Probing the elusive $\kappa/K_0^*(700)$ resonance in semileptonic D decays

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Introduction

Light scalar meson (S_0)

- S_0 with a mass below 1 GeV

$f_0/f_0(980)$, $\sigma_0/f_0(500)$, $a_0/a_0(980)$, $\kappa_0/K_0^*(700)$

- Regarded as

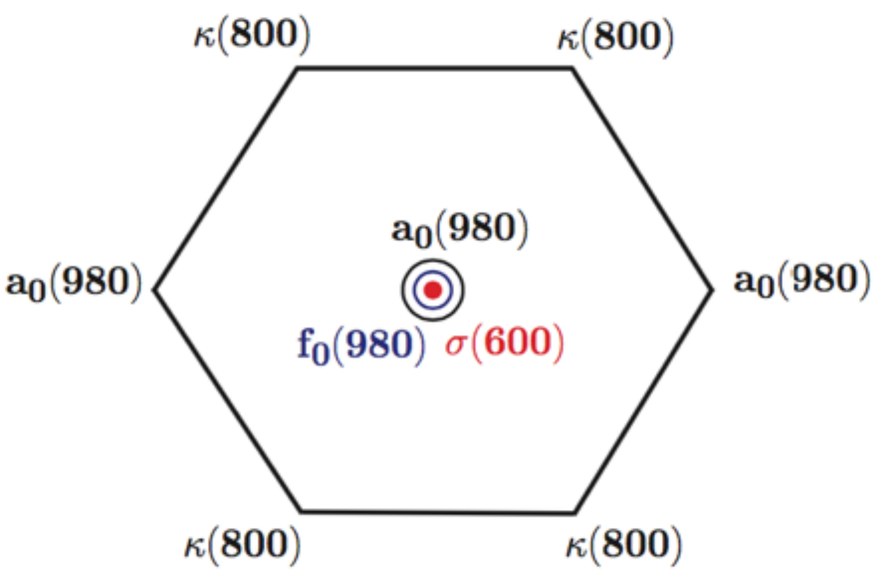
a normal p-wave meson ($q\bar{q}$), or

an exotic tetraquark [compact $q^2\bar{q}^2$ bound state or

molecular M_1M_2 bound state.]

- By excluding S_0 as a normal $q\bar{q}$ state,

confirm the existence of a charmless tetraquark.



Semileptonic D decays

• useful for testing the internal structure of a light scalar meson:

1. Wang, Lu, PRD82, 034016 (2010), *"Distinguishing two kinds of scalar mesons from heavy meson decays."*

2. Achasov, Kiselev, Shestakov, PRD102, 016022 (2020), *"Semileptonic decays* $D \to \pi^+\pi^- e^+\nu_e$ *and* $D_s \to \pi^+\pi^- e^+\nu_e$ *as the probe of constituent quark-antiquark pairs in the light scalar mesons"*

3. Achasov, Kiselev, Shestakov, PRD104, 016034 (2021), *"Semileptonic decays* $D \to \eta\pi e\nu$ *in the* $a_0(980)$ *region,"* taking $a_0(980)$ as a tetraquark.

• $D \to S_0$ form factor $f^+(q^2)$ plays a key role.

Extractable: $f^+(0)|V_{cs}| = 0.504 \pm 0.017 \pm 0.035$

in resonant decay channel $D_s^+ \to f_0 e^+ \nu_e, f_0 \to \pi^+\pi^-$

[BESIII, PRL132, 141901 (2024).]

• $SU(3)_f$ relations for $q\bar{q}$ and $q^2\bar{q}^2$,

leading to different mixing scenarios for a_0, f_0, and σ_0:

$|\theta| \simeq 20^\circ$ for $q\bar{q}$, $|\theta| < 10^\circ$ for $q^2\bar{q}^2$,

FFs for $q\bar{q}$ and $q^2\bar{q}^2$ can be very different.

- $SU(3)_f$ relations also applied to

1. $D \to S_0 P$ [H. Y. Cheng, PRD67, 034024 (2003)],

"Hadronic D decays involving scalar mesons,"

2. $D, B \to S_0 e^+ \nu$ [Wang, Lu, PRD82, 034016 (2010)],

"Distinguishing two kinds of scalar mesons from heavy meson decays,"

3. $\bar{B}^0_{(s)} \to J/\psi(f_0, \sigma_0)$ [Stone, Zhang, PRL (2013)], *"Use of* $B \to J/\psi f_0$ *decays to discern the* $q\bar{q}$ *or tetraquark nature of scalar mesons,"*

Extracted form factors

Form factor	$\lvert F^{D_s^+\to f_0}\rvert$	$\lvert F^{D_s^+\to\sigma_0}\rvert$	$\lvert F^{D^0\to a_0^-}\rvert$	$\lvert F^{D^+\to f_0}\rvert$	$\lvert F^{D^+\to\sigma_0}\rvert$
our work: $(q\bar{q})$	0.52 ± 0.02	0.22 ± 0.01	0.66 ± 0.03	0.19 ± 0.01	0.43 ± 0.02
our work: $(q^2\bar{q}^2)$	0.53 ± 0.02	0.037 ± 0.032	0.44 ± 0.03	0.28 ± 0.03	0.46 ± 0.03
Fit to $\mathcal{B}(D \to SP)$ [47]	0.52 ± 0.04			0.26 ± 0.02	0.42 ± 0.05
CQM [81]					0.57 ± 0.09
Fit to E791 data [82]					0.38 ± 0.06
CLFD (DR) [83]	0.45 (0.46)			0.21 (0.22)	
QCDSR [32]	0.50 ± 0.13			0.53 ± 0.15	
QCDSR [33]	0.46 ± 0.03			0.54 ± 0.05	
	(0.27 ± 0.02)			(0.32 ± 0.03)	
LFQM [84]	0.43			0.22	
CCQM [36]	0.39 ± 0.02		0.55 ± 0.02	0.45 ± 0.02	
LCSR [34]	0.30 ± 0.03				
LCSR [43]			0.88 ± 0.14		
LCSR [85]			$0.85^{+0.10}_{-0.11}$		
LCSR [86]			$1.070^{+0.066}_{-0.033}$		
AdS/QCD [87]			0.72 ± 0.09		

decay channel	our work: $(q\bar{q}, q^2\bar{q}^2)$	experimental data
$10^4\mathcal{B}(D_s^+ \to f_0 e^+\nu_e, f_0 \to \pi^+\pi^-)$	$(16.9 \pm 1.2 \pm 0.7,\ 17.2 \pm 1.3 \pm 0.7)$	17.1 ± 1.6 [48]
$10^4\mathcal{B}(D_s^+ \to f_0 e^+\nu_e, f_0 \to \pi^0\pi^0)$	$(8.4 \pm 0.6 \pm 0.4,\ 8.6 \pm 0.7 \pm 0.4)$	7.9 ± 1.4 [49]
$10^4\mathcal{B}(D_s^+ \to \sigma_0 e^+\nu_e, \sigma_0 \to \pi^+\pi^-)$	$(20.3 \pm 1.8 \pm 0.5,\ 0.58^{+1.43}_{-0.57} \pm 0.01)$	< 3.3 [48]
$10^4\mathcal{B}(D_s^+ \to \sigma_0 e^+\nu_e, \sigma_0 \to \pi^0\pi^0)$	$(10.7 \pm 1.0 \pm 0.3,\ 0.29^{+0.72}_{-0.29} \pm 0.01)$	< 7.3 [49] (< 1.7 [48])†
$10^4\mathcal{B}(D^0 \to a_0^- e^+\nu_e, a_0^- \to \pi^-\eta)$	$(1.8 \pm 0.2 \pm 0.2,\ 0.8 \pm 0.1 \pm 0.1)$	1.3 ± 0.3 [8]
$10^4\mathcal{B}(D^+ \to a_0^0 e^+\nu_e, a_0^0 \to \pi^0\eta)$	$(2.4 \pm 0.2 \pm 0.3,\ 1.0 \pm 0.1 \pm 0.1)$	1.7 ± 0.7 [8]
$10^5\mathcal{B}(D^+ \to f_0 e^+\nu_e, f_0 \to \pi^+\pi^-)$	$(1.3 \pm 0.1 \pm 0.1, 2.9 \pm 0.7 \pm 0.2)$	< 2.8 [88]
$10^4\mathcal{B}(D^+ \to \sigma_0 e^+\nu_e, \sigma_0 \to \pi^+\pi^-)$	$(5.4 \pm 0.5 \pm 0.1,\ 6.2 \pm 0.9 \pm 0.1)$	6.3 ± 0.5 [88]

- Resonant effects ($S_0 \to M_1 M_2$) are taken into account,

without using the narrow-width approximation.

- With predicted branching fractions, we test S_0 as a non-$q\bar{q}$ state.
- Molecule scenario provides suppressed branching fractions.

[T. Sekihara and E. Oset, PRD92, 054038 (2015).]

For example, $\mathcal{B}(D_s^+ \to \hat{f}_0 e^+\nu_e, \hat{f}_0 \to \pi^+\pi^-) < 2.5 \times 10^{-4}$,

unable to interpret the data [$(17.1 \pm 1.6) \times 10^{-4}$].

[Hsiao, Yang, Wei, Ke, JHEP 12 (2025) 226.]

Absence of κ resonance in $D \to K\pi\ell\nu_\ell$

branching fraction	data
$10^2\mathcal{B}(D^+ \to K^-\pi^+e^+\nu_e)$	3.77 ± 0.09 [12]
$10^3\mathcal{B}(D^+ \to (K^-\pi^+)_{\rm s-wave}e^+\nu_e)$	2.28 ± 0.11 [2, 12]
$10^2\mathcal{B}(D^+ \to \bar{K}^{*0}e^+\nu_e, \bar{K}^{*0} \to K^-\pi^+)$	3.54 ± 0.09 [12]
$10^3\mathcal{B}(D^+ \to \bar{\kappa}^0e^+\nu_e, \bar{\kappa}^0 \to K^-\pi^+)$	ignored [12]
$10^3\mathcal{B}(D^+ \to (K^-\pi^+)_{\rm NR}e^+\nu_e)$	2.28 ± 0.11 [12]

[2] pdg

[12] BESIII, PRD94, 032001 (2016).

- $f^+(q^2)$ of the $D \to \kappa$ transition not extractable.
- Indicating a serious $SU(3)_f$ breaking.

Absence of κ resonance in $D \to K\pi\ell\nu_\ell$

- $D^+ \to K^-\pi^+e^+\nu_e$ via $D \to K_J\ell\nu_\ell, K_J \to K\pi$

K_J: $\kappa/K^*(700)$, $K^*/K_0^*(892)$, $K^*(1410)$, $K_0^*(1430)$, $K_2^*(1430)$

[BaBar, PRD83, 072001 (2011).]

K^*: 95% of $\mathcal{B}_\mathrm{T} = (3.77 \pm 0.03 \pm 0.08) \times 10^{-2}$,

$\bar{K}^*(1410)^0$: $\mathcal{B} = (0 \pm 0.5) \times 10^{-5}$,

$\bar{K}_2^*(1430)^0$: $\mathcal{B} = (3.7 \pm 2.5) \times 10^{-5}$,

[BESIII, PRD94, 032001 (2016).]

$K_0^*(1430)$: $\mathcal{B} < 0.64\%$ of $\mathcal{B}_\mathrm{T}$ ($\mathcal{B} < 2.5 \times 10^{-4}$).

[FOCUS, PLB621, 72 (2005).]

- partial wave analysis:

p-wave components account for 95% of $\mathcal{B}_{\mathrm{T}}$.

s-wave component contributes to 5% of the total rate,

attributed to a non-resonant scalar component.

Since a simulation using the LASS parametrization of the non-resonant contribution is sufficient to reproduce the data, we exclude a possible κ contribution from further consideration. [FOCUS, PLB621, 72 (2005).]

space available. We also assume the contribution from the κ to be negligible, as follows from the FOCUS results [23]. Possible contributions from the $\bar{K}^*(1410)^0$ and $\bar{K}_2^*(1430)^0$ are searched. [BESIII, PRD94, 032001 (2016).]

- κ resonance systematically excluded.

1. κ remains the most elusive menber of the light scalar nonet.

2. recently confirmed through $\pi K \to \pi K$ and $\pi\pi \to K\bar{K}$.

[Peláez and A. Rodas, PRL124, 172001 (2020).]

3. Any κ contribution must manifest in the s-wave channel,

and thus warrants careful investigation.

4. To address this, we perform a partial-wave analysis,

incorporating $D \to K\pi\pi, K\pi\rho$.

A theoretical re-examination

- Amplitude of $D \to K\pi e\nu_e$

$$\mathcal{M} = \frac{G_F}{\sqrt{2}} V_{cs} \langle K\pi|(\bar{s}c)_{V-A}|D\rangle (\bar{u}_\nu v_e)_{V-A}$$

- Resonant matrix elements

$$\langle K\pi|(\bar{s}c)_{V-A}|D\rangle_{\rm RE} = \langle K\pi|K_J\rangle D^{-1}_{K_J} \langle K_J|(\bar{s}c)_{V-A}|D\rangle$$

1. K^* contribution:

$$\langle K^*|(\bar{s}c)_{V-A}|D\rangle = \epsilon_{\mu\alpha\beta\gamma}\varepsilon^\alpha p_D^\beta p_{K^*}^\gamma \frac{2V}{m_D+m_{K^*}} - i\left[\varepsilon_\mu - \frac{\varepsilon\cdot p_D}{q^2} q_\mu\right](m_D + m_{K^*})A_1$$
$$-i\frac{\varepsilon\cdot p_D}{q^2} q_\mu (2m_{K^*})A_0 + i\left[(p_D + p_{K^*})_\mu - \frac{m_D^2 - m_{K^*}^2}{q^2} q_\mu\right](\varepsilon\cdot p_D)\frac{A_2}{m_D+m_{K^*}},$$

$$D_{K^*} = m_{K^*}^2 - t - i\, m_{K^*}\Gamma_{K^*}(t),$$

$$\langle K\pi|K^*\rangle = g_{K^*K\pi}\varepsilon\cdot(p_K - p_\pi).$$

2. κ contribution:

$$\langle\kappa|(\bar{s}c)_{V-A}|D\rangle = if^{+}(p_D+p_\kappa)_\mu + i(f^0-f^+)\frac{m_D^2-m_\kappa^2}{q^2}q_\mu,$$

$$D_\kappa = m_\kappa^2 - t - \Gamma_\kappa^2(t)/4 - i\,m_\kappa\Gamma_\kappa(t),$$

$$\langle K\pi|\kappa\rangle = g_{\kappa K\pi}.$$

- Non-resonant matrix element

$$\langle K\pi|(\bar{s}c)_{V-A}|D\rangle_{\mathrm{NR}} = h\epsilon_{\mu\alpha\beta\gamma}p_D^\alpha(p_K+p_\pi)^\beta(p_K-p_\pi)^\gamma$$

$$+ir(p_D-p_K-p_\pi)_\mu + iw_+(p_K+p_\pi)_\mu + iw_-(p_K-p_\pi)_\mu.$$

partial-wave analysis

$$d\Gamma = \frac{1}{3}\frac{G_F^2|V_{cs}|^2}{(4\pi)^5 m_D^3} X\alpha_{\mathbf{M}}\alpha_{\mathbf{L}}\left\{2|F_{10}|^2 + \frac{2}{3}\left[|F_{11}|^2 + |F_{21}|^2 + |F_{31}|^2\right]\right\}dsdt,$$

$$F_{10} = X\left[\left(w_+ + \frac{m_K^2-m_\pi^2}{t}w_-\right) + 2\hat{f}^+\right],$$

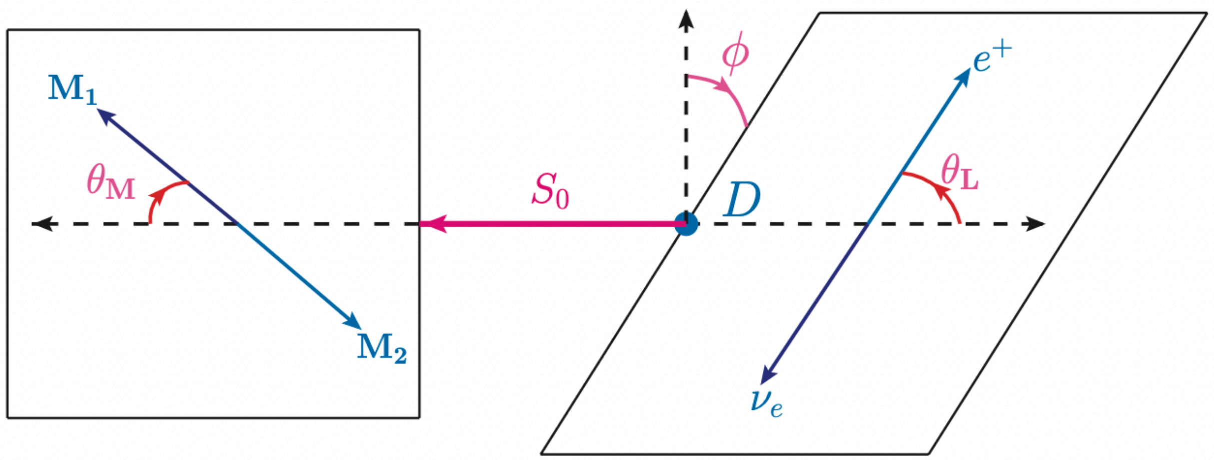
$$F_{11} = \alpha_{\mathbf{M}}\left\{\frac{1}{2}(m_D^2 - s - t)[w_- + (m_D + m_{K^*})\hat{A}_1] - 2X^2\frac{\hat{A}_2}{m_D+m_{K^*}}\right\},$$

$$F_{21} = \alpha_{\mathbf{M}}\sqrt{2st}[w_- + (m_D + m_{K^*})\hat{A}_1],$$

$$F_{31} = \alpha_{\mathbf{M}}X\sqrt{2st}\left(h - \frac{2\hat{V}}{m_D+m_{K^*}}\right).$$

C. L. Y. Lee, M. Lu and M. B. Wise,

"B(ℓ4) and D(ℓ4) decay," PRD46, 5040 (1992).



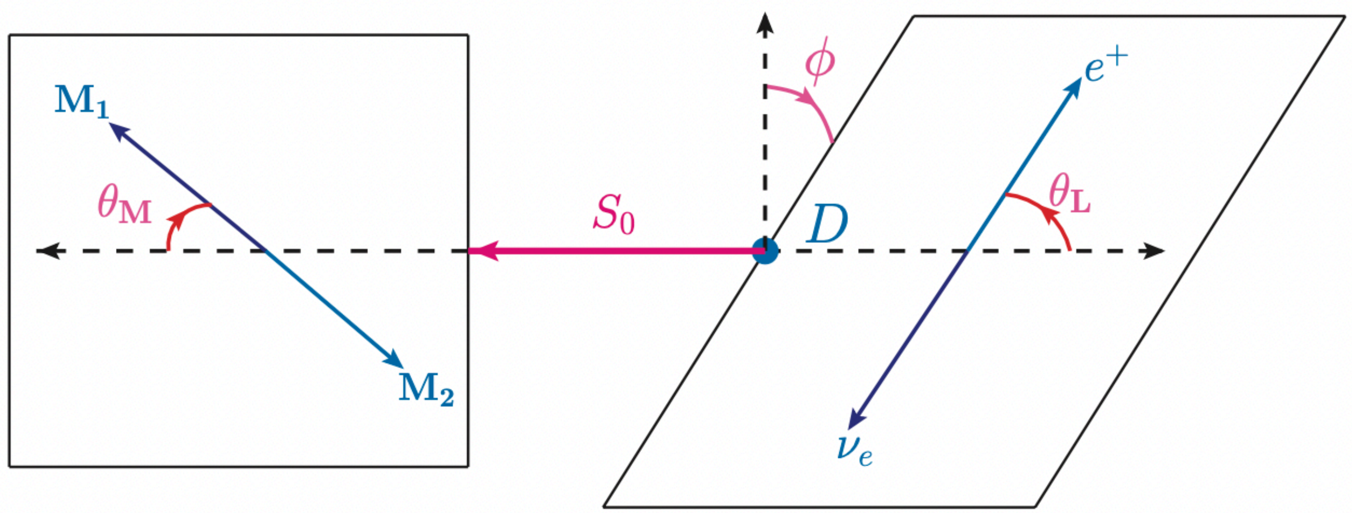
Phase space $D(p_D) \to S_0(p_{S_0})e^+(p_e)\nu(p_\nu),\, S_0(p_{S_0}) \to M_1(p_1)M_2(p_2)$

$$d\Gamma = \frac{|\bar{\mathcal{M}}|^2}{4(4\pi)^6 m_D^3} X \alpha_{\mathbf{M}} \alpha_{\mathbf{L}}\, ds\, dt\, d\cos\theta_{\mathbf{M}}\, d\cos\theta_{\mathbf{L}}\, d\phi$$

$$X = [(m_D^2 - s - t)^2/4 - st]^{1/2},$$

$$\alpha_{\mathbf{M}} = \lambda^{1/2}(t, m_1^2, m_2^2)/t,\ \alpha_{\mathbf{L}} = \lambda^{1/2}(s, m_e^2, m_\nu^2)/s$$

$$\lambda(a,b,c) = a^2 + b^2 + c^2 - 2ab - 2bc - 2ca.$$



The allowed regions of the variables:

$(m_e + m_\nu)^2 \le s \le (m_D - \sqrt{t})^2,$

$(m_1 + m_2)^2 \le t \le (m_D - m_e - m_\nu)^2,$

$0 \le \theta_{\mathbf{M,L}} \le \pi$, and $0 \le \phi \le 2\pi$.

- Form factors

$(\hat{V}, \hat{A}_1, \hat{A}_2) = g_{K^*K\pi} D_{K^*}^{-1} \times (V, A_1, A_2)$,

$\hat{f}^+ = g_{\kappa K\pi} D_\kappa^{-1} \times f^+$,

$F_{\rm NR}$: $w_\pm$ and h.

- Momentum dependence:

$V(q^2) = \frac{V(0)}{1-\frac{q^2}{M_1^2}}$, $A_1(q^2) = \frac{A_1(0)}{1-\frac{q^2}{M_2^2}}$, $A_2(q^2) = \frac{A_2(0)}{1-\frac{q^2}{M_2^2}}$,

$f^+(q^2) = \frac{f(0)}{(1-\frac{q^2}{m_A^2})^n}$, $w_\pm(q^2) = \frac{w_\pm(0)}{(1-\frac{q^2}{m_A^2})^n}$, $h(q^2) = \frac{h(0)}{(1-\frac{q^2}{m_V^2})^n}$,

$n = 1$ (single pole) or $n = 2$ (double pole).

V, A_1, and A_2, experimentally extracted.

f^+ and $(w_\pm, h)$, to be determined in this study.

• Non-leptonic decay channels $D \to K\pi\pi$ and $D \to K\pi\rho$

$\mathcal{M}_{\pi,\rho} \simeq \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} a_1 \langle \pi, \rho | (\bar{u}d)_{V-A} | 0 \rangle \langle K\pi | (\bar{s}c)_{V-A} | D \rangle$,

$\langle \pi, \rho | (\bar{u}d)_{V-A} | 0 \rangle = (i f_\pi p_\pi^\mu, m_\rho f_\rho \varepsilon^\mu)$,

branching fraction	data
$10^2 \mathcal{B}(D^+ \to \pi^+ (K^- \pi^+)_{\rm NR})$	1.2 ± 0.7 [2, 41]
$10^2 \mathcal{B}(D^+ \to \pi^+ \bar{\kappa}^0, \bar{\kappa}^0 \to K^- \pi^+)$	4.5 ± 1.2 [2, 41]
$10^2 \mathcal{B}(D^0 \to \pi^+ (K^- \pi^0)_{\rm NR})$	1.15 ± 0.60 [2]
$10^2 \mathcal{B}(D^+ \to \pi^+ (K_S^0 \pi^0)_{\rm NR} + \pi^+ (\bar{\kappa}^0 \to) K_S^0 \pi^0)$	1.37 ± 0.40 [2, 8]
$10^3 \mathcal{B}(D^+ \to \rho^+ (K_S^0 \pi^0)_{\rm NR})$	4.8 ± 0.5 [2]

[2] pdg

[8] BESIII, PRD89, 052001 (2014).

[41] E791, PRL89, 121801 (2002).

- χ^2-fit

$$\chi^2 = \Sigma_i[(\mathcal{B}^i_{\rm th} - \mathcal{B}^i_{\rm ex})/\delta\mathcal{B}^i_{\rm ex}]^2 + \Sigma_j[(F^j_{\rm th} - F^j_{\rm ex})(\delta F^j_{\rm ex})]^2,$$

$$(V(0), A_1(0), A_2(0)) = (0.83 \pm 0.05, 0.59 \pm 0.02, 0.47 \pm 0.03),$$

$$(M_1, M_2) = (2.0, 2.6)\ \text{GeV}.$$

- Assumption

$$w(0) = w_\pm(0),$$

δ_f: a relative phase between f^+ and $F_{\rm NR}$.

- fit results

$$f(0) = 0.38 \pm 0.07,\ \delta_f = (31.7 \pm 37.3)^\circ,$$

$$w(0) = (2.34 \pm 0.64)\ \text{GeV}^{-1},\ h(0) = (8.86 \pm 1.00)\ \text{GeV}^{-3}.$$

$$\chi^2/\text{n.d.f.} = 0.7\ (\text{n.d.f.} = 3).$$

branching fraction	our work	data
$10^2\mathcal{B}(D^+ \to K^-\pi^+e^+\nu_e)$	3.8 ± 0.2	$(3.77 \pm 0.09)^*$ [12]
$10^3\mathcal{B}(D^+ \to (K^-\pi^+)_{\text{s-wave}}e^+\nu_e)$	2.3 ± 1.4	$(2.28 \pm 0.11)^*$ [2, 12]
$10^2\mathcal{B}(D^+ \to \bar{K}^{*0}e^+\nu_e, \bar{K}^{*0} \to K^-\pi^+)$	3.5 ± 0.1	3.54 ± 0.09 [12]
$10^3\mathcal{B}(D^+ \to \bar{\kappa}^0e^+\nu_e, \bar{\kappa}^0 \to K^-\pi^+)$	2.7 ± 1.1	ignored [12]
$10^3\mathcal{B}(D^+ \to (K^-\pi^+)_{\rm NR}e^+\nu_e)$	0.7 ± 0.2	2.28 ± 0.11 [12]
$10^2\mathcal{B}(D^+ \to \pi^+(K^-\pi^+)_{\rm NR})$	$1.0 \pm 0.5 \pm 0.2$	$(1.2 \pm 0.7)^*$ [2, 41]
$10^2\mathcal{B}(D^+ \to \pi^+\bar{\kappa}^0, \bar{\kappa}^0 \to K^-\pi^+)$	$4.3 \pm 1.8 \pm 0.8$	$(4.5 \pm 1.2)^*$ [2, 41]
$10^2\mathcal{B}(D^0 \to \pi^+(K^-\pi^0)_{\rm NR})$	$0.4 \pm 0.2 \pm 0.1$	$(1.15 \pm 0.60)^*$ [2]
$10^2\mathcal{B}(D^+ \to \pi^+(K^0_S\pi^0)_{\rm NR} + \pi^+(\bar{\kappa}^0 \to)K^0_S\pi^0)$	$1.7 \pm 0.7 \pm 0.3$	$(1.37 \pm 0.40)^*$ [2, 8]
$10^3\mathcal{B}(D^+ \to \pi^+(K^0_S\pi^0)_{\rm NR})$	$2.5 \pm 1.2 \pm 0.5$	3 ± 4 [2, 8]
$10^3\mathcal{B}(D^+ \to \pi^+\bar{\kappa}^0, \bar{\kappa}^0 \to K^0_S\pi^0)$	$10.7 \pm 4.5 \pm 2.0$	6^{+5}_{-4} [2, 8]
$10^3\mathcal{B}(D^+ \to \rho^+(K^0_S\pi^0)_{\rm NR})$	$4.8 \pm 1.0 \pm 0.9$	$(4.8 \pm 0.5)^*$ [2]

- s-wave contributions:

1. $\mathcal{B}(D^+ \to (K^-\pi^+)_{\text{s-wave}}e^+\nu_e) = (2.3 \pm 1.4) \times 10^{-3}$,

agreeing with the data.

2. $\mathcal{B}(D^+ \to \bar{\kappa}^0e^+\nu_e, \bar{\kappa}^0 \to K^-\pi^+) = (2.7 \pm 1.1) \times 10^{-3}$,

which could be in-negligible.

3. $\mathcal{B}(D^+ \to (K^-\pi^+)_{\rm NR}e^+\nu_e) = (0.7 \pm 0.2) \times 10^{-3}$,

which could not fully interpret the s-wave branching fraction.

- In comparison with the $SU(3)_f$ results

[Hsiao, Yang, Wei, Ke, JHEP 12 (2025) 226.]

$$F_{q\bar{q}}^{D^+\to\bar{\kappa}^0} = \sqrt{2}(f_K/f_\pi)F^{D^+\to S_n},\ S_n = |(u\bar{u}+d\bar{d})/\sqrt{2}\rangle,$$

$$F_{q^2\bar{q}^2}^{D^+\to\bar{\kappa}^0} = \sqrt{2}(f_\pi/f_K)F^{D^+\to S_{ns}},\ S_{ns} = |(u\bar{u}+d\bar{d})s\bar{s}/\sqrt{2}\rangle,$$

$f_K/f_\pi = 1.2$ and $f_\pi/f_K = 0.8$.

Using $F^{D^+\to S_n} = 0.47 \pm 0.02$ and $F^{D^+\to S_{ns}} = 0.31 \pm 0.02$,

we obtain $F_{q\bar{q}}^{D^+\to\bar{\kappa}^0} = 0.82 \pm 0.05$, $F_{q^2\bar{q}^2}^{D^+\to\bar{\kappa}^0} = 0.36 \pm 0.02$.

The tetraquark scenario agrees with our extraction:

$f(0) = 0.38 \pm 0.07$.

Summary

• Using the information on semi-leptonic and non-leptonic D decays,

we have extracted the form factor f^+ of the $D^+ \to \bar{\kappa}^0$ transition.

• Its value $f(0) = 0.38 \pm 0.07$ agrees with $F_{q^2\bar{q}^2}^{D^+\to\bar{\kappa}^0} = 0.36 \pm 0.02$

obtained from the tetraqurk scenario under the $SU(3)_f$ relations.

• We thus obtained $\mathcal{B}(D^+ \to \bar{\kappa}^0 e^+ \nu_e, \bar{\kappa}^0 \to K^- \pi^+) = (2.7 \pm 1.1) \times 10^{-3}$,

previously neglected in measurements.

• $\mathcal{B}(D^+ \to (K^- \pi^+)_{\rm NR} e^+ \nu_e) = (0.7 \pm 0.2) \times 10^{-3}$

has been determined as well,

not fully interpret the s-wave branching fraction.

Thank You