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Recent (selected) theory progress on (selected) heavy quark exotics

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All known heavy quark exotics

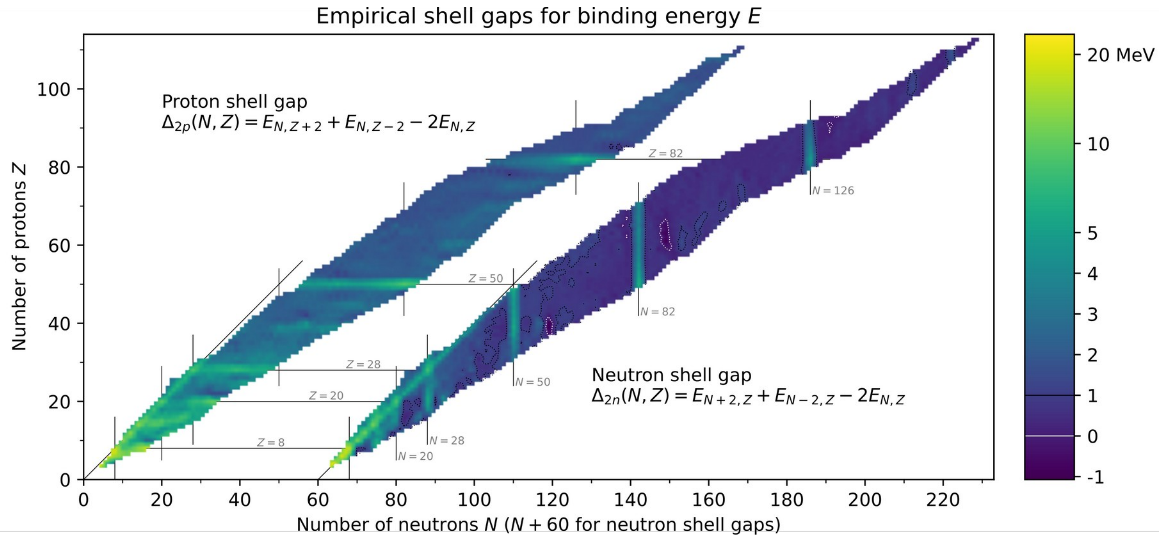
Category			States / Candidates
Meson-like (incl. tetraquarks)	Hidden Charm	$I=0$	$\chi-$like: $\underline{\chi_{c1}(3872)}$, $\chi_{c0}(3860)$, $\underline{\chi_{c0}(3915)}$, $\underline{\chi_{c2}(3930)}$, $X(3940)$
			$\psi-$like: $\underline{\psi(4230)}$, $\underline{\psi(4360)}$, $\psi(4660)$
			with $s\bar{s}$: $\underline{\chi_{c1}(4140)}$, $\underline{\chi_{c1}(4274)}$, $\chi_{c1}(4685)$, $\underline{\chi_{c1}(4500)}$, $\chi_{c1}(4700)$ $X(4150)$, $X(4630)$, $X(4740)$
		$I=1$	seen in e^+e^-: $\underline{T_{c\bar{c}1}(3900)^{+/0}}$, $\underline{T_{c\bar{c}}(4020)^{+/0}}$, $T_{c\bar{c}}(4055)^+$
			seen in B decays: $T_{c\bar{c}}(4050)^+$, $T_{c\bar{c}}(4100)^+$, $T_{c\bar{c}1}(4200)^+$, $T_{c\bar{c}}(4240)^+$, $T_{c\bar{c}}(4250)^+$, $\underline{T_{c\bar{c}1}(4430)^+}$
		$I={}^1\!/_2$	$T_{c\bar{c}s}(3985)^-$, $T_{c\bar{c}s1}(4000)^{-/0}$, $T_{c\bar{c}s1}(4220)^-$
	Hidden Bottom	$I=0$	$\underline{\Upsilon(10753)}$, $\underline{\Upsilon(10860)}$, $\underline{\Upsilon(11020)}$
		$I=1$	$\underline{T_{b\bar{b}1}(10610)^+}$, $\underline{T_{b\bar{b}1}(10650)^+}$
	Hidden Double Charm		$T_{c\bar{c}c\bar{c}}(6550)$, $\underline{T_{c\bar{c}c\bar{c}}(6900)}$, $T_{c\bar{c}c\bar{c}}(7290)$
	Open Single Charm		D_s^*-like: $\underline{D_{s0}^*(2317)^+}$, $\underline{D_{s1}(2460)^+}$
			$T_{cs/c\bar{s}}$: $T_{cs0}(2900)^0$, $T_{c\bar{s}0}(2900)^{0/++}$, $T_{cs1}(2900)^0$
	Open Double Charm		$T_{cc}(3875)^+$
Baryon-like (incl. pentaquarks)	Hidden Charm	$I={}^1\!/_2({}^3\!/_2)$	$P_{c\bar{c}}(4312)^+$, $P_{c\bar{c}}(4440)^+$, $P_{c\bar{c}}(4457)^+$ $P_{c\bar{c}}(4380)^+$, $P_{c\bar{c}}(4337)^+$
		$I=0(1)$	$P_{c\bar{c}s}(4458)^0$, $P_{c\bar{c}s}(4338)^0$

Current and past experiments contributing to the studies of heavy exotics

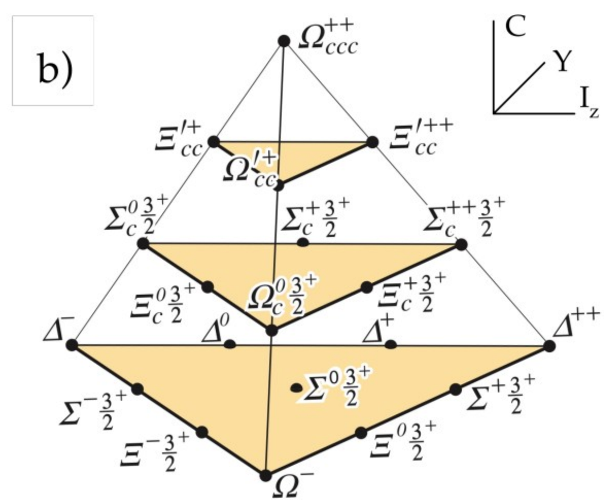
Experiment	Collisions	Energy (c.m.)	Production modes	Data Taking
BaBar Belle Belle II	e^+e^-	$10.58 - 11.2\,\text{GeV}$	e^+e^-, $\gamma\gamma$, B-decays	$1999 - 2008$ $1999 - 2010$ $2018 - 2035$
CDF-II D0 (Run2)	$p\bar{p}$	$1.96\,\text{TeV}$	$p\bar{p}$, b-decays	$2001 - 2011$
CLEO-c BESIII	e^+e^-	$3.97 - 4.26\,\text{GeV}$ $1.8 - 4.96\,\text{GeV}$	e^+e^-, $\gamma\gamma$	$2003 - 2008$ $2008 - 2030+$
ATLAS CMS LHCb	pp (+ pPb, PbPb)	$7 - 13.6\,\text{TeV}$	pp, b-decays	$2010 - 2041$
COMPASS AMBER	μp, πp	$p_{beam} = 160 - 200\,\text{GeV}/c$	diffractive	$2002 - 2021$ $2023 - 2032+$
GlueX	γN	$E_\gamma = 8.0 - 11.5\,\text{GeV}$	photoproduction	$2015 - 2028+$

arXiv:2410.06923

Good theories (Explanatory power, systematicity, and predictive power）



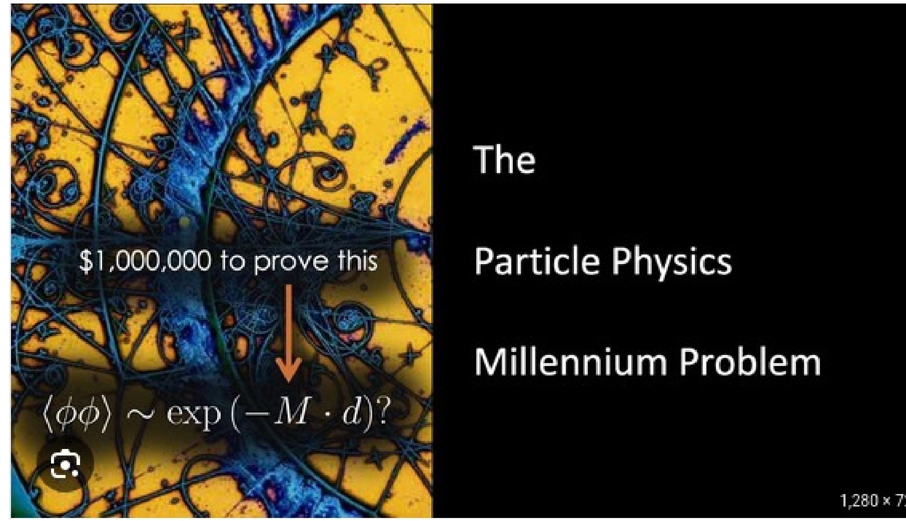
- **Nuclear Shell Model**



- **Quark Model**

Color Confinement

- **One of Seven Millennium Prize Problems**

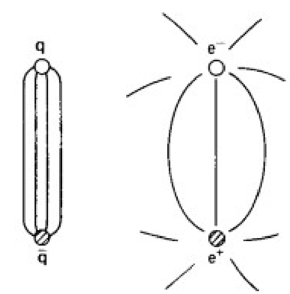


- **QCD as fundamental theory of Strong interaction but not been fully understood**

 1)Quark confining potential can not derived analytically; support from Lattice QCD

2)Exploring infrared effective theory of quark confinement:
 such as dual superconductivity theory

$$V(\boldsymbol{r}) = -\frac{Q^2}{4\pi}\frac{e^{-m_B r}}{r} + \frac{Q^2 {m_B}^2}{4\pi} K_0\left(\frac{\sqrt{2}\, m_B}{m_\phi}\right) r \,.$$

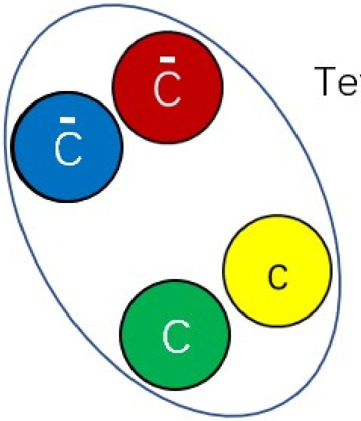


(a)

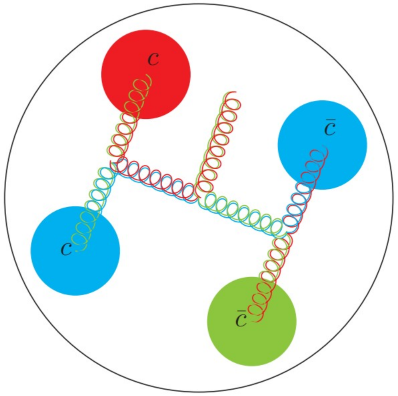
(1) Hidden double charm meson-like exotics

$$T_{c\bar{c}c\bar{c}}(6550),\ \underline{T_{c\bar{c}c\bar{c}}(6900)},\ T_{c\bar{c}c\bar{c}}(7290)$$

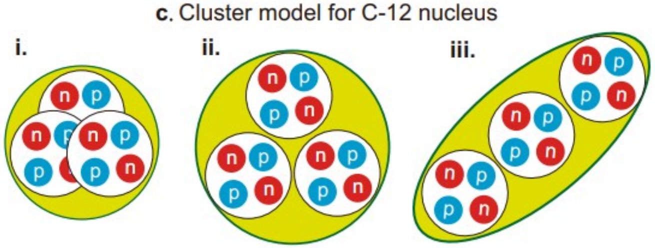
How to explain these exotic states?

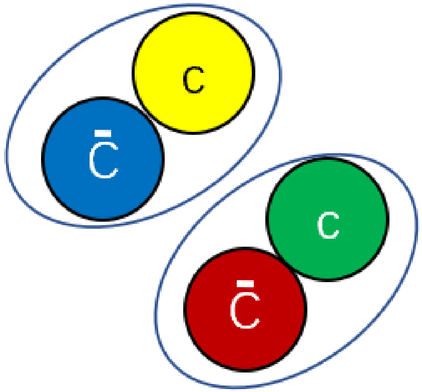


Tetraquark

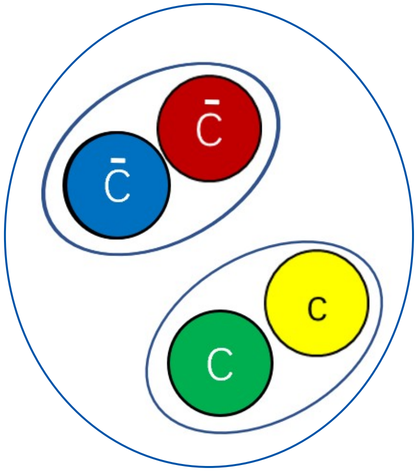


Gluonic Tetracharm Hybrid





Charmonia Molecule



Diquark-antidiquark

Similar to alpha cluster;
Different in diquark-antidiquark (color-confining)

Previous theoretical studies

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Y.Q. Ma, H.F. Zhang, arXiv:2009.08376； R.Zhu,2010.09082
……

Diquark-antidiquark or charmonium molecule?

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Fully-heavy tetraquark spectra and production at hadron colliders

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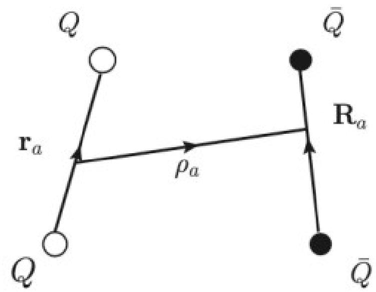
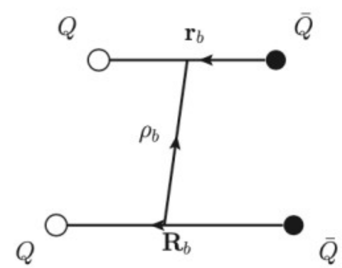
Abstract

Motivated by the observation of exotic structure around 6900 MeV in the J/ψ-pair mass spectrum using proton-proton collision data by the LHCb collaboration, we study the spectra of fully-heavy tetraquarks within Bethe-Salpeter equation and Regge trajectory relation. The $X(6900)$ may be explained as a radially excited state with quark content $cc\bar{c}\bar{c}$ and spin-parity $0^{++}(3S)$ or $2^{++}(3S)$ or an orbitally excited $2P$ state. New $cc\bar{c}\bar{c}$ structures around 6.0 GeV, 6.5 GeV, and 7.1 GeV are predicted together. Other $bb\bar{b}\bar{b}$ and $bc\bar{b}\bar{c}$ structures which may be experimentally prominent are discussed. On the other hand, the fully-heavy S-wave tetraquark production at hadron colliders is investigated and their cross sections are obtained.

$$\bar{3}_c \otimes 3_c = 1_c \text{ and } 6_c \otimes \bar{6}_c = 1_c.$$



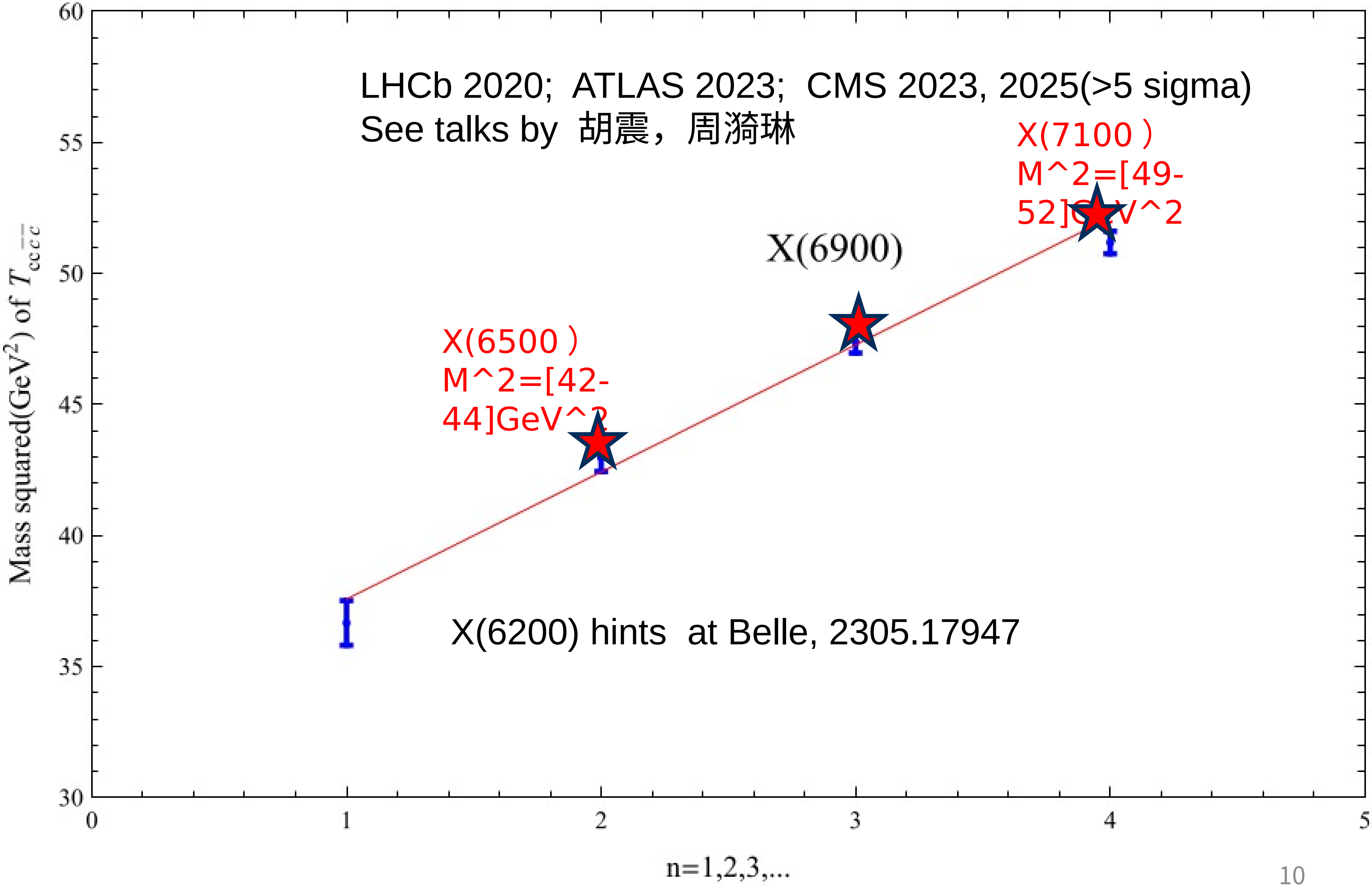
$$|\bar{\mathbf{3}},\mathbf{3}\rangle = |(Q_1Q_2)_{\bar{\mathbf{3}}};(\bar{Q}_3\bar{Q}_4)_{\mathbf{3}}\rangle,$$
$$|\mathbf{6},\bar{\mathbf{6}}\rangle = |(Q_1Q_2)_{\mathbf{6}};(\bar{Q}_3\bar{Q}_4)_{\bar{\mathbf{6}}}\rangle,$$



$$|\mathbf{1},\mathbf{1}\rangle = |(Q_1\bar{Q}_3)_{\mathbf{1}};(Q_2\bar{Q}_4)_{\mathbf{1}}\rangle,$$
$$|\mathbf{8},\mathbf{8}\rangle = |(Q_1\bar{Q}_3)_{\mathbf{8}};(Q_2\bar{Q}_4)_{\mathbf{8}}\rangle,$$

The masses (6.5GeV, 7.1GeV) are predicted previously and confirmed by ATLAS/CMS.

Fully charm tetraquark family: Linear Regge trajectories?



Fully charm tetraquark production

QCD factorization:

$$\sigma\left(pp \to T_{4c} + X\right) = \sum_{i,j=q,g} \int_0^1 dx_1 dx_2 f_{i/p}\left(x_1, \mu_F\right) f_{j/p}\left(x_2, \mu_F\right) \hat{\sigma}(i + j \to T_{4c} + X)\left(\hat{s}, \mu_F\right)$$

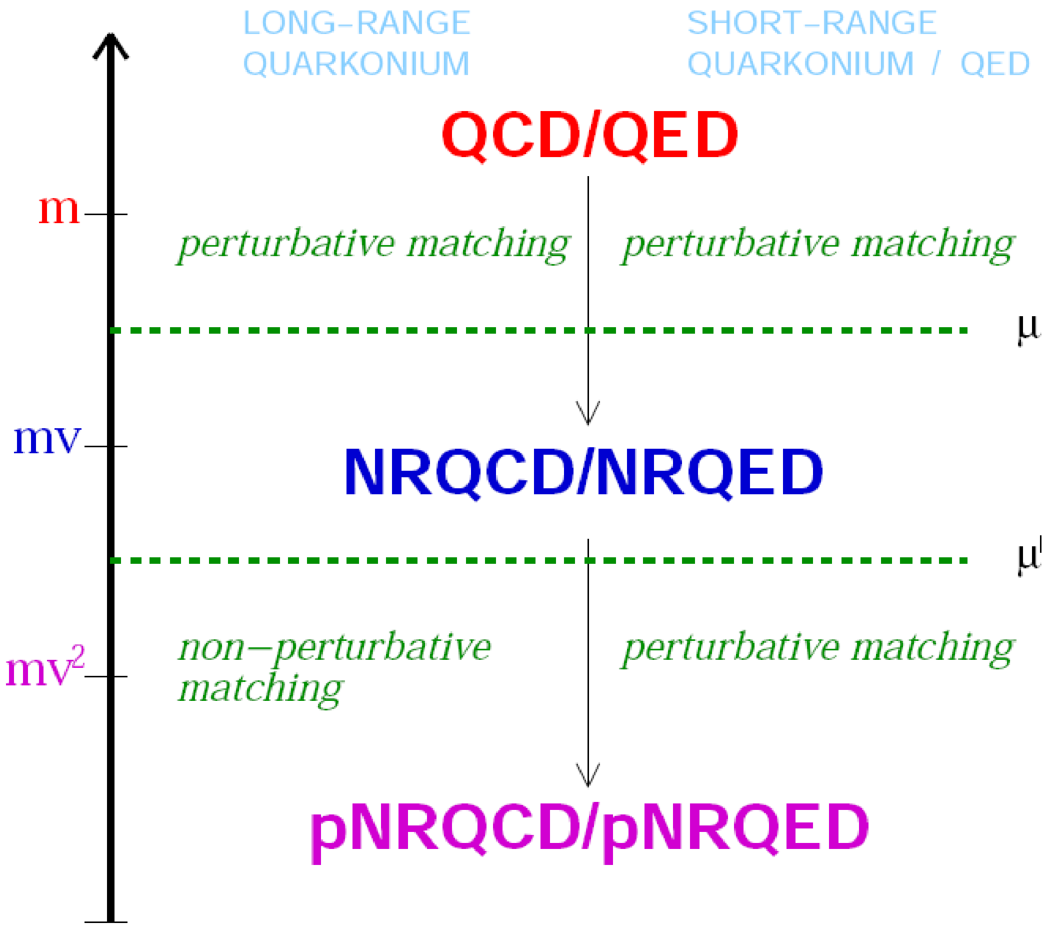
Partonic processes:

LO+NLO virtual: $g + g \to T_{4c}$ and $q + \bar{q} \to T_{4c}$.

NLO real: $g + g \to T_{4c} + g,\ q + \bar{q} \to T_{4c} + g$

$q + g \to T_{4c} + q,\ \bar{q} + g \to T_{4c} + \bar{q}$

NRQCD/pNRQCD factorization



$$\alpha_s(mv) \sim v$$

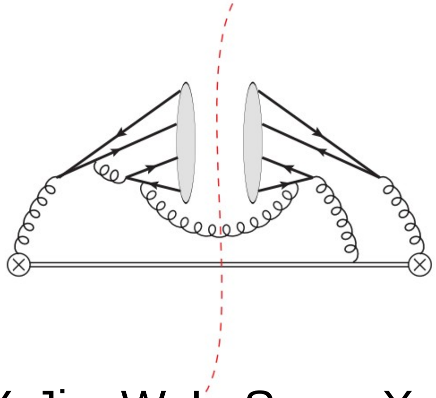
$$v^2 \approx 0.1 \text{ for the } \Upsilon$$

Bodwin-Braaten-Lapage
1995

Pineda-Soto-Brambilla-Vairo
2000

Previous studies (selected)

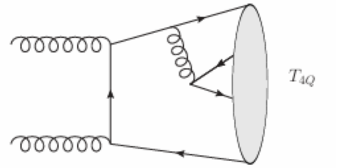
- **Fragmentation mechanism (based on NRQCD), equally considering the NLO real diagrams (valid for large Pt)**



F. Feng, Y. Huang, Y. Jia, W. L. Sang, X. Xiong and J. Y.Zhang, arXiv:2009.08450; F. Feng et al, 2304.11142
Y.Q. Ma, H.F. Zhang, arXiv:2009.08376
I. Belov, A. Giachino, and E. Santopinto, arXiv:2409.12070

- **NRQCD LO (Pt=0)**

R.L. Zhu, arXiv: 2010.09082



Differential cross section in NRQCD

$$\frac{\mathrm{d}\hat{\sigma}(T_{4c}^{(J)}+X)}{\mathrm{d}\hat{t}} = \frac{2M_{T_{4c}}}{m_c^{14}}\left[F_{3,3}^{(J)}\left\langle O_{3,3}^{(J)}\right\rangle + 2F_{3,6}^{(J)}\left\langle O_{3,6}^{(J)}\right\rangle + F_{6,6}^{(J)}\left\langle O_{6,6}^{(J)}\right\rangle\right],$$

$$O_{3,3}^{(J)} = \mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{(J)}\sum_X |T_{4c}^J+X\rangle\langle T_{4c}^J+X|\mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{(J)\dagger},$$

$$O_{6,6}^{(0)} = \mathcal{O}_{\mathbf{6}\otimes\bar{\mathbf{6}}}^{(0)}\sum_X |T_{4c}^0+X\rangle\langle T_{4c}^0+X|\mathcal{O}_{\mathbf{6}\otimes\bar{\mathbf{6}}}^{(0)\dagger},$$

$$O_{3,6}^{(0)} = \mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{(0)}\sum_X |T_{4c}^0+X\rangle\langle T_{4c}^0+X|\mathcal{O}_{\mathbf{6}\otimes\bar{\mathbf{6}}}^{(0)\dagger},$$

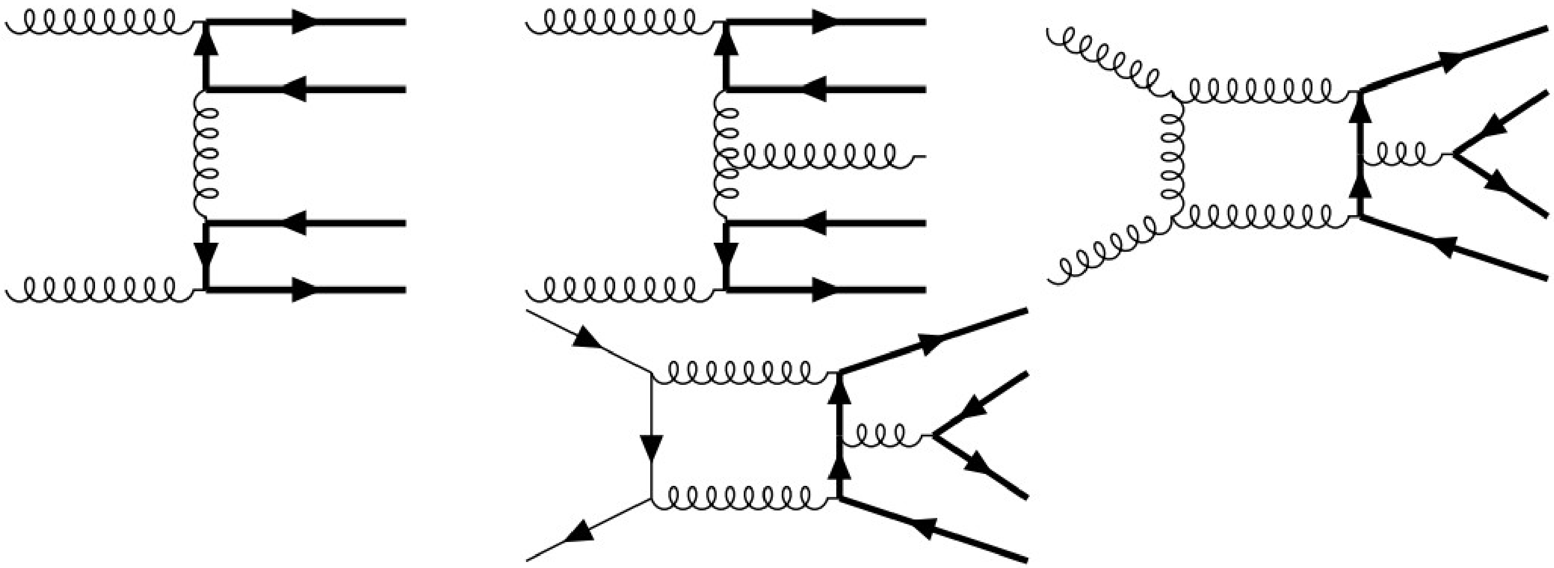
$$\mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{(0)} = -\frac{1}{\sqrt{3}}[\psi_a^T(i\sigma^2)\sigma^i\psi_b][\chi_c^\dagger\sigma^i(i\sigma^2)\chi_d^*]\;\mathcal{C}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{ab;cd},$$

$$\mathcal{O}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{ij;(2)} = [\psi_a^T(i\sigma^2)\sigma^m\psi_b][\chi_c^\dagger\sigma^n(i\sigma^2)\chi_d^*]\;\Gamma^{ij;mn}\;\mathcal{C}_{\bar{\mathbf{3}}\otimes\mathbf{3}}^{ab;cd},$$

$$\mathcal{O}_{\mathbf{6}\otimes\bar{\mathbf{6}}}^{(0)} = [\psi_a^T(i\sigma^2)\psi_b][\chi_c^\dagger(i\sigma^2)\chi_d^*]\;\mathcal{C}_{\mathbf{6}\otimes\bar{\mathbf{6}}}^{ab;cd}.$$

Full NLO calculation in NRQCD

Wang,**Zhu**, arXiv:2510.02085



LO:64(gg)， 4(qqbar)

NLO Virtual:2008(gg)， 170(qqbar)

NLO Real: 618(gg)， 98(qqbar, qg)

Exact IR cancellation

$$K^{\rm LH3}_{\rm gg,virtual} = \frac{3}{\epsilon^2} - \frac{1}{\epsilon}\left(3\log\left(\frac{\mu}{4m_c}\right)^2 - \frac{n_l}{3} + \frac{11}{2}\right) - \frac{3}{2}\log^2\left(\frac{\mu}{4m_c}\right)^2$$
$$+\left(\frac{11}{2} - \frac{2n_h+n_l}{3}\right)\log\left(\frac{\mu}{m_c}\right)^2 + \frac{4719}{256}\left(\mathrm{Li}_2\left(2\sqrt{2}-2\right) + \mathrm{Li}_2\left(-2\sqrt{2}-2\right)\right)$$
$$+\cdots$$

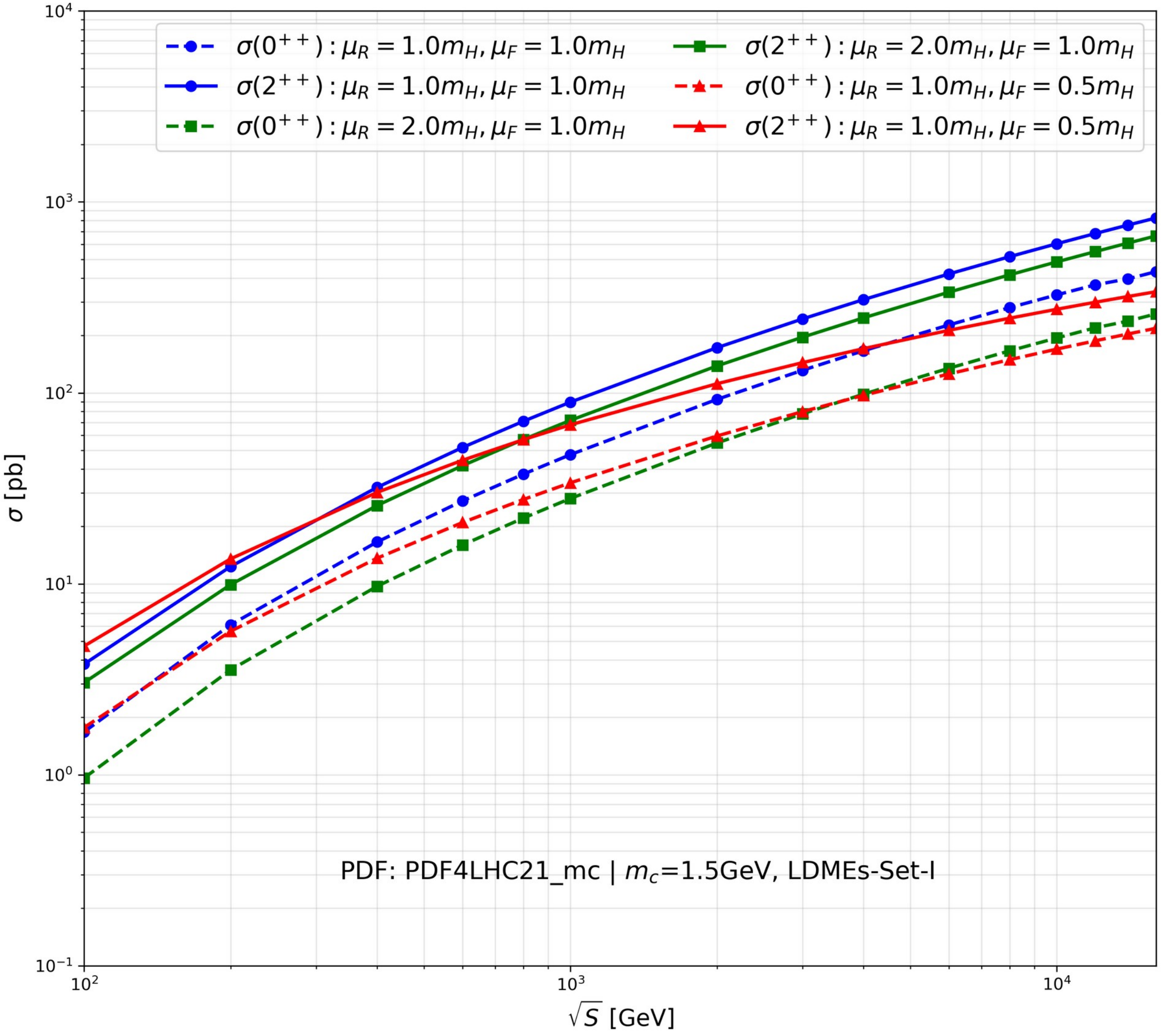
There are soft divergences when $k_g = 0$ or $z = 1$, and collinear divergences when $y_\theta = \cos\theta_{k_n k_g} = \pm 1$ with $k_n = k_1, k_2, k_T$.

$$\begin{aligned}\hat{\sigma}_{\rm soft}\ (i+j\to T_{4c}+k) &= -C\frac{1}{2\epsilon_{\rm IR}}\delta(1-z)\frac{4^{-\epsilon}\Gamma(1-\epsilon)\Gamma(-\epsilon)}{\Gamma(1-2\epsilon)}\mathcal{B}_{ij}\left(z=1,y_\theta\right)\\ &= \Gamma(1+\epsilon)\left(\frac{4\pi\mu_R^2}{\hat{s}}\right)^\epsilon \frac{\alpha_s}{\pi}\hat{\sigma}^{(0)}_{ij}\delta(1-z)\left(\frac{1}{\epsilon^2_{\rm IR}} - \frac{\pi^2}{3}\right)C_{ij}\end{aligned}$$

$$\begin{aligned}&\hat{\sigma}_{\text{hard col. } y_\theta=\pm1}\,(i+j\to T_{4c}+k)\\ &= -C\frac{4^{-\epsilon}}{2\epsilon_{\rm IR}}\left[\left(\frac{1}{1-z}\right)_+ - 2\epsilon\left(\frac{\log(1-z)}{1-z}\right)_+\right]\mathcal{B}_{ij}\left(z,y_\theta=\pm1\right)\\ &= -C\frac{4^{-\epsilon}}{2\epsilon_{\rm IR}}\left[\left(\frac{1}{1-z}\right)_+ - 2\epsilon\left(\frac{\log(1-z)}{1-z}\right)_+\right]\mathcal{B}_{ij}\left(z=1,y_\theta\right)\frac{b^{\rm collinear}_{ij}}{z^3}\\ &= \Gamma(1+\epsilon)\left(\frac{4\pi\mu_R^2}{\hat{s}}\right)^\epsilon\frac{\alpha_s}{\pi}\hat{\sigma}^{(0)}_{ij}\delta(1-z)\left[-2C_{ij}b^{\rm collinear}_{ij}\left(\left(\frac{1}{1-z}\right)_+\frac{1}{\epsilon_{\rm IR}} - 2\left(\frac{\log(1-z)}{1-z}\right)_+\right)\right]\end{aligned}$$

$$\hat{\sigma}^{\rm AP-CT} = \frac{1}{\epsilon}\frac{\alpha_s}{2\pi}\left(\frac{4\pi\mu_R^2}{\mu_F^2}\right)^\epsilon\Gamma(1+\epsilon)\hat{\sigma}^{(0)}zP_{ij}(z),$$

Collider energy dependence for different spin and scales



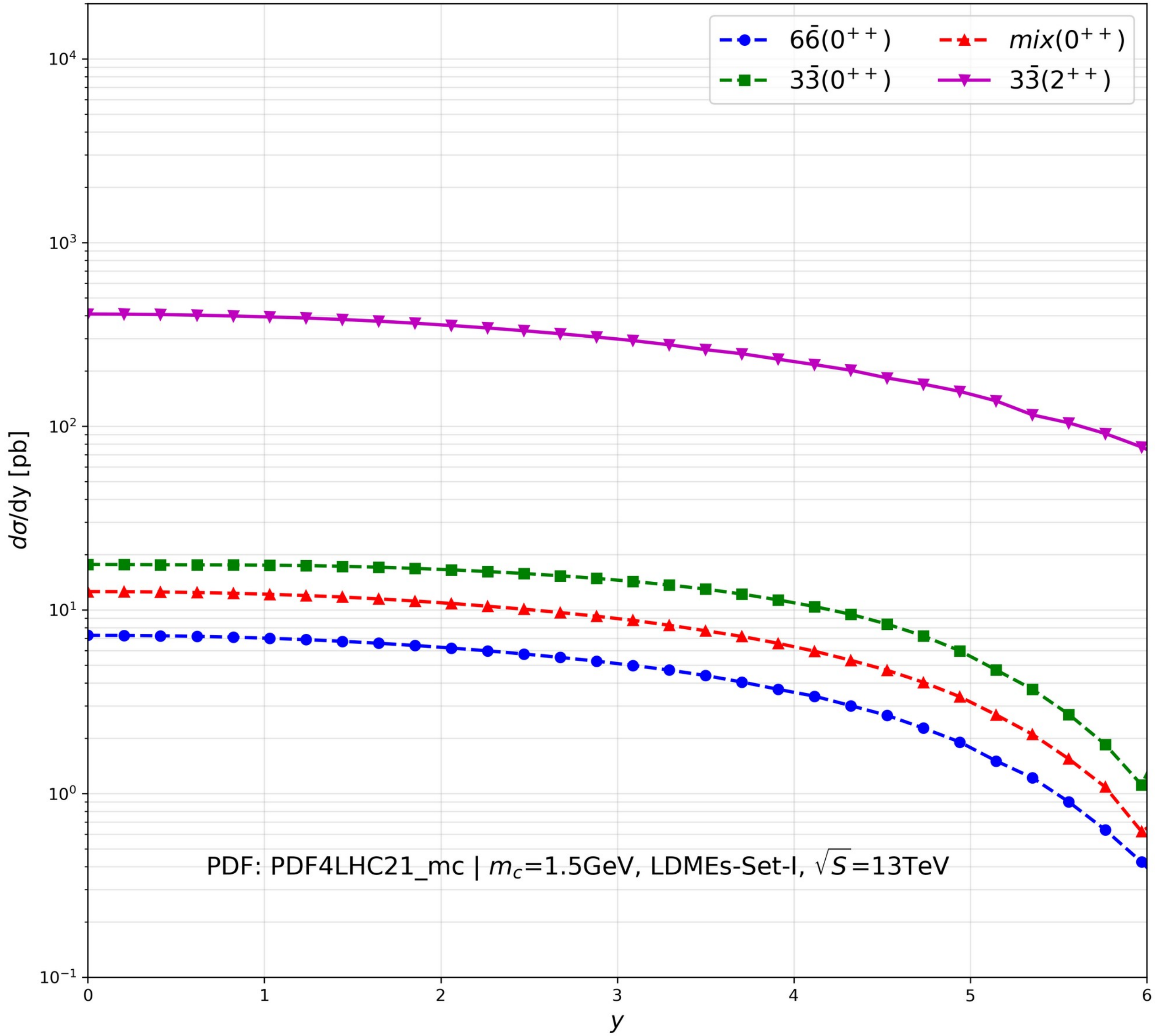
Reference at 7TeV:
σ(eta_c)~10^6nb

σ(2J/psi)~10nb

σ(T_4c)
~[0.1-0.5]nb

LDMEs extracted
from LHCb data

Rapidity dependence for different diquark configurations



[3 3_bar]
plays a major role

Soft gluon radiation produces large logarithms

$$\frac{d\sigma}{dyd^2p_t}\Big|^{gg}_{p_t\ll M} = \sigma_0(T^{J,i}_{4c})\frac{\alpha_s C_A}{2\pi^2}\int f(x)dx f(x')dx'$$
$$\times\frac{1}{p_t^2}\left[\frac{2(1-\xi_1+\xi_1^2)^2}{(1-\xi_1)_+}\delta(1-\xi_2)+\frac{2(1-\xi_2+\xi_2^2)^2}{(1-\xi_2)_+}\delta(1-\xi_1)\right.$$
$$\left.+\left(2\ln\frac{M^2}{p_t^2}\right)\delta(1-\xi_2)\delta(1-\xi_1)\right],$$

At low P_perp, the soft gluon radiations generate the divergence, which should be resumed to obtain reliable predictions.

Collins-Soper-Sterman resummation

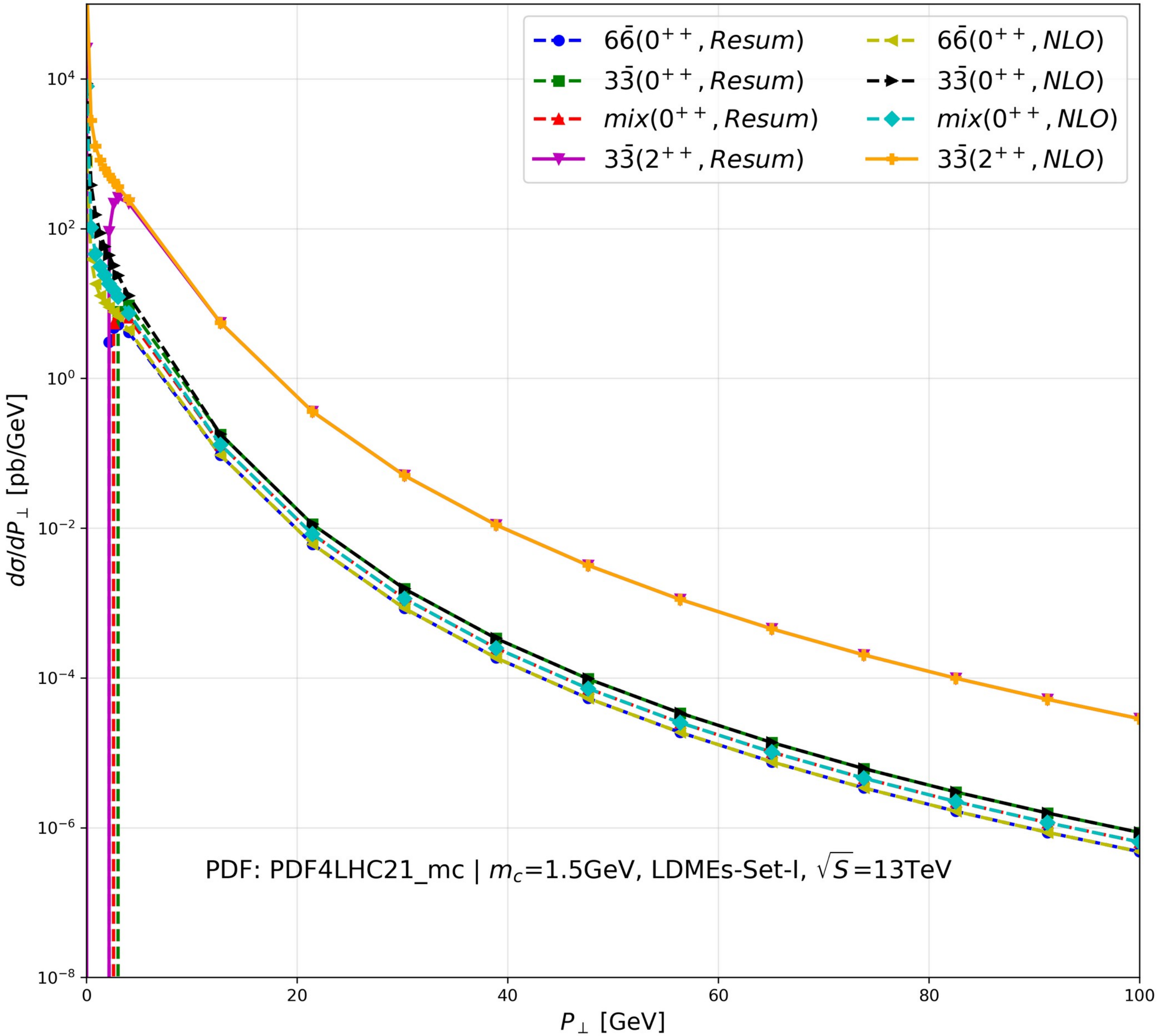
$$\frac{d\sigma}{dp_t^2} \sim \frac{(L+1)}{p_t^2}\Big\{\alpha_s + \alpha_s^2 L^2 + \alpha_s^3 L^4 + \alpha_s^4 L^6 + \alpha_s^2 + \alpha_s^3 L^2 + \alpha_s^4 L^4 + \alpha_s^3 + \alpha_s^4 L^2 \cdots\Big\},$$

$$\frac{\partial W(b,M^2)}{\partial \ln M^2} = (K+G')W(b,M^2)\ ,$$

Collins, Soper 81
Collins, Soper, Sterman 85

$$\frac{d\sigma}{dydp_t^2} \sim \int d^2b e^{ip_t\cdot b} \times \sum_a \int_{x_A}^1 \frac{d\xi_A}{\xi_A} f_{a/A}(\xi_A;1/b) \sum_b \int_{x_B}^1 \frac{d\xi_B}{\xi_B} f_{b/B}(\xi_B;1/b) \times \exp\left\{-\int_{1/b^2}^{Q^2} \frac{d\bar{\mu}^2}{\bar{\mu}^2}\left[\ln\left(\frac{Q^2}{\bar{\mu}^2}\right) A\left(g(\bar{\mu})\right) + B\left(g(\bar{\mu})\right)\right]\right\} \times C_{ja}\left(\frac{x_A}{\xi_A};g(1/b)\right) C_{jb}\left(\frac{x_B}{\xi_B};g(1/b)\right) + Y(p_t;Q,x_A,x_B).$$

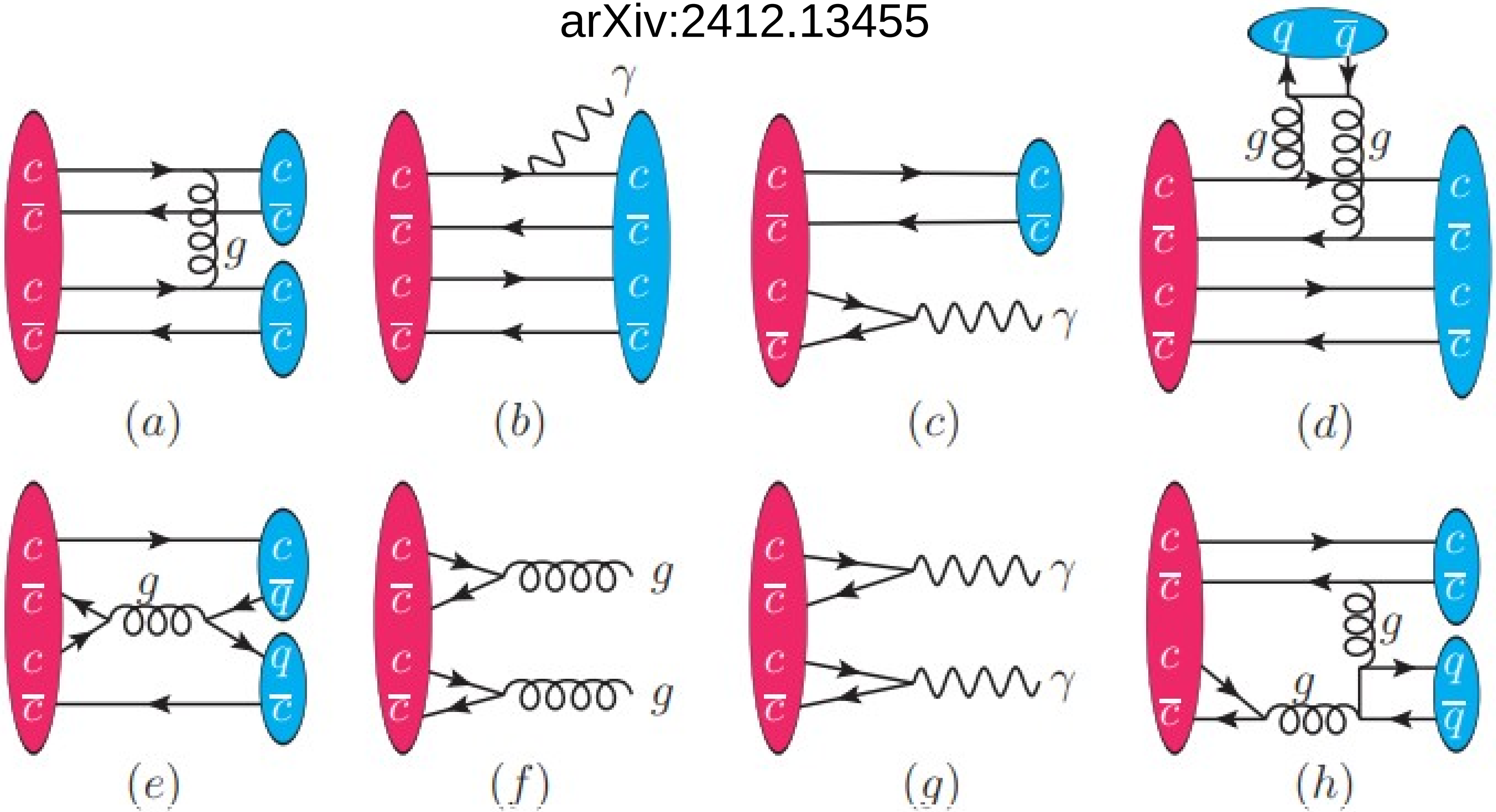
NLL Resummation results



Typical p_t around 2 GeV

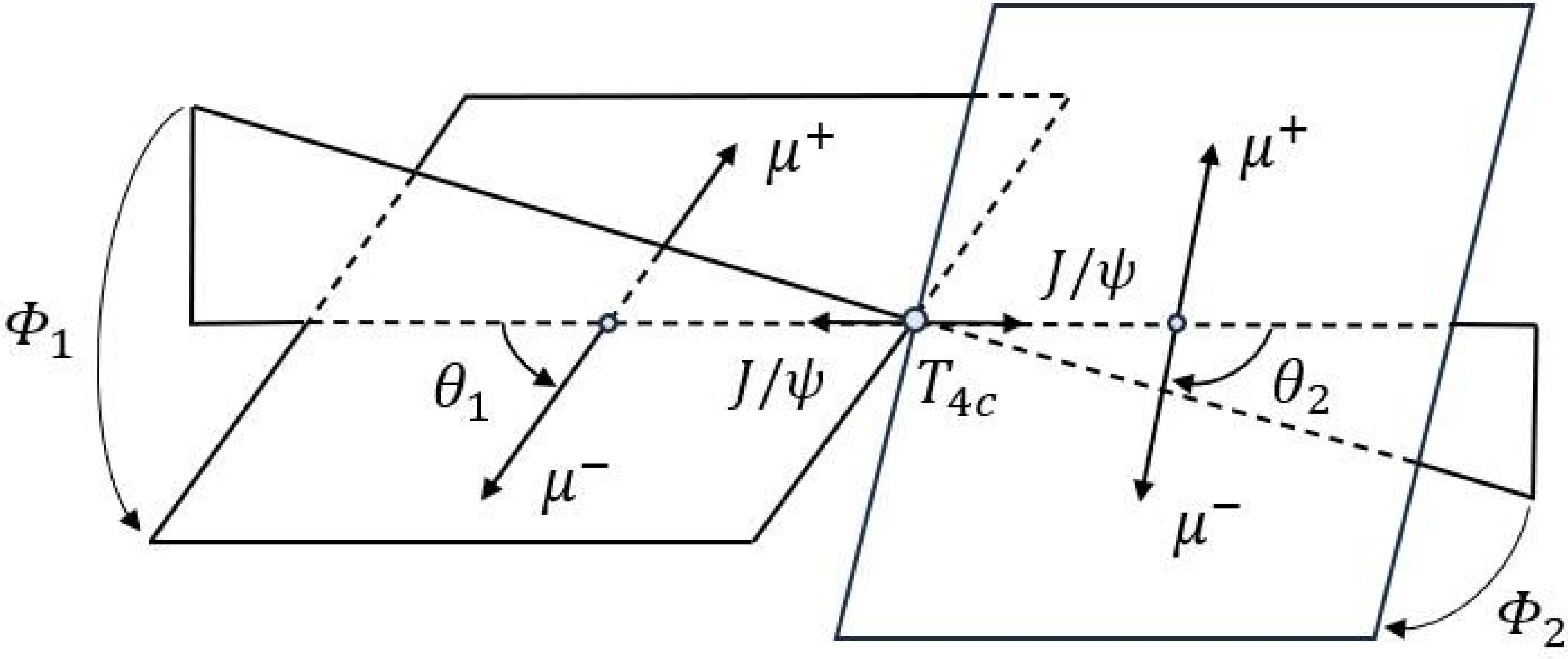
Fully charm tetraquark decay properties

Chen-Liu-Zhao-Zhong-**Zhu**-Zou,
arXiv:2412.13455



Major decay modes

Cascade decay angles



Helicity amplitudes

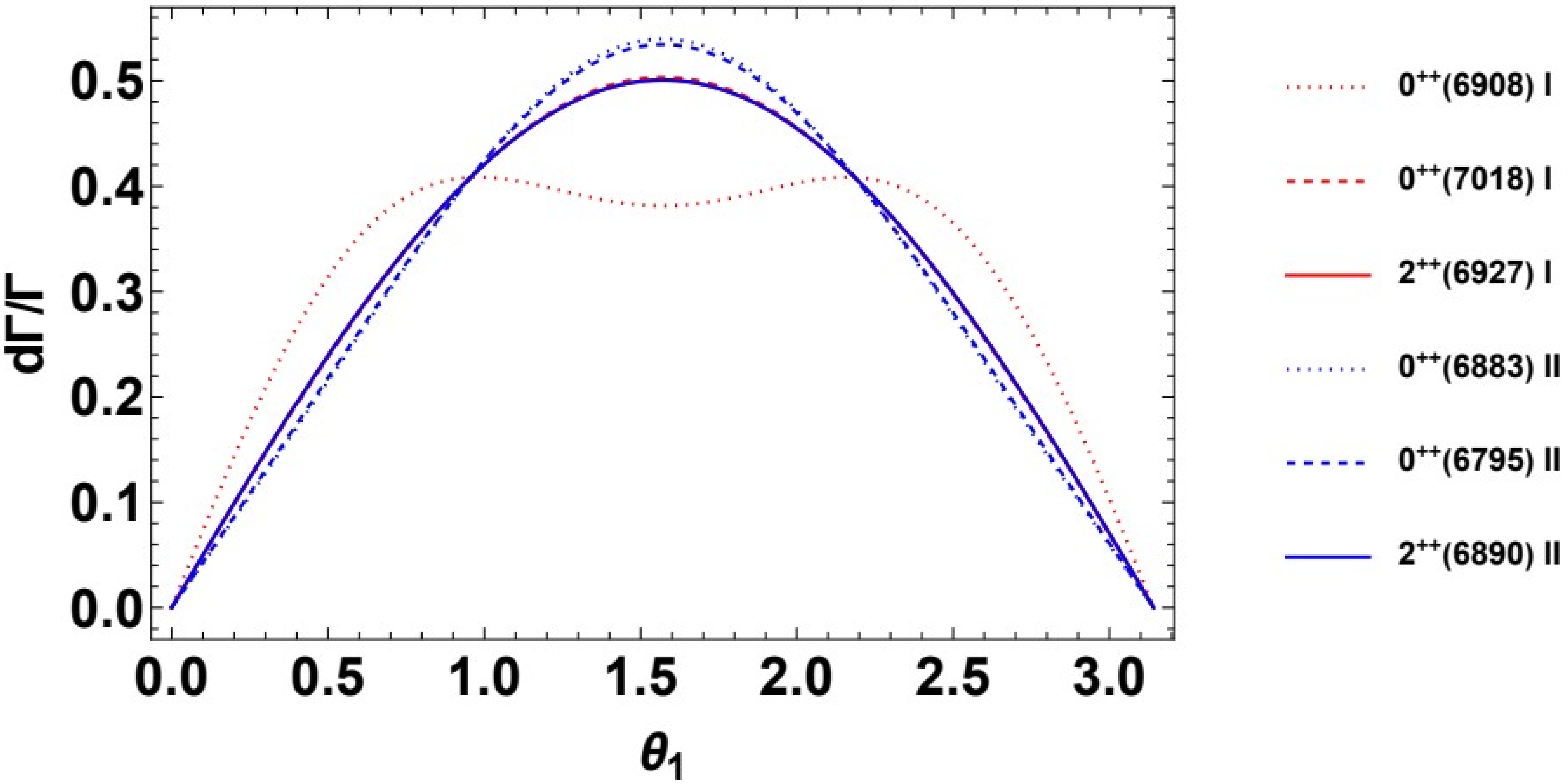
$$\langle \vec{p}_1 \lambda_{a_1}, -\vec{p}_1 \lambda_{a_2} | H_1 | s_1 \lambda_1 \rangle = 4\pi \sqrt{\frac{\omega_1}{p_1}} \langle \Omega_1 \lambda_{a_1} \lambda_{a_2} | H_1 | s_1 \lambda_1 \rangle$$
$$= 4\pi \sqrt{\frac{\omega_1}{p_1}} \sum_{\lambda'_{a_1} \lambda'_{a_2}} \langle \Omega_1 \lambda_{a_1} \lambda_{a_2} | | s_1 \lambda_1 \lambda'_{a_1} \lambda'_{a_2} \rangle \langle s_1 \lambda_1 \lambda'_{a_1} \lambda'_{a_2} | H_1 | s_1 \lambda_1 \rangle$$
$$= 4\pi \sqrt{\frac{\omega_1}{p_1}} N_{s_1} D^{s_1 *}_{\lambda_1 \lambda_a}(-\Phi_1, \theta_1, \Phi_1) \langle s_1 \lambda_1 \lambda_{a_1} \lambda_{a_2} | H_1 | s_1 \lambda_1 \rangle$$
$$= N_{s_1} F^{s_1}_{\lambda_{a_1} \lambda_{a_2}} D^{s_1 *}_{\lambda_1 \lambda_a}$$

Symmetry constraint:
Parity conservation;
Identical particles

$$F^s_{\lambda_1 \lambda_2} = \eta \eta_1 \eta_2 (-1)^{s - s_1 - s_2} F^s_{-\lambda_1 - \lambda_2}$$

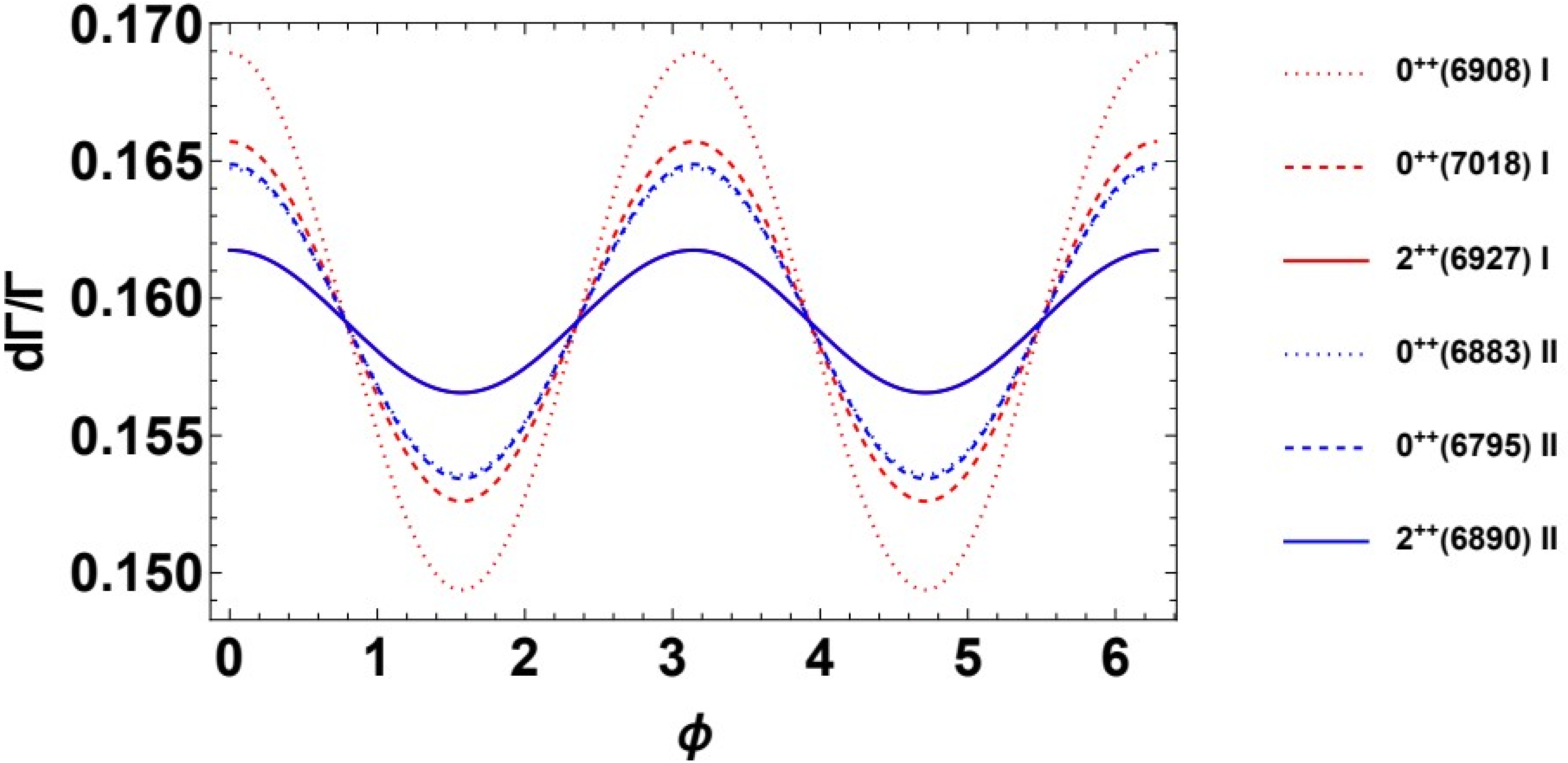
$$F^s_{\lambda_1 \lambda_2} = (-1)^s F^s_{\lambda_2 \lambda_1}$$

Polar angular distribution

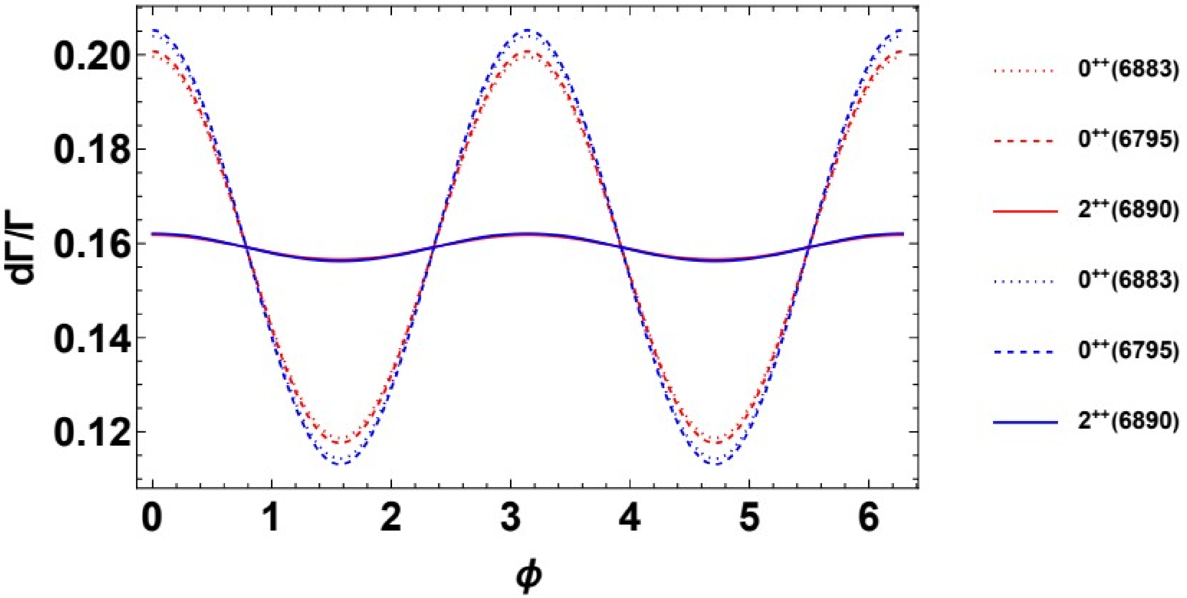


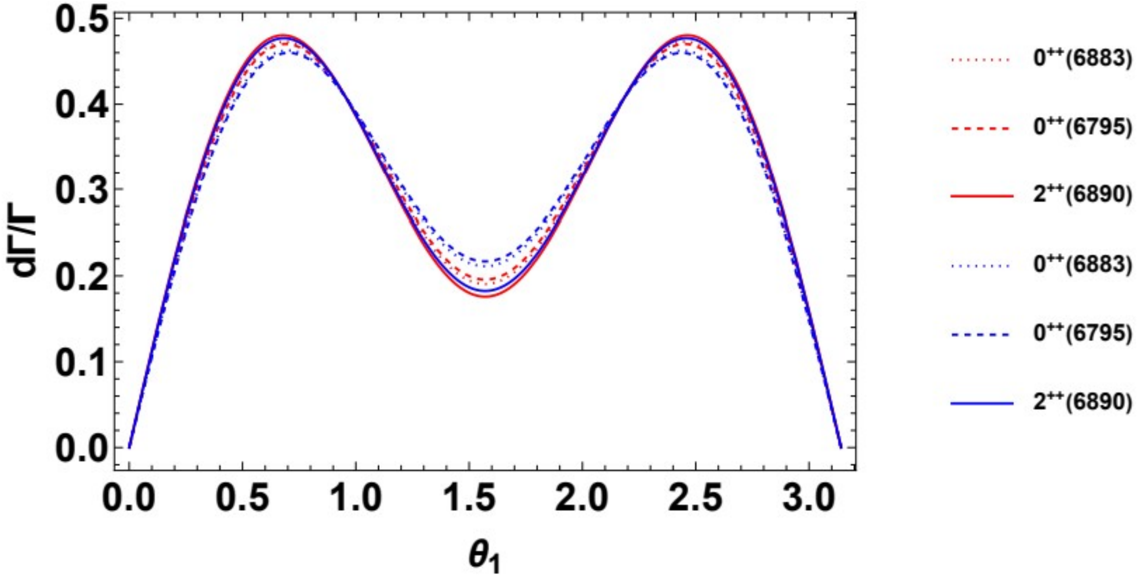
Model I: quark model (QM);
Model II: diquark model+heavy quark effective theory(HQET)

Plane angular distribution



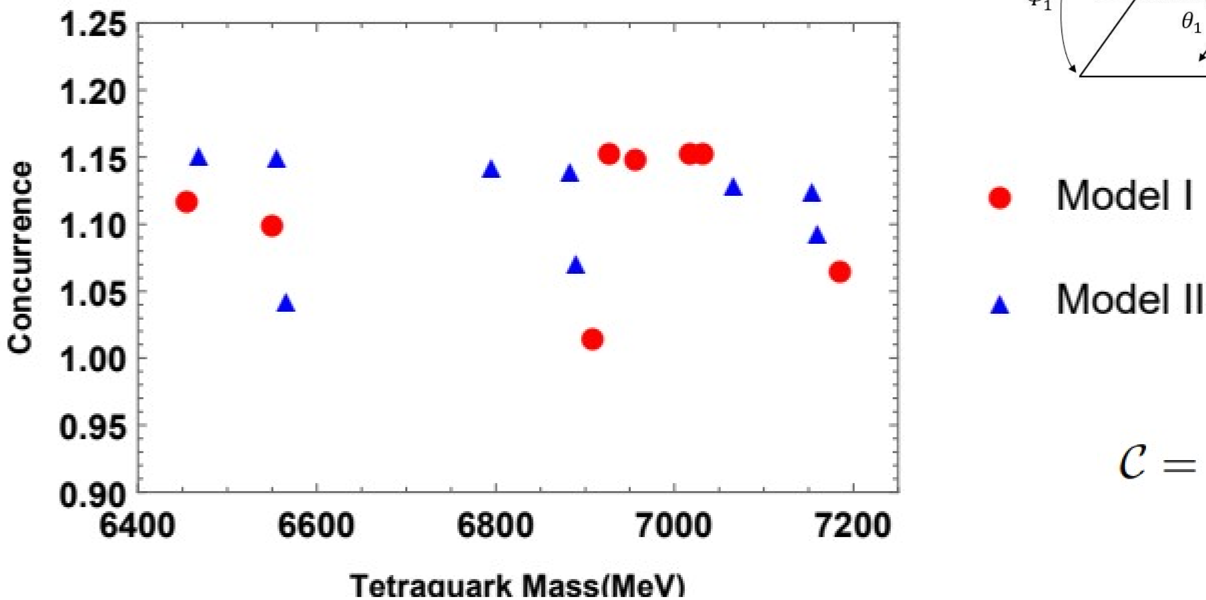
Charmed meson pair channel

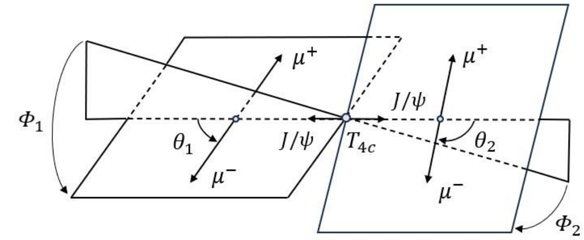




The θ_1 and Φ distributions for various tetraquarks near 6.9 GeV into $D^*(\to D\pi)$ and $\bar{D}^*(\to \bar{D}\pi)$ using Model II (HQET).

Quantum entanglement





$$\mathcal{C} = \sqrt{2(1 - \mathrm{Tr}\rho_A^2)},$$

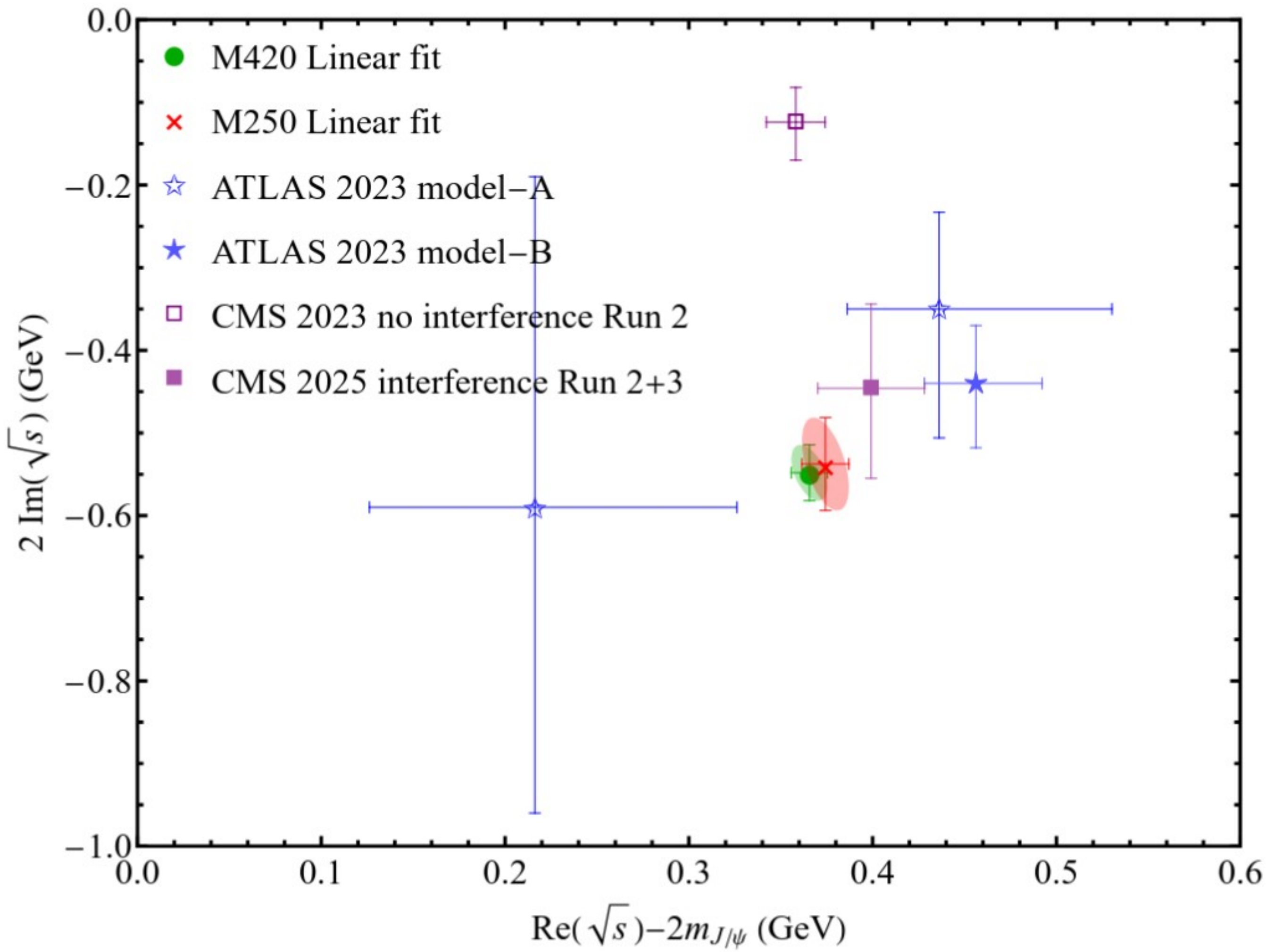
$$max\left(0, \sqrt{\frac{1}{3}}\left[\frac{h_{00}^{00} + 2h_{01}^{01} + 4h_{11}^{00} + 2h_{11}^{11}}{N} - 1\right]\right) \le \sqrt{2(1 - \frac{\left(h_{00}^{00}\right)^2 + 2\left(h_{11}^{11}\right)^2}{N^2})} \le \frac{2}{\sqrt{3}}.$$

$$max\,(0, LB_2) \le \sqrt{2(1 - \left(\frac{h_{00}^{00} + 2h_{10}^{10}}{N}\right)^2 - \left(\frac{h_{10}^{10} + h_{11}^{11} + h_{1-1}^{1-1}}{N}\right)^2 - \left(\frac{h_{10}^{10} + h_{1-1}^{1-1} + h_{11}^{11}}{N}\right)^2)} \le \frac{2}{\sqrt{3}}.$$

If assume quantum entanglement as a basic principle, then a constraint formula for helicity amplitudes.

Recent results from Lattice QCD

Li,Shi,Chen,Sun,arXiv:2505.24213;2505.23220;
Meng,Liu,Tuo,Yan,Zhang,arXiv:2411.11533;



X(6500) resonance state is observed in LQCD

See talk by 李更

(2)Hidden charm /open double charm meson-like exotics

χ−like: $\underline{\chi_{c1}(3872)}$,

$\underline{\chi_{c0}(3915)}$, $\underline{\chi_{c2}(3930)}$, $X(3940)$

$T_{cc}(3875)^+$

Unified study of T_{cc} and X(3872) from model, experimental data and Lattice data

Ren,Wang,Yang,Wu,PRD110,074007(2024);PRD 110,114029(2024);……;
Meng,Liu,Tuo,Yan,Zhang,arXiv:2411.11533;

TABLE III. Branching ratio of different channels $\Gamma_i/\Gamma_{\text{total}}$

Decay channels	**PDG** [60]	**Ref.** [68]	**Ours**	
$\chi_{c1}(3872) \to \pi^+\pi^- J/\psi$	0.035 ± 0.009	$4.1^{+1.9}_{-1.1}\%$	0.035	✓
$\chi_{c1}(3872) \to \rho^0 J/\psi$	$2.8 \pm 0.7\%$	–	4.1%	∼
$\chi_{c1}(3872) \to \omega J/\psi$	$4.1 \pm 1.4\%$	$4.4^{+2.3}_{-1.3}\%$	13.7%	H
$\chi_{c1}(3872) \to \pi\pi\pi J/\psi$	not seen	–	6.2%	∼
$\chi_{c1}(3872) \to D^0\bar{D}^0\pi^0$	$45 \pm 21\%$	–	11.4%	L
$\chi_{c1}(3872) \to \bar{D}^{*0}D^0$	$34 \pm 12\%$	$52.4^{+25.3}_{-14.3}\%$	58.5%	✓
$\chi_{c1}(3872) \to \pi^0\chi_{c2}$	$< 4\%$	–	1.8%	✓
$\chi_{c1}(3872) \to \pi^0\chi_{c1}$	$3.1^{+1.5}_{-1.3}\%$	$3.6^{+2.2}_{-1.6}\%$	2.3%	✓
$\chi_{c1}(3872) \to \pi^0\chi_{c0}$	$< 13\%$	–	1.5%	✓
$\chi_{c1}(3872) \to \gamma D^+D^-$	$< 3.5\%$	–	0.1%	✓
$\chi_{c1}(3872) \to \gamma\bar{D}^0 D^0$	$< 6\%$	–	6.4%	∼
$\chi_{c1}(3872) \to \gamma J/\psi$	$(7.8 \pm 2.9) \times 10^{-3}$	$1.1^{+0.6}_{-0.3}\%$	$< 5 \times 10^{-3}$	?
$\chi_{c1}(3872) \to \gamma\psi(2S)$	possibly seen	$2.4^{+1.3}_{-0.8}\%$	7×10^{-3}	✓

$$H = H_0 + H_I$$

$$H_I = \hat{g} + \hat{v}$$

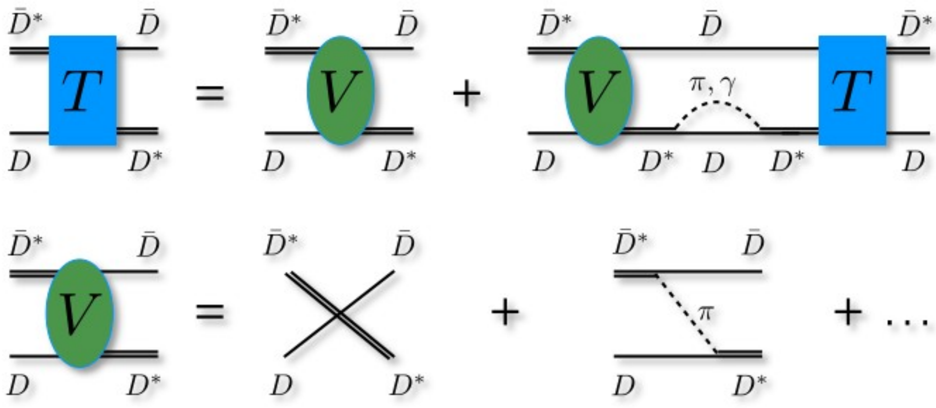
$$\hat{g} = \sum_{\alpha} \sum_{i=1,n} \left[\left|\alpha(k_\alpha)\right\rangle g^{+}_{i,\alpha} \left\langle B_i\right| + \left|B_i\right\rangle g_{i,\alpha} \left\langle \alpha(k_\alpha)\right| \right]$$

$$\hat{v} = \sum_{\alpha,\beta} \left|\alpha(k_\alpha)\right\rangle v_{\alpha,\beta} \left\langle \beta(k_\beta)\right|$$

They found **X(3872)** is
A mixing state
With bare charmoinum,
And DD*_bar+D*D_bar
molecule；
Tcc as DD*+D*D
molecule

Predicting isovector charmonium-like states from X(3872) properties

Zhang, Ji, Dong, Guo, Hanhart, Meißner, Rusetsky, JHEP 08 (2024) 130;PRD 111 (2025) 014031;arXiv:2502.04458



- Pole position of the $X(3872)$ determined to be (relative to the $D^0\bar{D}^{*0}$ threshold)

$$E_X = \left(-160^{+43+38}_{-47-57} - 125^{+18+15}_{-25-28} i\right)\,\mathrm{keV}$$

- Compositeness of $X(3872)$: 0.97(2)
- Existence of an isovector $W_{c1}(3880)$
  - Signal of $W_{c1}(3880)^0$ predicted to be more visible in $B^0 \to K^0[D^0\bar{D}^0\pi^0, J/\psi\pi^+\pi^-]$
  - Signal of $W_{c1}(3880)^{\pm}$: threshold cusp at $D^+\bar{D}^{*0}$ threshold in $J/\psi\pi^{\pm}\pi^0$

Summary and outlook

- ✓ Overview the mass spectrum, the hadron-production and decay properties for hidden double charm meson-like exotics
- ✓ Overview the recent studies for X(3872) and Tcc(3875)

Outlook: measuring the (differential) cross section (or) and the decay angular distribution shall tell us the inner structure of exotics; Color confinement has not been really understood, and its deep investigation is needed.

Thank you a lot!