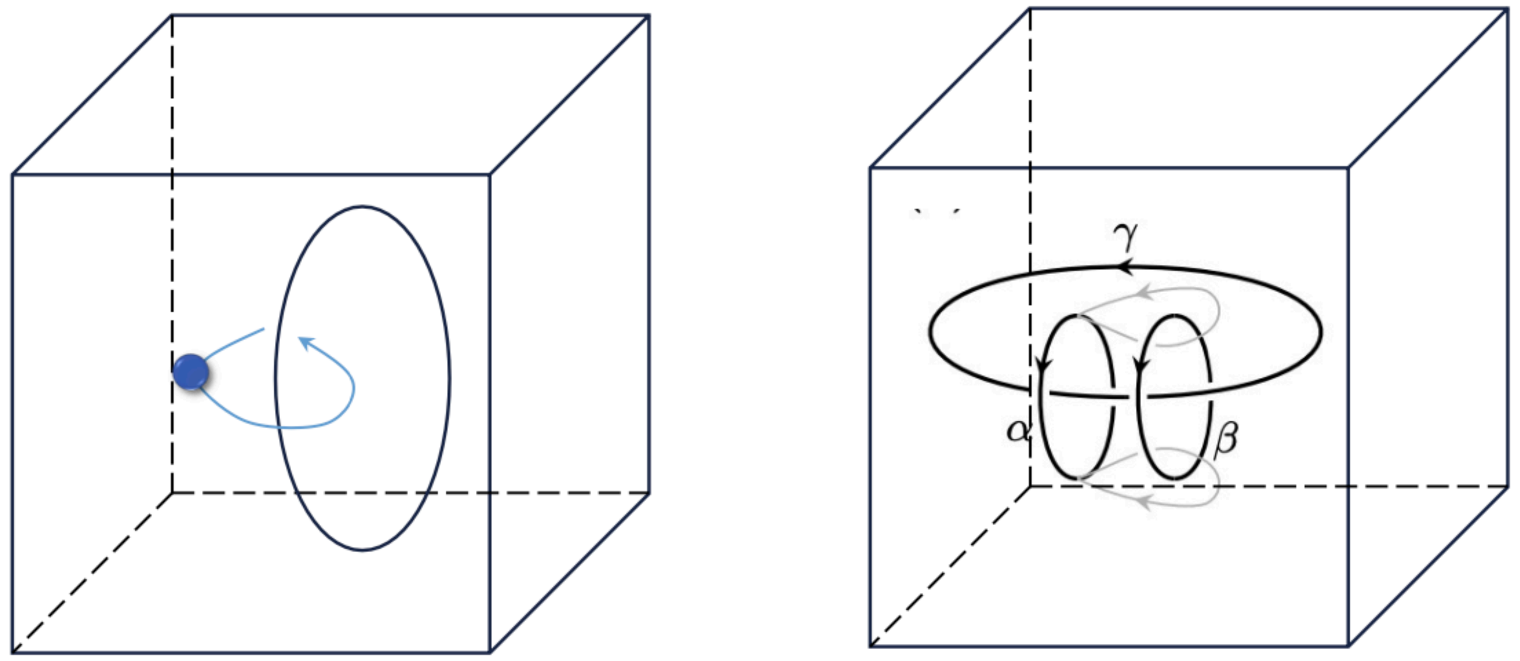


Flux loop in deconfined phase of
G gauge theories
(G topological order)

Higher dimensional excitations/defects



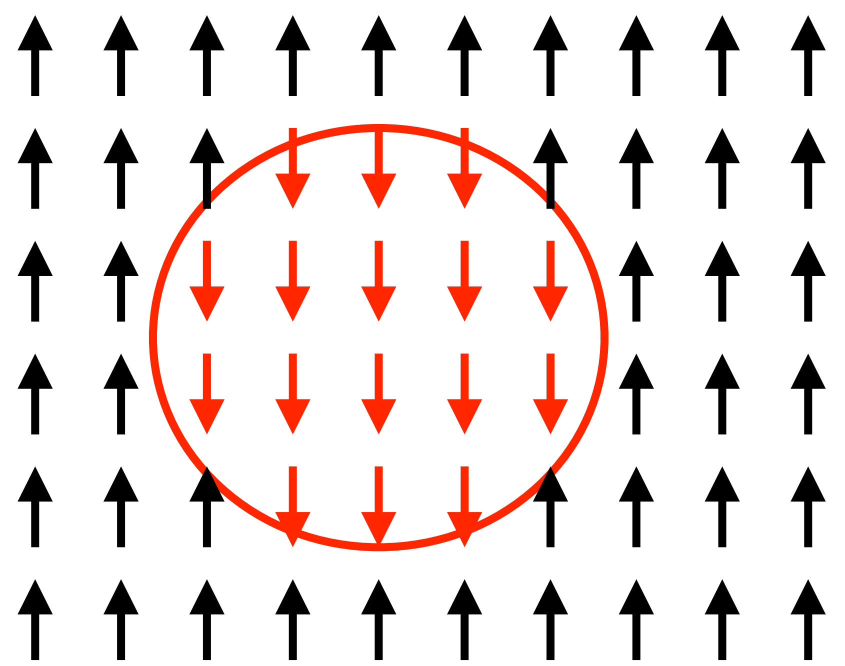
Domain wall
in Ising Ferromagetic phase



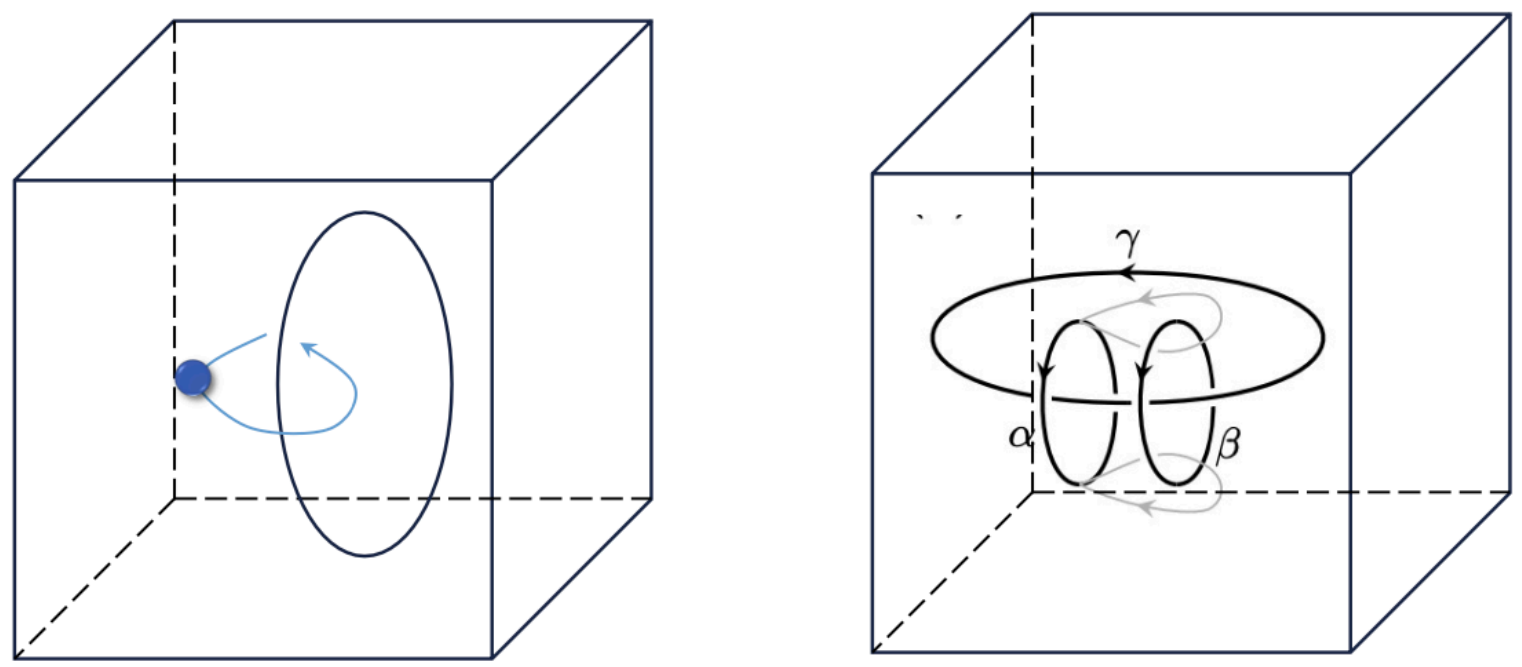
Flux loop in G gauge theories

- How to generate? 1. Modify the Hamiltonian, 2. A unitary operator on the ground state

Higher dimensional excitations/defects



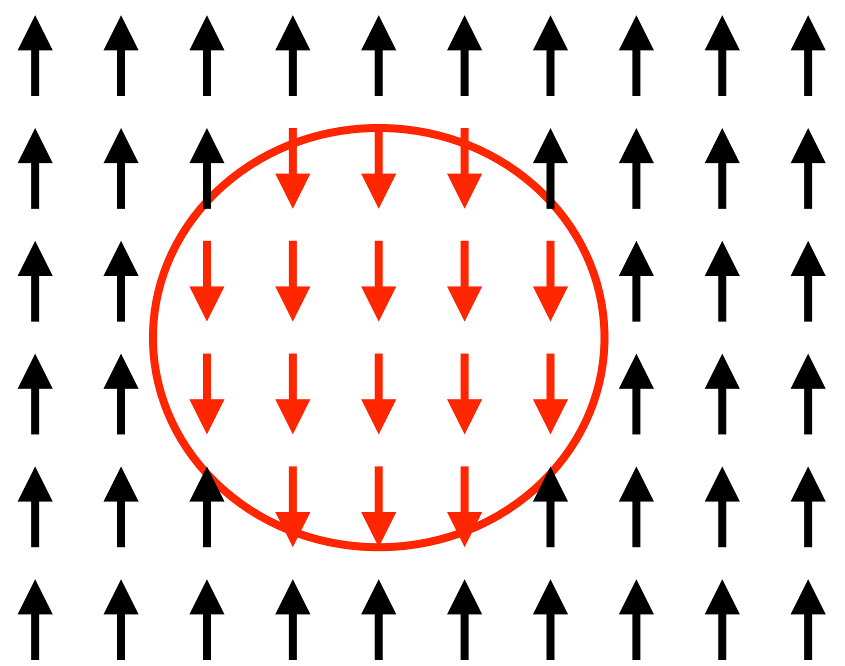
Domain wall
in Ising Ferromagetic phase



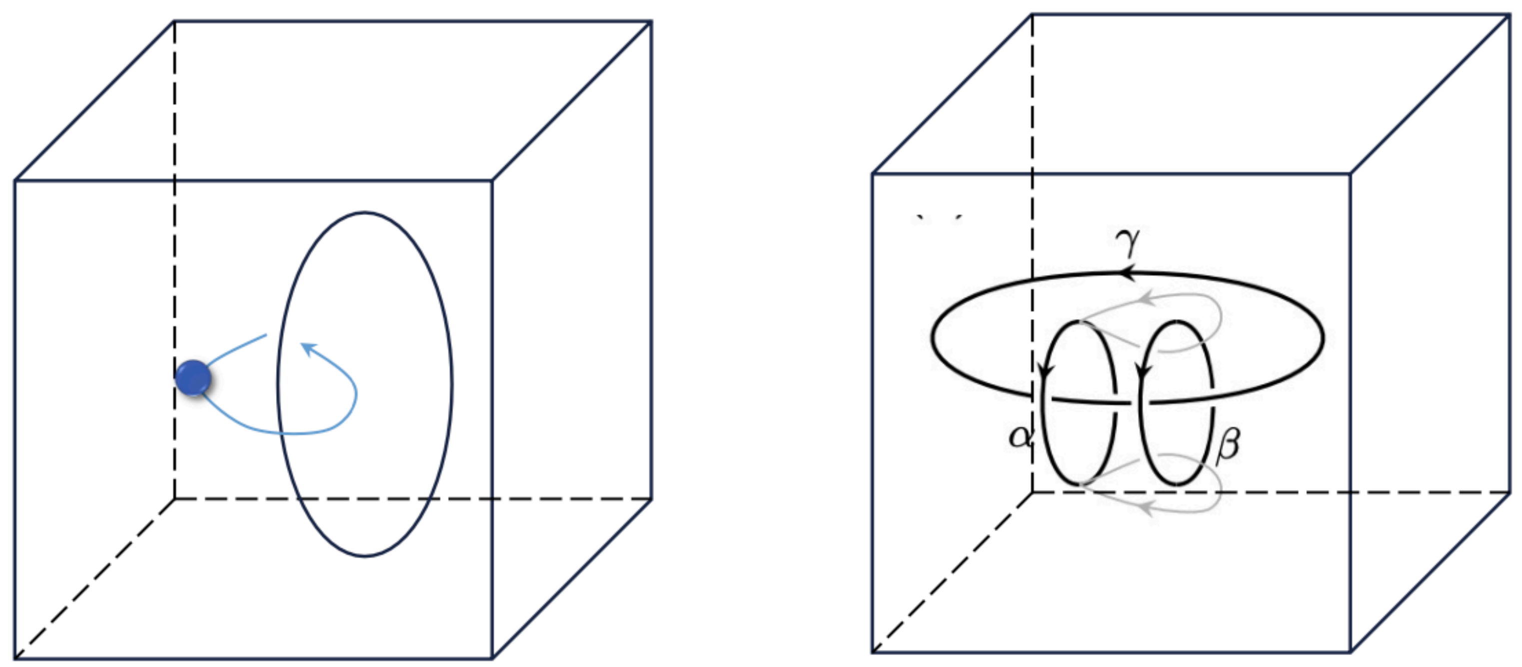
Flux loop in G gauge theories

- How to generate? 1. Modify the Hamiltonian, 2. A unitary operator on the ground state
- Not necessarily the energy eigenstate

Higher dimensional excitations/defects



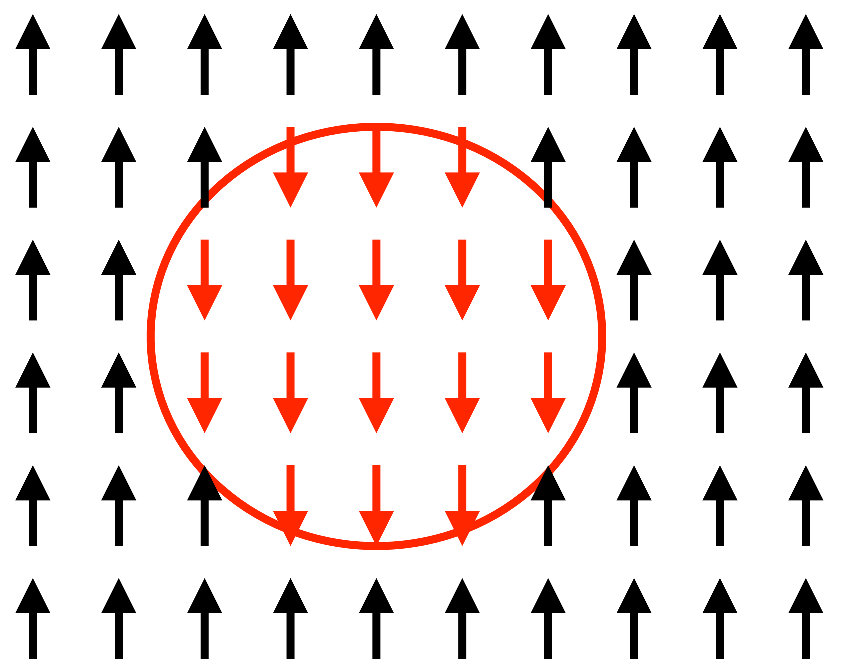
Domain wall
in Ising Ferromagetic phase



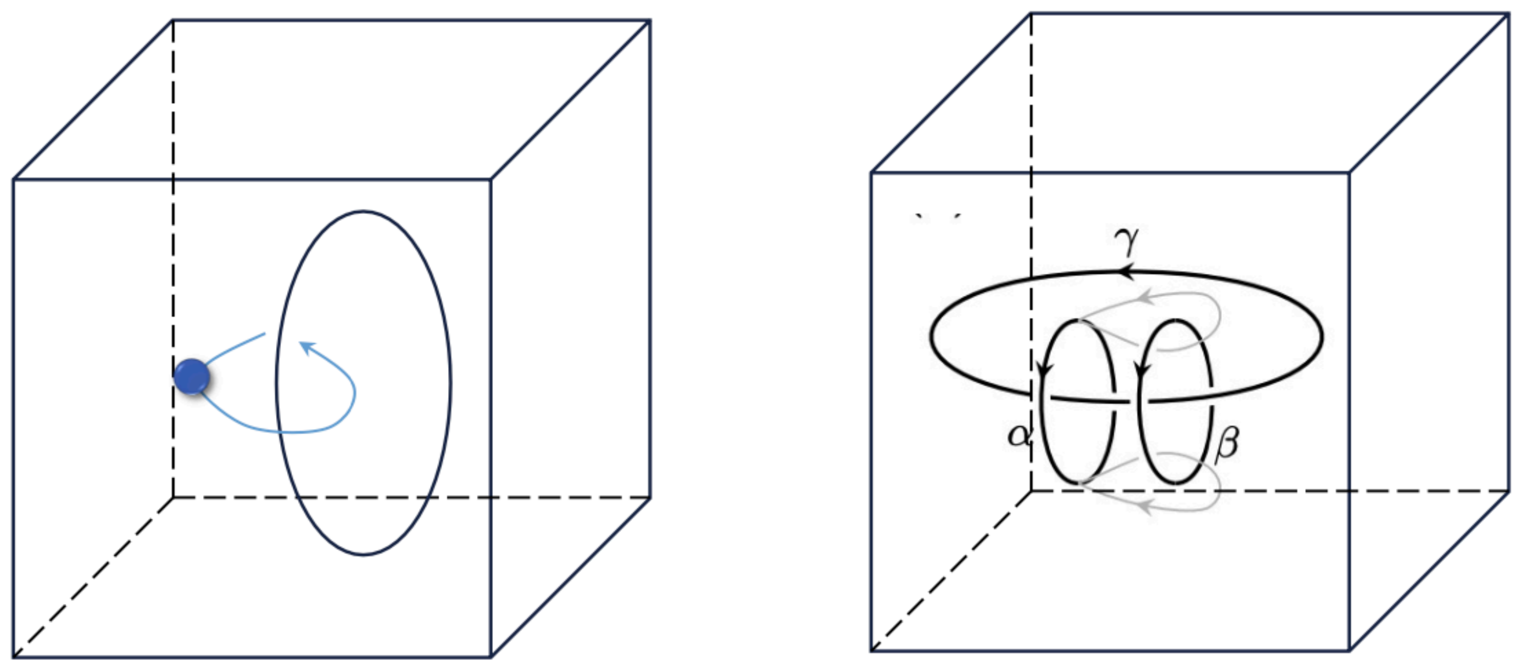
Flux loop in G gauge theories

- How to generate? 1. Modify the Hamiltonian, 2. A unitary operator on the ground state
- Not necessarily the energy eigenstate
- Good enough if maintaining its form for an amount of time

Higher dimensional excitations/defects



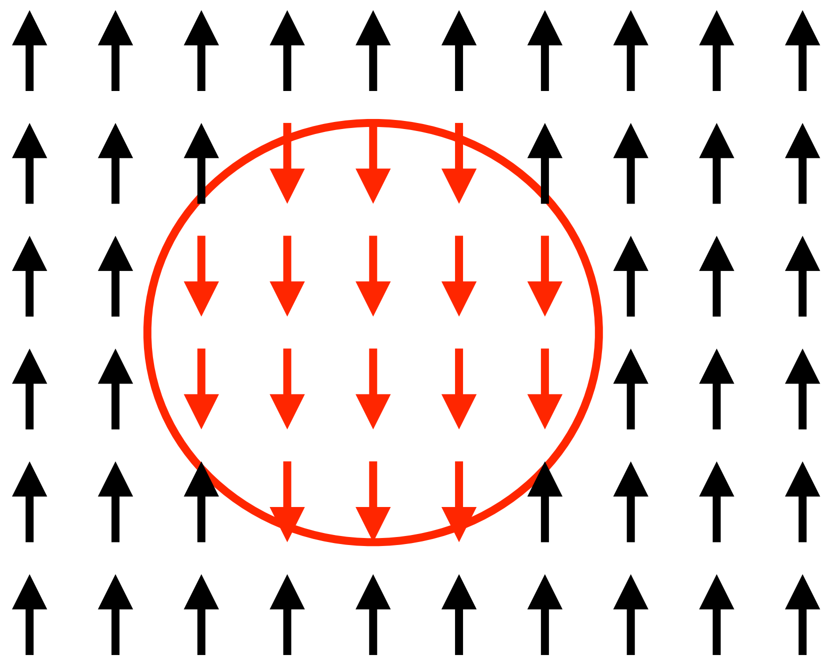
Domain wall
in Ising Ferromagetic phase



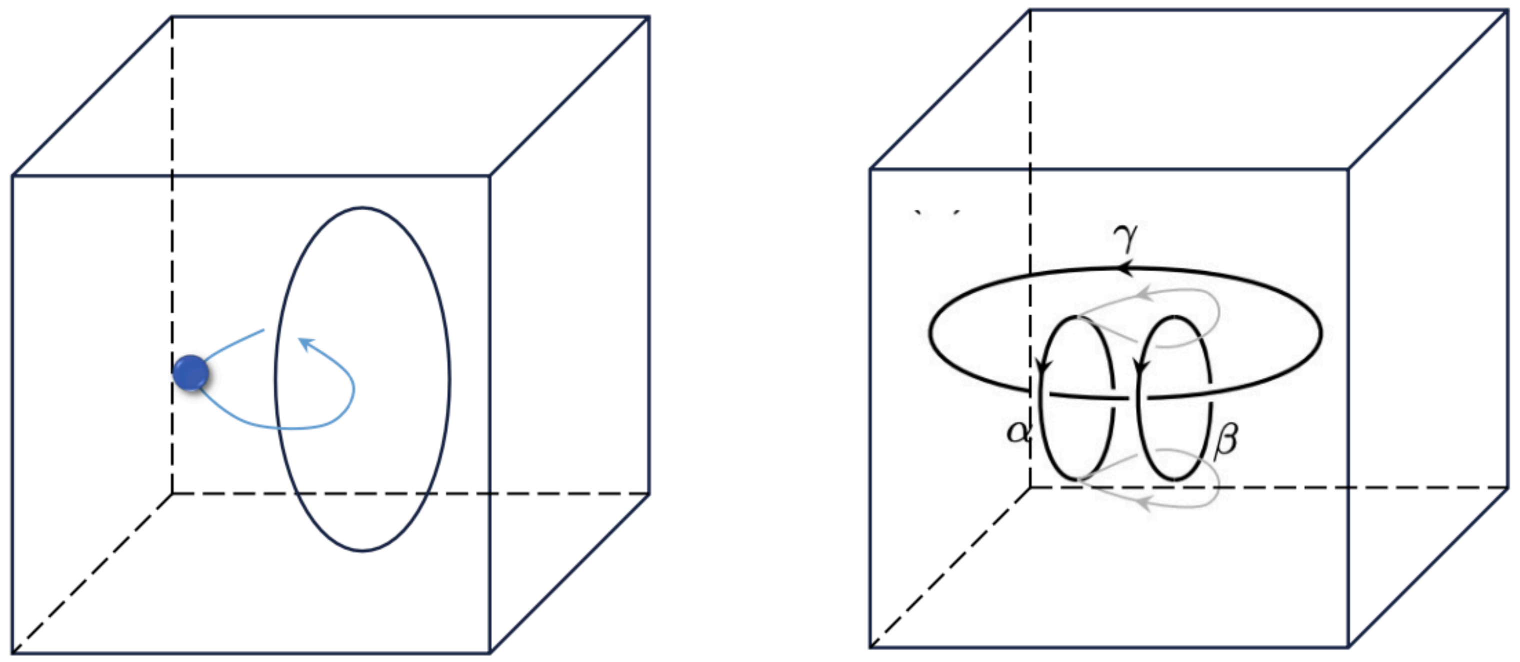
Flux loop in G gauge theories

- How to generate? 1. Modify the Hamiltonian, 2. A unitary operator on the ground state
- Not necessarily the energy eigenstate
- Good enough if maintaining its form for an amount of time
- **Low entanglement** (area law since modified Hamiltonian has a gap)

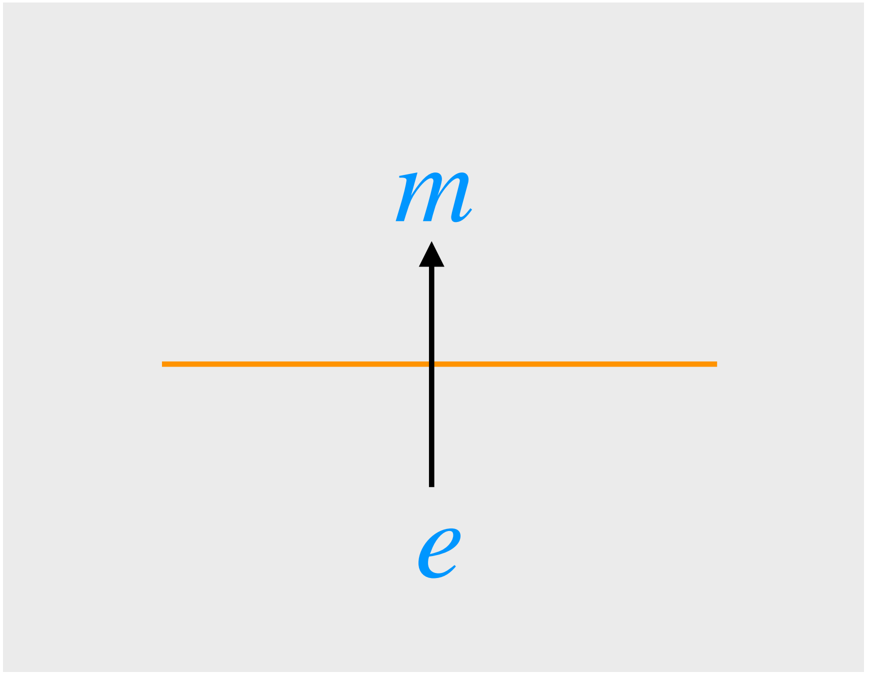
Higher dimensional excitations/defects



Domain wall
in Ising Ferromagetic phase



Flux loop in G gauge theories

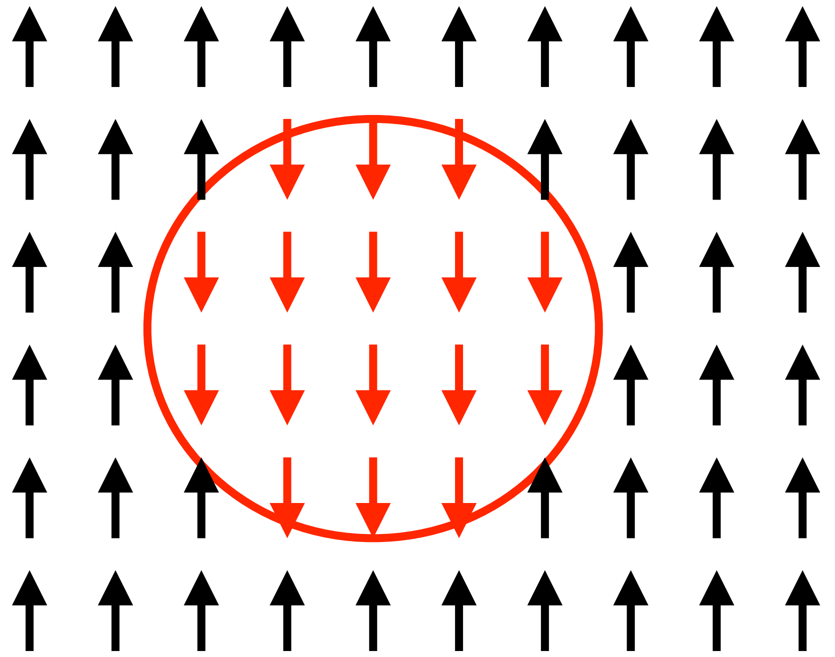


e-m exchange defect
in toric code

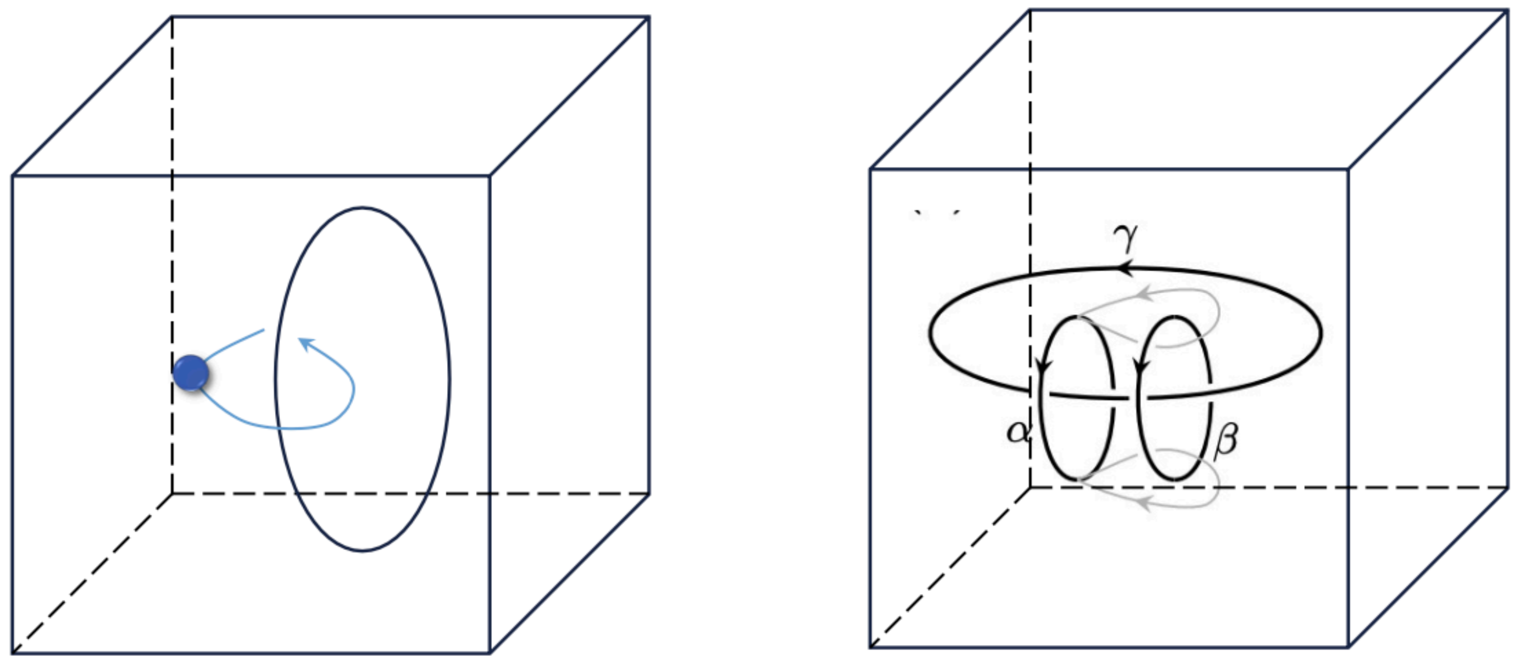
- How to generate? 1. Modify the Hamiltonian, 2. A unitary operator on the ground state
- Not necessarily the energy eigenstate
- Good enough if maintaining its form for an amount of time
- **Low entanglement** (area law since modified Hamiltonian has a gap)

Defects can share many of these properties

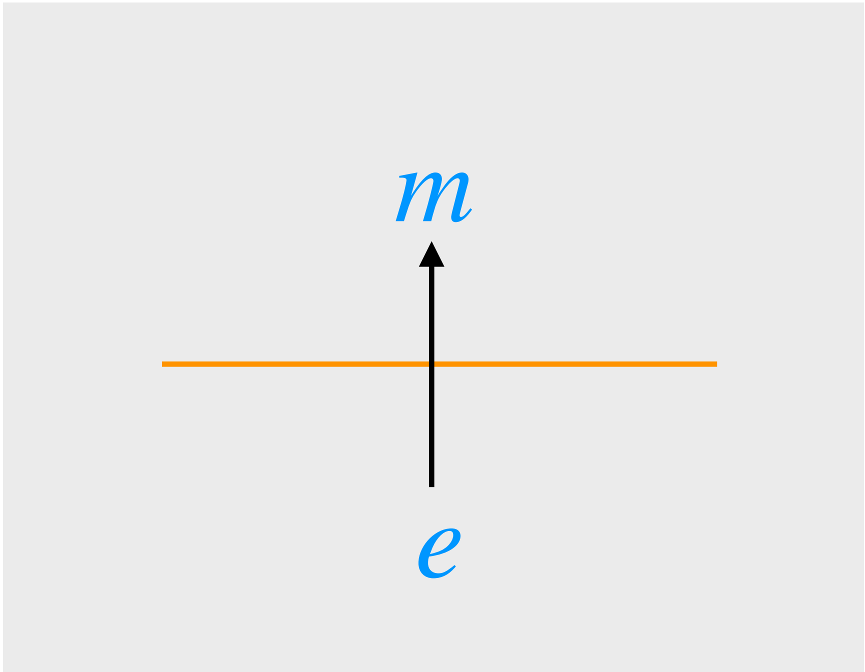
Higher dimensional excitations/defects



Domain wall
in Ising Ferromagetic phase



Flux loop in G gauge theories



e-m exchange defect
in toric code

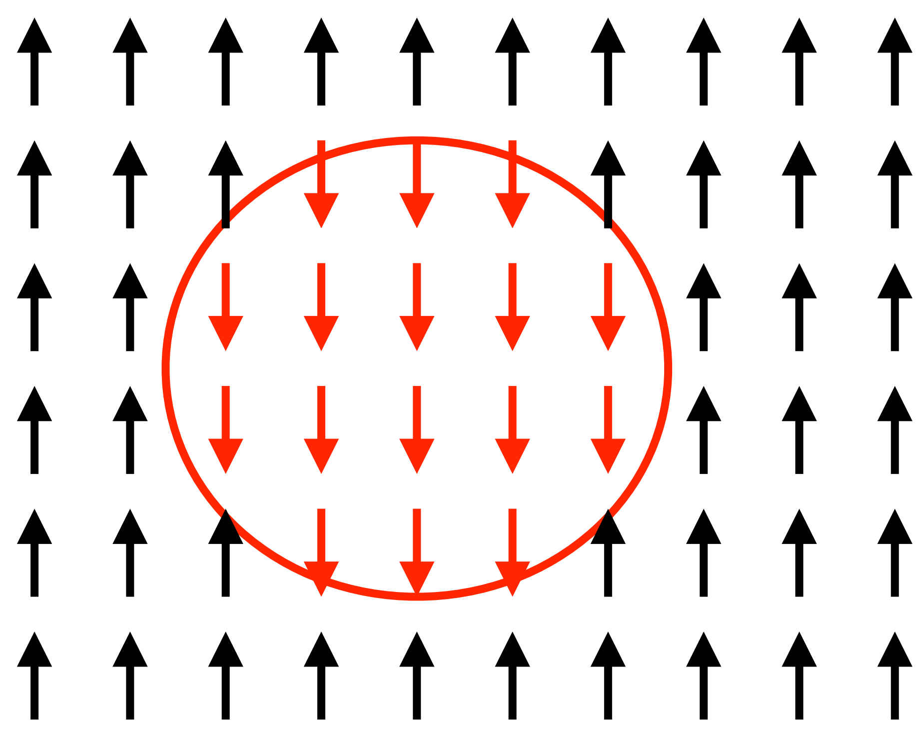
- How to generate? 1. Modify the Hamiltonian, 2. A unitary operator on the ground state
- Not necessarily the energy eigenstate
- Good enough if maintaining its form for an amount of time
- **Low entanglement** (area law since modified Hamiltonian has a gap)

Defects can share many of these properties

A unified definition for excitations/defects ? Especially, those with low entanglement.

Low entanglement excitations

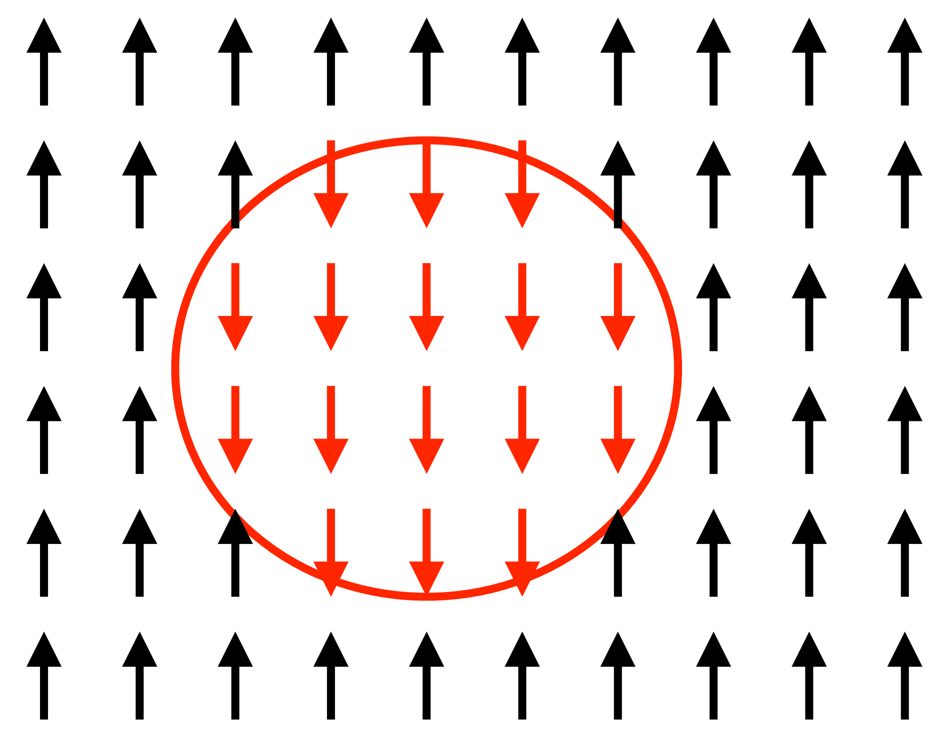
Use **quantum circuits** to create/define *k*-dimentional excitations



Low entanglement excitations

Use **quantum circuits** to create/define *k*-dimentional excitations

In the ground state of a gapped Hamiltonian in d dimensions, a *k*-**dim. excitation**

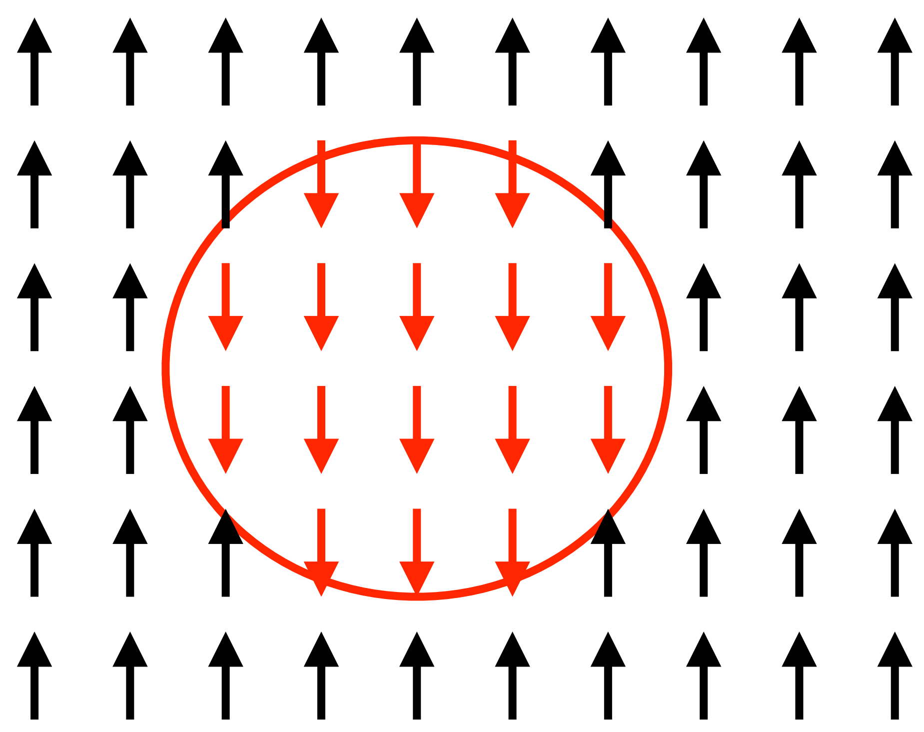


- Gapped ground state of the Hamitonian modified **only along the excitation**

Low entanglement excitations

Use **quantum circuits** to create/define k-dimentional excitations

In the ground state of a gapped Hamiltonian in d dimensions, a k**-dim. excitation**



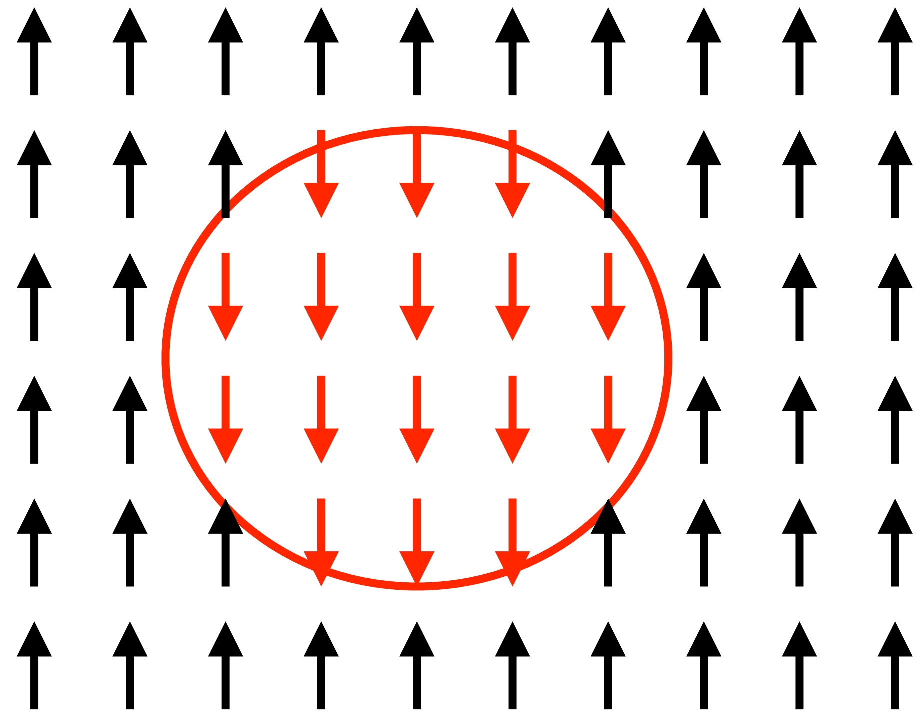
- Gapped ground state of the Hamitonian modified **only along the excitation**
- **Equivalence condition?**

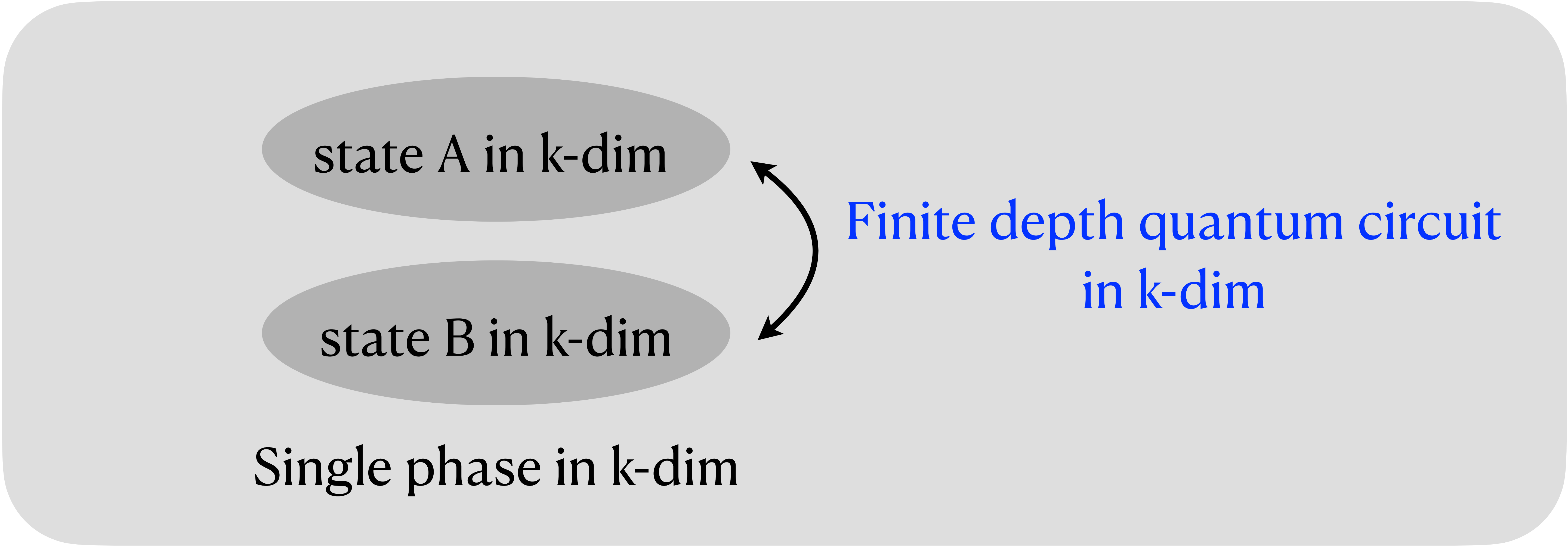
Low entanglement excitations

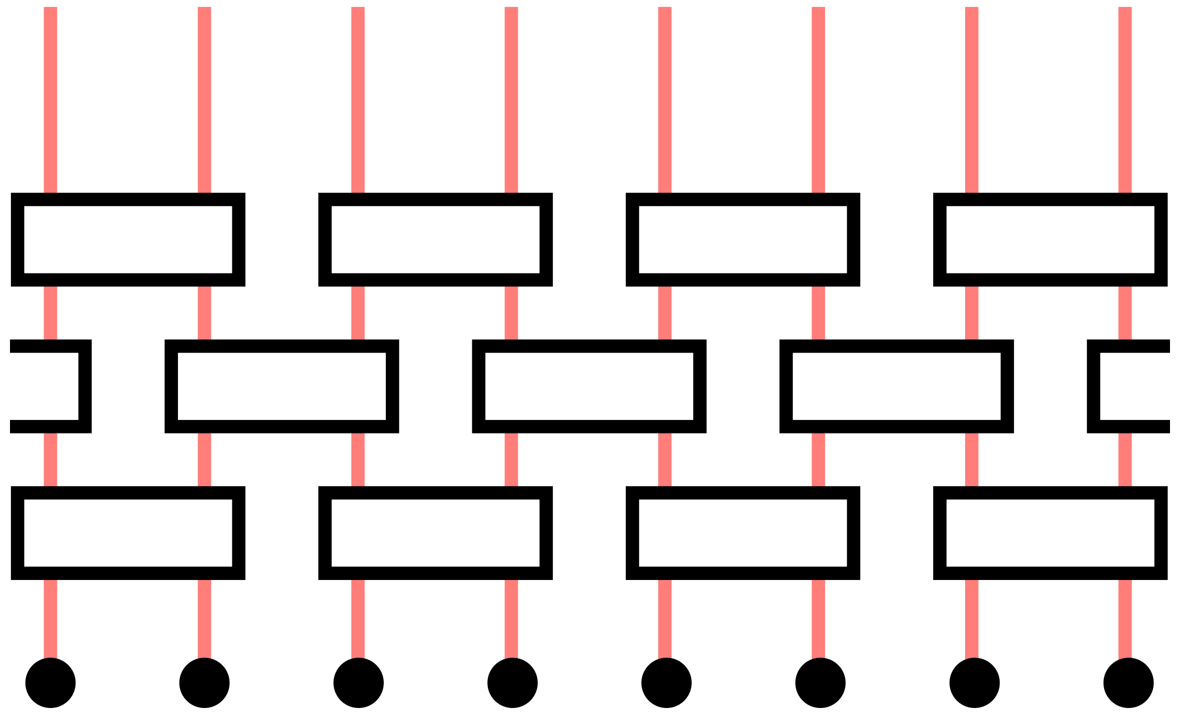
Use **quantum circuits** to create/define *k*-dimentional excitations

In the ground state of a gapped Hamiltonian in d dimensions, a *k*-**dim. excitation**

- Gapped ground state of the Hamitonian modified **only along the excitation**
- **Equivalence condition?**







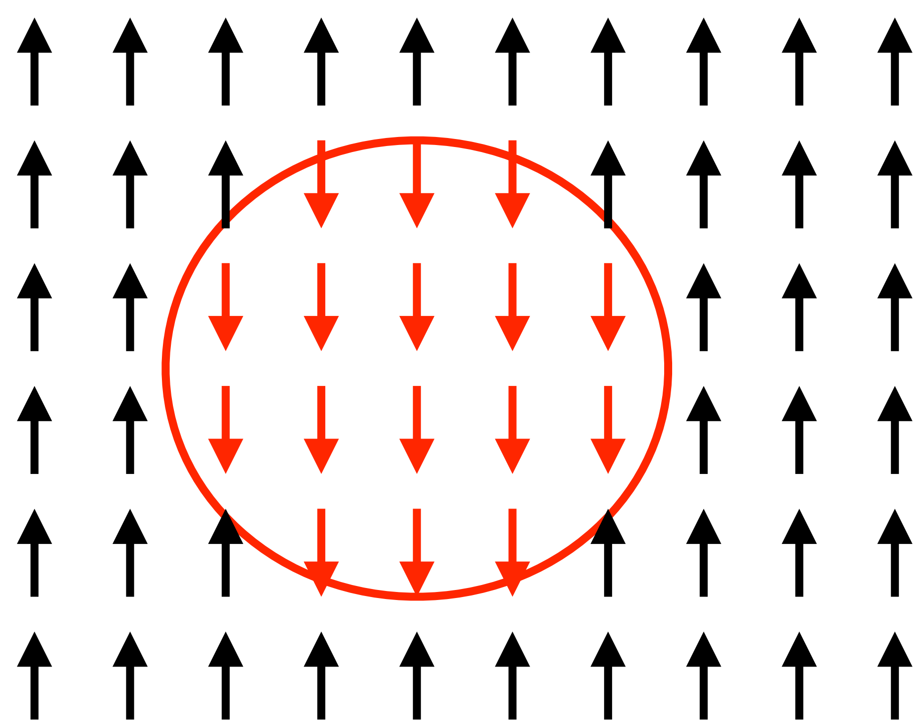
Chen-Gu-Wen

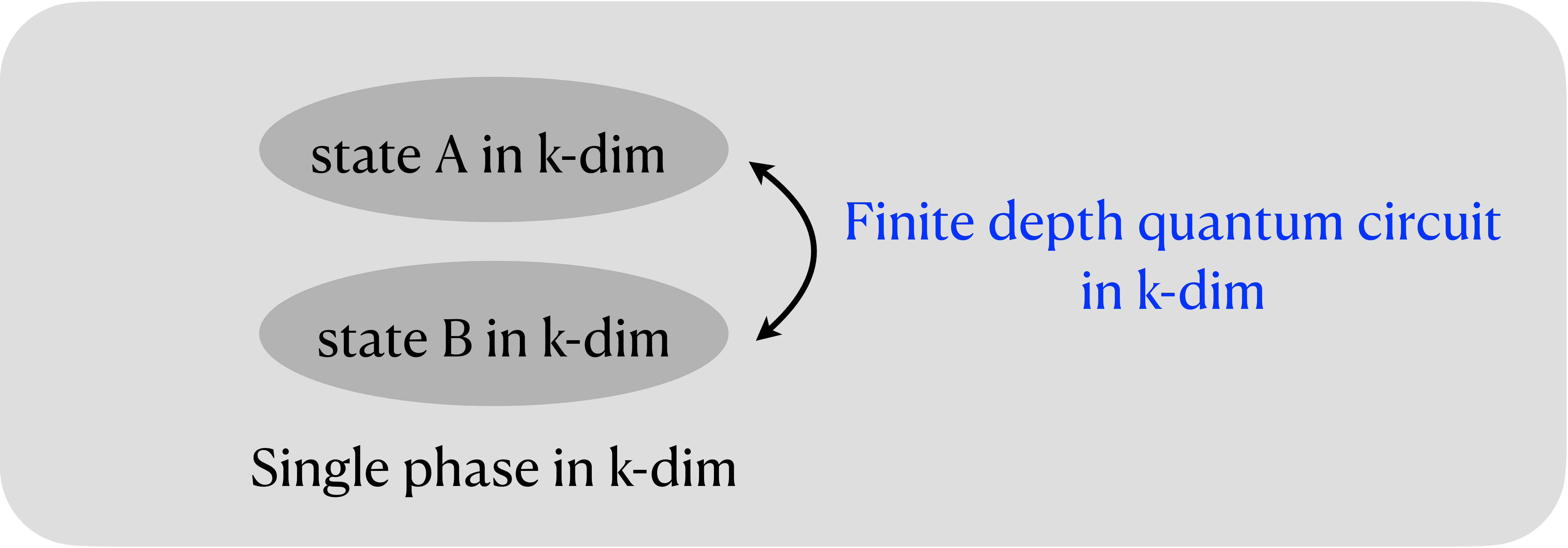
Low entanglement excitations

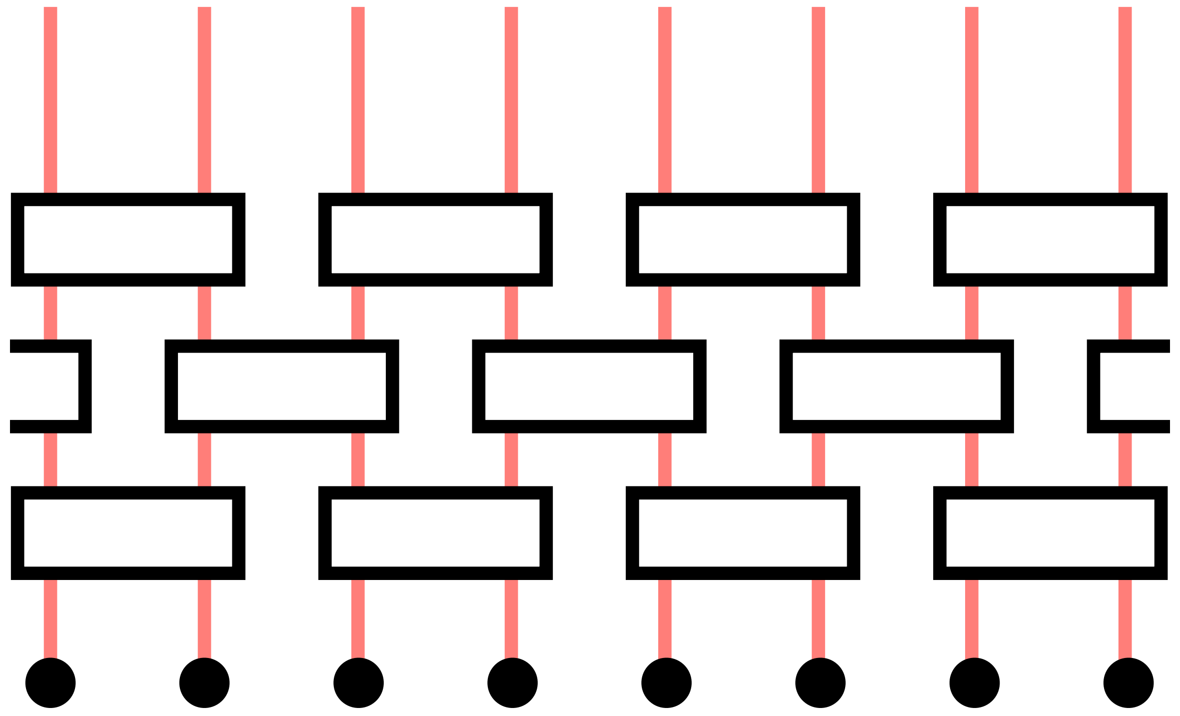
Use **quantum circuits** to create/define *k*-dimentional excitations

In the ground state of a gapped Hamiltonian in d dimensions, a *k*-**dim. excitation**

- Gapped ground state of the Hamitonian modified **only along the excitation**
- Two are **equivalent** if related by a **k-dim** (symmetric) **FDQC**.







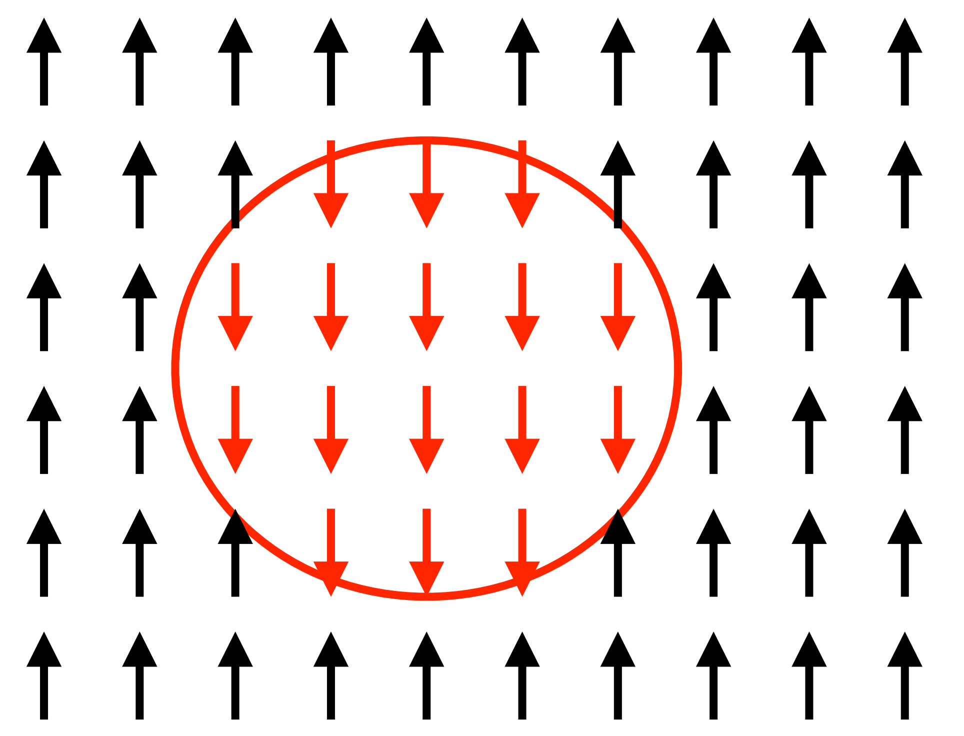
Chen-Gu-Wen

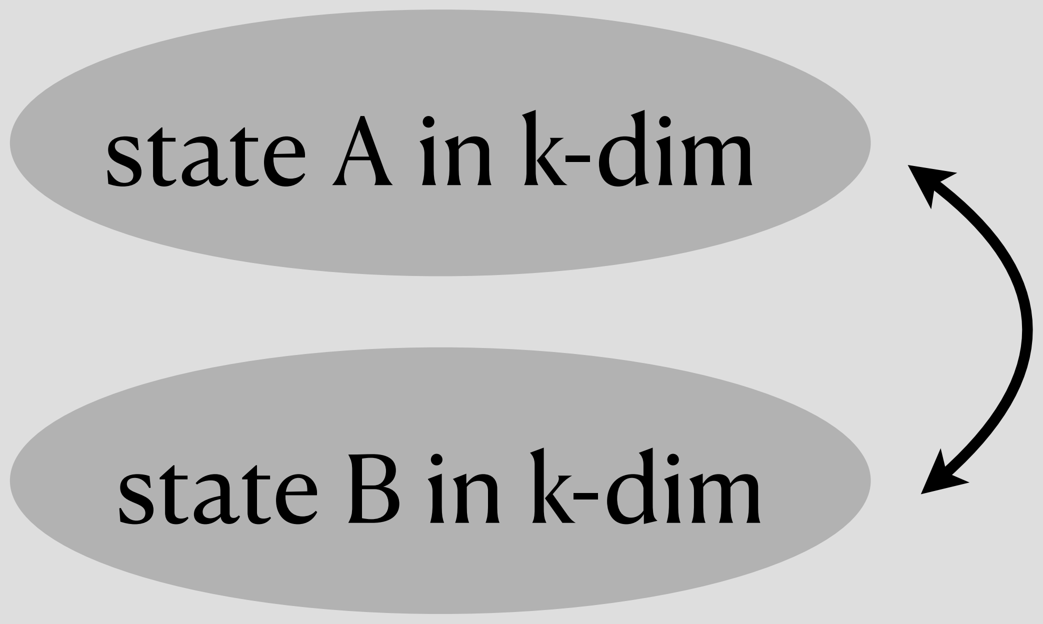
Low entanglement excitations

Use **quantum circuits** to create/define *k*-dimentional excitations

In the ground state of a gapped Hamiltonian in d dimensions, a *k*-**dim. excitation**

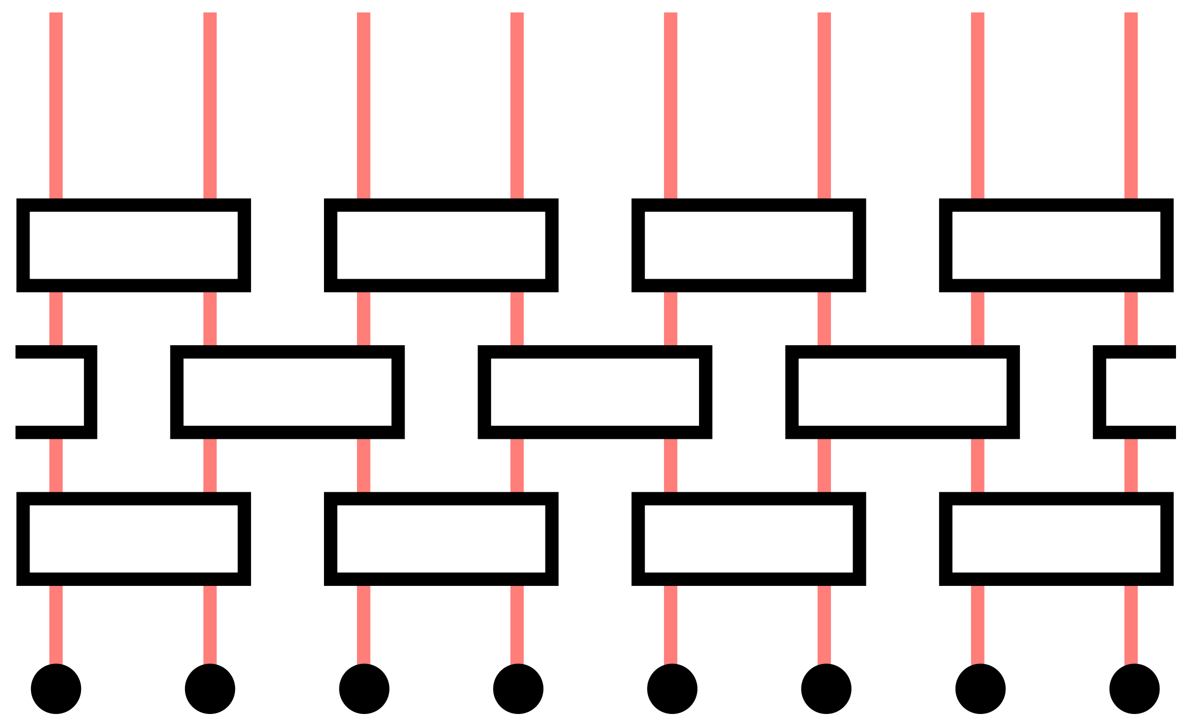
- Gapped ground state of the Hamitonian modified **only along the excitation**
- Two are **equivalent** if related by a **k-dim** (symmetric) **FDQC**.
- **Trivial types** are created from bulk ground state by a **k-dim** (symmetric) **FDQC**.





Finite depth quantum circuit in k-dim

Single phase in k-dim

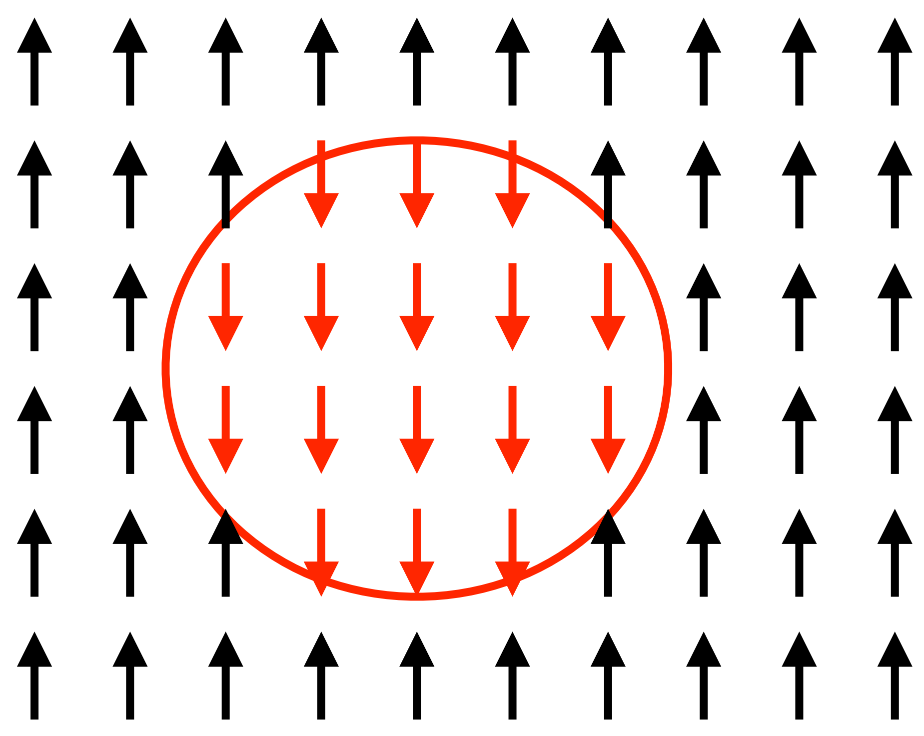


Chen-Gu-Wen

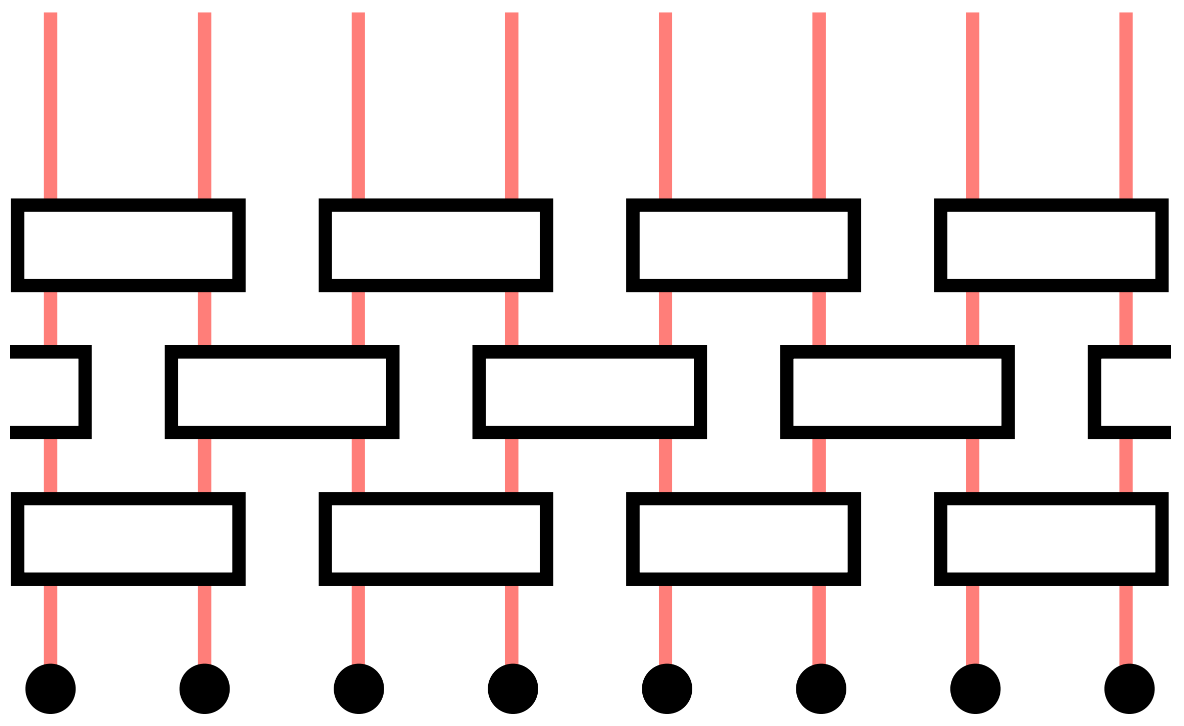
Low entanglement excitations

Use **quantum circuits** to create/define *k*-dimentional excitations

In the ground state of a gapped Hamiltonian in d dimensions, a *k*-**dim. excitation**



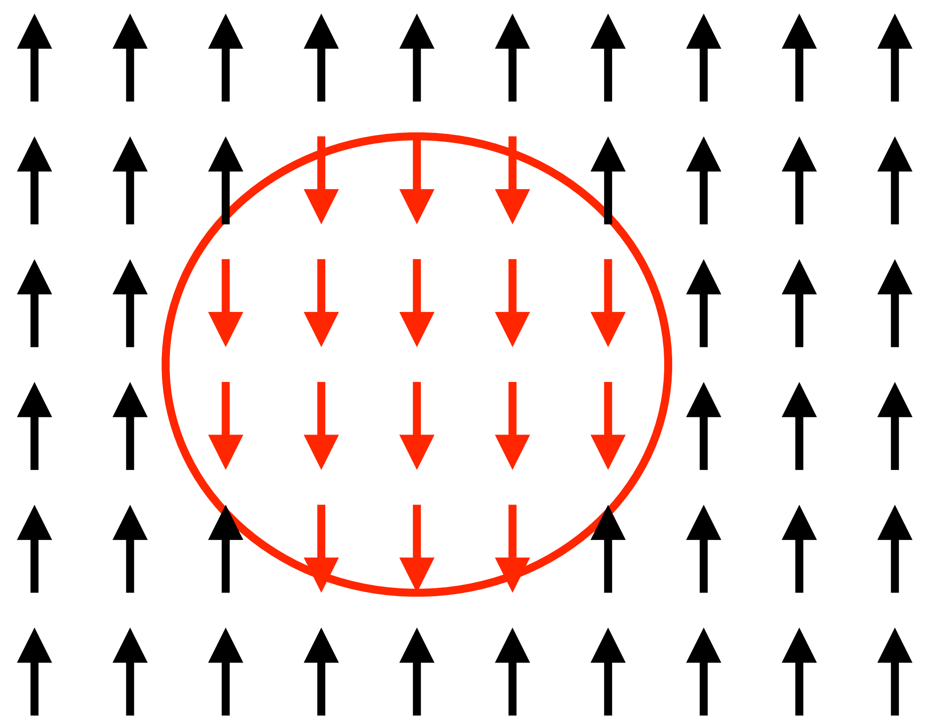
- Gapped ground state of the Hamitonian modified **only along the excitation**
- Two are **equivalent** if related by a **k-dim** (symmetric) **FDQC**.
- **Trivial types** are created from bulk ground state by a **k-dim** (symmetric) **FDQC**.
- **Non-trivial ones?**



Low entanglement excitations

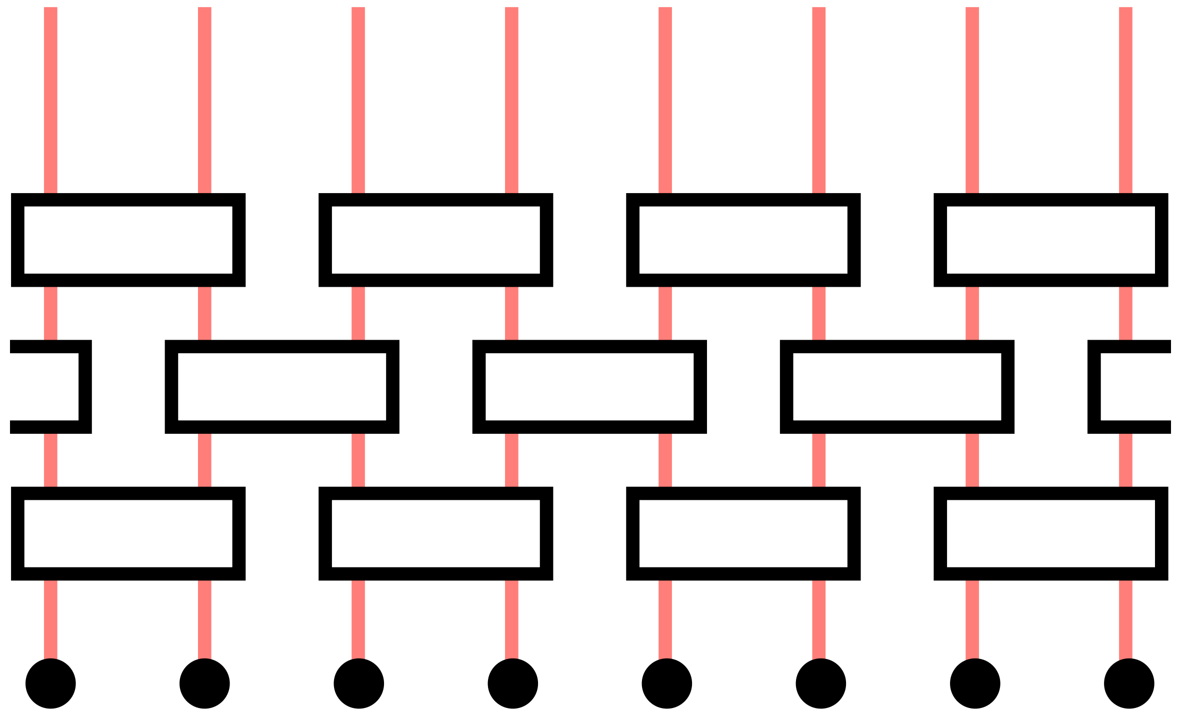
Use **quantum circuits** to create/define *k*-dimentional excitations

In the ground state of a gapped Hamiltonian in d dimensions, a ***k*-dim. excitation**

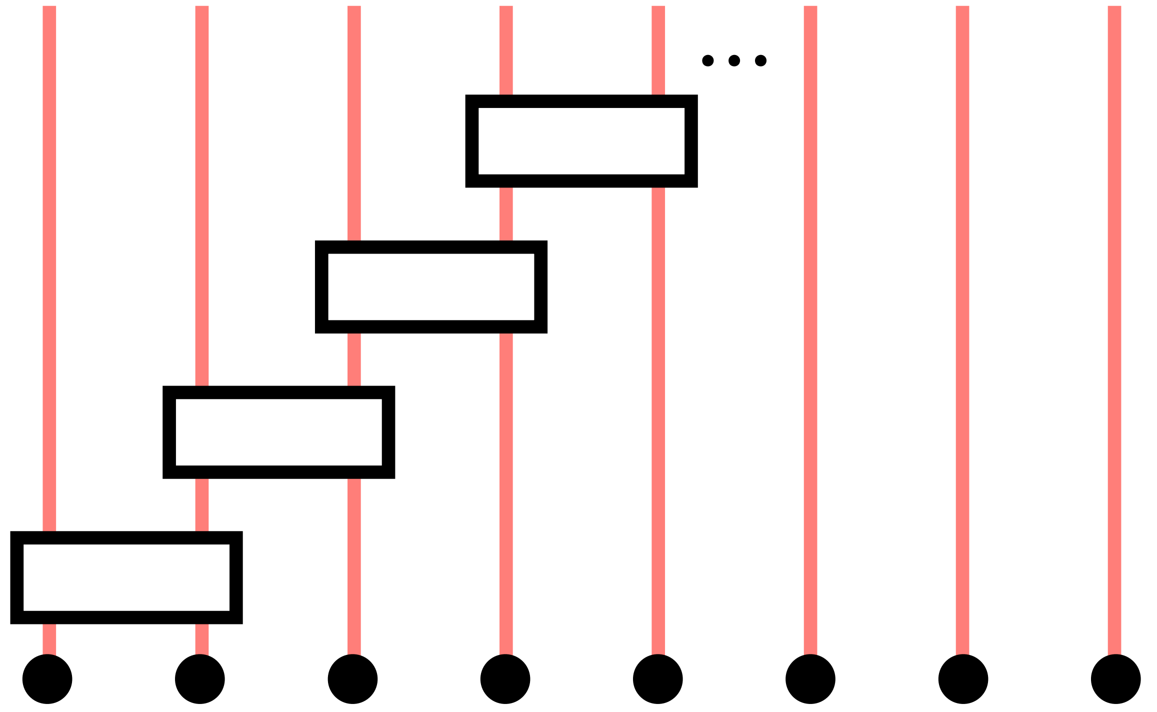


- Gapped ground state of the Hamitonian modified **only along the excitation**
- Two are **equivalent** if related by a **k-dim** (symmetric) **FDQC**.
- **Trivial types** are created from bulk ground state by a **k-dim** (symmetric) **FDQC**.
- **Non-trivial ones?** i. k+1 dim circuit ii. Linear depth circuit

finite depth quantum circuit



linear depth sequential circuit



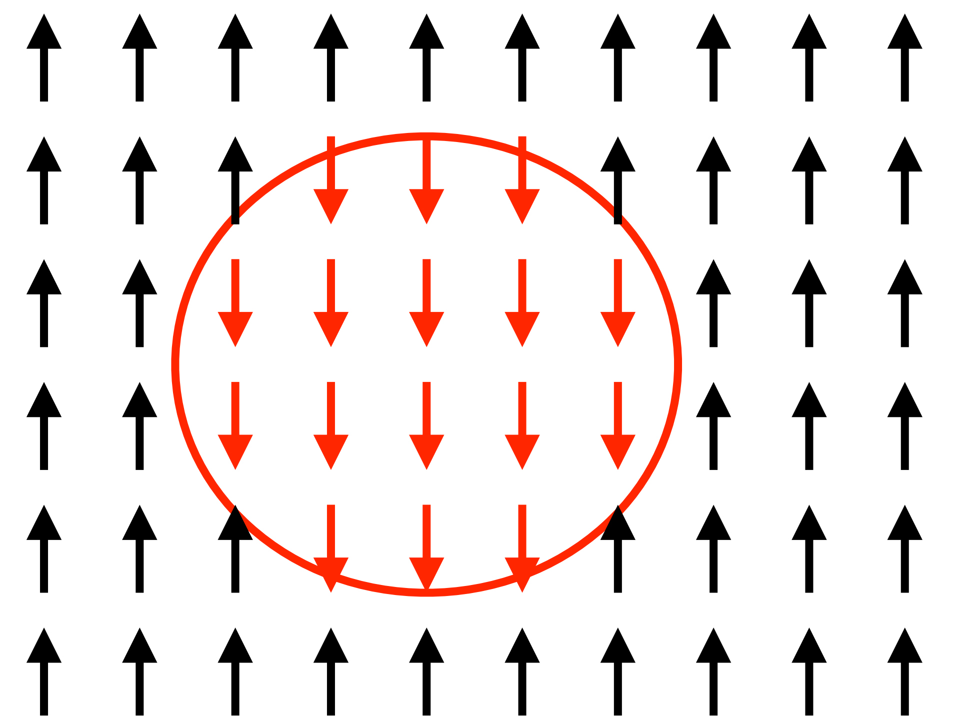
Schon-Solano-Verstraete-Cirac-Wolf, 2005; Ho-Hsieh, 2019

Chen-Dua-Hermele-Stephen-Tantivasadakarn-Vanhove-Zhao, 2023

Low entanglement excitations

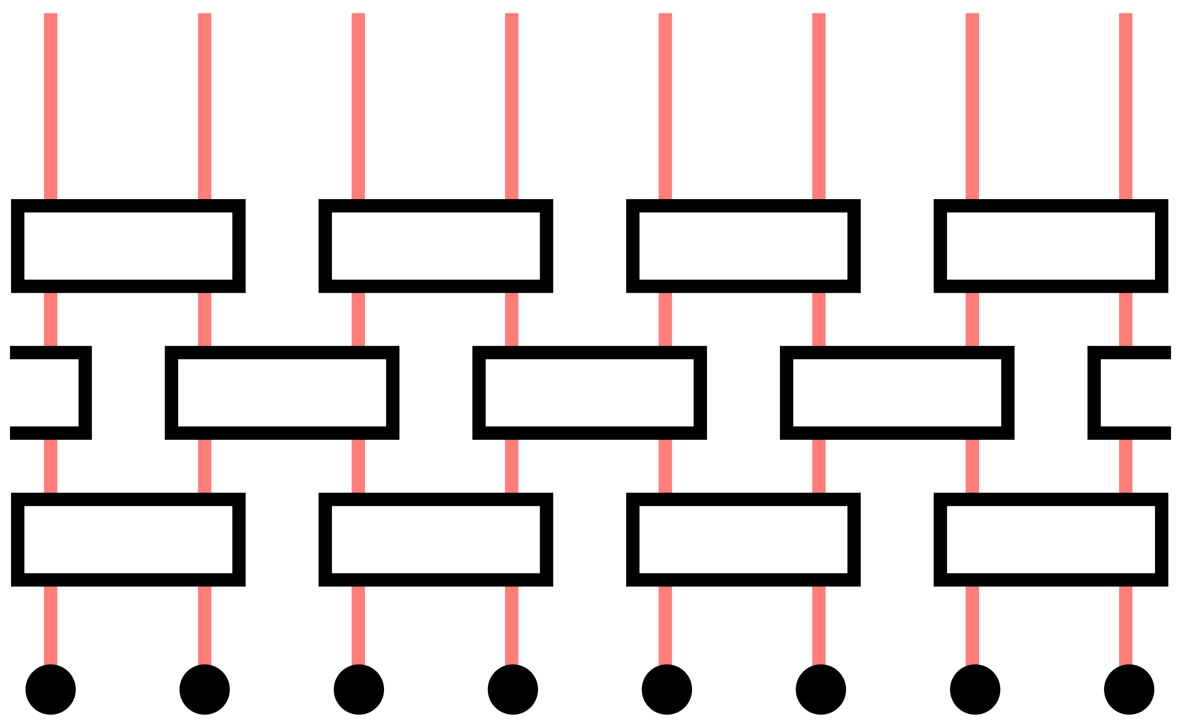
Use **quantum circuits** to create/define *k*-dimentional excitations

In the ground state of a gapped Hamiltonian in d dimensions, a ***k*-dim. excitation**

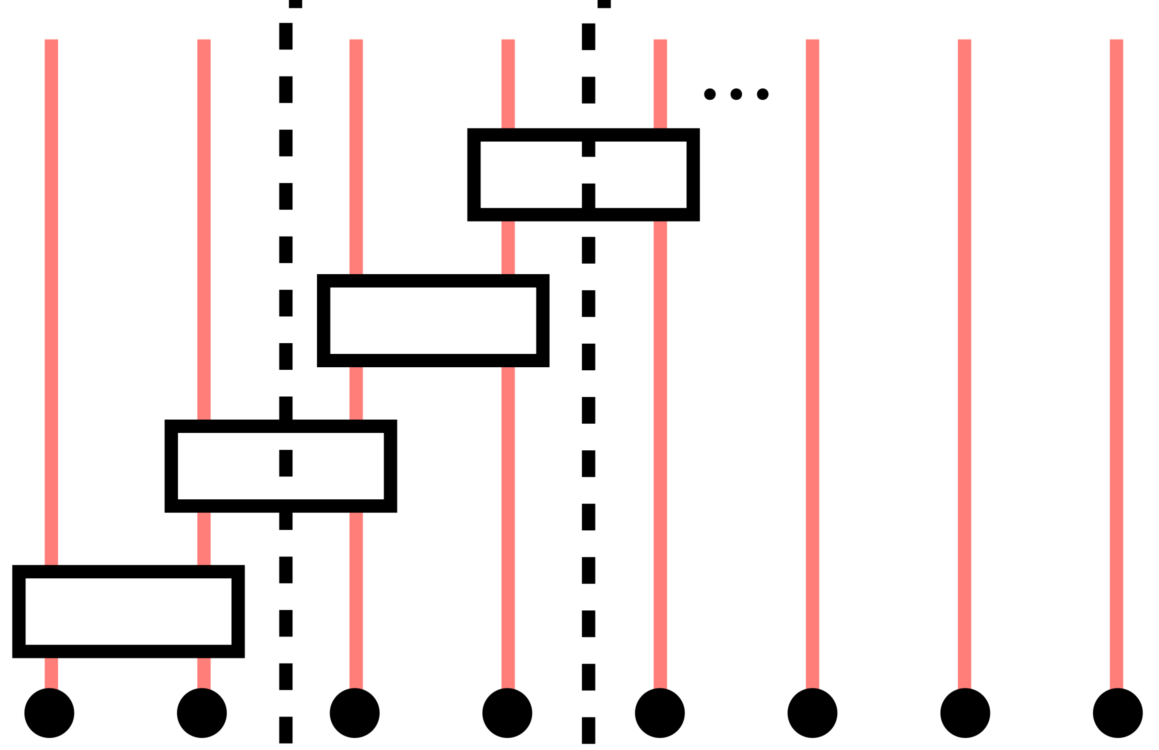


- Gapped ground state of the Hamitonian modified **only along the excitation**
- Two are **equivalent** if related by a **k-dim** (symmetric) **FDQC**.
- **Trivial types** are created from bulk ground state by a **k-dim** (symmetric) **FDQC**.
- **Non-trivial ones?** i. k+1 dim circuit ii. Linear depth circuit

finite depth quantum circuit



linear depth sequential circuit



Schon-Solano-Verstraete-Cirac-Wolf, 2005; Ho-Hsieh, 2019

Chen-Dua-Hermele-Stephen-Tantivasadakarn-Vanhove-Zhao, 2023

Low entanglement excitations

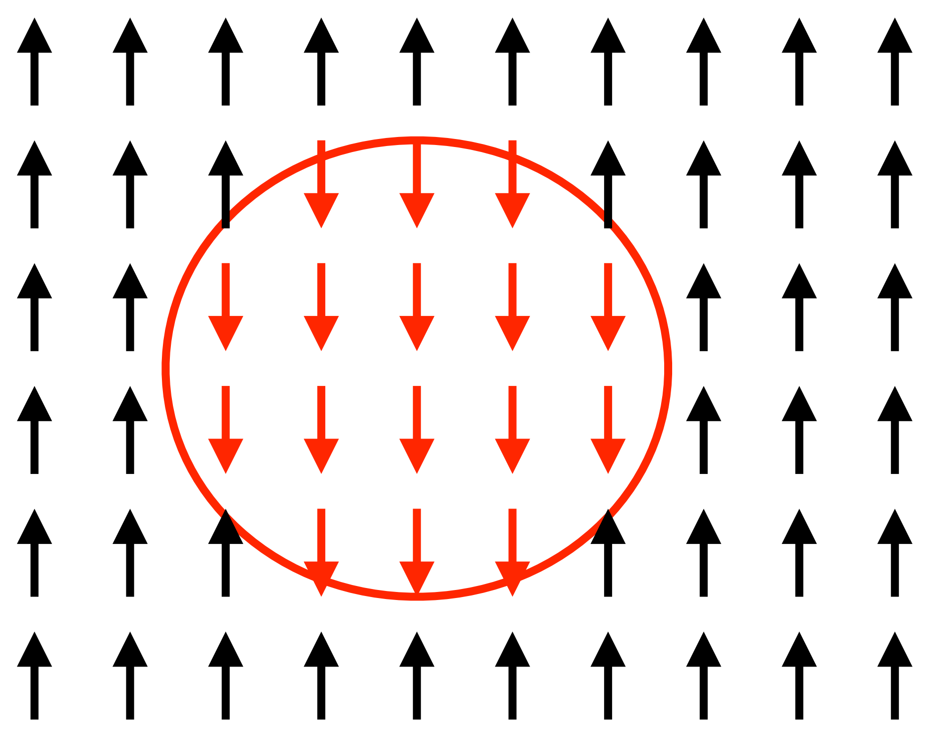
Use **quantum circuits** to create/define *k*-dimentional excitations

In the ground state of a gapped Hamiltonian in d dimensions, a ***k*-dim. excitation**

- Gapped ground state of the Hamitonian modified **only along the excitation**
- Two are **equivalent** if related by a **k-dim** (symmetric) **FDQC**.
- **Trivial types** are created from bulk ground state by a **k-dim** (symmetric) **FDQC**.
- **Non-trivial types** cannot be created by a k-dim (symmetric) FDQC;

  can be created with a **(k+1)-dim** (symmetric) **quantum circuit**,

  or a **k-dim** (symmetric) **linear depth squential circuit**.

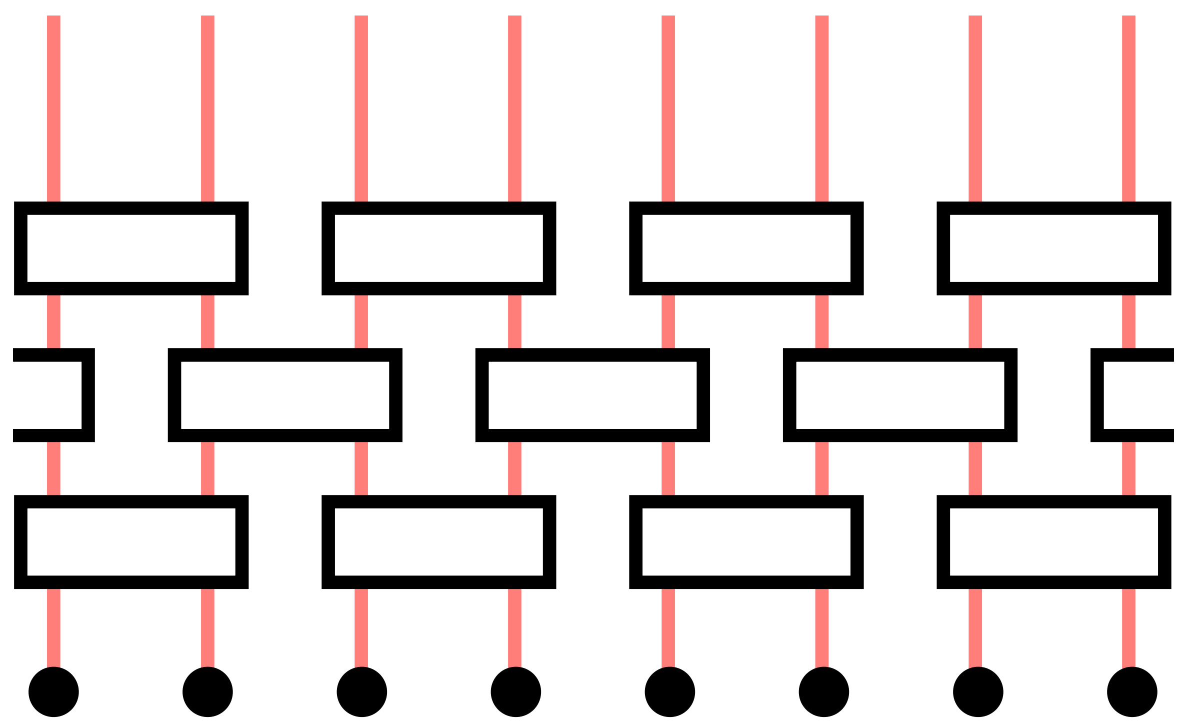


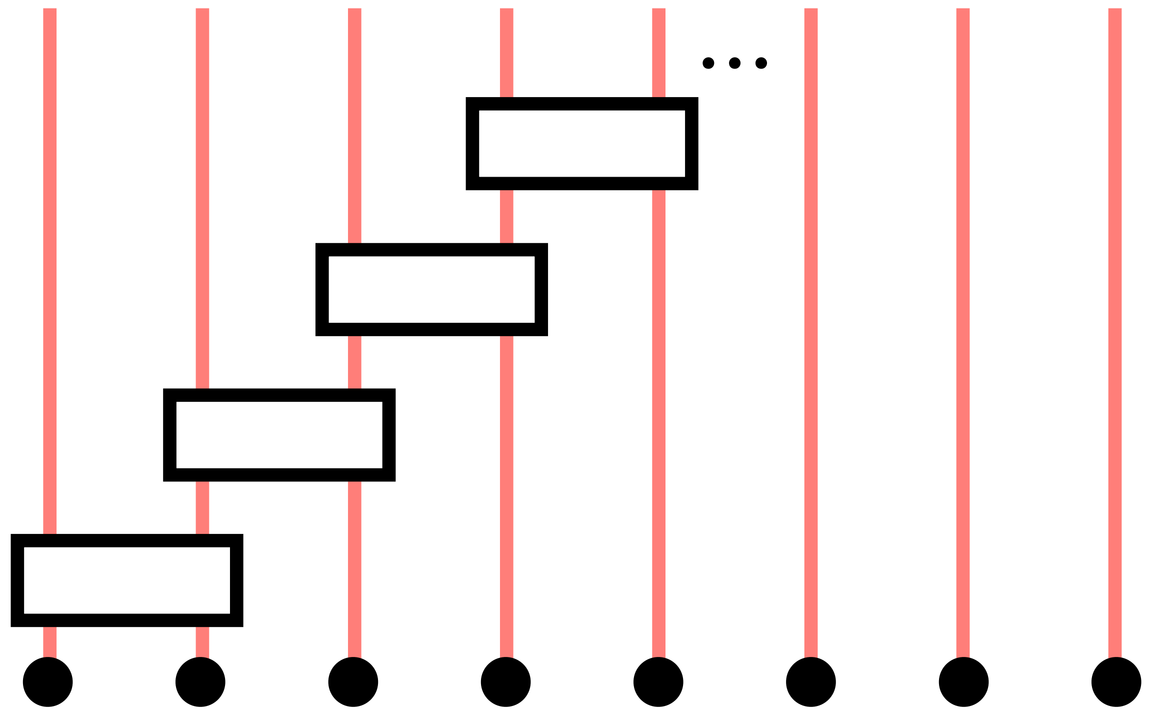
1d excitations

Hamiltonian modified

along a loop

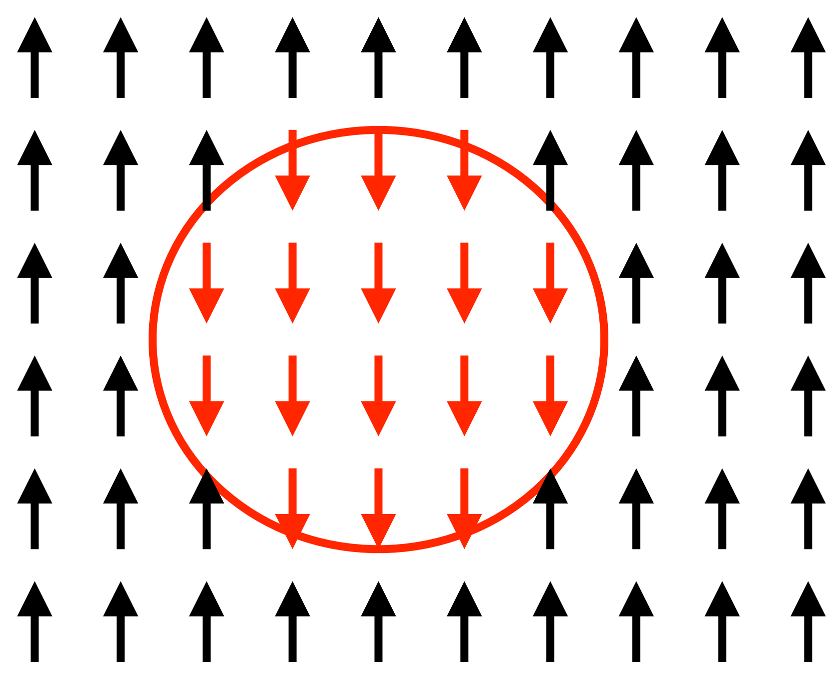
2d FDLU



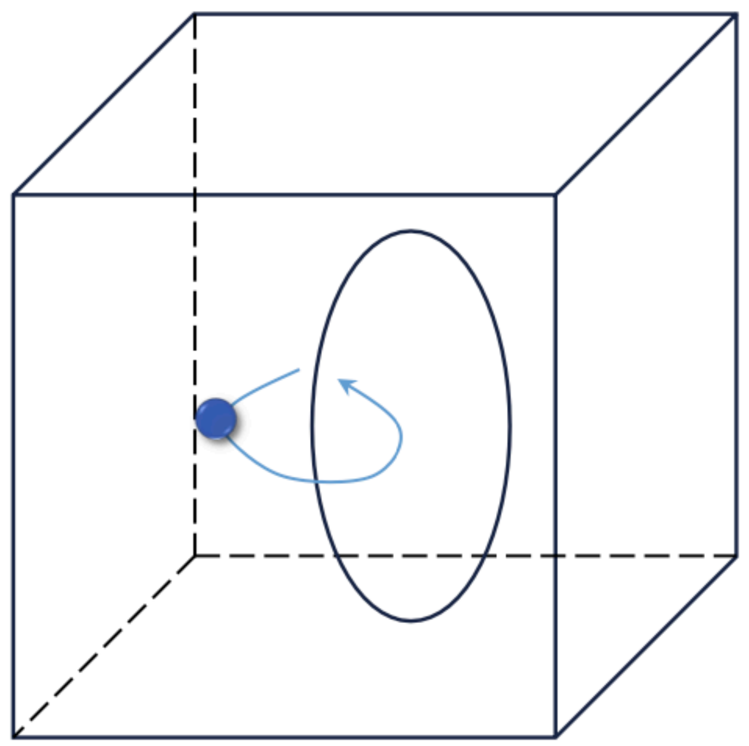


Low entanglement excitations

Line excitations created by 2d FDQC



1d excitations
Hamiltonian modified
along a loop
2d FDLU

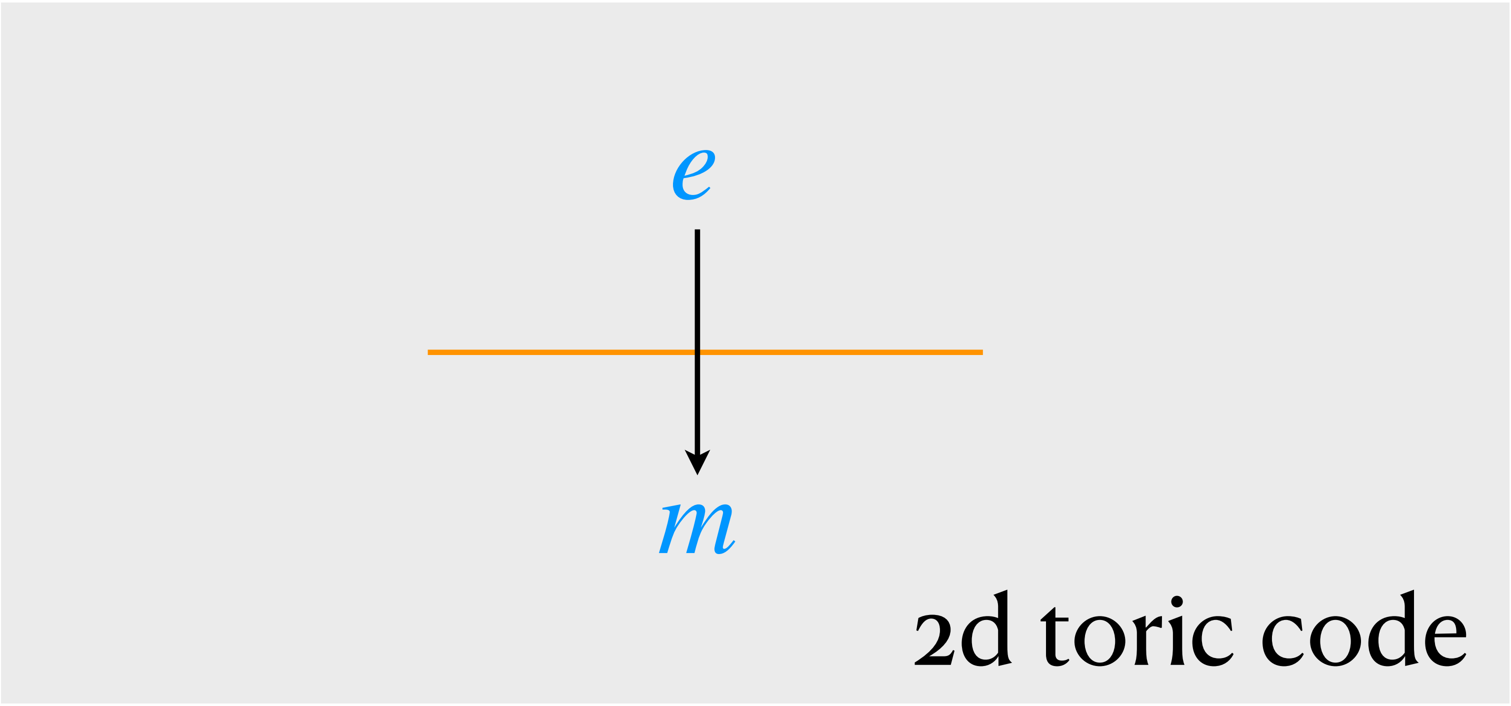


1d excitations
Hamiltonian modified
along a loop
2d FDLU

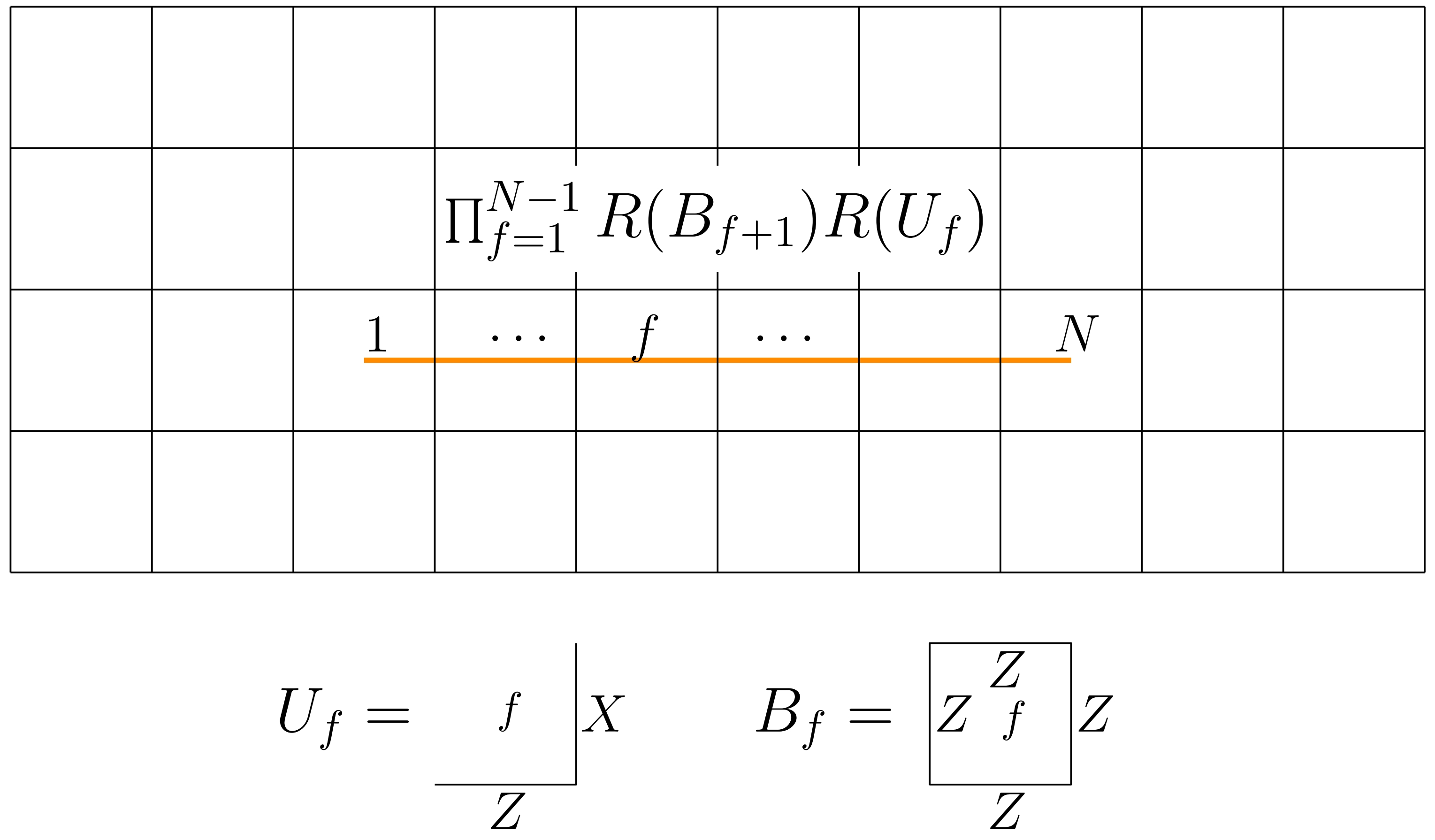
Low entanglement excitations

Line defect exchanging anyons

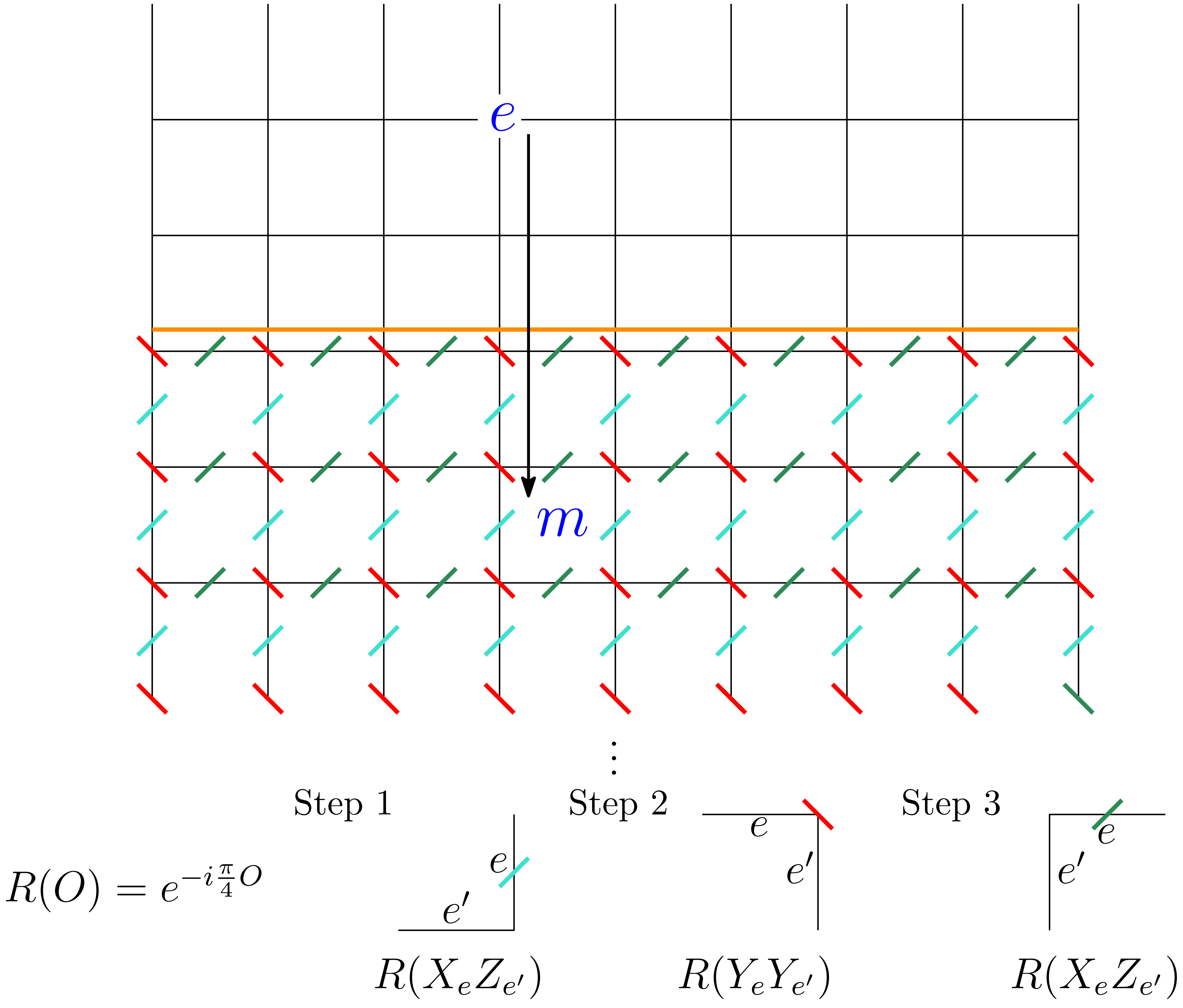
e-m exchange defect in toric code



1D linear depth sequential circuit



2D FDLU ("pump" unitary)



Tantivasadakarn-Chen, 2023 Barkarshli et al, 2023

Low entanglement excitations

Line defect in a trivial product state

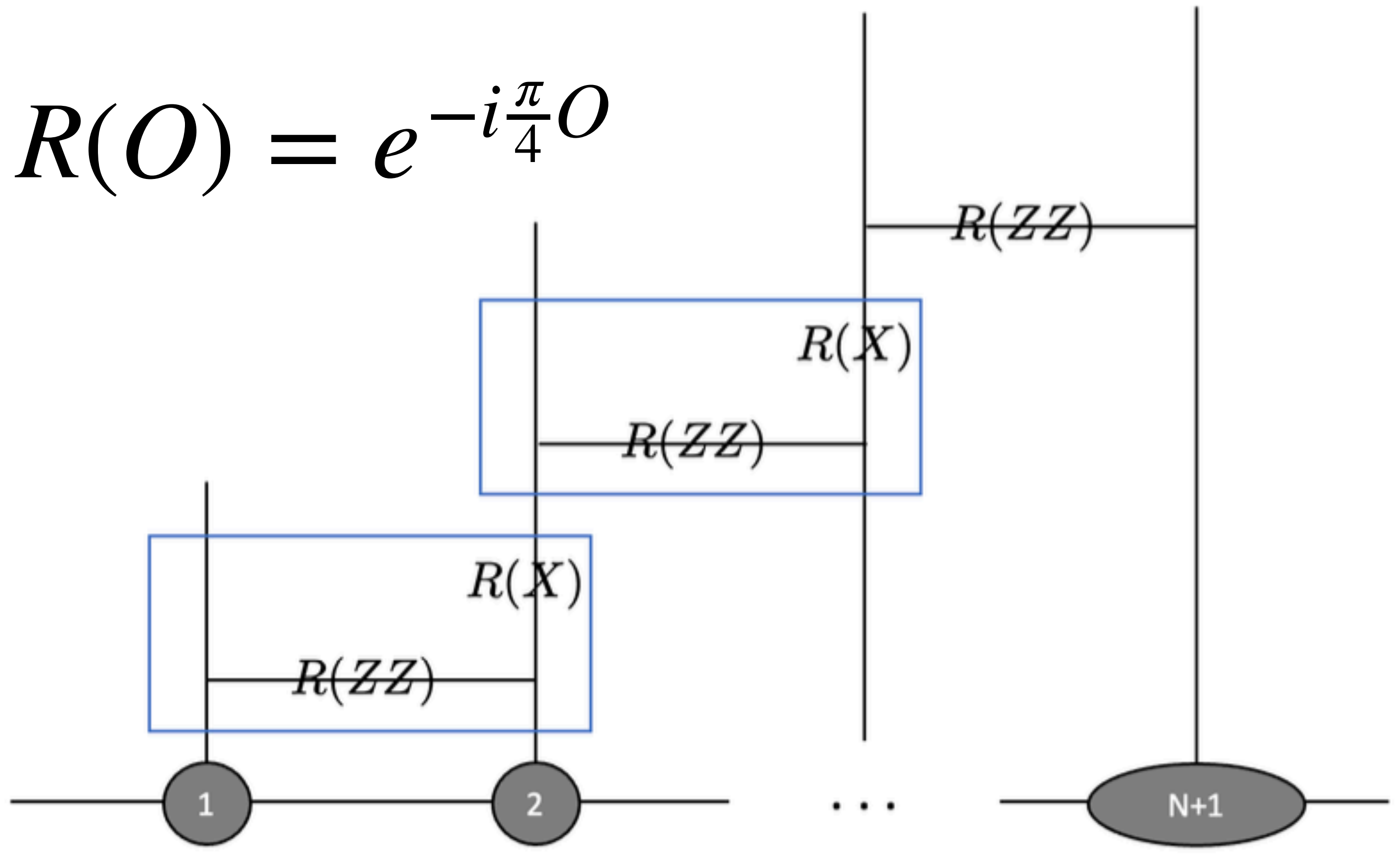
1d GHZ state (SSB)

$$|\cdots 000 \cdots\rangle + |\cdots 111 \cdots\rangle$$

2d symmetric state $|+\rangle^{\otimes N}$

1D sequential circuit

$$R(O) = e^{-i\frac{\pi}{4}O}$$



$$X_i \to Z_{i-1}Z_i$$

$$Z_iZ_{i+1} \to X_i$$

Schon-Solano-Verstraete-Cirac-Wolf, 2005; Ho-Hsieh, 2019

Chen-Dua-Hermele-Stephen-Tantivasadakarn-Vanhove-Zhao, 2023

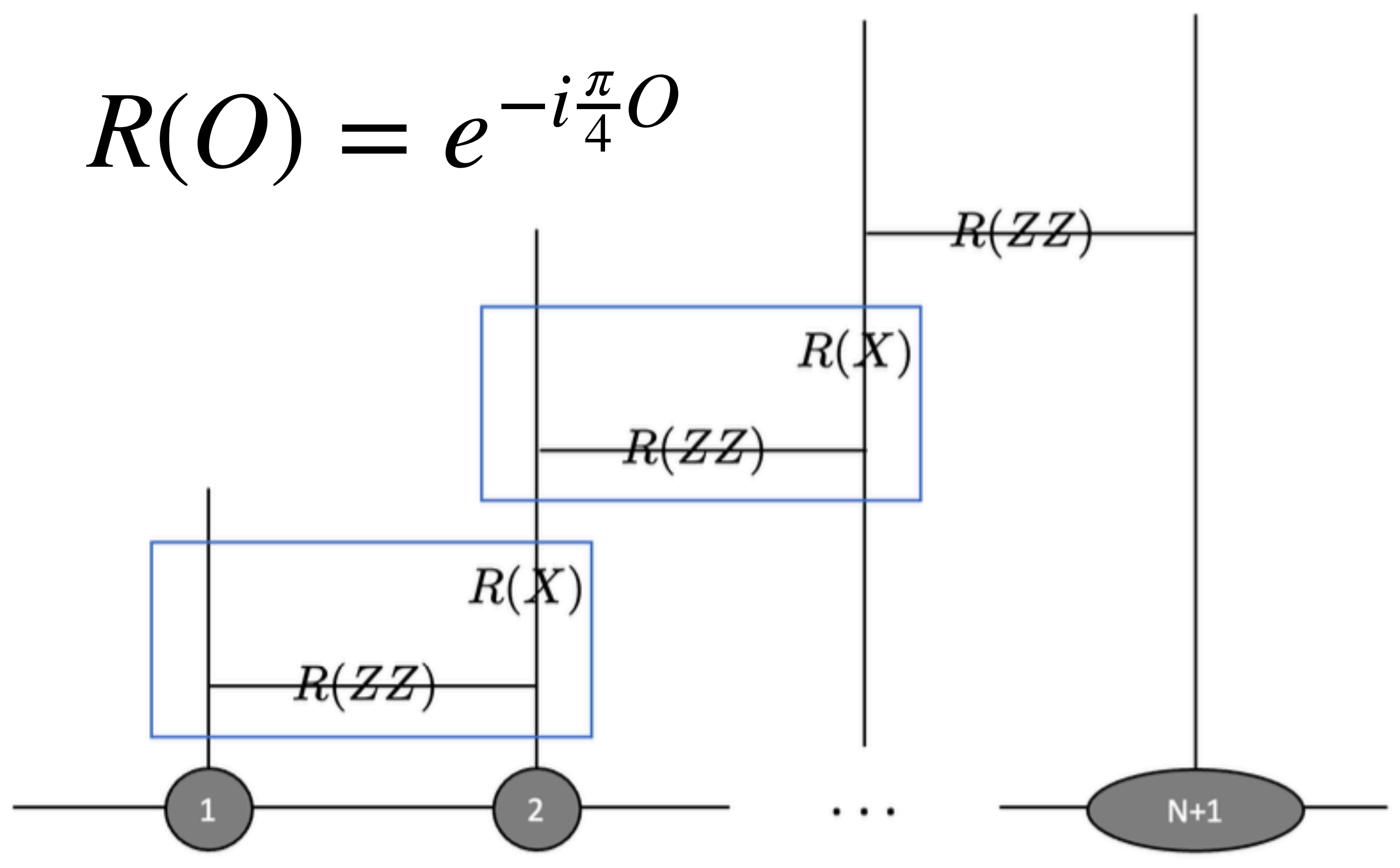
Classifying Low entanglement excitations?

Low entanglement excitations

1d GHZ state (SSB)

2d symmetric state

$$R(O) = e^{-i\frac{\pi}{4}O}$$



All excitations in a 2d trivial product state

Ground state		2d $\mathbb{Z}_2$ paramagnet
Excitations	0d	$\mathbb{Z}_2$ charge (1d FDLU)
	1d	SSB state (1d SQC)

What about an entangled ground state?

LEE in an Invertible phase

Claim: Low entanglement excitations in an invertible phase and those in a product state have a one-to-one correspondence.

LEE in an Invertible phase

Claim: Low entanglement excitations in an invertible phase and those in a product state have a one-to-one correspondence.

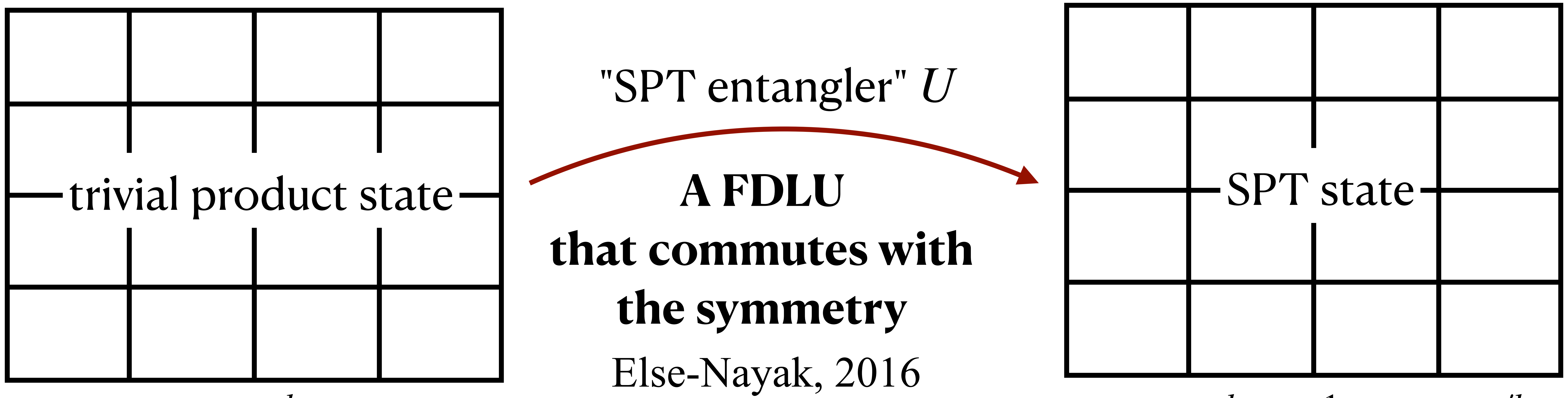
Ground state		**2d $\mathbb{Z}_2$ paramagnet**	**2d $\mathbb{Z}_2$ SPT**
Excitations	0d	$\mathbb{Z}_2$ charge 1d FDLU	$\mathbb{Z}_2$ charge 1d FDLU
	1d	1d $\mathbb{Z}_2$ SSB 1d SQC	1d $\mathbb{Z}_2$ SSB 1d SQC

Bulk excitations in an Invertible phase

Claim: Low entanglement excitations in an invertible phase and those in a product state have a one-to-one correspondence.

For many examples, already easy to show.

Example SPT phases



$O(\Omega^k)$ $\quad$ $UO(\Omega^k)U^{-1} = O'(\Omega'^k)$

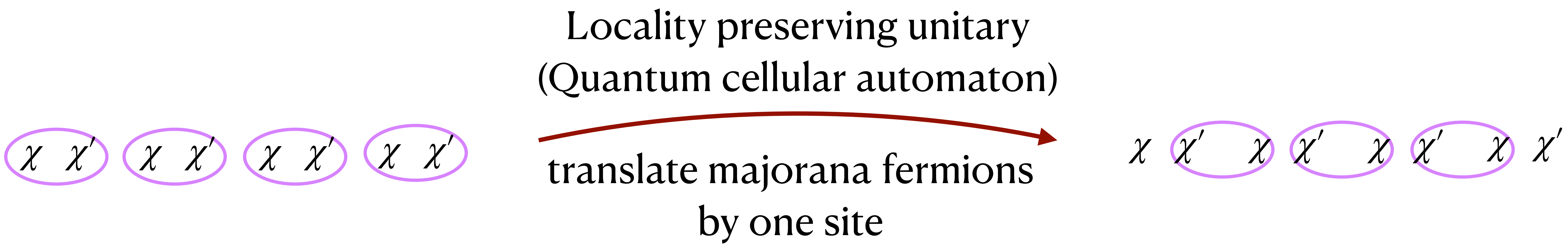
- Symmetric operators have one-to-one correspondence, including LEE

Bulk excitations in an Invertible phase

Claim: Low entanglement excitations in an invertible phase and those in a product state have a one-to-one correspondence.

For many examples, already easy to show.

Example 1d Kitaev chain (fermion parity symmetry)



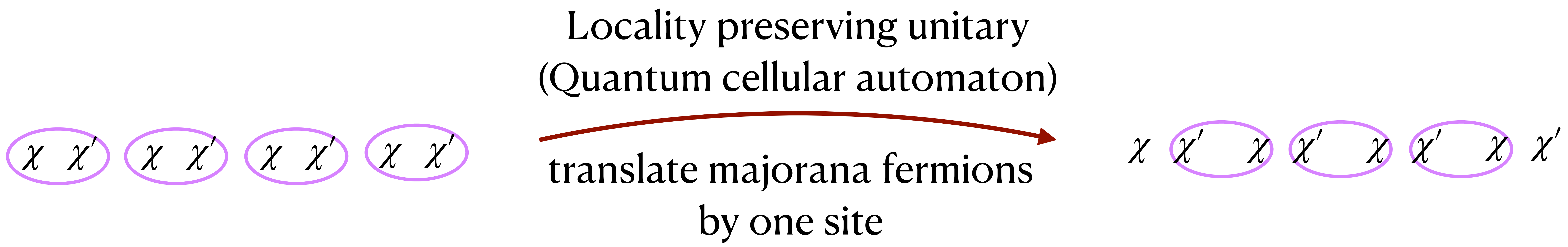
- Symmetric operators have one-to-one correspondence, including LEE

Bulk excitations in an Invertible phase

Claim: Low entanglement excitations in an invertible phase and those in a product state have a one-to-one correspondence.

For many examples, already easy to show.

Example 1d Kitaev chain (fermion parity symmetry)



- Symmetric operators have one-to-one correspondence, including LEE

SPT entangler approach does not apply to examples such as $p + ip$ **superconductor**

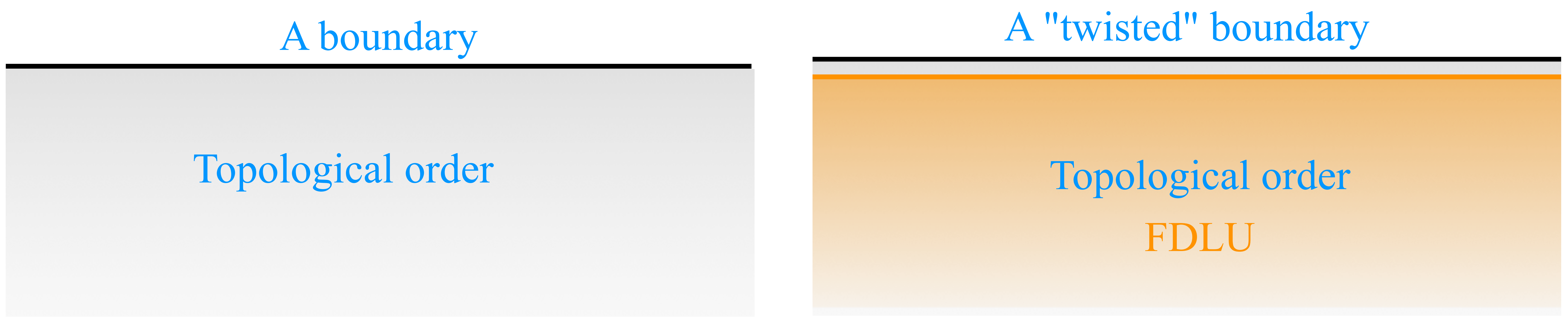
Bulk excitations in an Invertible phase

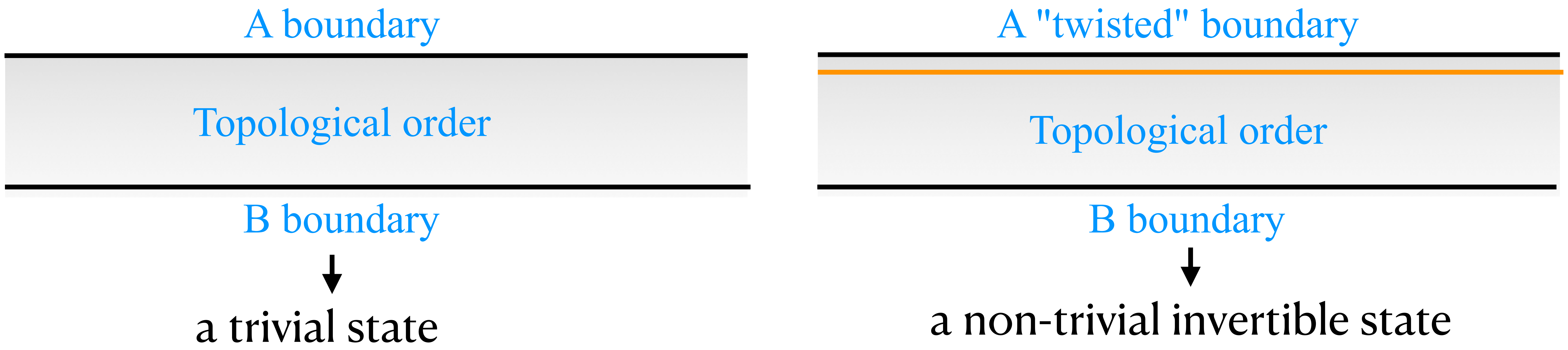
Claim: Low entanglement excitations in an invertible phase and those in a product state have a one-to-one correspondence.

Arguement using "Symmetry TO/TFT"

Arguement using "Symmetry TO/TFT"

Step 1: Compare Boundaries of topological orders

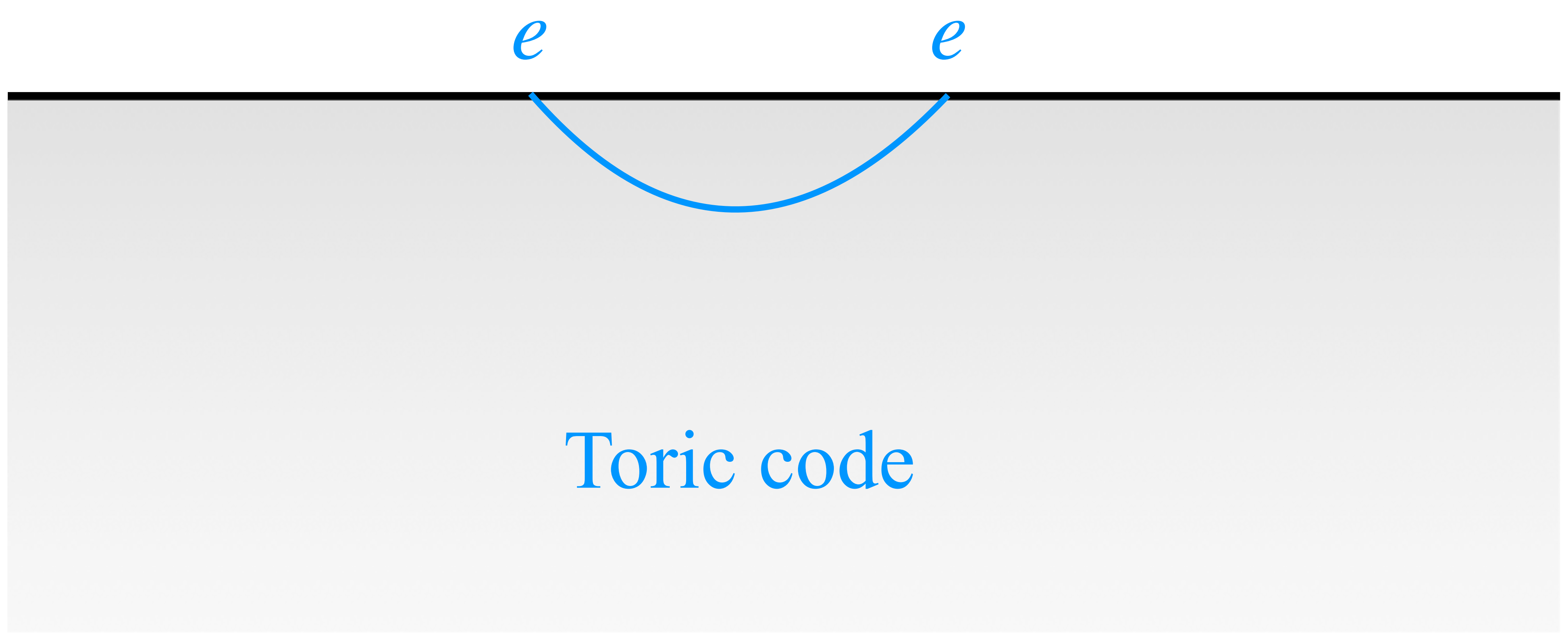


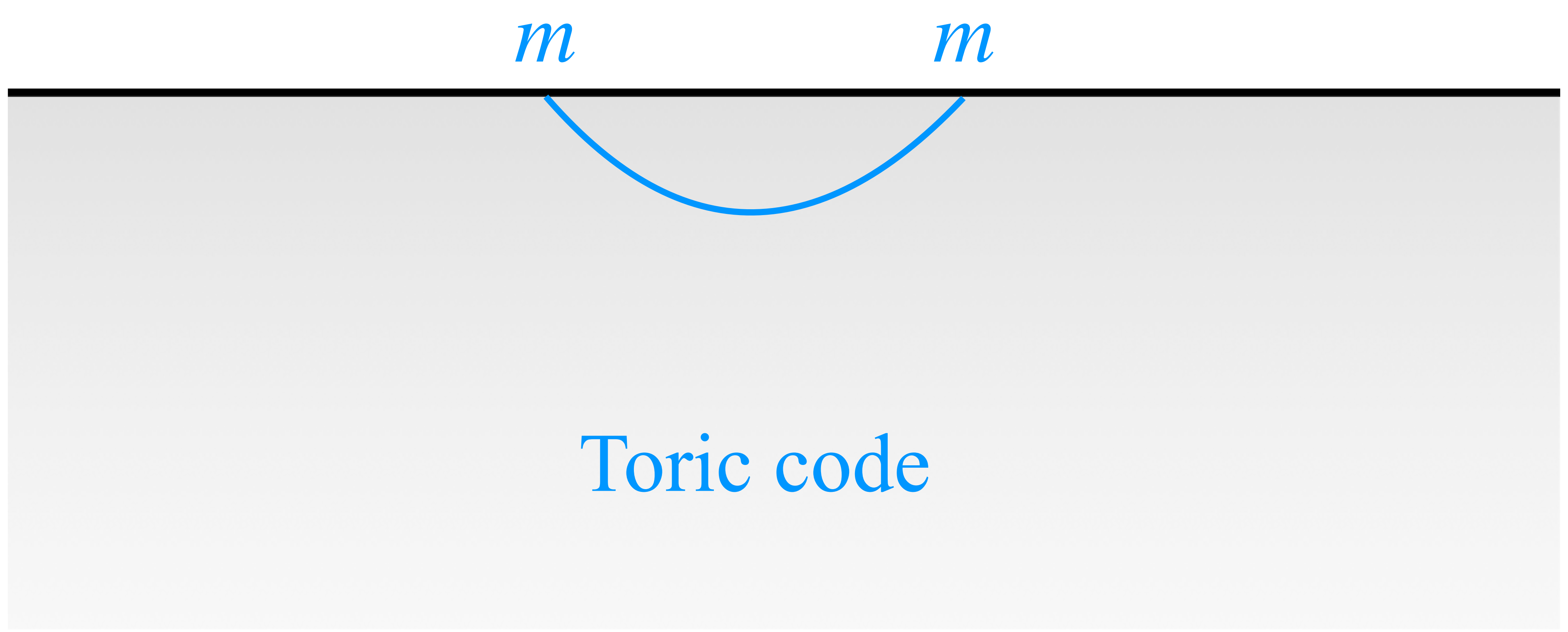


Step 3: Compare excitations

Topological order with a boundary

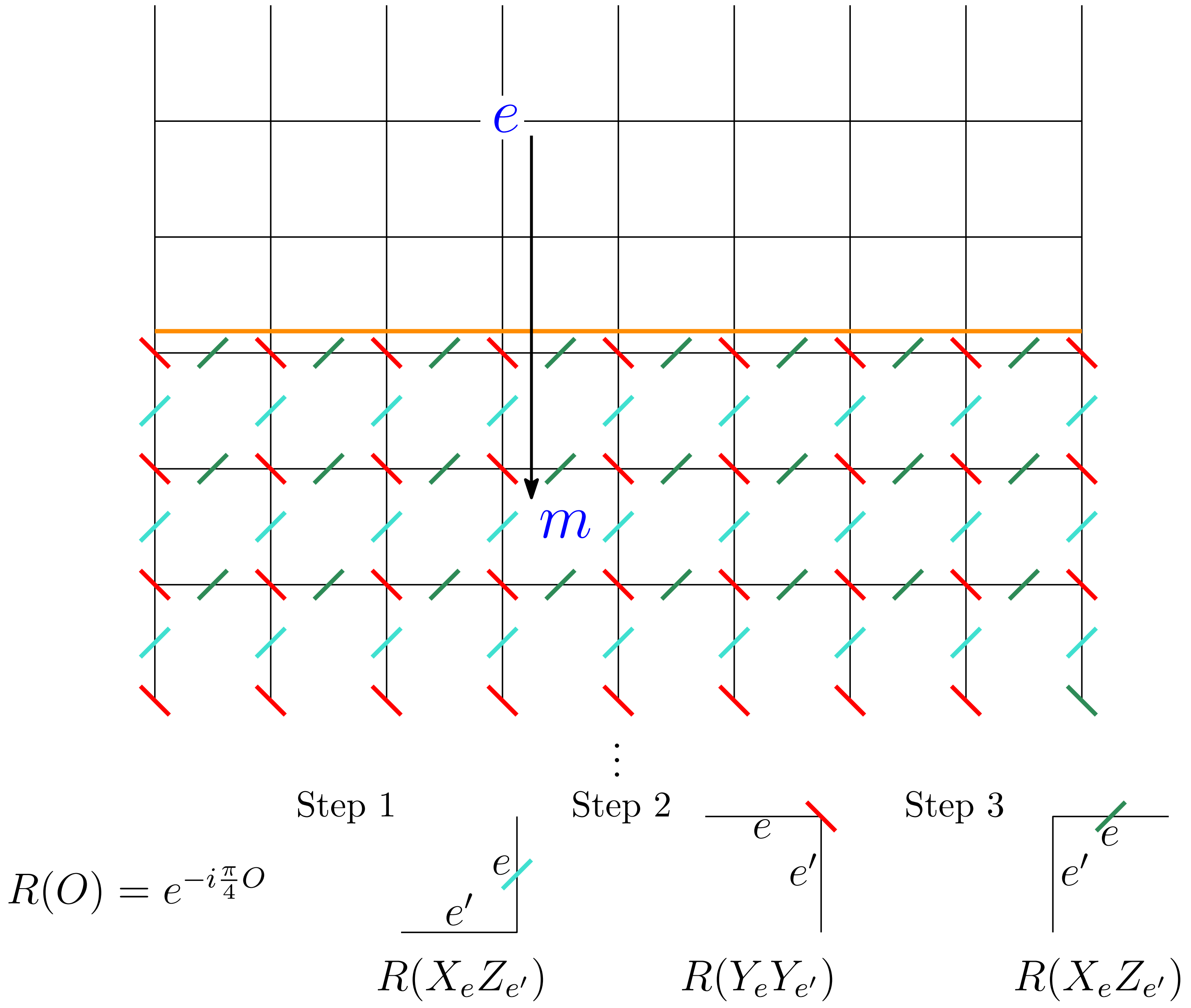
Topological order with a boundary





Emergent symmetry in toric code bulk:

anyon exchange symmetry $e \leftrightarrow m$

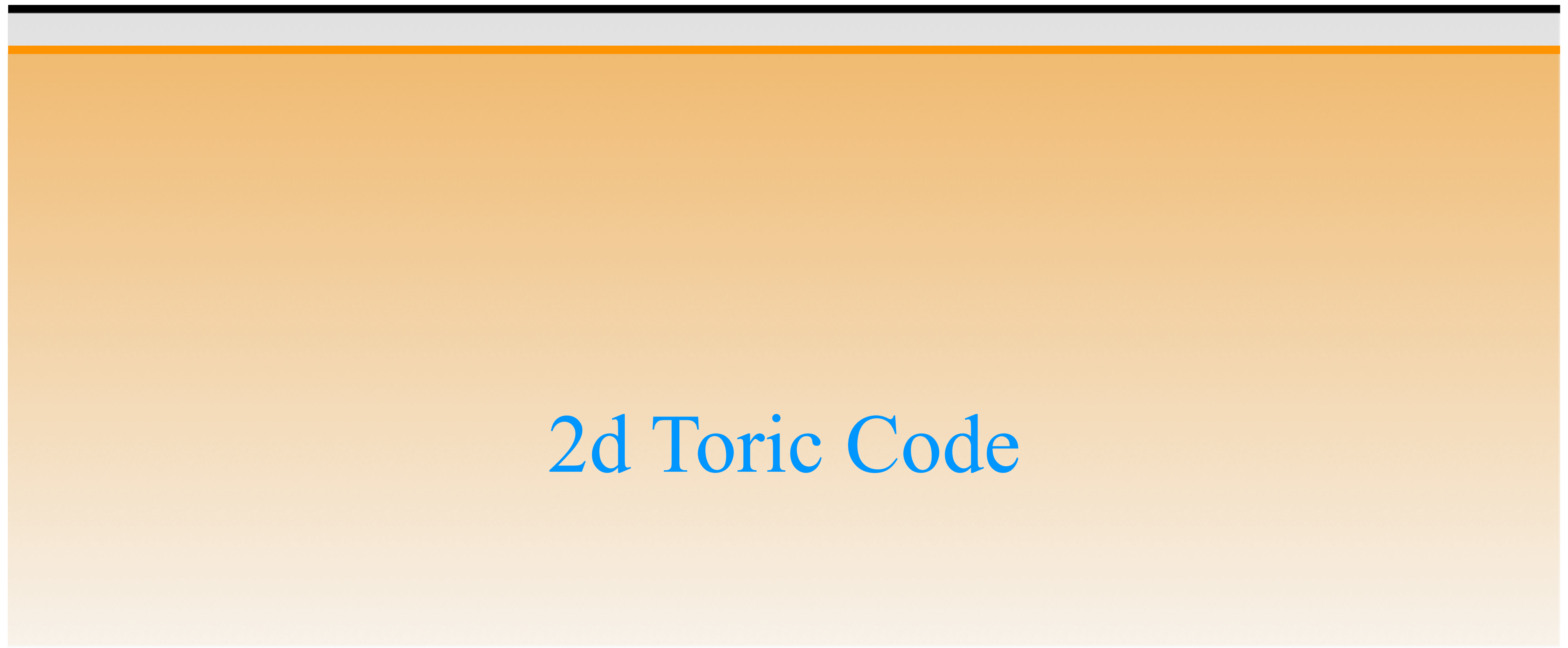


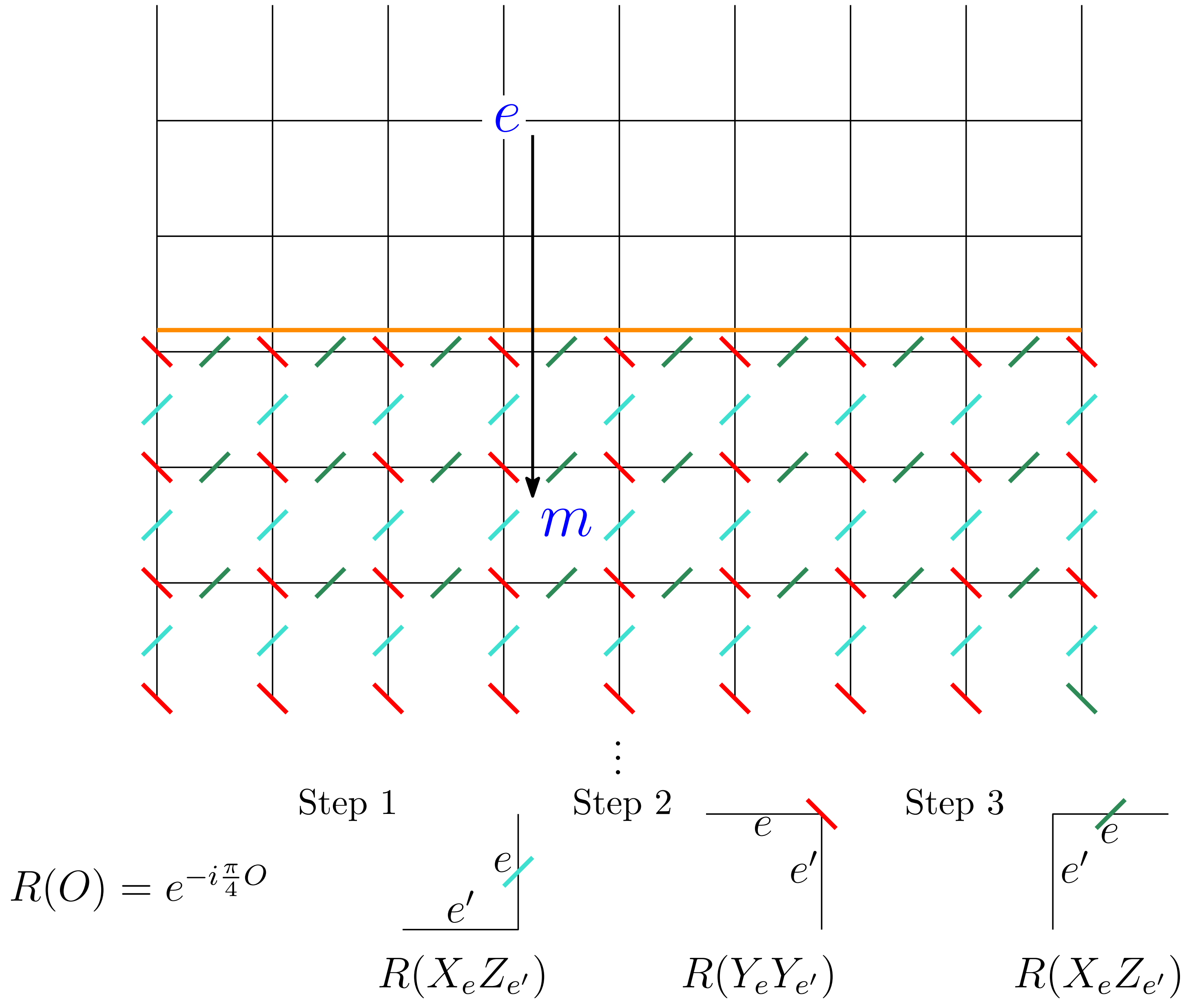
Barkeshili et al, 2022

Topological order with a boundary

e condensed

m condensed





U: Sweeping the e-m exchange defect
2d FDQC

U generates an $\mathbb{Z}_2$ emergent symmetry
only modify the boundary terms non-trivially

Barkeshili et al, 2022

Topological order with a boundary

No experiments near the boundary can pin down the boundary type.

e condensed

$O(\Omega^k)$

Toric Code

m condensed

$UO(\Omega^k)U^{-1} = O'(\Omega'^k)$

Toric Code

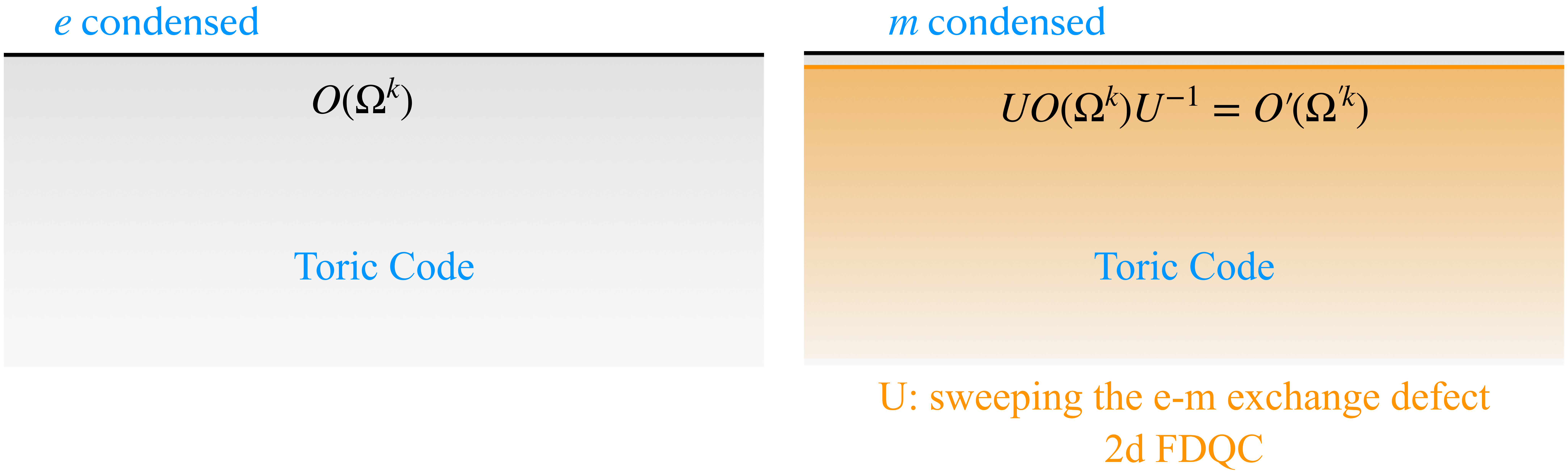
U: sweeping the e-m exchange defect

2d FDQC

Topological order with a boundary

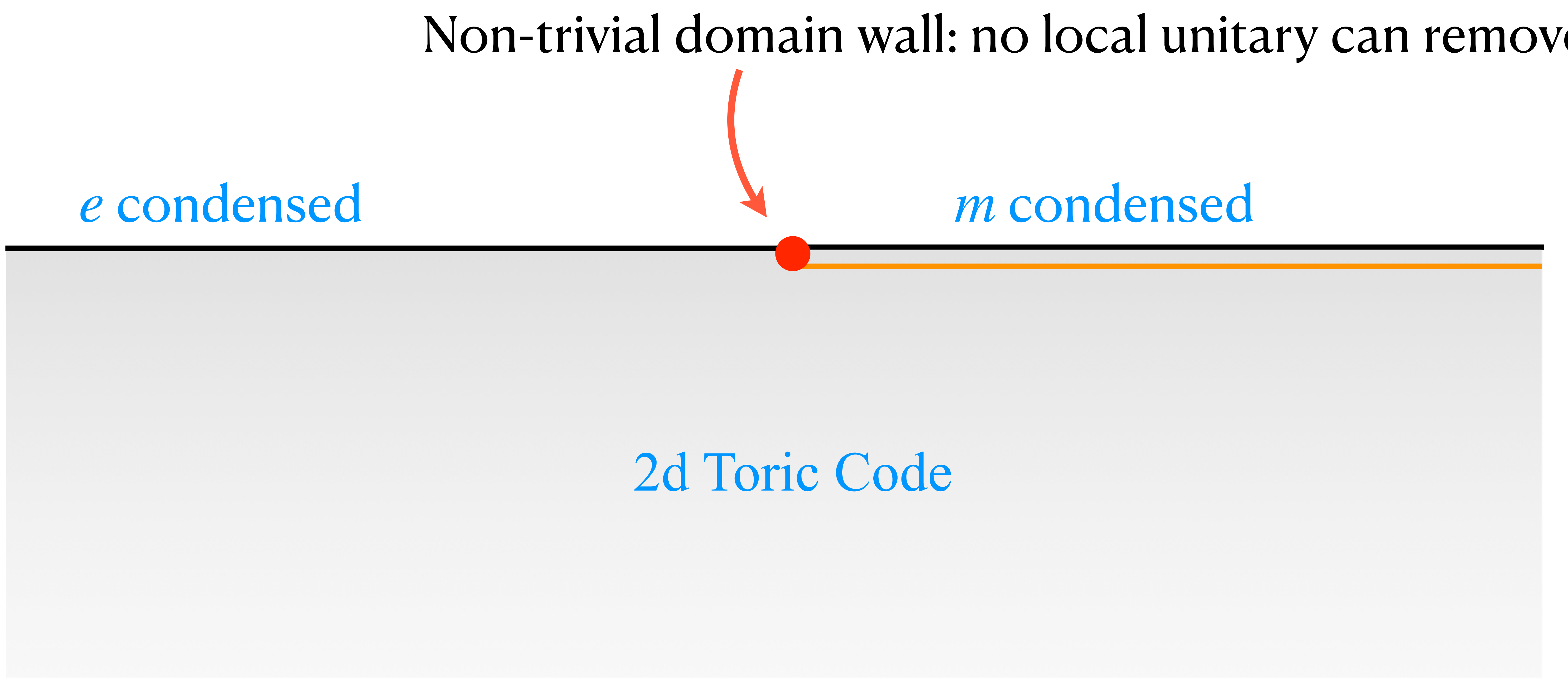
No absolute distinction

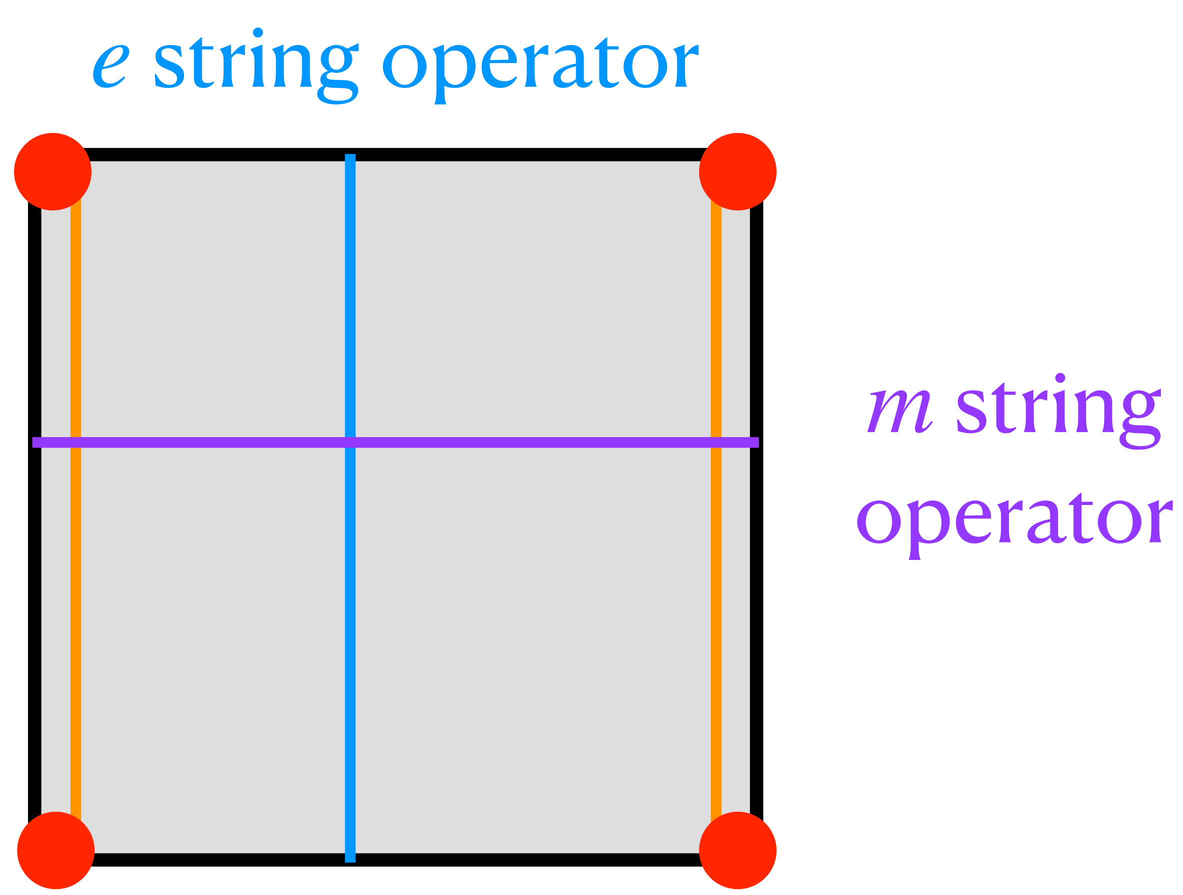
No experiments near the boundary can pin down the boundary type.



Topological order with a boundary

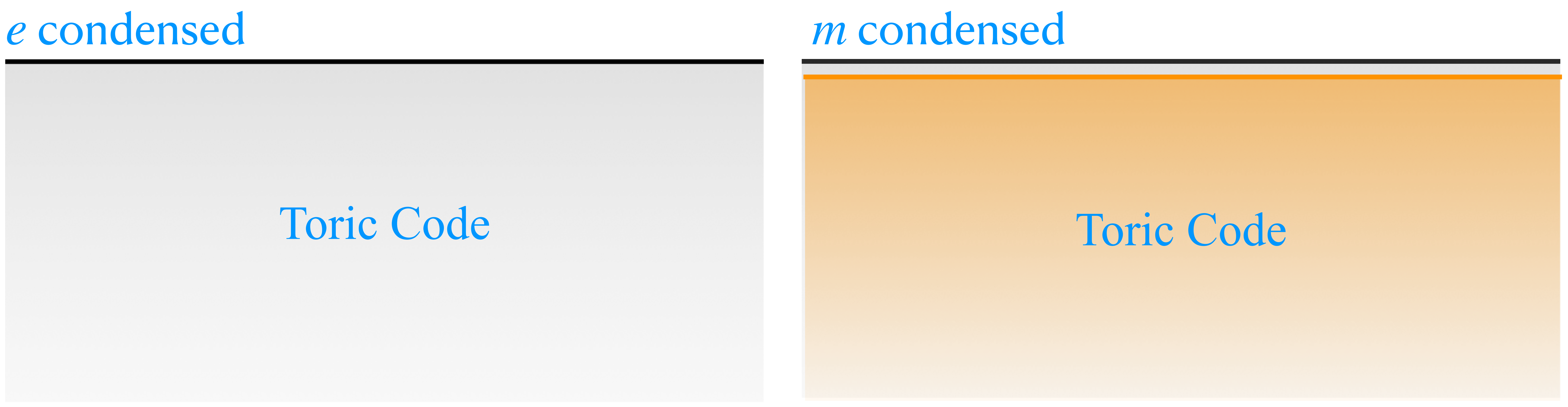
Relative distinction





There is a logical qubit.

Topological order with a boundary



Sweep invertible line defect ⟶ emergent symmetry of the bulk ⟶ change boundary types

generated by **FDQC**

only relatively distinguishable

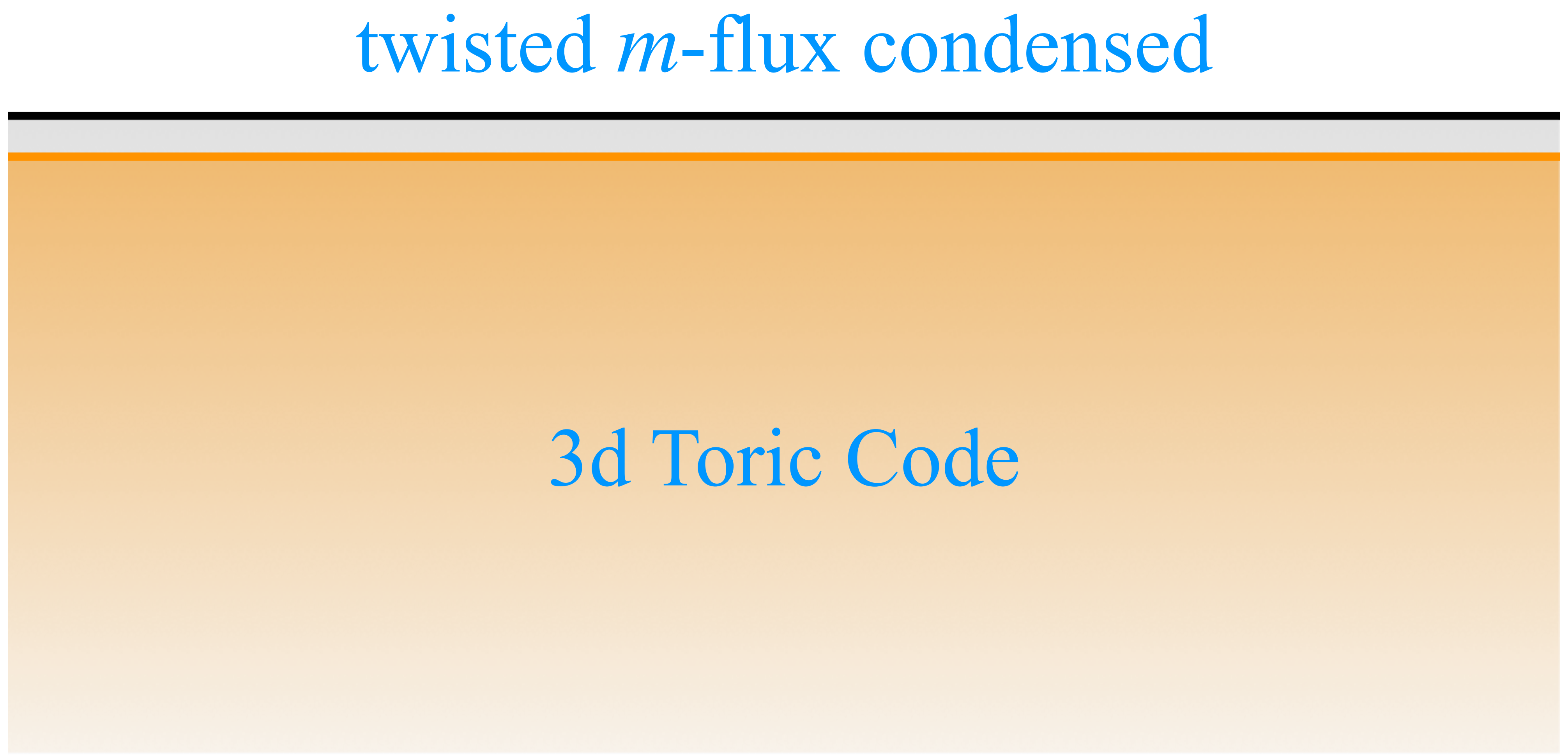
Boundaries related by bulk FDQC are "twins".

Topological order with a boundary

Example II

Topological order with a boundary

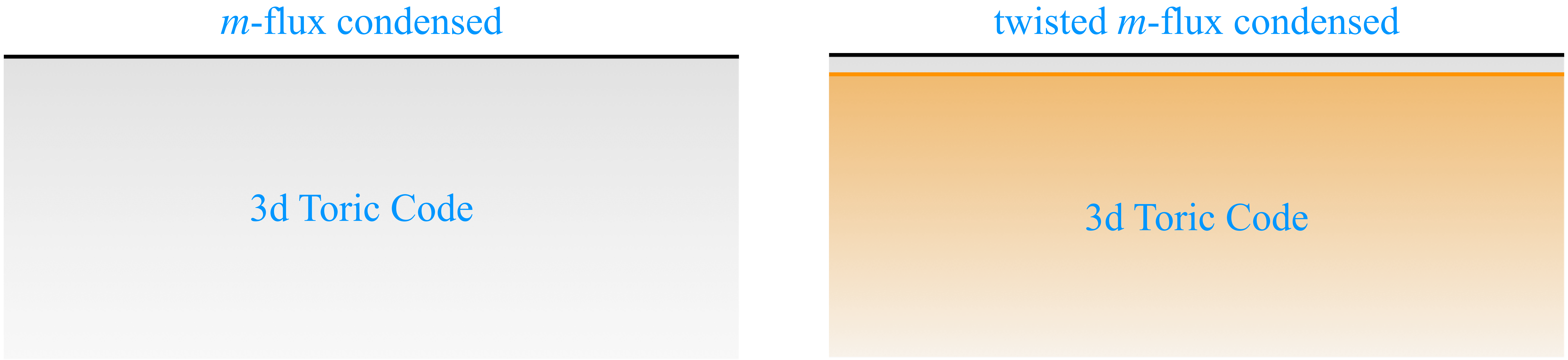
Example II



Barkeshli-Chen-Hsin-Kobayashi, 2022;
WJ-Tantivasadakarn-Xu, 2023

Topological order with a boundary

Example II



Emergent $\mathbb{Z}_2$ symmetry: Sweep the "gauged $\mathbb{Z}_2$ SPT" defect (3d FDQC)

$[U, H_{\text{tc}}] = 0$ in ground state subspace

Barkeshli-Chen-Hsin-Kobayashi, 2022;
WJ-Tantivasadakarn-Xu, 2023

Topological order with a boundary

No absolute distinction

m- flux condensed

$O(\Omega^k)$

3d Toric Code

twisted m- flux condensed

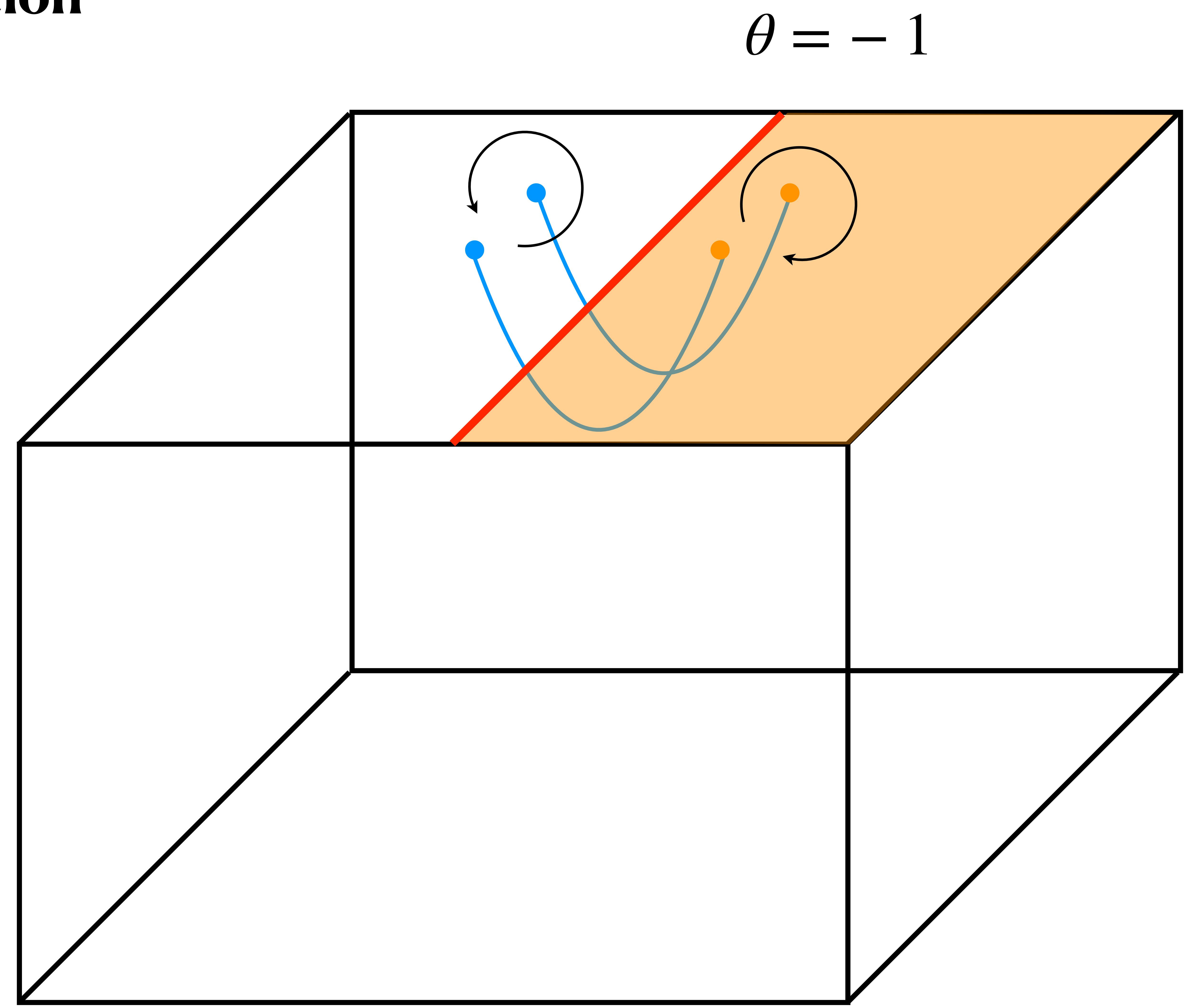
$UO(\Omega^k)U^{-1} = O'(\Omega'^k)$

3d Toric Code

3d FDQC

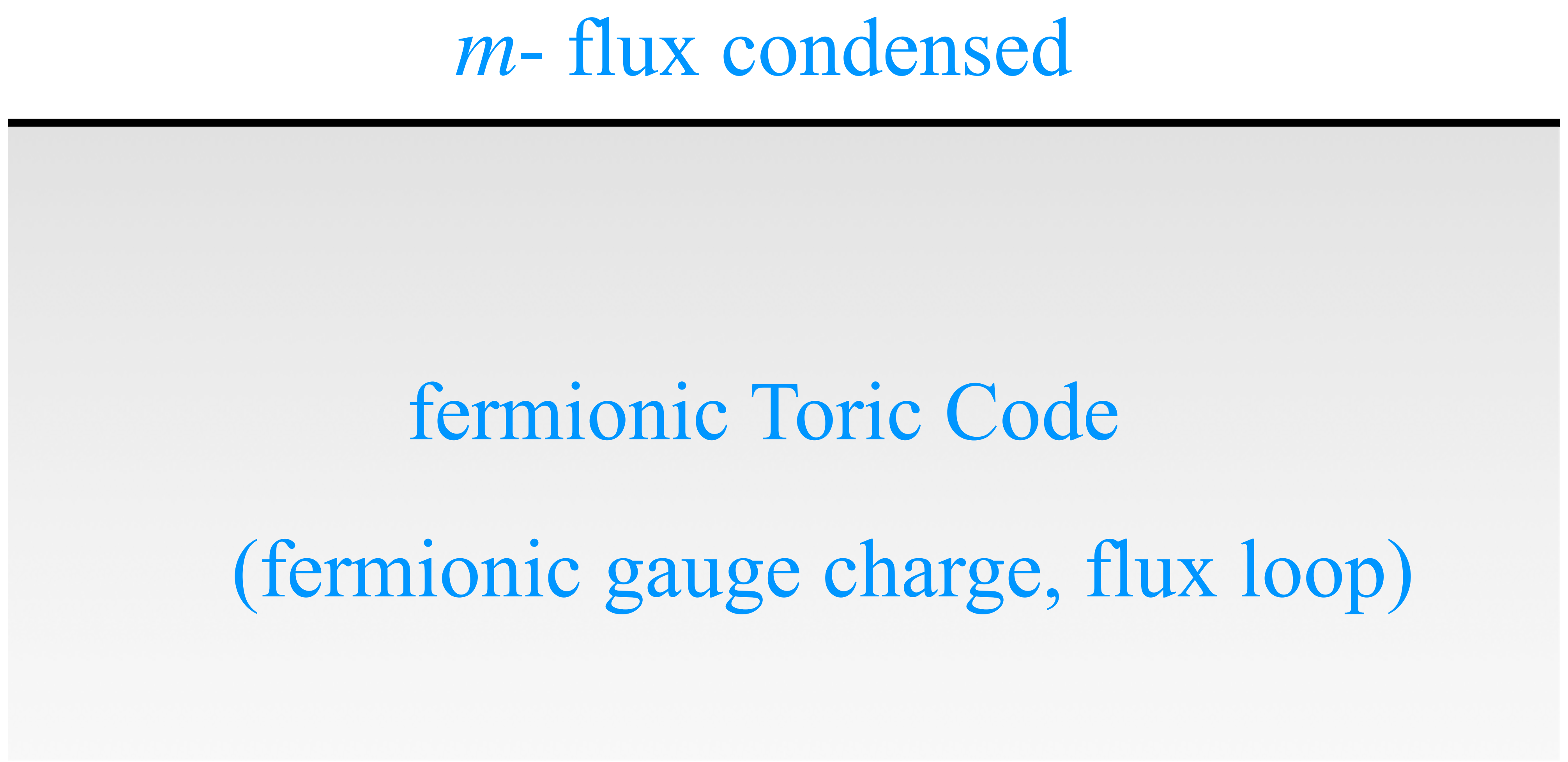
Sweep the "gauged $\mathbb{Z}_2$ SPT" defect

Relative distinction



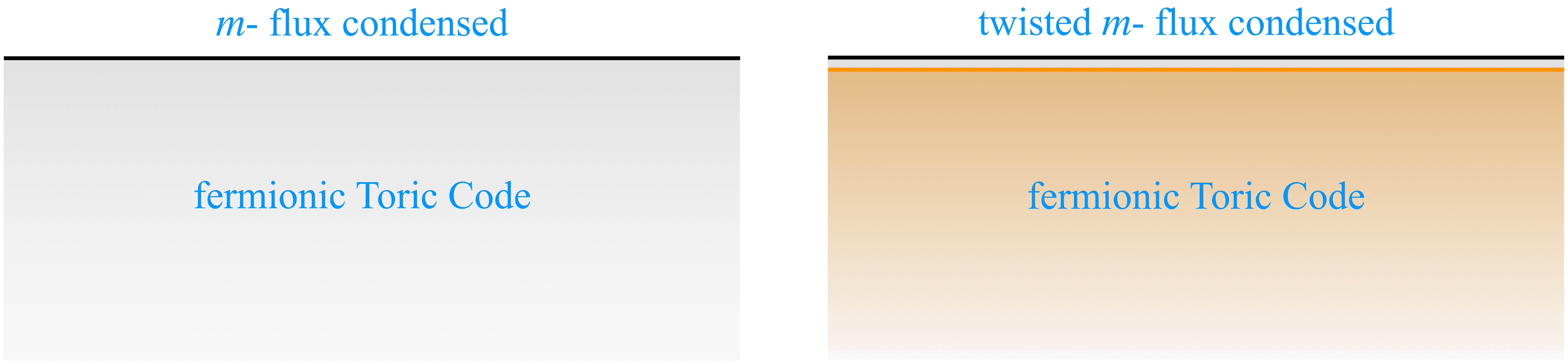
Topological order with a boundary

Example III



Topological order with a boundary

Example III



Sweep the "p+ip" defect

Hastings-Fidkowski, 2023

Topological order with a boundary

Example III

Sweep the "p+ip" defect

3d FDQLU preserving $(-1)^F$

Hastings-Fidkowski, 2023

Topological order with a boundary

Example III

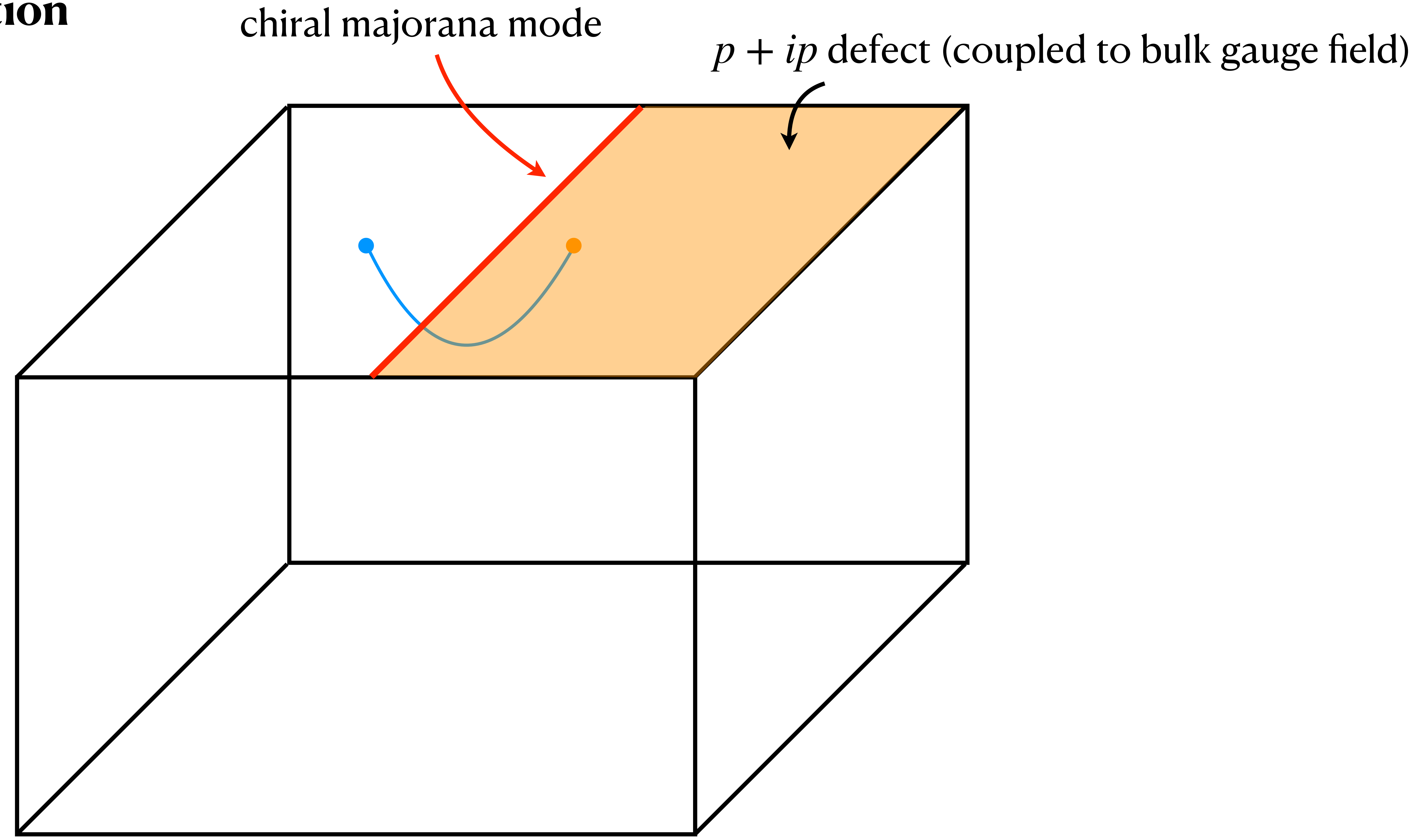
No absolute distinction

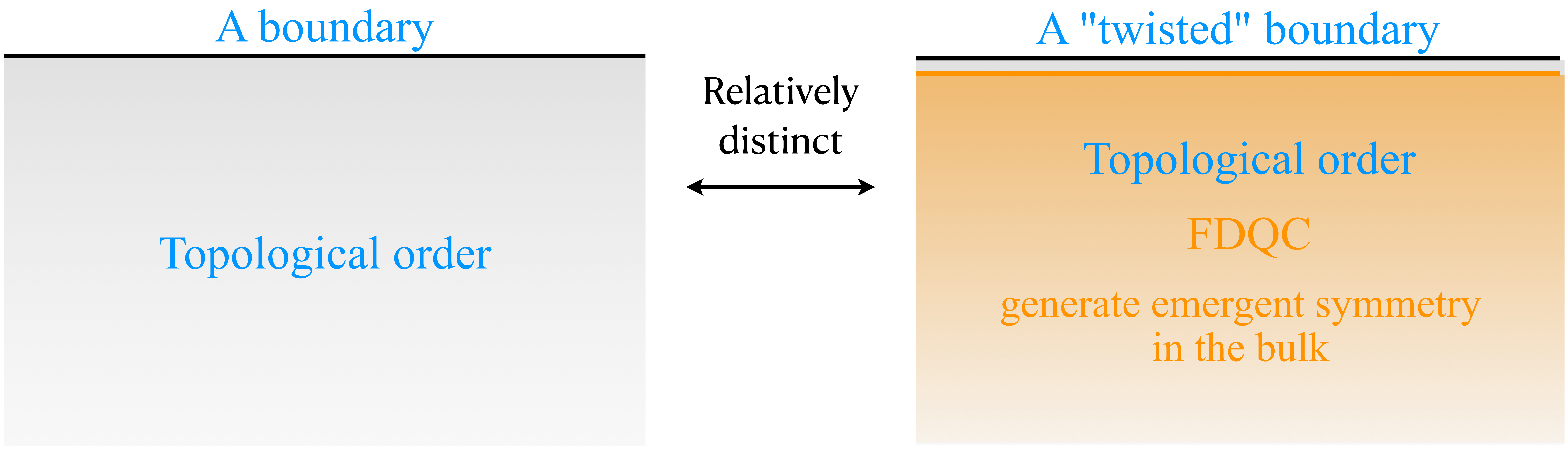
Sweep the "p+ip" defect

3d FDQLU preserving $(-1)^F$

Hastings-Fidkowski, 2023

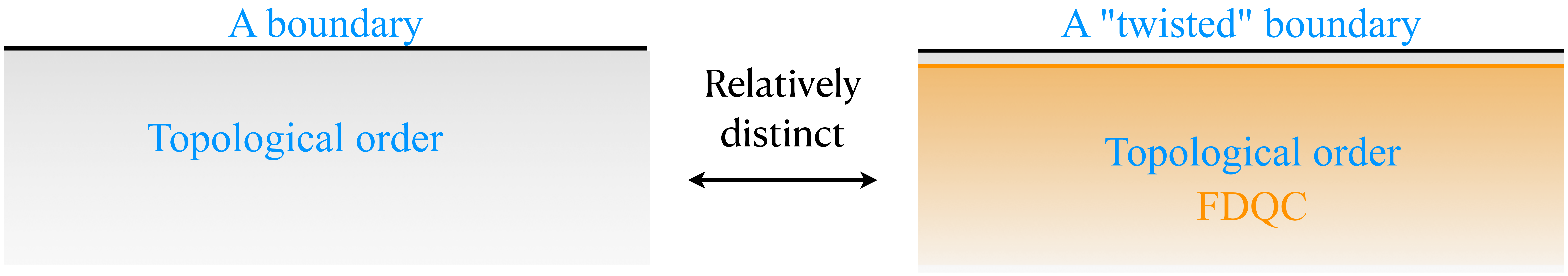
Relative distinction




A boundary
Topological order
Relatively
distinct
A "twisted" boundary
Topological order
FDQC
generate emergent symmetry
in the bulk

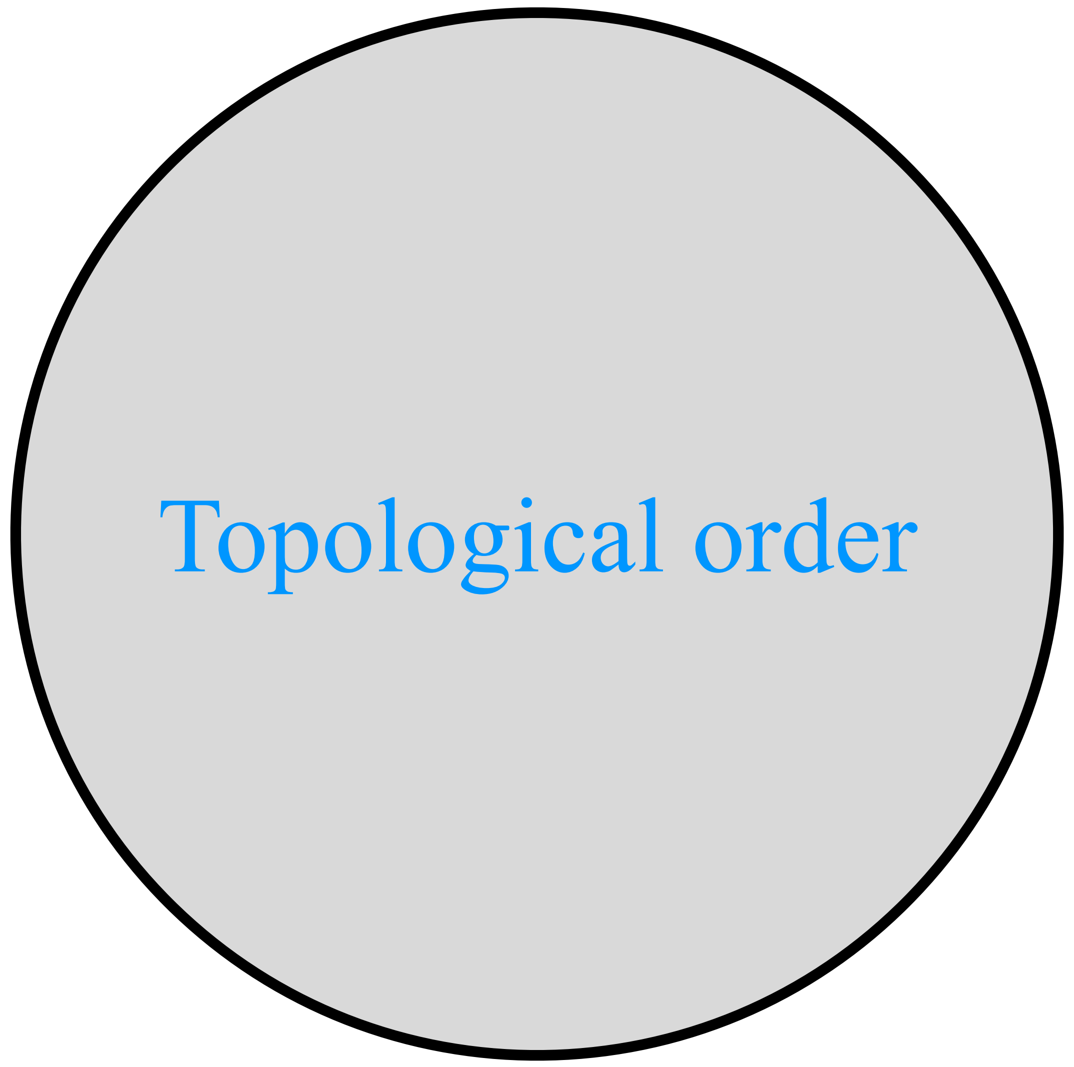
Arguement using "Symmetry TO/TFT"

Step 1:

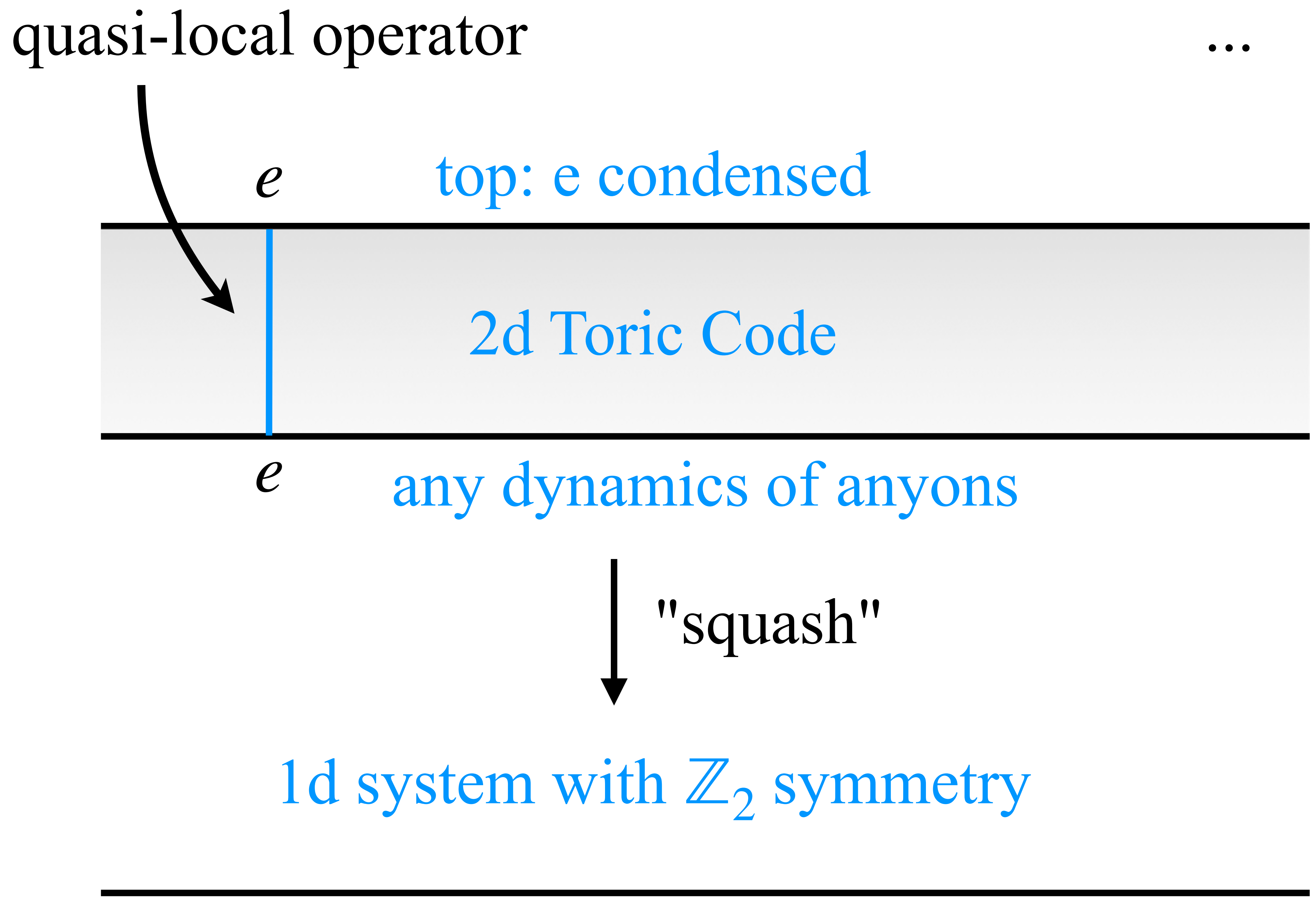


Invertible phases from symmetry TO/TFT construction

topological holography,
topological symmetry,
"sandwich construction"
...

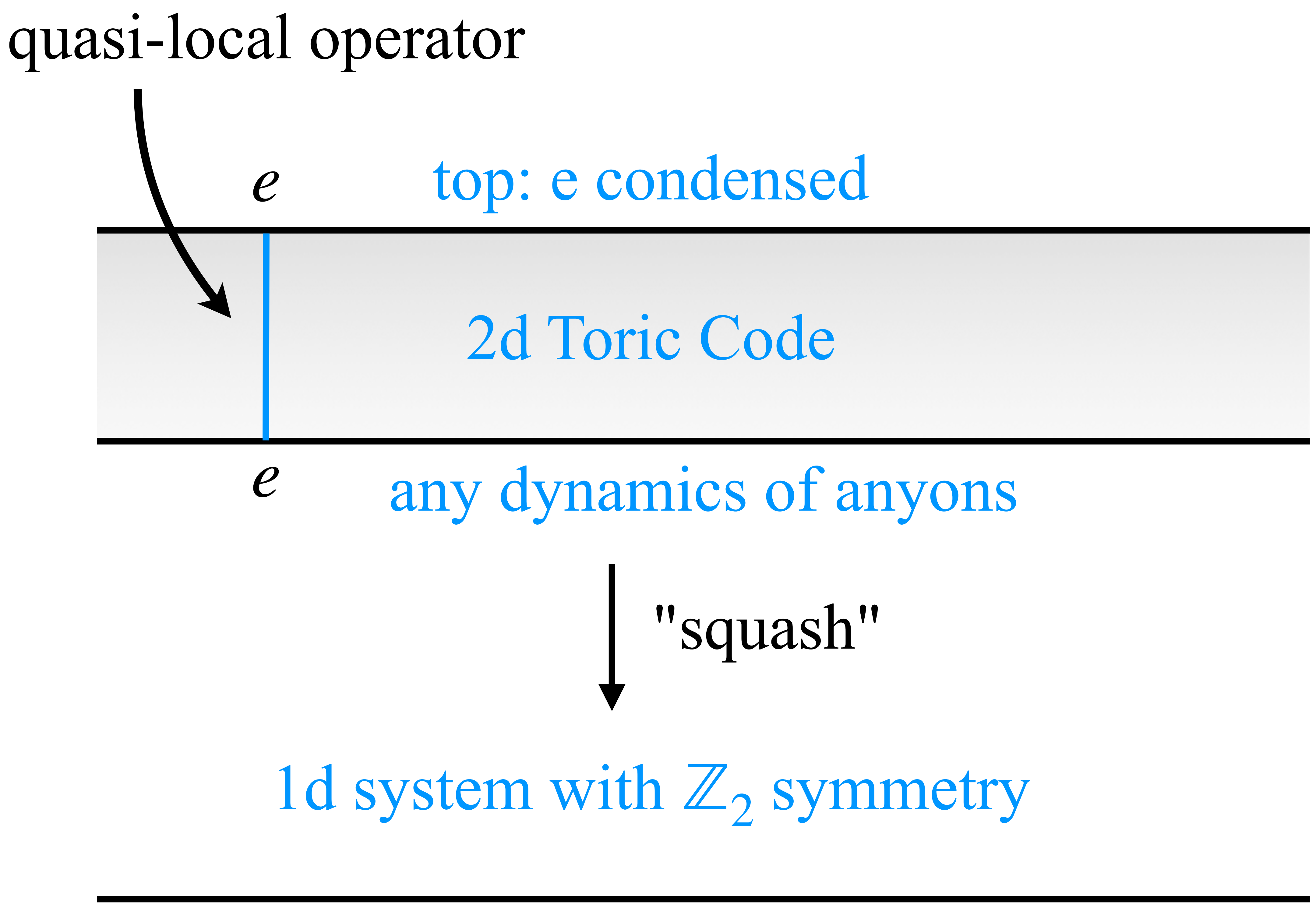


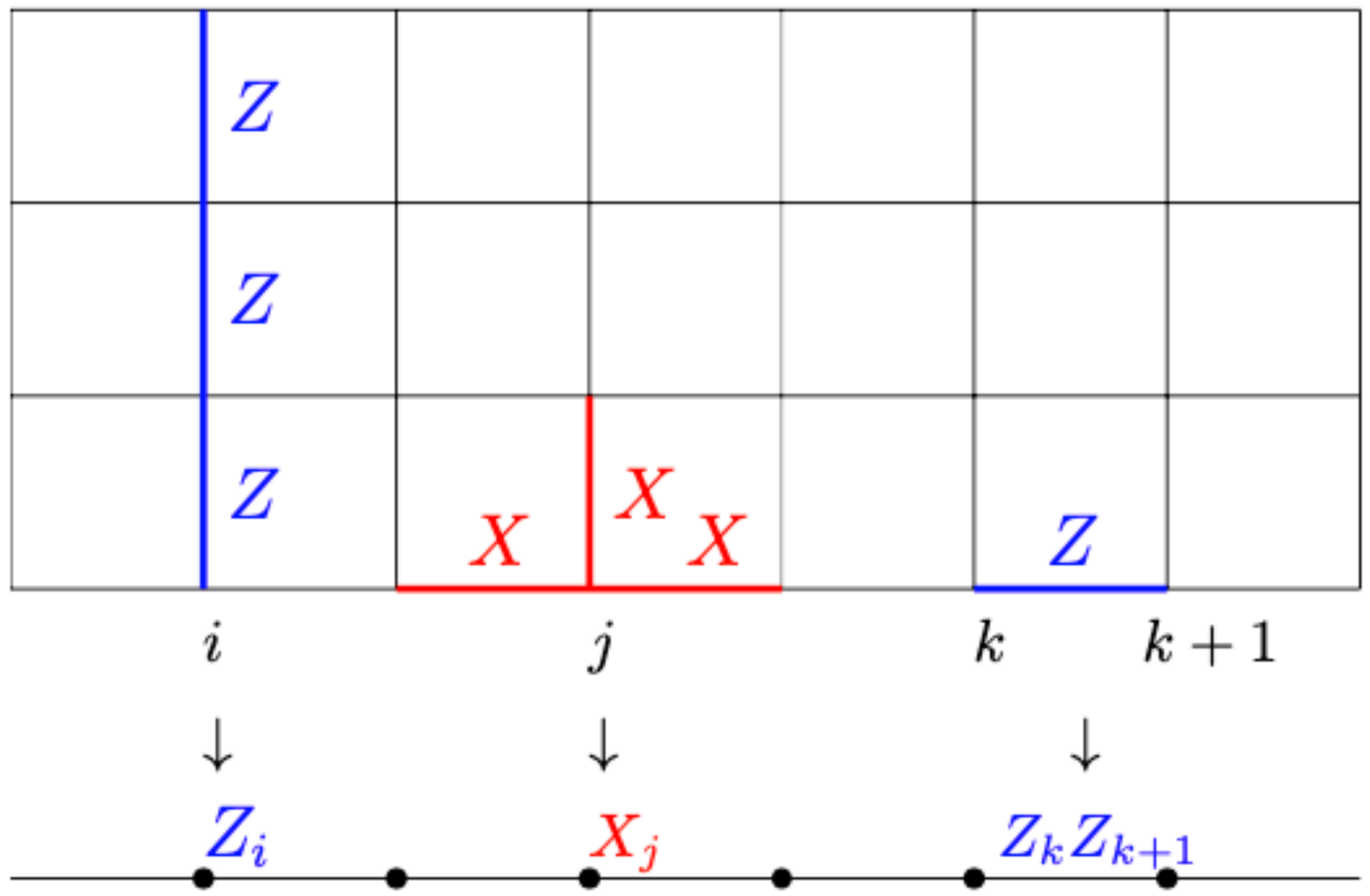
Boundary: symmetric sector of
d-1 dimensional system
with a global symmetry



WJ-Wen, 2019; Lichtman, et al, 2020; Kong, et al, 2020; Kulp, 2020; Freed-Moore-Teleman, 2021; Apruzzi, et al, 2021 …

Invertible phases from symmetry TO/TFT construction





WJ-Wen, 2019; Lichtman, et al, 2020; Kong, et al, 2020; Kulp, 2020; Freed-Moore-Teleman, 2021; Apruzzi, et al, 2021 …

Invertible phases from symmetry TO/TFT construction

top: e condensed

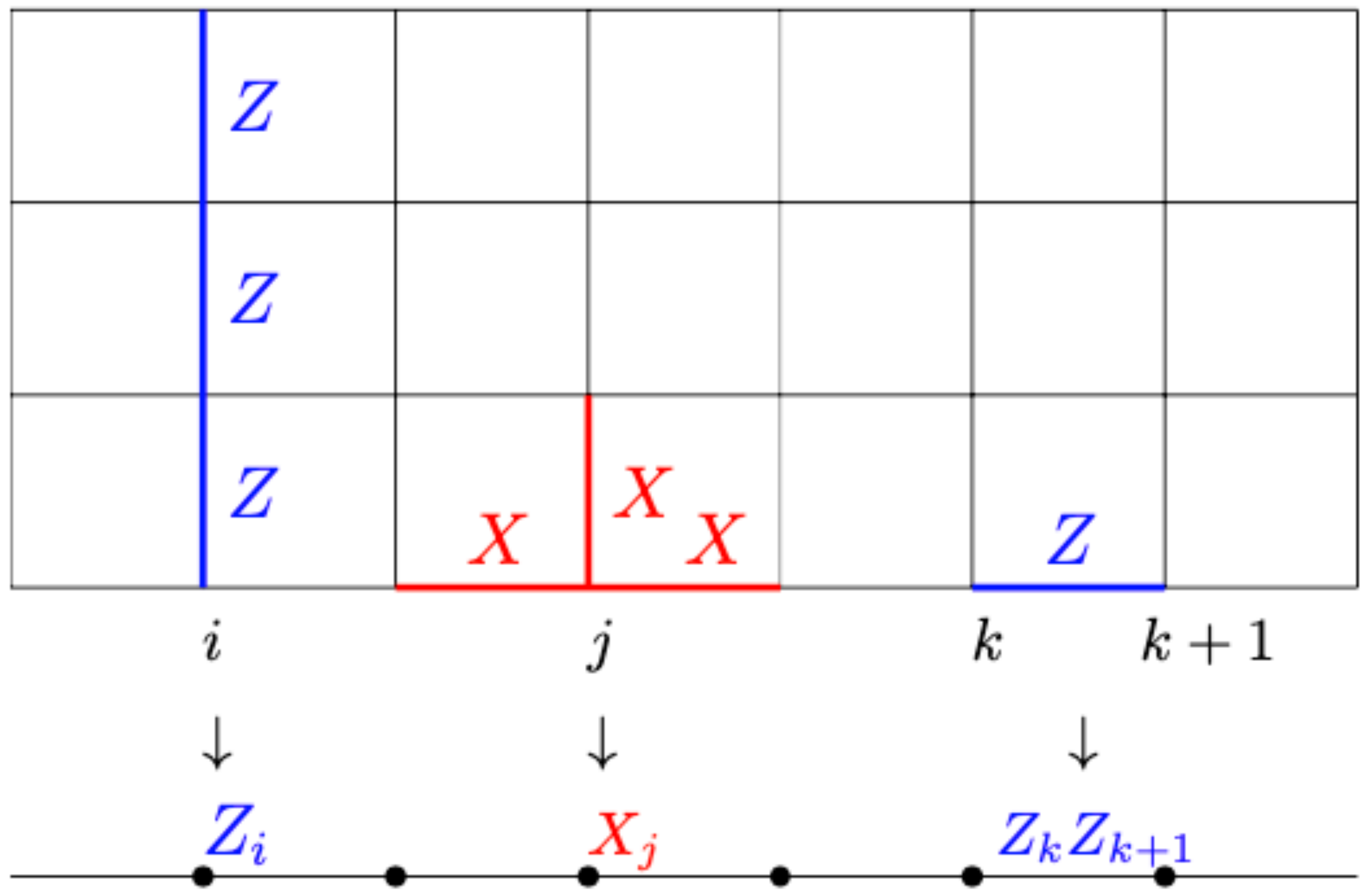
2d Toric Code

bottom: e condensed

"squash"

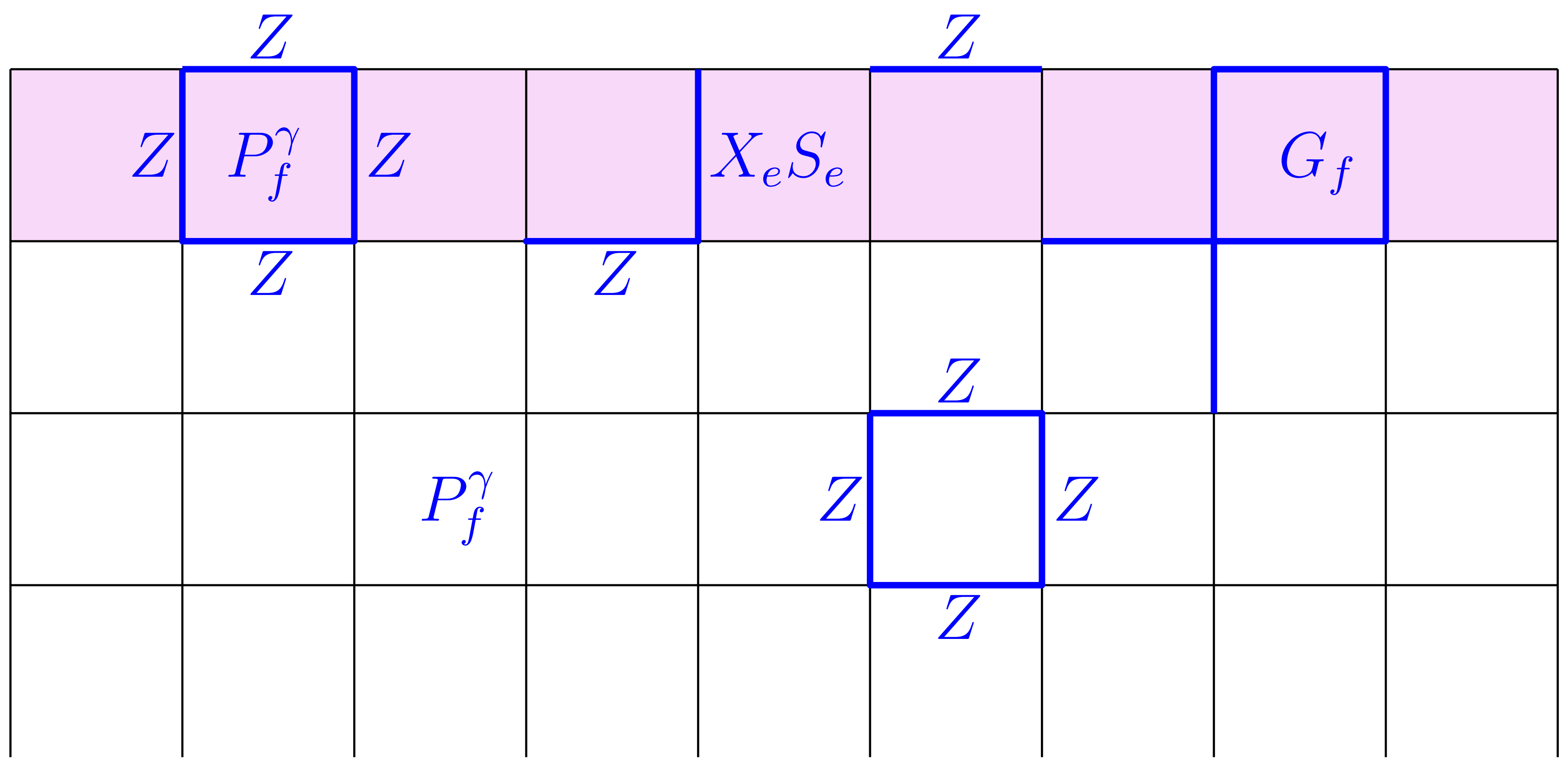
1d system with $\mathbb{Z}_2$ symmetry

symmetry breaking

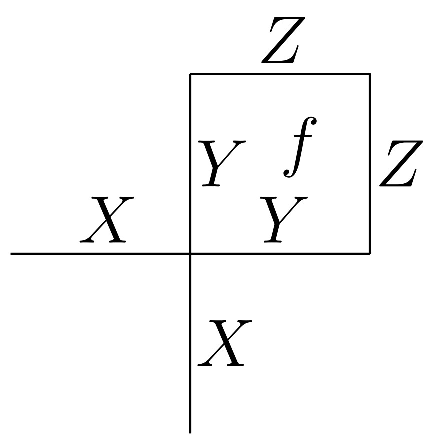


WJ-Wen, 2019; Lichtman, et al, 2020; Kong, et al, 2020; Kulp, 2020; Freed-Moore-Teleman, 2021; Apruzzi, et al, 2021 …

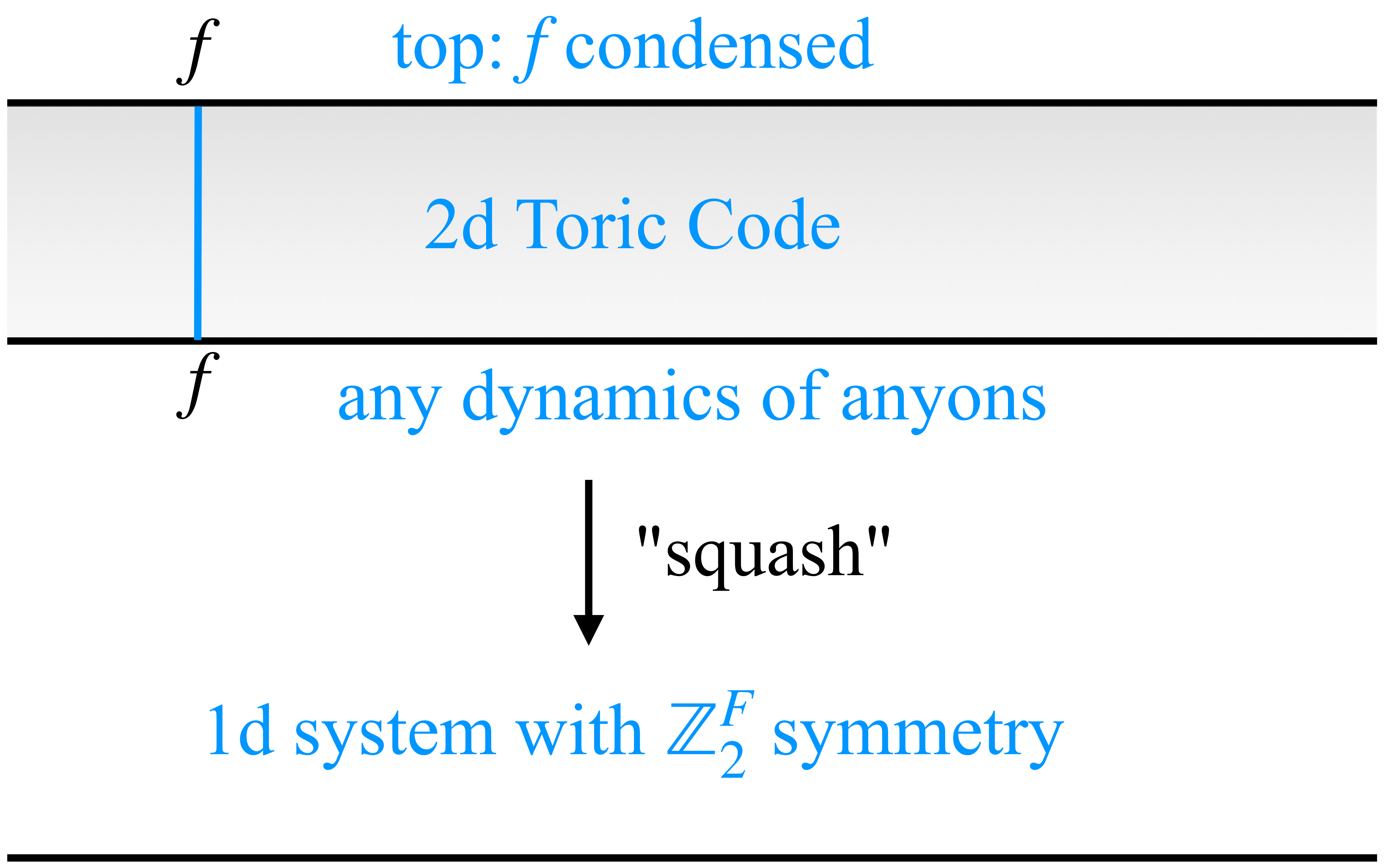
Invertible phases from symmetry TO/TFT construction

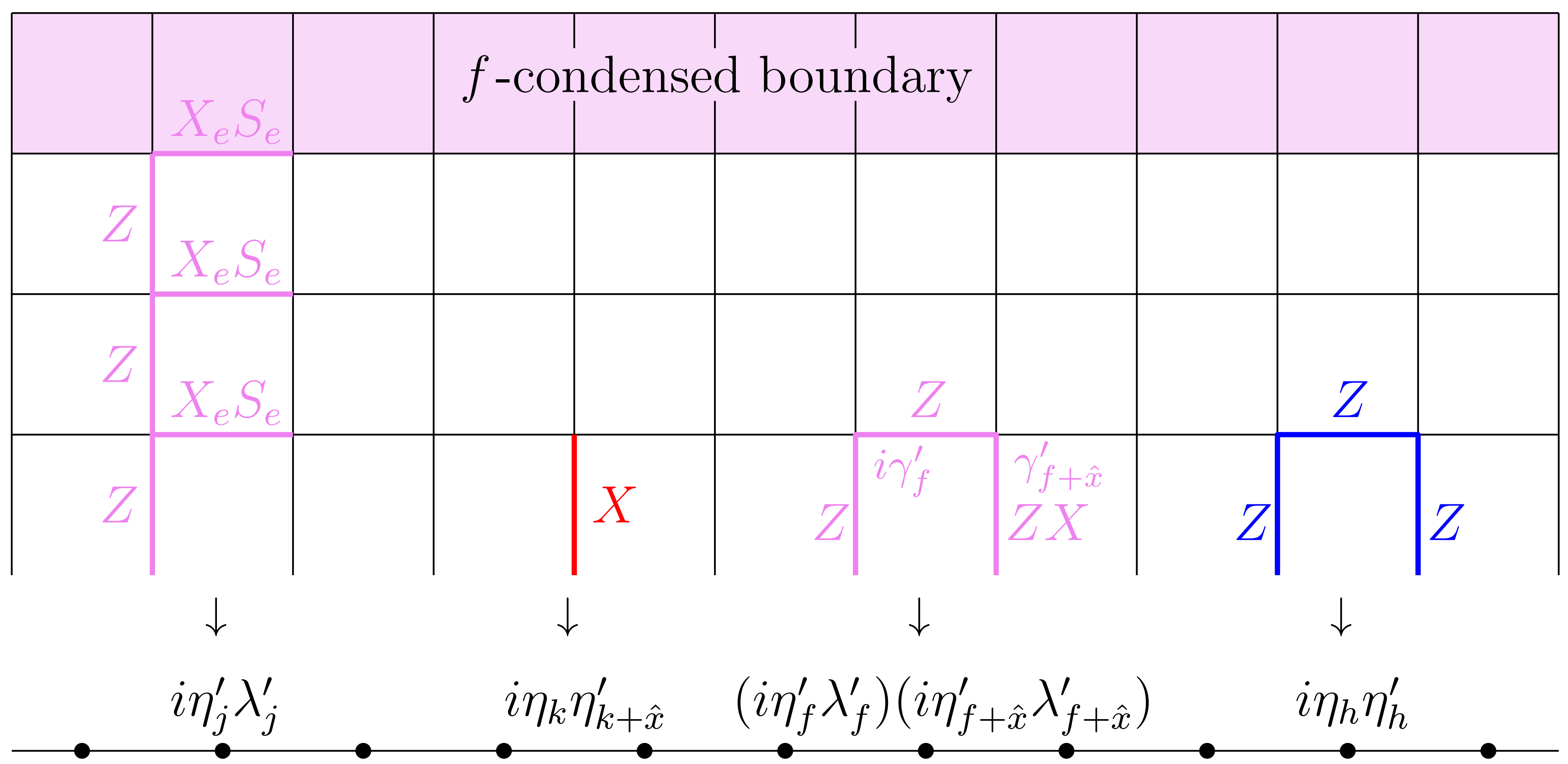


$P_f^\gamma = -i\gamma_f\gamma_f'$ $\quad S_e = i\gamma_{e_L}'\gamma_{e_R}$ $\quad G_f =$

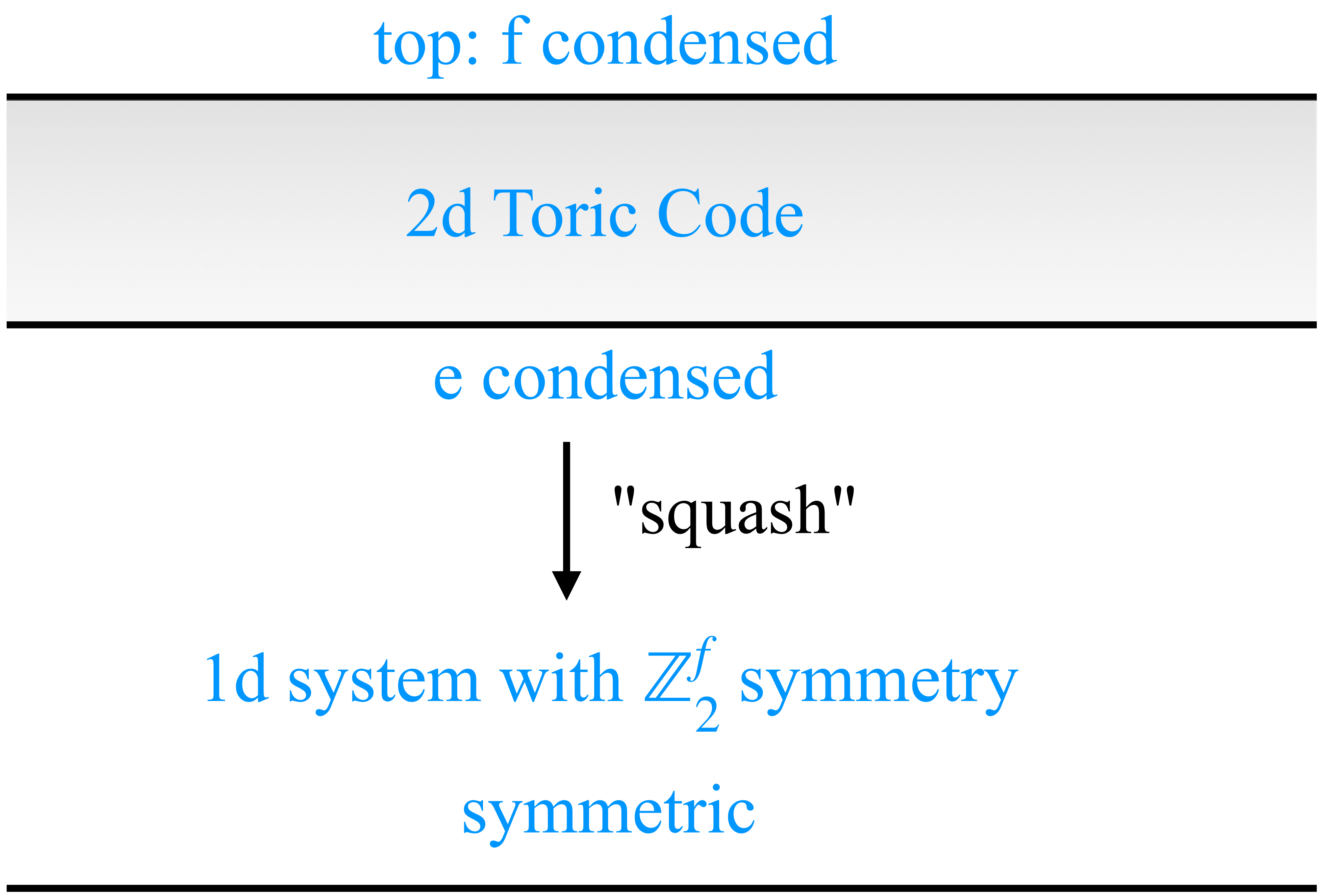


Invertible phases from symmetry TO/TFT construction

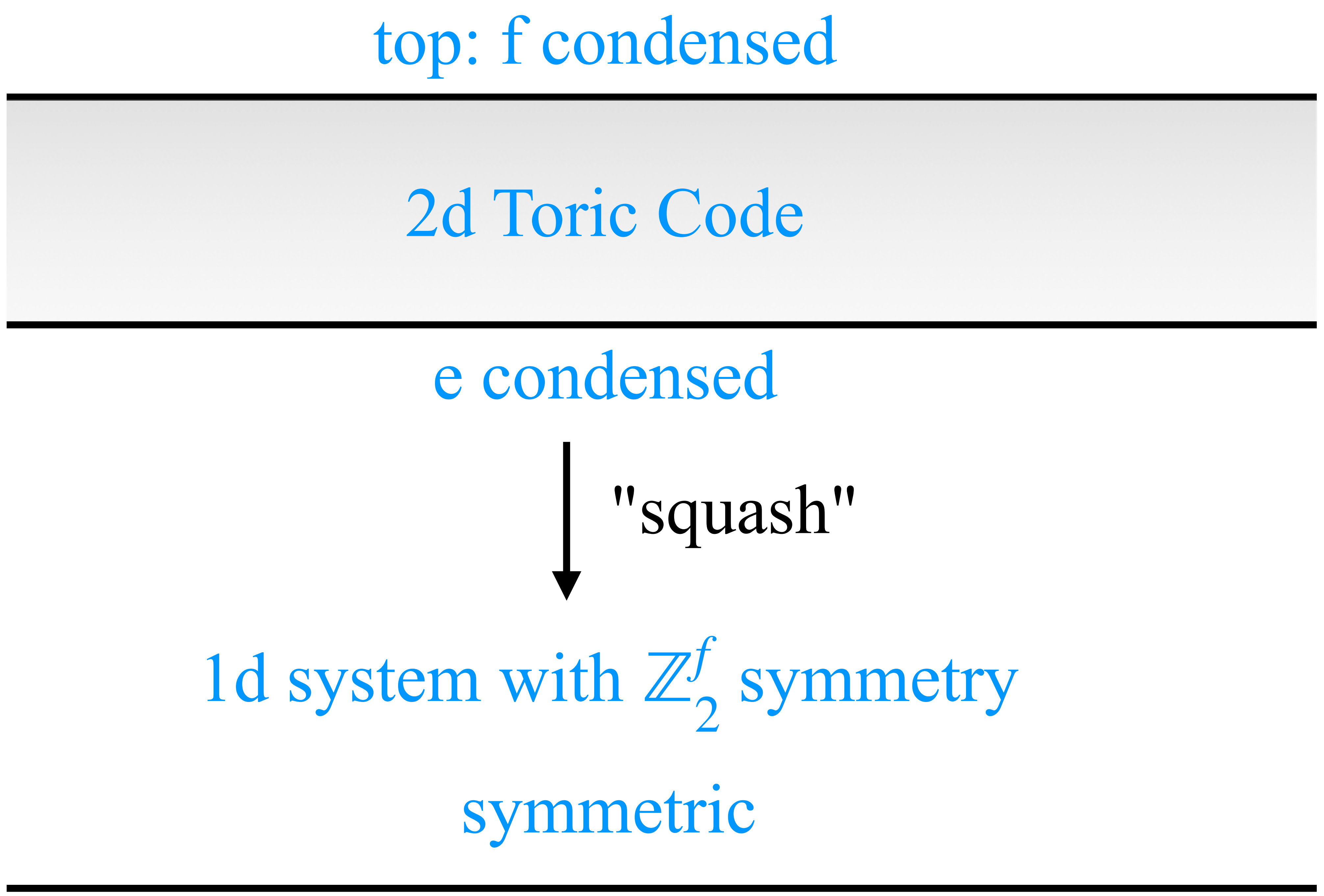


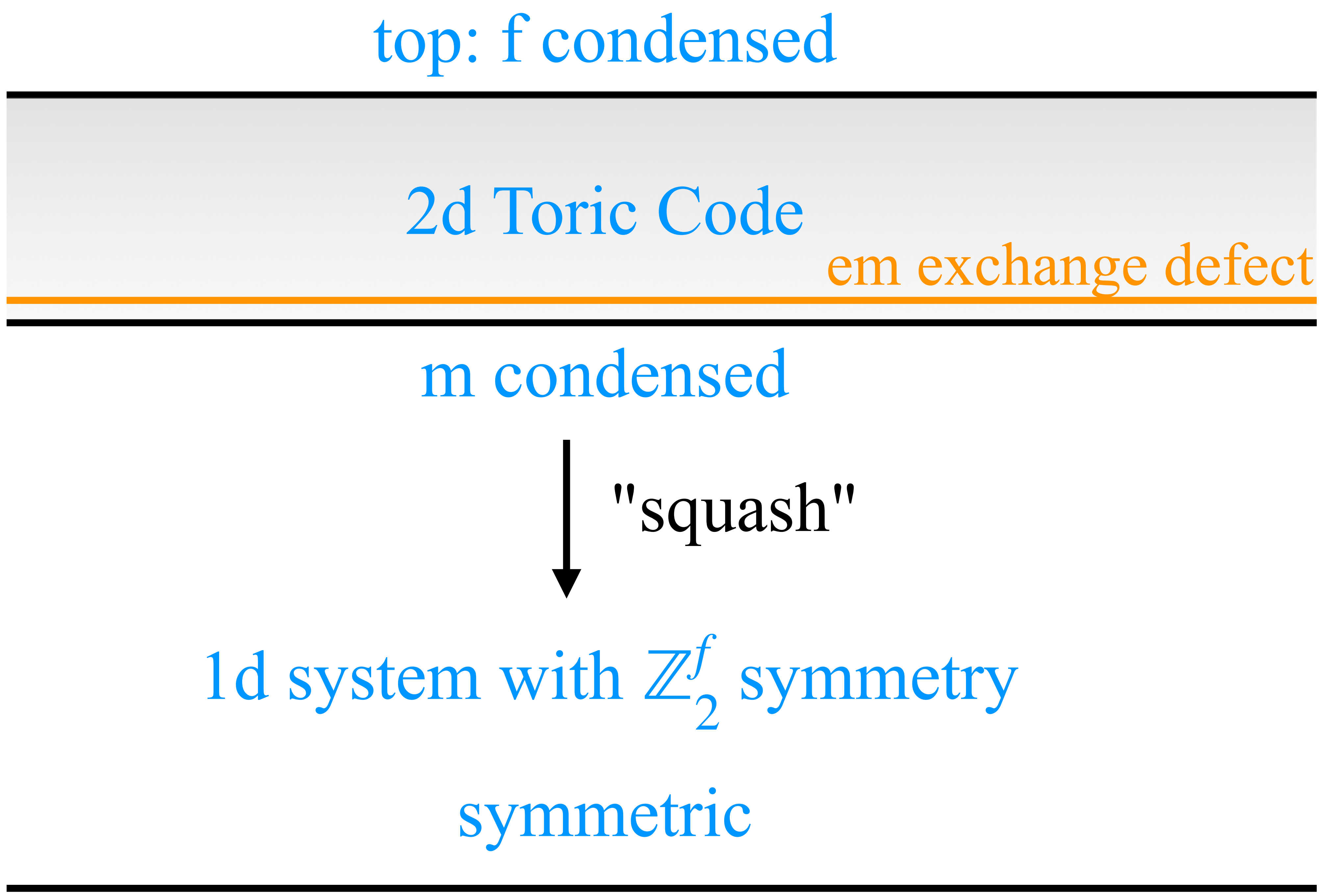


Invertible phases from symmetry TFT construction



Invertible phases from symmetry TFT construction

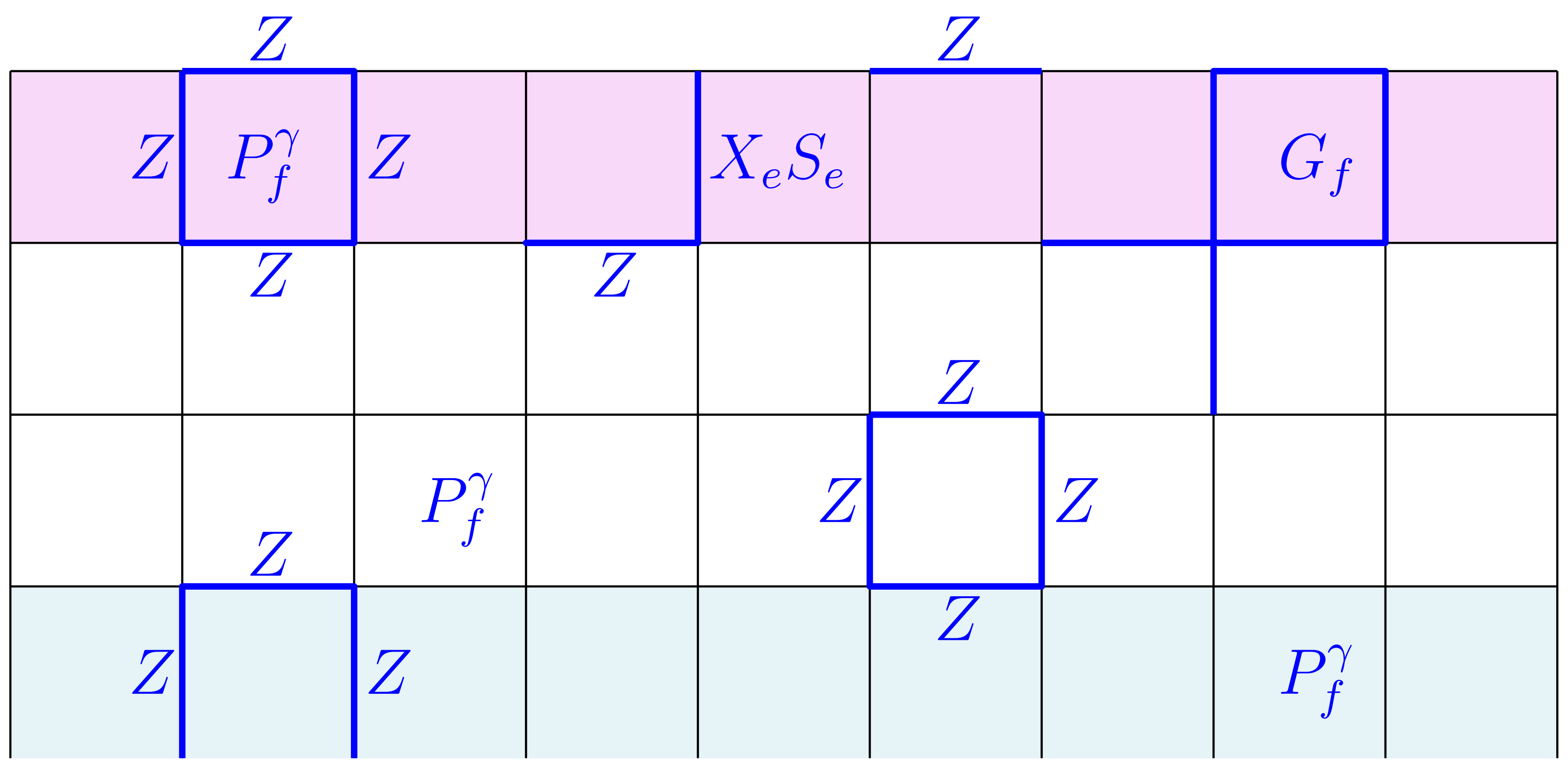


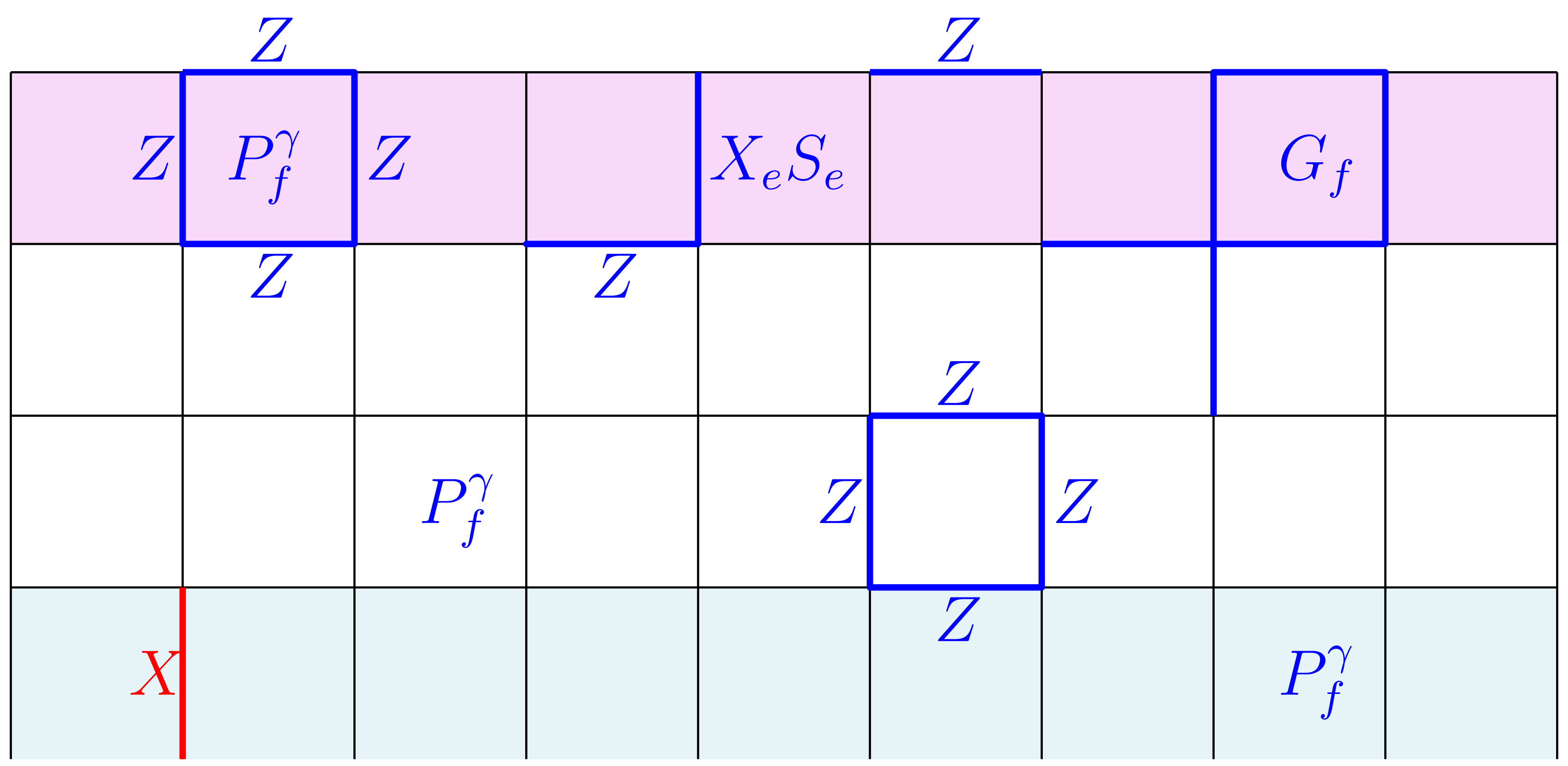


Invertible phases from symmetry TFT construction

(i) trivial superconductor (ii) Kitaev chain

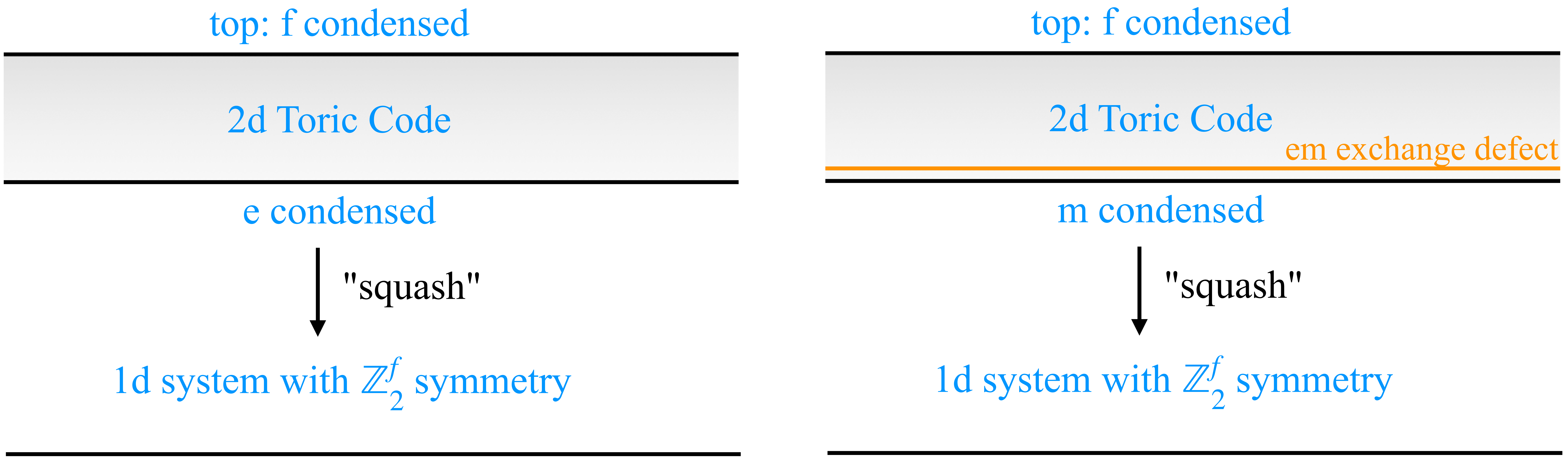
Invertible phases from symmetry TFT construction





(i) trivial superconductor (ii) Kitaev chain

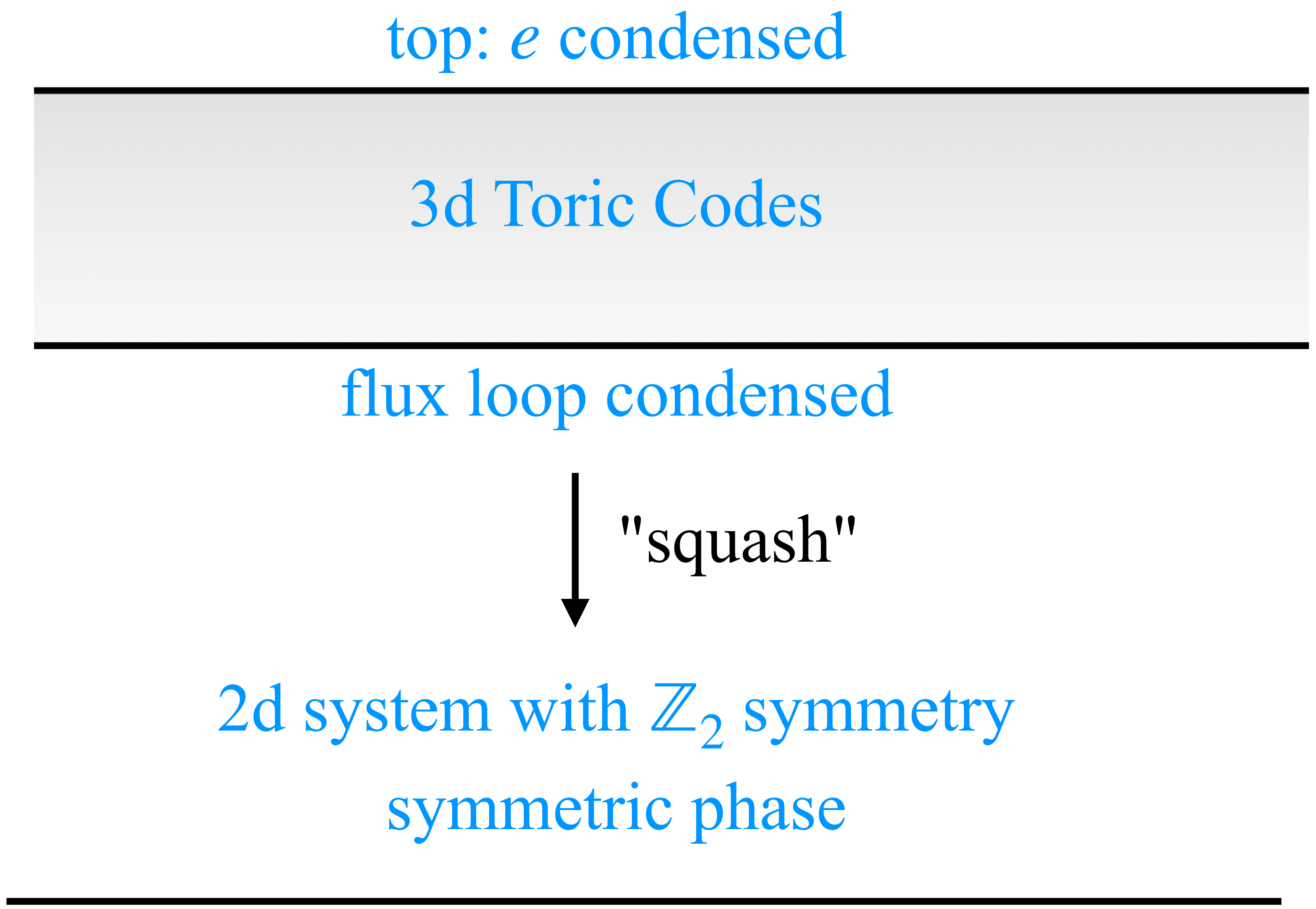
Invertible phases from symmetry TFT construction



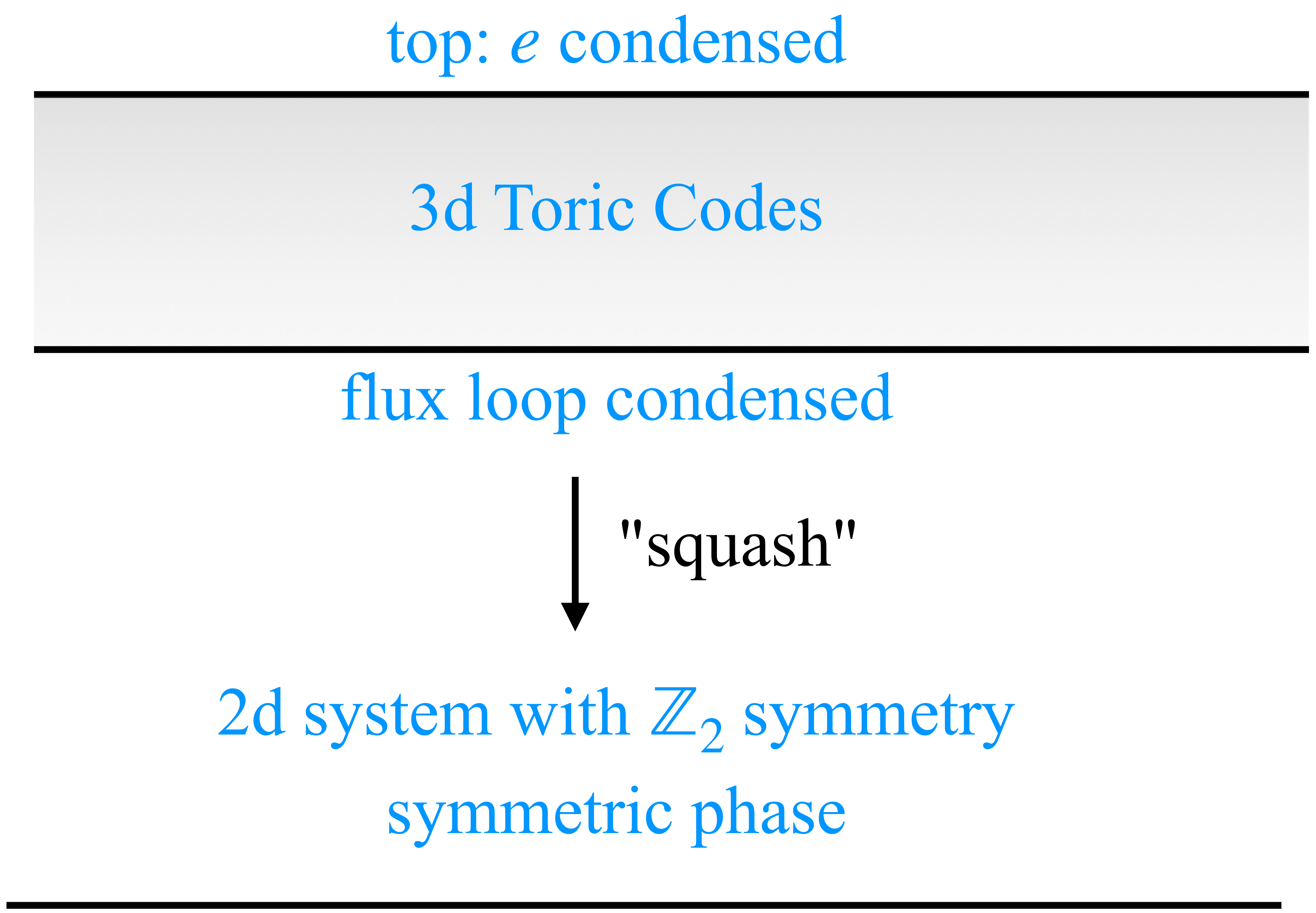
(i) trivial superconductor (ii) Kitaev chain

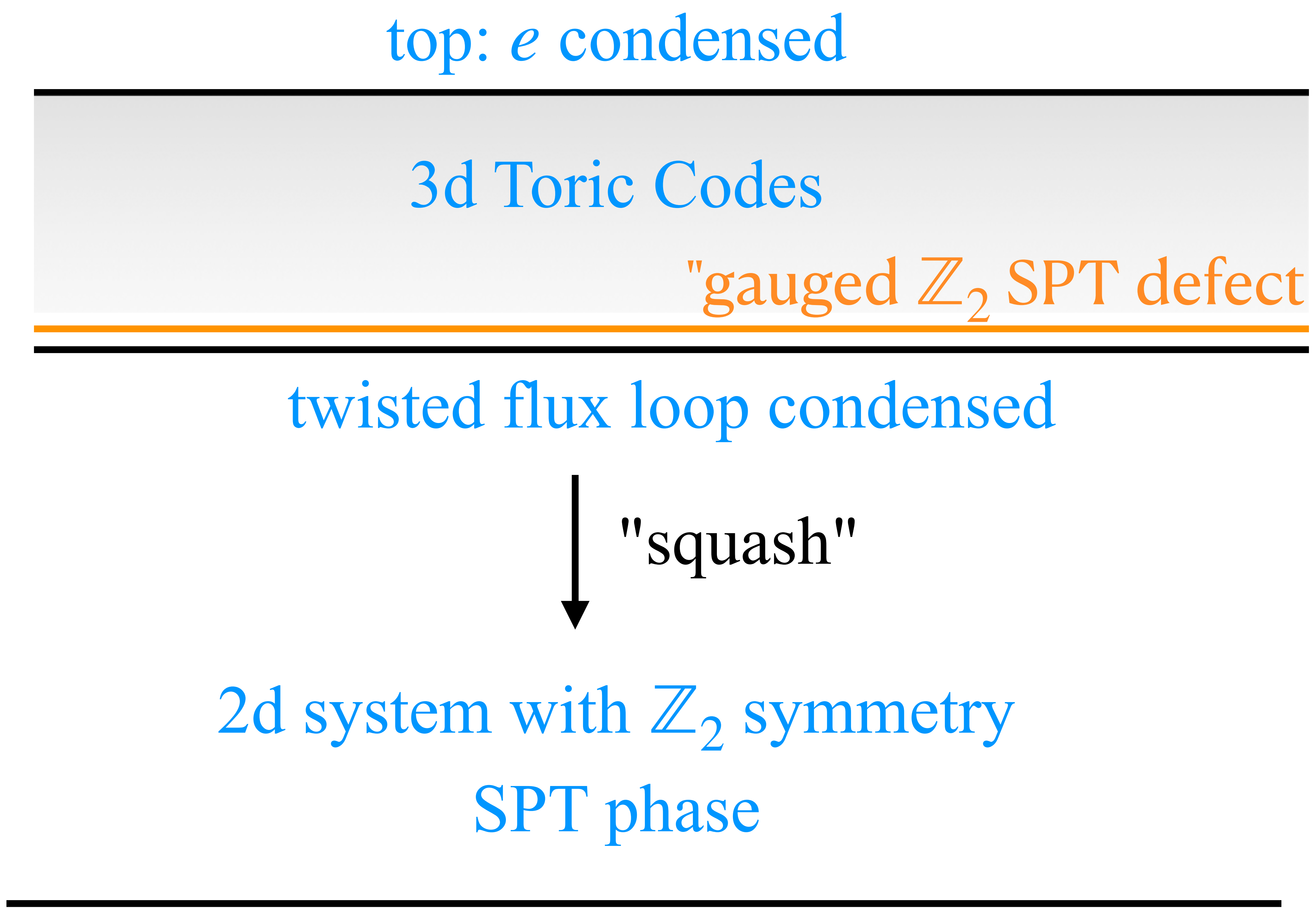
- Symmetric operators have one-to-one correspondence
- **Low entanglement excitations of the two have one-to-one correspondence**
- Only using Hamiltonians, and symmetric operators, one cannot distinguish two phases.

Invertible phases from symmetry TFT construction

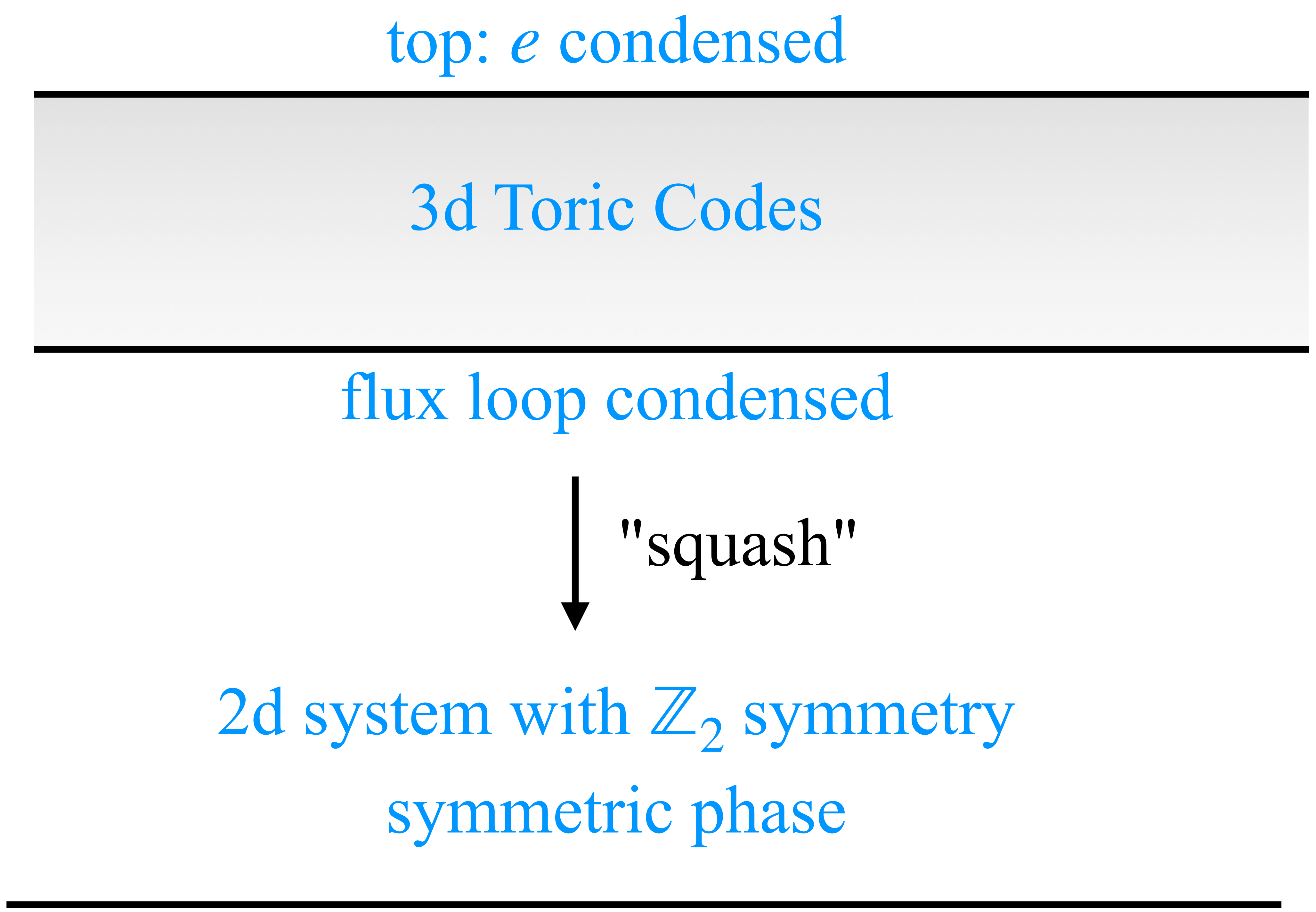


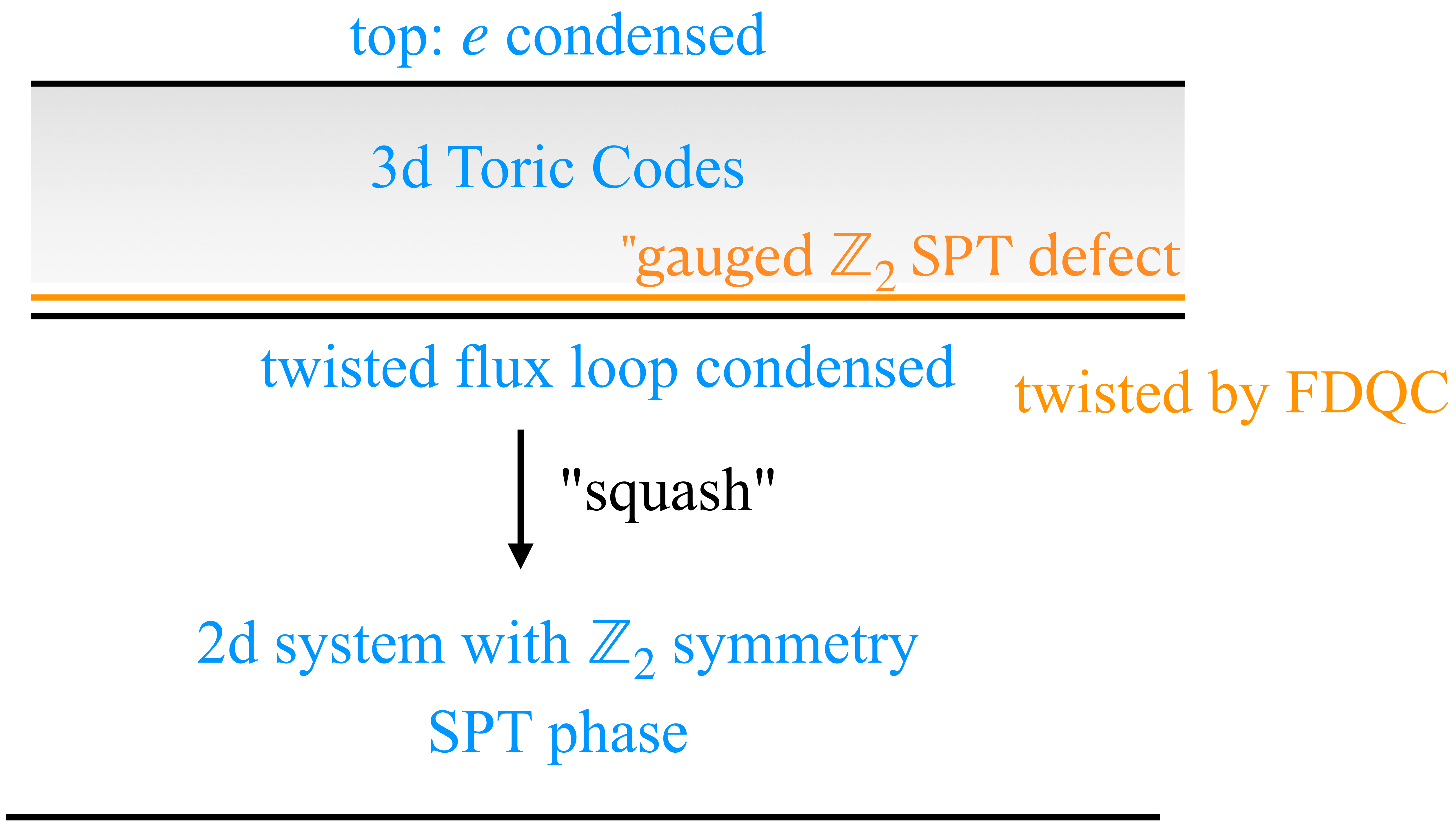
Invertible phases from symmetry TFT construction





Invertible phases from symmetry TFT construction





- Symmetric operators have one-to-one correspondence
- **Low entanglement excitations of the two have one-to-one correspondence**
- Only using Hamiltonians, and symmetric operators, one cannot distinguish two phases.

"Sandwiches" with fermionic Toric Code

top: fermion condensed

3d fermionic Toric Code

m- flux condensed

↓ "squash"

2d phase with fermion parity symmetry

trivial superconductor

top: fermion condensed

3d fermionic Toric Code

$p + ip$ defect

twisted m- flux condensed

↓ "squash"

2d phase with fermion parity symmetry

$p + ip$ superconductor

		2d trivial superconductor	**$p + ip$ superconductor**
non-trivial defect/excitation	0d	single fermion 1d QC	single fermion 1d QC
	1d	a Kitaev chain defect 1d SQC	fermion parity twist line 1d SQC

		2d $\mathbb{Z}_2$ paramagnet	**2d $\mathbb{Z}_2$ SPT**
non-trivial defect/excitation	0d	$\mathbb{Z}_2$ charge 1d FDLU	$\mathbb{Z}_2$ charge 1d FDLU
	1d	1d $\mathbb{Z}_2$ SSB 1d SQC	1d $\mathbb{Z}_2$ SSB 1d SQC

Conclusion & Further questions

Low entanglement excitations in an invertible phase and those in a product state have a one-to-one correspondence

SPTs, Kitaev chain, $p + ip$ superconductor

- Higher form SPTs
- Integer quantum Hall
- Non-invertible defects
- Implications for dynamics

$\cdots$

Conclusion & Further questions

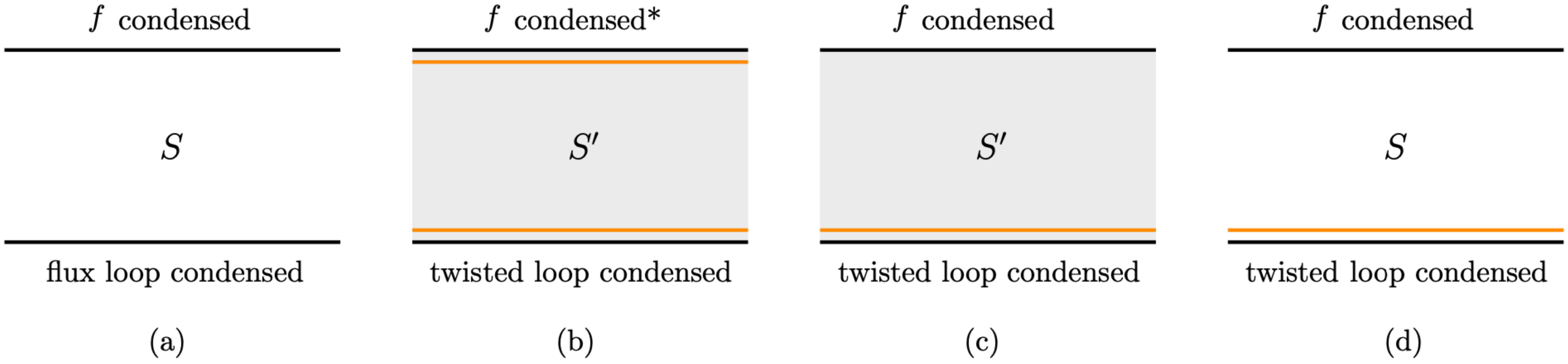
Low entanglement excitations in an invertible phase and those in a product state have a one-to-one correspondence

SPTs, Kitaev chain, $p + ip$ superconductor

- Higher form SPTs
- Integer quantum Hall
- Non-invertible defects
- Implications for dynamics

$\cdots$

Thank you !


f condensed
S
flux loop condensed
(a)
f condensed*
S′
twisted loop condensed
(b)
f condensed
S′
twisted loop condensed
(c)
f condensed
S
twisted loop condensed
(d)