第二十二届全国重味物理和CP破坏研讨会

Recent Progress on PQCD for hadronic B decays

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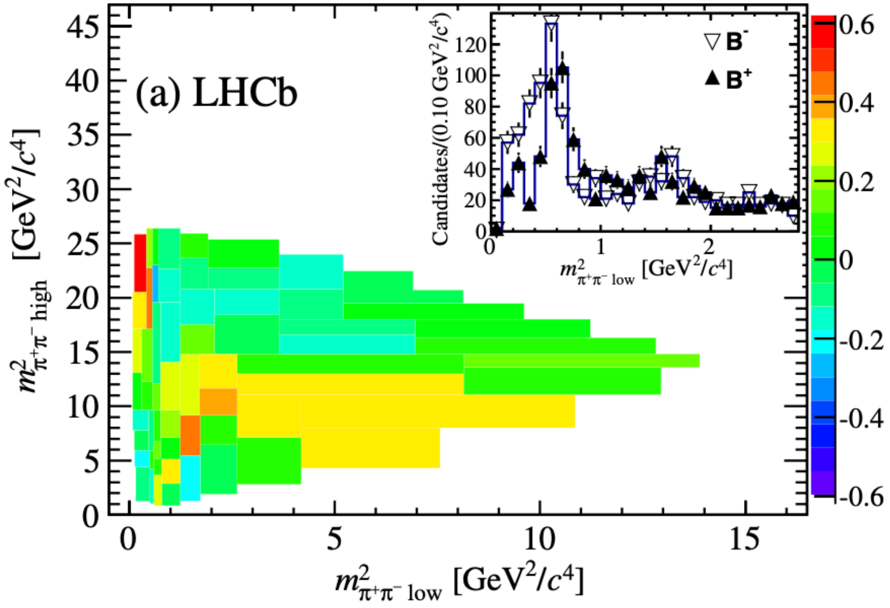
Outline

01 Motivation

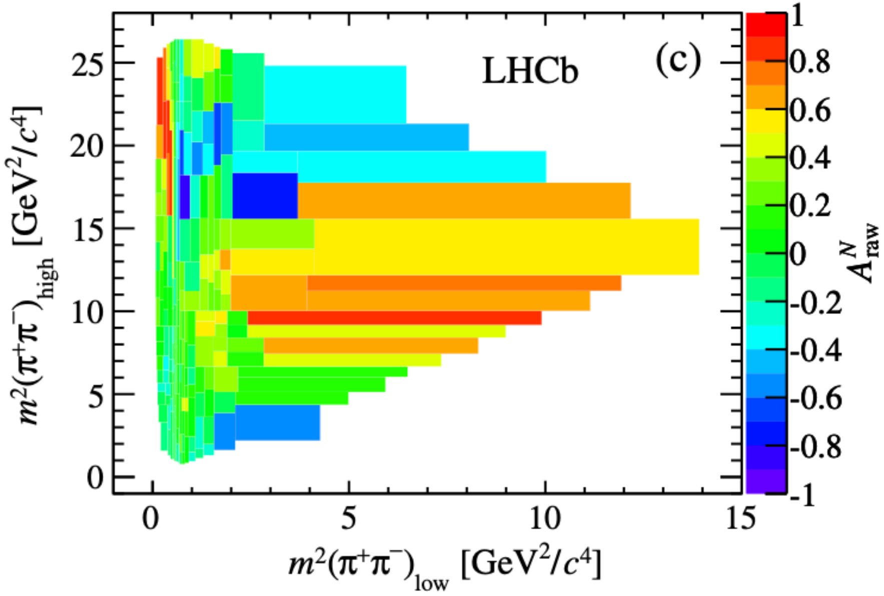
- CP violation (CPV) is a key requirement for generating the matter-antimatter asymmetry in the universe.
- CPVs in K-, B-, D-meson have been confirmed by experiments.
- CPV has been observed in baryons for the first time by LHCb .
- The SM provides the CKM mechanism for generating CPVs, which is, however, too weak to account for the excess of matter.
- Localized CPV and angular-distribution asymmetries in multi-body decays can also reveal potential signals of CPV.

Three-body charmless B decay

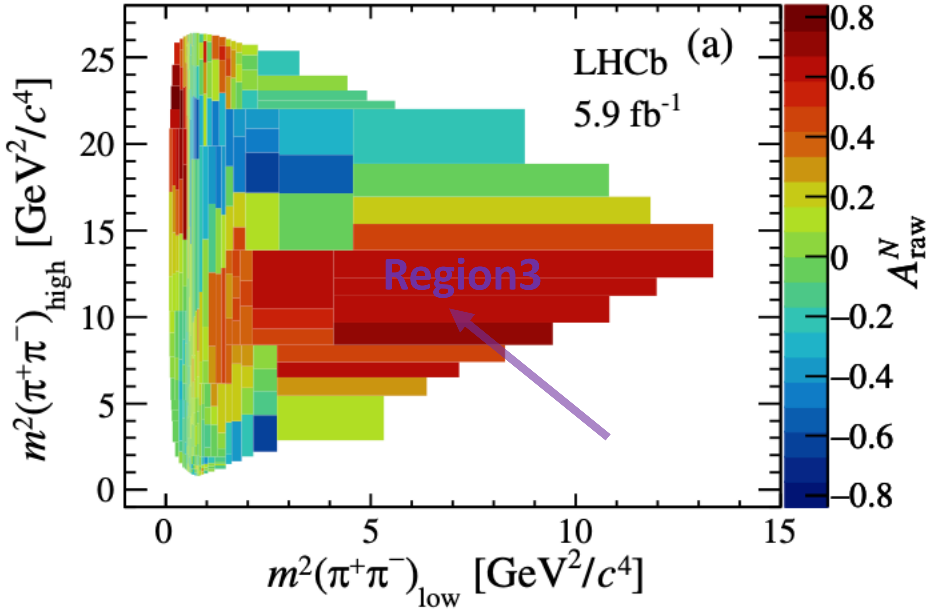
- **Localized A_{CP} in $B \to 3\pi, 3K, KK\pi, K\pi\pi$**

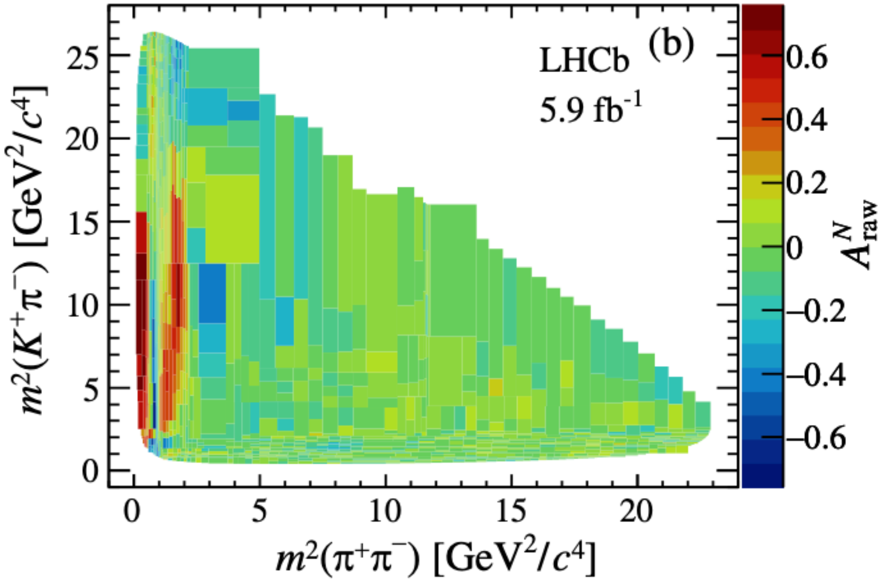


Phys. Rev. Lett.112, 011801(2014)



Phys. Rev. D 90,112004(2014)





Phys. ReV. D 108,012008(2023)

$B^\pm \to \pi^\pm\pi^+\pi^-$	A_{CP}
Region1	$+0.303 \pm 0.009 \pm 0.004 \pm 0.003$
Region2	$-0.284 \pm 0.017 \pm 0.007 \pm 0.003$
Region3	$\mathbf{+0.745 \pm 0.027 \pm 0.018 \pm 0.003}$

- Significant A_{CP}: $B \to 3\pi, 3K$ for the first time
- Large Localized A_{CP}

Several Collaborations have observed B meson decays into various four-body charmless hadronic final states in certain two body invariant mass regions.

$B_s^0 \to (K^+K^-)(K^-\pi^+)$	JHEP11(2013)092
$B_s^0 \to (K^+\pi^-)(K^-\pi^+)$	JHEP03(2018)140
$B^0 \to (\pi^+\pi^-)(K^+\pi^-)$	JHEP05(2019)026
$B_{(s)}^0 \to (K^+\pi^-)(K^-\pi^+)$	JHEP07(2019)032
$B_s^0 \to (K^+K^-)(K^+K^-)$	JHEP12(2019)155
$B^+ \to (\pi^+\pi^-)(K_S^0\pi^+)$	arXiv:2508.13563

Multi-body B meson decays are rich in CPV phenomena in the quark sector.

- Large numbers of different multi-body final states.
- A non-trivial kinematic multiplicity as opposed to two-body decays where the kinematics is fixed by the masses.
- Rich resonance structures and interference between different resonances might induce lager CPV.
- Angular analysis could provide a lot of meaningful CP asymmetries.

Opportunities for CPV searches but also modelling challenges

- More complicated strong dynamics than two-body ones.
- Receive both resonant and nonresonant contributions.
- Suffer substantial final-state interactions.
- A factorization formalism in full phase space is not yet available.

In the multi-body sector

- The leading-power regions of a Dalitz plot.
  [C.-H. Chen and H.-N. Li, PLB 561 (2003) 258]
- Quasi-two-body mechanism. The validity of factorization.
  [NPB 899 (2015) 247,NPB 555 (1999) 231]
- Two-meson distribution amplitudes (TMDAs).
  [Sov. J. Nucl. Phys. 38 (1983) 289, PRD 91 (2015) 094024,PLB 763 (2016) 29…]
- The scenario has been successful applied to three-body decay, encourages us to extended it to the four-body ones.

02 Framework

In the standard model, low-energy effective Hamiltonian is given:

Fermi coupling constant

Local four-quark operators

$$\mathcal{H}_{eff} = \frac{G_F}{\sqrt{2}}\{\sum_{q=u,c} V_{qb}V^{\star}_{qD}[C_1O_1^q + C_2O_2^q] - V_{tb}V^{\star}_{tD}[\sum_{i=3}^{10,7\gamma,8g} C_iO_i]\}\,.$$

CKM matrix elements

Wilson coefficient

For two-body nonleptonic B meson decays, the key step is to calculate the hadronic matrix elements

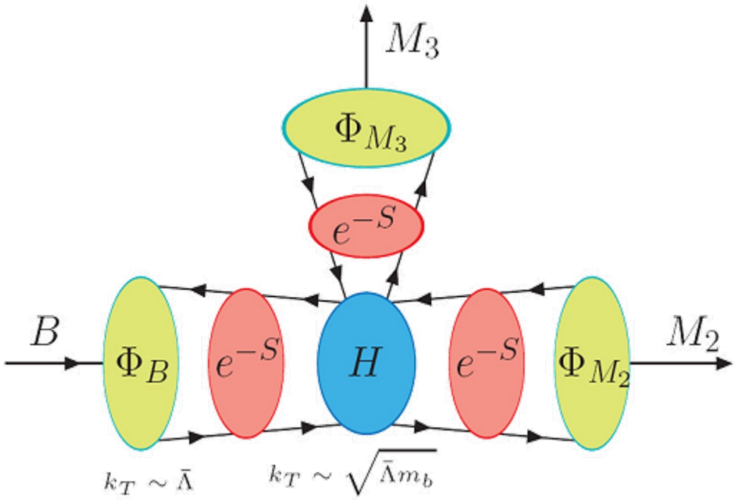
$$\mathcal{A}(B \to M_1M_2) = \langle M_1M_2 \mid \mathcal{H}_{eff} \mid B\rangle = \frac{G_F}{\sqrt{2}}\sum_i V_iC_i(\mu)\underline{\langle M_1M_2 \mid O_i(\mu) \mid B\rangle}$$

Theoretical approach

Factorization Approach	H.Y. Cheng, C.K. Chua, Y. Li,…
QCD Factorization	A. Furman, B.El Bennich, R. Kaminski, T. Mannel, X.H. Guo, Y.D. Yang, X.Q. Li, Z.H. Zhang,…
PQCD	H.n. Li, C.D. Lü, Z.J. Xiao, W. Wang, W.F. Wang, R. Zhou, …
Symmetry	X.G. He, G.N. Li, D. Xu, J.L. Rosner, M. Gronau,…
Sum rules	Ulf-G. Meißner, Y.M. Wang, Y.L. Sheng, S. Cheng A. Khodjamirian (QCD)

PQCD approach： k_T factorization

The full amplitudes of two-body decay $B \to M_1 M_2$ can be factorized as



References:
PPNP51-85(2003);
arXiv:0707.1294;
0907.4940;
1406.7689

$$A = \phi_B \otimes C \otimes H \otimes J_t \otimes S \otimes \phi_{M_2} \otimes \phi_{M_3}$$

Hard Kernel	Wave Function	Sudakov Factor	Jet Function	Wilson Coefficients

k_T **resummation**　　　　**Threshold resummation**

➢PQCD approach

- Transition form factor is dominated from the perturbative region.
- Nonfactorizable and annihilation diagrams can be calculated in PQCD.
- Large strong phase from annihilation diagrams which plays an important role in the CP asymmetry.

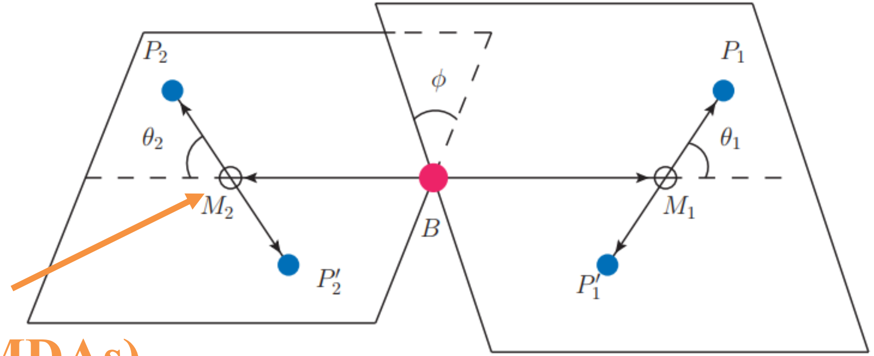
$\mathrm{C}_{\pi\pi}(B^0 \to \pi^+\pi^-)\%$	$\mathrm{A}_{\mathrm{CP}}(B^0 \to K^+\pi^-)\%$
~ -40(Lü,Ukai,Yang,2000)	~ -18(Keum,Li,Sanda,2000)
$-30 \pm 25 \pm 4$ [BaBar,2002]	$-19 \pm 10 \pm 3$ [BaBar,2001]
$-12.8^{+3.48}_{-3.29}$(Chai,Cheng,Ju,Yan, Lü,Xiao,2022)	$-5.43^{+2.25}_{-2.34}$(Chai,Cheng,Ju,Yan, Lü,Xiao,2022)
-31.4 ± 3(PDG)	-8.31 ± 0.31(PDG)

PQCD

- **Semileptonic B meson decays**
- **Two-body B meson decays (pure annihilation decays)**
- ***b*-baryon decays** see talks from Rui Zhou, Jia-Jie Han and Zhi-Tian Zou
- **multi-body decays (three-body and four-body decays)**

$B \to VV \to P_1P_2P_3P_4$ decays

Based on the quasi-two-body mechanism, four-body processes are assumed to proceed dominantly with two intermediate resonances



two-meson DAs (TMDAs)

Figure 1. Helicity angles θ_1, θ_2 and ϕ for the $B \to M_1M_2$ decay, with each intermediate resonance decaying into two pseudoscalars, $M_1 \to P_1P_1'$ and $M_2 \to P_2P_2'$.

A decay amplitude is written as

$$\boxed{\mathcal{A} \propto \Phi_B \otimes H \otimes \Phi_{P_1P_1'} \otimes \Phi_{P_2P_2'},}$$

TMDAs

$B \to VV \to P_1P_2P_3P_4$ decays

- **Analyze angular distribution in four-body decays**

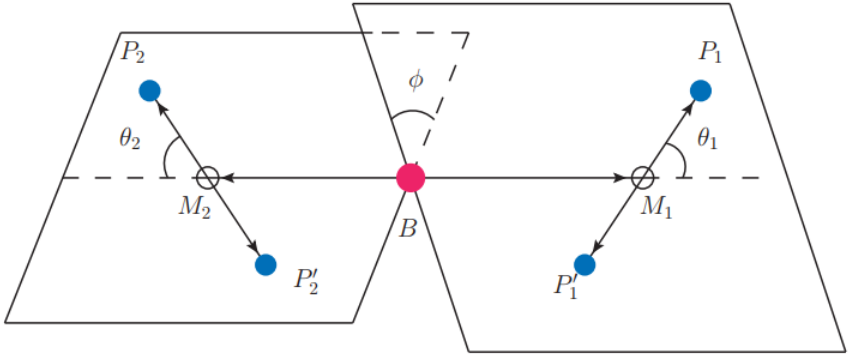


Figure 1. Helicity angles θ_1, θ_2 and ϕ for the $B \to M_1M_2$ decay, with each intermediate resonance decaying into two pseudoscalars, $M_1 \to P_1P_1'$ and $M_2 \to P_2P_2'$.

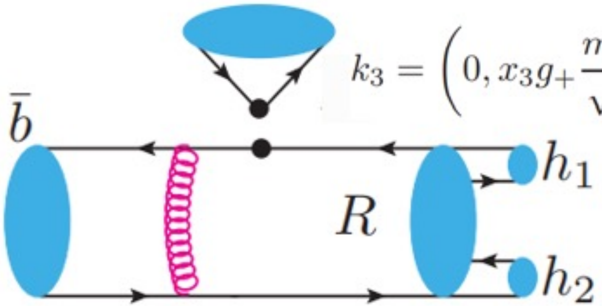
$$
\begin{aligned}
A \;=\; & A_0 \frac{2\zeta_1 - 1}{\sqrt{1+4\alpha_1}} \frac{2\zeta_2 - 1}{\sqrt{1+4\alpha_2}} + A_\parallel 2\sqrt{2}\sqrt{\frac{\zeta_1(1-\zeta_1)+\alpha_1}{1+4\alpha_1}}\sqrt{\frac{\zeta_2(1-\zeta_2)+\alpha_2}{1+4\alpha_2}}\cos\phi \\
& + iA_\perp 2\sqrt{2}\sqrt{\frac{\zeta_1(1-\zeta_1)+\alpha_1}{1+4\alpha_1}}\sqrt{\frac{\zeta_2(1-\zeta_2)+\alpha_2}{1+4\alpha_2}}\sin\phi + A_{VS}\frac{2\zeta_1 - 1}{\sqrt{1+4\alpha_1}} + A_{SV}\frac{2\zeta_2 - 1}{\sqrt{1+4\alpha_2}} + A_{SS}.
\end{aligned}
$$

Improved Framework: *corrections from the final-state meson masses*

The meson momenta in light-cone coordinates

$$p_3 = \frac{m_B}{\sqrt{2}}(g_-, g_+, \mathbf{0}_\mathrm{T})$$

$$k_3 = \left(0, x_3 g_+ \frac{m_B}{\sqrt{2}}, \mathbf{k}_{3\mathrm{T}}\right)$$



$$p_B = \frac{m_B}{\sqrt{2}}(1, 1, \mathbf{0}_\mathrm{T})$$

$$p = \frac{m_B}{\sqrt{2}}(f_+, f_-, \mathbf{0}_\mathrm{T})$$

$$k_B = \left(0, x_B \frac{m_B}{\sqrt{2}}, \mathbf{k}_{B\mathrm{T}}\right) \qquad k = \left(z f_+ \frac{m_B}{\sqrt{2}}, 0, \mathbf{k}_\mathrm{T}\right)$$

$$f_\pm = \frac{1}{2}\left(1 + \eta - r_3 \pm \sqrt{(1-\eta)^2 - 2r_3(1+\eta) + r_3^2}\right)$$

$$g_\pm = \frac{1}{2}\left(1 - \eta + r_3 \pm \sqrt{(1-\eta)^2 - 2r_3(1+\eta) + r_3^2}\right)$$

with the ratio $r_3 = m_{P_3}^2/m_{B_{(s)}}^2$ and $\eta = \omega^2/m_{B_{(s)}}^2$ $\qquad \omega^2 = p^2$

$$p = p_1 + p_2 \qquad \zeta = p_1^+/p^+$$

$$\begin{aligned}
p_1 &= \left((\zeta + \frac{r_1 - r_2}{2\eta})f_+ \frac{m_B}{\sqrt{2}}, (1 - \zeta + \frac{r_1 - r_2}{2\eta})f_- \frac{m_B}{\sqrt{2}}, \mathbf{p}_\mathrm{T}\right), \\
p_2 &= \left((1 - \zeta - \frac{r_1 - r_2}{2\eta})f_+ \frac{m_B}{\sqrt{2}}, (\zeta - \frac{r_1 - r_2}{2\eta})f_- \frac{m_B}{\sqrt{2}}, -\mathbf{p}_\mathrm{T}\right), \\
p_\mathrm{T}^2 &= \zeta(1-\zeta)\omega^2 + \frac{(m_{P_1}^2 - m_{P_2}^2)^2}{4\omega^2} - \frac{m_{P_1}^2 + m_{P_2}^2}{2},
\end{aligned}$$

$p_i^2 = m_{P_i}^2$, $i = 1, 2$, with the mass ratios $r_{1,2} = m_{P_1,P_2}^2/m_B^2$

For a P-wave meson pair, we introduce the longitudinal polarization vector

$$\epsilon = \frac{1}{\sqrt{2\eta}}(f_+, -f_-, \mathbf{0}_T)$$

Improved Framework: *global determination of TMDAs*

Standard nonlinear least-χ^2 (lsq) method

$$\chi^2 = \sum_{i=1}^{n} \left(\frac{v_i - v_i^{\text{th}}}{\delta v_i} \right)^2 .$$

experimental data: $v_i \pm \delta v_i$

theoretical values: v_i^{th}

- Measured branching ratios and polarization fractions of both three-body and four-body B decays (Measurements with significance larger than 3σ)

- Fitted Gegenbauer moments

$\pi\pi$	:	twist-2 $\phi_{\pi\pi}^{0}(a_{\rho}^{0})$ and twist -3 $\phi_{\pi\pi}^{s,t}(a_{\rho}^{s,t})$	
$K\pi$	:	twist-2 $\phi_{K\pi}^{0}(a_{1K*}^{\parallel}, a_{2K*}^{\parallel})$ and $\phi_{K\pi}^{T}(a_{1K*}^{\perp}, a_{2K*}^{\perp})$	
KK	:	twist-2 $\phi_{KK}^{0}(a_{2\phi}^{0})$ and $\phi_{KK}^{T}(a_{2\phi}^{T})$	

- The Gegenbauer moments in the twist-3 $KK, K\pi$ DAs and the transverse $\pi\pi$ DAs need to be further constrained from the global analysis of the multi-body B meson decays, when more precise measurements are available in the future.

$B \to VV \to P_1P_2P_3P_4$ decays

The differential rate for the decays

$$\frac{d^5\Gamma}{d\Omega} = \frac{k(\omega_1)k(\omega_2)k(\omega_1,\omega_2)}{16(2\pi)^6 M^2}|A|^2, \quad \text{with } \Omega \equiv \{\theta_1, \theta_2, \phi, \omega_1, \omega_2\}$$

$$k(\omega) = \frac{\sqrt{[\omega^2-(m_K+m_\pi)^2][\omega^2-(m_K-m_\pi)^2]}}{2\omega}, \quad k(\omega_1,\omega_2) = \frac{\sqrt{[M^2-(\omega_1+\omega_2)^2][M^2-(\omega_1-\omega_2)^2]}}{2M},$$

The transtormation connecting the B meson rest frame and the meson pair rest frame leads to the relations between ζ and θ

$$2\zeta_i - 1 = \sqrt{1+4\alpha_i}\cos\theta_i, \quad \zeta_i \in \left[\frac{1-\sqrt{1+4\alpha_i}}{2}, \frac{1+\sqrt{1+4\alpha_i}}{2}\right]$$

$$\alpha_i = \frac{(r_i - r_i')^2}{4\eta_i^2} - \frac{r_i + r_i'}{2\eta_i}.$$

$$\frac{d^5\mathcal{B}}{d\zeta_1 d\zeta_2 d\omega_1 d\omega_2 d\varphi} = \frac{\tau_{B_{(s)}} k(\omega_1)k(\omega_2)k(\omega_1,\omega_2)}{4(2\pi)^6 m_{B_{(s)}}^2 \sqrt{1+4\alpha_1}\sqrt{1+4\alpha_2}}|A|^2$$

The CP-averaged branching ratios of each component can be defined as follows

$$\mathcal{B}_h^{\text{avg}} = \frac{1}{2}(\mathcal{B}_h + \bar{\mathcal{B}}_h).$$

The polarization fractions f_λ with $\lambda = 0, \|, \perp$ in the $B \to VV$ decays are described as

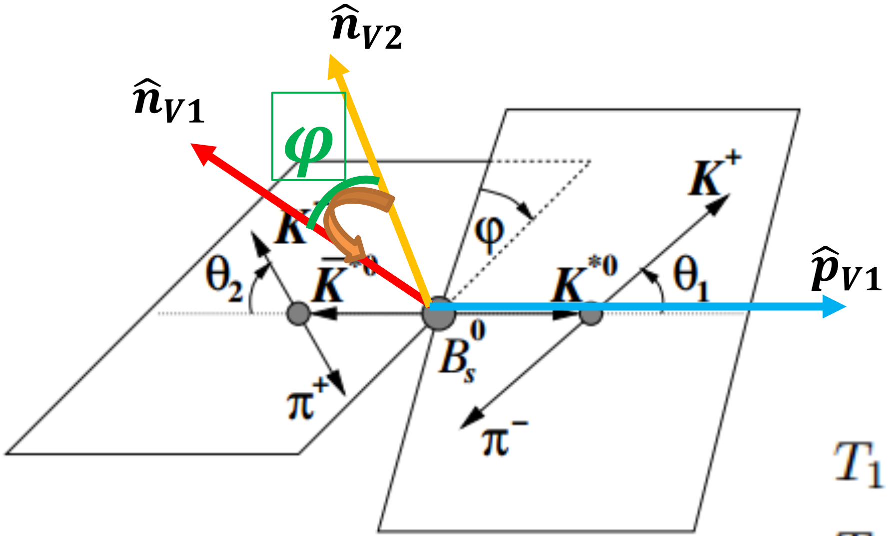
$$f_\lambda = \frac{\mathcal{B}_\lambda}{\mathcal{B}_0 + \mathcal{B}_\| + \mathcal{B}_\perp}$$

The direct CP asymmetry in each component can be defined as

$$\mathcal{A}_h^{\text{dir}} = \frac{\bar{\mathcal{B}}_h - \mathcal{B}_h}{\bar{\mathcal{B}}_h + \mathcal{B}_h} \quad \propto \boldsymbol{Sin\Delta\phi\ Sin\Delta\delta}$$

$\Delta\phi(\Delta\delta)$weak(strong)phase difference

In addition to direct CPV, one can predict triple product asymmetries (TPAs)



$$A_T \equiv \frac{\Gamma(TP>0)-\Gamma(TP<0)}{\Gamma(TP>0)+\Gamma(TP<0)},$$

$$\textbf{TP} \equiv \vec{p}_1 \times \vec{p}_2 \cdot \vec{p}_3.$$

$$T_1 = (\hat{n}_{V_1} \times \hat{n}_{V_2}) \cdot \hat{p}_{V_1} = \sin\varphi \quad \text{and}$$

$$T_2 = 2(\hat{n}_{V_1} \cdot \hat{n}_{V_2})(\hat{n}_{V_1} \times \hat{n}_{V_2}) \cdot \hat{p}_{V_1} = \sin 2\varphi,$$

where $\hat{n}_{V_i}(i=1,2)$ is a unit vector perpendicular to the V_i decay plane and $\hat{p}_{V_1}$ is a unit vector in the direction of V_1 in the $B_{(s)}$ rest frame.

Note that a strong phase difference yields a TPA even in the absence of weak phases, so a nonzero TPA does not necessarily signal CP violation.

$$A_T^1 = \frac{\Gamma((2\zeta_1-1)(2\zeta_2-1)\sin\phi>0)-\Gamma((2\zeta_1-1)(2\zeta_2-1)\sin\phi<0)}{\Gamma((2\zeta_1-1)(2\zeta_2-1)\sin\phi>0)+\Gamma((2\zeta_1-1)(2\zeta_2-1)\sin\phi<0)} = -\frac{2\sqrt{2}}{\pi\mathcal{D}}\int d\omega_1 d\omega_2 k(\omega_1)k(\omega_2)k(\omega_1,\omega_2)Im[A_\perp A_0^*],$$

$$A_T^2 = \frac{\Gamma(\sin(2\phi)>0)-\Gamma(\sin(2\phi)<0)}{\Gamma(\sin(2\phi)>0)+\Gamma(\sin(2\phi)<0)} = -\frac{4}{\pi\mathcal{D}}\int d\omega_1 d\omega_2 k(\omega_1)k(\omega_2)k(\omega_1,\omega_2)Im[A_\perp A_\parallel^*],$$

$$\boxed{Im(A_\perp A_h^*) = |A_\perp||A_h^*|\sin(\Delta\phi+\Delta\delta)}$$

Strong phase difference $\Delta\delta$ can produce a nonzero value

To identify a true CP violation signal, one has to compare the TPAs in B and $\bar{B}$ meson decays.

$$\mathcal{A}_{\text{T-true}}^{1(2),\text{ave}} \equiv \frac{[\Gamma(T_{1(2)}>0)+\bar{\Gamma}(\bar{T}_{1(2)}>0)]-[\Gamma(T_{1(2)}<0)+\bar{\Gamma}(\bar{T}_{1(2)}<0)]}{[\Gamma(T_{1(2)}>0)+\bar{\Gamma}(\bar{T}_{1(2)}>0)]+[\Gamma(T_{1(2)}<0)+\bar{\Gamma}(\bar{T}_{1(2)}<0)]} = B_{1(2)}\int d\omega_1 d\omega_2 k(\omega_1)k(\omega_2)k(\omega_1,\omega_2) \times \text{Im}\left[\frac{A_\perp A_{0(\parallel)}^* - \bar{A}_\perp \bar{A}_{0(\parallel)}^*}{(\mathcal{D}+\bar{\mathcal{D}})}\right] \propto \boxed{\sin\Delta\phi\cos\Delta\delta,}$$

$$\mathcal{A}_{\text{T-fake}}^{1(2),\text{ave}} \equiv \frac{[\Gamma(T_{1(2)}>0)-\bar{\Gamma}(\bar{T}_{1(2)}>0)]-[\Gamma(T_{1(2)}<0)-\bar{\Gamma}(\bar{T}_{1(2)}<0)]}{[\Gamma(T_{1(2)}>0)+\bar{\Gamma}(\bar{T}_{1(2)}>0)]+[\Gamma(T_{1(2)}<0)+\bar{\Gamma}(\bar{T}_{1(2)}<0)]} = B_{1(2)}\int d\omega_1 d\omega_2 k(\omega_1)k(\omega_2)k(\omega_1,\omega_2) \times \text{Im}\left[\frac{A_\perp A_{0(\parallel)}^* + \bar{A}_\perp \bar{A}_{0(\parallel)}^*}{(\mathcal{D}+\bar{\mathcal{D}})}\right] \propto \boxed{\cos\Delta\phi\sin\Delta\delta,}$$

03

Recent progress

JHEP 05 (2021) 082; Eur. Phys. J. C 81 (2021) 9, 806; Phys. Rev. D 105 (2022) 5, 053002; Phys. Rev. D 105 (2022) 9, 093001; Eur. Phys. J. C 83 (2023) 10, 974; Eur. Phys. J. C 84 (2024) 7, 754 ; Eur. Phys. J. C 85 (2025) 4, 444

PQCD predictions for Brs of the four-body $B_{(s)} \to (K\pi)(K\pi)$ decays

Modes	$\mathcal{B}(10^{-6})$	$f_0(\%)$	$f_\perp(\%)$
$B_s^0 \to K^{*0}\bar{K}^{*0}$	$13.2^{+5.2}_{-3.7}$	$28.2^{+8.8}_{-9.5}$	$35.7^{+4.7}_{-4.2}$
PQCD-I	$5.4^{+3.0}_{-2.4}$	$38.3^{+12.1}_{-10.5}$	$30.0^{+5.3}_{-6.1}$
PQCD-NLO	$6.7^{+2.9}_{-2.2}$	$43.4^{+12.7}_{-12.9}$	$23.5^{+5.8}_{-5.9}$
QCDF	6.6 ± 2.2	56^{+22}_{-27}	$\cdots$
SCET	8.6 ± 3.1	44.9 ± 18.3	24.9 ± 11.1
FAT	14.9 ± 3.6	34.3 ± 12.6	33.2 ± 6.9
Data	11.1 ± 2.7	24 ± 4	38 ± 12
$B_s^0 \to K^{*+}K^{*-}$	$13.7^{+5.4}_{-3.7}$	$32.3^{+10.5}_{-10.6}$	$33.9^{+5.3}_{-5.2}$
PQCD-I	$5.4^{+3.3}_{-2.3}$	$42.0^{+14.2}_{-11.2}$	$27.7^{+5.2}_{-7.0}$
PQCD-NLO	$6.5^{+2.8}_{-2.1}$	$48.1^{+9.7}_{-8.9}$	$23.9^{+4.4}_{-5.2}$
QCDF	$7.6^{+2.5}_{-2.1}$	52^{+20}_{-21}	$\cdots$
SCET	11.0 ± 3.3	55 ± 14	20.3 ± 8.6
FAT	15.9 ± 3.5	30.9 ± 10.4	34.9 ± 5.8
$B^0 \to K^{*0}\bar{K}^{*0}$	$0.60^{+0.22}_{-0.14}$	$81.1^{+3.2}_{-5.2}$	$9.6^{+2.7}_{-1.7}$
PQCD-I	$0.34^{+0.16}_{-0.15}$	58 ± 8	$19.7^{+4.0}_{-3.6}$
QCDF	$0.6^{+0.2}_{-0.3}$	52 ± 48	24 ± 24
SCET	0.48 ± 0.16	50 ± 16	22.9 ± 10.0
FAT	0.61 ± 0.17	58.3 ± 11.1	20.8 ± 6.0
Data	0.83 ± 0.24	74 ± 5	$\cdots$
$B^0 \to K^{*+}K^{*-}$	$1.24^{+0.38}_{-0.32}$	~ 100	~ 0.0
PQCD-I	0.21 ± 0.10	~ 100	~ 0.0
QCDF	0.1 ± 0.1	~ 100	~ 0.0
FAT	1.43 ± 0.96	$\cdots$	$\cdots$
Data	< 2.0	$\cdots$	$\cdots$
$B^+ \to K^{*+}\bar{K}^{*0}$	$0.71^{+0.34}_{-0.16}$	$83.5^{+5.0}_{-3.8}$	$8.5^{+1.5}_{-3.3}$
PQCD-I	$0.56^{+0.26}_{-0.22}$	74^{+4}_{-5}	$12.9^{+1.7}_{-2.4}$
QCDF	0.6 ± 0.3	45^{+55}_{-38}	27^{+19}_{-27}
SCET	0.52 ± 0.18	50 ± 16	22.9 ± 10.0
FAT	0.66 ± 0.18	58.3 ± 11.1	20.8 ± 6.0
Data	0.91 ± 0.29	82^{+15}_{-21}	$\cdots$

$$\mathcal{B}(B \to R_1R_2 \to (P_1P_2)(P_3P_4)) = \mathcal{B}(B \to R_1R_2)\times\mathcal{B}(R_1 \to P_1P_2)\times \mathcal{B}(R_2 \to P_3P_4)$$

- $\omega \in [m_{K^*} - 0.15, m_{K^*} + 0.15]$GeV
- Theoretical uncertainties are added in quadrature:
  - B meson LCDAs
  - Improved two-meson DAs
  - Hard scale
- $b \to s$ transitions $\sim 10^{-6}$
- $b \to d$ transitions $\sim 10^{-7}$

PQCD-I:PRD91,054033(2015)
PQCD-NLO:NPB935-17(2018)
QCDF:PRD80,114026(2009)
SCET:PRD96,073004(2017)
FAT:EPJC77,333(2017)

Modes	$\mathcal{B}(10^{-6})$	$f_0(\%)$	$f_\perp(\%)$
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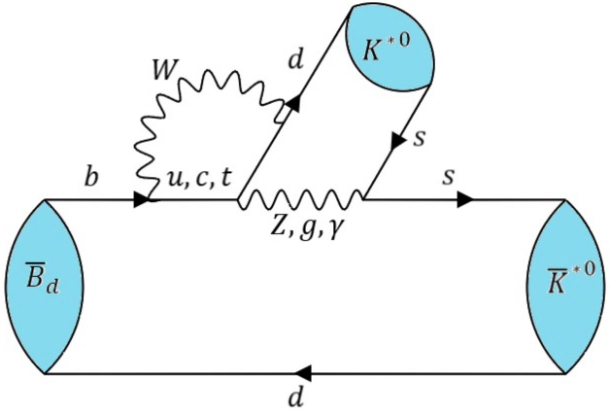
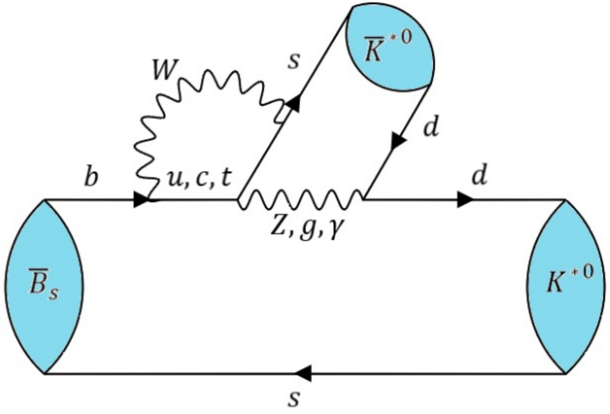
SCET	8.6 ± 3.1	44.9 ± 18.3	24.9 ± 11.1
FAT	14.9 ± 3.6	34.3 ± 12.6	33.2 ± 6.9
Data	11.1 ± 2.7	24 ± 4	38 ± 12

$B^0 \to K^{*0}\bar{K}^{*0}$	$0.60^{+0.22}_{-0.14}$	$81.1^{+3.2}_{-5.2}$	$9.6^{+2.7}_{-1.7}$
PQCD-I	$0.34^{+0.16}_{-0.15}$	58 ± 8	$19.7^{+4.0}_{-3.6}$
QCDF	$0.6^{+0.2}_{-0.3}$	52 ± 48	24 ± 24
SCET	0.48 ± 0.16	50 ± 16	22.9 ± 10.0
FAT	0.61 ± 0.17	58.3 ± 11.1	20.8 ± 6.0
Data	0.83 ± 0.24	74 ± 5	$\cdots$

A new observable $L_{K^*\overline{K}^*}$ defined as the ratio of the longitudinal branching ratios of $\overline{B}_s \to K^{*0}\overline{K}^{*0}$ versus $\overline{B}_d \to K^{*0}\overline{K}^{*0}$





$$L_{K^*\bar{K}^*} = \underline{\rho(m_{K^{*0}}, m_{K^{*0}})} \frac{\mathcal{B}(\bar{B}_s \to K^{*0}\bar{K}^{*0})}{\mathcal{B}(\bar{B}_d \to K^{*0}\bar{K}^{*0})} \boxed{\frac{f_L^{B_s}}{f_L^{B_d}}} = \frac{|A_0^s|^2 + |\bar{A}_0^s|^2}{|A_0^d|^2 + |\bar{A}_0^d|^2}$$

phase-space factor

longitudinal polarisation fractions

$$L_{K^*\bar{K}^*} = \frac{|A_0^s|^2 + |\bar{A}_0^s|^2}{|A_0^d|^2 + |\bar{A}_0^d|^2}$$

$L_{K^*\bar{K}^*}$	QCDF (2301.10542)	PQCD (2209.13389)	PQCD-updated (2501.15150)	exp
	$19.53^{+9.14}_{-6.64}$	$12.7^{+5.6}_{-3.2}$	$7.7^{+2.9}_{-2.6}$	4.43 ± 0.92

Modes	$\mathcal{B}(10^{-6})$	$f_0(\%)$
$B_s^0 \to K^{*0}\bar{K}^{*0}$	$13.2^{+5.2}_{-3.7}$	$28.2^{+8.8}_{-9.5}$
PQCD-I	$5.4^{+3.0}_{-2.4}$	$38.3^{+12.1}_{-10.5}$
PQCD-NLO	$6.7^{+2.9}_{-2.2}$	$43.4^{+12.7}_{-12.9}$
QCDF	6.6 ± 2.2	56^{+22}_{-27}
SCET	8.6 ± 3.1	44.9 ± 18.3
FAT	14.9 ± 3.6	34.3 ± 12.6
Data	11.1 ± 2.7	24 ± 4

Modes	$\mathcal{B}(10^{-6})$	$f_0(\%)$
$B^0 \to K^{*0}\bar{K}^{*0}$	$0.60^{+0.22}_{-0.14}$	$81.1^{+3.2}_{-5.2}$
PQCD-I	$0.34^{+0.16}_{-0.15}$	58 ± 8
QCDF	$0.6^{+0.2}_{-0.3}$	52 ± 48
SCET	0.48 ± 0.16	50 ± 16
FAT	0.61 ± 0.17	58.3 ± 11.1
Data	0.83 ± 0.24	74 ± 5

PQCD predictions of the four-body $B_{(s)} \to (\pi\pi)(\pi\pi)$ decays

Modes	$\mathcal{B}(10^{-6})$	$f_0(\%)$	$f_\perp(\%)$
$B^+ \to \rho^+\rho^0$ T	$12.7^{+4.5}_{-3.3}$	$97.7^{+0.8}_{-0.9}$	$1.3^{+0.5}_{-0.4}$
PQCD [26]	$13.5^{+5.1}_{-4.1}$	98 ± 1	$0.46^{+0.08}_{-0.06}$
QCDF [23]	$20.0^{+4.5}_{-2.1}$	96 ± 2	$\cdots$
SCET [28]	22.1 ± 3.7	100	$\cdots$
FAT [29]	21.7 ± 5.1	95.5 ± 1.1	2.22 ± 0.64
Data [1]	24.0 ± 1.9	95.0 ± 1.6	$\cdots$
$B^0 \to \rho^+\rho^-$ T	$27.0^{+10.7}_{-7.5}$	$92.2^{+3.6}_{-4.3}$	$4.5^{+2.3}_{-1.9}$
PQCD [26]	$26.0^{+10.3}_{-8.3}$	95 ± 1	$2.42^{+0.21}_{-0.19}$
QCDF [23]	$25.5^{+1.9}_{-3.0}$	92^{+1}_{-3}	$\cdots$
SCET [28]	27.7 ± 4.1	99.1 ± 0.3	0.40 ± 0.18
FAT [29]	29.5 ± 6.5	92.6 ± 1.6	3.65 ± 0.91
Data [1]	27.7 ± 1.9	$99.0^{+2.1}_{-1.9}$	$\cdots$
$B^0 \to \rho^0\rho^0$ C	$0.35^{+0.12}_{-0.07}$	$37.9^{+9.4}_{-3.2}$	$33.9^{+2.6}_{-5.7}$
PQCD [26]	$0.27^{+0.12}_{-0.10}$	12^{+16}_{-2}	$45.9^{+1.1}_{-8.2}$
QCDF [23]	$0.9^{+1.9}_{-0.45}$	92^{+7}_{-37}	$\cdots$
SCET [28]	1.00 ± 0.29	87 ± 5	5.81 ± 2.84
FAT [29]	0.94 ± 0.59	81.7 ± 10.8	9.21 ± 5.50
Data [1]	0.96 ± 0.15	71^{+8}_{-9}	$\cdots$
$B_s^0 \to \rho^+\rho^-$ A	$1.35^{+0.84}_{-0.43}$	$99.4^{+0.6}_{-0.7}$	$0.3^{+0.1}_{-0.2}$
PQCD [26]	$1.5^{+0.7}_{-0.6}$	~ 1.0	~ 0.0
QCDF [23]	$0.68^{+0.73}_{-0.53}$	~ 1.0	$\cdots$
FAT [29]	0.10 ± 0.06	$\cdots$	$\cdots$
$B_s^0 \to \rho^0\rho^0$ A	$0.68^{+0.42}_{-0.22}$	$99.4^{+0.6}_{-0.7}$	$0.3^{+0.1}_{-0.2}$
PQCD [26]	$0.74^{+0.45}_{-0.28}$	~ 1.0	~ 0.0
QCDF [23]	$0.34^{+0.36}_{-0.26}$	~ 1.0	$\cdots$
FAT [29]	0.05 ± 0.03	$\cdots$	$\cdots$
Data [1]	< 320	$\cdots$	$\cdots$

PQCD:PRD91,054033(2015)
QCDF:PRD80,114026(2009)
SCET:PRD96,073004(2017)
FAT:EPJC77,333(2017)

Direct CP asymmetries (10^{-2})

- The direct CPVs arises from the interference between the tree and penguin amplitudes.
- The total direct CP asymmetry can be well approximated by the weighted sum of the three asymmetries.

$$\mathcal{A}_{CP}^{\text{dir}} \approx f_0 \mathcal{A}_{CP}^{0} + f_{||} \mathcal{A}_{CP}^{||} + f_{\perp} \mathcal{A}_{CP}^{\perp},$$

$$\propto \boldsymbol{Sin\Delta\phi\ Sin\Delta\delta}$$

Modes	$\mathcal{A}^{0}_{\rm CP}$	$\mathcal{A}^{\parallel}_{\rm CP}$	$\mathcal{A}^{\perp}_{\rm CP}$	$\mathcal{A}^{\rm dir}_{\rm CP}$
$B^+ \to \rho^+\rho^0 \to (\pi^+\pi^0)(\pi^+\pi^-)$	$0.4^{+0.2}_{-0.1}(97.7\%)$	$-0.1^{+0.4}_{-0.6}(1.0\%)$	$0.5^{+0.2}_{-0.6}(1.3\%)$	$0.4^{+0.2}_{-0.1}$
Data	$\cdots$	$\cdots$	$\cdots$	-5 ± 5
$B^0 \to \rho^+\rho^- \to (\pi^+\pi^0)(\pi^-\pi^0)$	$-3.7^{+0.9}_{-2.0}(92.2\%)$	$43.4^{+11.7}_{-17.7}(3.3\%)$	$38.4^{+12.0}_{-16.9}(4.5\%)$	$-0.3^{+2.9}_{-2.4}$
Data	$\cdots$	$\cdots$	$\cdots$	0 ± 9
$B^0 \to \rho^0\rho^0 \to (\pi^+\pi^-)(\pi^+\pi^-)$	$61.7^{+12.3}_{-14.8}(37.9\%)$	$64.9^{+6.1}_{-8.4}(28.2\%)$	$76.9^{+8.8}_{-9.0}(33.9\%)$	$67.8^{+7.1}_{-9.9}$
Data	$\cdots$	$\cdots$	$\cdots$	20 ± 90
$B^0_s \to \rho^+\rho^- \to (\pi^+\pi^0)(\pi^-\pi^0)$	$5.1^{+4.3}_{-2.5}(99.4\%)$	$4.3^{+4.6}_{-3.9}(0.3\%)$	$5.9^{+8.6}_{-5.3}(0.3\%)$	$5.1^{+4.2}_{-2.6}$
$B^0_s \to \rho^0\rho^0 \to (\pi^+\pi^-)(\pi^+\pi^-)$	$5.1^{+4.3}_{-2.5}(99.4\%)$	$4.3^{+4.6}_{-3.9}(0.3\%)$	$5.9^{+8.6}_{-5.3}(0.3\%)$	$5.1^{+4.2}_{-2.6}$
$B^+ \to \rho^+K^{*0} \to (\pi^+\pi^0)(K^+\pi^-)$	$-0.2^{+1.9}_{-3.4}(48.7\%)$	$1.6^{+0.9}_{-1.3}(25.6\%)$	$1.7^{+0.2}_{-0.6}(25.7\%)$	$0.8^{+0.8}_{-1.8}$
Data	$\cdots$	$\cdots$	$\cdots$	-1 ± 16
$B^+ \to \rho^0K^{*+} \to (\pi^+\pi^-)(K^0\pi^+)$	$32.8^{+2.7}_{-8.5}(57.8\%)$	$0.8^{+4.4}_{-4.6}(25.3\%)$	$-56.2^{+9.7}_{-9.4}(16.9\%)$	$9.6^{+7.1}_{-7.7}$
Data	$\cdots$	$\cdots$	$\cdots$	31 ± 13
$B^0 \to \rho^0K^{*0} \to (\pi^+\pi^-)(K^+\pi^-)$	$0.1^{+5.3}_{-7.3}(33.2\%)$	$-34.5^{+9.2}_{-12.2}(25.6\%)$	$12.8^{+0.8}_{-1.4}(41.2\%)$	$-3.5^{+2.2}_{-2.7}$
Data	$\cdots$	$\cdots$	$\cdots$	-6 ± 9
$B^0 \to \rho^-K^{*+} \to (\pi^-\pi^0)(K^0\pi^+)$	$57.2^{+5.8}_{-10.1}(48.0\%)$	$-26.5^{+5.3}_{-4.7}(25.8\%)$	$-30.7^{+4.6}_{-4.9}(26.2\%)$	$12.6^{+12.0}_{-9.5}$
Data	$\cdots$	$\cdots$	$\cdots$	21 ± 15
$B^0_s \to \rho^+K^{*-} \to (\pi^+\pi^0)(\bar{K}^0\pi^-)$	$-14.2^{+3.4}_{-3.8}(89.4\%)$	$73.7^{+12.5}_{-15.4}(5.3\%)$	$75.2^{+11.5}_{-14.1}(5.3\%)$	$-4.7^{+3.1}_{-2.9}$
$B^0_s \to \rho^0\bar{K}^{*0} \to (\pi^+\pi^-)(K^-\pi^+)$	$21.2^{+19.6}_{-16.5}(59.4\%)$	$73.2^{+15.4}_{-25.0}(19.1\%)$	$81.8^{+11.6}_{-21.8}(21.5\%)$	$44.2^{+11.5}_{-13.9}$
$B^0_s \to \rho^0\phi \to (\pi^+\pi^-)(K^+K^-)$	$-2.4^{+9.0}_{-6.9}(82.9\%)$	$-18.3^{+5.2}_{-4.2}(8.2\%)$	$-16.8^{+4.0}_{-2.9}(8.9\%)$	$-5.0^{+8.2}_{-5.8}$
$B^+ \to K^{*+}\phi \to (K^0\pi^+)(K^+K^-)$	$-5.7^{+11.2}_{-3.7}(54.6\%)$	$2.8^{+0.3}_{-2.9}(22.3\%)$	$-2.3^{+1.3}_{-1.0}(23.1\%)$	$-3.3^{+7.6}_{-1.6}$
$B^+ \to K^{*+}\bar{K}^{*0} \to (K^0\pi^+)(K^+\pi^-)$	$-21.5^{+8.7}_{-10.8}(83.5\%)$	$-9.0^{+1.4}_{-1.3}(8.0\%)$	$7.9^{+1.7}_{-1.5}(8.5\%)$	$-19.4^{+7.6}_{-9.2}$
$B^0 \to K^{*+}K^{*-} \to (K^0\pi^+)(\bar{K}^0\pi^-)$	$19.1^{+2.8}_{-1.5}(\sim 100\%)$	$-29.5^{+17.3}_{-16.0}(\sim 0)$	$10.2^{+6.7}_{-5.1}(\sim 0)$	$19.1^{+2.8}_{-1.5}$

PQCD predictions for TPAs (%) of the four-body B(s) → (Kπ)(Kπ) decays

Asymmetries	$B_s^0 \to (K^+\pi^-)(K^-\pi^+)$	$B_s^0 \to (K^0\pi^+)(\bar{K}^0\pi^-)$	$B^0 \to (K^-\pi^+)(K^+\pi^-)$	$B^0 \to (K^0\pi^+)(\bar{K}^0\pi^-)$	$B^+ \to (K^0\pi^+)(K^+\pi^-)$
A_T^1	$11.8^{+0.8}_{-1.1}$	$9.7^{+0.5}_{-0.6}$	$10.6^{+1.3}_{-1.7}$	~ 0	$8.5^{+0.9}_{-0.3}$
$\bar{A}_T^1$	$-11.8^{+0.8}_{-1.1}$	$-9.9^{+0.5}_{-0.6}$	$-10.6^{+1.3}_{-1.7}$	~ 0	$-9.2^{+3.5}_{-0.2}$
A_T^1(true)	0	$-0.1^{+0.0}_{-0.1}$	0	~ 0	$-0.2^{+0.2}_{-0.1}$
A_T^1(fake)	$11.8^{+0.8}_{-1.1}$	$9.8^{+0.3}_{-0.5}$	$10.6^{+1.3}_{-1.7}$	~ 0	$8.7^{+1.5}_{-0.1}$
A_T^2	$0.2^{+0.1}_{-0.1}$	$0.2^{+0.1}_{-0.1}$	$0.3^{+0.1}_{-0.1}$	~ 0	$0.2^{+0.0}_{-0.1}$
$\bar{A}_T^2$	$-0.2^{+0.1}_{-0.1}$	$-0.2^{+0.0}_{-0.1}$	$-0.3^{+0.1}_{-0.1}$	~ 0	$0.3^{+0.2}_{-0.2}$
A_T^2(true)	0	0	0	~ 0	$0.25^{+0.10}_{-0.10}$
A_T^2(fake)	$0.2^{+0.1}_{-0.1}$	$0.2^{+0.1}_{-0.1}$	$0.3^{+0.1}_{-0.1}$	~ 0	$-0.05^{+0.00}_{-0.00}$
A_T^3	$2.1^{+1.0}_{-0.6}$	$3.3^{+0.4}_{-0.7}$	$2.0^{+0.9}_{-1.0}$	~ 0	$3.6^{+0.3}_{-0.7}$
$\bar{A}_T^3$	$-2.1^{+1.0}_{-0.6}$	$-1.0^{+0.0}_{-0.1}$	$-2.0^{+0.9}_{-1.0}$	~ 0	$-0.8^{+0.1}_{-0.2}$
A_T^3(true)	0	$1.2^{+0.5}_{-0.2}$	0	~ 0	$1.6^{+0.6}_{-0.1}$
A_T^3(fake)	$2.1^{+1.0}_{-0.6}$	$2.2^{+0.7}_{-1.0}$	$2.0^{+0.9}_{-1.0}$	~ 0	$2.3^{+0.1}_{-0.5}$
A_T^4	$-4.6^{+2.0}_{-1.5}$	$-5.1^{+2.0}_{-1.4}$	$-5.1^{+2.3}_{-1.9}$	~ 0	$-2.7^{+0.5}_{-0.5}$
$\bar{A}_T^4$	$4.6^{+2.0}_{-1.5}$	$4.1^{+1.6}_{-1.4}$	$5.1^{+2.3}_{-1.9}$	~ 0	$1.0^{+0.5}_{-0.5}$
A_T^4(true)	0	$-0.5^{+0.2}_{-0.2}$	0	~ 0	$-0.9^{+0.5}_{-0.5}$
A_T^4(fake)	$-4.6^{+2.0}_{-1.5}$	$-4.6^{+1.8}_{-1.5}$	$-5.1^{+2.3}_{-1.9}$	~ 0	$-1.9^{+0.3}_{-0.9}$

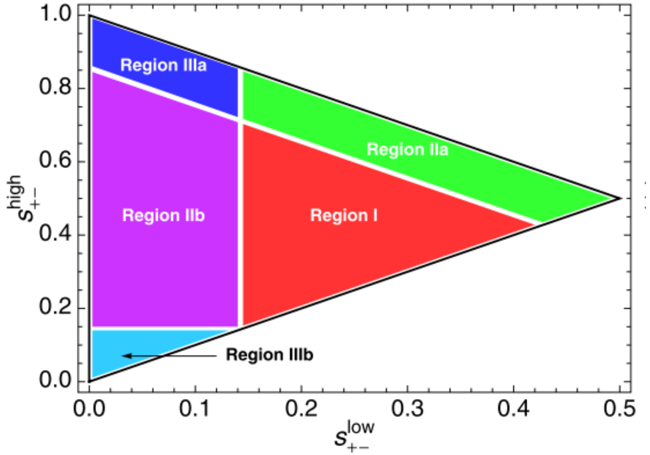
LHCb[JHEP07(2015)166]

Asymmetry	Data
A_T^1(true)	$0.003 \pm 0.041 \pm 0.009$
A_T^2(true)	$0.009 \pm 0.041 \pm 0.009$
A_T^3(true)	$0.019 \pm 0.041 \pm 0.008$
A_T^4(true)	$-0.040 \pm 0.041 \pm 0.008$

The LHCb measurements show no manifest deviation from zero.

04 Outlook

- Develop a systematic theoretical approach to 3-body hadronic B decays in the whole phase space



- Extraction of the SM parameters through multi-body B decays
- NLO corrections to multi-body B decays

Thank you