



第22届重味物理与CP破坏研讨会

Study of the decays $D^0 \rightarrow K^+ K^- \eta$ and $\pi^+ \pi^- \eta$ with the role of $a_0(980)$

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arXiv: [2503.02224](https://arxiv.org/abs/2503.02224)

2025.10. 北京

Outline

1. Introduction
2. Formalism
3. Results
4. Summary

§ 1. Introduction

$a_0(980)$

$$I^G(J^{PC}) = 1^-(0^{++})$$



VALUE (MeV)

980 ± 20 OUR ESTIMATE

$\eta\pi$ FINAL STATE ONLY

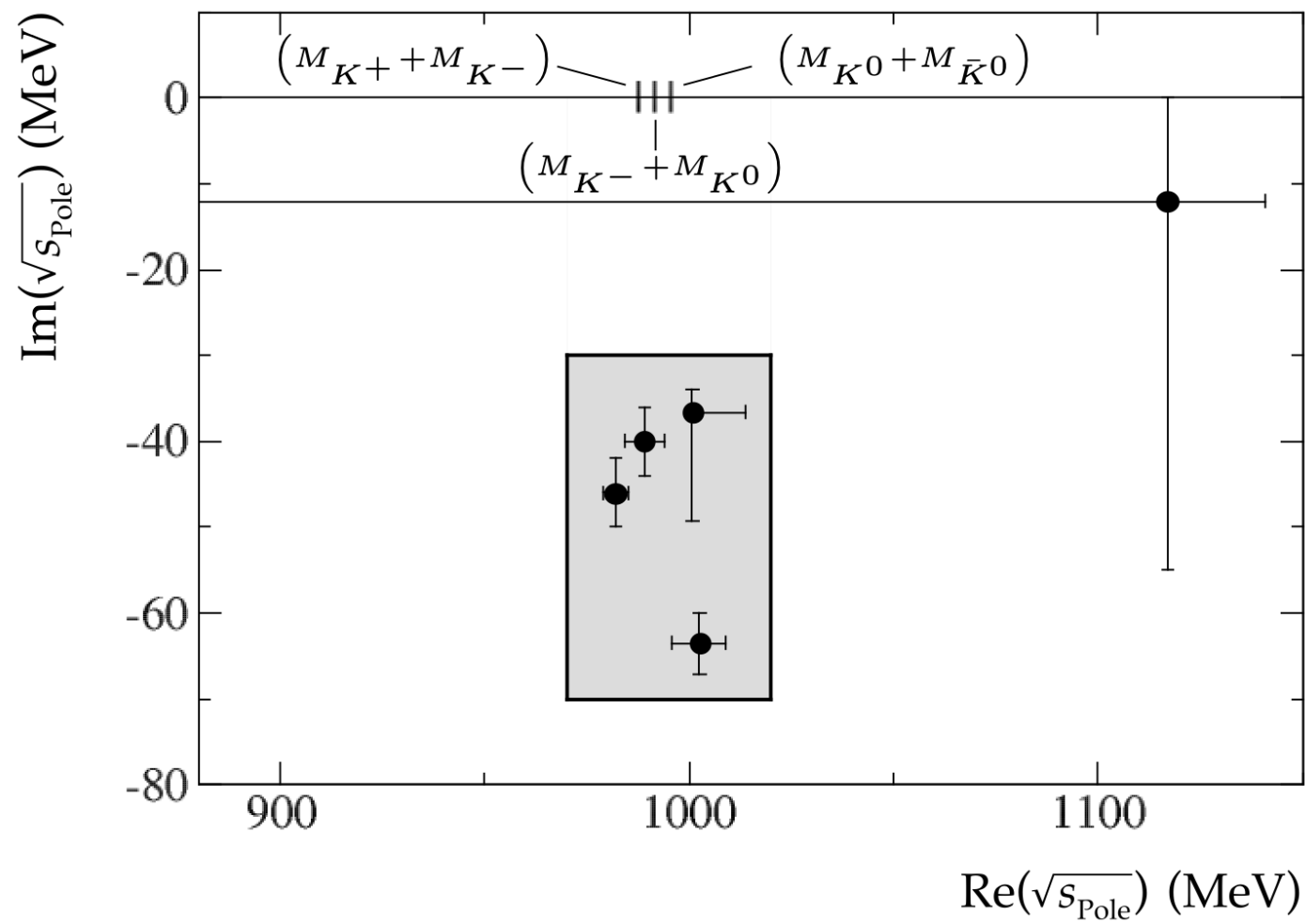
VALUE (MeV)	EVTs	DOCUMENT ID	TECN	CHG	COMMENT
50 to 100 OUR ESTIMATE		Width determination very model dependent. Peak width in $\eta\pi$ is about 60 MeV, but decay width can be much larger.			



- A. Astier, L. Montanet, M. Baubillier and J. Duboc, Phys. Lett. B 25, 294 (1967).
 R. Ammar et al., Phys. Rev. Lett. 21, 1832 (1968).
 C. Defoix, P. Rivet, J. Siaud, B. Conforto, M. Widgoff and F. Shively, Phys. Lett. B 28, 353 (1968).

64. Scalar Mesons below 1 GeV

Revised August 2023 by T. Gutsche (Tübingen U.), C. Hanhart (FZ Jülich), R.E. Mitchell (Indiana U.) and S. Spanier (Tennessee U.).



$$\sqrt{s_{\text{Pole}}}^{a_0(980)} = (970 - 1020) - i(30 - 70) \text{ MeV}$$



Conventional scalar $q\bar{q}$ states:

S. Godfrey and N. Isgur, Phys. Rev. D 32, 189-231 (1985).

D. Morgan and M. R. Pennington, Phys. Rev. D 48, 1185-1204 (1993).

N. A. Tornqvist and M. Roos, Phys. Rev. Lett. 76, 1575-1578 (1996).

Tetraquarks, composed of four quarks $qq\bar{q}\bar{q}$ states:

R. L. Jaffe, Phys. Rev. D 15, 267 (1977).

J. D. Weinstein and N. Isgur, Phys. Rev. Lett. 48, 659 (1982).

N. N. Achasov, Nucl. Phys. A 728, 425-438 (2003).

Molecular state, composed of $K\bar{K}$:

B. S. Zou and D. V. Bugg, Phys. Rev. D 50, 591-594 (1994).

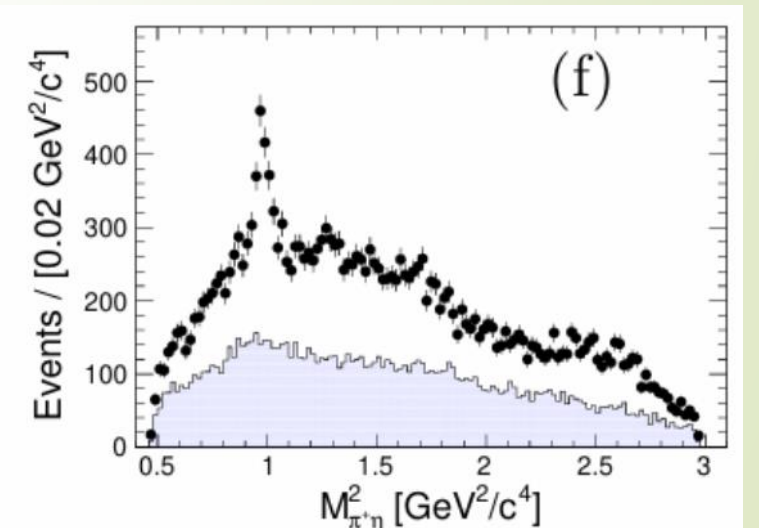
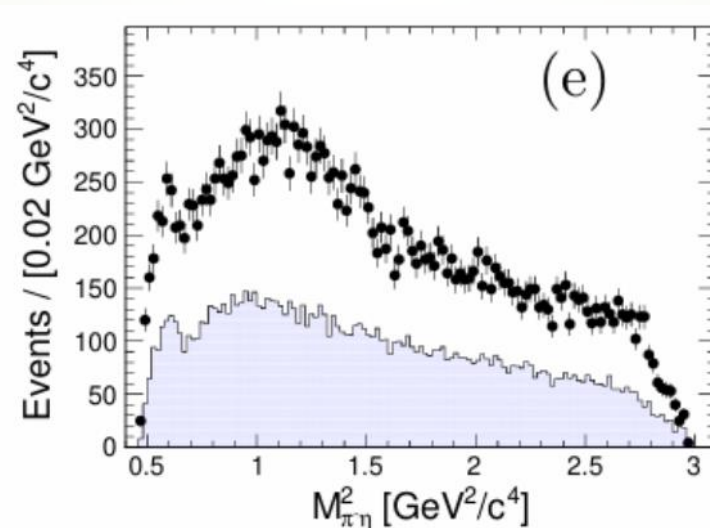
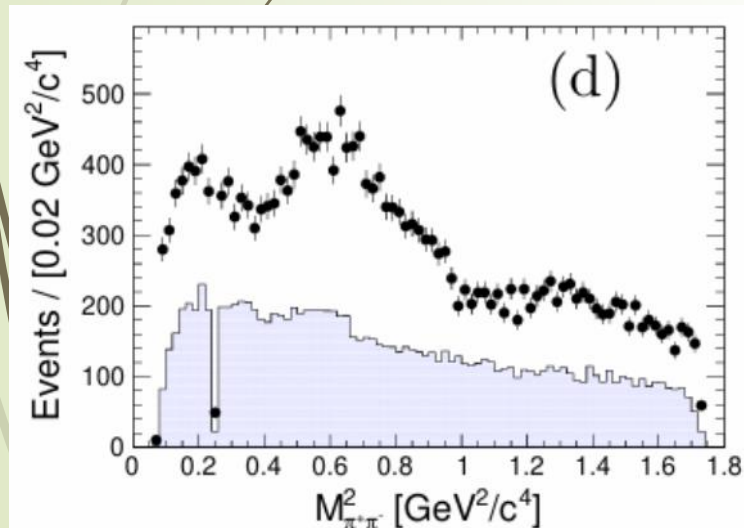
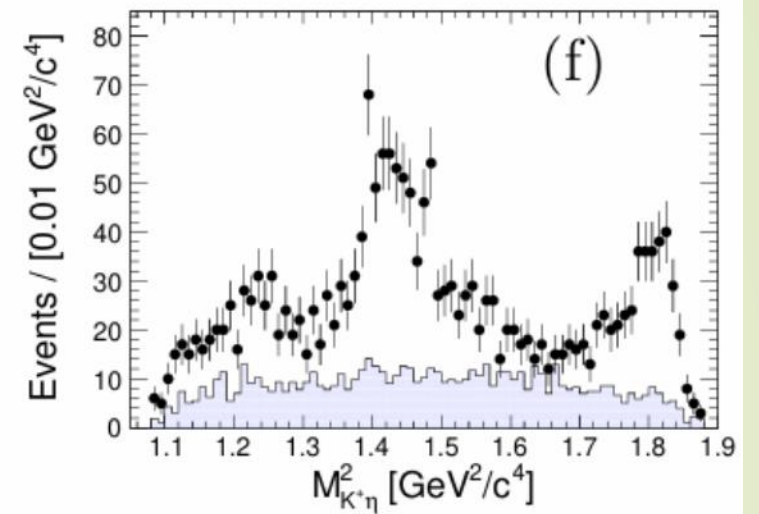
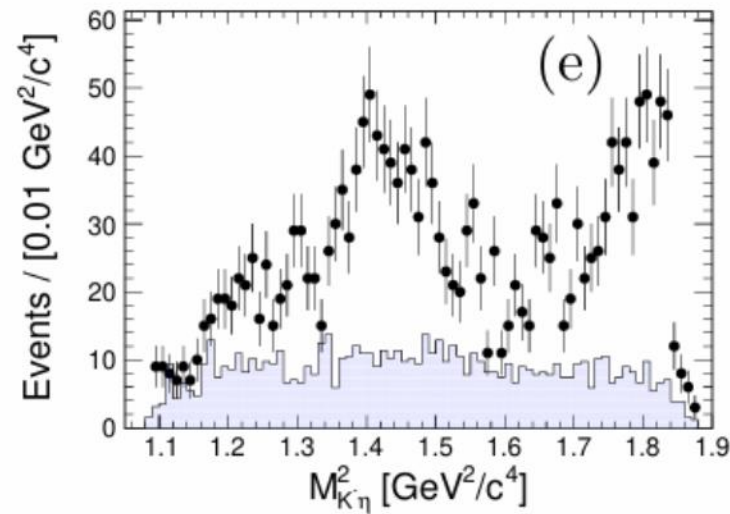
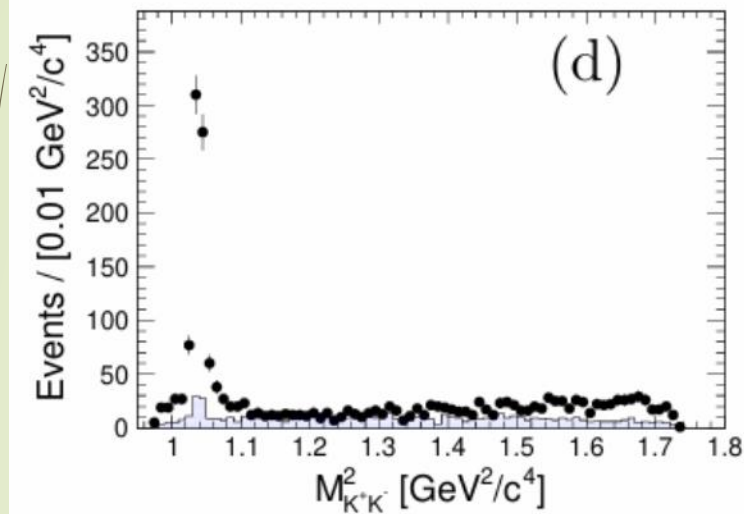
G. Janssen, B. C. Pearce, K. Holinde and J. Speth, Phys. Rev. D 52, 2690-2700 (1995).

J. A. Oller and E. Oset, Nucl. Phys. A 620, 438-456 (1997) [erratum: Nucl. Phys. A 652, 407-409 (1999)].

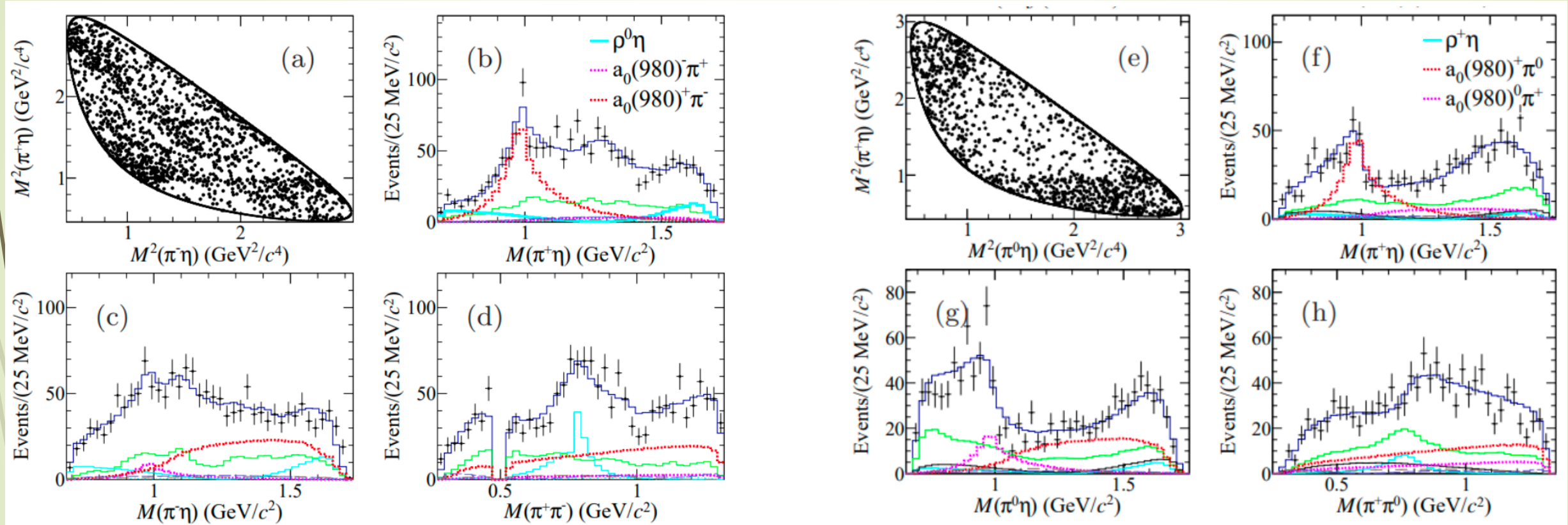
M. P. Locher, V. E. Markushin and H. Q. Zheng, Eur. Phys. J. C 4, 317-326 (1998).

J. A. Oller, E. Oset and J. R. Pelaez, Phys. Rev. Lett. 80, 3452-3455 (1998).

$$D^0 \rightarrow K^+ K^- \eta, \quad \pi^+ \pi^- \eta$$



$$D^0 \rightarrow \pi^+ \pi^- \eta, \quad D^+ \rightarrow \pi^+ \pi^0 \eta$$



M. Ablikim et al. [BESIII], Phys. Rev. D 110, L111102 (2024)

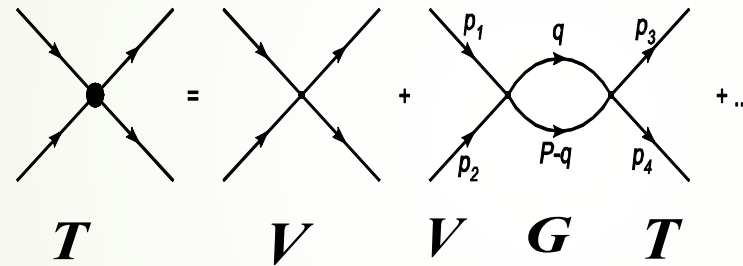
What can we learn from the invariant mass distributions of the experimental measurement?

§2. Formalism

(1) Coupled channel interaction from the chiral unitary approach

- **Chiral Unitary Approach**: solving Bethe-Salpeter equations, which take on-shell approximation for the loops.

$$T = V + V G T, \quad T = [1 - V G]^{-1} V$$



D. L. Yao, L. Y. Dai, H. Q. Zheng
and Z. Y. Zhou, Rept. Prog. Phys.
84, 076201 (2021)

where **V** matrix (potentials) can be evaluated from the interaction Lagrangians.

$I = 0$ sector

$$\pi^+ \pi^-, \pi^0 \pi^0, K^+ K^-, K^0 \bar{K}^0, \eta \eta$$

$I = 1$ sector

$$K^+ K^-, K^0 \bar{K}^0, \pi^0 \eta$$

$I = 1/2$ sector

$$K^+ \pi^-, K^0 \pi^0, K^0 \eta$$

J. A. Oller and E. Oset, Nucl. Phys. A
620 (1997) 438

E. Oset and A. Ramos, Nucl. Phys. A
635 (1998) 99

J. A. Oller and U. G. Meißner, Phys.
Lett. B 500 (2001) 263

G is a diagonal matrix with the loop functions of each channels:

$$G_{ll}(s) = i \int \frac{d^4 q}{(2\pi)^4} \frac{1}{(P-q)^2 - m_{l1}^2 + i\varepsilon} \frac{1}{q^2 - m_{l2}^2 + i\varepsilon}$$

The coupled channel scattering amplitudes **T matrix satisfy the unitary** :

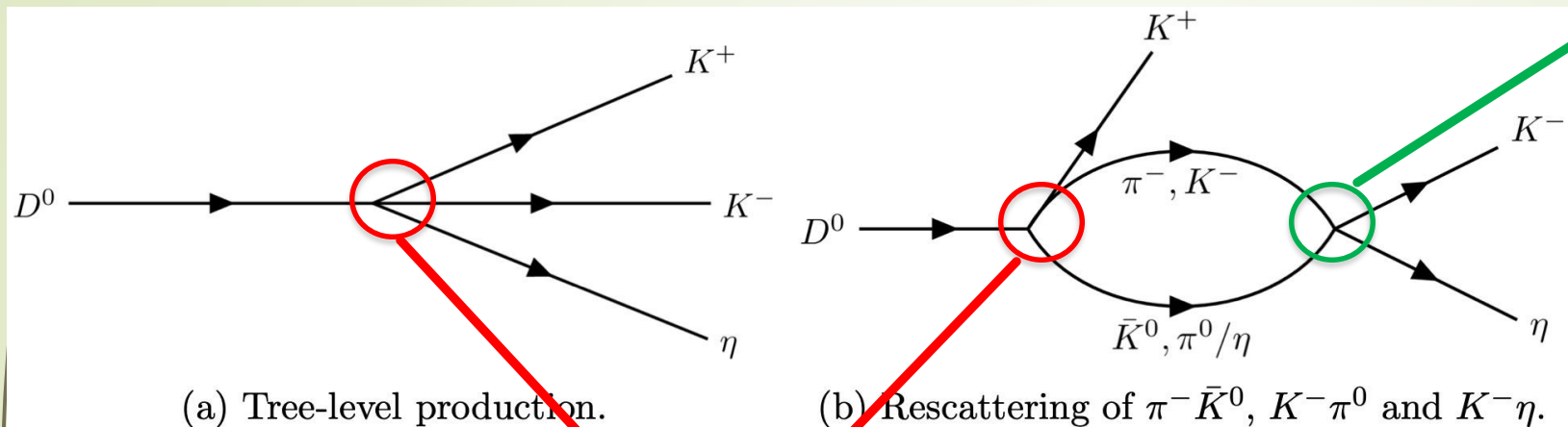
$$\text{Im } T_{ij} = T_{in} \sigma_{nn} T_{nj}^*$$

$$\sigma_{nn} \equiv \text{Im } G_{nn} = - \frac{q_{cm}}{8\pi\sqrt{s}} \theta(s - (m_1 + m_2)^2)$$

To search the poles of the resonances, we should extrapolate the scattering amplitudes to the second Riemann sheets:

$$G_{ll}^{II}(s) = G_{ll}^I(s) + i \frac{q_{cm}}{4\pi\sqrt{s}}$$

(2) Final state interaction

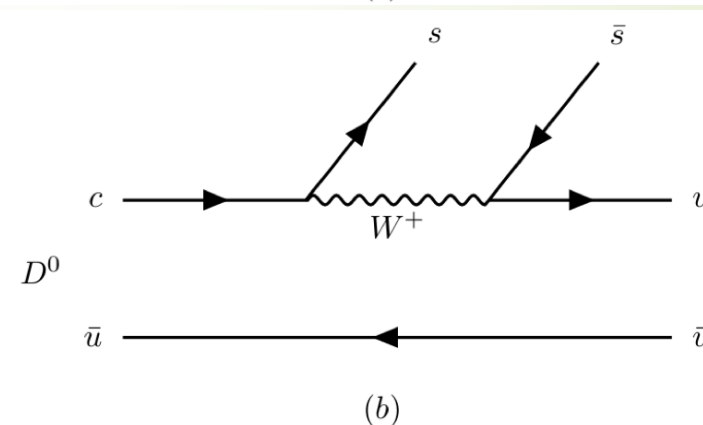
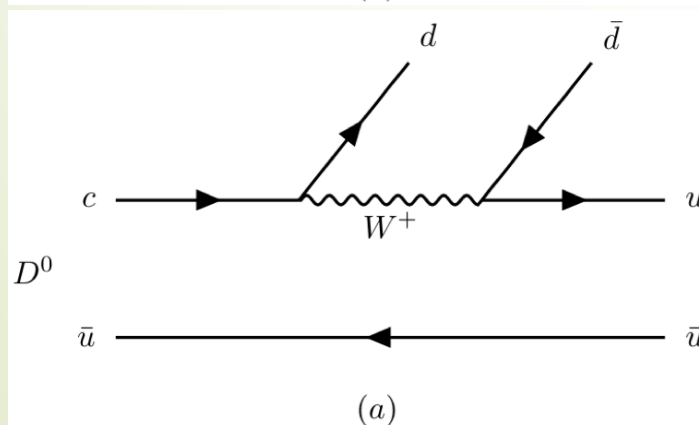
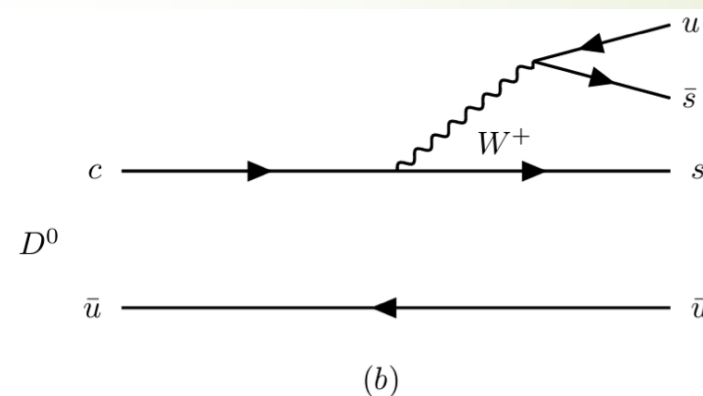
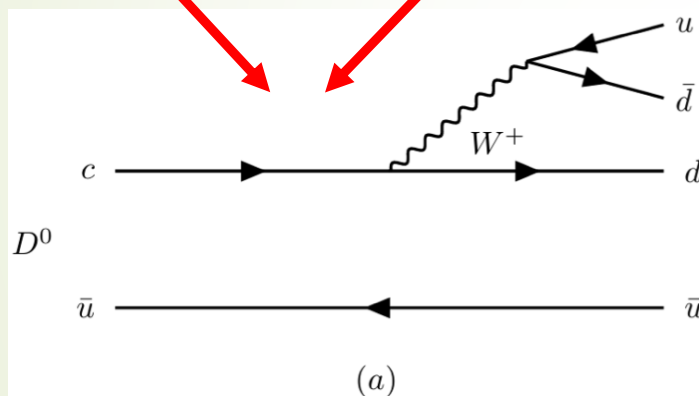


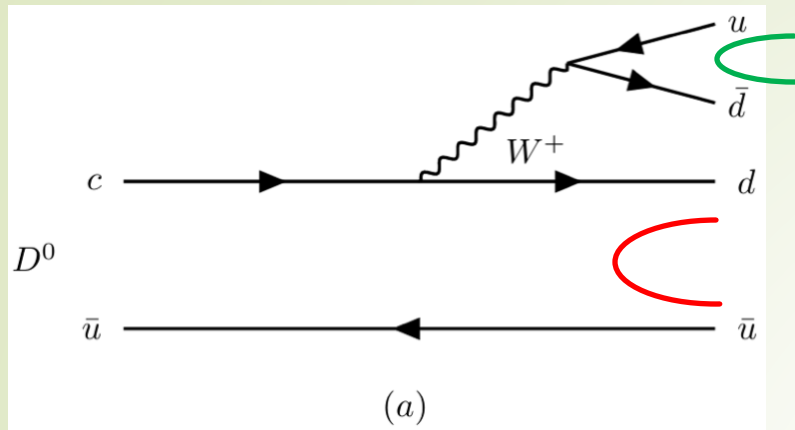
$$T = [1 - VG]^{-1} V$$

S-wave

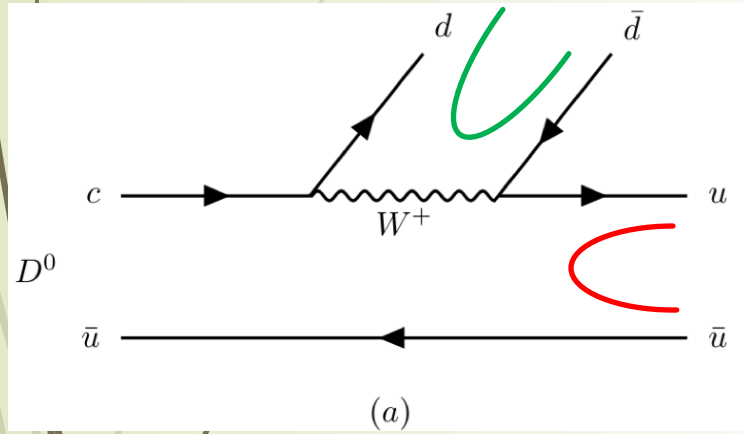
The final state interaction at the hadron level

The weak decay process at the quark level





$$\begin{aligned}
 H^{(1a)} &= V_P V_{cd} V_{ud} \{ (u\bar{d} \rightarrow \pi^+) [\underline{d\bar{u} \rightarrow d\bar{u} (u\bar{u} + d\bar{d} + s\bar{s})}] \\
 &\quad + (d\bar{u} \rightarrow \pi^-) [\underline{u\bar{d} \rightarrow u\bar{d} (u\bar{u} + d\bar{d} + s\bar{s})}] \} \\
 &= V_P V_{cd} V_{ud} \{ (u\bar{d} \rightarrow \pi^+) [M_{21} \rightarrow (M \cdot M)_{21}] \\
 &\quad + (d\bar{u} \rightarrow \pi^-) [(M \cdot M)_{12}] \},
 \end{aligned}$$



$$\begin{aligned}
 H^{(2a)} &= \beta V_P V_{cd} V_{ud} \left[\left(d\bar{d} \rightarrow -\frac{1}{\sqrt{2}} \pi^0 \right) [\underline{u\bar{u} \rightarrow u\bar{u} (u\bar{u} + d\bar{d} + s\bar{s})}] \right. \\
 &\quad + \left(d\bar{d} \rightarrow \frac{1}{\sqrt{6}} \eta \right) [\underline{u\bar{u} \rightarrow u\bar{u} (u\bar{u} + d\bar{d} + s\bar{s})}] \\
 &\quad + \left(u\bar{u} \rightarrow \frac{1}{\sqrt{2}} \pi^0 \right) [\underline{d\bar{d} \rightarrow d\bar{d} (u\bar{u} + d\bar{d} + s\bar{s})}] \\
 &\quad \left. + \left(u\bar{u} \rightarrow \frac{1}{\sqrt{6}} \eta \right) [\underline{d\bar{d} \rightarrow d\bar{d} (u\bar{u} + d\bar{d} + s\bar{s})}] \right] \\
 &= \beta V_P V_{cd} V_{ud} \left[\left(d\bar{d} \rightarrow -\frac{1}{\sqrt{2}} \pi^0 \right) [M_{11} \rightarrow (M \cdot M)_{11}] \right. \\
 &\quad + \left(d\bar{d} \rightarrow \frac{1}{\sqrt{6}} \eta \right) [M_{11} \rightarrow (M \cdot M)_{11}] \\
 &\quad + \left(u\bar{u} \rightarrow \frac{1}{\sqrt{2}} \pi^0 \right) [M_{22} \rightarrow (M \cdot M)_{22}] \\
 &\quad \left. + \left(u\bar{u} \rightarrow \frac{1}{\sqrt{6}} \eta \right) [M_{22} \rightarrow (M \cdot M)_{22}] \right],
 \end{aligned}$$

$$M = \begin{pmatrix} u\bar{u} & u\bar{d} & u\bar{s} \\ d\bar{u} & d\bar{d} & d\bar{s} \\ s\bar{u} & s\bar{d} & s\bar{s} \end{pmatrix} = \begin{pmatrix} \frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & \pi^+ & K^+ \\ \pi^- & -\frac{1}{\sqrt{2}}\pi^0 + \frac{1}{\sqrt{6}}\eta & K^0 \\ K^- & \bar{K}^0 & -\frac{2}{\sqrt{6}}\eta \end{pmatrix}$$



$$(M \cdot M)_{12} = \frac{2}{\sqrt{6}}\pi^+\eta + K^+\bar{K}^0$$

$$(M \cdot M)_{21} = \frac{2}{\sqrt{6}}\pi^-\eta + K^0K^-$$

$$(M \cdot M)_{11} = \frac{1}{2}\pi^0\pi^0 + \frac{1}{6}\eta\eta + \frac{1}{\sqrt{3}}\pi^0\eta + \pi^+\pi^- + K^+K^-$$

$$(M \cdot M)_{22} = \pi^+\pi^- + \frac{1}{2}\pi^0\pi^0 + \frac{1}{6}\eta\eta - \frac{1}{\sqrt{3}}\pi^0\eta + K^0\bar{K}^0$$



$$H^{(1a)} = V_P V_{cd} V_{ud} \left(\frac{4}{\sqrt{6}} \pi^+\pi^-\eta + \pi^+K^0K^- + \pi^-K^+\bar{K}^0 \right)$$

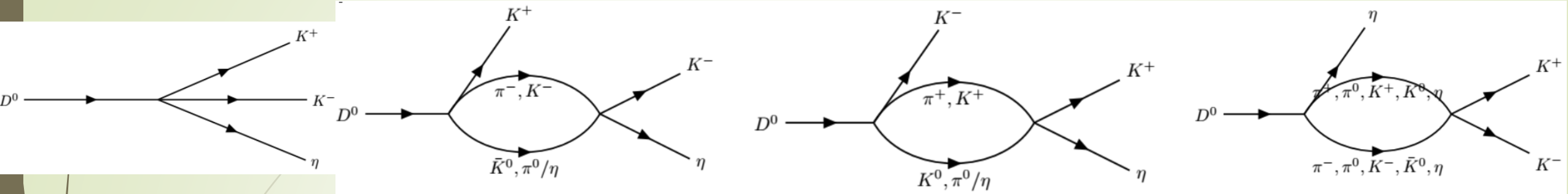
$$H^{(2a)} = \beta V_P V_{cd} V_{ud} \left(\frac{1}{3\sqrt{6}}\eta\eta\eta - \frac{1}{\sqrt{2}}\pi^0K^+K^- + \frac{2}{\sqrt{6}}\pi^+\pi^-\eta + \frac{1}{\sqrt{6}}K^+K^-\eta \right. \\ \left. - \frac{1}{\sqrt{6}}\pi^0\pi^0\eta + \frac{1}{\sqrt{2}}\pi^0K^0\bar{K}^0 + \frac{1}{\sqrt{6}}K^0\bar{K}^0\eta \right),$$

$D^0 \rightarrow \pi^+ \pi^- \eta$ and $K^+ K^- \eta$, having

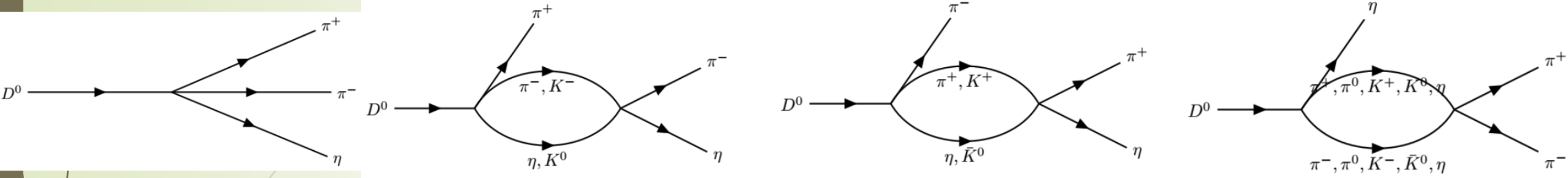


$$H = H^{(1a)} + H^{(1b)} + H^{(2a)} + H^{(2b)}$$

$$\begin{aligned} &= \frac{2}{\sqrt{6}}(2C_1 + \beta C_1 + \beta C_2) \pi^+ \pi^- \eta + (C_1 - C_2) \pi^+ K^0 K^- + (C_1 - C_2) \pi^- K^+ \bar{K}^0 \\ &\quad - \frac{1}{\sqrt{2}}(2C_2 + \beta C_1 + \beta C_2) K^+ K^- \pi^0 + \frac{1}{\sqrt{6}}(2C_2 + \beta C_1 + \beta C_2) K^+ K^- \eta \\ &\quad + \frac{1}{3\sqrt{6}}\beta(C_1 - C_2)\eta\eta\eta + \frac{1}{\sqrt{6}}\beta(C_2 - C_1)\pi^0\pi^0\eta + \frac{1}{\sqrt{2}}\beta(C_1 - C_2)\pi^0 K^0 \bar{K}^0 \\ &\quad + \frac{1}{\sqrt{6}}\beta(C_1 - C_2)K^0 \bar{K}^0 \eta, \end{aligned}$$

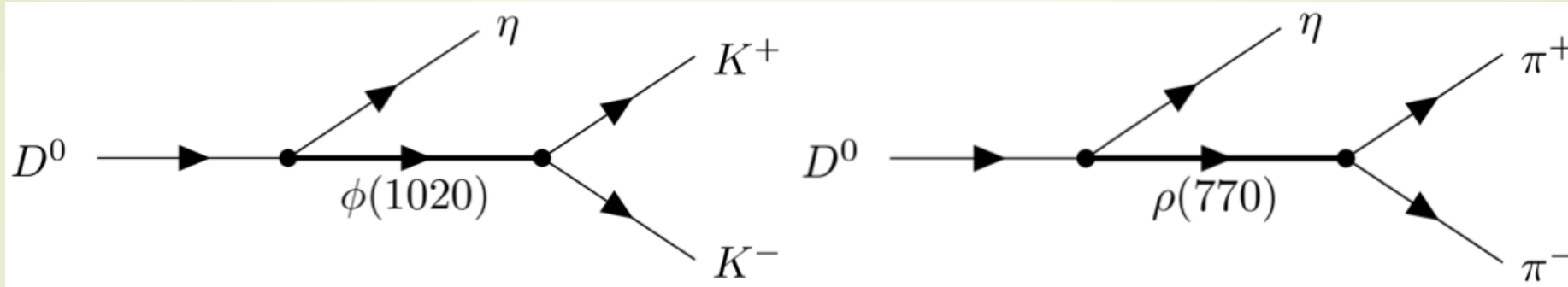


$$\begin{aligned}
 t_{D^0 \rightarrow K^+ K^- \eta}(s_{12}, s_{23}) = & \frac{2}{\sqrt{6}}(2C_1 + \beta C_1 + \beta C_2)G_{\pi^+ \pi^-}(s_{12})T_{\pi^+ \pi^- \rightarrow K^+ K^-}(s_{12}) \cdot \\
 & + \frac{1}{\sqrt{6}}(2C_2 + \beta C_1 + \beta C_2) + \frac{1}{\sqrt{6}}(2C_2 + \beta C_1 + \beta C_2)G_{K^+ K^-}(s_{12})T_{K^+ K^- \rightarrow K^+ K^-}(s_{12}) \\
 & + \frac{1}{3\sqrt{6}}\beta(C_1 - C_2)G_{\eta\eta}(s_{12})T_{\eta\eta \rightarrow K^+ K^-}(s_{12}) - \frac{1}{\sqrt{6}}\beta(C_1 - C_2)G_{\pi^0 \pi^0}(s_{12})T_{\pi^0 \pi^0 \rightarrow K^+ K^-}(s_{12}) \\
 & + \frac{1}{\sqrt{6}}\beta(C_1 - C_2)G_{K^0 \bar{K}^0}(s_{12})T_{K^0 \bar{K}^0 \rightarrow K^+ K^-}(s_{12}) + (C_1 - C_2)G_{\pi^- \bar{K}^0}(s_{23})T_{\pi^- \bar{K}^0 \rightarrow K^- \eta}(s_{23}) \\
 & - \frac{1}{\sqrt{2}}(2C_2 + \beta C_1 + \beta C_2)G_{K^- \pi^0}(s_{23})T_{K^- \pi^0 \rightarrow K^- \eta}(s_{23}) + \frac{1}{\sqrt{6}}(2C_2 + \beta C_1 + \beta C_2)G_{K^- \eta}(s_{23})T_{K^- \eta \rightarrow K^- \eta}(s_{23}) \\
 & + (C_1 - C_2)G_{\pi^+ K^0}(s_{13})T_{\pi^+ K^0 \rightarrow K^+ \eta}(s_{13}) - \frac{1}{\sqrt{2}}(2C_2 + \beta C_1 + \beta C_2)G_{K^+ \pi^0}(s_{13})T_{K^+ \pi^0 \rightarrow K^+ \eta}(s_{13}) \\
 & + \frac{1}{\sqrt{6}}(2C_2 + \beta C_1 + \beta C_2)G_{K^+ \eta}(s_{13})T_{K^+ \eta \rightarrow K^+ \eta}(s_{13}),
 \end{aligned}$$



$$\begin{aligned}
 t_{D^0 \rightarrow \pi^+ \pi^- \eta}(s_{12}, s_{23}) = & \frac{2}{\sqrt{6}}(2C_1 + \beta C_1 + \beta C_2) + \frac{2}{\sqrt{6}}(2C_1 + \beta C_1 + \beta C_2)G_{\pi^+ \pi^-}(s_{12})T_{\pi^+ \pi^- \rightarrow \pi^+ \pi^-}(s_{12}) \\
 & + \frac{1}{\sqrt{6}}(2C_2 + \beta C_1 + \beta C_2)G_{K^+ K^-}(s_{12})T_{K^+ K^- \rightarrow \pi^+ \pi^-}(s_{12}) + \frac{1}{3\sqrt{6}}\beta(C_1 - C_2)G_{\eta\eta}(s_{12})T_{\eta\eta \rightarrow \pi^+ \pi^-}(s_{12}) \\
 & - \frac{1}{\sqrt{6}}\beta(C_1 - C_2)G_{\pi^0 \pi^0}(s_{12})T_{\pi^0 \pi^0 \rightarrow \pi^+ \pi^-}(s_{12}) + \frac{1}{\sqrt{6}}\beta(C_1 - C_2)G_{K^0 \bar{K}^0}(s_{12})T_{K^0 \bar{K}^0 \rightarrow \pi^+ \pi^-}(s_{12}) \\
 & + \frac{2}{\sqrt{6}}(2C_1 + \beta C_1 + \beta C_2)G_{\pi^- \eta}(s_{23})T_{\pi^- \eta \rightarrow \pi^- \eta}(s_{23}) + \frac{2}{\sqrt{6}}(2C_1 + \beta C_1 + \beta C_2)G_{\pi^+ \eta}(s_{13})T_{\pi^+ \eta \rightarrow \pi^+ \eta}(s_{13}) \\
 & + (C_1 - C_2)G_{K^0 K^-}(s_{23})T_{K^0 K^- \rightarrow \pi^- \eta}(s_{23}) + (C_1 - C_2)G_{K^+ \bar{K}^0}(s_{13})T_{K^+ \bar{K}^0 \rightarrow \pi^+ \eta}(s_{13}),
 \end{aligned}$$

(3) Contributions from P-wave



$$M_{\phi}(s_{12}, s_{23}) = \frac{D_{\phi} e^{i\alpha_{\phi}}}{s_{12} - m_{\phi}^2 + im_{\phi}\Gamma_{\phi}} (s_{23} - s_{13})$$

$$M_{\rho}(s_{12}, s_{23}) = \frac{D_{\rho} e^{i\alpha_{\rho}}}{s_{12} - m_{\rho}^2 + im_{\rho}\Gamma_{\rho}} (s_{23} - s_{13})$$

$$\frac{d^2\Gamma}{ds_{12}ds_{23}} = \frac{1}{(2\pi)^3} \frac{1}{32m_{D^0}^3} |t_{D^0 \rightarrow \pi^+\pi^-\eta}(s_{12}, s_{23}) + M_{\rho}|^2$$

$$\frac{d^2\Gamma}{ds_{12}ds_{23}} = \frac{1}{(2\pi)^3} \frac{1}{32m_{D^0}^3} |t_{D^0 \rightarrow K^+K^-\eta}(s_{12}, s_{23}) + M_{\phi}|^2$$

$$G(s)T(s) = G(s_{cut})T(s_{cut})e^{-\alpha(\sqrt{s}-\sqrt{s_{cut}})}, \quad \text{for } \sqrt{s} > \sqrt{s_{cut}}, \quad \sqrt{s_{cut}} = 1.1 \text{ GeV}$$

• free parameters

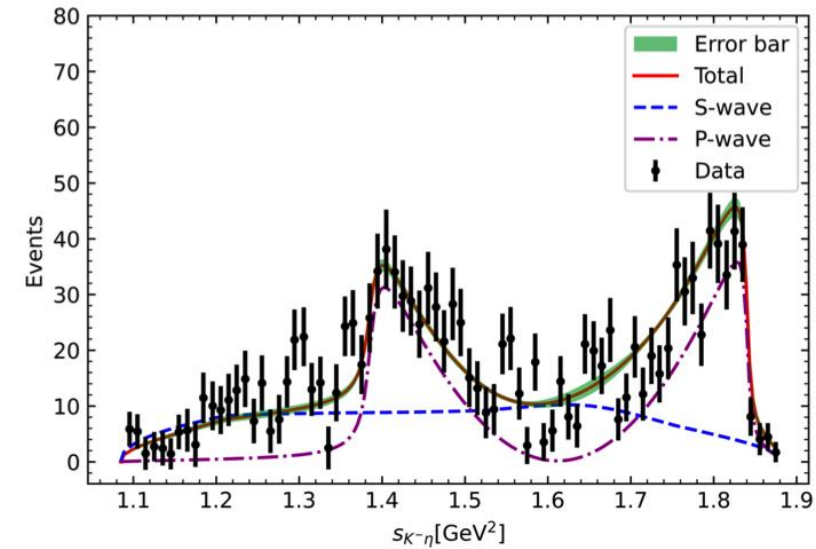
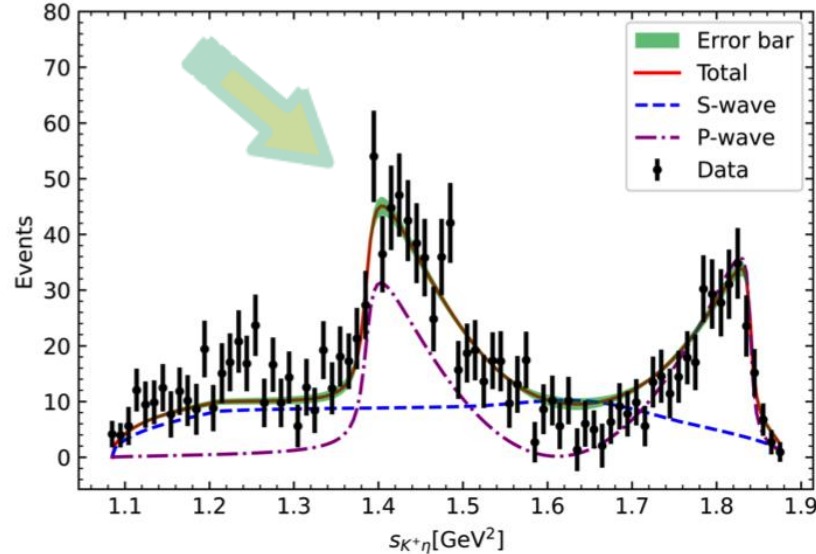
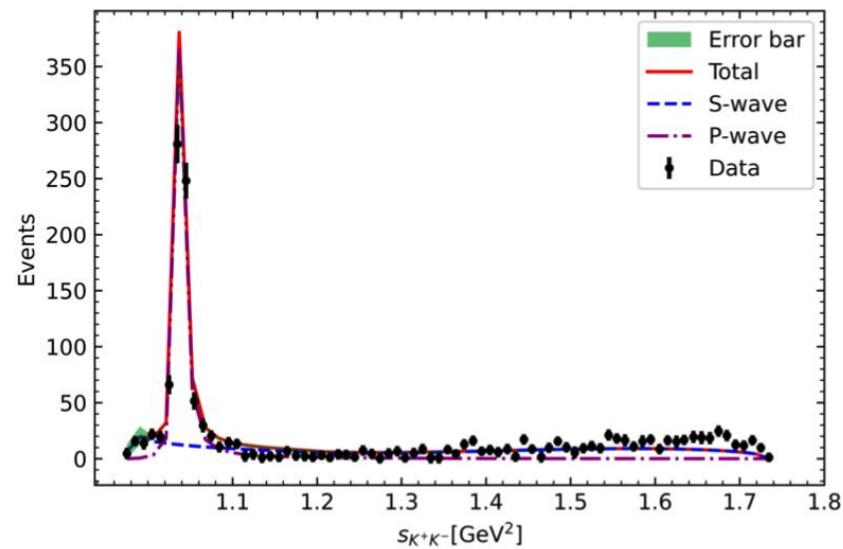
S-wave: C_1, C_2, β, α

P-wave: $D_{\rho}, \alpha_{\rho}, D_{\phi}, \alpha_{\phi}$

§3. Results

(1) $D^0 \rightarrow K^+K^-\eta$ results (Belle)

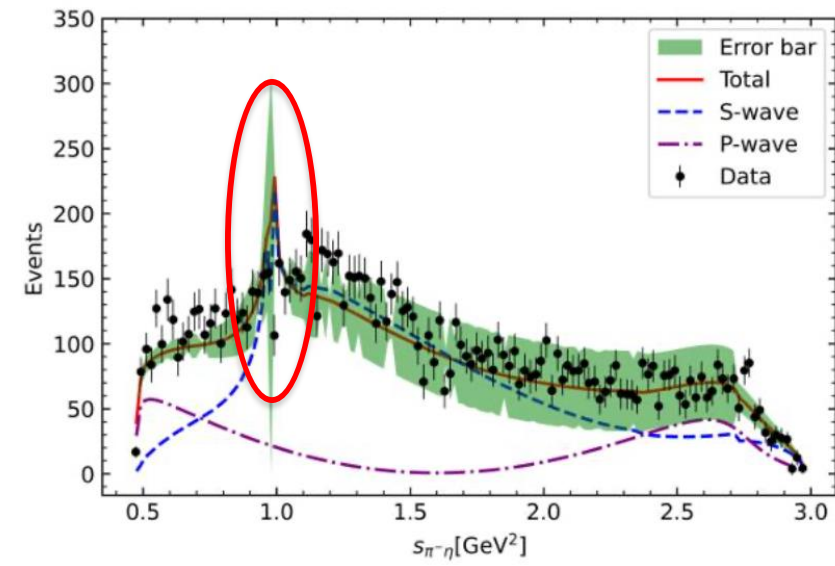
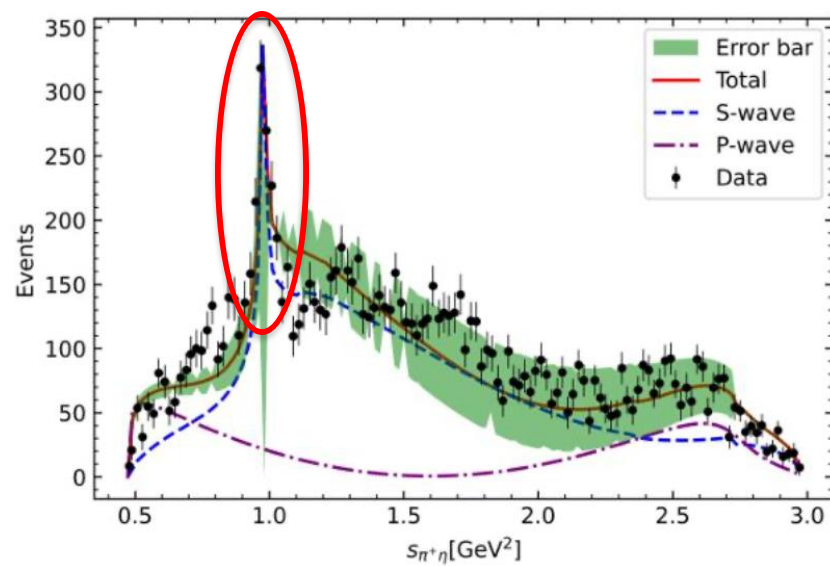
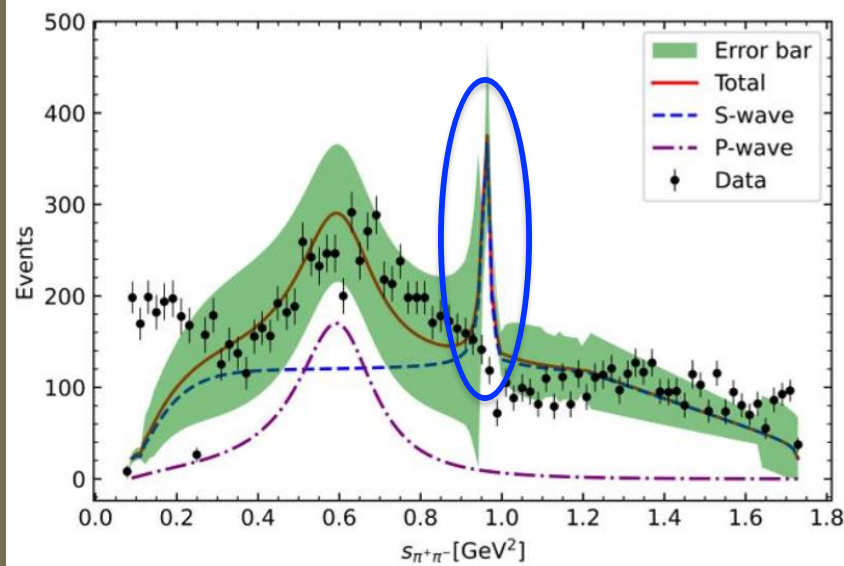
Parameters	C_1	C_2	β	α	$D_{\phi(1020)}$	$\alpha_{\phi(1020)}$	$\chi^2/dof.$
Fit	-3448.24 ± 221.07	99.07 ± 91.70	0.77 ± 0.04	6.28 ± 0.05	123.75 ± 2.02	2.89 ± 0.12	1.48



L. K. Li et al. [Belle], JHEP 09, 075 (2021)

(2) $D^0 \rightarrow \pi^+\pi^-\eta$ results (Belle)

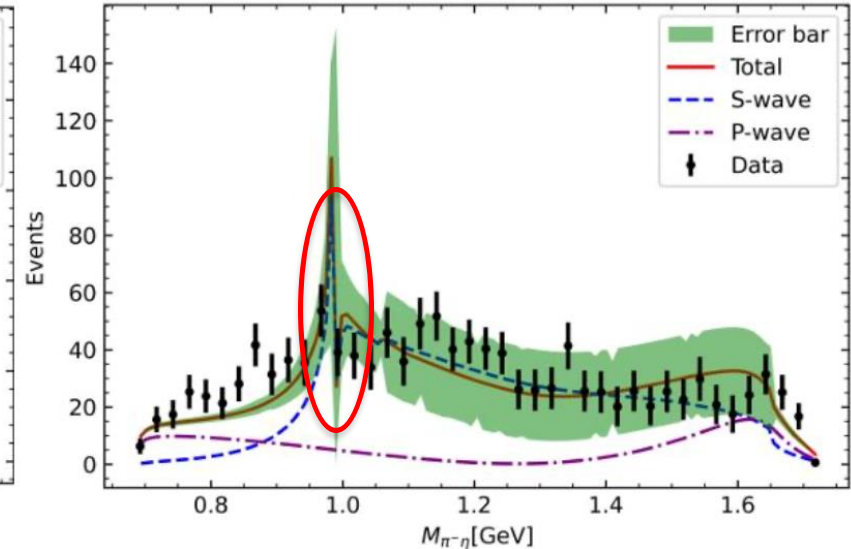
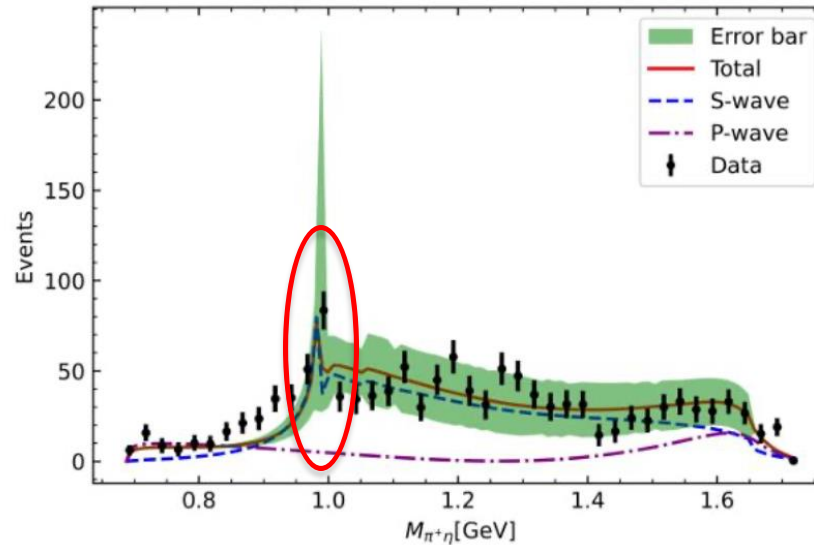
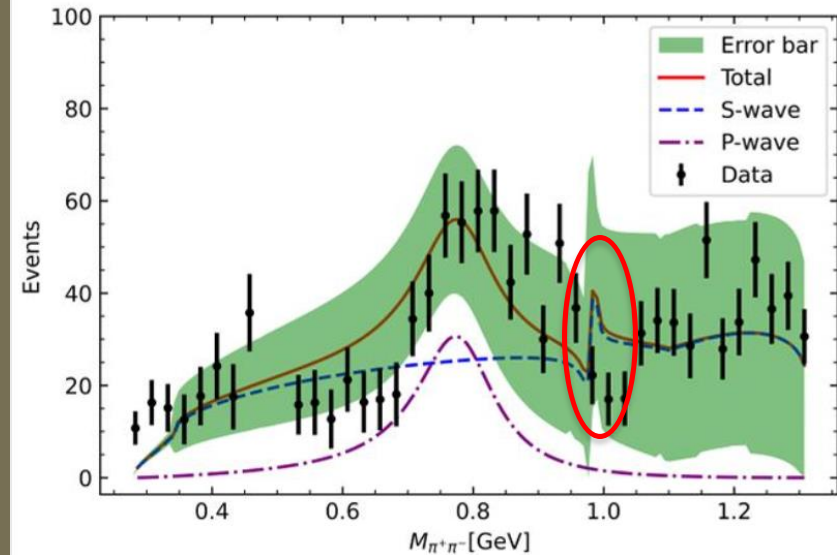
Parameters	C_1	C_2	β	α	$D_{\rho(770)}$	$\alpha_{\rho(770)}$	$\chi^2/dof.$
Fit	419.79 ± 23.61	2196.24 ± 57.25	1.00 ± 0.006	0.38 ± 0.10	-170.97 ± 2.90	2.20 ± 0.03	5.23



L. K. Li et al. [Belle], JHEP 09, 075 (2021)

(3) $D^0 \rightarrow \pi^+\pi^-\eta$ results (BESIII)

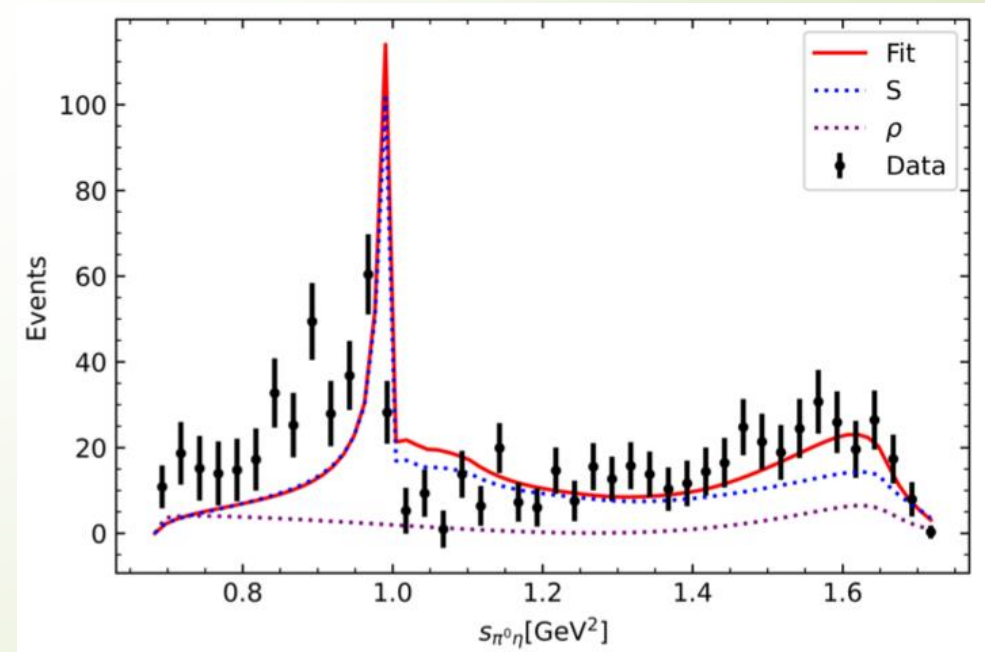
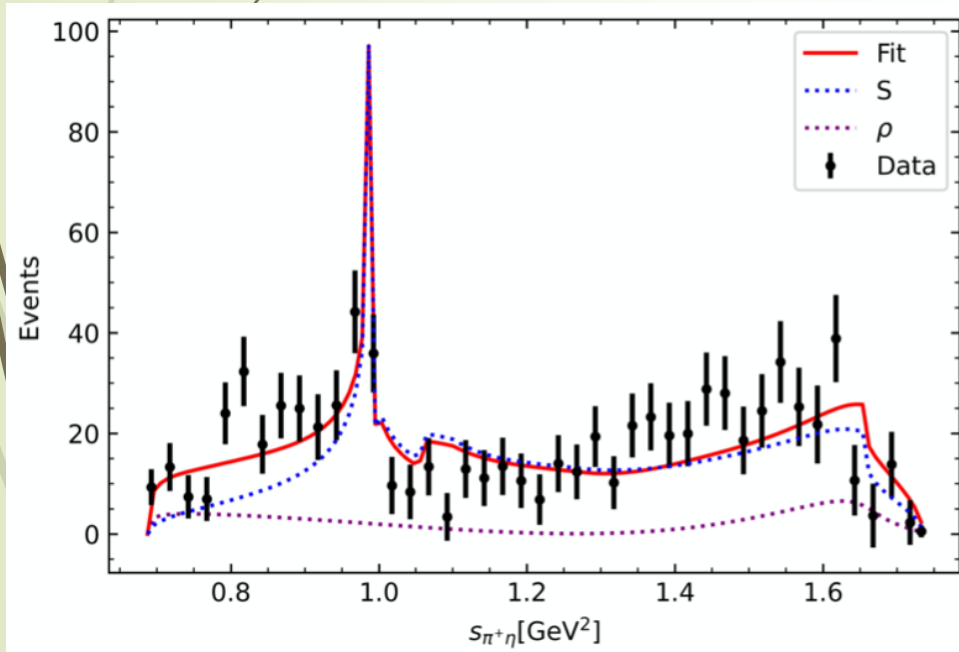
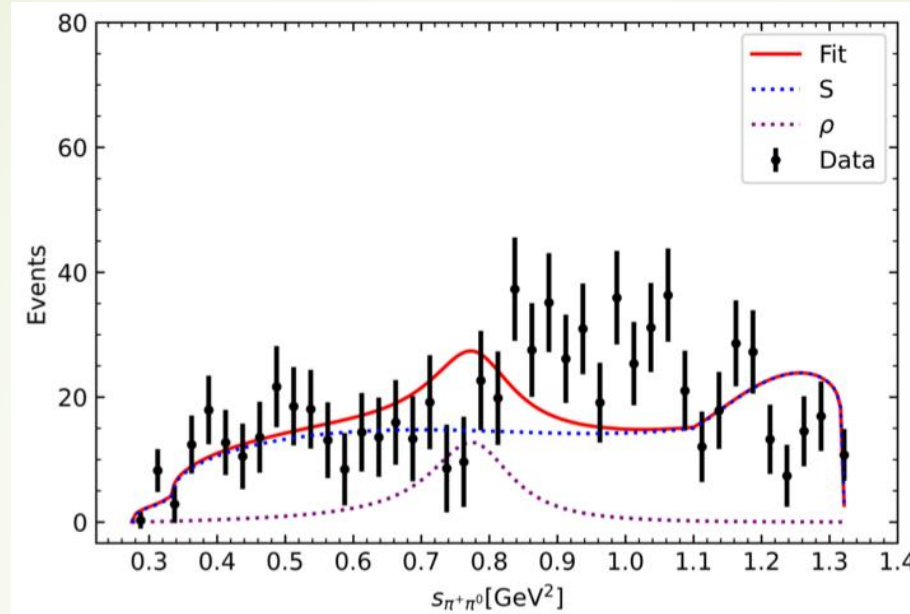
Parameters	C_1	C_2	β	α	$D_{\rho(770)}$	$\alpha_{\rho(770)}$	$\chi^2/dof.$
Fit	282.74 ± 15.25	-516.33 ± 26.32	1.00 ± 0.03	2.47 ± 0.44	58.41 ± 2.98	4.36 ± 0.13	2.52



M. Ablikim et al. [BESIII], Phys. Rev. D 110, L111102 (2024)

(4) $D^+ \rightarrow \pi^+\pi^0\eta$ results (BESIII)

M. Ablikim et al. [BESIII],
Phys. Rev. D 110, L111102
(2024)



§4. Summary

- We use the chiral unitary approach to dynamically generate the state $a_0(980)$
- Taking the data from Belle, we fit the invariant mass distributions of the decays $D^0 \rightarrow K^+ K^- \eta, \pi^+ \pi^- \eta$
- Taking the data from BESIII, we fit the invariant mass distributions of the decays $D^0 \rightarrow \pi^+ \pi^- \eta, D^+ \rightarrow \pi^+ \pi^0 \eta$
- Indicating the molecular nature of the $a_0(980)$



Thanks for your attention!

感谢大家的聆听！