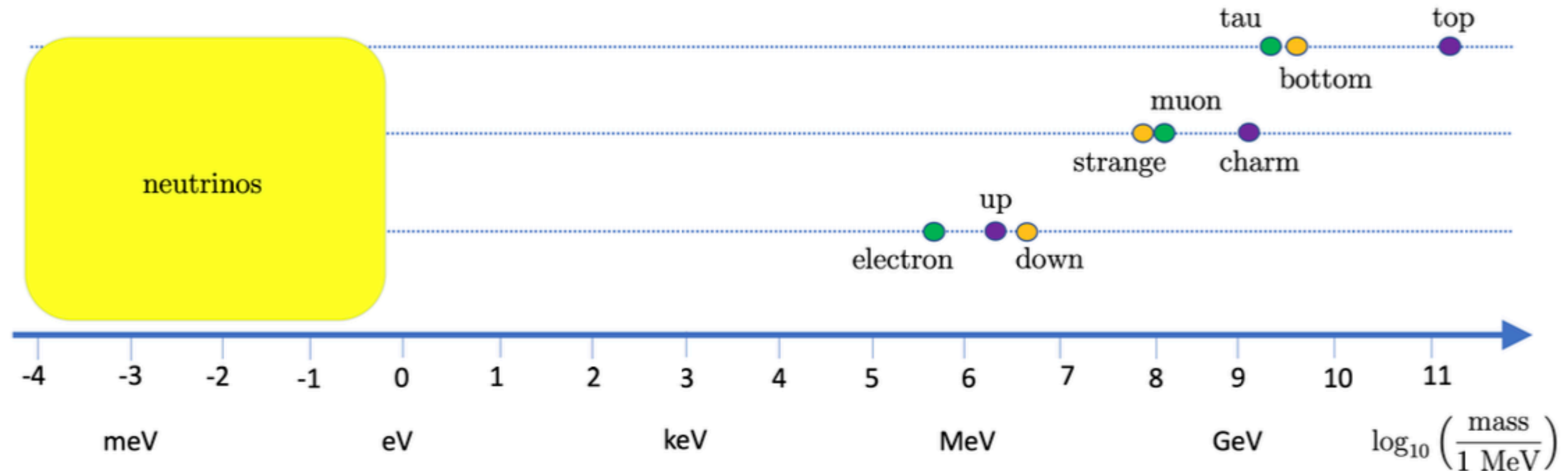


Theory of “toponium”

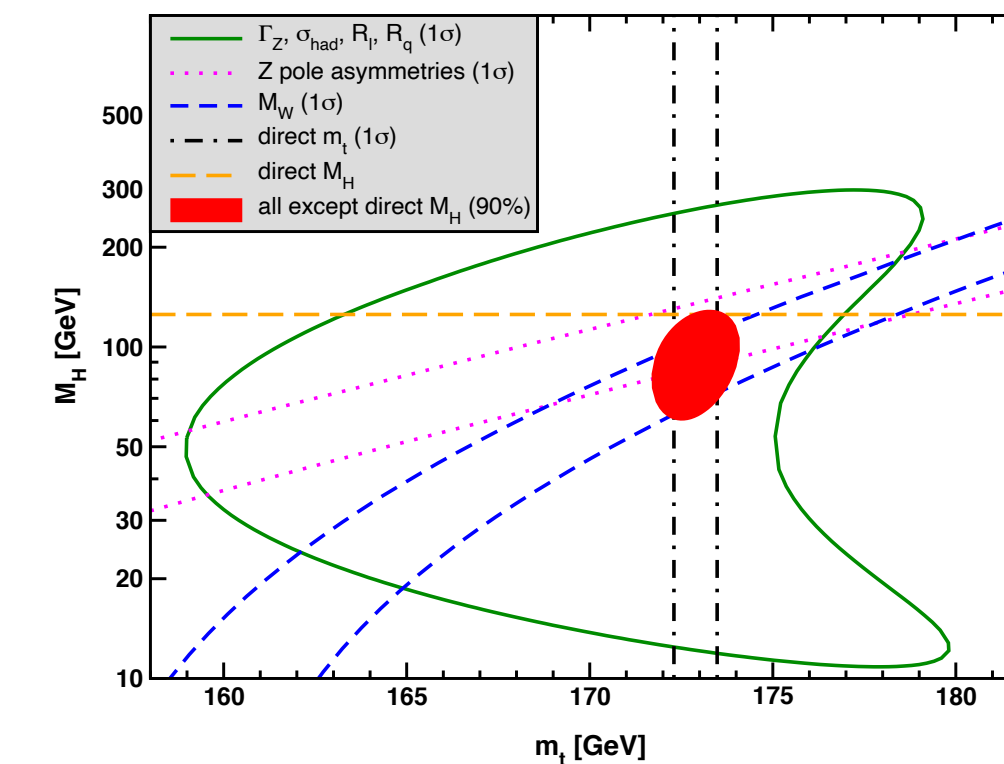
Li Lin Yang
Zhejiang University



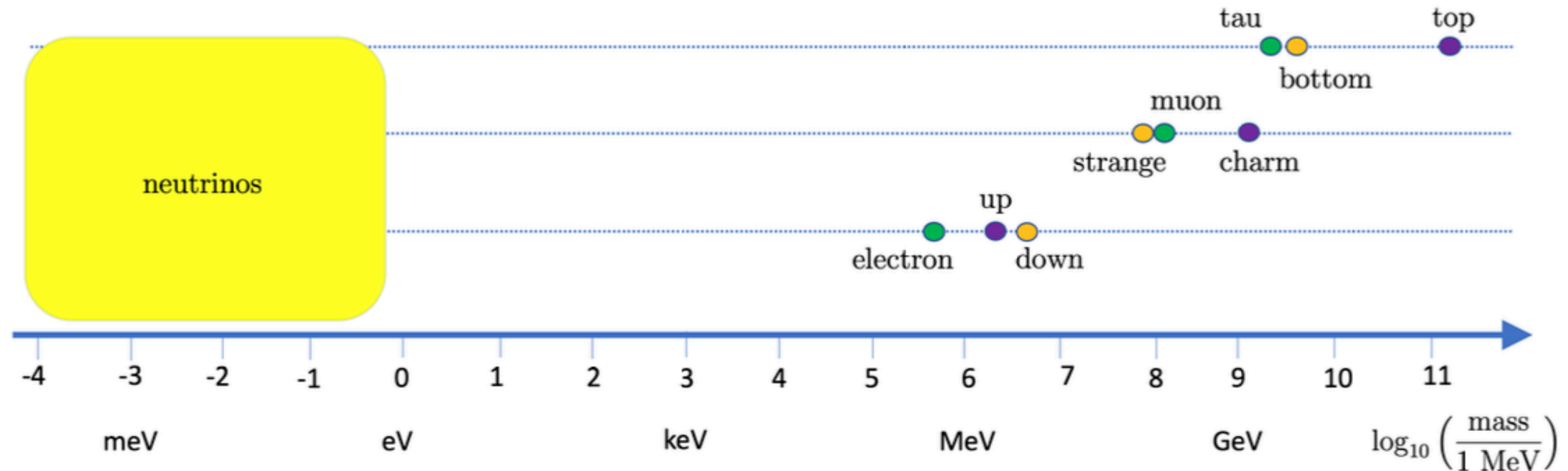
The top quark



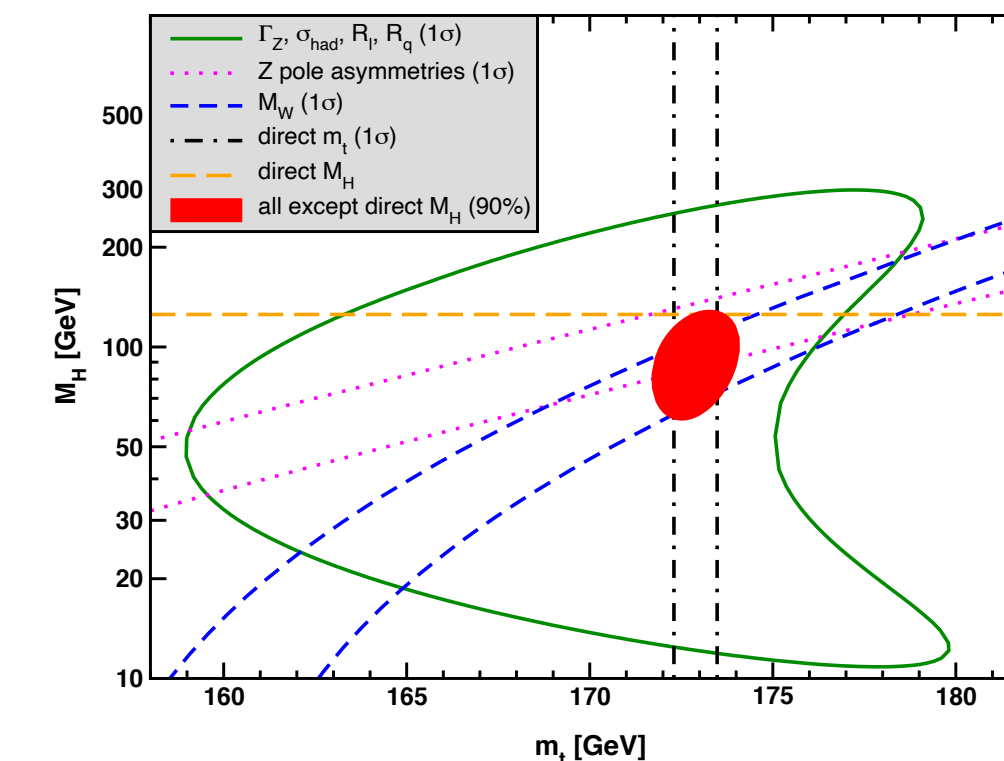
- The heaviest fundamental particle
- Large Yukawa coupling with the Higgs field
- Top quark mass: a crucial parameter of the SM
- Decay before hadronization: “bare quark”



The top quark

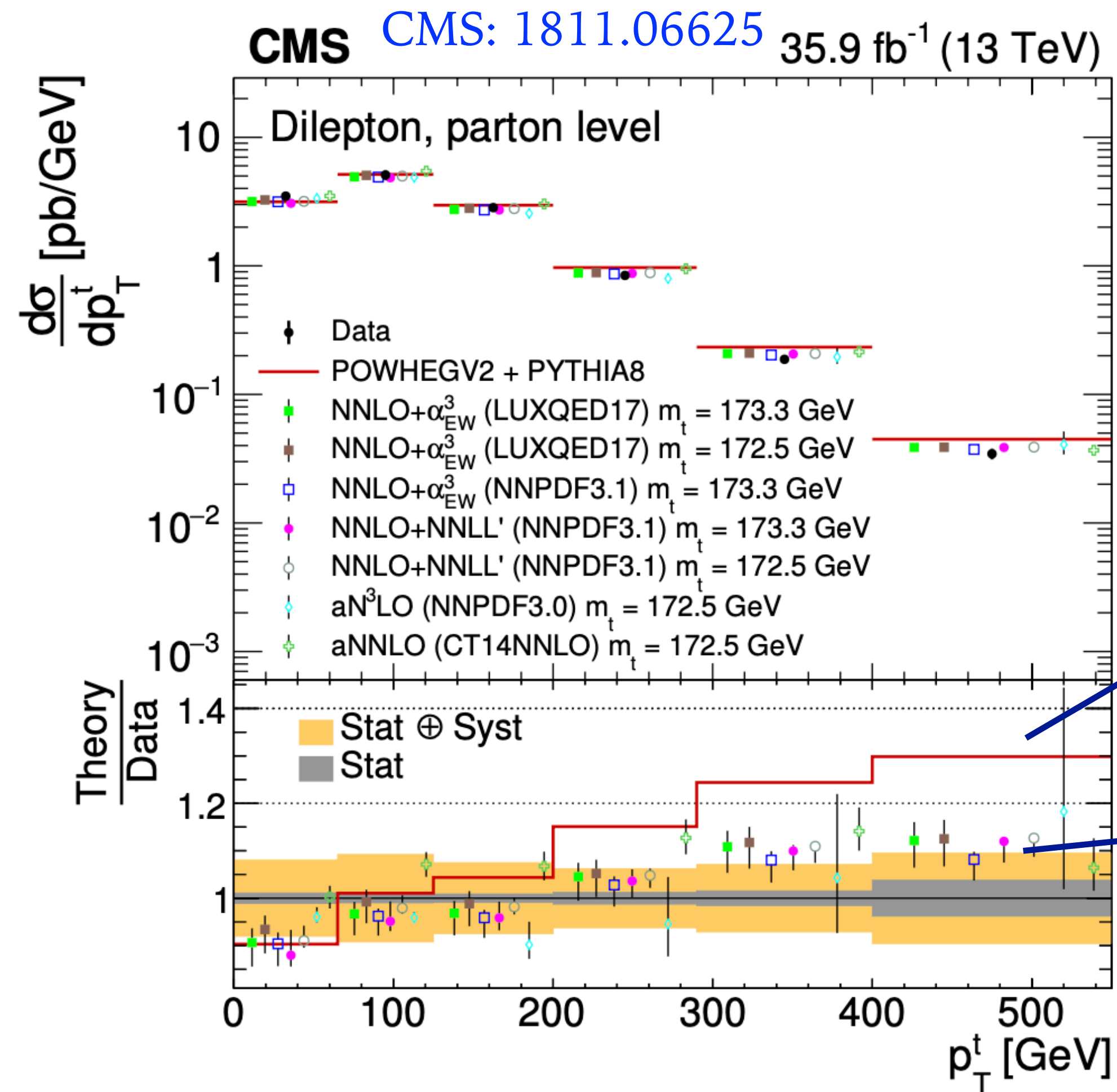


- The heaviest fundamental particle
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What is the “toponium” then?

Top quark pair production



State-of-the-art theoretical prediction
NNLO+NNLL' in QCD + NLO in EW

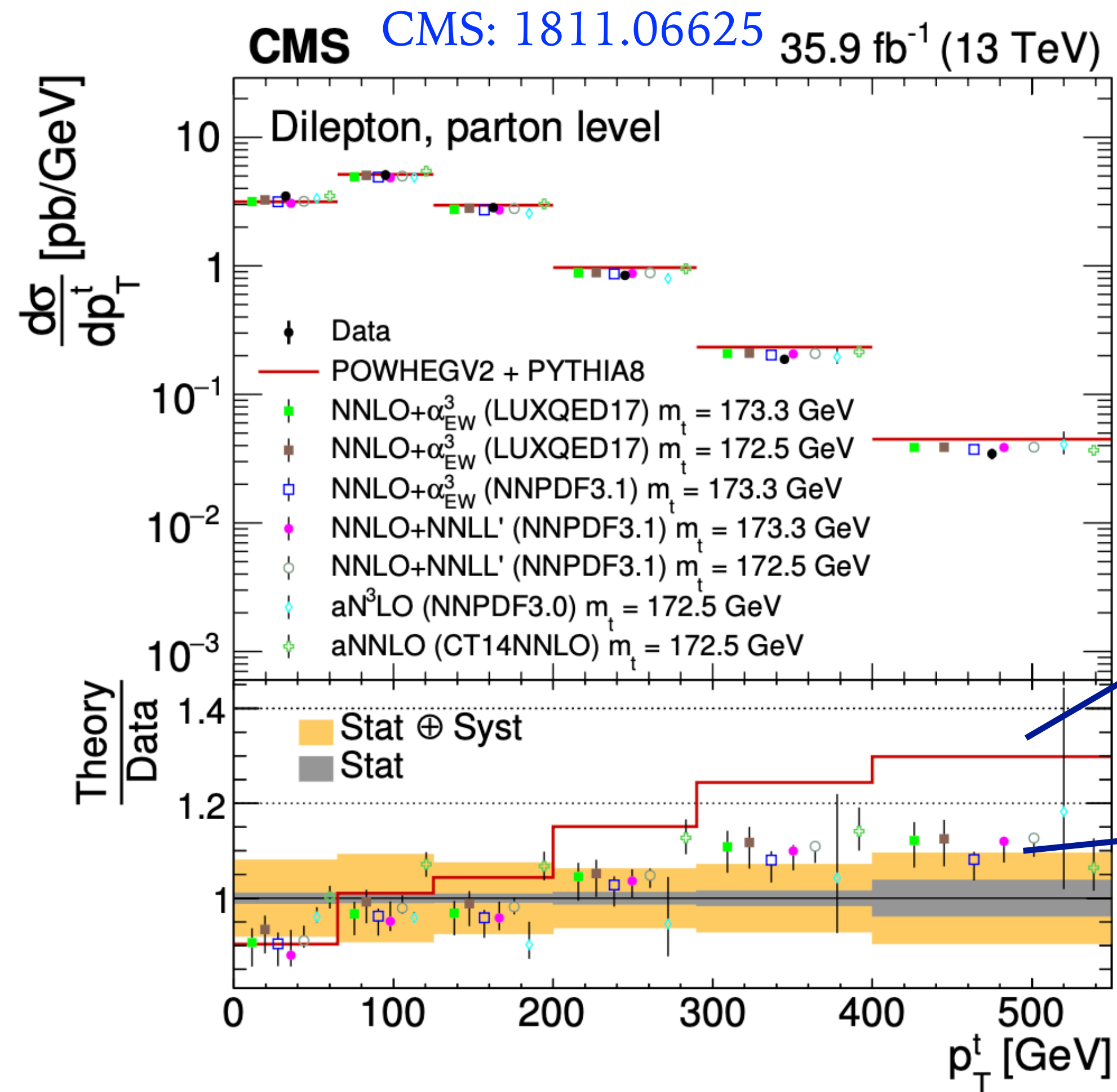
Pecjak, Scott, Wang, LLY: 1601.07020

Czakon et al.: 1803.07623, 1901.08281

NLO alone cannot describe data

Including higher-order corrections

Top quark pair production



State-of-the-art theoretical prediction

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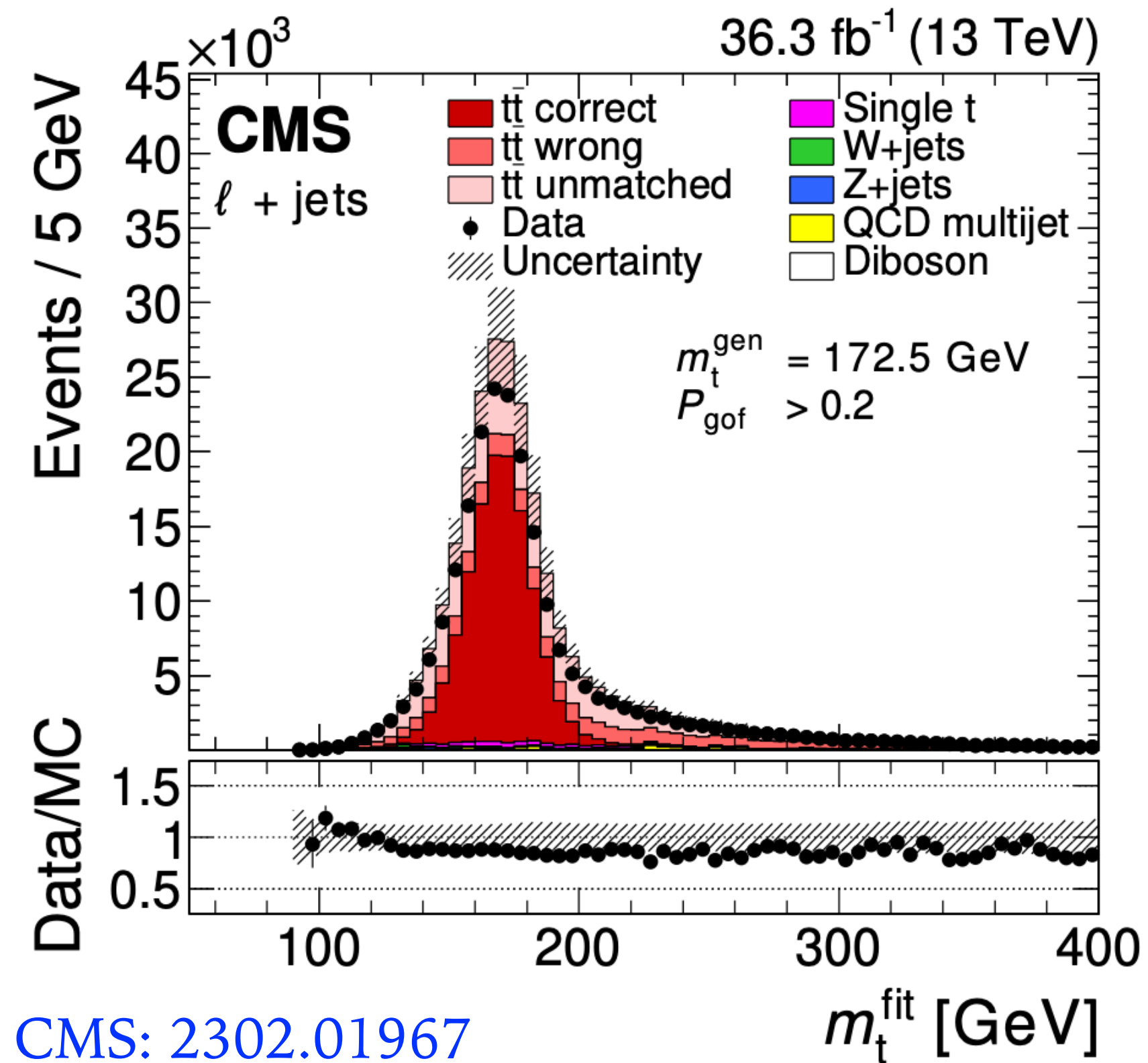
NLO alone cannot describe data

Including higher-order corrections

With such precision data and predictions, one would like to extract m_t from them...

Top quark mass measurement

Direct measurement

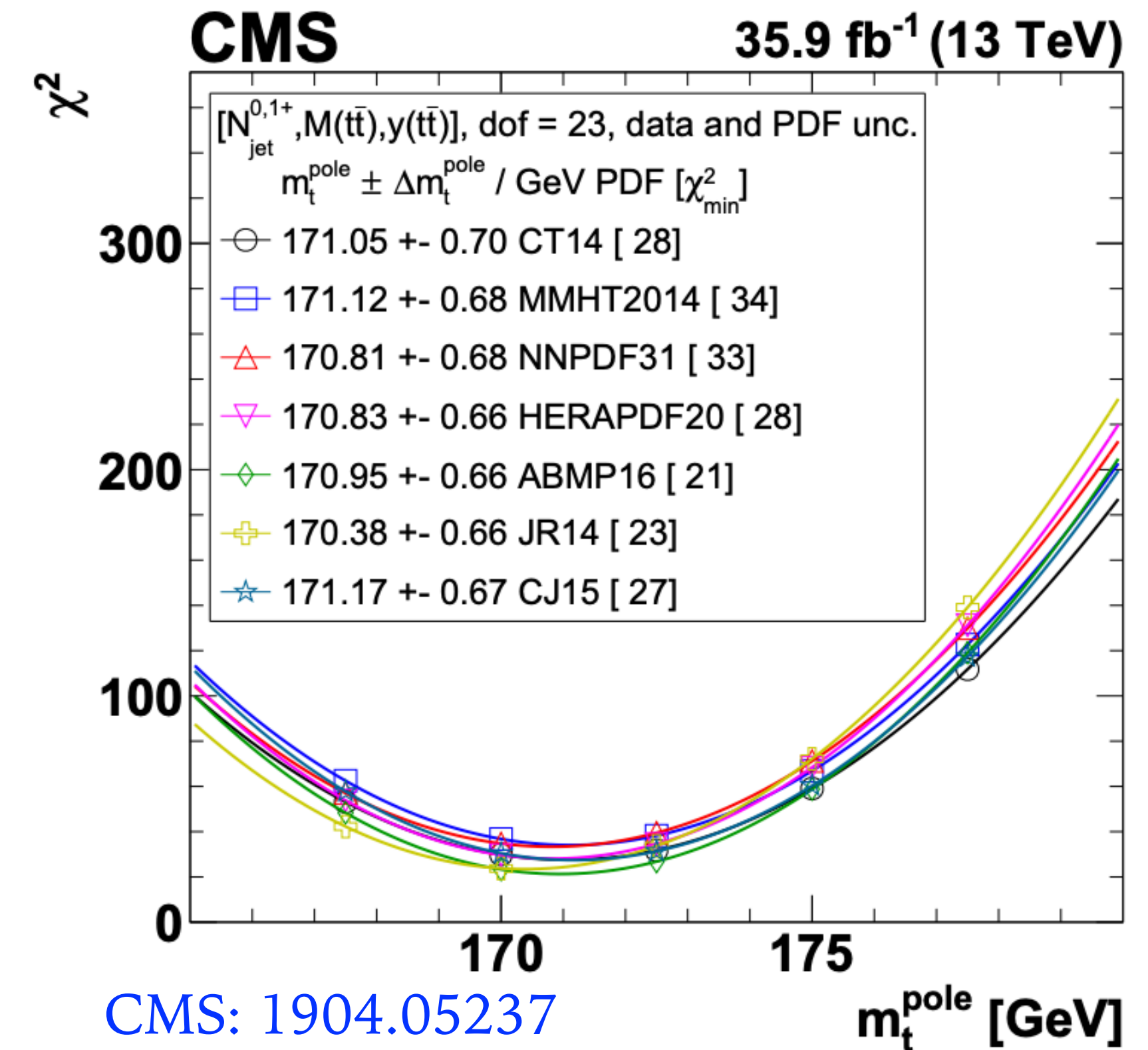


CMS: 2302.01967

Reconstruct the invariant mass
of decay products

VS

Indirect measurement

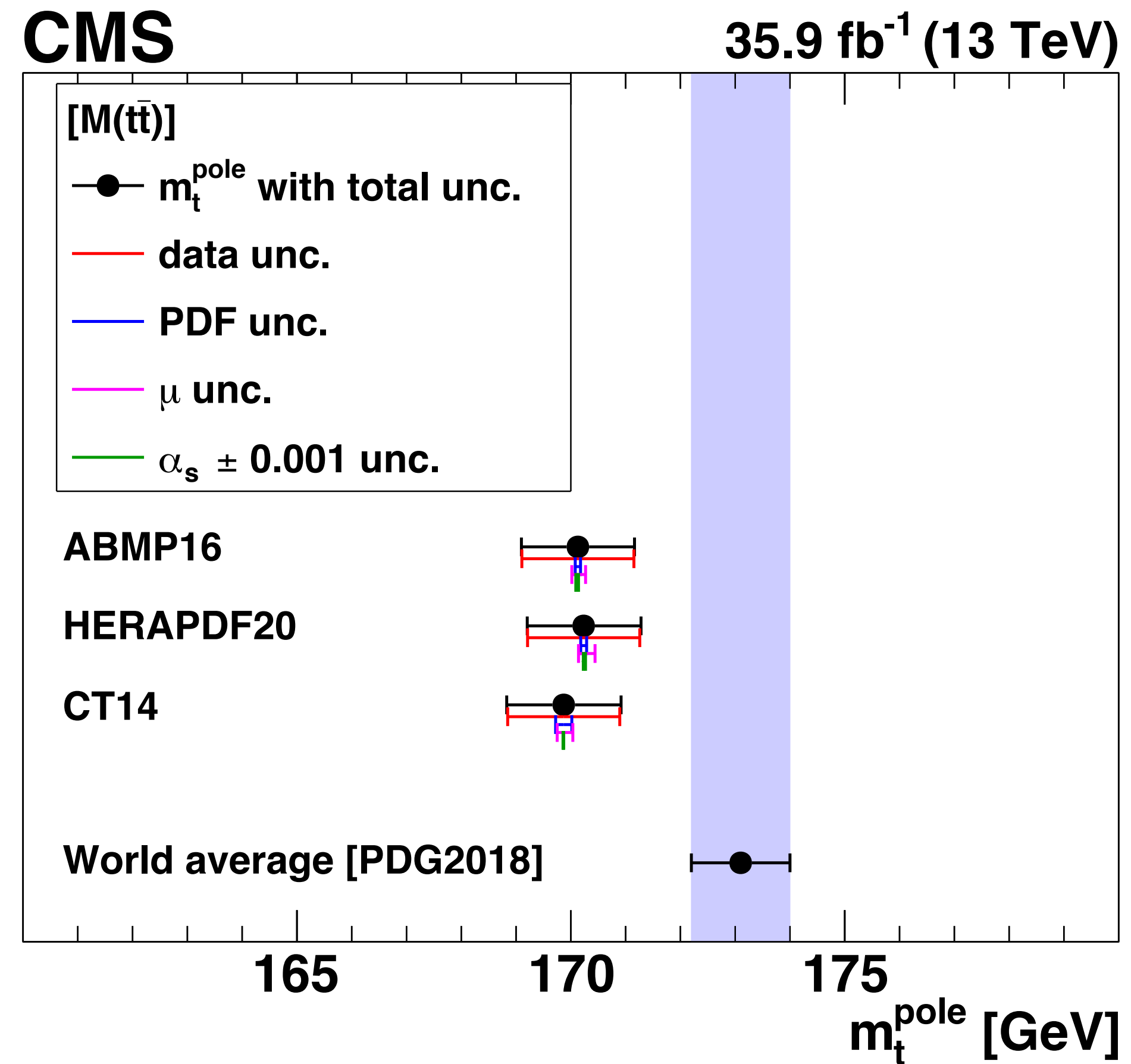


$$\frac{d\sigma}{d\mathcal{O}}^{\text{TH}}(m_t) = \frac{d\sigma}{d\mathcal{O}}^{\text{Exp}}$$

Top quark mass measurement

ATLAS: 1905.02302

CMS: 1904.05237

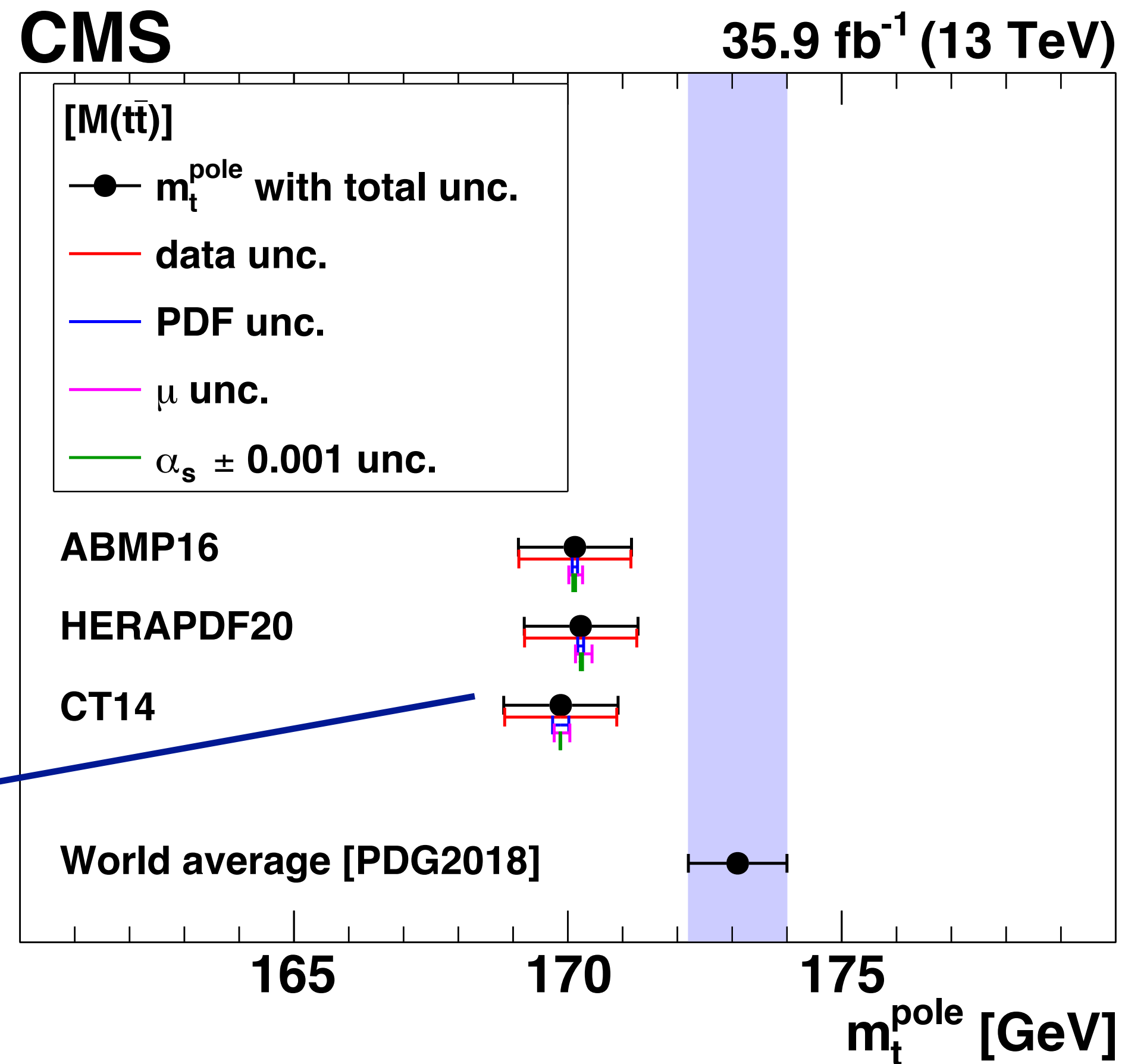


Top quark mass measurement

ATLAS: 1905.02302

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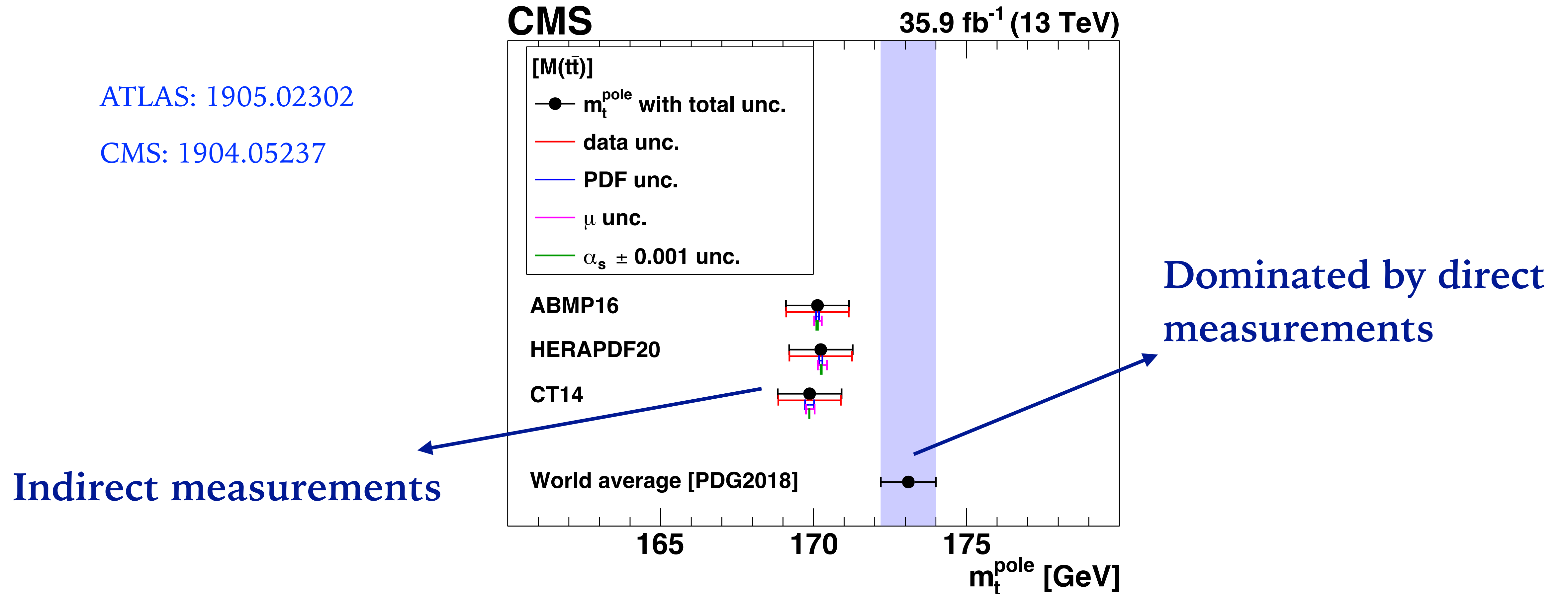
Indirect measurements



Top quark mass measurement

ATLAS: 1905.02302

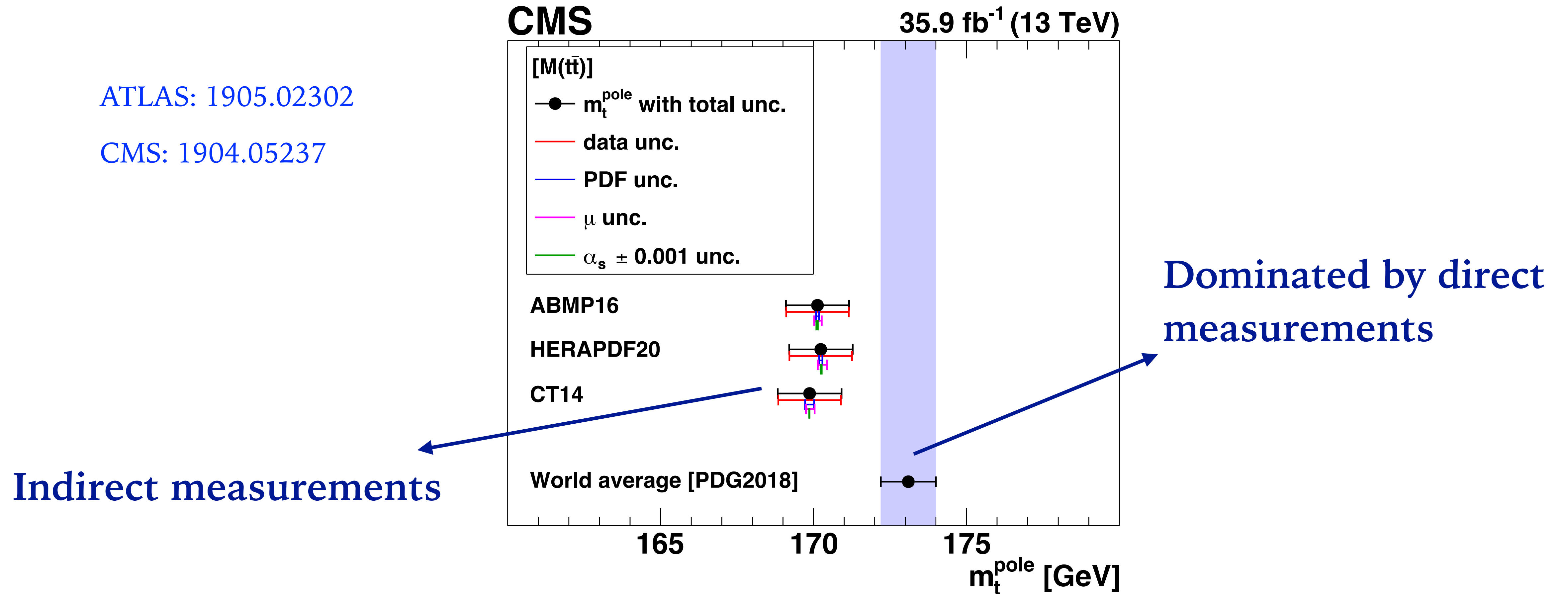
CMS: 1904.05237



Top quark mass measurement

ATLAS: 1905.02302

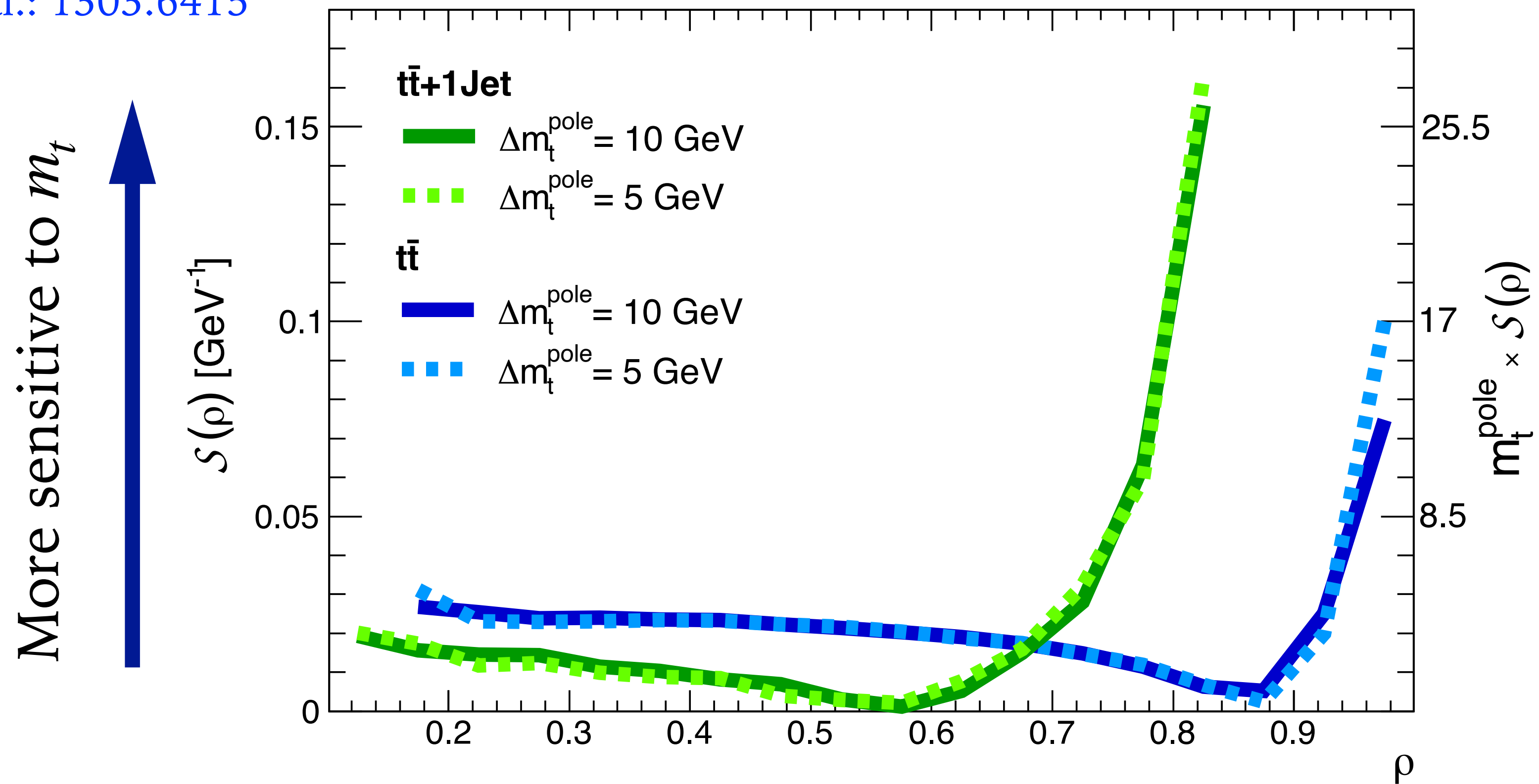
CMS: 1904.05237



Tension! Why?

Threshold region and mass measurement

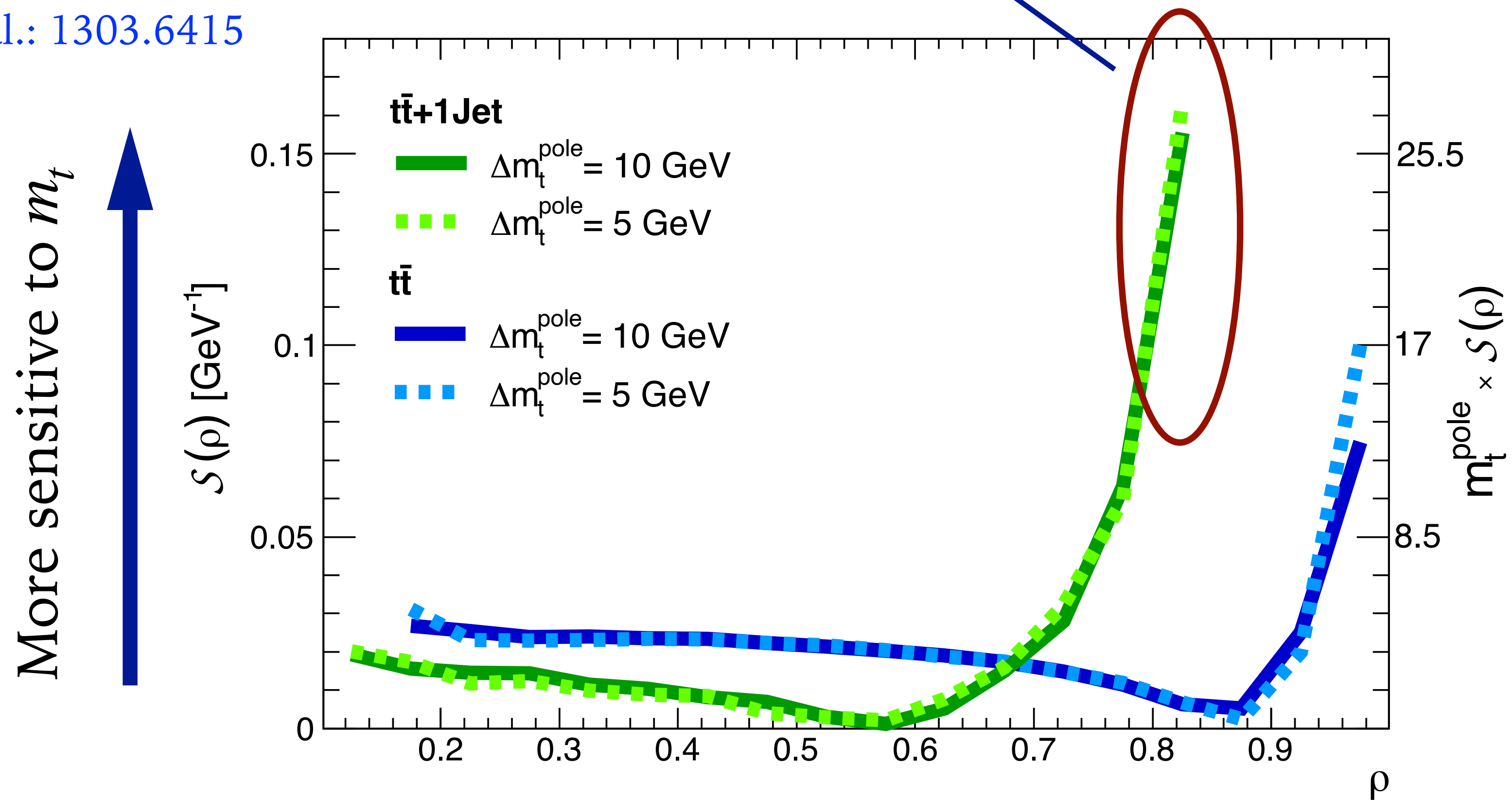
Alioli et al.: 1303.6415



Threshold region and mass measurement

$M_{t\bar{t}} \rightarrow 2m_t$ in $t\bar{t}$ +jets production

Alioli et al.: 1303.6415

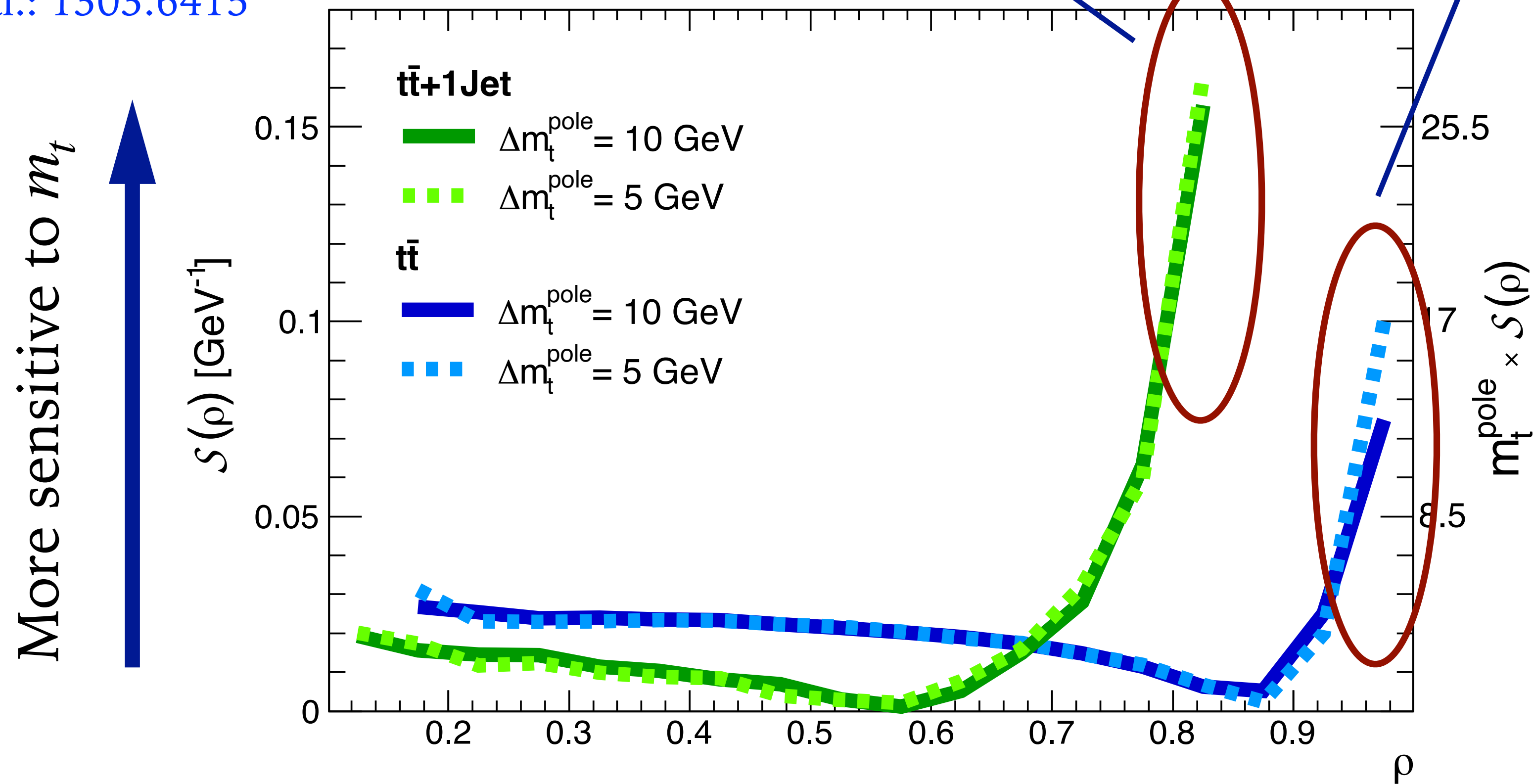


Threshold region and mass measurement

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$M_{t\bar{t}} \rightarrow 2m_t$ in $t\bar{t}$ production

Alioli et al.: 1303.6415

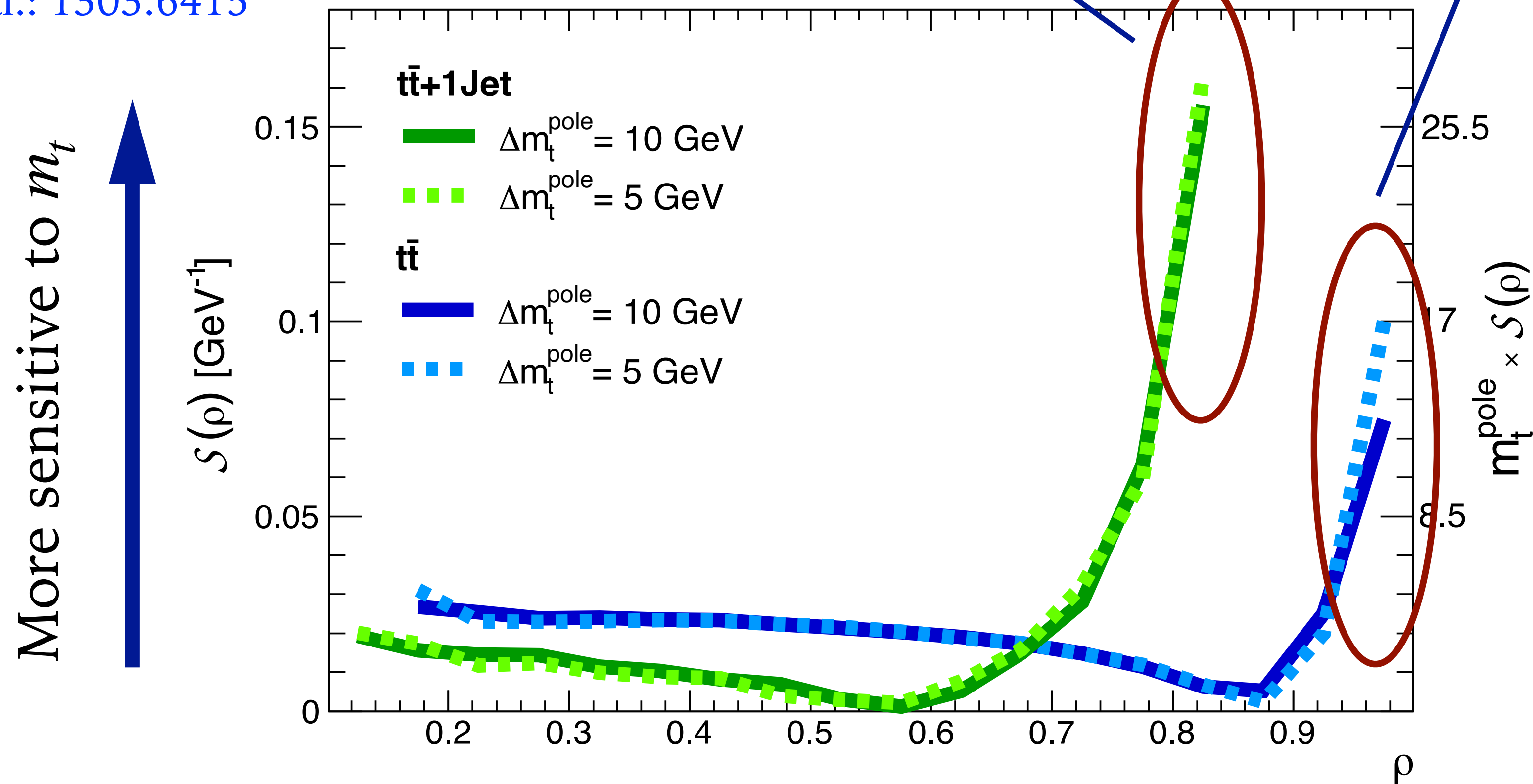


Threshold region and mass measurement

$M_{t\bar{t}} \rightarrow 2m_t$ in $t\bar{t}$ +jets production

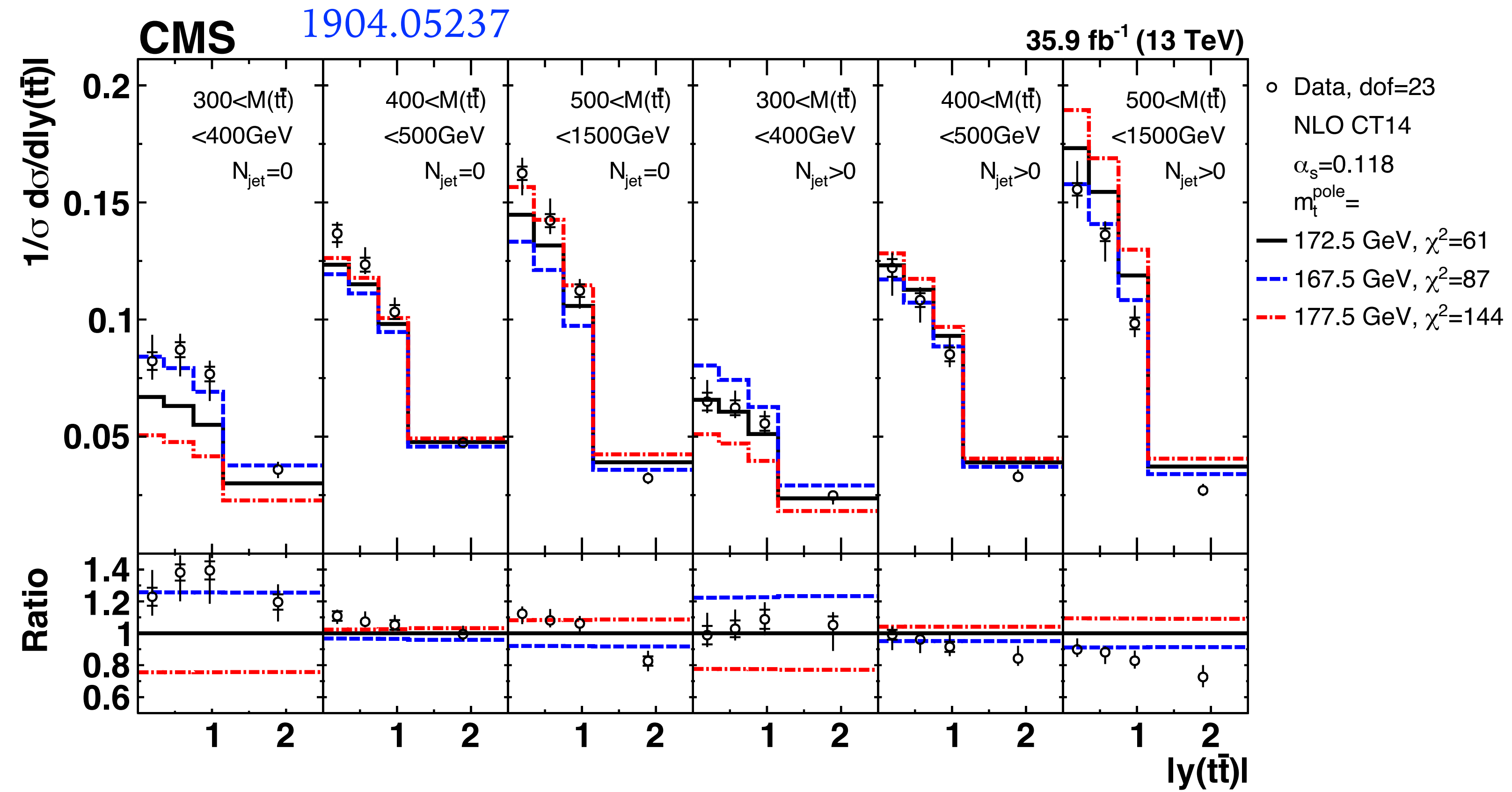
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Alioli et al.: 1303.6415

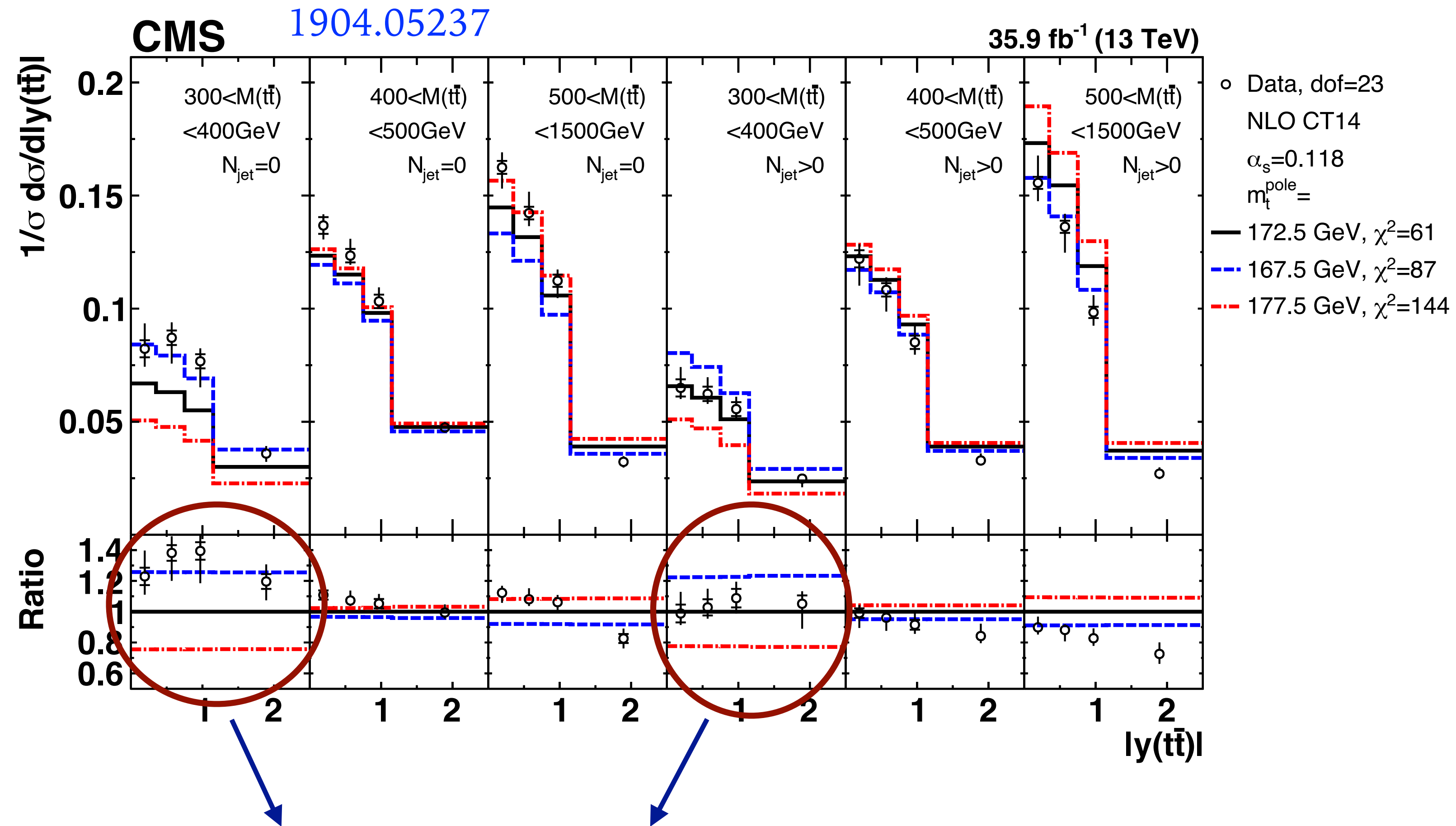


It is well-known that threshold region is most sensitive to m_t

Threshold region and mass measurement

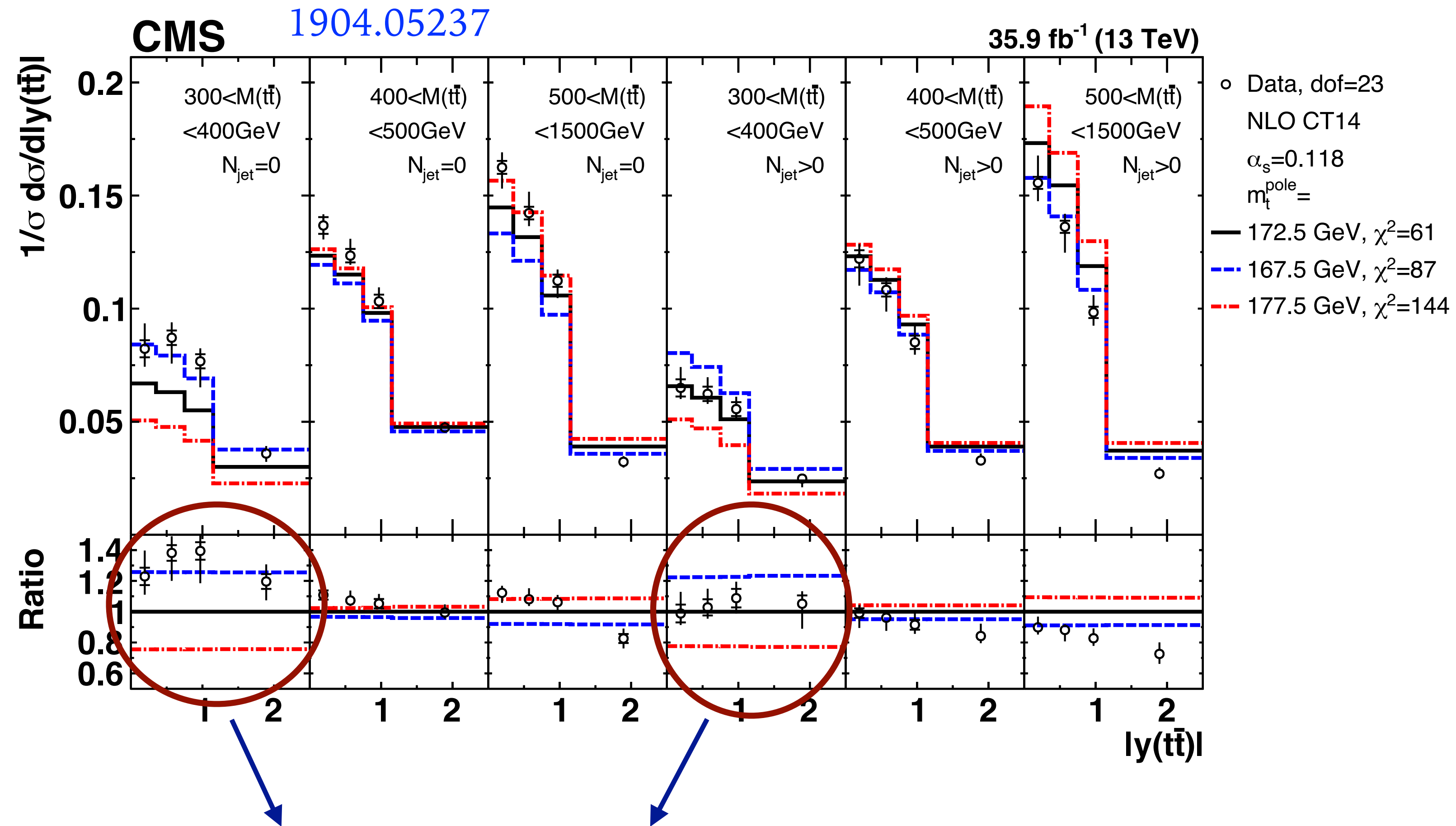


Threshold region and mass measurement



Data also shows that most of the mass-sensitivity comes from the low $M_{t\bar{t}}$ region

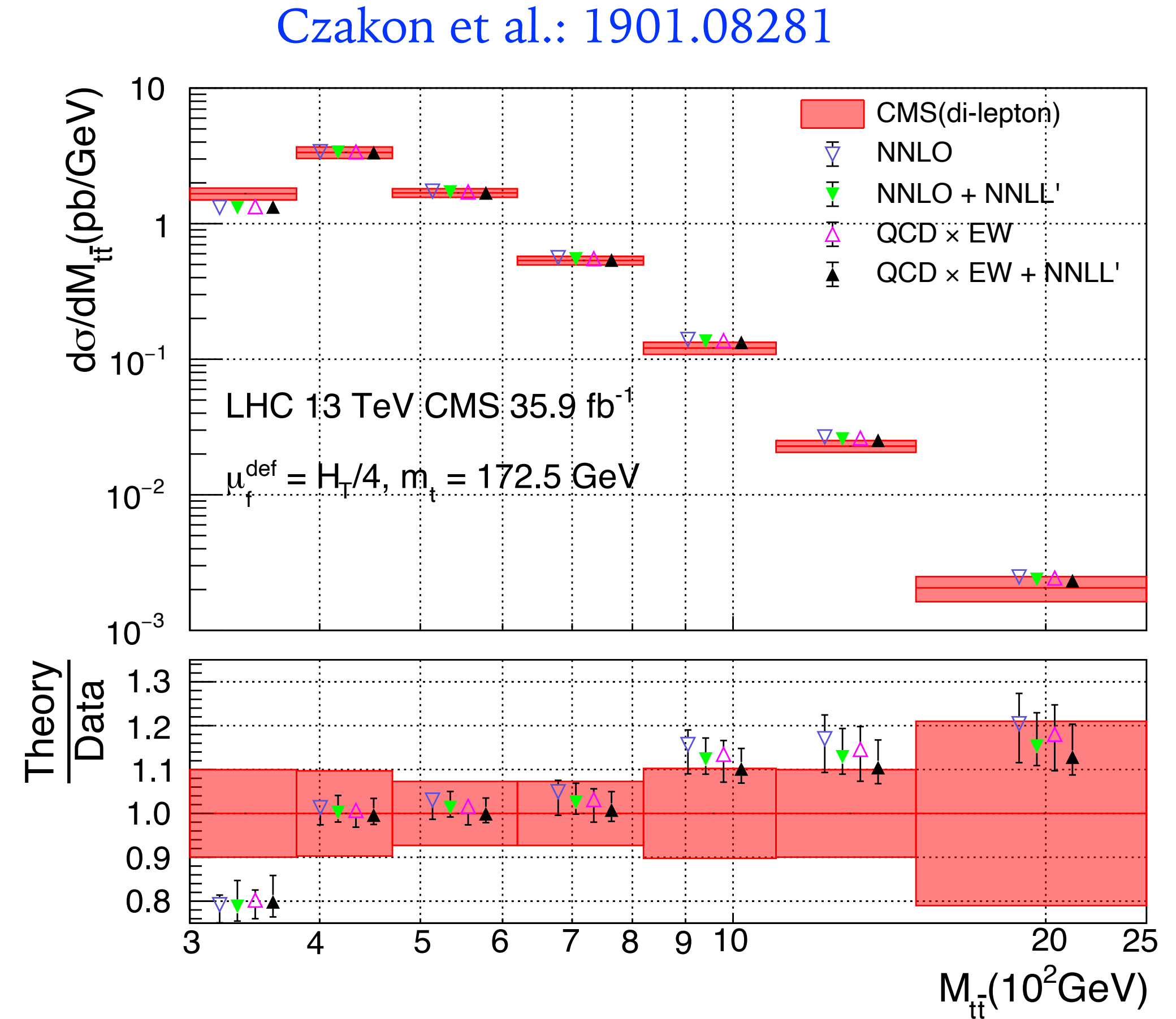
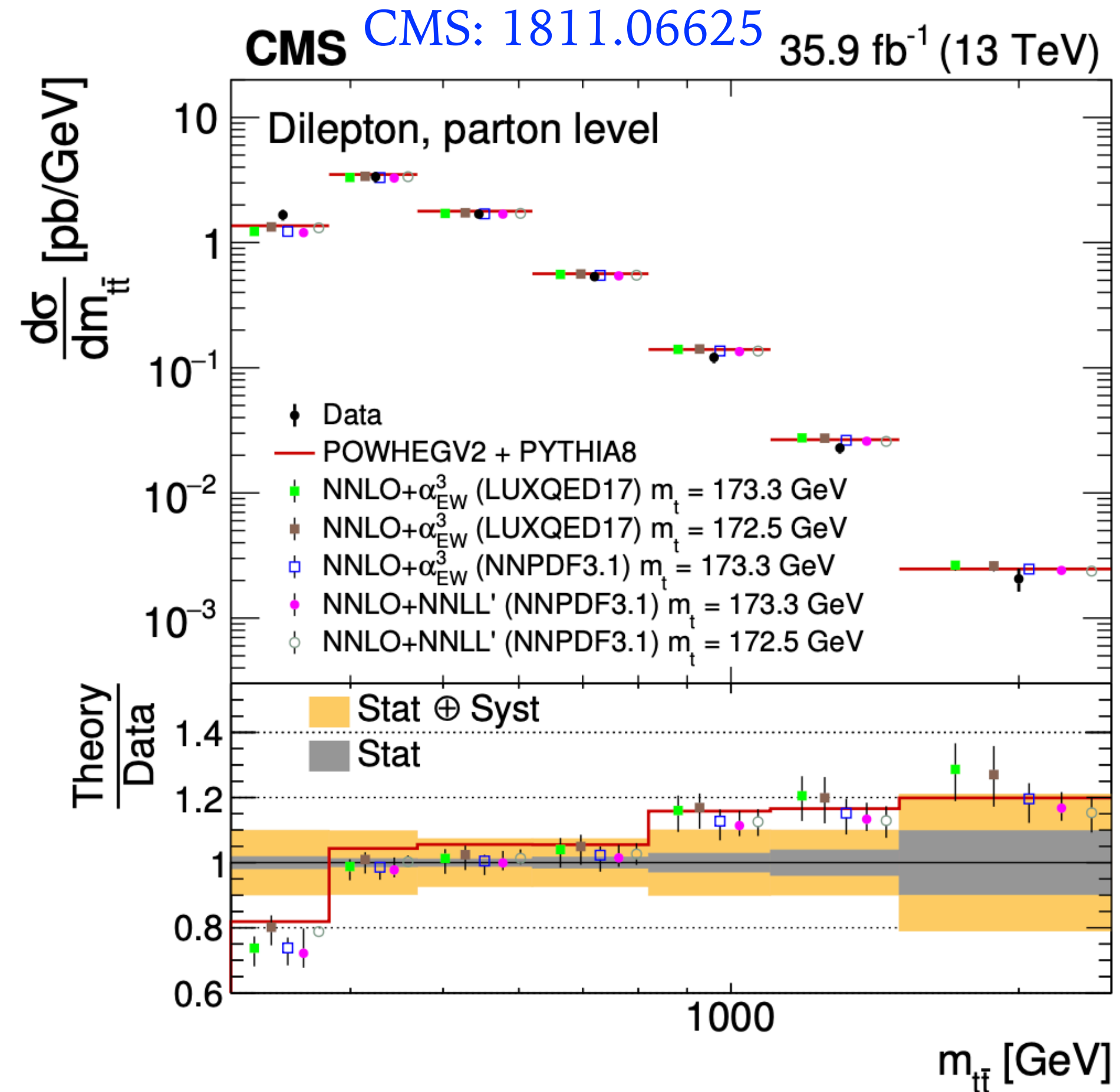
Threshold region and mass measurement



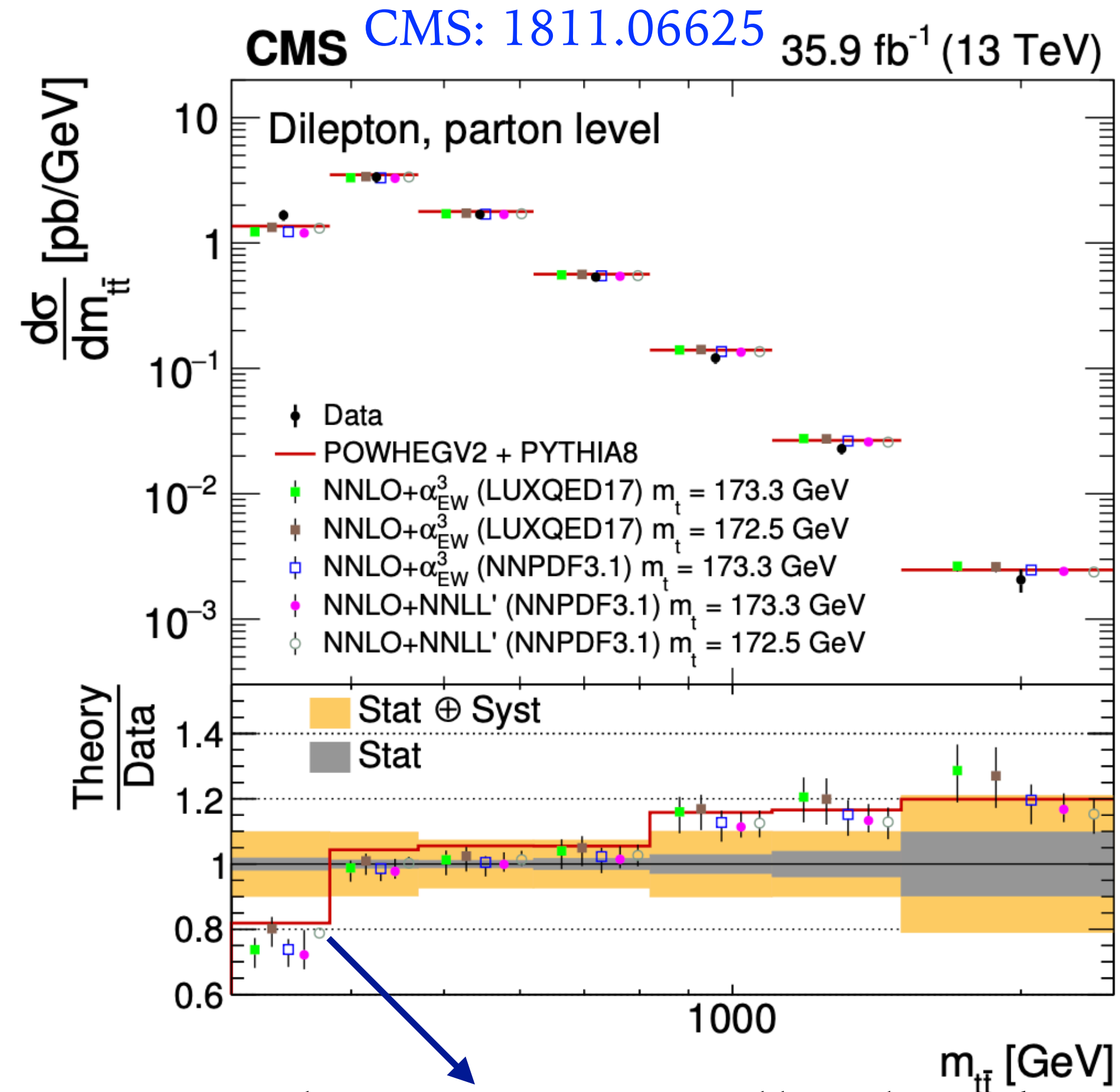
Data also shows that most of the mass-sensitivity comes from the low $M_{t\bar{t}}$ region

What's going on here?

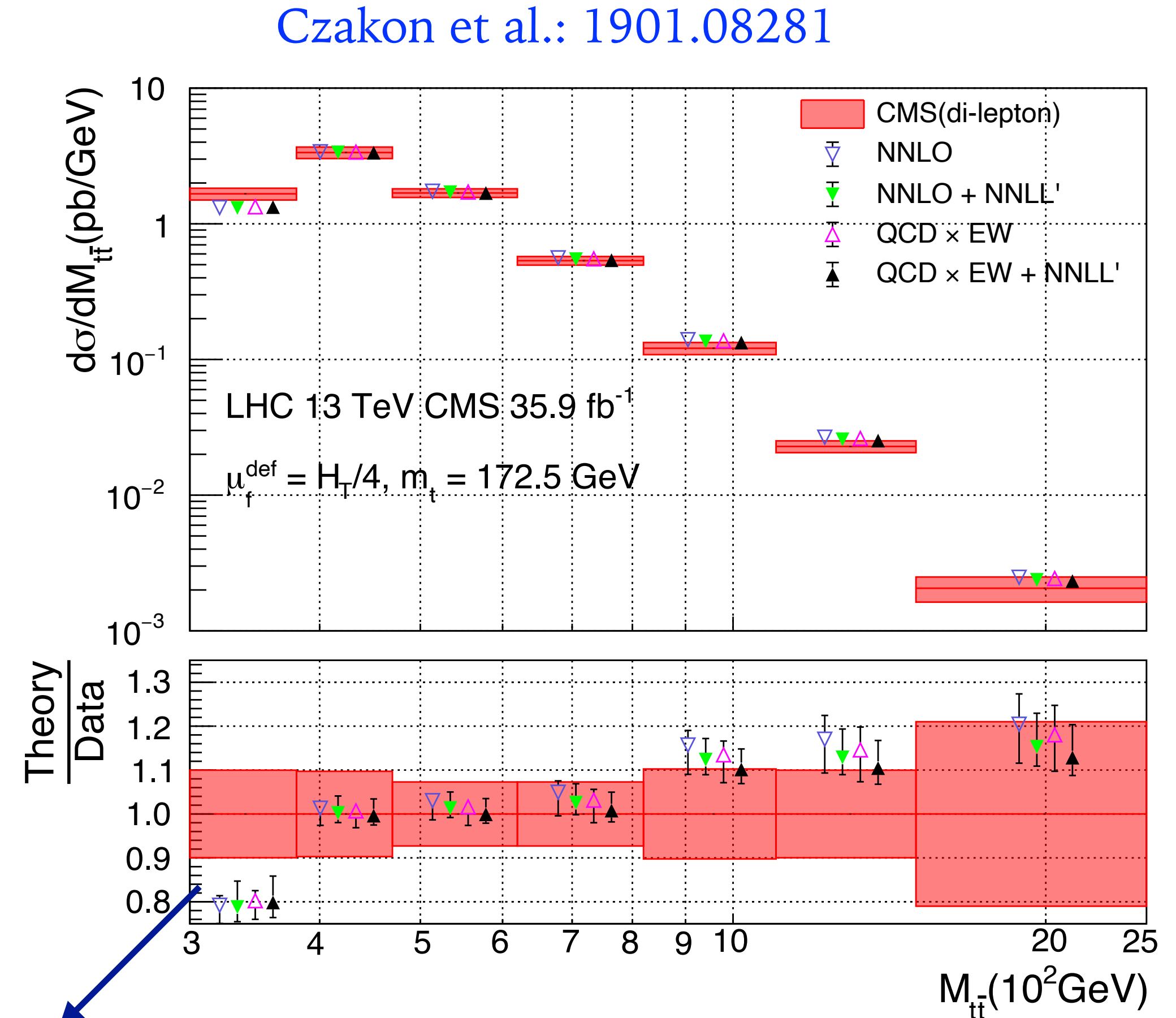
Theoretical predictions vs experimental data



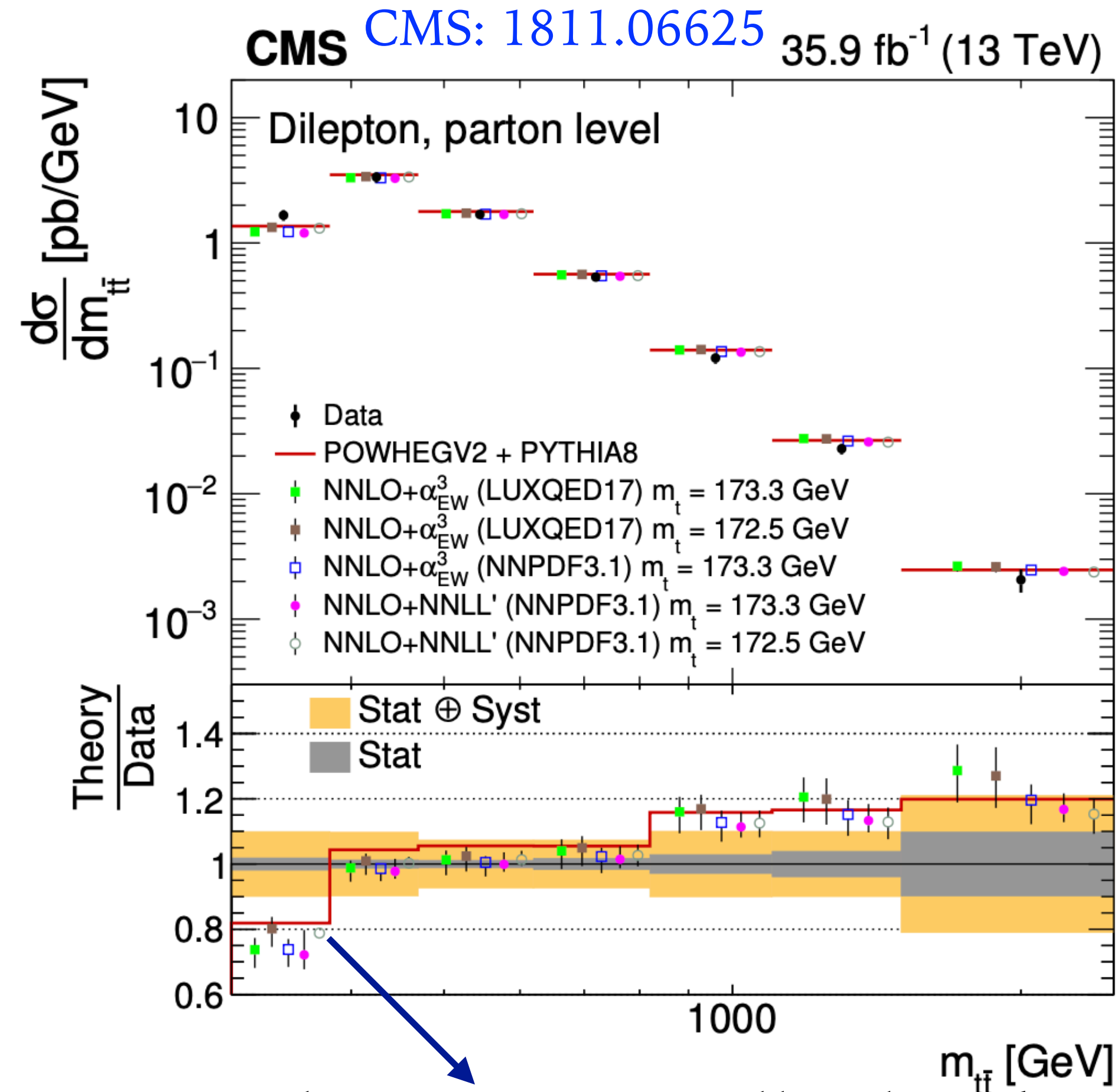
Theoretical predictions vs experimental data



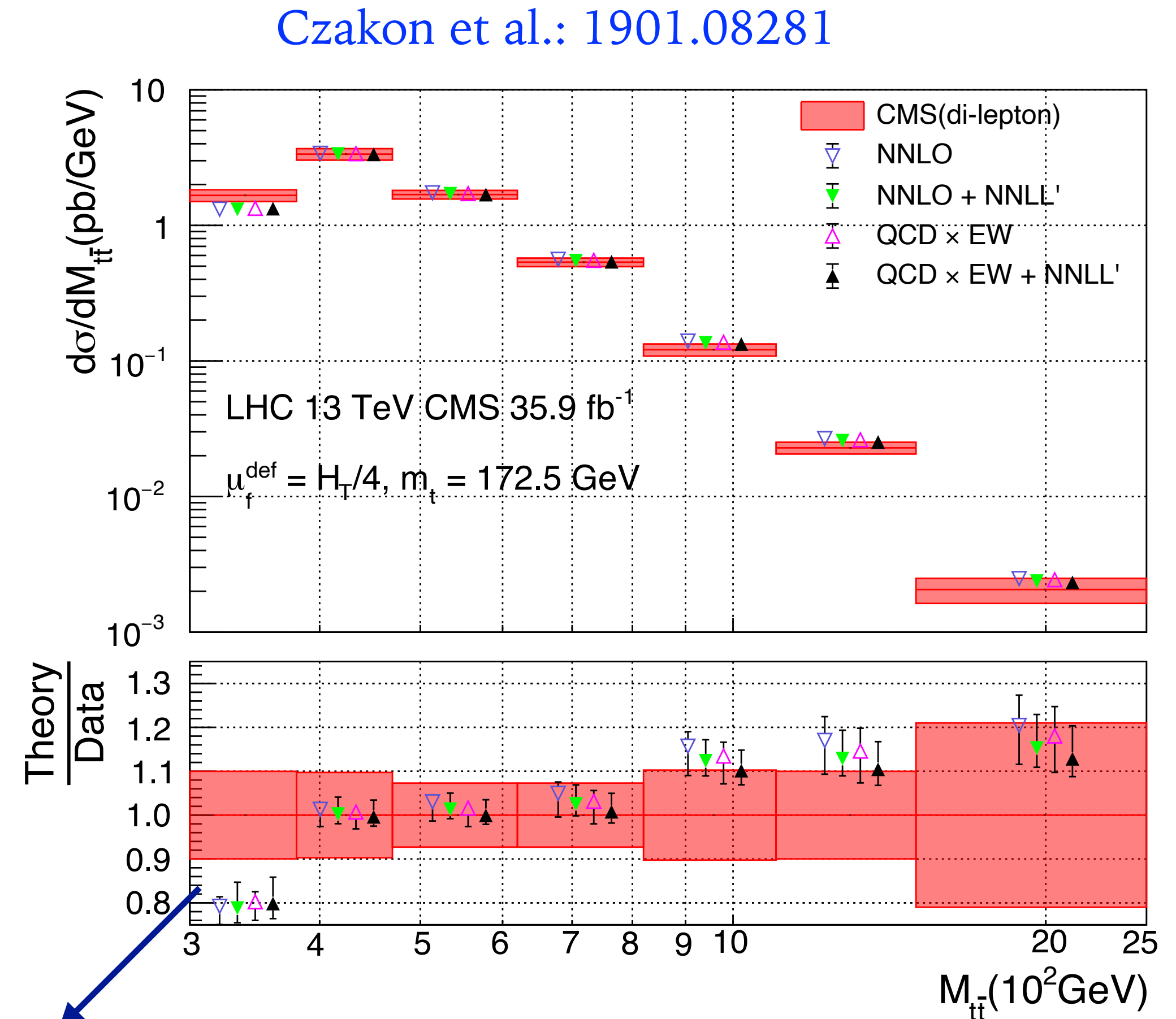
There was actually already a discrepancy between the state-of-the-art theoretical predictions and experimental data in the threshold region!



Theoretical predictions vs experimental data

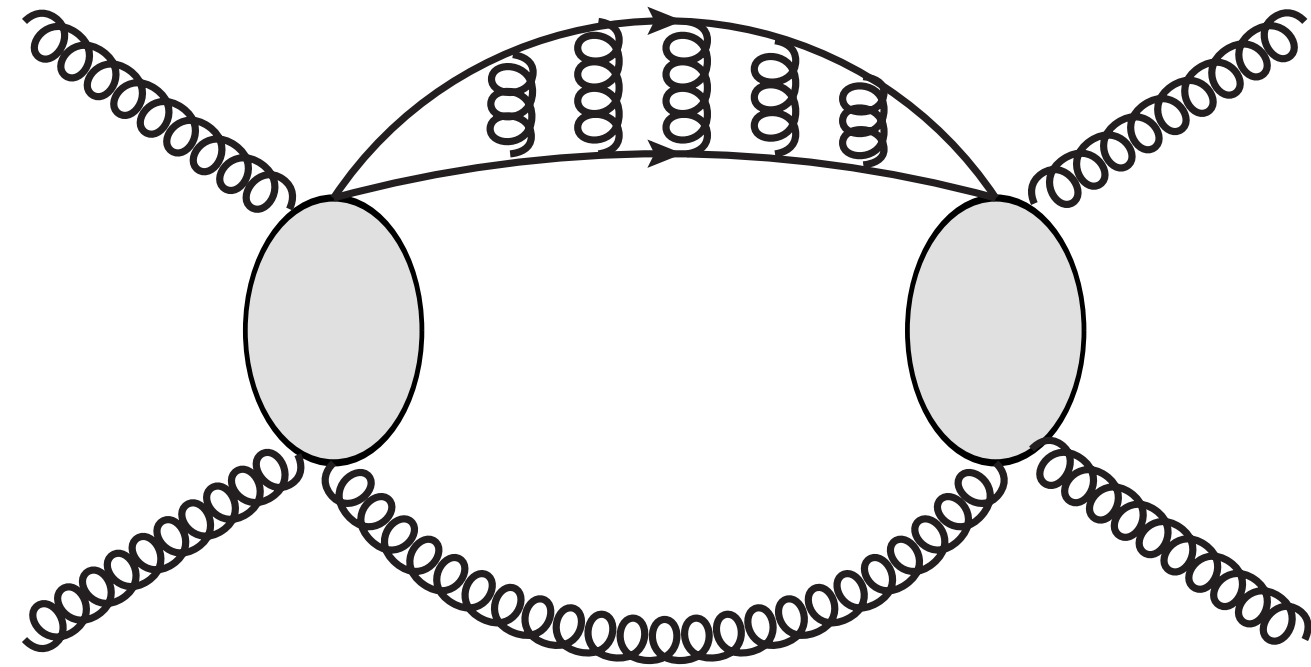


There was actually already a discrepancy between the state-of-the-art theoretical predictions and experimental data in the threshold region!



More corrections? New physics?

Non-relativistic Sommerfeld enhancement

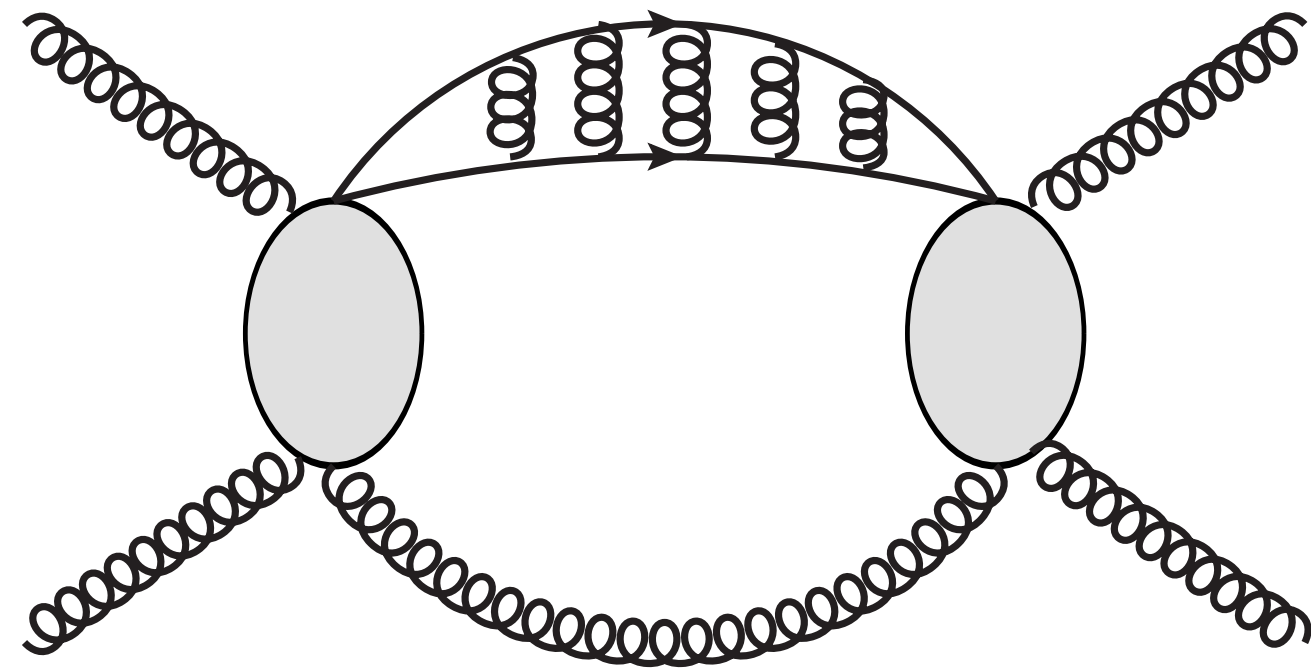


$$\frac{d\sigma}{dM_{t\bar{t}}d\Theta} \sim \beta \left(1 + \frac{\alpha_s}{\beta} + \frac{\alpha_s^2}{\beta^2} + \frac{\alpha_s^3}{\beta^3} + \dots \right)$$

$$\beta = \sqrt{1 - \frac{4m_t^2}{M_{t\bar{t}}^2}} \quad \beta \sim \alpha_s \ll 1$$

3-velocity of the top quark in
the $t\bar{t}$ center-of-mass frame

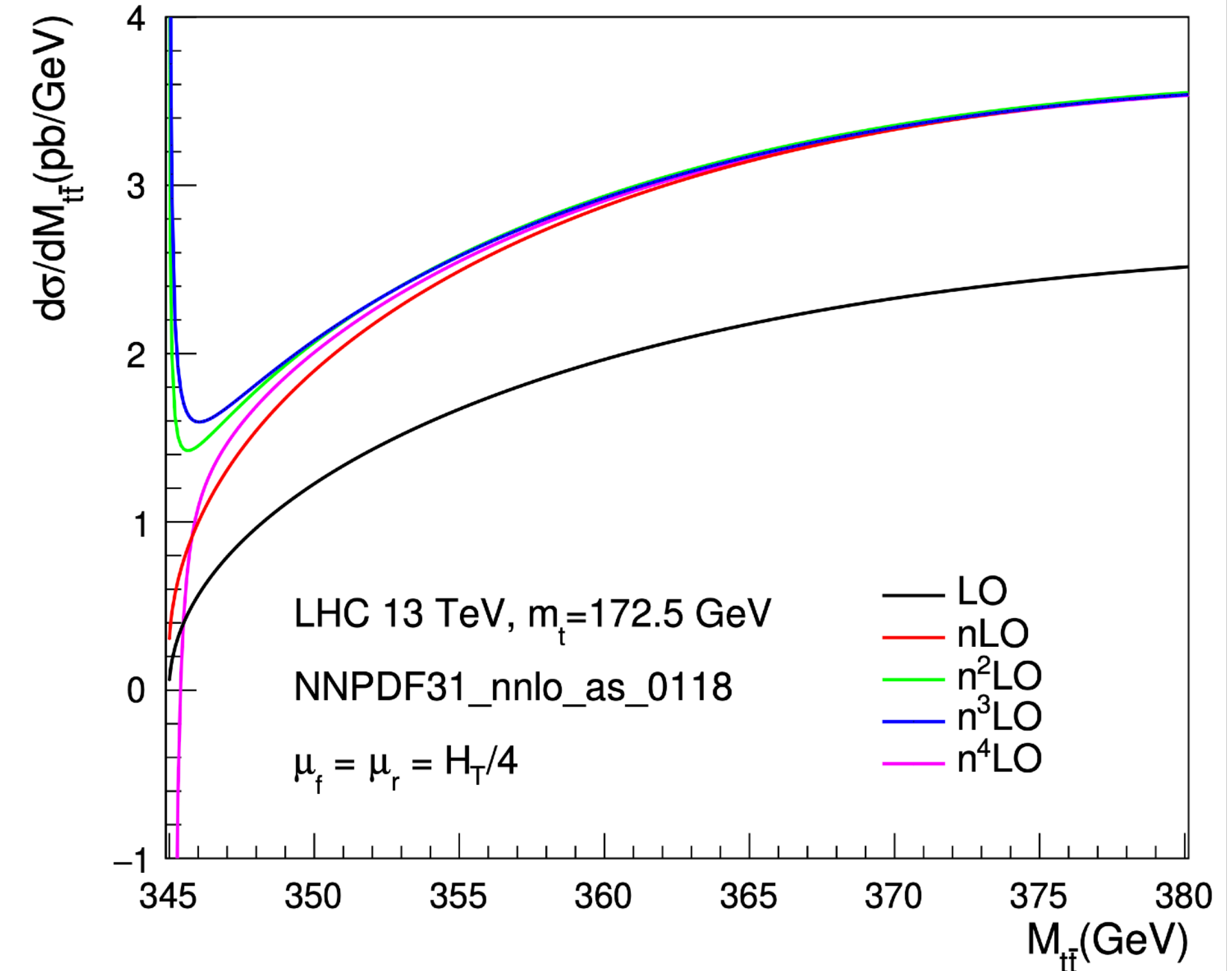
Non-relativistic Sommerfeld enhancement



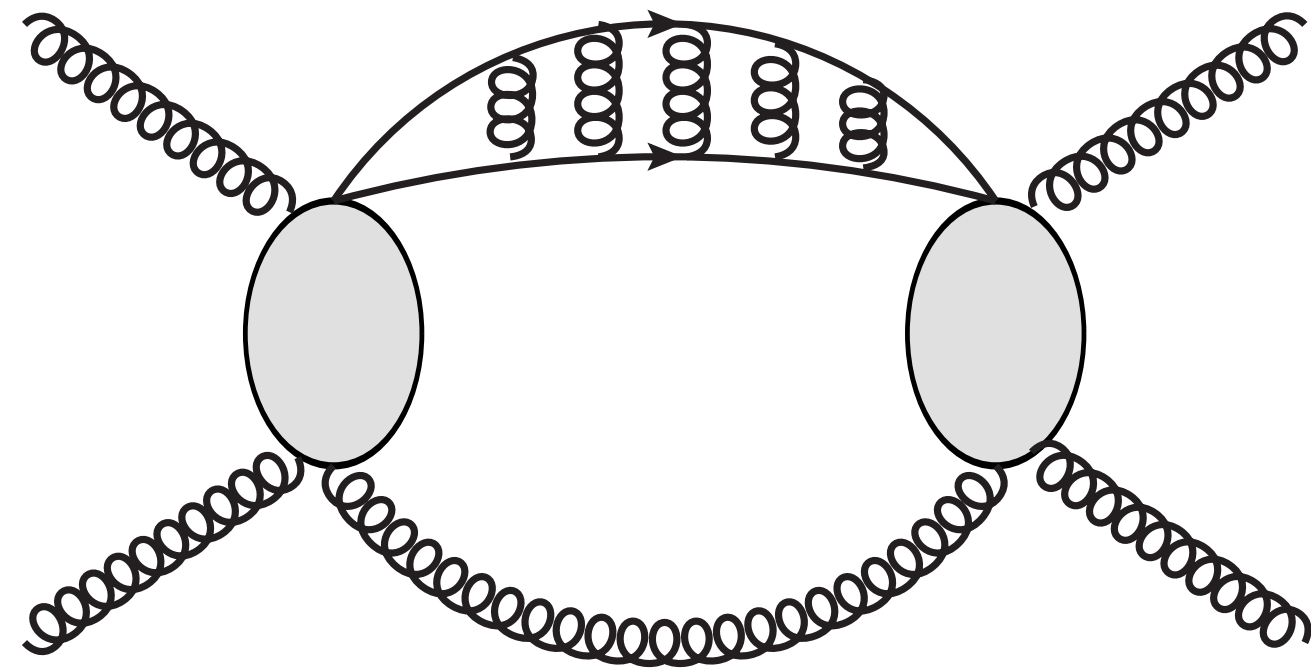
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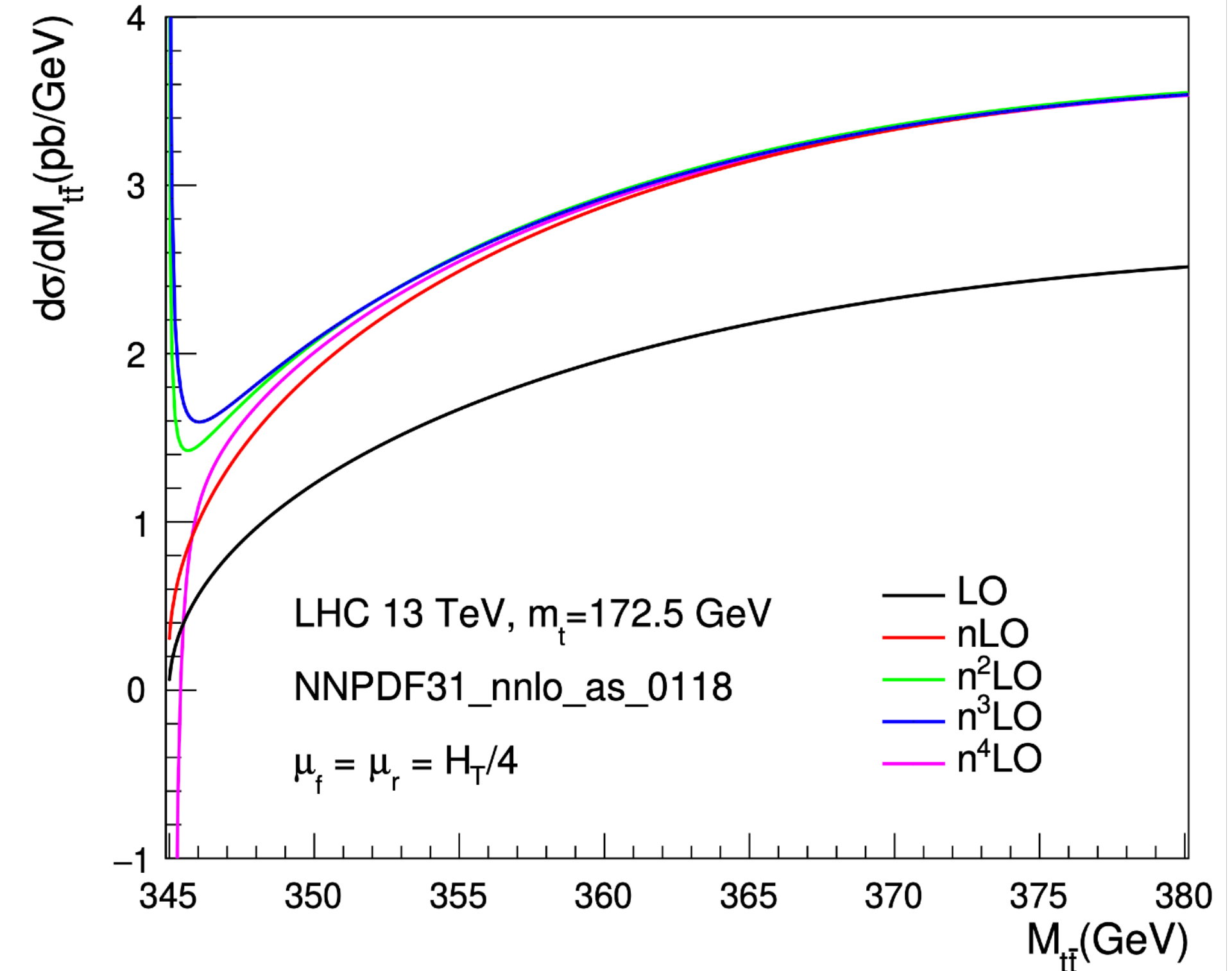
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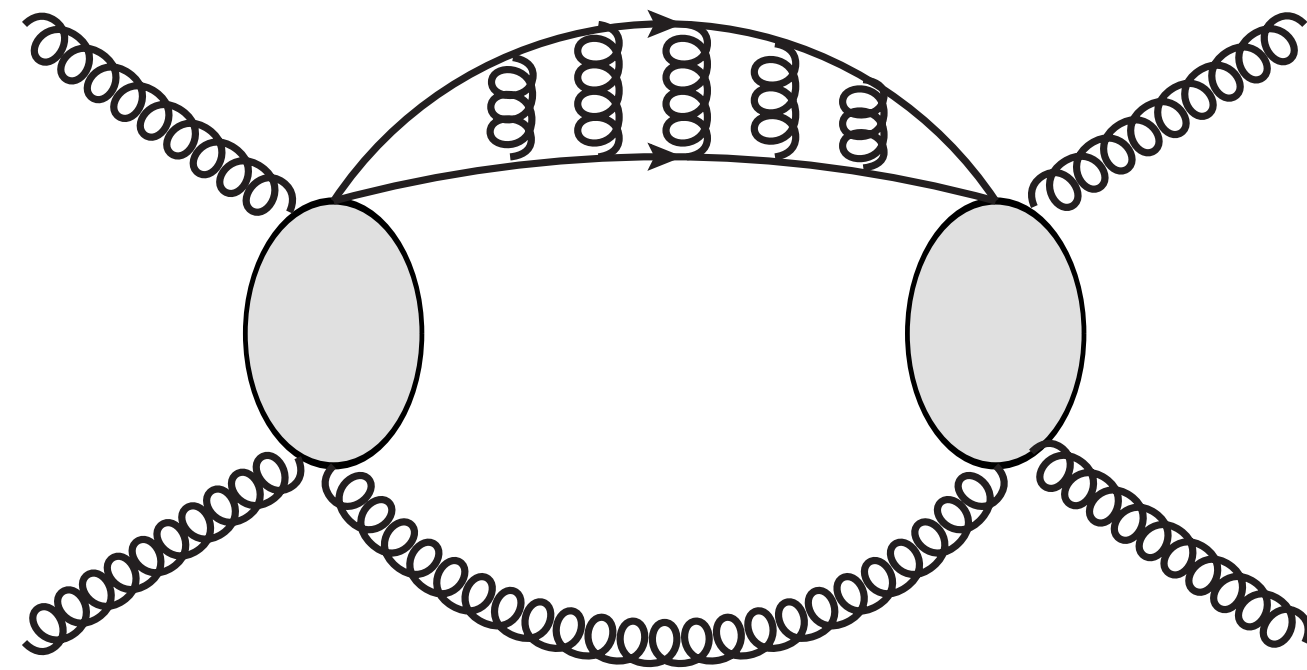
3-velocity of the top quark in the $t\bar{t}$ center-of-mass frame



- No big deal at NLO
- Integrable at NNLO
- Subtleties beyond NNLO

$$dM_{t\bar{t}} \sim \beta d\beta$$

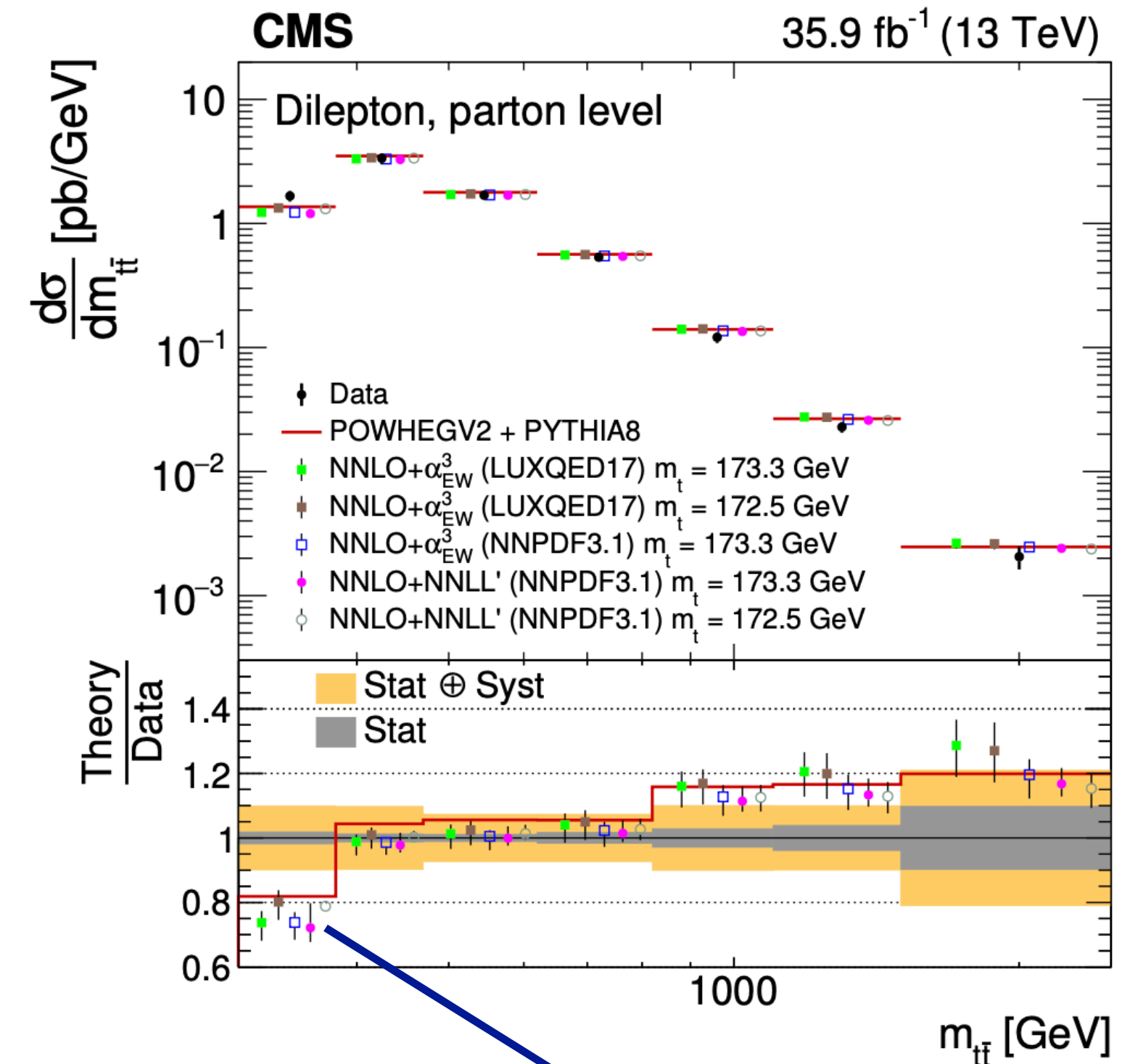
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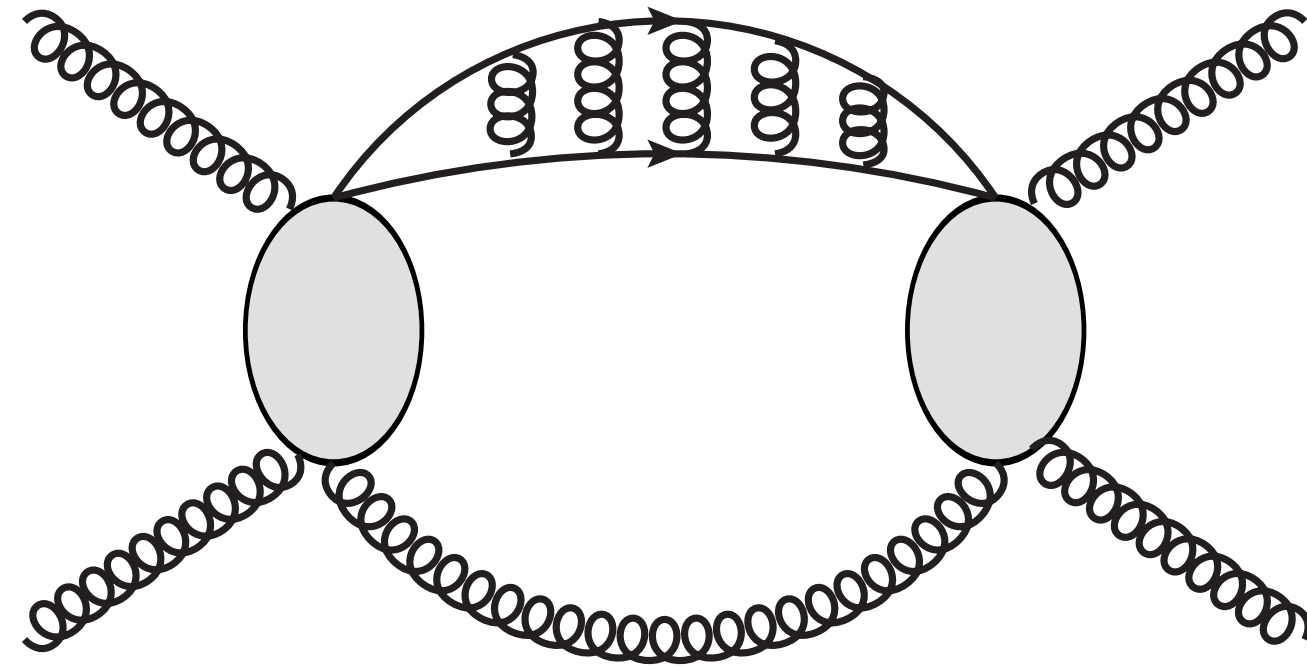
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3-velocity of the top quark in the $t\bar{t}$ center-of-mass frame



It's not “new” effects — already included in the NNLO+NNLL' calculations, but only up to α_s^2

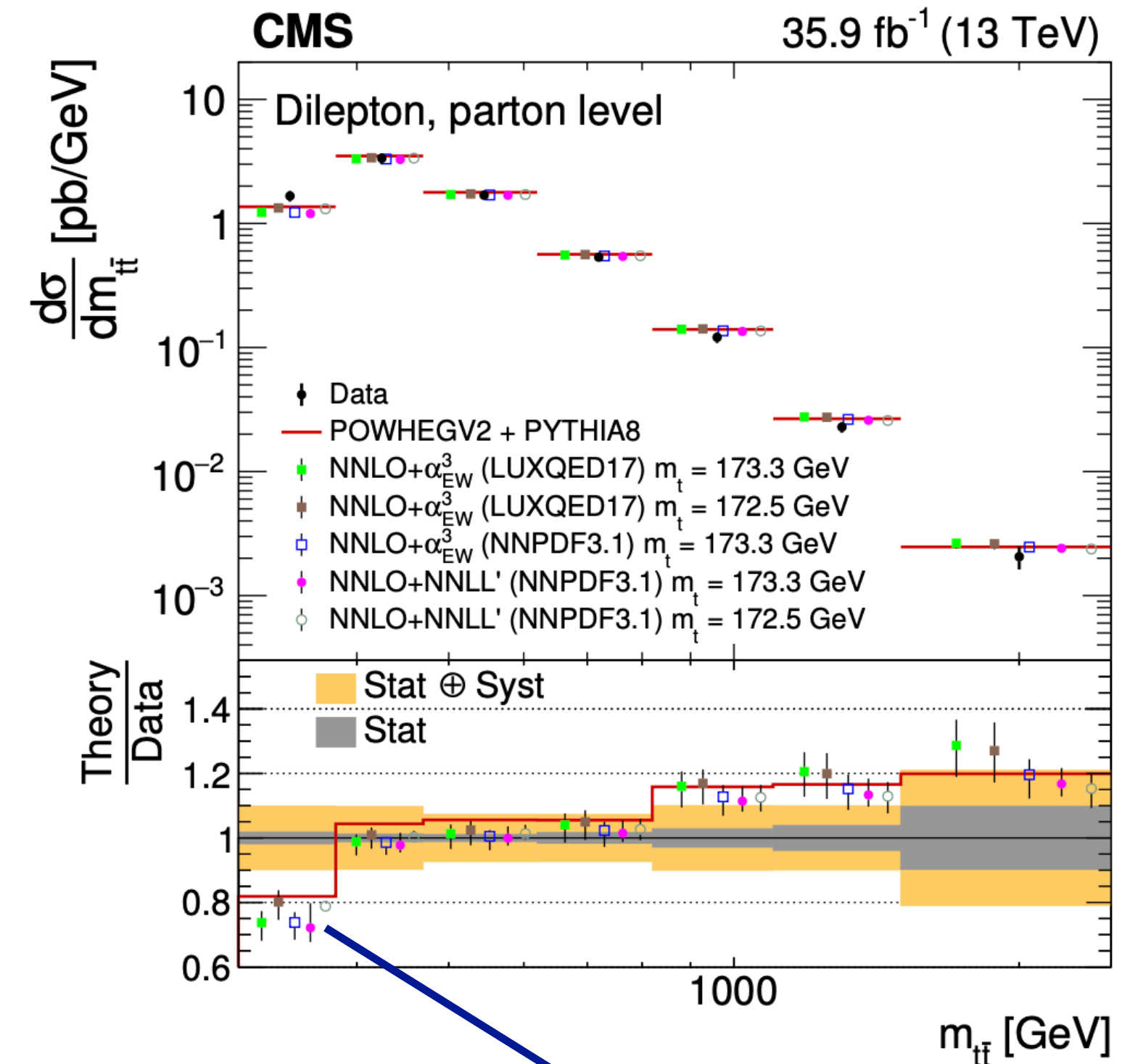
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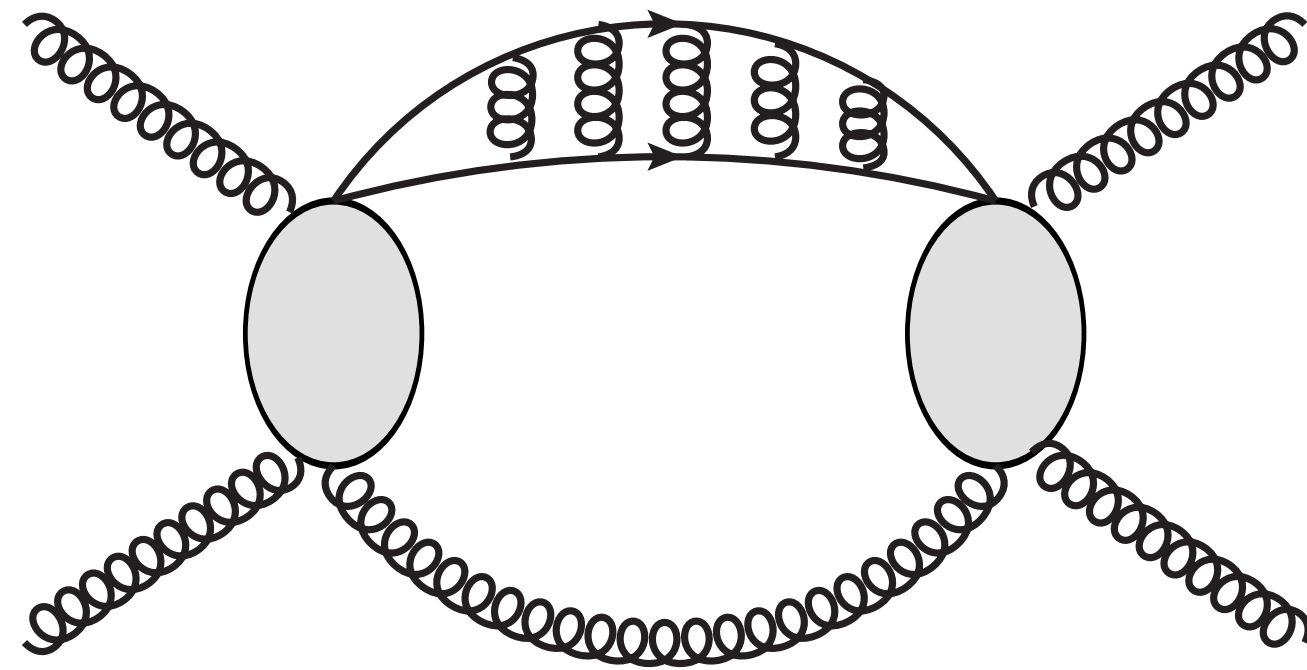
3-velocity of the top quark in the $t\bar{t}$ center-of-mass frame



It's not “new” effects — already included in the NNLO+NNLL' calculations, but only up to α_s^2

Higher order terms are important!

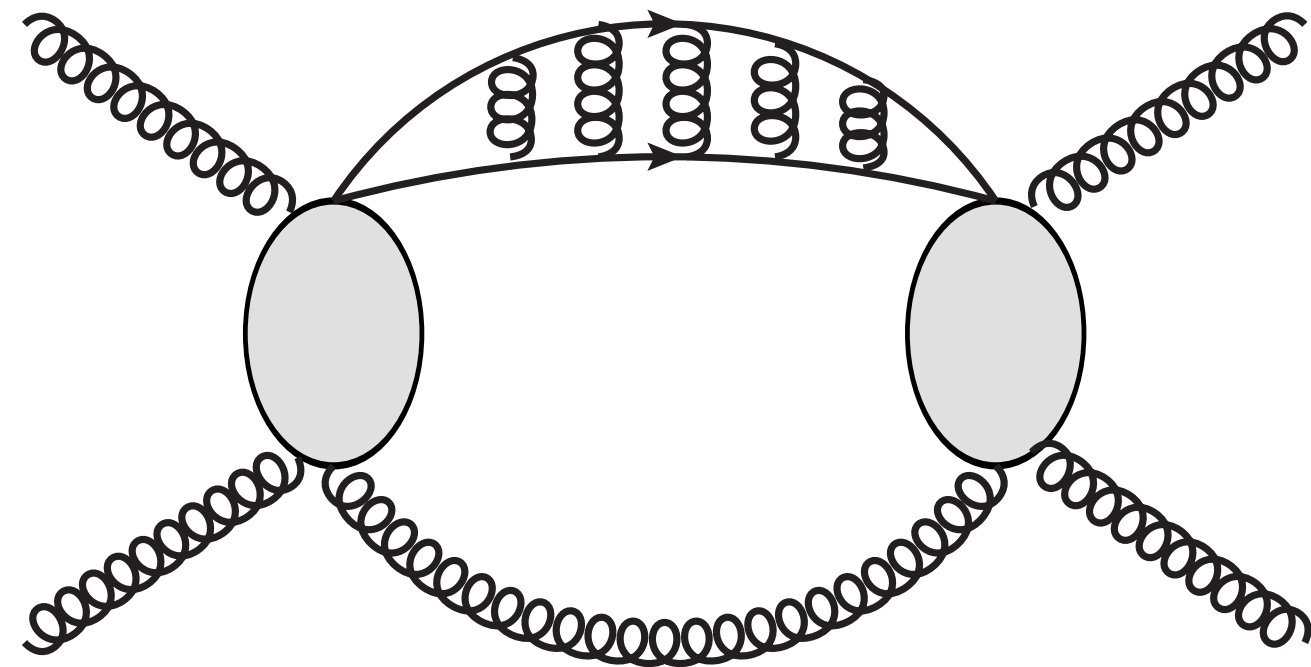
The “toponium”



$$\frac{d\sigma}{dM_{t\bar{t}}d\Theta} \sim \beta \left(1 + \frac{\alpha_s}{\beta} + \frac{\alpha_s^2}{\beta^2} + \frac{\alpha_s^3}{\beta^3} + \dots \right)$$

- Similar effects well-studied in quarkonium systems (hence the name “toponium”)
- However, there are important differences!

The “toponium”

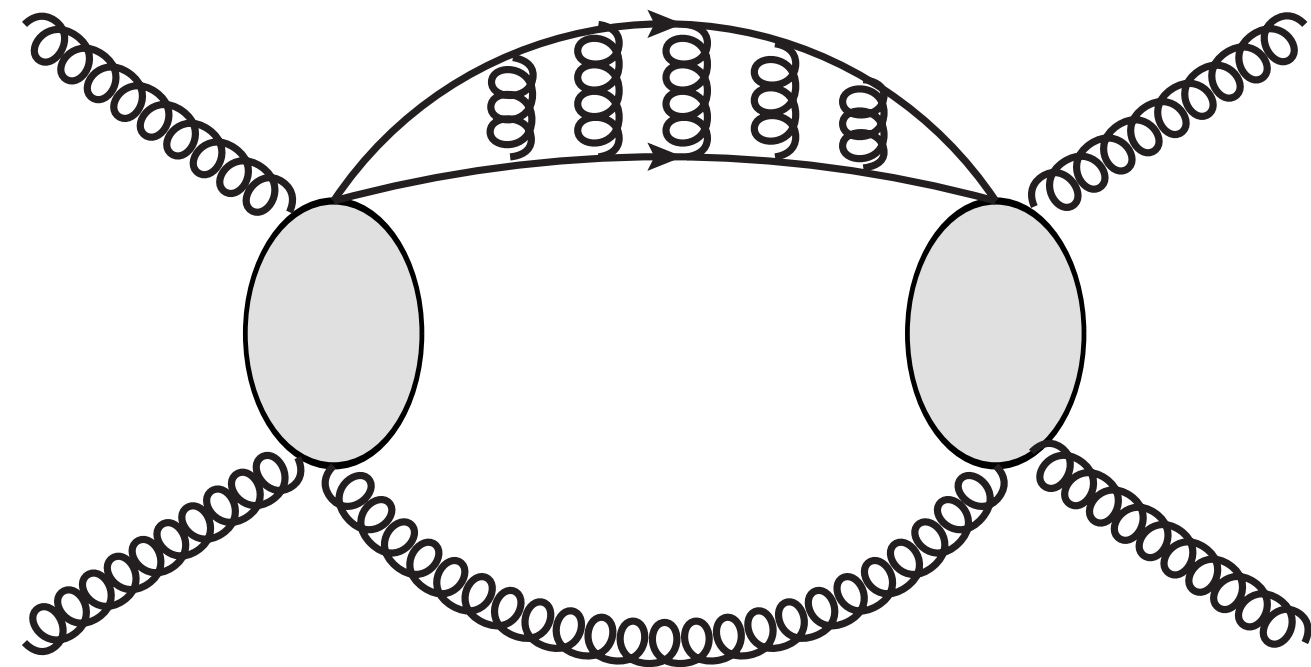


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η_c, η_b	Lifetime $\sim 10^{-22}$ s
$J/\psi, \Upsilon$	Lifetime $\sim 10^{-20}$ s
Decay mainly through strong interactions (annihilation)	
$t\bar{t}$	Lifetime $\sim 10^{-25}$ s
	Decay mainly through weak interactions (individually)

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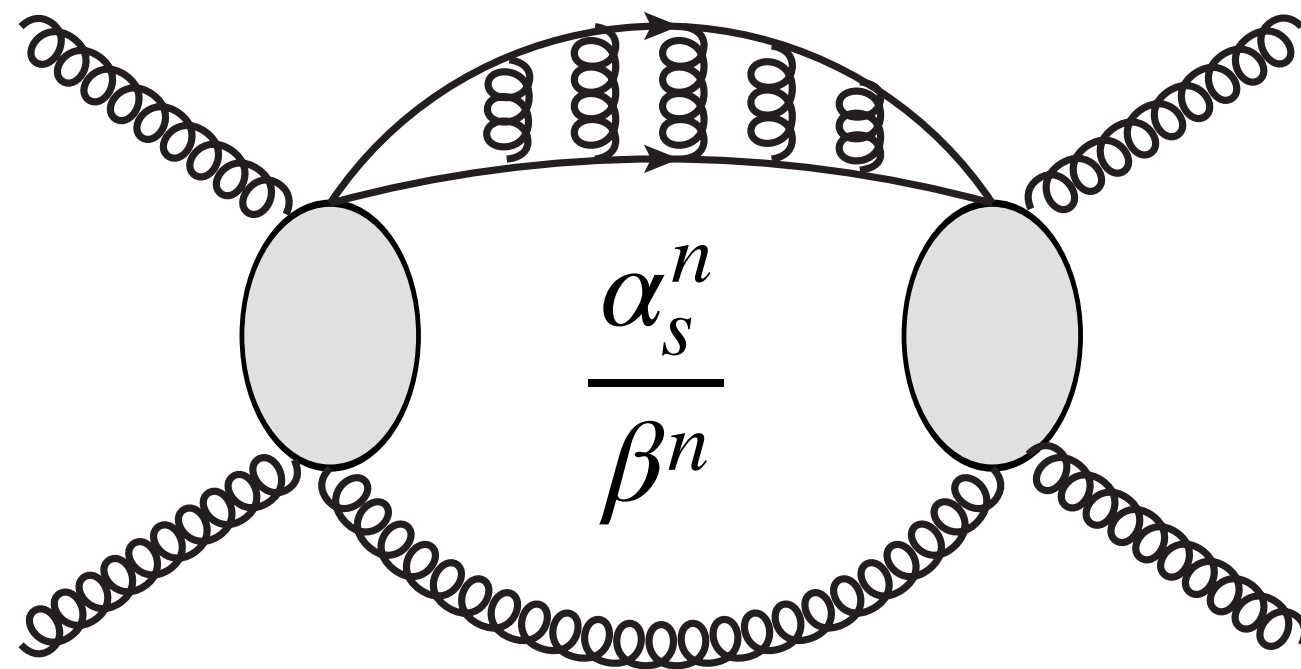
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------------	----------------------------

Decay mainly through weak interactions (individually)

“quasi-bound state”

Resummation of non-relativistic enhancement



See also:

Fadin et al.: 1987~1990

Kiyo et al.: 0812.0919

Sumino and Yokoya: 1007.0075

Beneke et al.: 1109.1536

Some further works:

Fuks et al.: 2102.11281, 2411.18962,
2509.03596

Garzelli et al.: 2412.16685

Wang et al.: 2411.17955

Fu et al.: 2412.11254, 2504.12634

Factorization in the $\beta \sim \alpha_s \ll 1$ limit

$$\frac{d\sigma}{dM_{t\bar{t}}d\Theta} \sim \int H \times J \times f \times f$$

PDFs

Hard functions

Potential functions in NRQCD

NNLO+NLP resummation

Ju, Wang, Wang, Xu, Xu, LLY:
1908.02179, 2004.03088

- Calculated NLO hard functions relevant for double differential distributions and dynamic scales
- Incorporated potential functions at NLP
- Combined with NNLO fixed-order results by matching
- First to point out this effect in Run 2 data

Resummation of non-relativistic enhancement

Ju, Wang, Wang, Xu, Xu, LLY:
1908.02179, 2004.03088

$$\frac{d\sigma^{\text{NNLO+NLP}}}{dM_{t\bar{t}}d\Theta} = \frac{d\sigma^{\text{NLP}}}{dM_{t\bar{t}}d\Theta} + \frac{d\sigma^{\text{NNLO}}}{dM_{t\bar{t}}d\Theta} - \frac{d\sigma^{\text{nnLO}}}{dM_{t\bar{t}}d\Theta}$$

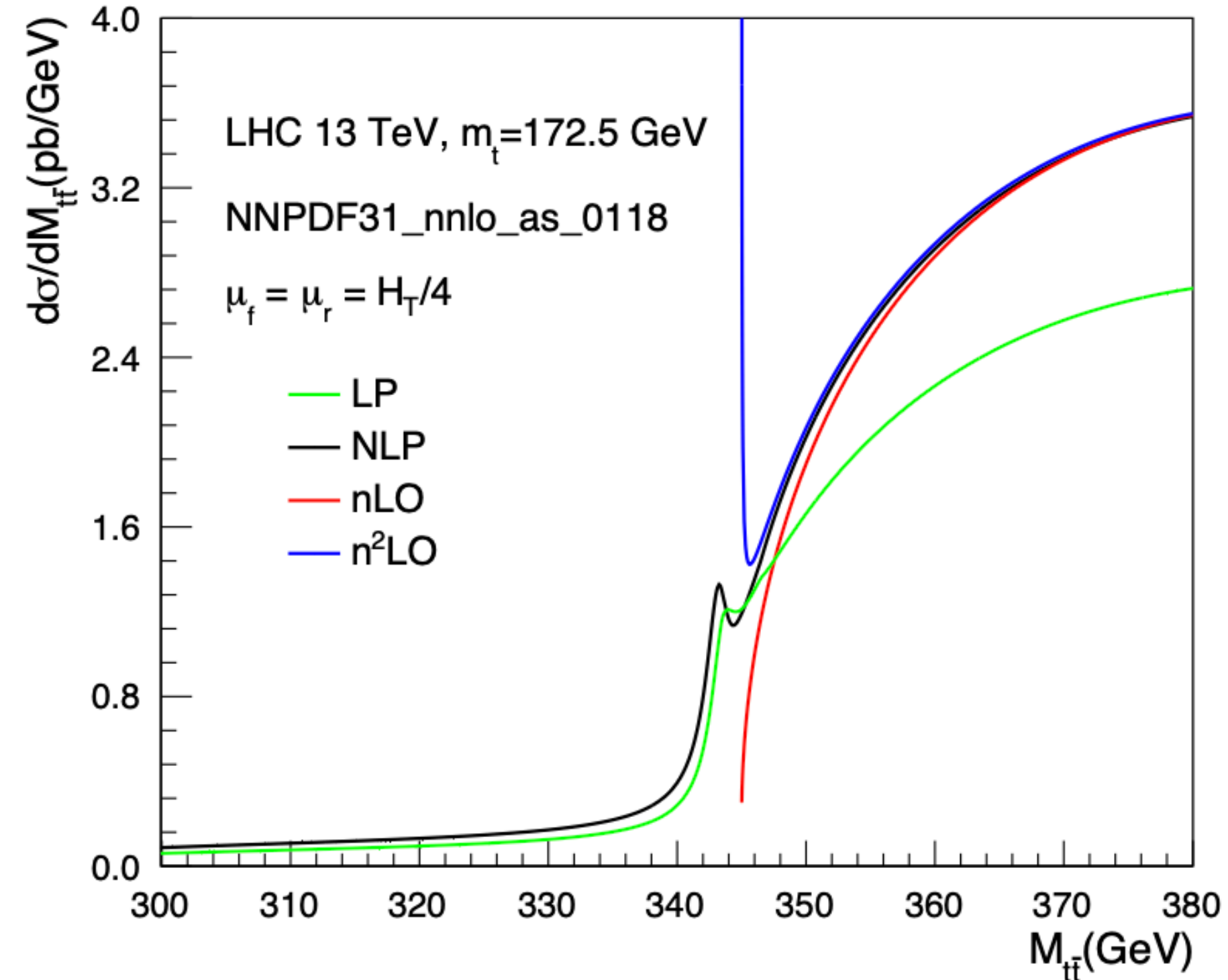
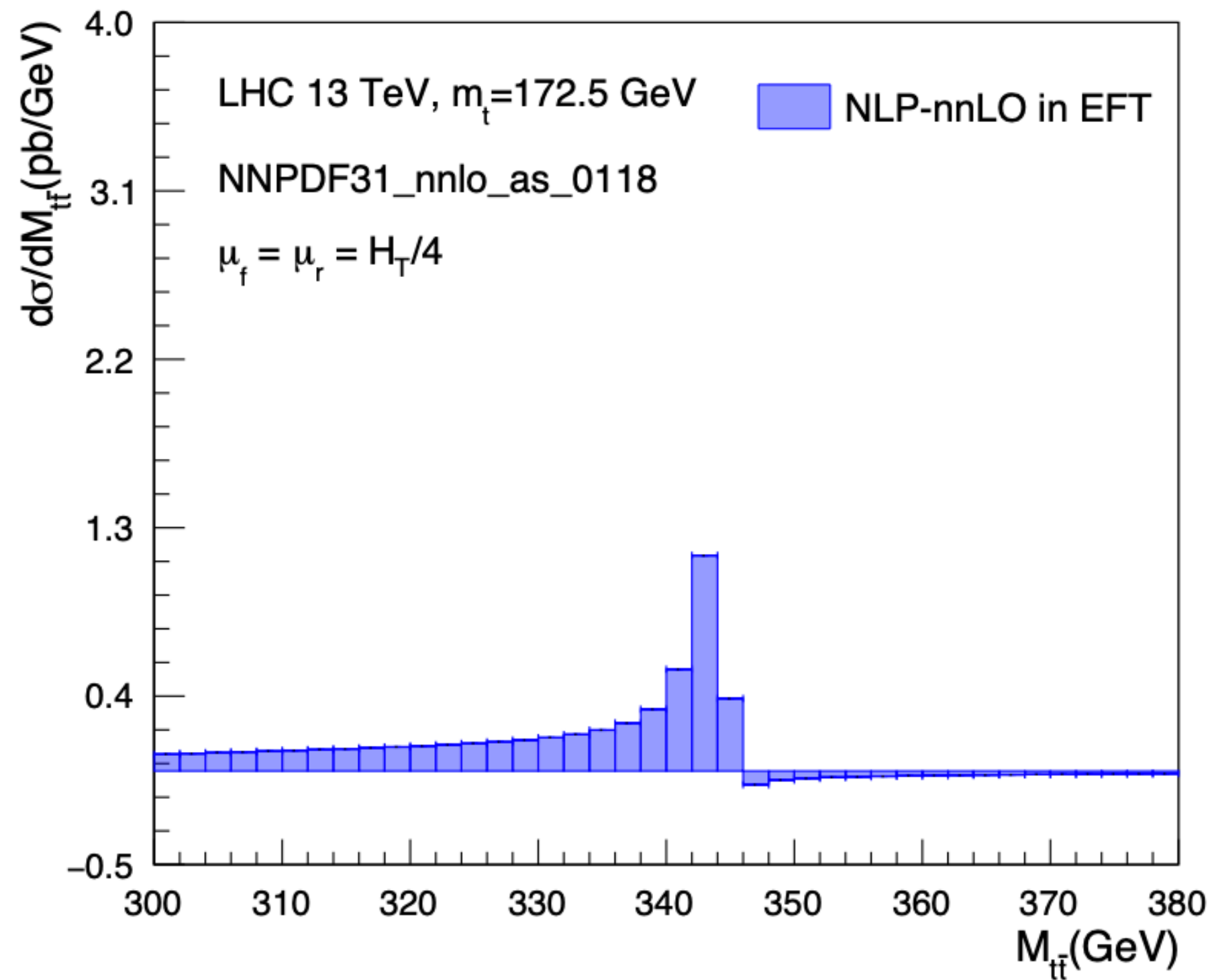
NLP re-expanded to NNLO

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NLP re-expanded to NNLO

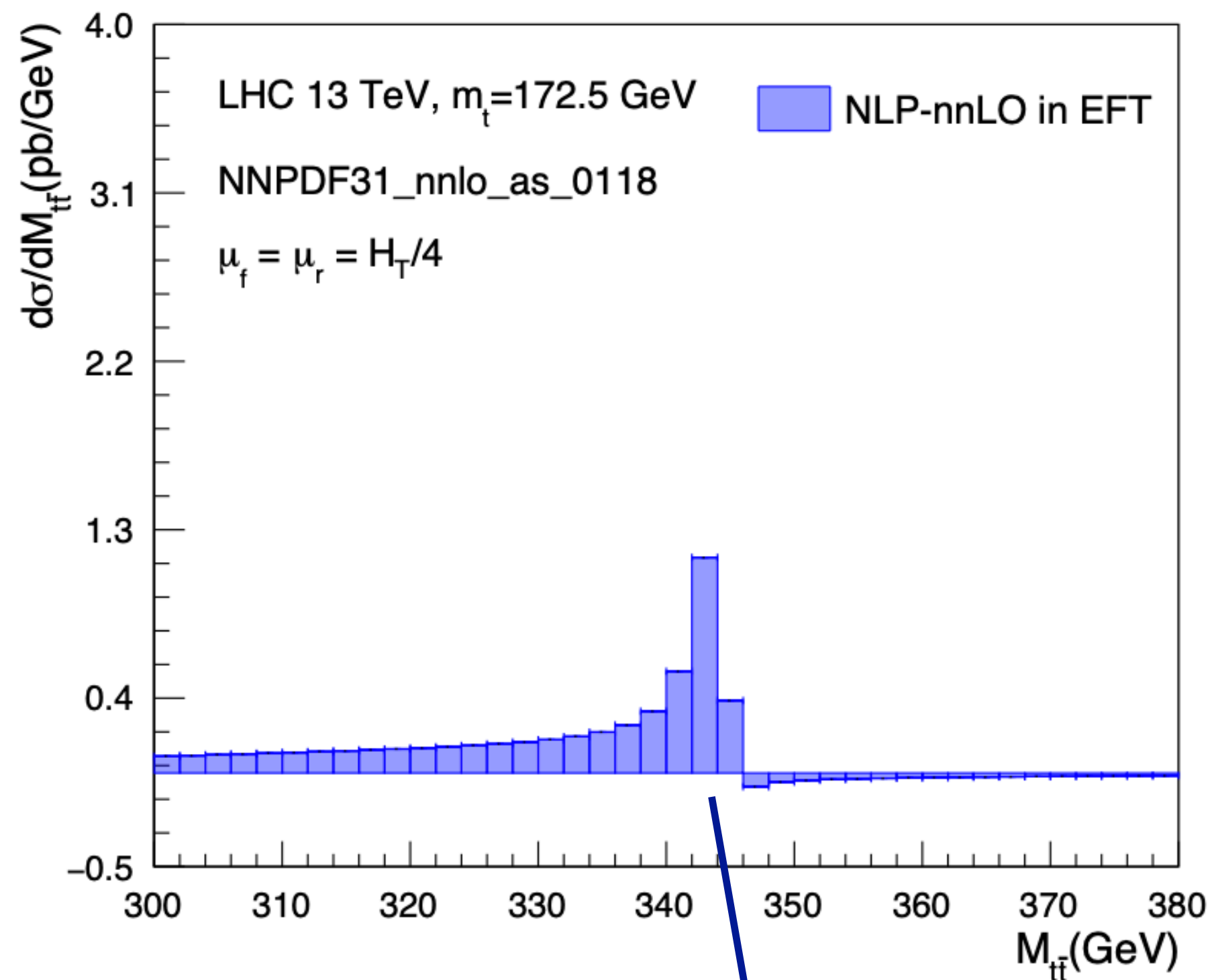


Resummation of non-relativistic enhancement

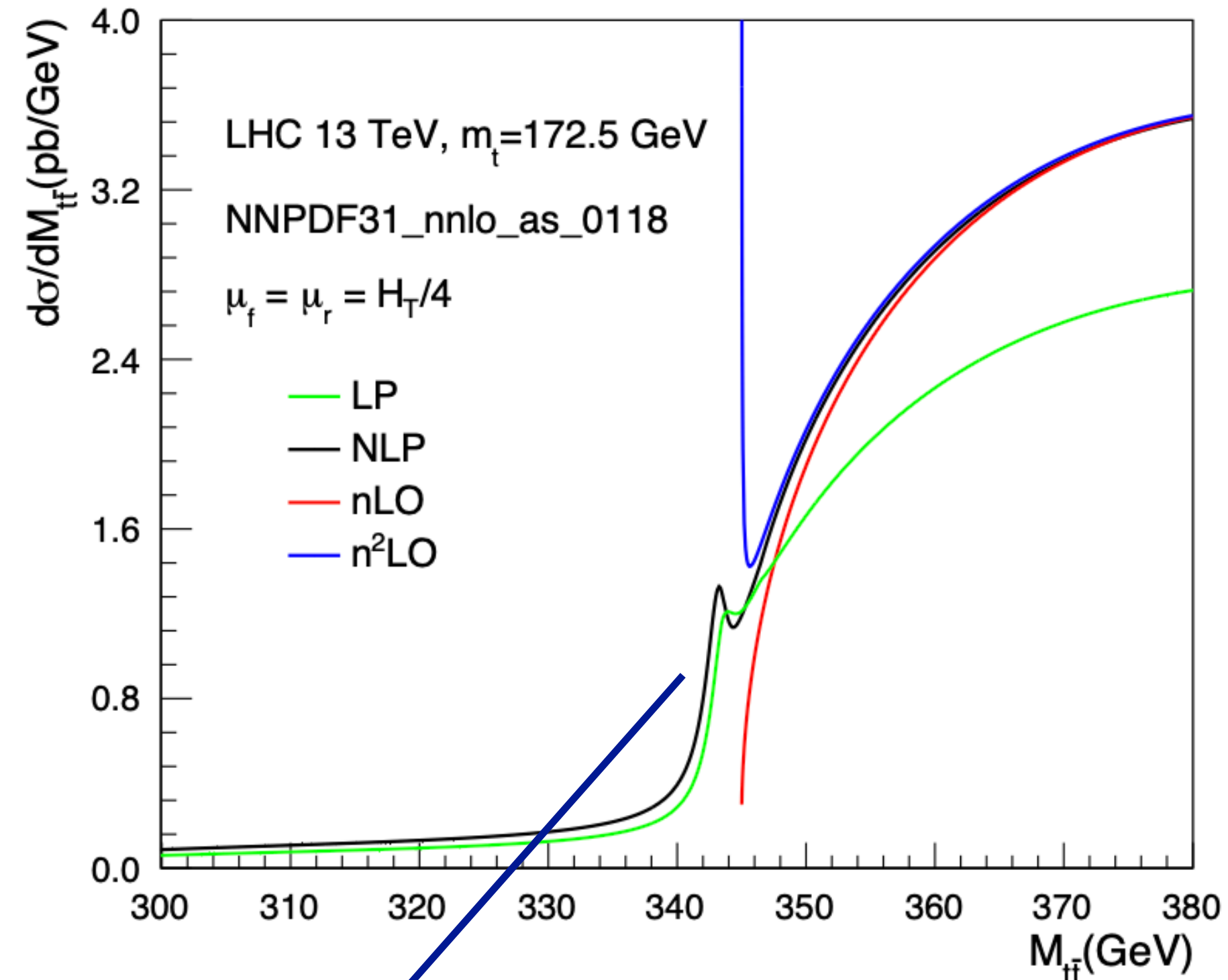
Ju, Wang, Wang, Xu, Xu, LLY:
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NLP re-expanded to NNLO



Enhancement near threshold

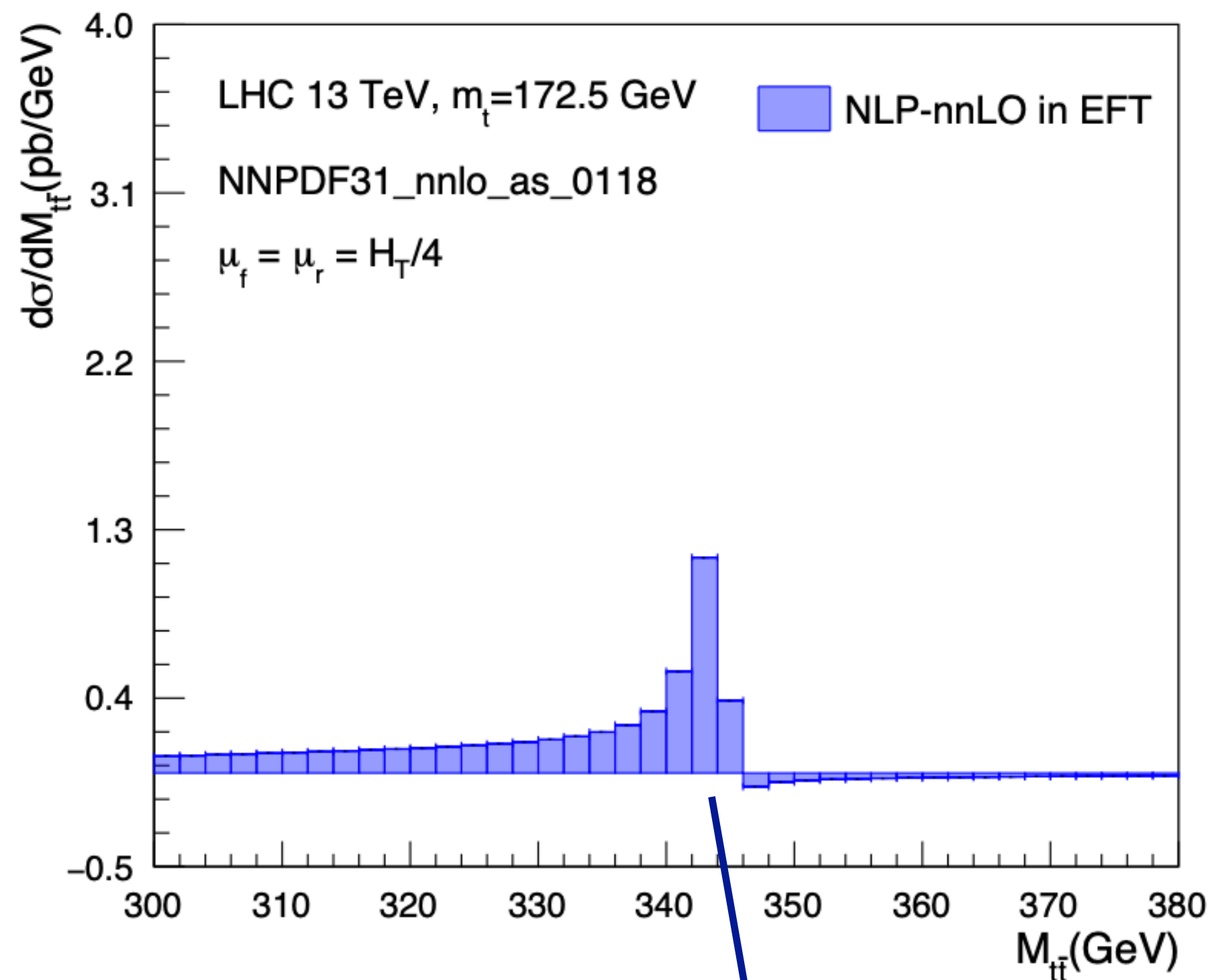


Resummation of non-relativistic enhancement

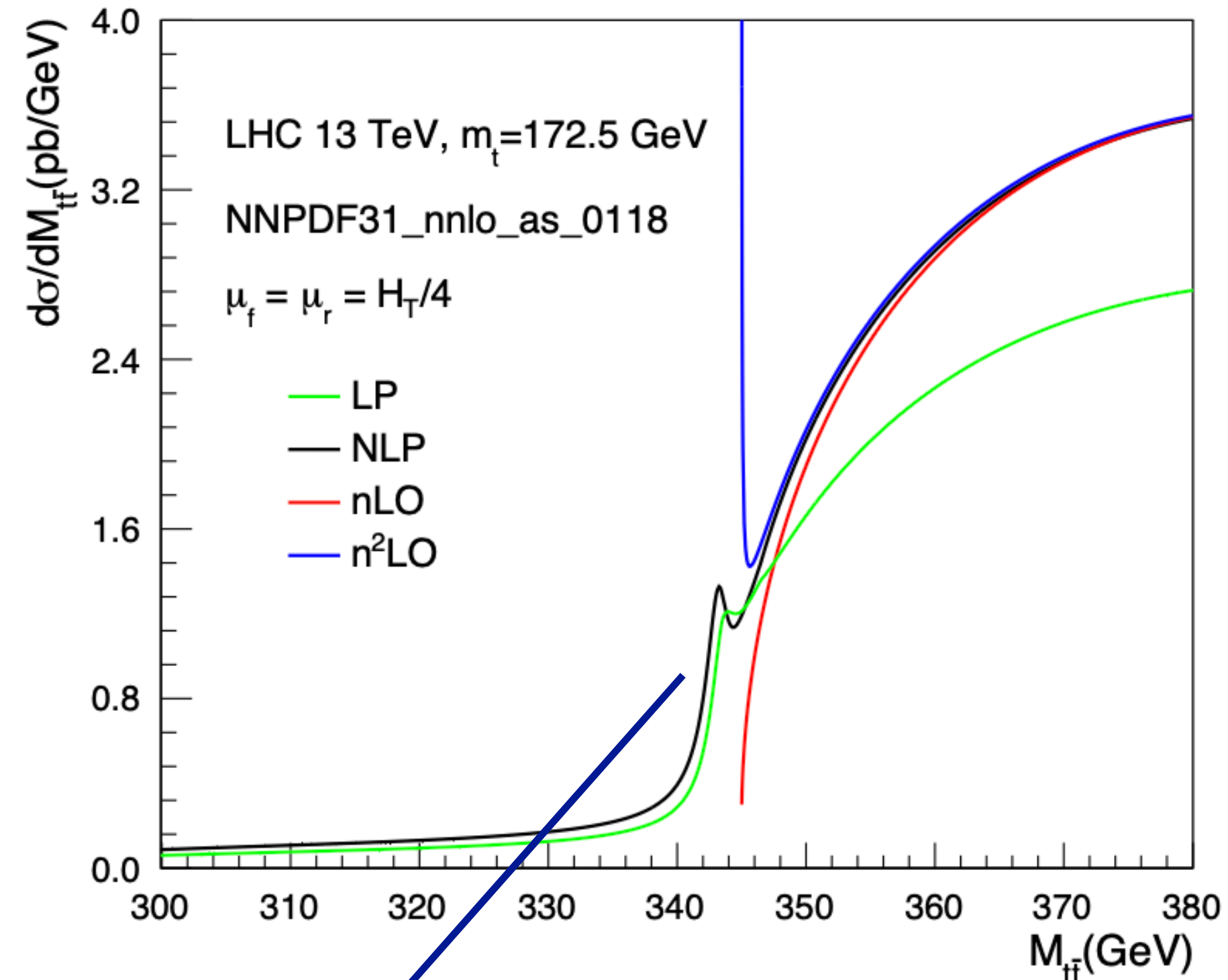
Ju, Wang, Wang, Xu, Xu, LLY:
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NLP re-expanded to NNLO



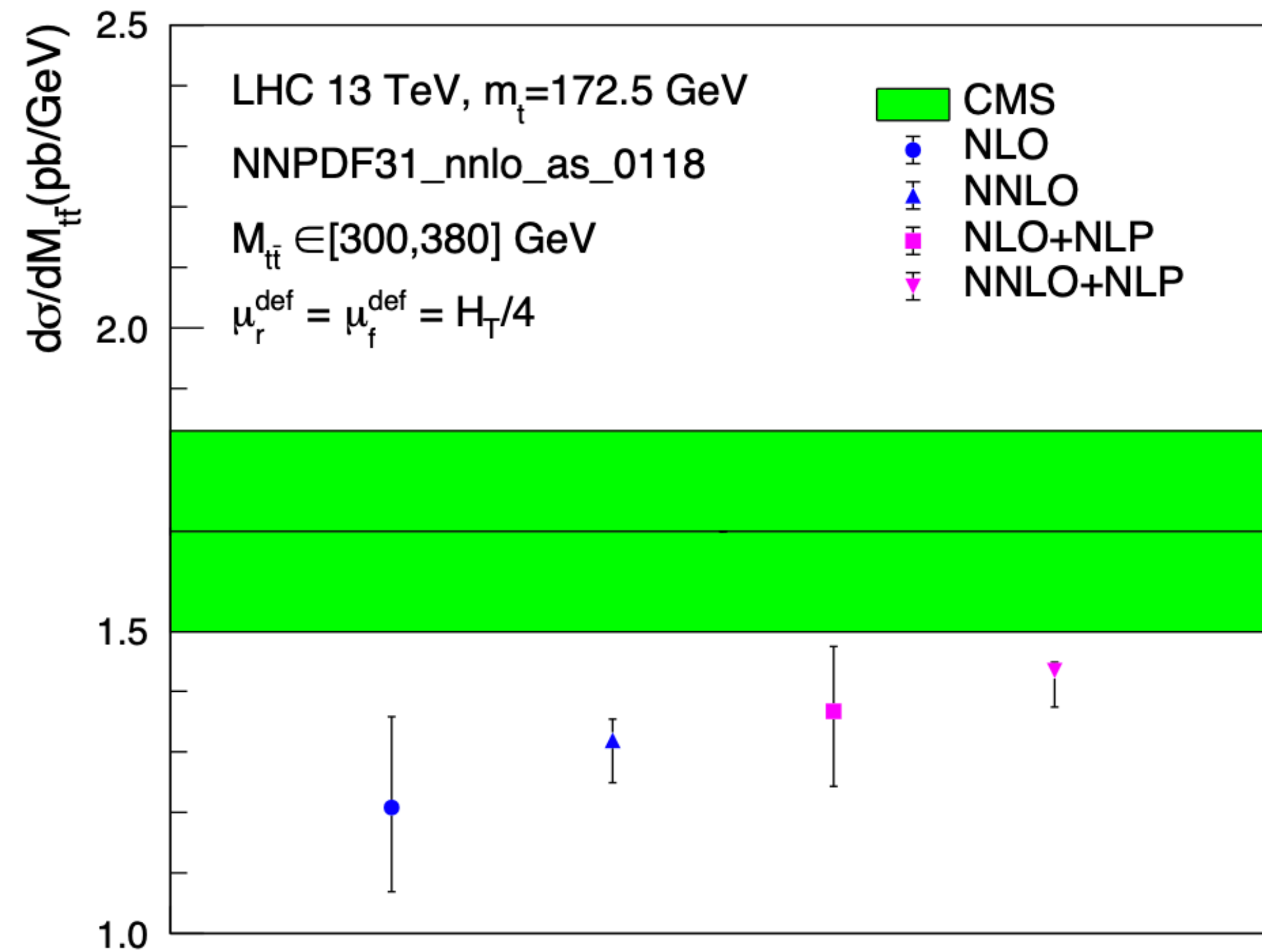
Enhancement near threshold



Note: include all color and spin configurations

Resummation of non-relativistic enhancement

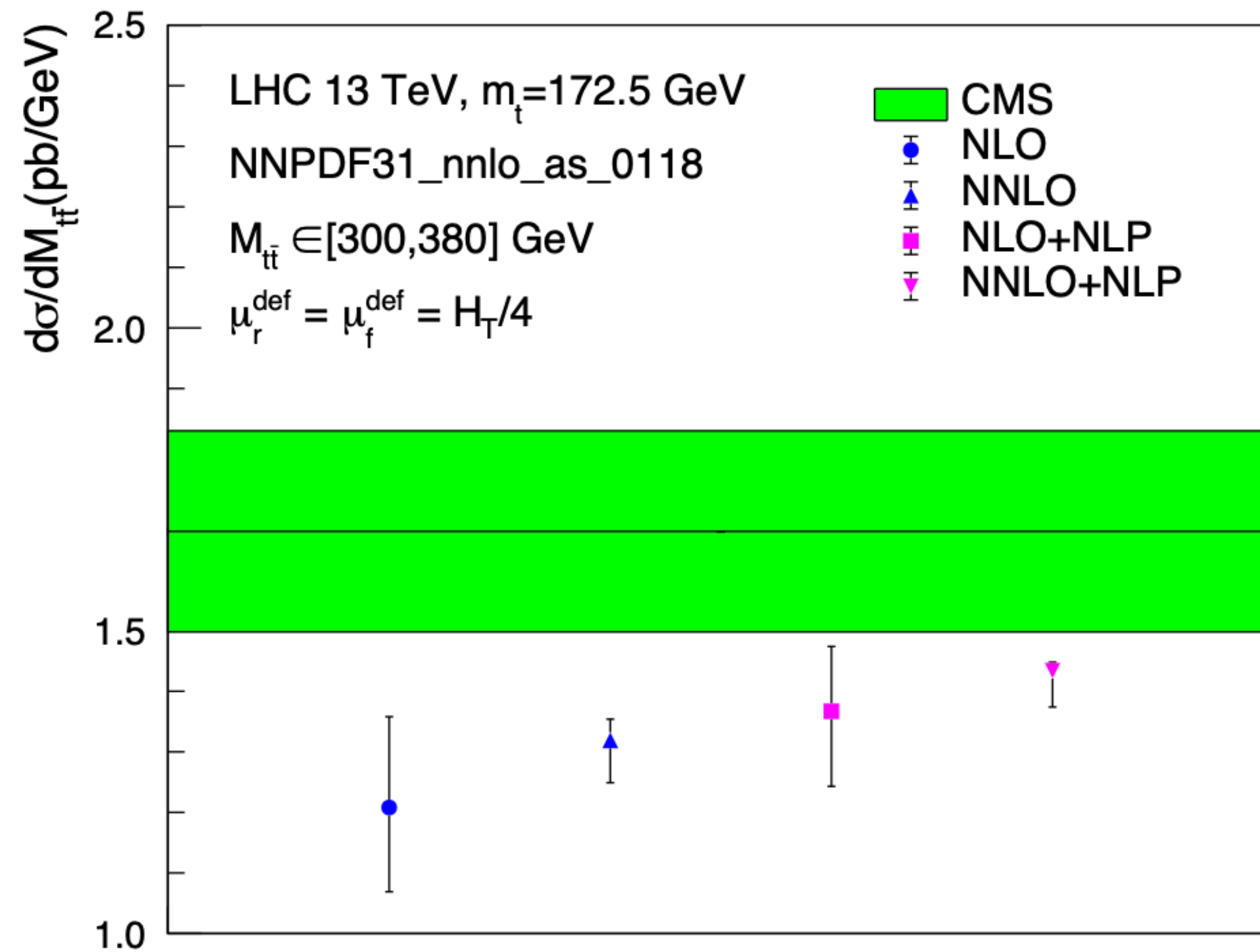
Ju, Wang, Wang, Xu, Xu, LLY:
1908.02179, 2004.03088



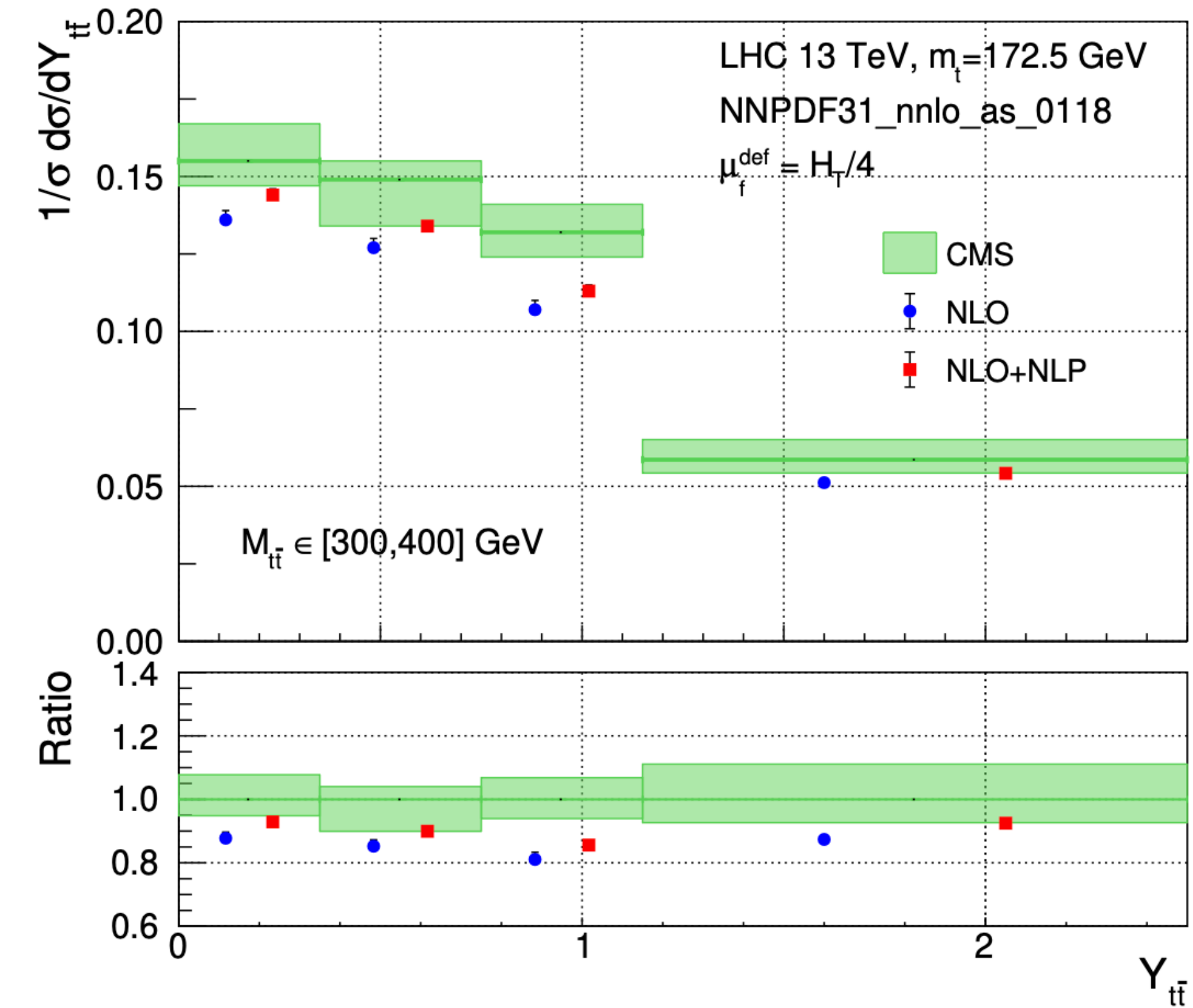
Enhancement ~ 10 pb

Resummation of non-relativistic enhancement

Ju, Wang, Wang, Xu, Xu, LLY:
1908.02179, 2004.03088



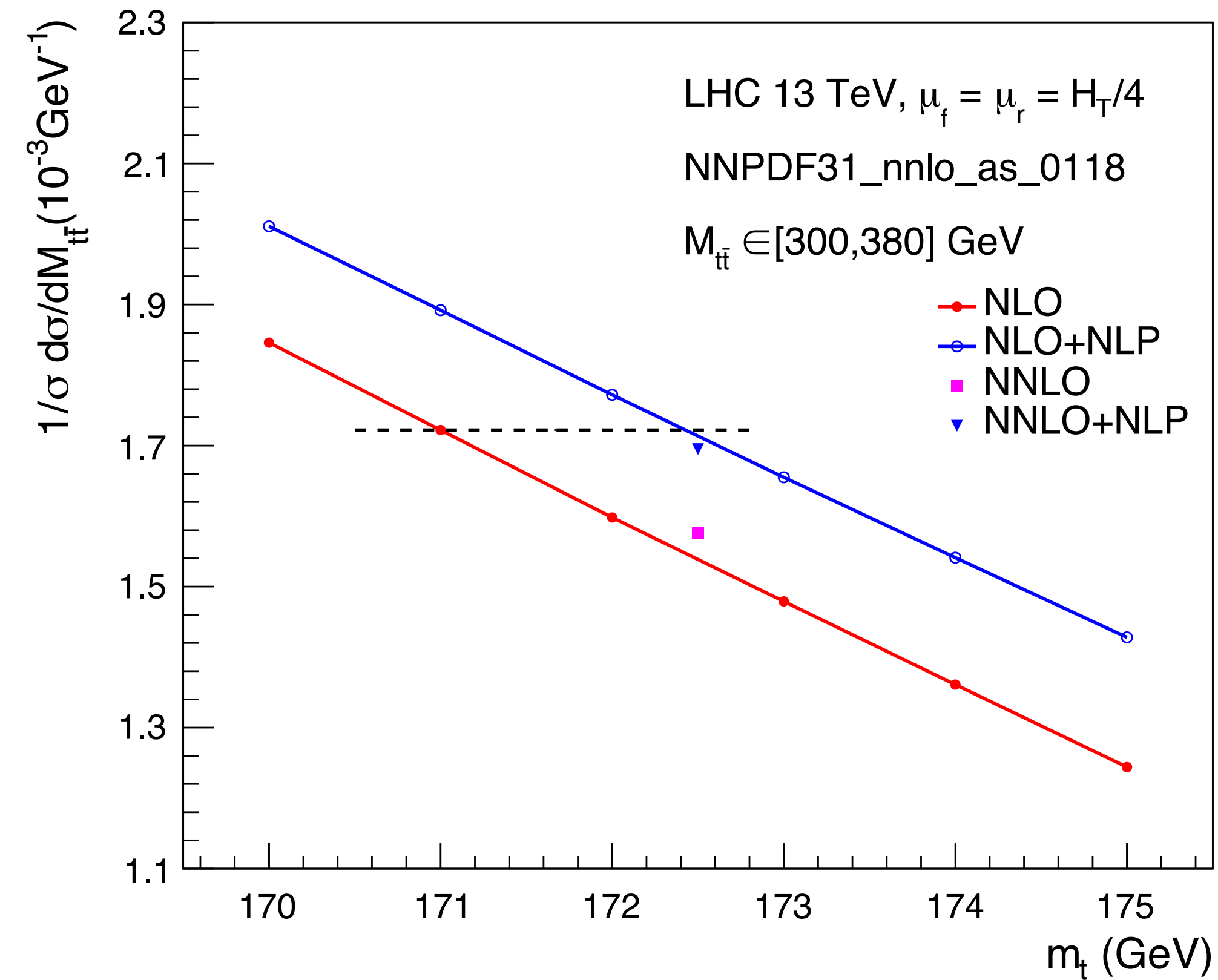
Enhancement ~ 10 pb



Similar effects in double distributions

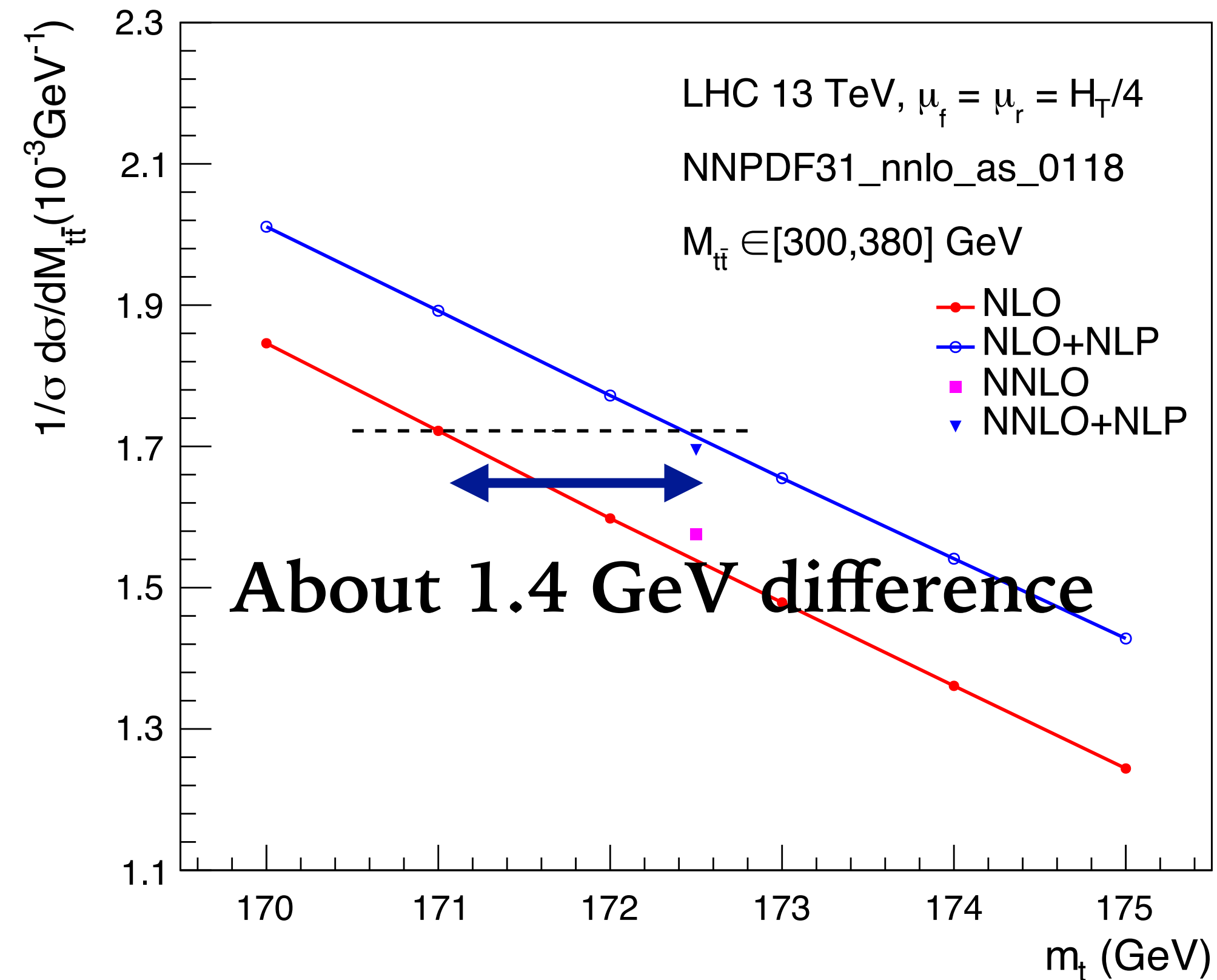
Resummation of non-relativistic enhancement

Ju, Wang, Wang, Xu, Xu, LLY:
1908.02179, 2004.03088



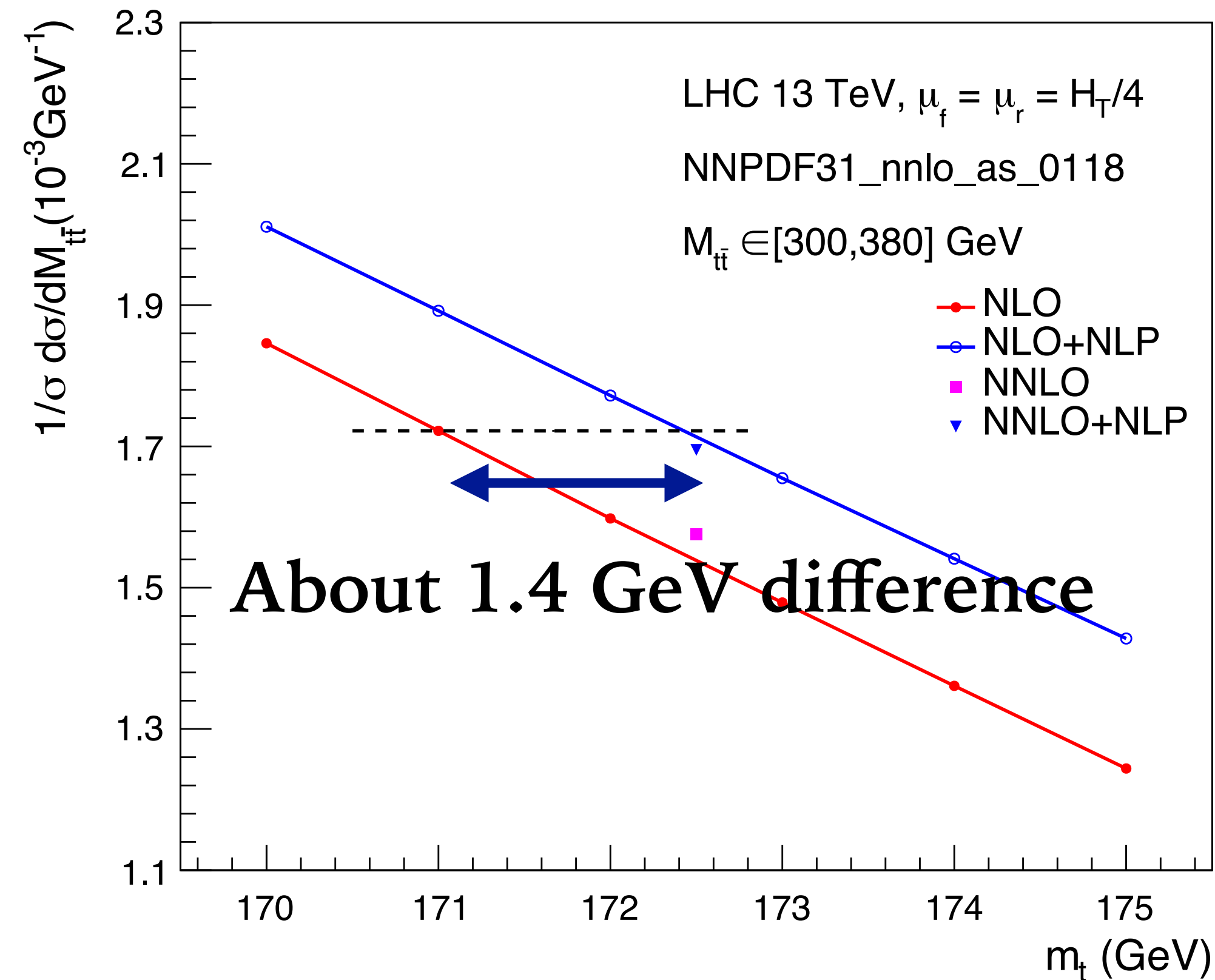
Resummation of non-relativistic enhancement

Ju, Wang, Wang, Xu, Xu, LLY:
1908.02179, 2004.03088



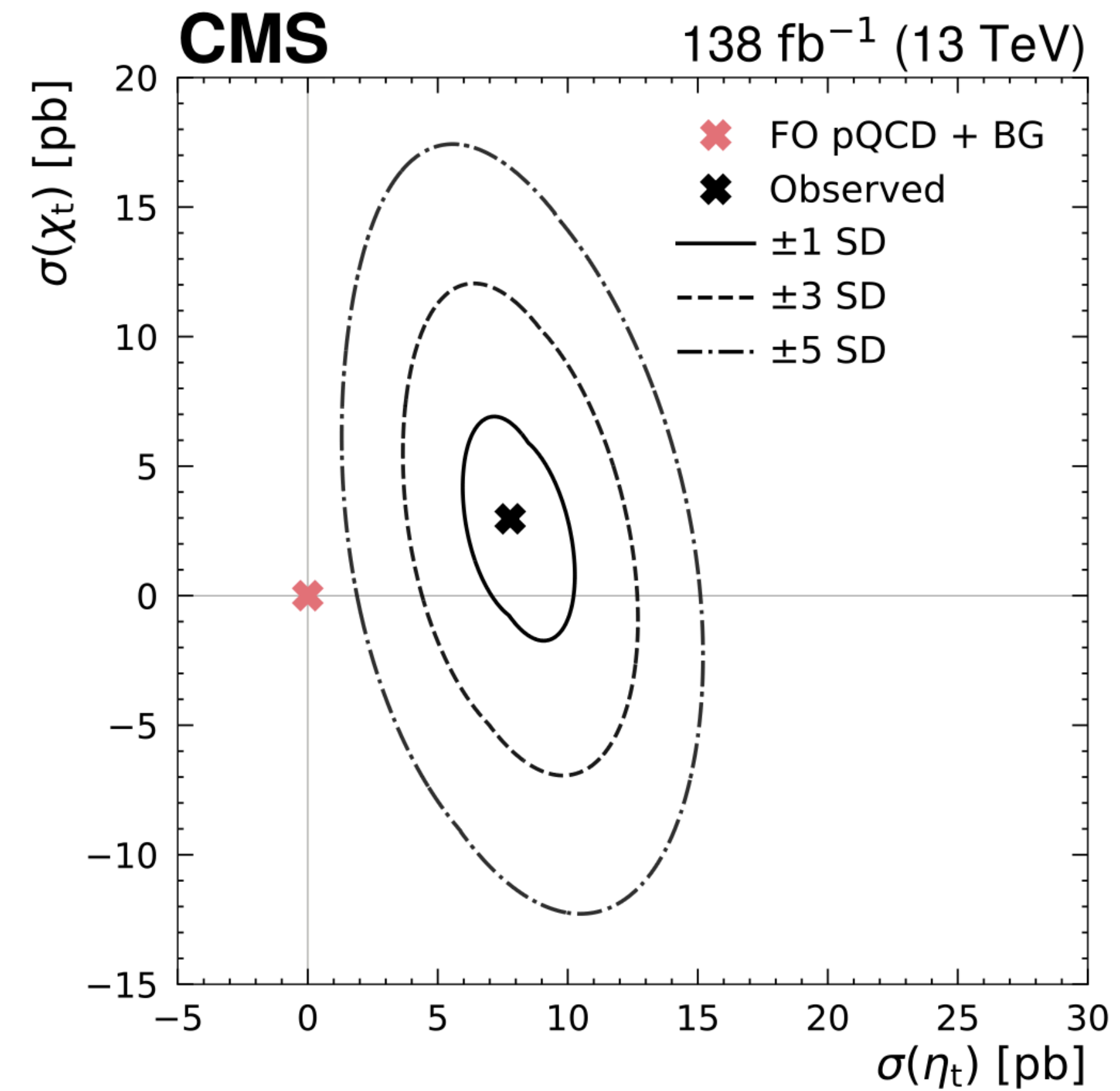
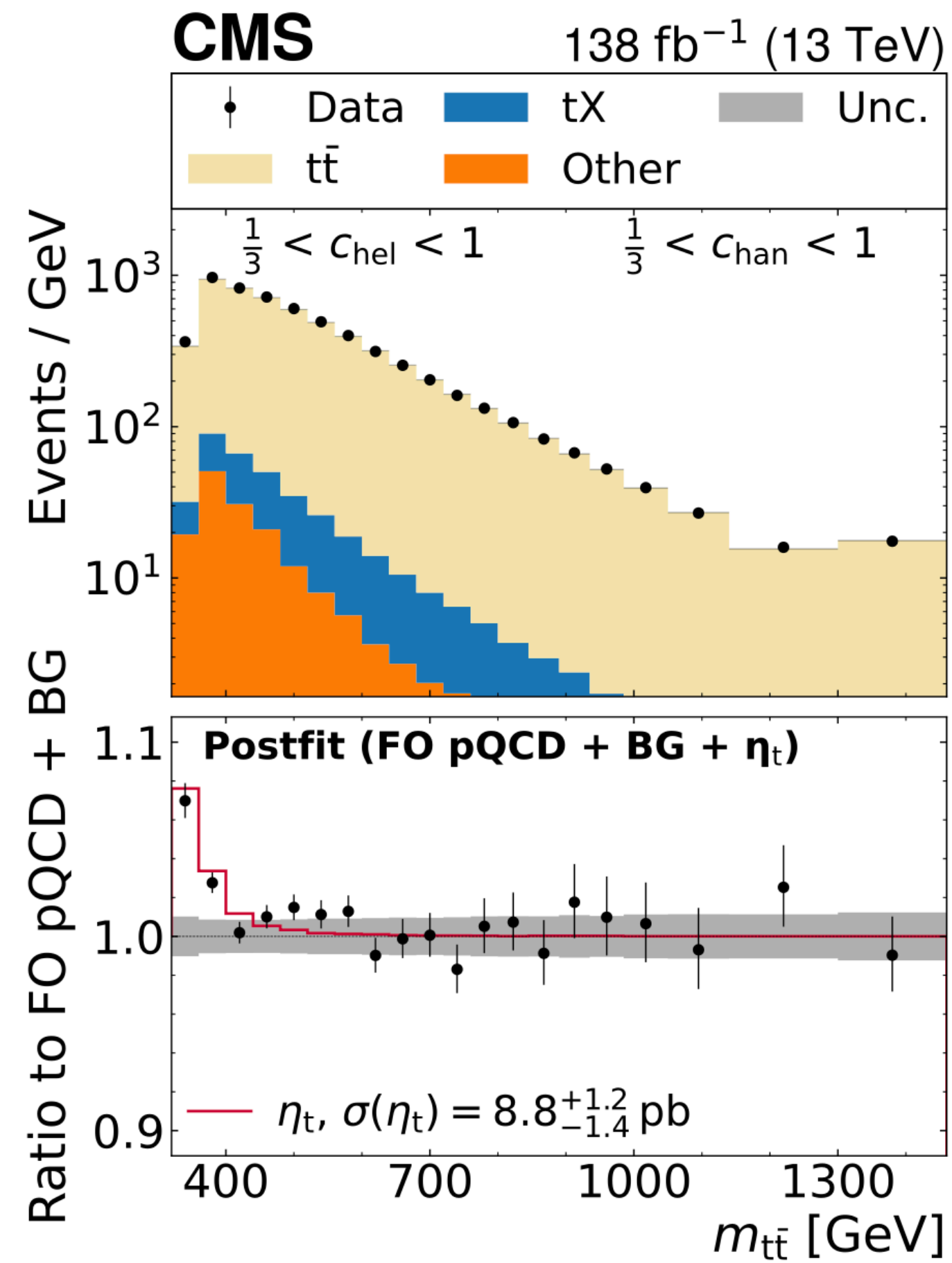
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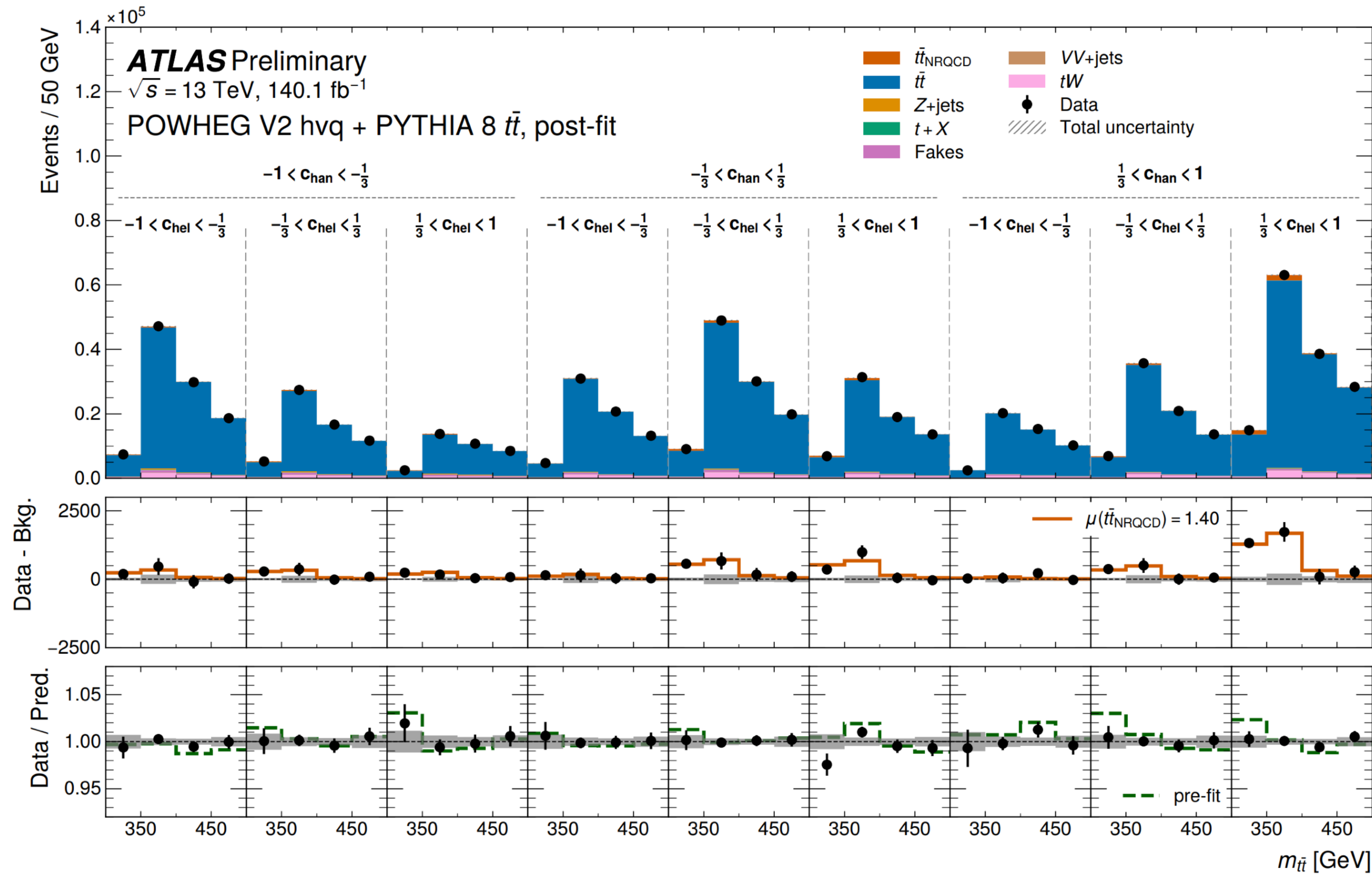
Account for most of the tension between
direct and indirect measurements!

Recent CMS measurement



CMS: 2503.22382

Recent ATLAS measurement



See talk by Hai-Feng Li

ATLAS-CONF-2025-008

Further discussions

Recent works often focus on color-singlet pseudo-scalar configuration “ η_t ”

Signatures of toponium formation in LHC run 2 data

2102.11281

Benjamin Fuks,^{1,2,*} Kaoru Hagiwara,^{3,†} Kai Ma,^{4,5,6,‡} and Ya-Juan Zheng^{7,§}

¹*Sorbonne Université, CNRS, Laboratoire de Physique Théorique et Hautes Énergies, LPTHE, F-75005 Paris, France*

²*Institut Universitaire de France, 103 boulevard Saint-Michel, 75005 Paris, France*

³*KEK Theory Center, Tsukuba 305-0801, Japan*

⁴*School of Fundamental Physics and Mathematical Science, Hangzhou Institute for Advanced Study, UCAS, Hangzhou 310024, Zhejiang, China*

⁵*International Centre for Theoretical Physics Asia-Pacific, Beijing/Hangzhou, China*

⁶*Department of Physics, Shaanxi University of Technology, Hanzhong 723000, Shaanxi, China*

⁷*Department of Physics and Astronomy, University of Kansas, Lawrence, KS 66045, U.S.A.*

Simulating toponium formation signals at the LHC

2411.18962

Benjamin Fuks  ^{a,1}, Kaoru Hagiwara ^{b,2}, Kai Ma  ^{c,3} and Ya-Juan Zheng  ^{d,4}

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Toponium: Implementation of a toponium model in FeynRules

2504.12634

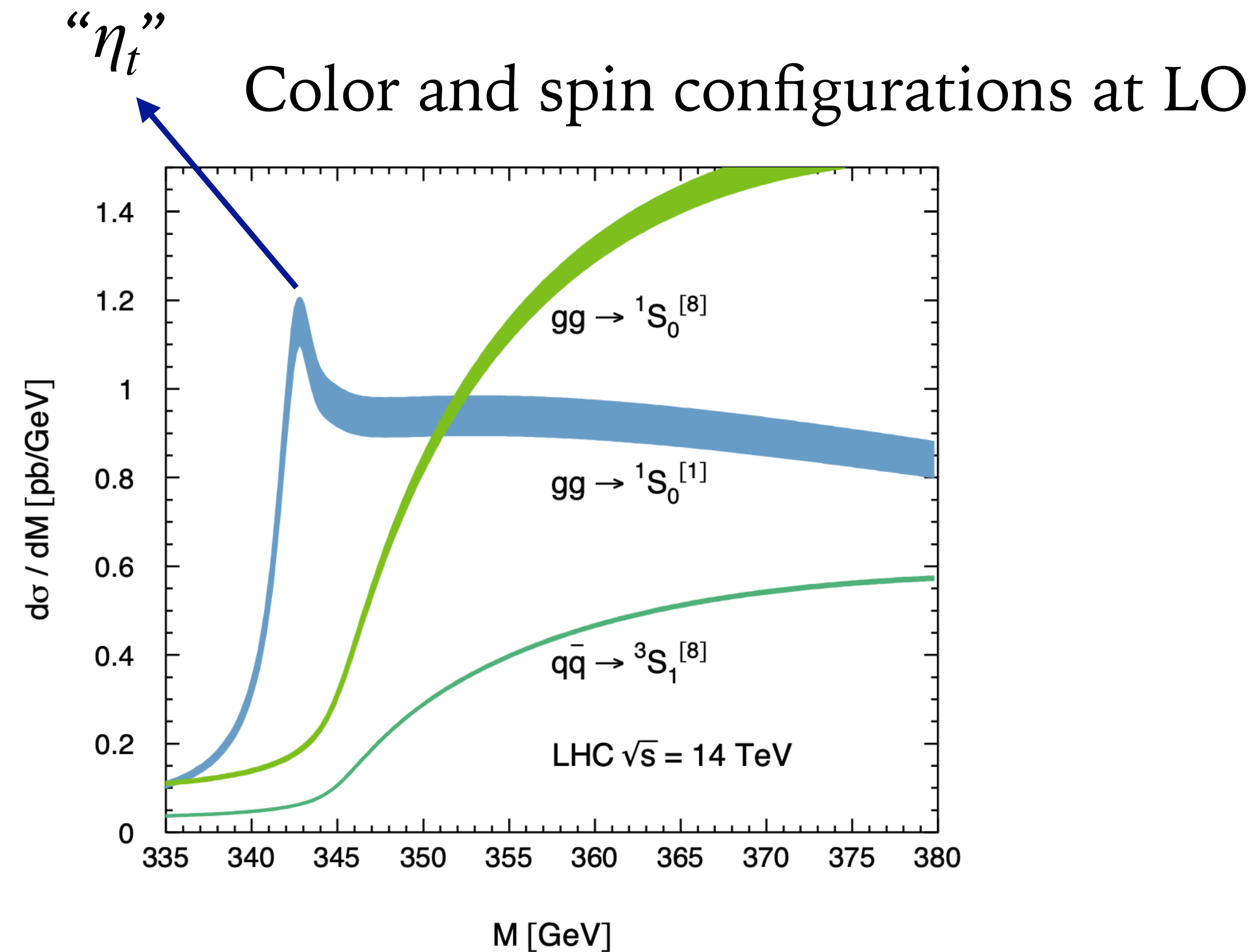
Jing-Hang Fu^a, Yu-Jie Zhang^a, Guang-Zhi Xu^b, Kui-Yong Liu^{c,*}

^a*School of Physics, Beihang University, Beijing, China.*

^b*School of Physics, Liaoning University, Shenyang, China.*

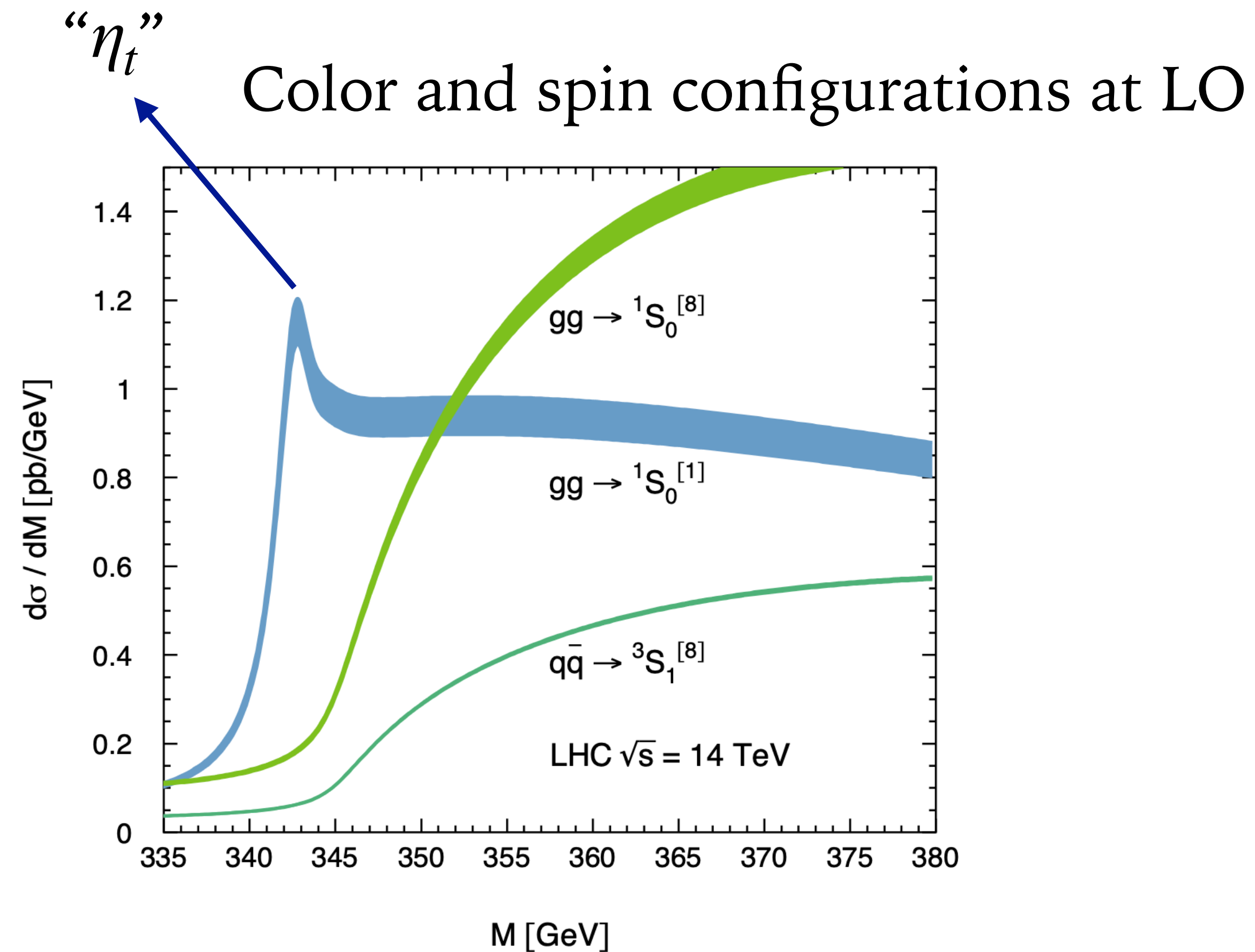
^c*School of Physics, Liaoning Normal University, Dalian, China.*

Further discussions

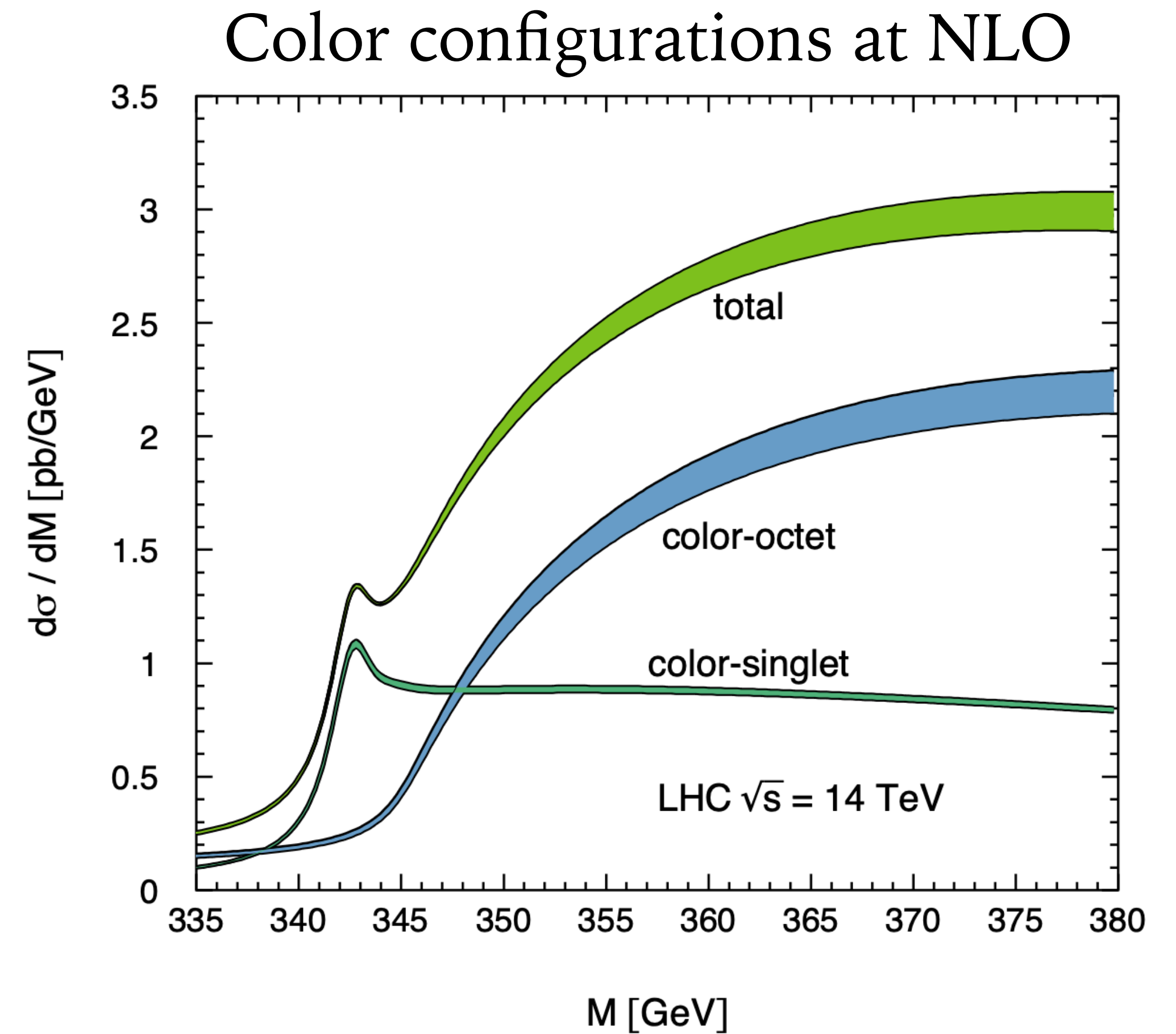


Kiyo et al.: 0812.0919

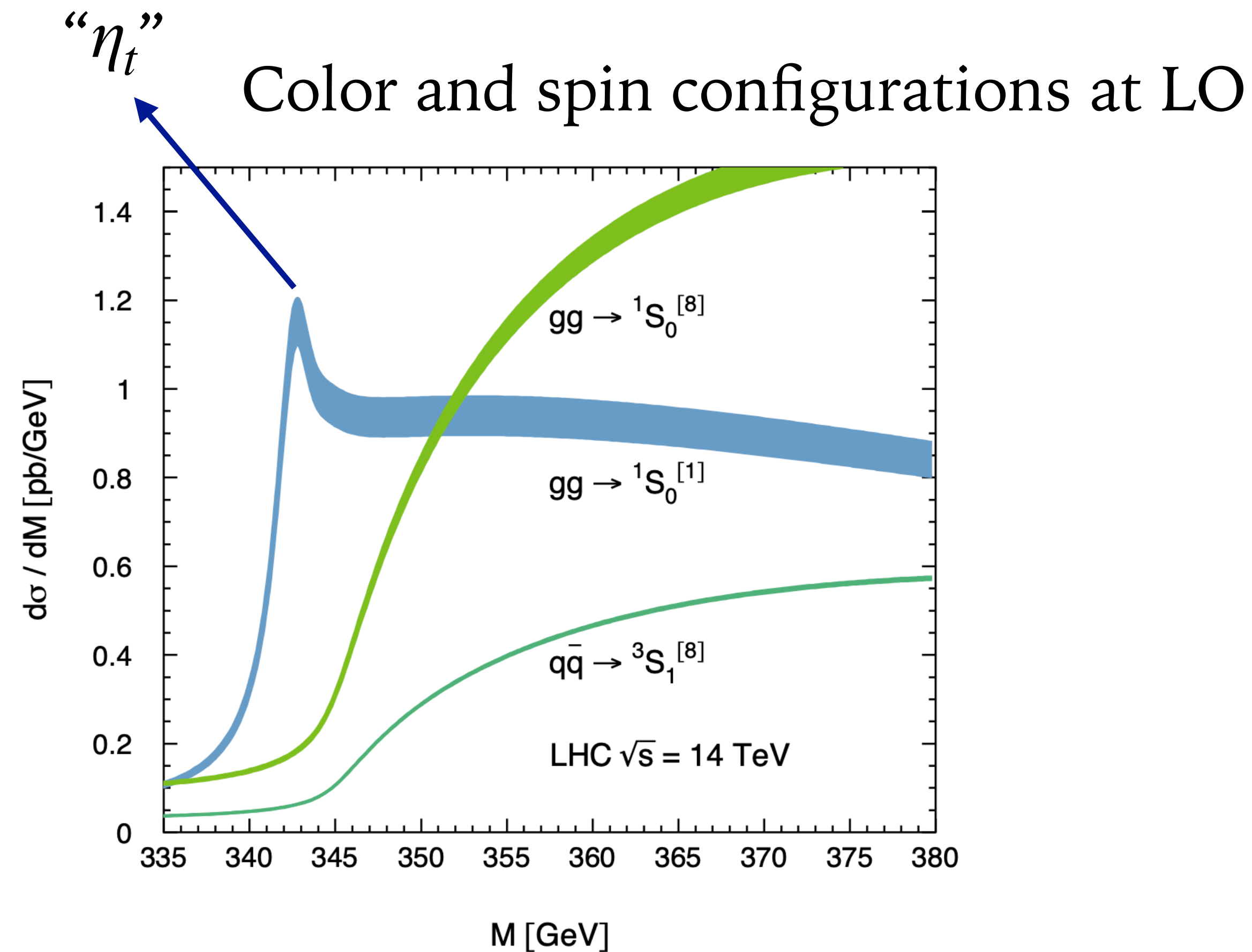
Further discussions



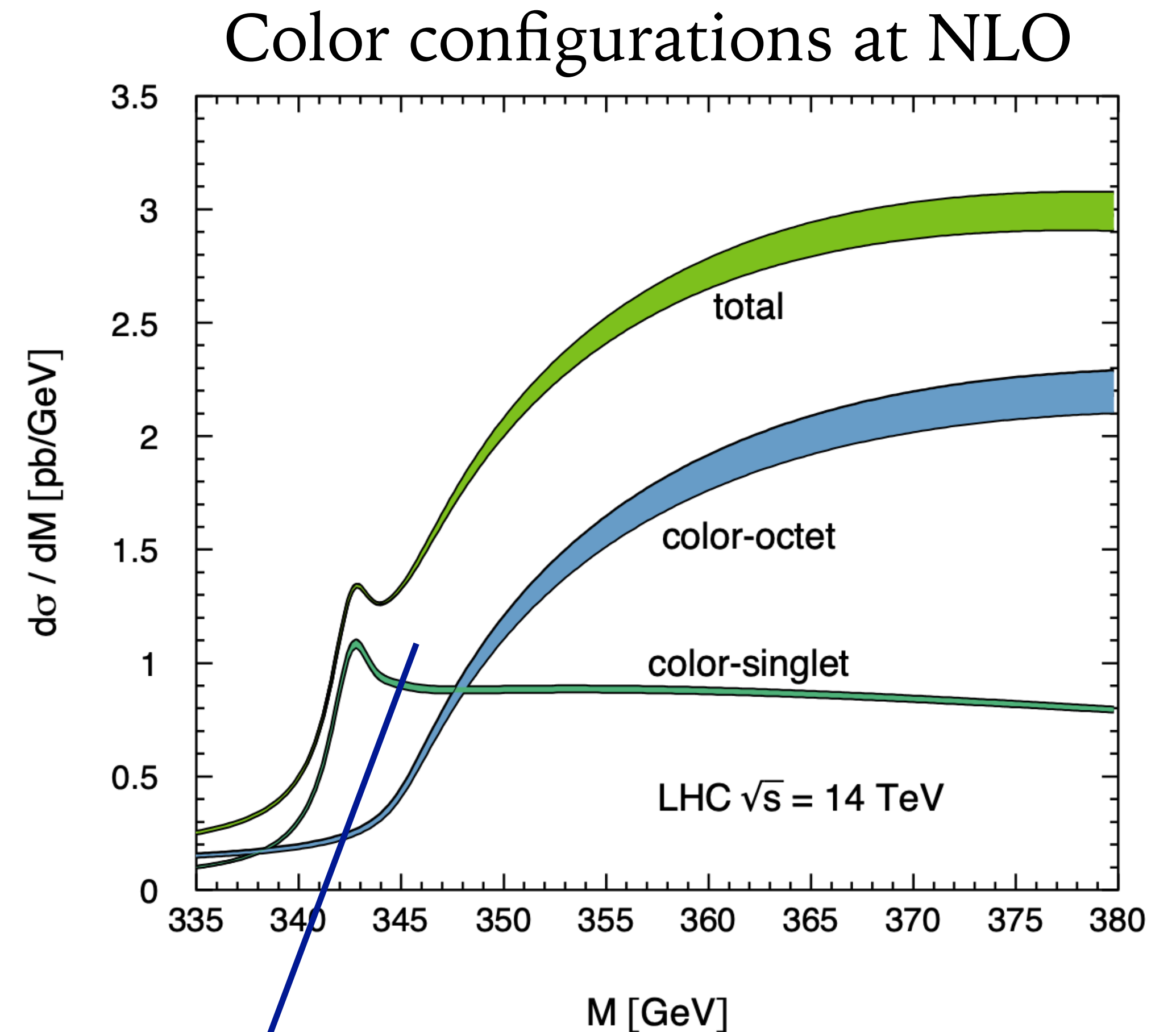
Kiyo et al.: 0812.0919



Further discussions

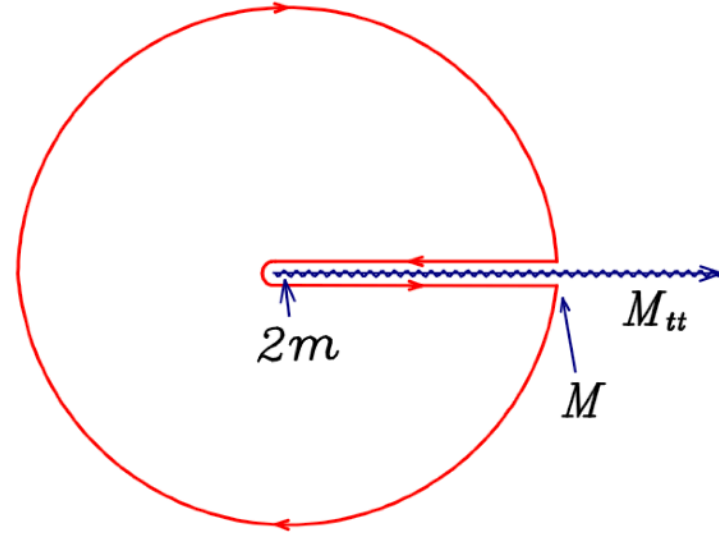


Kiyo et al.: 0812.0919



Color-octet contributions are non-negligible

Further discussions

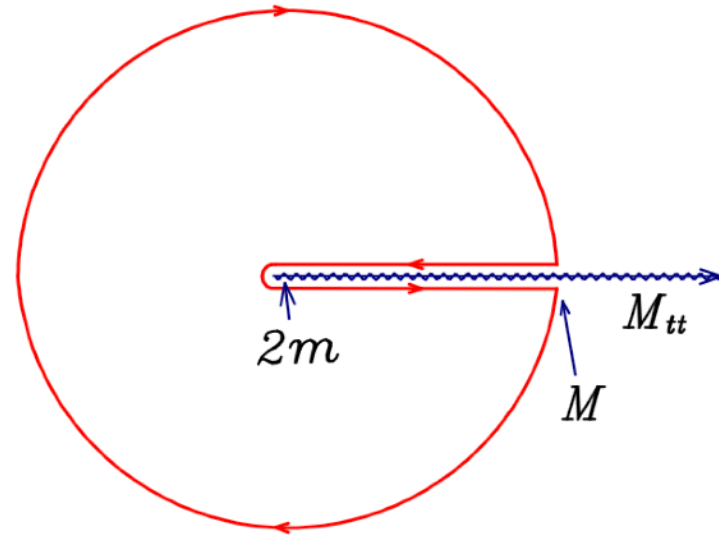


Typical value of β depends on the upper edge of the first bin

Nason et al.: 2505.00096

See also Beneke et al.: 1606.02434

Further discussions



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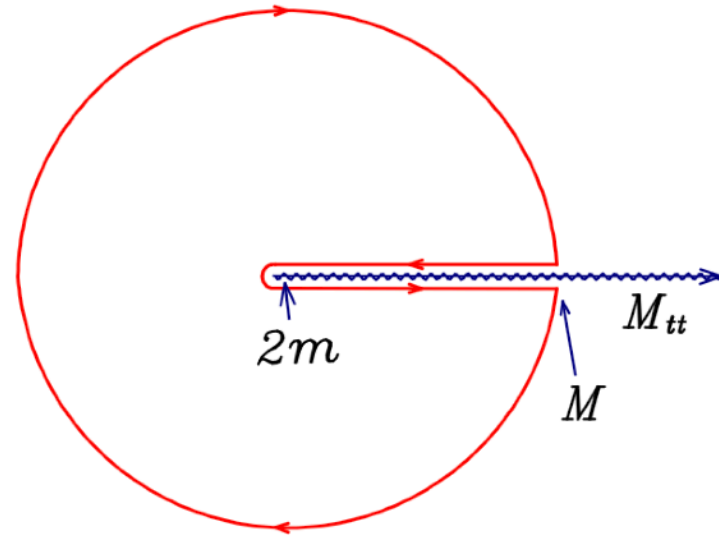
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$$\frac{d\sigma}{dM_{t\bar{t}}d\Theta} \sim \int H \times J \times f \times f \sim \beta \left(1 + \frac{\alpha_s}{\beta} + \frac{\alpha_s^2}{\beta^2} + \frac{\alpha_s^3}{\beta^3} + \dots \right)$$

Further discussions



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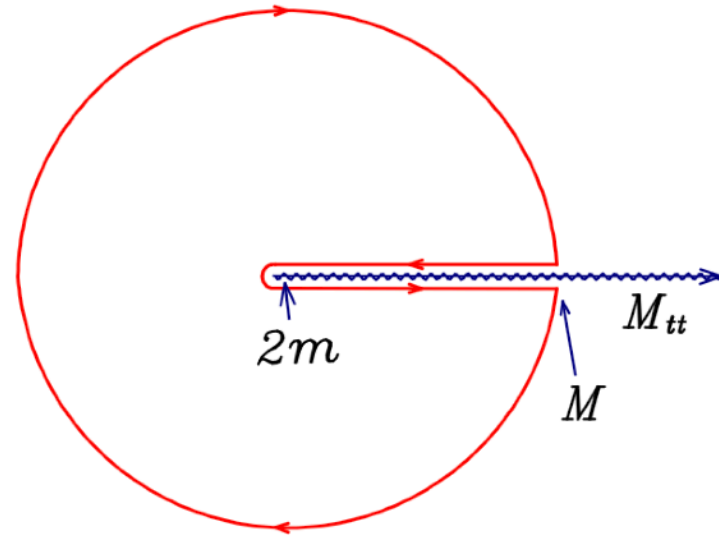
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$M_{t\bar{t}}$ (GeV)	β
400	0.50
380	0.42
360	0.28
350	0.16

Further discussions



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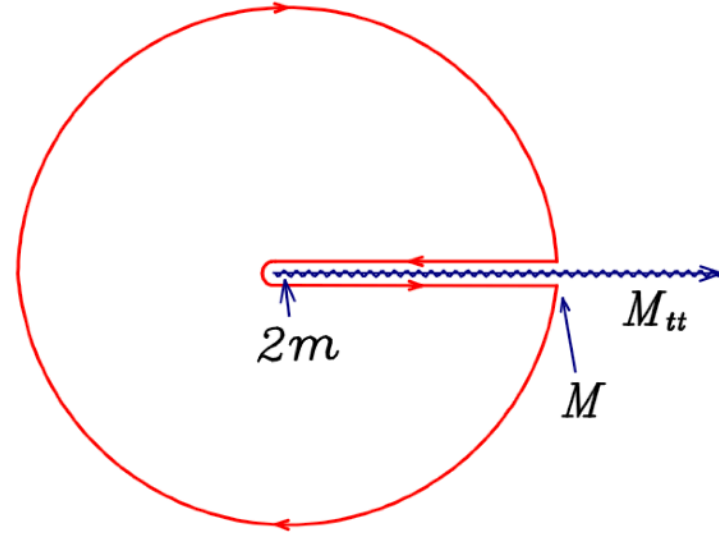
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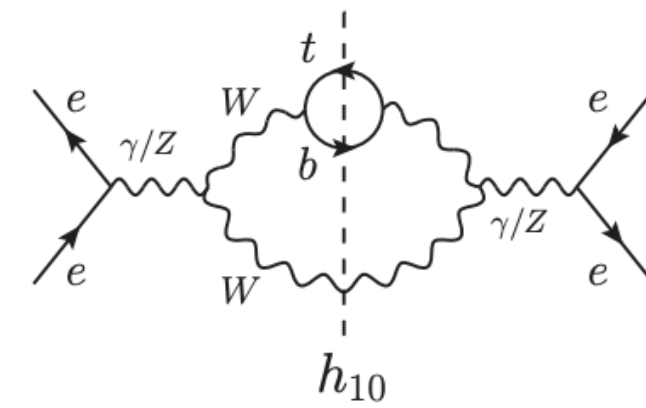
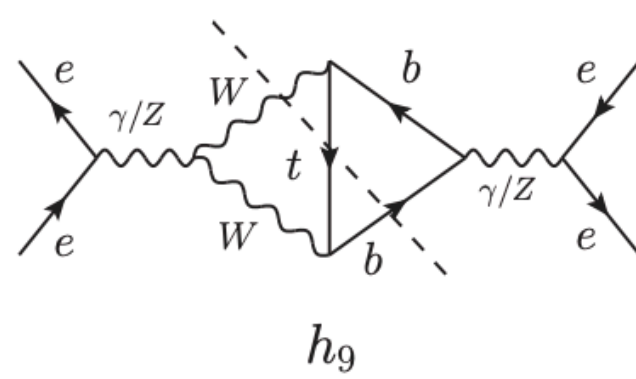
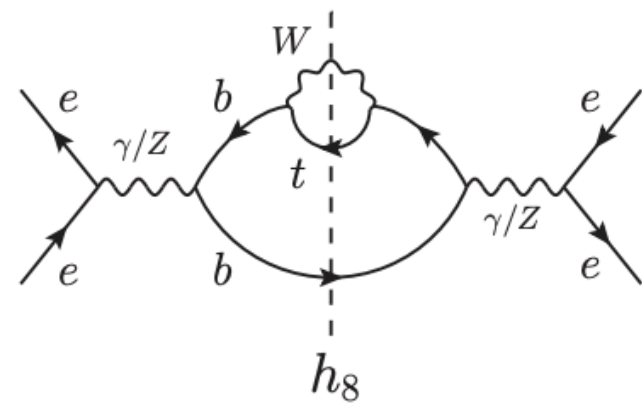
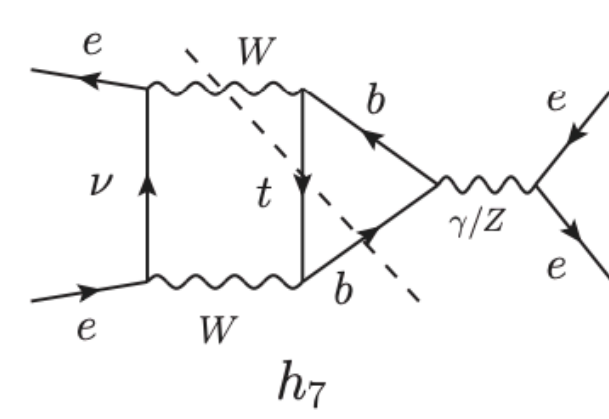
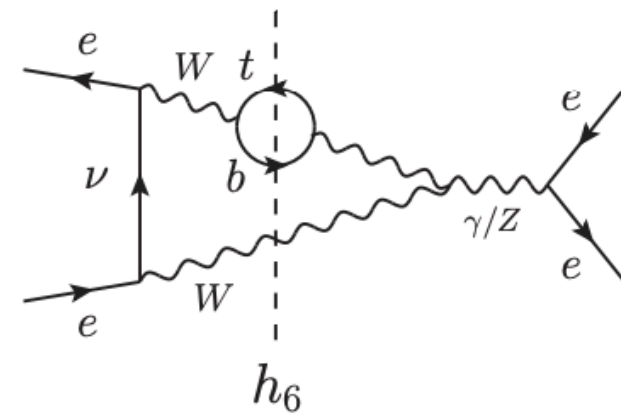
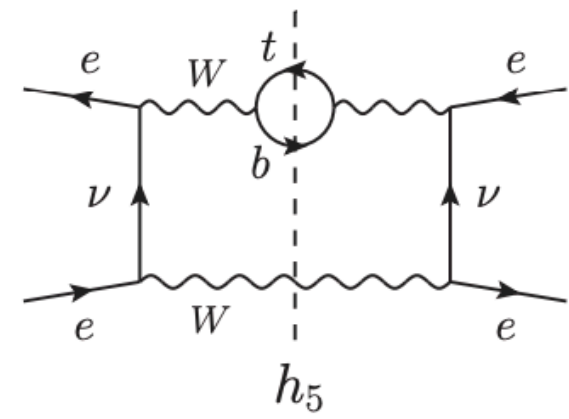
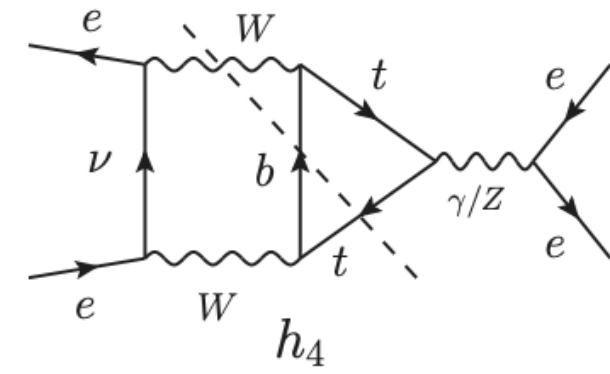
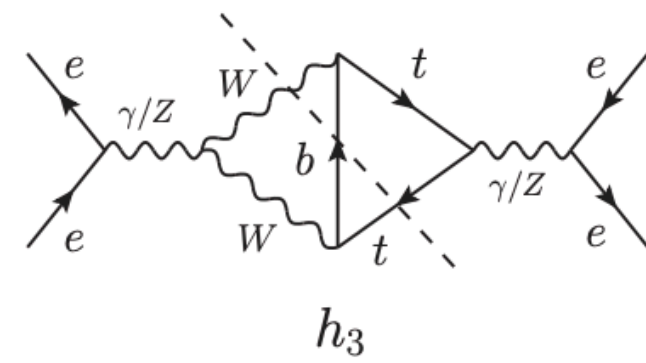
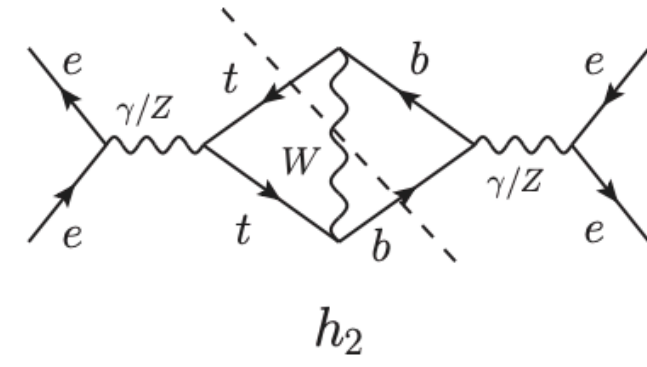
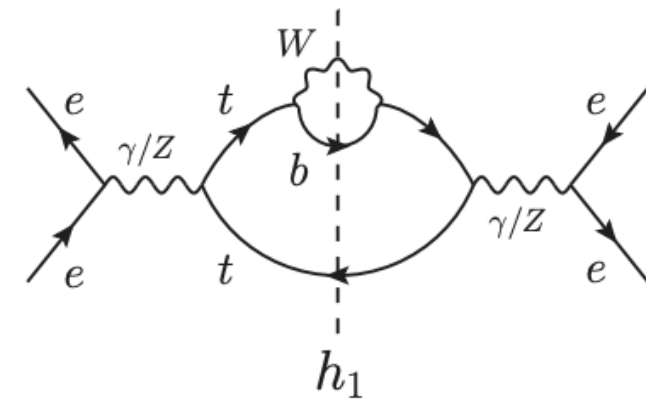
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Note also: $M_{t\bar{t}}\beta^2$ is still a perturbative scale

$$E = M_{t\bar{t}} - 2m_t + i\Gamma_t \sim \frac{1}{2}M_{t\bar{t}}\beta^2 + i\Gamma_t$$

Further discussions



Top quark decay and non-resonant effects?

Existing studies only focus on lepton colliders

See, e.g., Beneke et al.: [1711.10429](#), Bach et al.: [1712.02220](#)

Ongoing: extending to hadron colliders

Summary

- Near-threshold production of top quark pairs is phenomenologically important
 - Indirect top quark mass measurements from differential cross sections
 - Spin correlations and quantum entanglement
- Indication of non-relativistic enhancement in LHC Run 2 data
 - Performed NLP resummation combined with NNLO fixed-order results for differential distributions
 - Enhancement ~ 10 pb, bridging the discrepancy between indirect and direct top quark mass measurements
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Thank you!