

Recent progress on heavy quark physics

——A personal perspective

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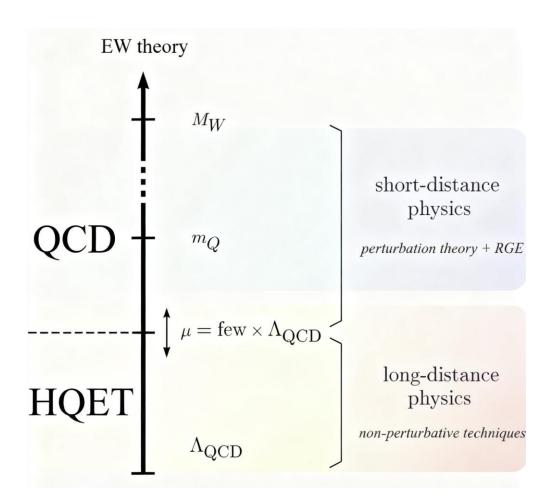
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第二十二届全国重味物理与CP破坏研讨会

Importance of heavy quark physics

B-meson dynamics is an especially simple way in principle of learning about the properties of single, constituent quarks.

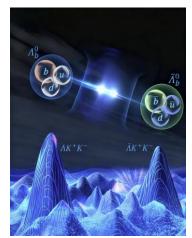
— Bjorken

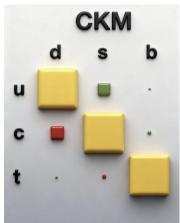


• Study on CP violation







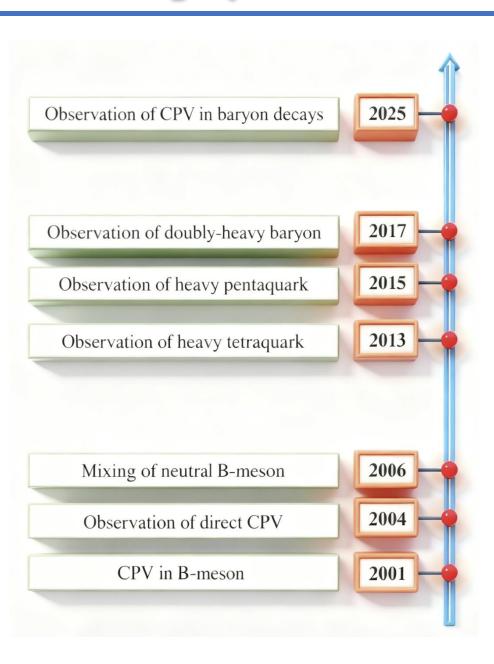


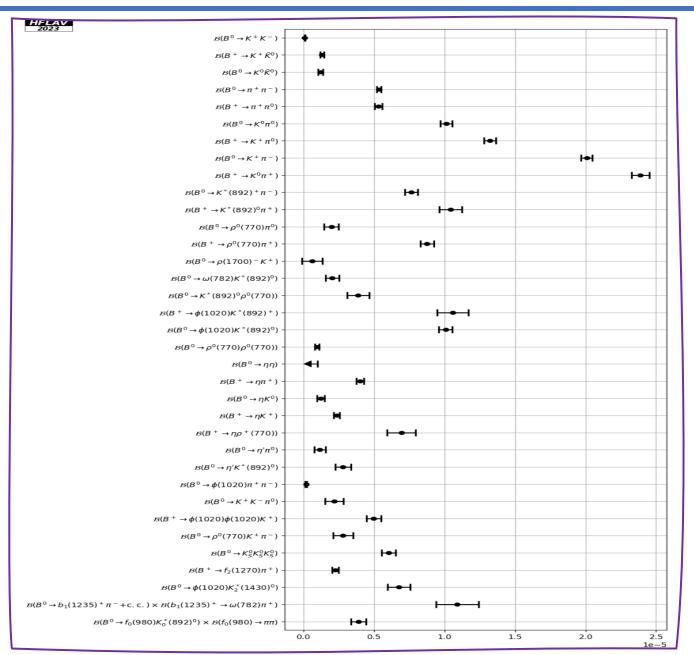


Era of high precision

B-meson dynamics is an especially simple way in principle (alas, not so much in experimental practice) of learning about the properties of single, constituent quarks. — Bjorken (1996)

Era of high precision



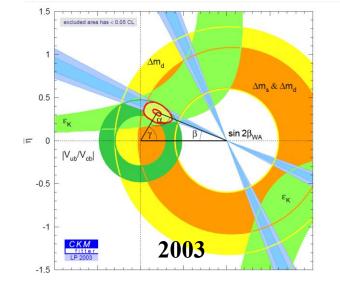


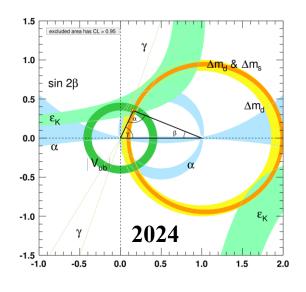
Era of high precision

Much progress achieved thanks to experiment and theory

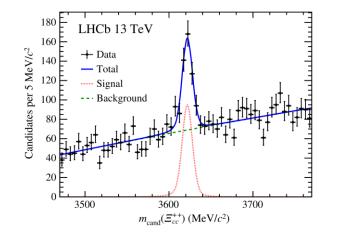
Test on Unitarity Triangle

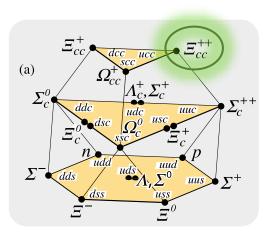






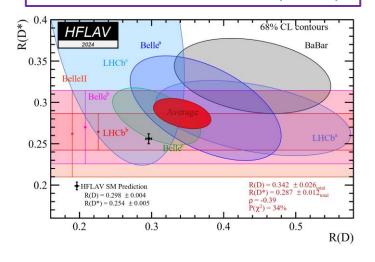
Discovery on doubly-heavy baryon (2017)



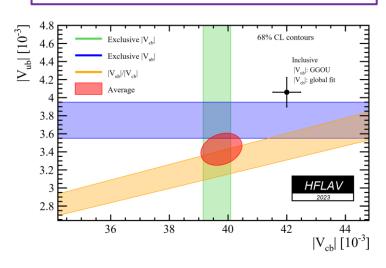


Flavor anomalies and puzzles

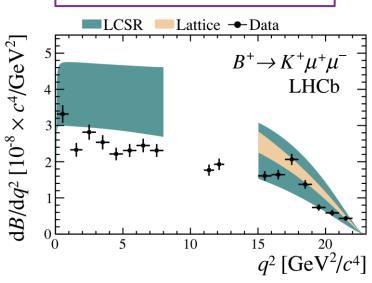
LFU violation in $R(D^{(*)})$



The $\left|V_{ub}\right|$ and $\left|V_{cb}\right|$ puzzle



$m{B} ightarrow m{K}^{(*)} \ell \ell$ anomoly



$B \rightarrow \pi \pi \text{ puzzle}$

$$Br(B^0 \to \pi^0 \pi^0) = (1.55 \pm 0.16) \times 10^{-6}$$

Much larger than theoretically calculated.

$B \rightarrow \pi K$ puzzle

$$\Delta A_{CP}(\pi K) = A_{CP}(\pi^0 K^-) - A_{CP}(\pi^+ K^-)$$

= (11.3 ± 1.2)%

 9σ deviation from isospin limit.

$$B \rightarrow K K$$
 puzzle

$$B \rightarrow D_q^{(*)} L$$
 puzzle

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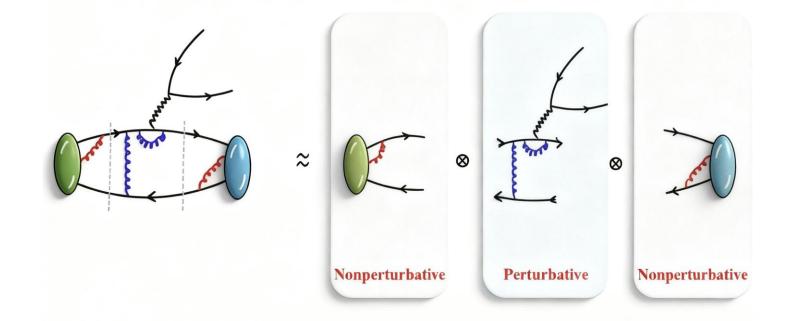
Weak Hamiltonian and hadronic matrix elements

Standard starting point for theoretical calculations.

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_i C_i O_i + \sum_i C_i' O_i'$$

- Wilson coefficients C_i : perturbatively calculable.
- Hadronic matrix elements $\langle f|O_i|B\rangle$: essentially nonperturbative quantities.

Factorization



Factorization methods

Naïve factorization and Generalized factorization

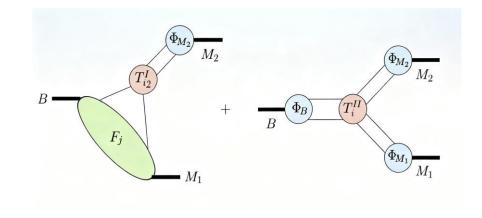
[BSW (1985)] [Cheng etc., Ali, Kramer, Lü (1998)]

$$\langle M_1 M_2 | O_i | \overline{B} \rangle = C_1 F^{B \to M_1} f_{M_2} + C_2 F^{B \to M_2} f_{M_1}$$

QCD factorization and SCET

[Bauer et.al, (2001)] [Beneke, Buchalla, Neubert, Sachrajda (1999)]

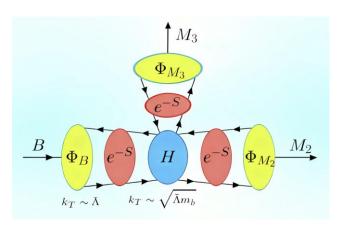
$$\langle M_1 M_2 | O_i | \overline{B} \rangle = F^{B \to M_1} T_i^I \otimes \phi_{M_2} + T_i^{II} \otimes \phi_B \otimes \phi_{M_1} \otimes \phi_{M_2}$$



Perturbative QCD (PQCD)

[Li, Lü, Xiao etc.]

$$\langle M_1 M_2 | O_i | \bar{B} \rangle = H \otimes \phi_B \otimes \phi_{M_1} \otimes \phi_{M_2} \otimes e^{-\sum S_i}$$



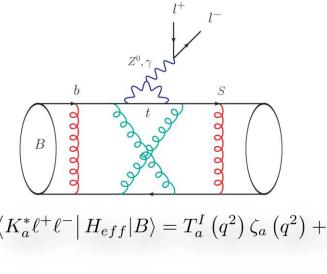
The LCDA of light meson

High precision predictions rely on reliable knowledge on nonperturbative inputs

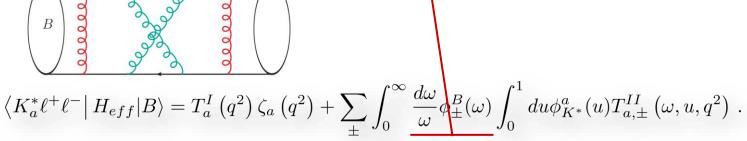
The $B \rightarrow P$ and $B \rightarrow V$ form factors

$$\Delta F_M(2E,\mu) = T(u,2E,\omega,\mu) \otimes \underline{\phi_B^+(\omega,\mu)} \otimes \underline{\phi_M(u,\mu)}.$$

The rare $B \to K\ell\ell$ decay

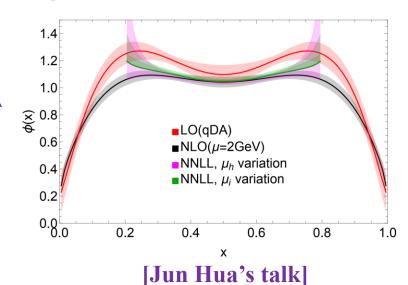


B-meson LCDA



The LCDA of light hadron

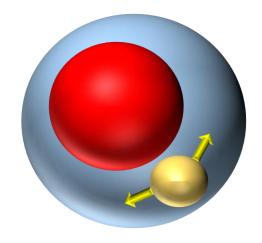
- π and K distribution amplitude from lattice QCD: Hua, Wang, **JX**, Zhang et.al, PRL 129 (2022)
- Distribution amplitude of K^* and ϕ from Lattice QCD: Hua, Wang, JX, Zhang et.al, PRL 127 (2021)
- NNLO QCD corrections to pion electromagnetic form factors: Chen, Chen, Feng, Jia, PRL 132 (2024)
- NNLO QCD Prediction for the Pion Form Factor: Ji, Wang, Wang, Wang et.al, PRL 134 (2025)
- Lattice QCD calculation of the pion distribution amplitude: Baker, Zhao et.al, JHEP 07 (2024)



Structure of heavy meson

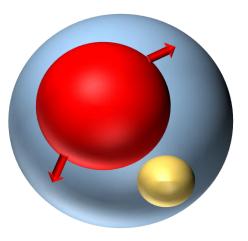
A heavy flavor meson consists of a pair of heavy and light quarks.

Light-cone distribution function



LCDA describes the momentum distribution amplitude of the light quark.

Shape function



SF characterizes the momentum distribution function of the heavy quark.

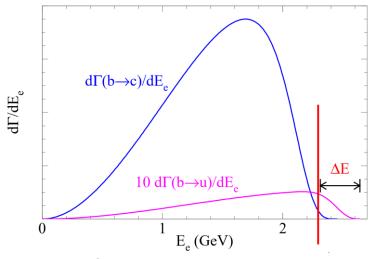
Together, they provide the most essential information about the profile of heavy mesons.

The $|V_{ub}|$ Puzzle

The $|V_{ub}|$ tension

$$|V_{ub}| = (4.13 \pm 0.12 \stackrel{+}{}_{-0.14}^{0.13} \pm 0.18) \times 10^{-3}$$
 (inclusive),
 $|V_{ub}| = (3.70 \pm 0.10 \pm 0.12) \times 10^{-3}$ (exclusive),

• The inclusive process: $B \to X_u \ell \nu$

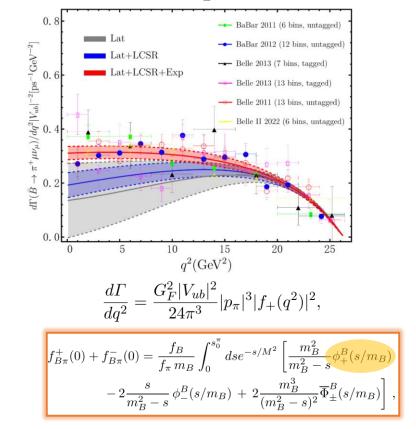


$$\frac{d\Gamma}{dq^2 dE_e dE_{\nu}} = \frac{|V_{jb}|^2 G_F^2}{2\pi^3} \left[W_1 q^2 + W_2 \left(2E_e E_{\nu} - \frac{1}{2} q^2 \right) + W_3 q^2 \left(E_e - E_{\nu} \right) \right],$$

$$W^{\mu\nu} = \sum_{i,j=1}^{3} H_{ij}(\bar{n} \cdot p) \operatorname{tr}\left(\bar{\Gamma}_{j}^{\mu} \frac{\not p_{-}}{2} \Gamma_{i}^{\nu} \frac{1+\not \psi}{2}\right) \int d\omega J(p_{\omega}^{2}) S(\omega)$$

via
$$B \to X_u \ell \nu$$
 [PDG (2024)]
via $B \to \pi \ell \nu$ [PDG (2024)]

• The exclusive process: $B \to \pi \ell \nu$



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Importance of heavy meson LCDA and SF

The form factors for $B \to K^*$ and $B \to \pi$

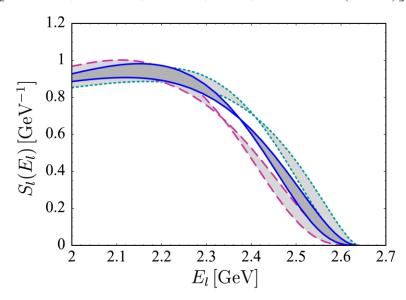
[Gao, Lü, Shen, Wang, Wei, PRD 101 (2020)] [Cui, Huang, Shen, Wang, Wang, JHEP 03 (2023)]

$$\mathcal{V}_{B\to K^*}(0) = 0.359^{+0.141}_{-0.085} \Big|_{\lambda_B} \begin{array}{c} +0.019_{-0.062} \Big|_{\mu} \\ -0.019_{-0.062} \Big|_{\mu} \\ +0.010_{-0.004} \Big|_{M^2} \begin{array}{c} +0.016_{-0.079} \Big|_{\varphi_{\pm}(\omega)}, \\ -0.079_{-0.079} \Big|_{\varphi_{\pm}(\omega)}, \end{array}$$

$$f_{B\to\pi}^{+}(0) = 0.122 \times \left[1 \pm 0.07 \Big|_{S_0^{\pi}} \pm 0.11 \Big|_{\Lambda_q} \pm 0.02 \Big|_{\lambda_E^2/\lambda_H^2 - 0.06} \Big|_{M^2} \pm 0.05 \Big|_{2\lambda_E^2 + \lambda_H^2} \pm 0.06 \Big|_{\mu_h} \pm 0.04 \Big|_{\mu_h} \pm 0.04 \Big|_{\mu_h} \pm 0.04 \Big|_{\mu_h} \pm 0.04 \Big|_{\lambda_B^{-0.56}} \Big|_{\lambda_B} + 0.25 \Big|_{-0.43} \Big|_{\sigma_1, \sigma_2} \right].$$

The lepton energy spectrum for $B \to X_u \ell \nu$ and Br for $B \to X_s \gamma$

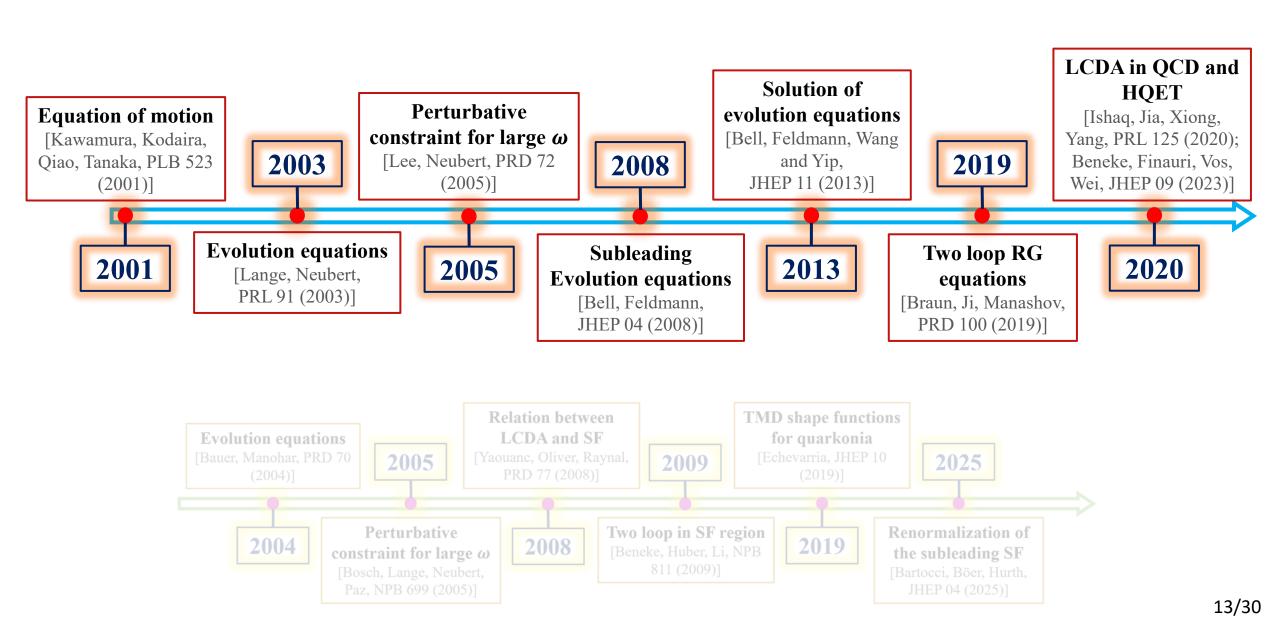
[Bosch, Lange, Neubert, Paz, NPB 699 (2004)] [Czakon, Fiedler, Huber, et.al, JHEP 04 (2015)]



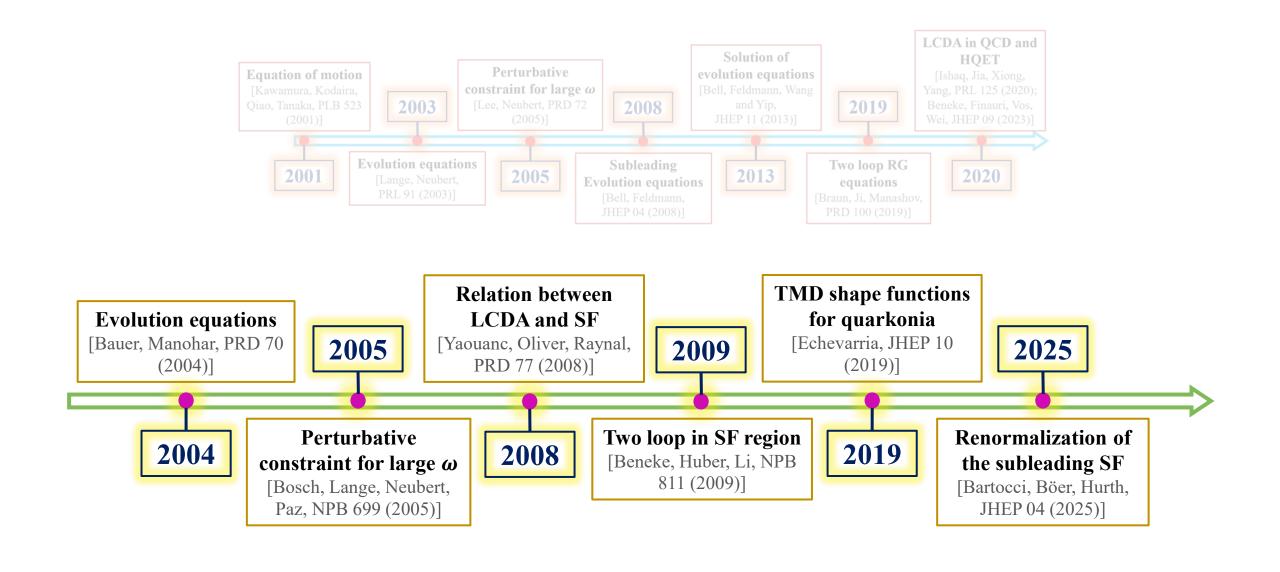
$$\mathcal{B}_{s\gamma}^{\text{SM}} = (3.36 \pm 0.23) \times 10^{-4}$$

The nonperturbative contribution dominated by the shape function is the largest source of uncertainty, accounting for $\sim 60-70\%$.

Our knowledge on LCDA



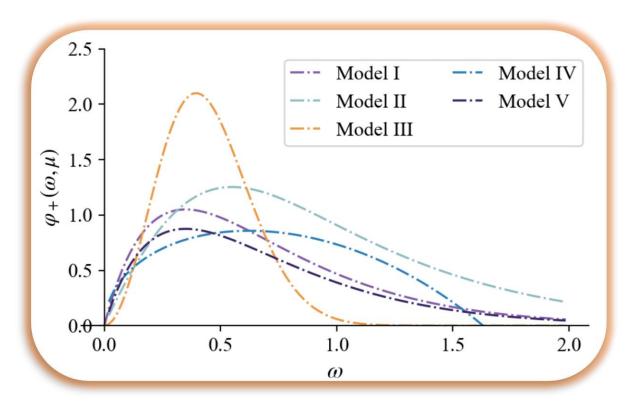
Our knowledge on SF



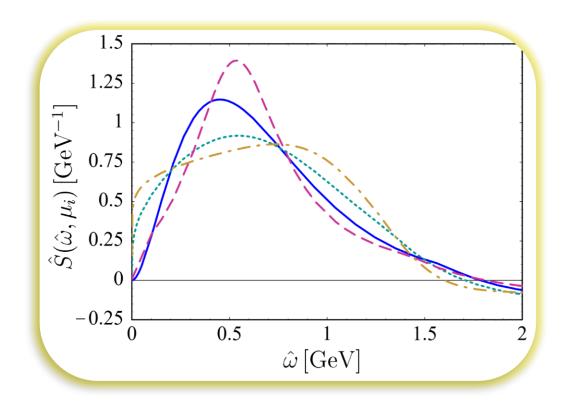
But we do not have the accurate LCDA and SF

Comparing several phenomenological models

Light-cone distribution amplitude

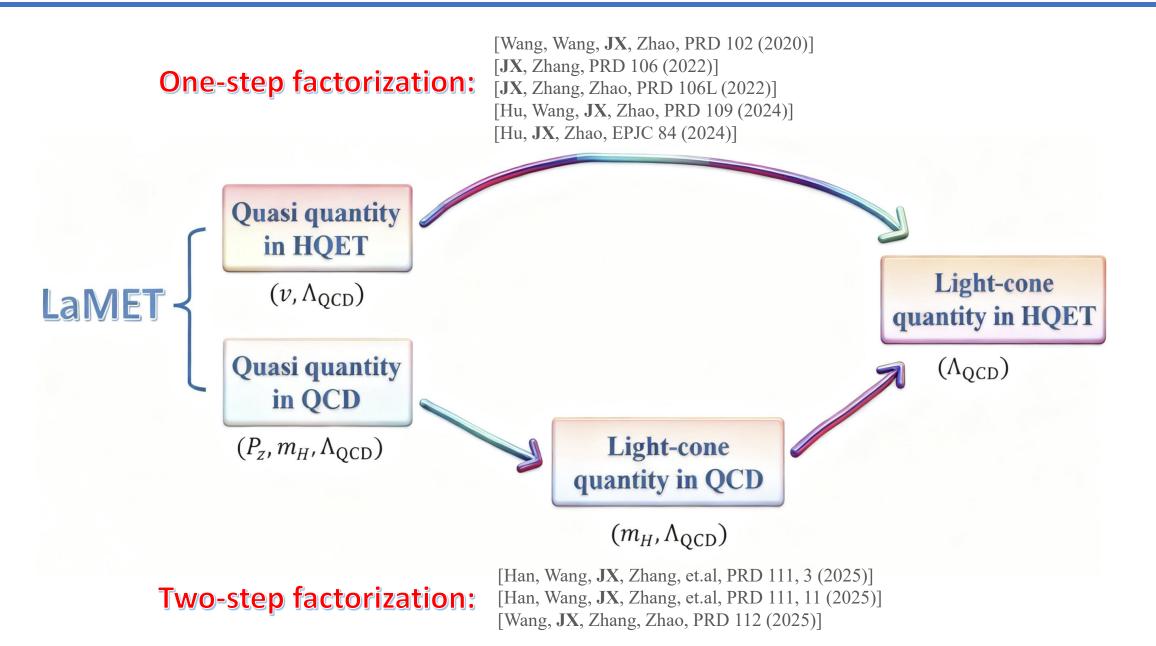


Shape function



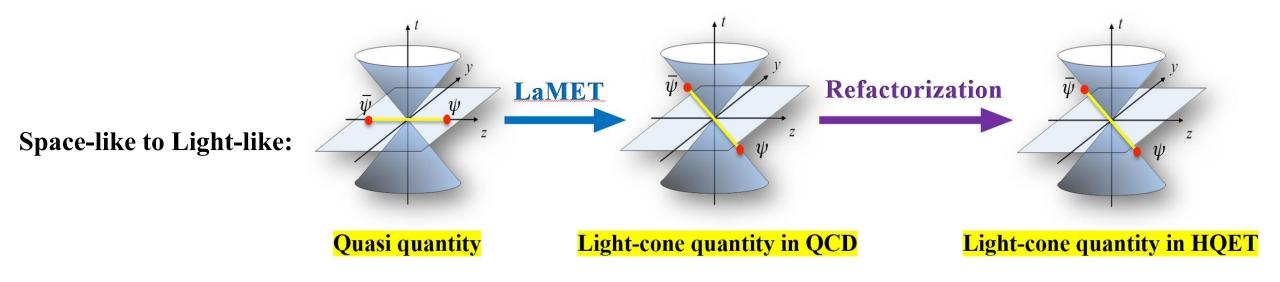
There are considerable differences between various models

Lattice QCD with LaMET

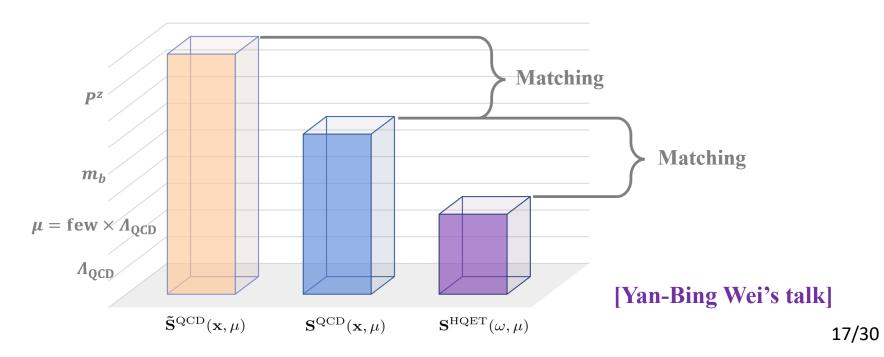


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Lattice QCD with LaMET



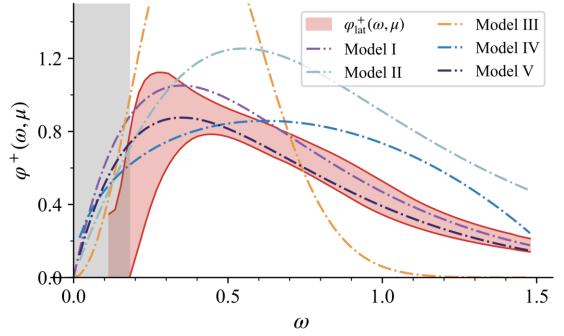
Integrating different scales:



Lattice QCD with LaMET

Light-cone distribution amplitude

- Matching between quasi-DA to LCDA in QCD ✓
- Matching between LCDA in QCD to HQET ✓
- Lattice simulation ✓



[Han, Wang, **JX**, Zhang, et.al, PRD 111, 3 (2025)] [Han, Wang, **JX**, Zhang, et.al, PRD 111, 11 (2025)]

Shape function

- Matching between quasi-SF to SF in QCD ✓
- Matching between SF in QCD to HQET ✓
- Lattice simulation

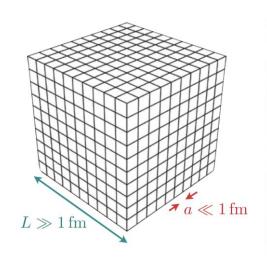
In progress

[Wang, JX, Zhang, Zhao, PRD 112 (2025)]

[Qi-An Zhang's talk]

Lattice QCD in heavy quark physics

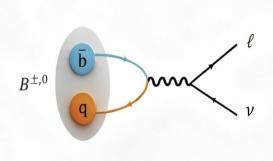
Offering the first-principle results of nonperturbative quantities in heavy quark physics

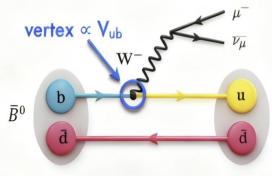


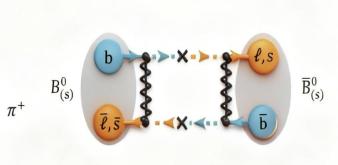




✓ Mixing parameter: $\langle \overline{1} | \mathcal{J}^{\Delta F=2} | 1 \rangle$







LFU violation in $R(D^{(*)})$

- $B \rightarrow D\ell\nu$ form factors at nonzero recoil: MILC collaboration and HPQCD collaboration, PRD 92 (2015)
- Semileptonic form factors for B → D*ℓν at nonzero recoil: MILC collaboration, EPJC 82 (2022)
- $B \to D^*$ and $B_s \to D_s^*$ form factors for the full q2 range: HPQCD collaboration, PRD 109 (2024)

The $|V_{ub}|$ and $|V_{cb}|$ puzzle

- $B_s \to D_s^*$ form factors for the full q^2 range from lattice QCD: HPQCD collaboration, PRD 105 (2022)
- $|V_{ub}|$ from $B \to \pi \ell \nu$ decays and (2+1)-flavor lattice QCD: Bailey et.al, PRD 92 (2015)
- $B_s \rightarrow D_s$ form factors form factors for the full q^2 range from lattice QCD: HPQCD collaboration, PRD 105 (2020)

$B \to K^{(*)} \ell \ell$ anomoly

- Rare decay $B \to K\ell\ell$ form factors from lattice QCD: HPQCD collaboration, PRD 88 (2013)
- SM predictions for $B \to K\ell\ell$ and $B \to K\nu\nu$ from Lattice QCD: Parrott, Bouchard, Davies, PRD 107 (2023)

[Lei-Yi Li's talk]
[Yu Meng's talk]

Precise theoretical prediction



Precise theoretical prediction: Loop corrections

NLO weak-annihilation correction to rare $B \to \{K, \pi\} \ell^+ \ell^-$ decays

- $B \to \{K, \pi\} \ell^+ \ell^-$ is an important probe for studying flavor-changing processes and new physics signals in the Standard Model.
- Using SCET, the contributions of weak annihilation processes have been systematically calculated at NLO.

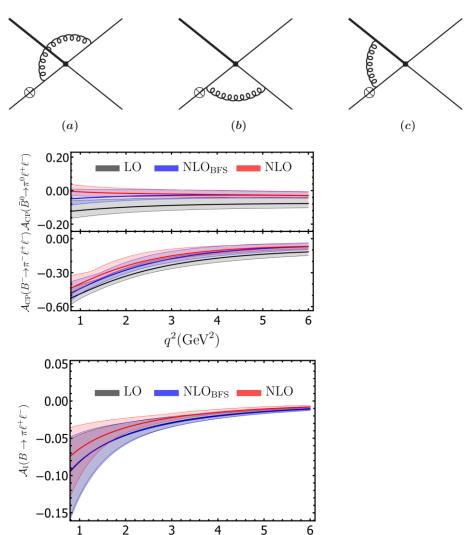
$$\mathcal{QCD} \to \mathcal{SCET}_{II} \to \mathcal{SCET}_{II}$$

$$\mathcal{T}_{P}^{(t,u)} = C_{P}^{(t,u)}(q^{2}) f_{BP}^{+}(q^{2}) - \frac{\pi^{2}}{N_{c}} \frac{\mathcal{F}_{B} f_{P}}{m_{B}} \sum_{m=\pm}$$

$$\int_{0}^{\infty} \frac{d\omega}{\omega} \int_{0}^{1} du T_{P,m}^{(t,u)}(\omega, u, \mu) \phi_{B,m}(\omega, \mu) \phi_{P}(u, \mu)$$

$$\Sigma C_{i} \mathbb{H}_{i} \otimes \mathbb{J}_{\parallel}$$

[Huang, Shen, Wang, Wang, PRL 134 (2025)]



 $q^2(\text{GeV}^2)$

Precise theoretical prediction: Power corrections

Deciphering the long-distance penguin contribution to $\overline{B}_{d,s} o \gamma \gamma$ decays

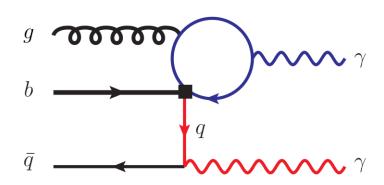
- These processes offer a remarkably clean environment to address the intricate strong interaction mechanism.
- The leading-power contribution has been investigated in QCD factorization approach. However, the long-distance penguin contribution has not been considered before.
- The power counting

$$m_b \gg m_c \sim \sqrt{m_b \Lambda_{QCD}} \gg \Lambda_{QCD}$$

A new B-meson LCDA

$$\langle 0|(\bar{q}_s S_n)(\tau_1 n) (S_n^{\dagger} S_{\bar{n}})(0) (S_{\bar{n}}^{\dagger} g_s G_{\mu\nu} S_{\bar{n}})(\tau_2 \bar{n}) \bar{n}^{\nu} \not n \gamma_{\perp}^{\mu} \gamma_5 (S_{\bar{n}}^{\dagger} h_v)(0) |\bar{B}_v \rangle$$

$$= 2 \tilde{f}_B(\mu) m_B \int_0^{\infty} d\omega_1 \int_0^{\infty} d\omega_2 \exp\left[-i(\omega_1 \tau_1 + \omega_2 \tau_2)\right] \Phi_{G}(\omega_1, \omega_2, \mu),$$



$$\mathcal{H}\cdot\mathcal{J}\starar{\mathcal{J}}\star\Phi_G$$

[Qin, Shen, Wang, Wang, PRL 131 (2023)]

Theoretical efforts and progress

LFU violation in $R(D^{(*)})$

- Shedding new light on R(D*): Cui, Huang,
 Wang, Zhao, PRD 108 (2023)
- On sum rules for semi-leptonic $b \rightarrow c$ and $b \rightarrow u$ decays: Li, Yang, et.al, JHEP 07 (2025)
- Probing semileptonic decay with new physics effects in the PQCD approach: Zhang, Sun, Xing, Zhu, PRD 111 (2025)
- Global fit to $b \rightarrow c\tau\nu$ anomalies as of Sping 2024: Iguro, Kitahara, Watanabe, PRD 110 (2024)
- Analysis of charmed B meson decays B →
 D: Ou-Yang, Li, Zhou, PRD 112 (2025)
- Semileptonic and nonleptonic $\bar{B}_s \to D_{sJ}$ decays: Qi, Li, Zhao, EPJC 85 (2025)

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$m{B} ightarrow m{K}^{(*)} \ell \ell$ anomoly

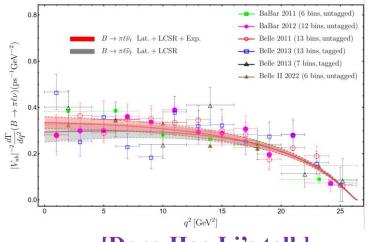
- NLO weak-annihilation correction to rare $B \to \{K, \pi\} \ell^+ \ell^-$ decays: Huang, Shen, Wang, Wang, PRL 134 (2025)
- Precision calculations of $B \to K^*$ form factors from SCET sum rules: Gao, Meißner, Shen, Li, PRD 112 (2025)
- Renormalization-group evolution for the bottom-meson soft function: Huang, Ji, Shen, Wang, Wang, Zhao, PRL 133 (2024)
- Deciphering the Long-Distance Penguin Contribution to $\bar{B}_{d,s} \rightarrow \gamma \gamma$ decays: Qin, Shen, Wang, Wang, PRL 131 (2023)
- The rare decay $B^+ \to K^+ \ell^+ \ell^- (\nu \bar{\nu})$ under the QCD sum rules approach: Tian, Fu, Zhong, Wang, Wu, PRD 111 (2025)

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The $|V_{ub}|$ and $|V_{cb}|$ puzzle

- Investigating the ratio of CKM matrix elements $|V_{ub}/V_{cb}|$: Zhong, Fu, Wu, PRD 105 (2022)
- Extracting IM of LCDA and $|V_{ub}|$ from $B \rightarrow P$ form factors: Li, Lü, Meißner, Gao, In progress

.



[Dong Hao Li's talk]

Theoretical efforts and progress

Leptonic *B* decays

- Factorization and Sudakov resummation in leptonic radiative B decay: Galda, Neubert, Wang, JHEP 07 (2022)
- QCD factorization for the $B \to \gamma \ell \nu$ decay beyond leading power: Cui, Shen, Wang, Wei, EPJC 85 (2025)
- QED corrections to $B_q \to \tau^+ \tau^-$: Huang, Shen, Zhao, Zhou, JHEP 10 (2023)

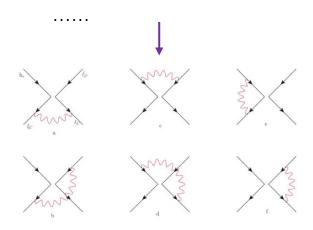


Figure 2. One-loop QED corrections to hard functions.

[Si-Hong Zhou's talk]

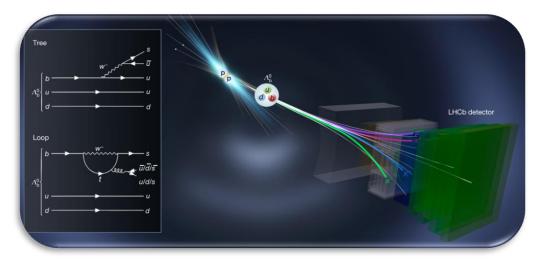
Nonleptonic B decays

- Study of the $B^+ \to \pi^+(\pi^+\pi^-)$ decay in PQCD approach: Chang, Yang, Zou, Li, EPJC 84 (2024)
- Systematic analysis of $B_s \rightarrow SP$ decays in perturbative QCD approach: Zou, He, Wang, Li, EPJC 85 (2025)
- NLO order QCD corrections to $B_c^* \rightarrow J/\psi$ form factors: Chang, Tao, Xiao, Zhu, PRD 111 (2025)
- Study of B → PP decays in the modified PQCD approach: Lü, Wang, Yang, PRD 110 (2024)
- On the equivalence of Flavor SU(3) analyses of $B \rightarrow PP$ decays: Shi, Wang, **JX**, ePrint: 2505.21283
- Form factors of light pseudoscalar mesons from the PQCD approach: Chai, Cheng, JHEP 06 (2025)

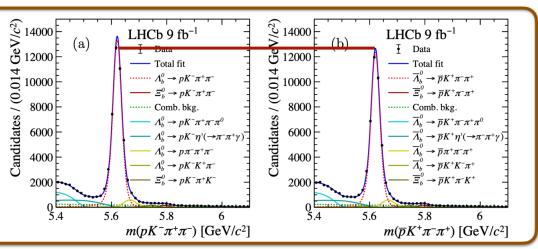
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Era of high precision, challenges and opportunities

A milestone in particle physics: First observation of baryon CP violation



$$\Lambda_b^0 o p K^- \pi^+ \pi^ \mathcal{A}_{CP} = (2.45 \pm 0.46 \pm 0.10)\%$$
 5.2σ



Experiment

$\Lambda_h^0 o p K_S^0 h^-$, LHCb, JHEP 04 (2014) $\Lambda_h^0 o p\pi^-\pi^+\pi^-$, LHCb, Nature Phys. 13 (2017) $\Lambda_h^0 \rightarrow p\pi^-K^+K^-$, LHCb, Nature Phys. 13 (2017) $\Lambda_h^0 \rightarrow ph^-$, LHCb, PRD 11 (2025) $\Lambda_h^0 \rightarrow \Lambda h^+ h^-$, LHCb, PRL 134 (2025) $\Lambda_c^+ \rightarrow ph^+h^-$, LHCb, JHEP 03 (2018) $\Xi_c^+ o pK^-\pi^+$, LHCb, EPJC 80 (2020) $\Sigma_0 \rightarrow \Lambda \pi^0$, BESIII, PRD 108 (2023) $\Sigma_0 \rightarrow \Lambda \gamma$, BESIII, PRL 133 (2024) $\Lambda_c^+ \to ph^+h^-, \Xi_c^+ \to \Sigma^+h^+h^-$, Belle II collaboration $\Lambda_c^+ \to \Lambda h^+, \Lambda_c^+ \to \Sigma^0 h^+, \quad \text{Belle, Sci.Bull. 68 (2023)}$

Theory

A novel strategy for searching for CPV in the baryon sector: Zhang, Guo, JHEP 07 (2021)

New extraction of CPV in b-baryon decays: Geng et.al, PLB 834 (2022)

Searching for CP violation in four-body decays of bottom and charmed baryons: Zhang, PRD 107 (2023)

Estimates of CPV observables in Λ_b decay: Zhou et.al, PRD 107 (2023)

Possible large CP violation in charmed Λ_b decays: Yu, Wang, Qin, PRD 108 (2023)

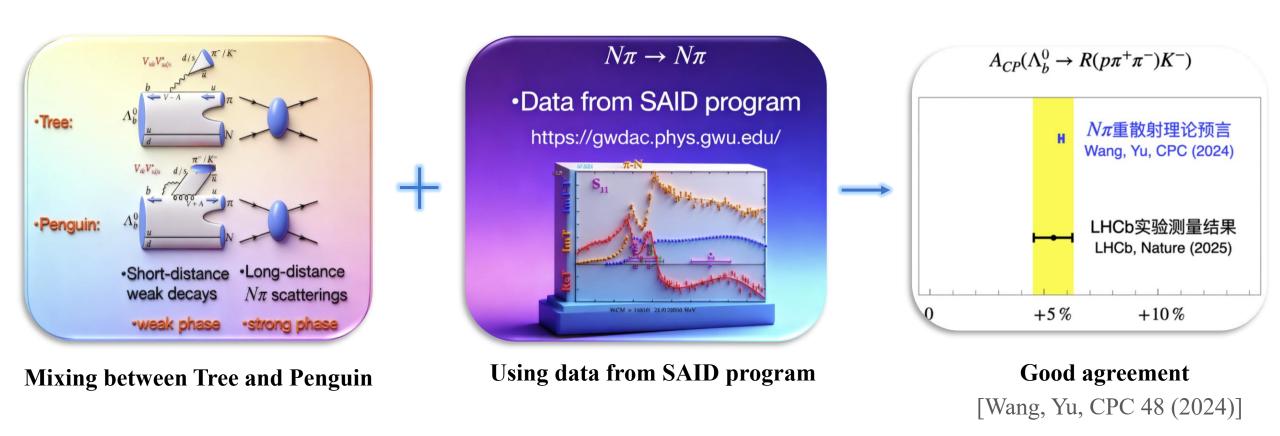
Complementary CPV induced by T-odd and T-even correlations: Wang, Qin, Yu, PRD 111 (2025)

Establishing CP Violation in b-Baryon Decays: Han, Li, Xiao, Yu et.al, PRL 134 (2025)

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[Zhi-Peng Xing's talk]
[Tian-Liang Feng's talk]

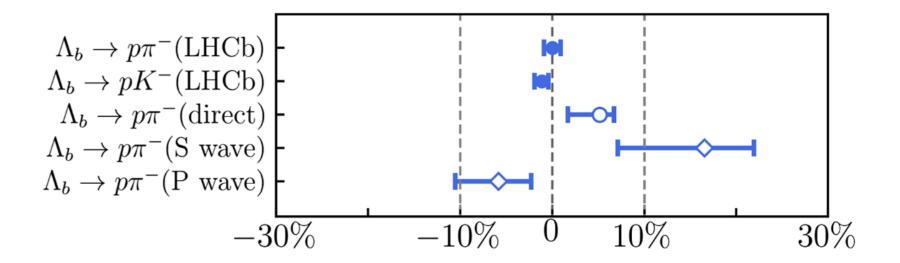
- Studying the processes of $\Lambda_b^0 \to (p\pi^+\pi^-)h^-$ and $(p\pi^0)h^-$ with $h^- = p$ or K.
- The global CPV of the above processes in the invariant mass regions of $N\pi$ scatterings are at the order of several percent.



Good agreement between theory and experiment

Cancellation between different partial wave CPVs.

	$A_{CP}^{ m dir}$	$A_{CP}^{S ext{-wave}}(\kappa_S)$	$A_{CP}^{P ext{-wave}}(\kappa_P)$
$\Lambda_b \to p\pi^-$	$0.05^{+0.02}_{-0.03}$	$0.17^{+0.05}_{-0.09}(49\%)$	$-0.06^{+0.04}_{-0.05}(51\%)$
$\Lambda_b \to pK^-$	$-0.06^{+0.03}_{-0.02}$	$-0.05^{+0.05}_{-0.04}(94\%)$	$-0.21^{+0.39}_{-0.46}(6\%)$



[Han, Yu, Li, Li, Wang, Xiao, Yu PRL 134 (2025)]

[Jia-Jie Han's talk]

Topics not included

- Multi-body B decays
- Inclusive decays of heavy quarks
- Hadronic B decays in PQCD
- The charm physics
- Heavy quarkonium
- Heavy quark exotics
- NP in heavy flavor physics

[Shan Cheng's talk]

[Qin Qin's talk]

[Rui Zhou's talk] [Ya Li's talk]

[Xiao-Hui Hu's talk]

[Xiang-Peng Wang's talk] [Zi-Qiang Chen's talk]

[Rui-Lin Zhu's talk]

[Xing-Bo Yuan's talk]

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Summary

- Heavy quark physics is now entering a precision era, thanks to advances in both experiment and theory.
- Several anomalies and puzzles between experiment data and theoretical calculations in SM are observed.
- Through efforts in nonperturbative inputs, loop corrections and power corrections, theoretical predictions will match the precision of experimental data. Providing crucial support for understanding QCD and hadron structure, as well as searching for new physics.

Back up

$$B \to \pi \pi \text{ puzzle}$$

$$Br(B^0 \to \pi^0 \pi^0) = (1.55 \pm 0.16) \times 10^{-6}$$

Much larger than expected. NNLO hard function does not work.

 $B \rightarrow \pi K$ puzzle

$$\Delta A_{CP} = A_{CP}(B^- \to K^- \pi^0) - A_{CP}(B^0 \to K^- \pi^+) = (11.3 \pm 1.2)\%$$

9 σ deviation from isospin limit.

$$B \rightarrow K K$$
 puzzle

$$R_{KK}^{sd} = \left| \frac{V_{td}}{V_{ts}} \right|^2 \frac{\Gamma(B_s \to K^0 \overline{K}^0)}{\Gamma(B_d \to K^0 \overline{K}^0)} = 0.62 \pm 0.14$$

Over 3σ deviation from SM prediction.

$$B \rightarrow D_q^{(*)} L$$
 puzzle

 $R_{\pi}^{(*)}$, R_K , $R_{S\pi}$: Over 4σ deviation from SM prediction.

• The definition on LCD defined in HQET

[Grozin, Neubert, PRD 55 (1997)]

$$i\tilde{f}_B(\mu)m_B\phi^+(\omega,\mu) = \int_{-\infty}^{+\infty} \frac{d\eta}{2\pi} e^{i\bar{n}\cdot v\omega\eta} \langle 0|\bar{q}(\eta\bar{n}) W(\eta\bar{n},0) \bar{\eta}\gamma_5 h_v(0)|B(v)\rangle.$$

Here h_v is the heavy quark field defined in HQET. The variable has support $0 < \omega < \infty$.

The definition on SF defined in HQET

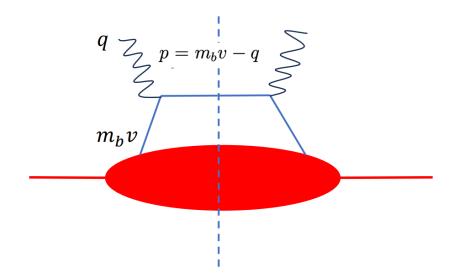
[Neubert, PRD 49 (1994)]

$$S(\omega,\mu) = \int_{-\infty}^{+\infty} \frac{dt}{2\pi} e^{i\omega v^{+}t} \frac{\langle B(v)|\bar{h}_{v}(0) W(0,tn_{+}) h_{v}(tn_{+})|B(v)\rangle}{\langle B(v)|\bar{h}_{v}(0) h_{v}(0)|B(v)\rangle}.$$

The variable has support $-\infty < \omega < \overline{\Lambda}$, with $\overline{\Lambda} = m_B - m_b$. "Fermi motion" of the heavy quark.

QCD analysis for inclusive B decays

Inclusive decay rate is related to the hadronic tensor



$$W^{\mu\nu} = rac{1}{\pi} \operatorname{Im} rac{\langle \bar{B}(v) | T^{\mu\nu} | \bar{B}(v) \rangle}{2M_B}$$

$$T^{\mu\nu} = i \int d^4x \, e^{iq\cdot x} \, \mathrm{T} \left\{ J^{\dagger\mu}(0), J^{\nu}(x) \right\}$$

$$J^{\mu} = \bar{u}\gamma^{\mu} (1 - \gamma_5) b$$

 $p = m_b v - q$: momentum of light partons

- When $p^{\mu} \sim m_b \gg \Lambda_{QCD}$: heavy quark expansion (local OPE)
- When $p^{\mu}=(p^+,p^-,p_{\perp})\sim E(\lambda,1,\sqrt{\lambda})$: twist expansion (light-cone OPE) $\lambda=\frac{\Lambda_{QCD}}{E}$ Infinite set of power corrections are resummed into the shape function

B meson inclusive decays and shape function

The differential decay rates are expressed in terms of shape function

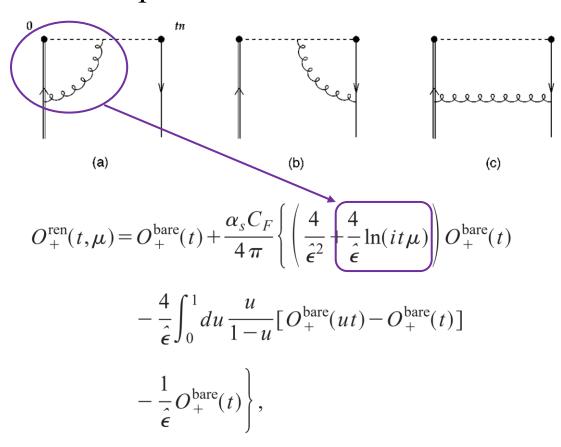
$$B \to X_u \,\ell \,\nu \qquad \frac{d\Gamma}{dE_\ell} = \frac{G_F^2 |V_{ub}|^2 m_b^4}{96\pi^3} \int d\omega \theta (m_b - 2E_\ell - \omega) S(\omega)$$

$$B \to X_s \gamma \qquad \frac{d\Gamma}{dE_\gamma} = \frac{G_F^2 |V_{tb}V_{ts}^*|^2 \alpha |C_7^{\text{eff}}|^2 m_b^5}{32\pi^4} S(E_\gamma)$$

Bauer, Manohar, 2003; Bosch, Lange, Neubert, Paz, 2004,...

- Shape function reflects the fermi motion of b-quark inside B-meson
- Shape function is universal
- Shape function is defined with matrix elements of nonlocal HQET operators

- The LCDAs are defined on the light-cone. [Grozin, Neubert, PRD 55, 272-290 (1997)]
- Non-negative moments $\int dk \ k^n \varphi_B^+(k)$ for n=0,1,2 ... are not related to matrix elements of local operators.



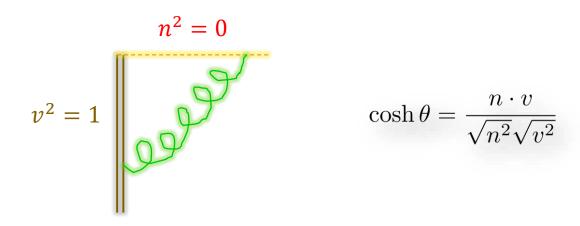
[Braun, Ivanov, Korchemsky, PRD 69, 034014 (2004)]

$$O_{+}^{\mathrm{ren}}(t,\mu) = O_{+}^{\mathrm{bare}}(t) + \frac{\alpha_{s}C_{F}}{4\pi} \left\{ \left(\frac{4}{\hat{\epsilon}^{2}} \underbrace{\frac{4}{\hat{\epsilon}} \ln(it\mu)} \right) O_{+}^{\mathrm{bare}}(t) \right. \\ \left. \left[\bar{q}(tn) \eta [tn,0] \Gamma h_{v}(0) \right]_{R} \neq \sum_{p=0}^{\infty} \frac{t^{p}}{p!} \left[\bar{q}(0) (\stackrel{\leftarrow}{D} \cdot n)^{p} h_{v}(0) \right]_{R} \right\} \\ \left. \left[\frac{1}{\hat{\epsilon}^{2}} \underbrace{\frac{4}{\hat{\epsilon}} \ln(it\mu)} \right] O_{+}^{\mathrm{bare}}(t) \\ \left. \left[\frac{1}{\hat{\epsilon}^{2}} \underbrace{\frac{4}{\hat{\epsilon}} \ln(it\mu)} \right] O_{+}^{\mathrm{bare}}(t) \right\} \\ \left. \left[\frac{1}{\hat{\epsilon}^{2}} \underbrace{\frac{4}{\hat{\epsilon}^{2}} \ln(it\mu)} \right] O_{+}^{\mathrm{bare}}(t) \\ \left. \left[\frac{1}{\hat{\epsilon}^{2}} \underbrace{\frac{4}{\hat{\epsilon}^{2}} \ln(it\mu)} \right] O_{+}^{\mathrm{bare}}(t) \right] \right\} \\ \left. \left[\frac{1}{\hat{\epsilon}^{2}} \underbrace{\frac{4}{\hat{\epsilon}^{2}} \ln(it\mu)} \right] O_{+}^{\mathrm{bare}}(t) \\ \left. \left[\frac{1}{\hat{\epsilon}^{2}} \underbrace{\frac{4}{\hat{\epsilon}} \ln(it\mu)} \right] O_{+}^{\mathrm{bare}}(t) \right] \right\} \\ \left. \left[\frac{1}{\hat{\epsilon}^{2}} \underbrace{\frac{4}{\hat{\epsilon}^{2}} \ln(it\mu)} \right] O_{+}^{\mathrm{bare}}(t) \\ \left. \left[\frac{1}{\hat{\epsilon}^{2}} \underbrace{\frac{4}{\hat{\epsilon}} \ln(it\mu)} \right] O_{+}^{\mathrm{bare}}(t) \right] \right] \\ \left. \left[\frac{1}{\hat{\epsilon}^{2}} \underbrace{\frac{4}{\hat{\epsilon}^{2}} \ln(it\mu)} \right] O_{+}^{\mathrm{bare}}(t) \\ \left. \left[\frac{1}{\hat{\epsilon}^{2}} \underbrace{\frac{4}{\hat{\epsilon}^{2}} \ln(it\mu)} \right] O_{+}^{\mathrm{bare}}(t) \right] \\ \left. \left[\frac{1}{\hat{\epsilon}^{2}} \underbrace{\frac{4}{\hat{\epsilon}^{2}} \ln(it\mu)} \right] O_{+}^{\mathrm{bare}}(t) \\ \left. \left[\frac{1}{\hat{\epsilon}^{2}} \underbrace{\frac{4}{\hat{\epsilon}^{2}} \ln(it\mu)} \right] O_{+}^{\mathrm{bare}}(t) \right] \right] \right\}$$

Cannot get φ_B^+ by their moments

$$\langle H(p_H)|\bar{h}_v(0)\eta_+\gamma_5[0,tn_+]q_s(tn_+)|0\rangle = -i\tilde{f}_H m_H n_+ \cdot v \int_0^\infty d\omega e^{i\omega t n_+ \cdot v} \varphi_+(\omega;\mu).$$

Cusp divergence:

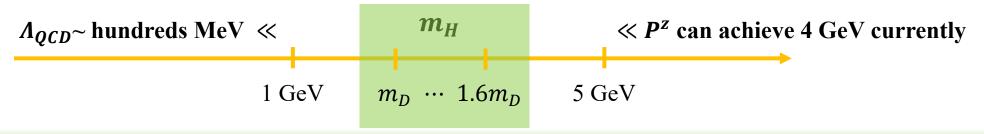


- \checkmark $n^2 \neq 0$, still heavy quark field h_v
- \checkmark $n^2 \neq 0$, and No h_v : QCD heavy quark

> Two-step factorization to access heavy meson LCDA.



 \Rightarrow Hierarchy $\Lambda_{\rm QCD} \ll m_H \ll P_z$:



A big challenge for lattice simulation but still calculable on the lattice

 \triangleright The factorization formula between $S^{\rm QCD}$ and $S^{\rm HQET}$.

$$S^{\text{QCD}}(x,\mu) = \begin{cases} Z_{\text{peak}}(x,\omega,\mu) \otimes S^{\text{HQET}}(\omega,\mu) \,, & 1-x \sim \mathcal{O}(\lambda) \,, & \text{(peak region)} \\ \\ Z_{\text{tail}}(x,\mu) \,, & 1-x \sim \mathcal{O}(1) \,, & \text{(tail region)} \end{cases}$$

Expand the shape functions and matching coefficient,

$$S^{\text{QCD}}(x,\mu) = S^{\text{QCD}(0)}(x,\mu) + \frac{\alpha_s C_F}{2\pi} S^{\text{QCD}(1)}(x,\mu) + \mathcal{O}(\alpha_s^2),$$

$$S^{\text{HQET}}(\omega,\mu) = S^{\text{HQET}(0)}(\omega,\mu) + \frac{\alpha_s C_F}{2\pi} S^{\text{HQET}(1)}(\omega,\mu) + \mathcal{O}(\alpha_s^2).$$

$$\begin{split} Z_{\text{peak}}(x,\omega,\mu) &= Z_{\text{peak}}^{(0)}(x,\omega,\mu) \\ &+ \frac{\alpha_s C_F}{2\pi} Z_{\text{peak}}^{(1)}(x,\omega,\mu) + \mathcal{O}(\alpha_s^2) \,, \\ Z_{\text{tail}}(x,\mu) &= Z_{\text{tail}}^{(0)}(x,\omega,\mu) \\ &+ \frac{\alpha_s C_F}{2\pi} Z_{\text{tail}}^{(1)}(x,\mu) + \mathcal{O}(\alpha_s^2) \,. \end{split}$$

The matching function at one-loop

$$Z_{\text{tail}}^{(1)}(x,\mu) = \frac{1}{m_b v^+} \frac{1+x^2}{1-x} \left[-1 + \ln \frac{\mu^2}{(1-x)^2 m_b^2} \right].$$

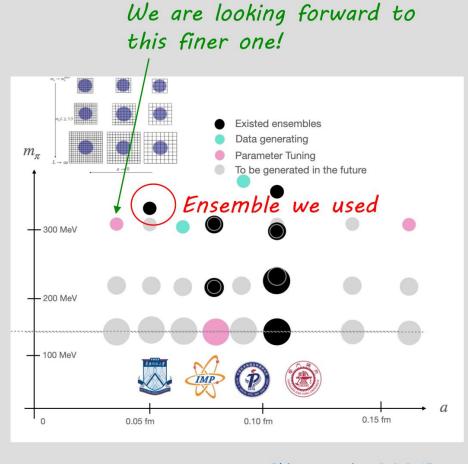
$$Z_{\text{peak}}^{(1)}(x,\omega,\mu) = \left(\frac{1}{2}\ln^2\frac{\mu^2}{m_b^2} - \frac{3}{2}\ln\frac{\mu^2}{m_b^2} + \frac{\pi^2}{12} - 2\right)\delta(xm_Bv^+ - m_bv^+ - \omega v^+).$$

- 1. This result implies the validity of the factorization formula.
- 2. The plus distributions cancel out in the matching, yielding remarkably simple form of factorization formula.
- 3. The "refactorization framework" developed in this work is in a similar spirit to LCDAs defined in QCD and HQET.

• Simulating on the finest CLQCD ensemble:

$$n_s^3 \times n_t = 48^3 \times 144, a \approx 0.052 \text{fm};$$

- $m_{\pi} \simeq 317 \text{MeV}, m_D \simeq 1.92 \text{GeV};$
- $P^z = \{2.99, 3.49, 3.98\} \text{GeV}$ up to about 4 GeV;
- Dispersion relation consistent with the relativistic one up to possible discretization error;
- The state-of-the-art techniques in renormalization and extrapolation on the lattice are adopted.



[Hu, et·al·, 2004]

> First inverse moment (IM)

$$\lambda_B^{-1}(\mu) \equiv \int_{-\infty}^{\infty} d\omega \frac{\phi_B^+(\omega, \mu)}{\omega} \,.$$

✓ We determine the IM by employing a model-independent parametrization formula.

μ		$\lambda_B \; ({ m GeV})$	$\sigma_B^{(1)}$
$1 {\rm GeV}$	N = 1	0.389(35)	1.63(8)
	Ref. [31]	> 0.24	
1GeV	Ref. [18]	0.383(153)	
	Ref. [7]	0.48(11)	1.6(2)
	Ref. [15]	0.46(11)	1.4(4)
	Ref. [1]	0.35(15)	
	Ref. [21]	$0.343^{+0.064}_{-0.079}$	
	Ref. [73]	0.338(68)	

[e-Print: 2403.17492]

[PRD 98, 112016 (2018)]

[JHEP 10, 043 (2020)]

[PRD 72, 094082 (2005)]

[PRD 69, 034014 (2004)]

[PRD 55, 272 (1997)]

[PRD 101, 7, 074035 (2020)]

[PLB 848, 138345 (2024)]

$$C_{9,P}^{(i)}(q^{2}) = C_{9} \, \delta^{it} + \frac{2m_{b}}{m_{B}} \, \frac{\mathcal{T}_{P}^{(i)}(q^{2})}{f_{BP}^{+}(q^{2})} \,. \tag{6}$$

$$\mathcal{J}_{P}^{(A0)} = \frac{\mathcal{F}_{B}m_{B}}{2} \int_{0}^{\infty} d\omega \, \frac{\mathbb{J}_{\parallel,-}(n \cdot q, \bar{n} \cdot q, \omega)}{\bar{n} \cdot q - \omega + i \, 0} \phi_{B,-}(\omega, \mu),$$

$$\mathcal{J}_{P}^{(B1)} = \frac{\mathcal{F}_{B}m_{B}}{2} \int_{0}^{\infty} d\omega \, \mathbb{J}_{\parallel,+}(n \cdot q, \bar{n} \cdot q, \omega, \tau) \phi_{B,+}(\omega, \mu).$$

$$\mathcal{A}_{CP} = \frac{\Gamma(\bar{B} \to \bar{P}\ell^{+}\ell^{-}) - \Gamma(B \to P\ell^{+}\ell^{-})}{\Gamma(\bar{B} \to \bar{P}\ell^{+}\ell^{-}) + \Gamma(B \to P\ell^{+}\ell^{-})},
\mathcal{A}_{I} = \frac{\kappa \Gamma(\bar{B}^{0} \to \bar{P}^{0}\ell^{+}\ell^{-}) - \Gamma(B^{-} \to P^{-}\ell^{+}\ell^{-})}{\kappa \Gamma(\bar{B}^{0} \to \bar{P}^{0}\ell^{+}\ell^{-}) + \Gamma(B^{-} \to P^{-}\ell^{+}\ell^{-})}, (20)$$

with the factor $\kappa = 2, 1$ for $P = \pi^0, K^0$. Un-

[3] for their explicit definitions). It turns out that such subleading power corrections can enhance the theory predictions for the mixing induced CP asymmetries $\mathcal{A}_{\mathrm{CP}}^{\mathrm{mix}, \parallel}$ and $\mathcal{A}_{\mathrm{CP}}^{\mathrm{mix}, \perp}$ by approximately an amount of $\mathcal{O}(30\,\%)$ with the default inputs, while yielding insignificant effects in the remaining observables numerically. Moreover, our

Multi-body decays of Λ_b

- Advantage: more resonances, more chances for large CPV
- •Disadvantage: Too many resonances, and with large uncertainties

N(1650)	1/2-	****
N(1675)	$5/2^-$	****
N(1680)	5/2 ⁺	••••
N(1700)	3/2-	***
N(1710)	1/2+	••••
N(1720)	3/2+	••••

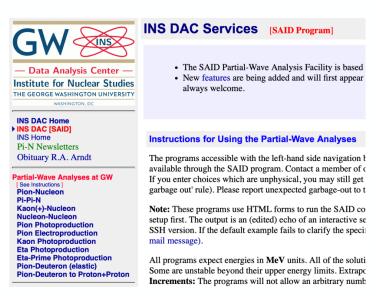
N(1700) Breit-Wigner Mass	1650 to $1800~(pprox1720)$ MeV
$\it N(1700)$ Breit-Wigner width	$100 ext{ to } 300 \ (pprox 200) ext{ MeV}$
$\it N(1710)$ Breit-Wigner Mass	$1680 ext{ to } 1740 \ (pprox 1710) ext{ MeV}$
$N\!(1710)$ Breit-Wigner width	$80 ext{ to } 200 \ (pprox 140) ext{ MeV}$
$N\!(1720)$ Breit-Wigner Mass	$1680 ext{ to } 1750 ext{ (}pprox 1720 ext{)}$ MeV
$\it N(1720)$ Breit-Wigner width	$150 ext{ to } 400 \ (pprox 250)$ MeV

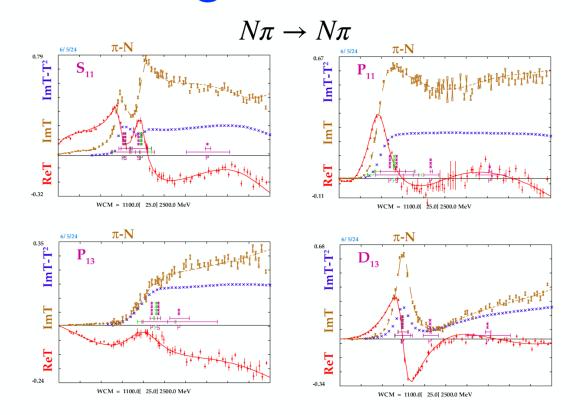
Close to each other, with large decay widths. No clear dominant one.

$N\pi$ scatterings

- • N^* usually from $N\pi$ scatterings
- Data from SAID program

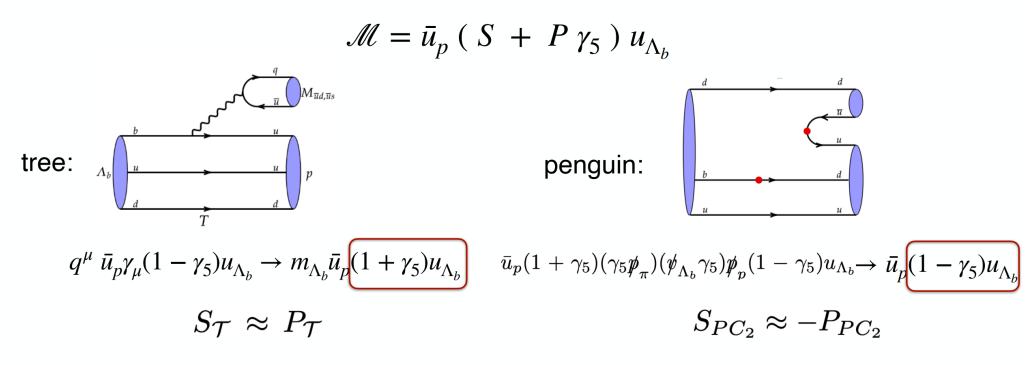
https://gwdac.phys.gwu.edu/





- Partial-wave amplitudes with strong phases!
- Data driven, model independent. Circumvent N*, more precise strong phases.

CPV cancelled between S- and P-waves



- •CPVs of S- and P-waves might be as large as B mesons, but cancelled with each other.
- Baryons have spinors and Dirac structures, and thus partial waves.