

Determination of B-meson distribution amplitudes from B to P form factors

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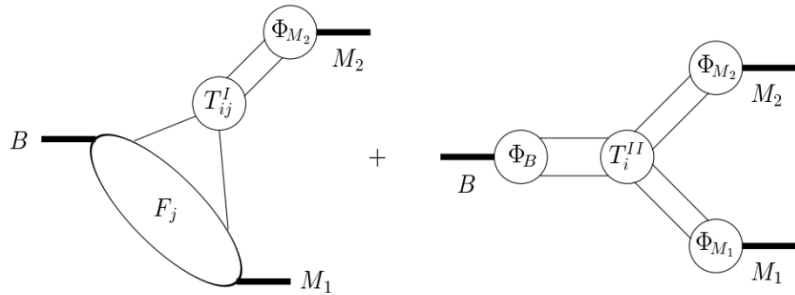
B-meson Light-cone Distribution Amplitude and Light-cone Sum Rules

LCDA: encodes the **momentum distribution** of the light spectator parton at soft-scale
fundamental ingredient for establishing factorization formula in exclusive process

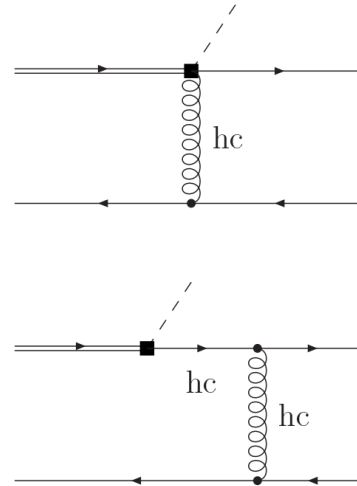
[Lepage and Brodsky, PLB (1979); Efremov and Radyushkin, PLB (1980)]

[Grozin and Neubert, PRD (1997); Beneke, Buchalla, Neubert and Sachrajda, PRL (1999)]

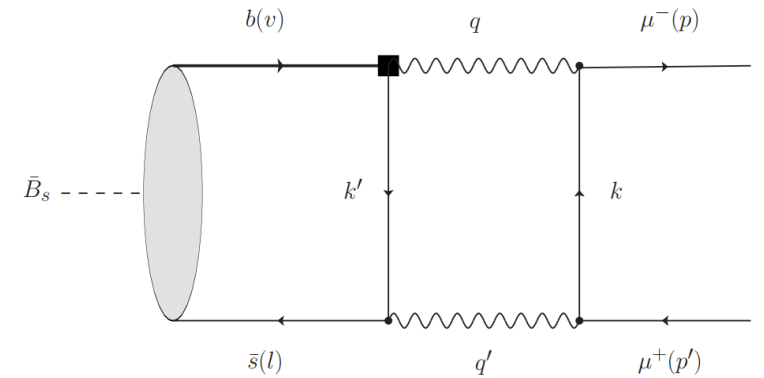
[Keum, H-N Li and Sanda, PRD (2001); Keum, Kurimoto, H-N Li and C-D Lu, PRD (2004)]



$$B \rightarrow \pi\pi/\pi K$$



$$B \rightarrow \pi/K \text{ or } B \rightarrow D$$



$$B_S \rightarrow \mu^+ \mu^-$$

B-meson Light-cone Distribution Amplitude and Light-cone Sum Rules

LCDA: renormalization group equation, perturbative QCD constraints, equations of motion, matching function and the model-independent determination of LCDA on the Euclidean lattice

$w \rightarrow 0$ or $w \rightarrow \infty$

[Lange, Neubert, PRL 2003; Bell, Feldmann, Wang, Yip, JHEP 2013]
[Braun, Ji, Manashov, JHEP 2017]
[Lee, Neubert, PRD 2005;Feldmann, Lange, Wang, PRD 2014]
[Wang, Wang, Xu, Zhao, PRD 2020; Beneke, Finauri, Vos, Wei, JHEP 2023]

Inverse moment: $\lambda_B = \int dw \phi_B^+(w)/w \sim \Lambda_{\text{QCD}}$

	QCD sum rule	LCSR $B \rightarrow \gamma$	LCSR $B \rightarrow \pi$	LCSR $B \rightarrow K$	LQCD LaMET	Traditional interval
λ_B [MeV]	460(110) 383(153)	310(60)-415(60) >214	303-389 460(160)	338^{+68}_{-9} 472^{+110}_{-41}	389(35)	350(150)
arxiv	hep-ph/0309330 2008.03935	2106.13616 1606.03080	1506.00667 0504091	2308.07033	2403.17492	

B-meson Light-cone Distribution Amplitude and Light-cone Sum Rules

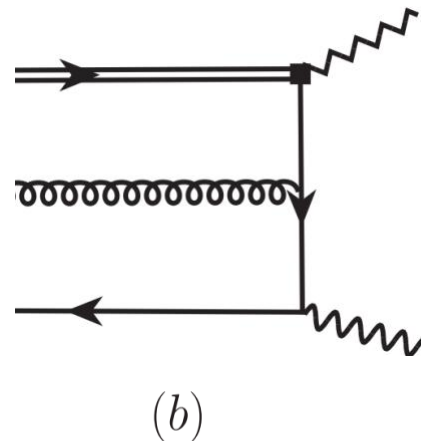
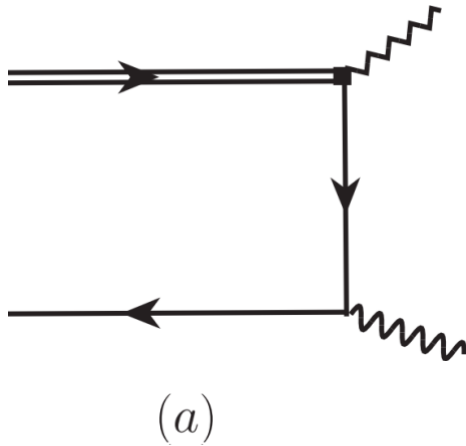
Higher order corrections in B decays: LCSRs with B-LCDA

Leading order results [Khodjamirian, Mannel, Offen, PLB 2005; PRD 2007]

Next-to-leading-logarithmic accuracy [Wang, Shen, NPB 2015; Wang, Wei, Shen, Lü, JHEP 2017]

higher-twist two-particle and three particle contributions [Gubernari, Kokulu, Dyk, JHEP 2019]

Next-to-leading power corrections [Cui, Huang, Shen, Wang, Wang, JHEP 2023; Gao, Huber, Ji, Wang, Wang, Wei, JHEP 2022]



B-meson Light-cone Distribution Amplitude and Light-cone Sum Rules

The starting point is two-point correlation function (2pt)

$$\Pi_\mu(n \cdot p, \bar{n} \cdot p) = i \int d^4x e^{ip \cdot x} \langle 0 | \mathcal{T} \{ \bar{d}(x) \not{n} \gamma_5 q_1(x), \bar{q}_1(0) \Gamma_\mu b(0) \} | \bar{B}(p_B) \rangle, \quad (1)$$

Calculated at the external momenta far below the hadronic thresholds

$$p^2 \ll m_\pi^2 \text{ and } q^2 \ll m_B^2, \quad n \cdot p = \frac{m_B^2 - q^2}{m_B} \sim \mathcal{O}(m_b), \quad \bar{n} \cdot p \sim \mathcal{O}(\Lambda_{\text{QCD}}),$$

Assuming the quark-hadron duality above the effective threshold ω_S

And applying the Borel transform $\bar{n} \cdot p \rightarrow \omega_M$, the sum rule of $f_{B\pi}^+$ can be obtained,

$$f_{B\pi}^+(q^2) = \frac{f_B m_B}{f_\pi n \cdot p} \text{Exp} \left[\frac{m_\pi^2}{n \cdot p w_M} \right] \int_0^{w_s} d\omega' \text{Exp} \left(-\frac{\omega'}{w_M} \right) \boxed{\phi_B^-(\omega, \mu)}.$$

Determination of the Inverse moment using B to P form factors

$$f_{B\pi}^+(q^2) = \frac{f_B m_B}{f_\pi n \cdot p} \text{Exp} \left[\frac{m_\pi^2}{n \cdot p w_M} \right] \int_0^{w_s} d\omega' \text{Exp} \left(-\frac{\omega'}{w_M} \right) \boxed{\phi_B^-(\omega, \mu)}$$

B-meson LCDA defined by non-local operator,

$$\langle 0 | \bar{d}_\beta(\tau \bar{n}) [\tau \bar{n}, 0] b_\alpha(0) | \bar{B}(p_B) \rangle = -\frac{i f_B m_B}{4} \left\{ \frac{1 + \not{p}}{2} \left[2 \tilde{\phi}_B^+(\tau) + \left(\tilde{\phi}_B^-(\tau) - \tilde{\phi}_B^+(\tau) \right) \not{n} \right] \gamma_5 \right\}_{\alpha\beta} \quad (9)$$

$$\phi_B^\pm(\omega', \mu) = \int_{-\infty}^{+\infty} \frac{d\tau}{2\pi} e^{i\omega'\tau} \tilde{\phi}_B^\pm(\tau, \mu), \quad \tilde{\phi}_B^\pm(\tau, \mu) = \int_0^{+\infty} d\omega' e^{-i\omega'\tau} \phi_B^\pm(\omega', \mu), \quad (10)$$

The key parameter in modeling LCDA is the inverse moments of the leading twist LCDA,

$$\frac{1}{\lambda_B(\mu)} = \int_0^\infty \frac{d\omega}{\omega} \phi_B^+(\omega, \mu), \quad \frac{\hat{\sigma}_n(\mu)}{\lambda_B(\mu)} = \int_0^\infty \frac{d\omega}{\omega} \ln^n \frac{e^{-\gamma_E} \lambda_B(\mu)}{\omega} \phi_B^+(\omega, \mu). \quad (11)$$

We adopt the three-parameter ansatz [M. Beneke, V. M. Braun, Y. Ji and Y. B. Wei, JHEP (2018)]

$$\phi_B^+(\omega) = \frac{\Gamma(\beta)}{\Gamma(\alpha)} \frac{\omega}{\omega_0^2} e^{-\omega/\omega_0} U(\beta - \alpha, 3 - \alpha, \omega/\omega_0)$$

Determination of the Inverse moment using B to P form factors

$$\phi_B^+(\omega) = \frac{\Gamma(\beta)}{\Gamma(\alpha)} \frac{\omega}{\omega_0^2} e^{-\omega/\omega_0} U(\beta - \alpha, 3 - \alpha, \omega/\omega_0)$$

The parameters α, β, ω_0 can be determined via the inverse moments

$$\lambda_B(\mu) = \frac{\alpha - 1}{\beta - 1} \omega_0,$$

$$\hat{\sigma}_1(\mu) = \psi(\beta - 1) - \psi(\alpha - 1) + \ln \frac{\alpha - 1}{\beta - 1},$$

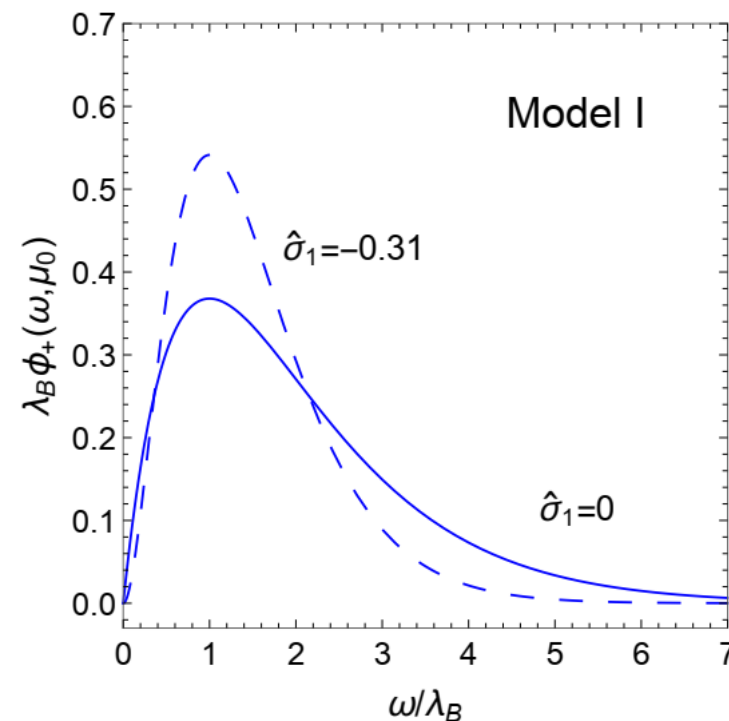
$$\hat{\sigma}_2(\mu) = \hat{\sigma}_1^2(\mu) + \psi'(\alpha - 1) - \psi'(\beta - 1) + \frac{\pi^2}{6},$$

With the choices

$$\lambda_B \in [200, 500] \text{ MeV}$$

$$\hat{\sigma}_1 \in [-0.7, 0.7]$$

$$\hat{\sigma}_2 \in [-6, 6] \quad \Leftrightarrow \text{Shape parameters, Model dependent}$$



[Beneke, Braun, Ji, Wei, JHEP 2018]

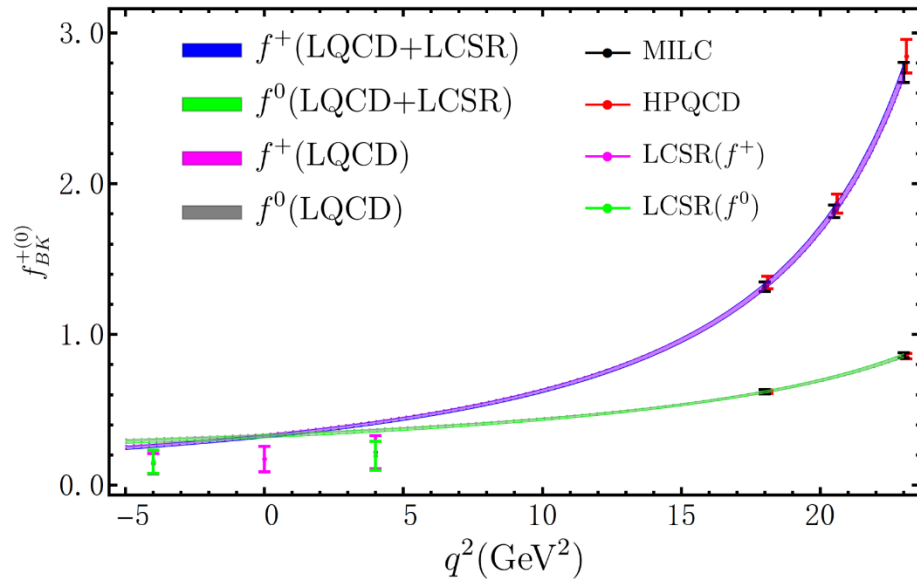
$$\hat{\sigma}_1 = 0, \hat{\sigma}_2 = \pi^2/6$$

corresponds to the exponential model

$$\phi_B^+(\omega) = \left(\frac{\omega}{\lambda_B^2} \right) \text{Exp} \left[-\frac{\omega}{\lambda_B} \right]$$

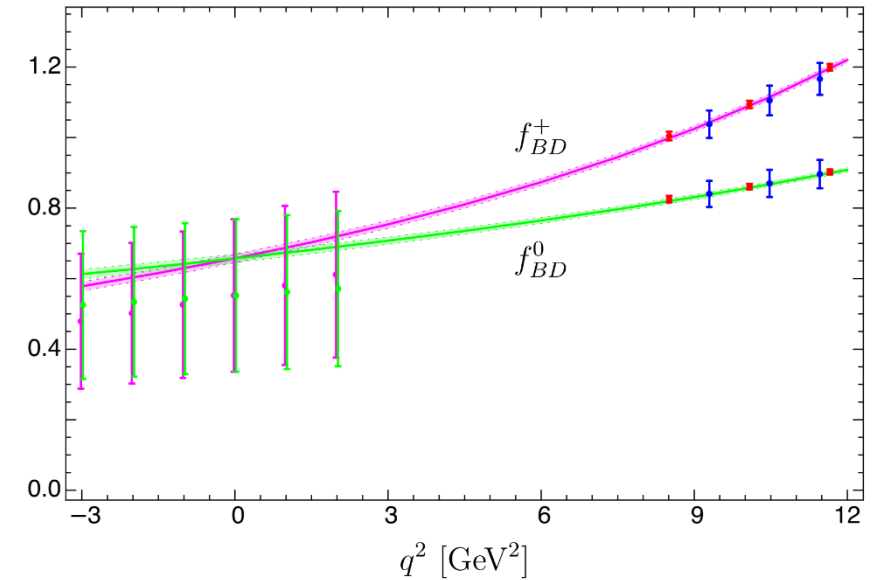
Determination of the Inverse moment using B to P form factors

$$\lambda_B \in [200, 500] \text{ MeV}, \hat{\sigma}_1 \in [-0.7, 0.7], \hat{\sigma}_2 \in [-6, 6]$$



$B \rightarrow K$

[Cui, Huang, Shen, Wang, Wang, JHEP 2023]



$B \rightarrow D$

[Gao, Huber, Ji, Wang, Wang, Wei, JHEP 2022]

Determination of the Inverse moment using B to P form factors

$$\chi^2 = \chi_{\text{LQCD}}^2 + \chi_{\text{LCSR}}^2 + \chi_{\text{exp}}^2$$

Inputs:

1. $B \rightarrow \pi, B \rightarrow K, B \rightarrow D$ form factors $f_{B\pi}^{+,0}$ $f_{BD}^{+,0}$ $f_{BK}^{+,0}$ f_{BK}^T from Lattice QCD
2. experimental measurements on q^2 -binned branching fractions of $B \rightarrow \pi \ell \nu$

Related by BCL parametrization

$$z(q^2, t_0) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}} \quad q^2 > (m_B + m_P)^2 \Rightarrow |z| = 1, \quad 0 < q^2 < (m_B - m_P)^2 \Rightarrow z < 1$$

$$f_{B_{q'}M}^+(q^2) = \frac{1}{1 - q^2/m_{B_q^*}^2} \sum_{k=0}^{N-1} b_k^+ \left[z(q^2, t_0)^k - (-1)^{k-N} \frac{k}{N} z(q^2, t_0)^N \right] \quad N = 3$$

22 adjustable parameters : BCL $b_k^i + V_{\text{ub}} + \lambda_B$

40 LQCD data points : $f_{B\pi}^{+,0}$ 6×2 , $f_{BK}^{+,0}$ f_{BK}^T 8×2 , $f_{BD}^{+,0}$ 6×2

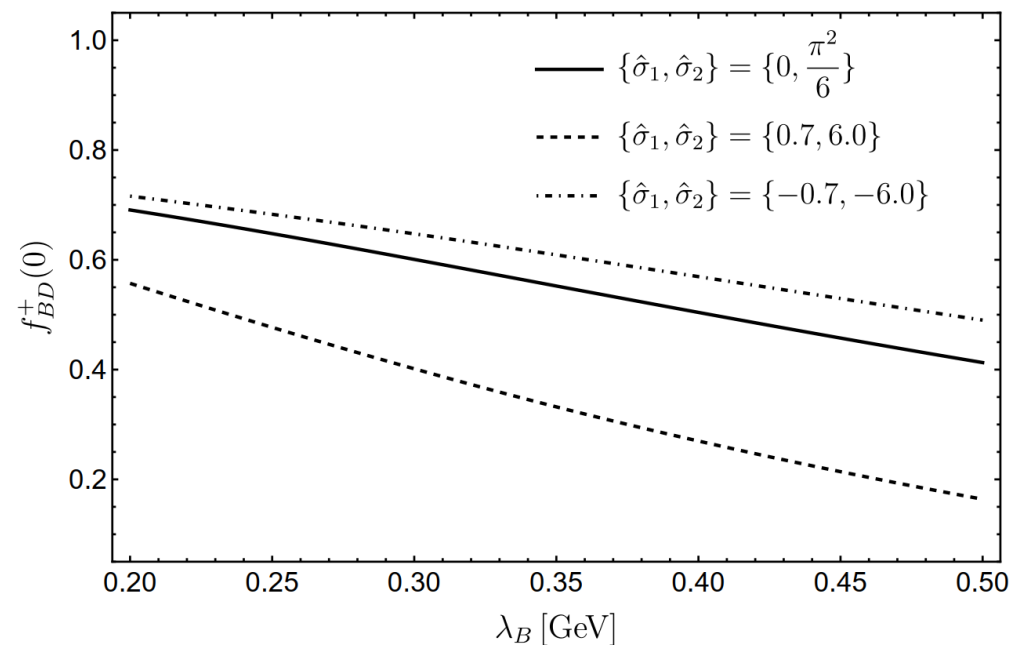
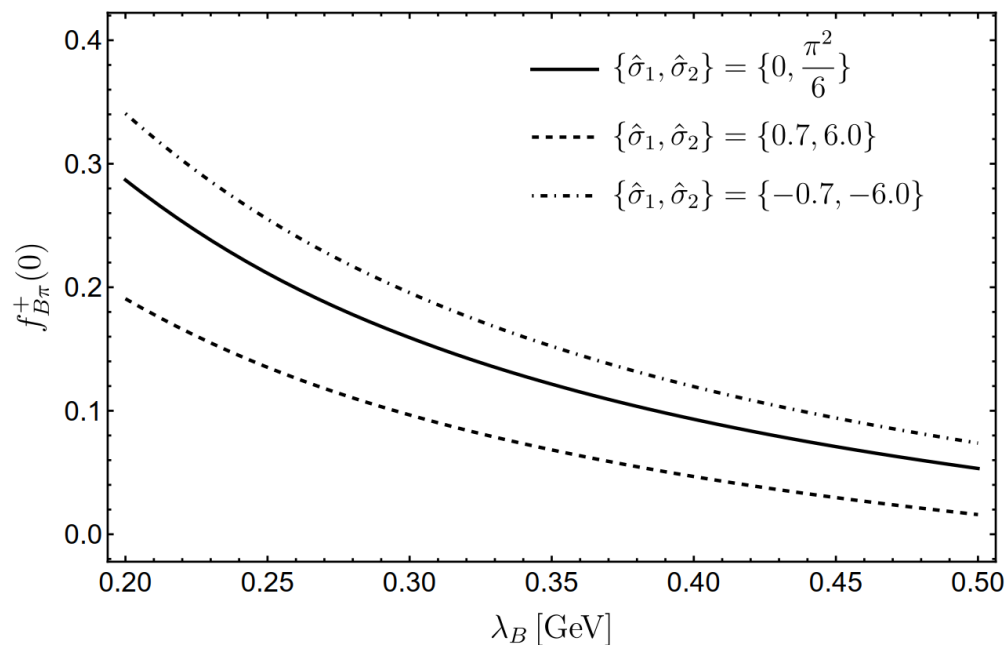
57 data points from experimental measurements

d.o.f = 75

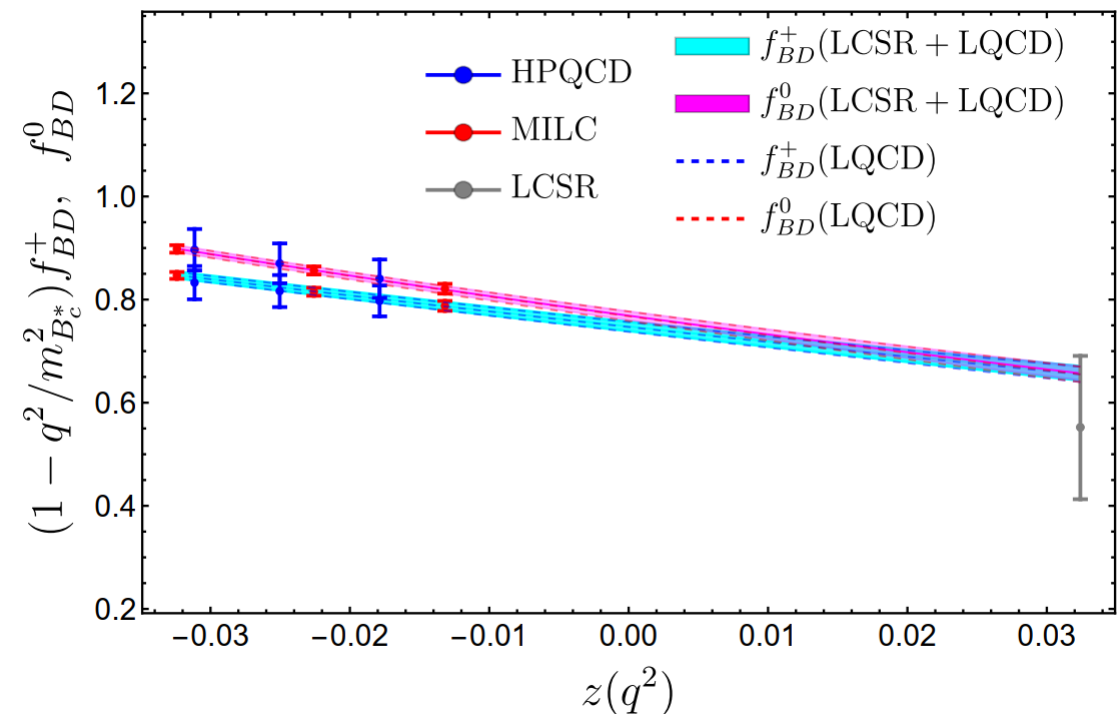
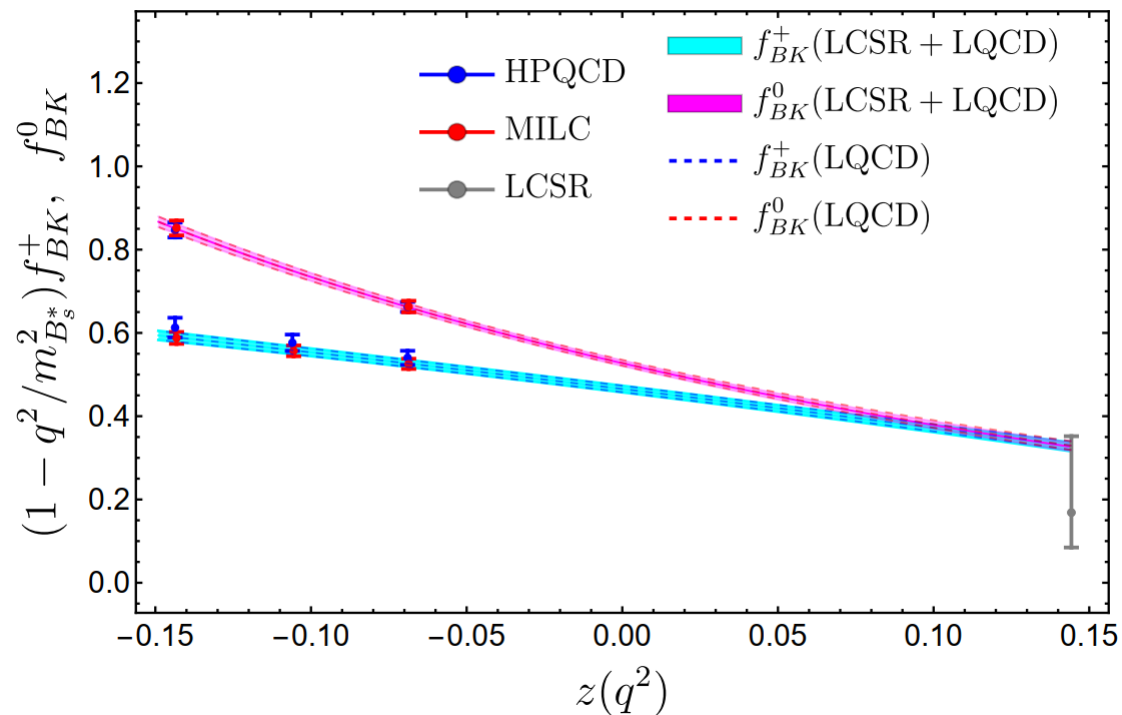
Determination of the Inverse moment using B to P form factors

$$\chi^2_{\text{LCSR}} = \sum_{j,k=1}^4 [f_j(0; b_i) - f_j(0; \lambda_B)] C_{jk}^{-1} [f_k(0; b_i) - f_k(0; \lambda_B)] \quad f_{B\pi}^+ \quad f_{BD}^+ \quad f_{BK}^+ \quad f_{BK}^T \text{ at } q^2 = 0$$

Covariance $C_{jk} = 10\% \times f_j$ uncorrelated + $10\% \times f_j^{\text{NLP}}$ correlated



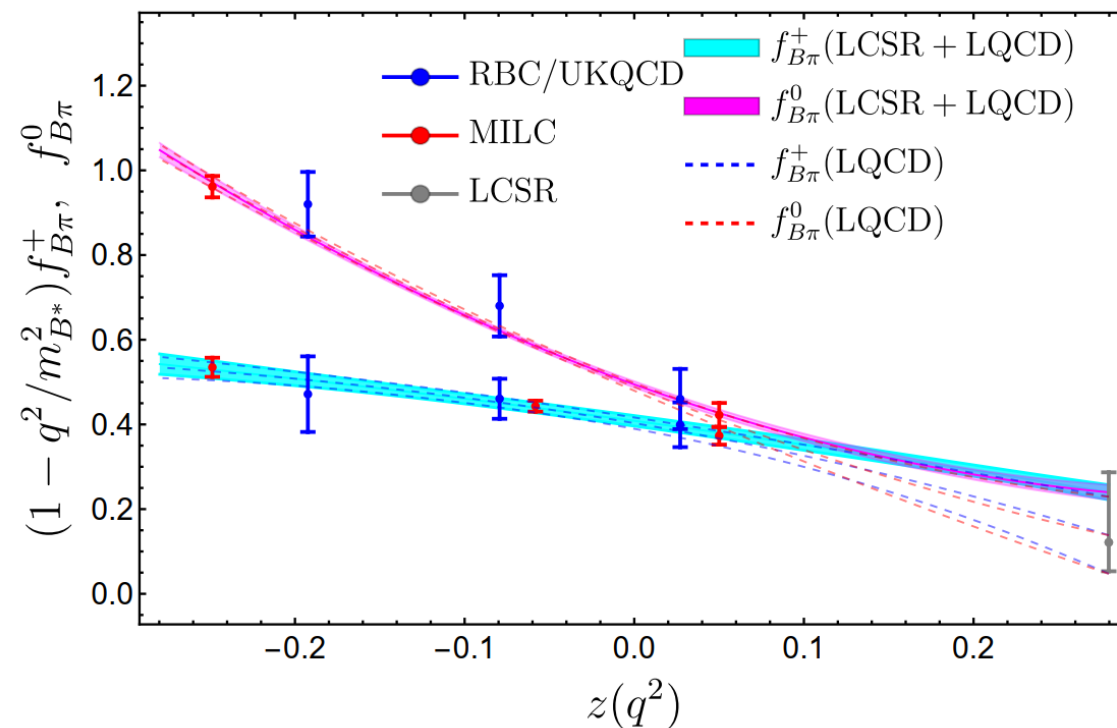
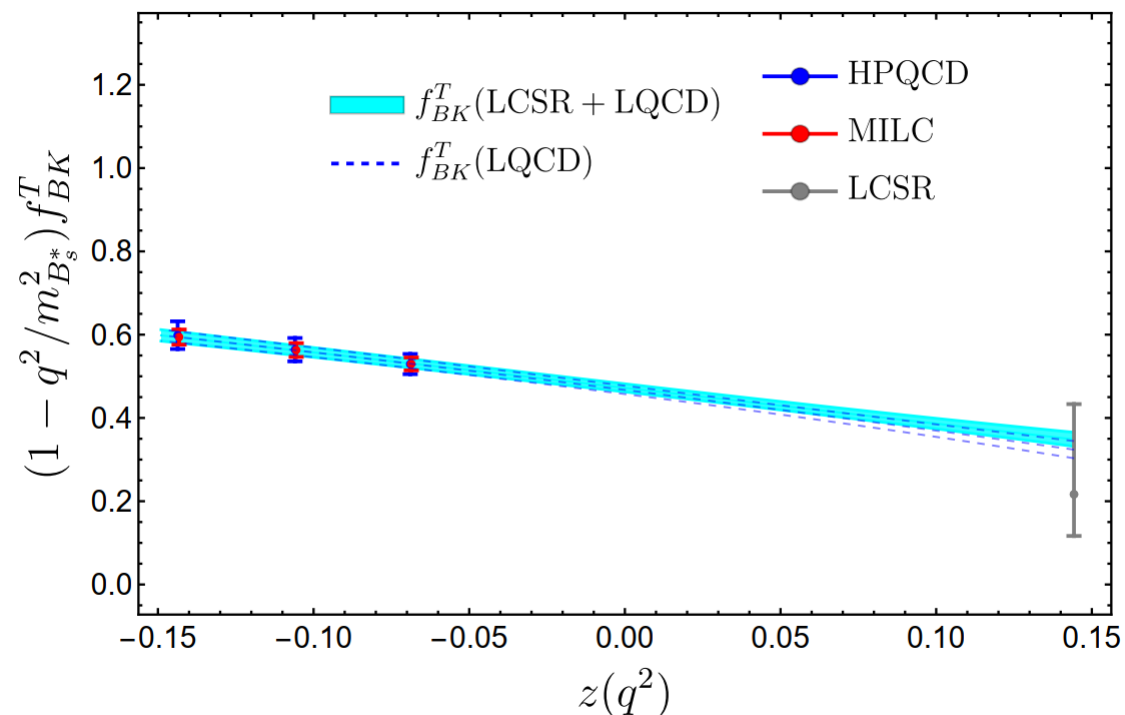
Analysis and Results



Gray line represents the LCSR calculated form factors with the interval $\lambda_B \in [200, 500]$ MeV and $\hat{\sigma}_1 = 0, \hat{\sigma}_2 = \pi^2/6$

The change of $\hat{\sigma}_1, \hat{\sigma}_2$ resulting an overall shift of the gray lines $\hat{\sigma}_1 \in [-0.7, 0.7]$
 $\hat{\sigma}_2 \in [-6, 6]$

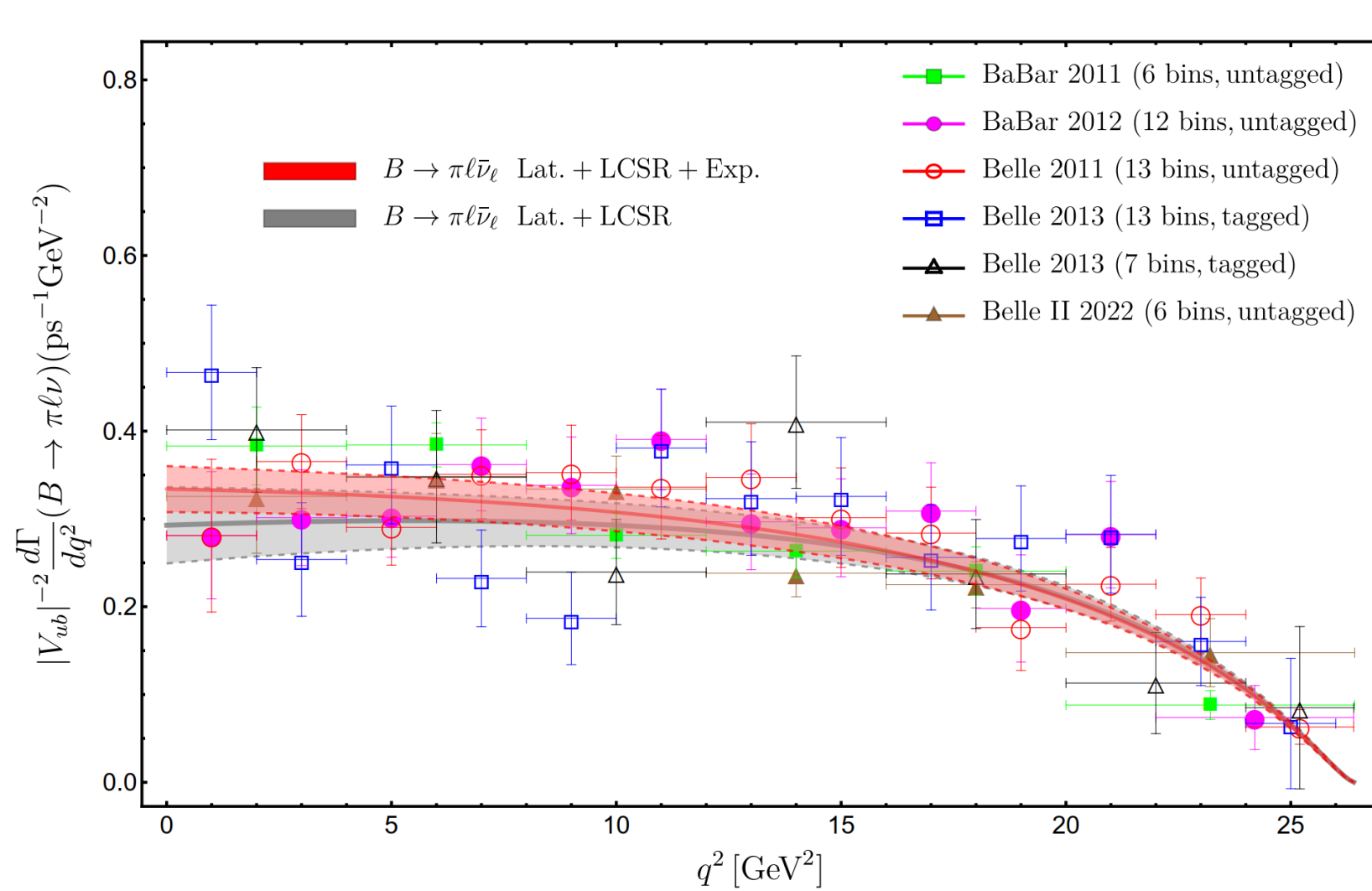
Analysis and Results



Gray line represents the LCSR calculated form factors with the interval $\lambda_B \in [200, 500]$ MeV and $\hat{\sigma}_1 = 0, \hat{\sigma}_2 = \pi^2/6$

The change of $\hat{\sigma}_1, \hat{\sigma}_2$ resulting an overall shift of the gray lines $\hat{\sigma}_1 \in [-0.7, 0.7]$
 $\hat{\sigma}_2 \in [-6, 6]$

Analysis and Results



Results:

$$\chi^2/\text{dof} = 1.1$$

$$\lambda_B = 221(9)_{-21}^{+87} \text{ MeV}$$

$$V_{ub} = 3.71(10)_{-1}^{+1} \times 10^{-3}$$

$$f_{B\pi}^+(0) = 0.256(10)_{-1}^{+1}$$

$$\hat{\sigma}_1 < 0.39$$

Constraint:

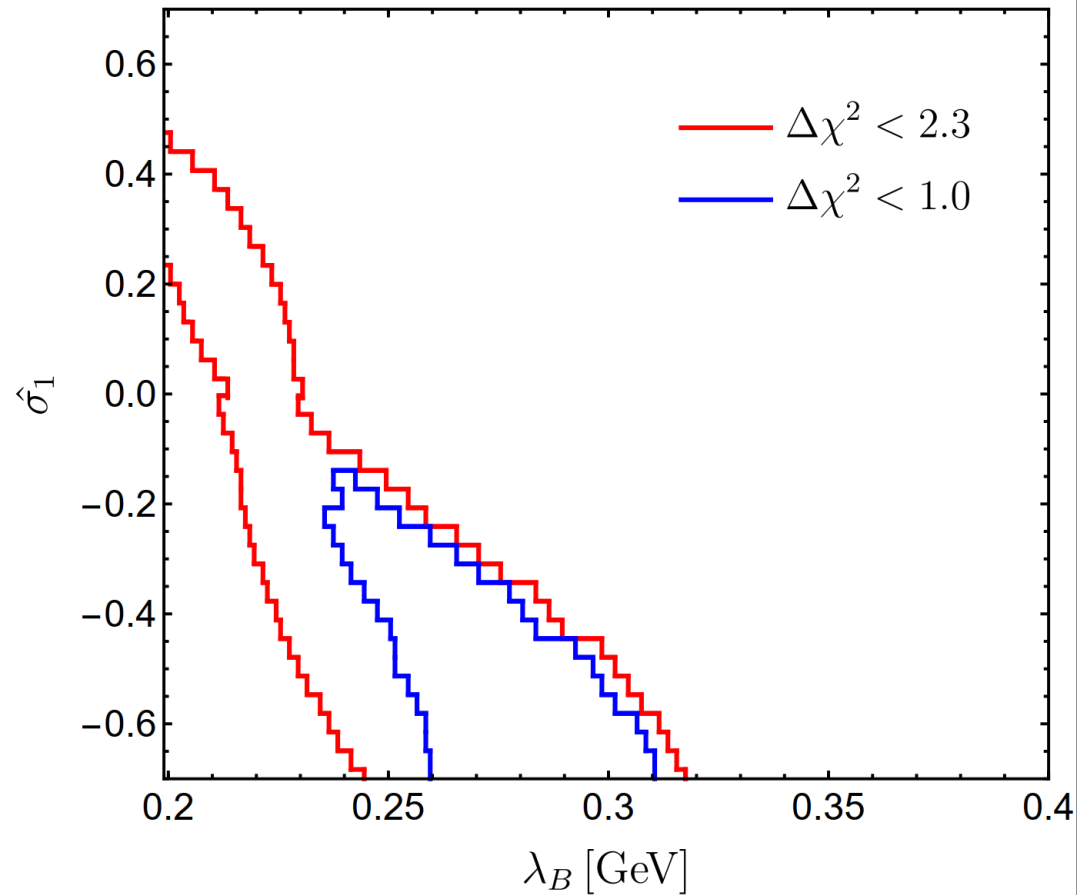
$$\lambda_B > 200 \text{ MeV}$$

$$\hat{\sigma}_1 \in [-0.7, 0.7]$$

$$\hat{\sigma}_2 \in [-6, 6]$$

Analysis and Results

$$\chi^2 - \chi_{min}^2 = \Delta\chi^2$$



Results:

$$\Delta\chi^2 < 1$$

$$\lambda_B = [240, 310] \text{ MeV}$$

$$\hat{\sigma}_1 = [-0.7, -0.16]$$

$$\hat{\sigma}_2 = [-4, 1]$$

Future Prospects:

1. Higher-twist corrections at $\mathcal{O}(\alpha_s)$
2. unitarity bound
3. $B \rightarrow \gamma, B \rightarrow \rho, K^*, D^*$ form factors

谢谢！

B介子LCDA和光锥求和规则方法介绍

以一个toy model为例，解释抽取 λ_B 的可行性

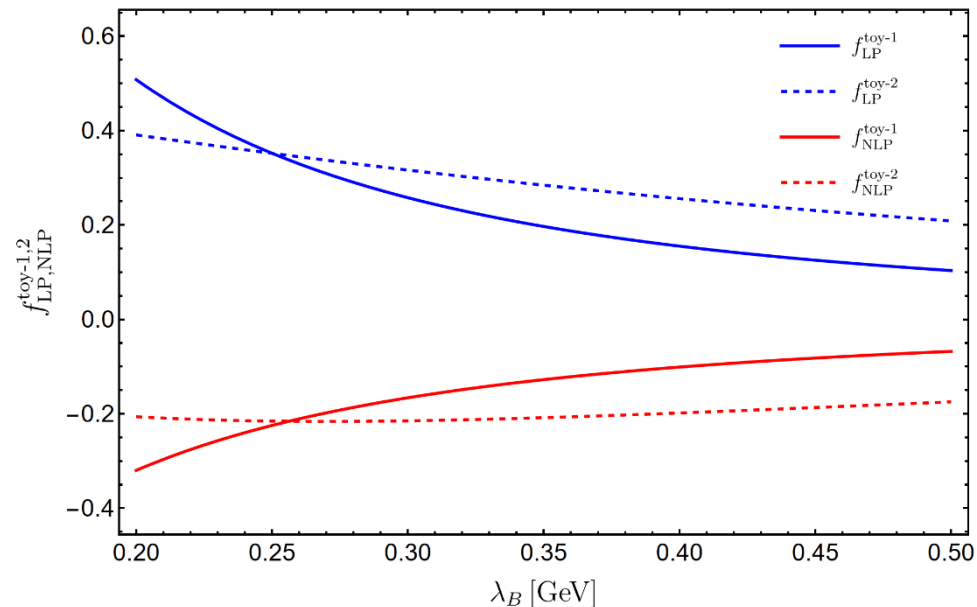
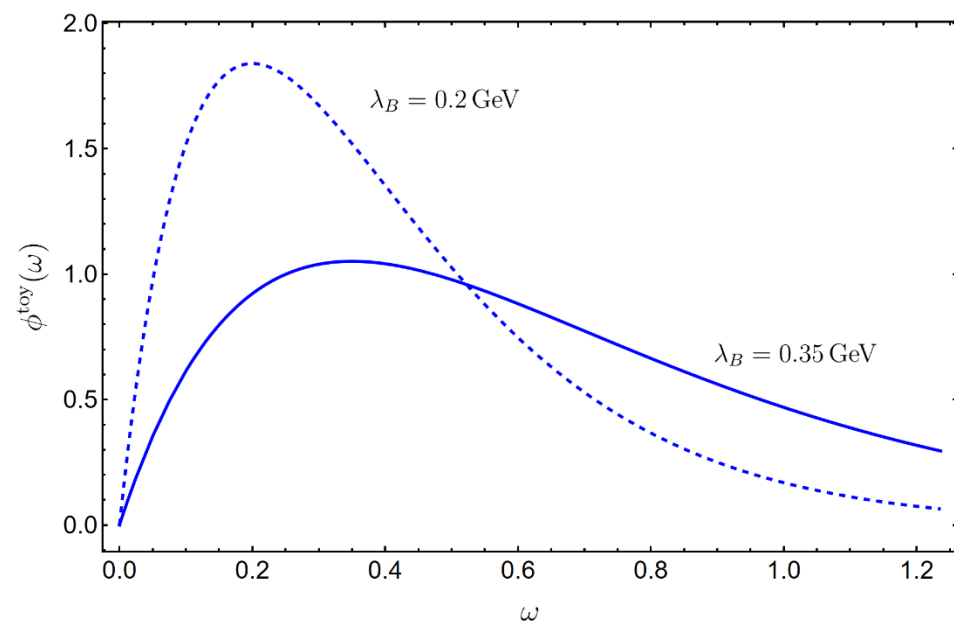
$$f_{\text{LP}}^{\text{toy}} \propto \int_0^{w_s} dw \text{Exp} \left[-\frac{w}{w_M} \right] \phi_B^+(w)$$

$$f_{\text{NLP}}^{\text{toy}} \propto - \int_0^{w_s} dw \frac{w}{n \cdot p} \text{Exp} \left[-\frac{w}{w_M} \right] \phi_B^+(w)$$

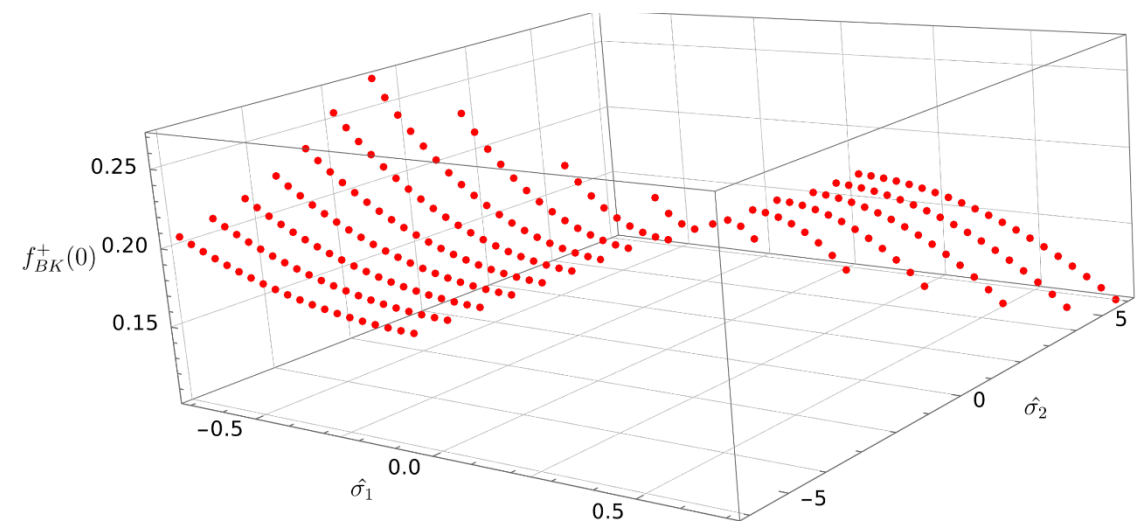
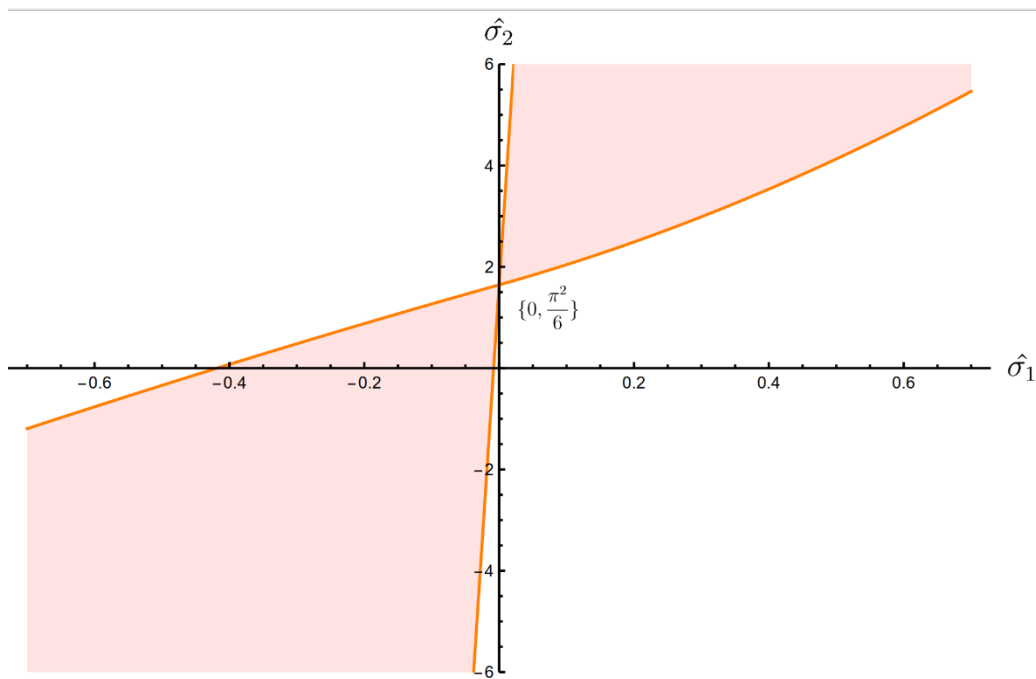
$$\phi_B^+(\omega) = \frac{\omega}{\lambda_B^2} e^{-\omega/\lambda_B}$$

Toy-1. $w_s = 0.13, w_M = 0.24$;

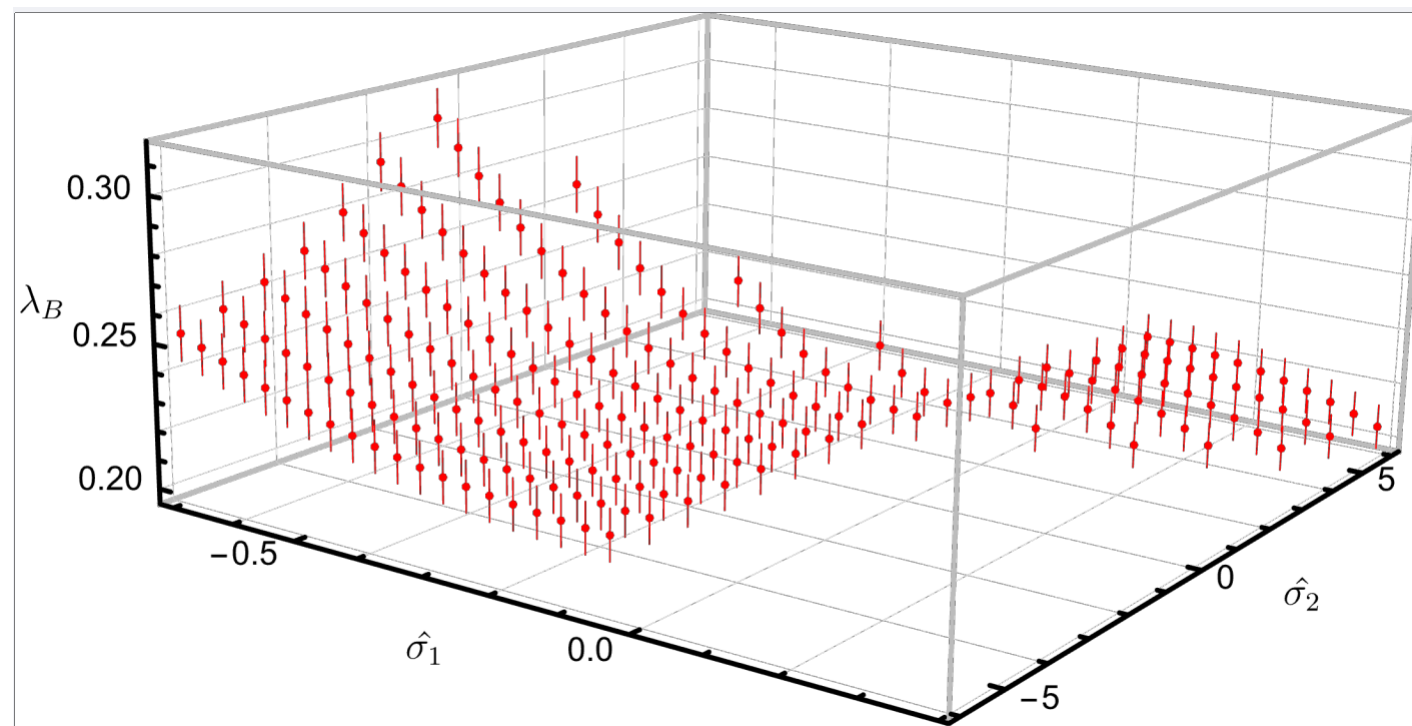
Toy-2. $w_s = 1.14, w_M = 0.85$;



B 介子 LCDA 和光锥求和规则方法介绍



强么正限
更多的道
更多的q2
更多的拟合参数



强么正限
更多的道
更多的 q^2
更多的拟合参数

