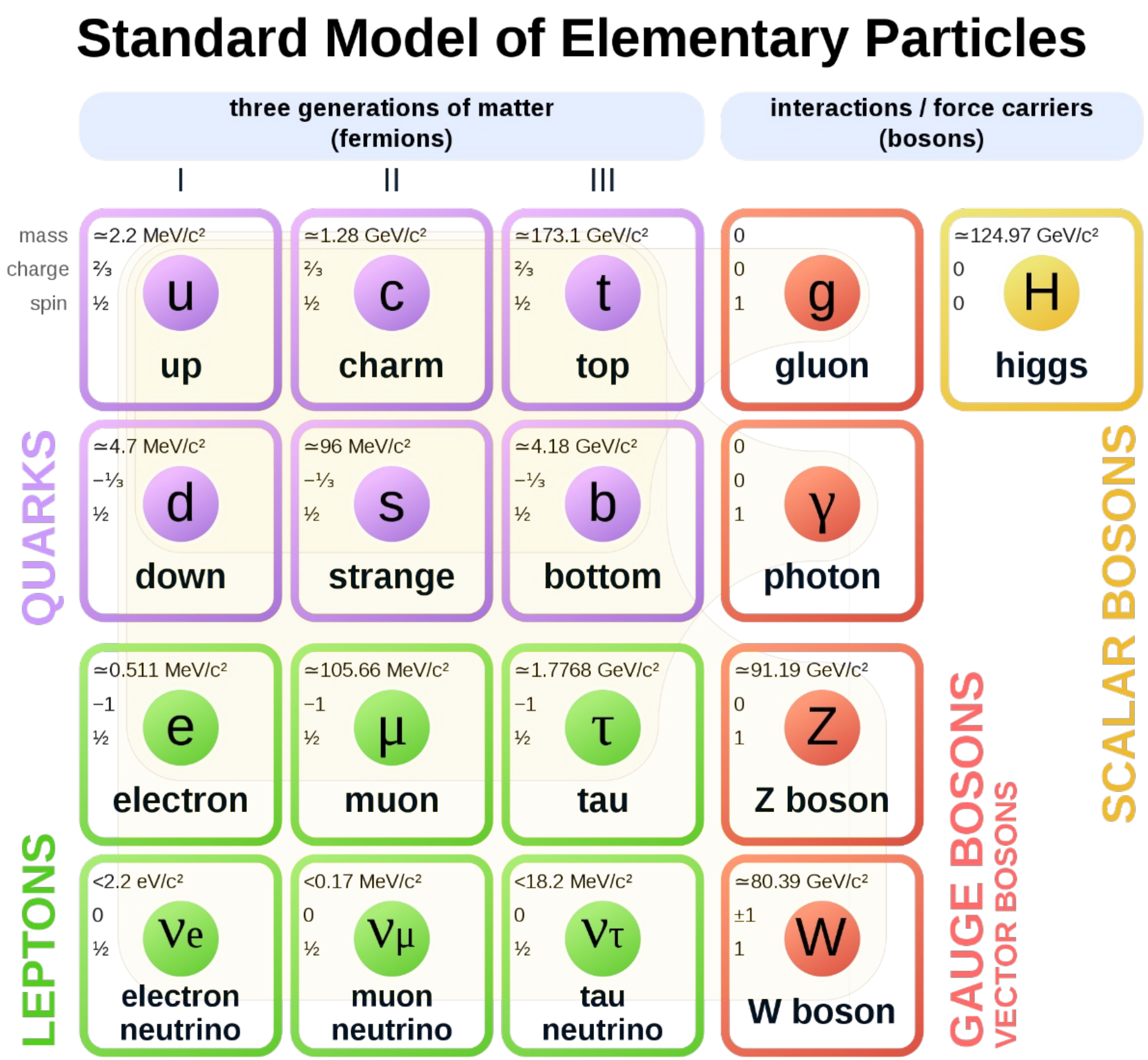
Nonleptonic Weak Decay of Bottom-Charmed Baryon

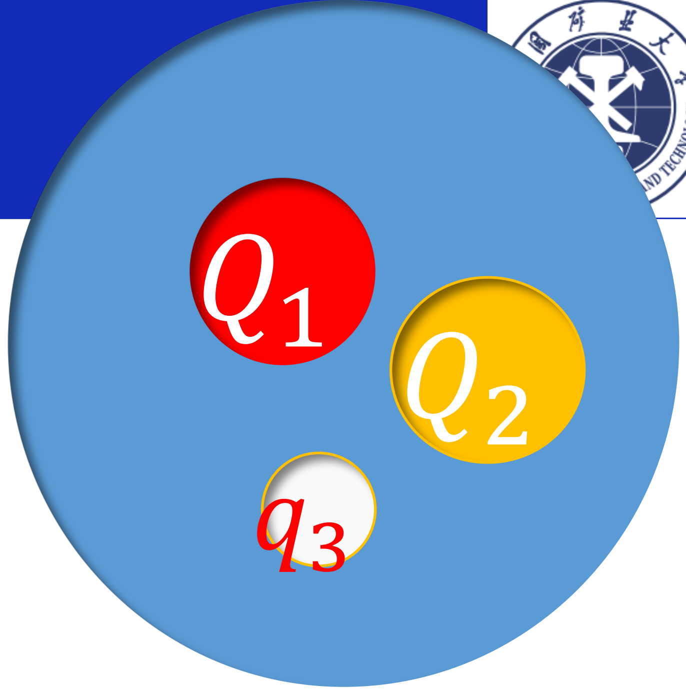
Xiao-Hui Hu
Email: huxiaohui@cumt.edu.cn
中国矿业大学
Collaborators: F. S. Yu, C. P. Jia, Y. Xing, Z. P. Xing
2025.10.27
第二十二届全国重味物理和CP破坏研讨会

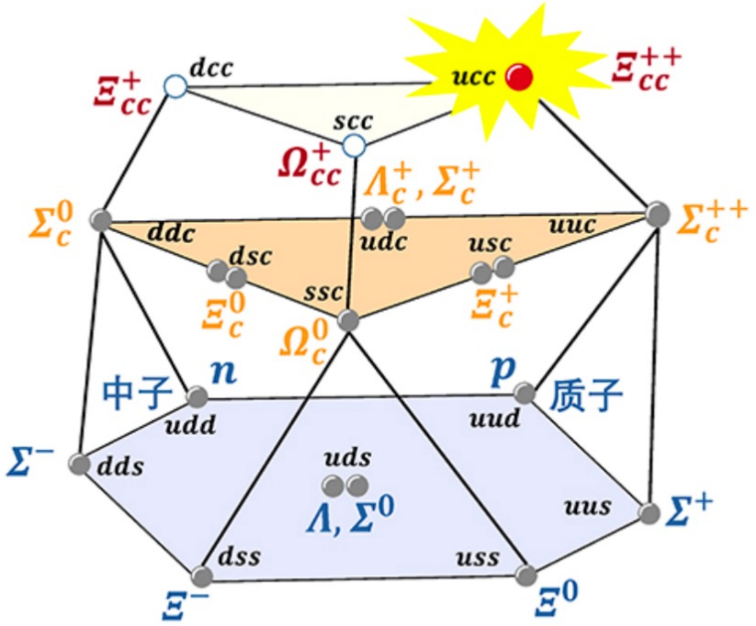
Contents

- **Research Motivation:** Experimental Status and Theoretical Progress
- **Research Scope:** Study of Weak Decays in Bottom-charmed Baryon
- **Results:** Branching ratio, Decay asymmetry parameters
- **Summary**

Doubly Heavy Baryons



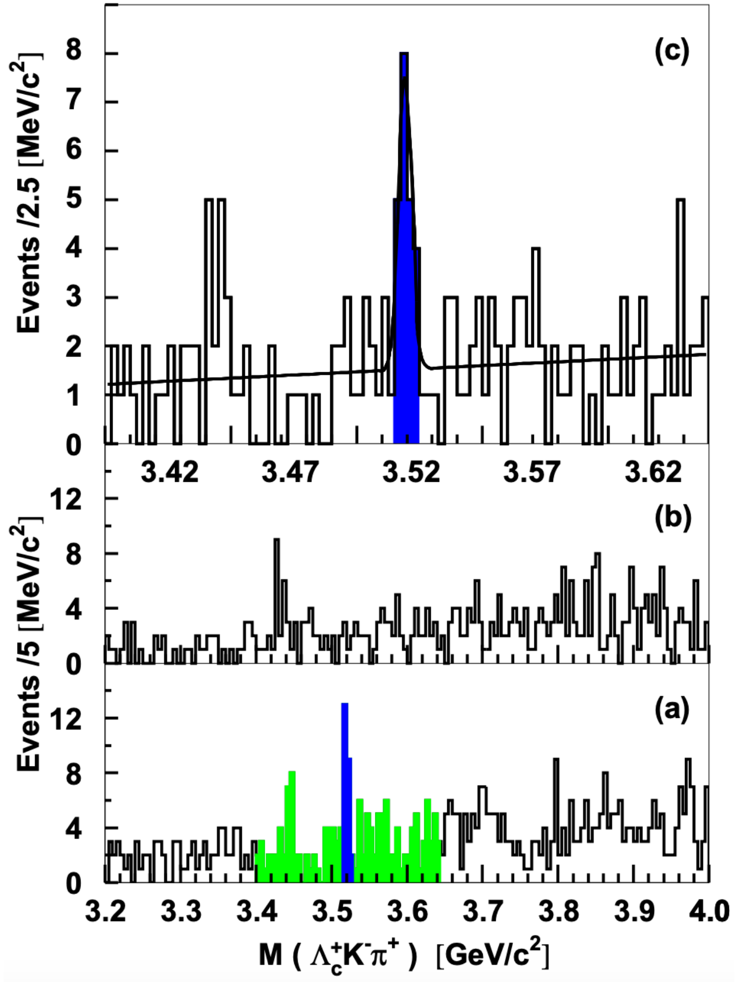




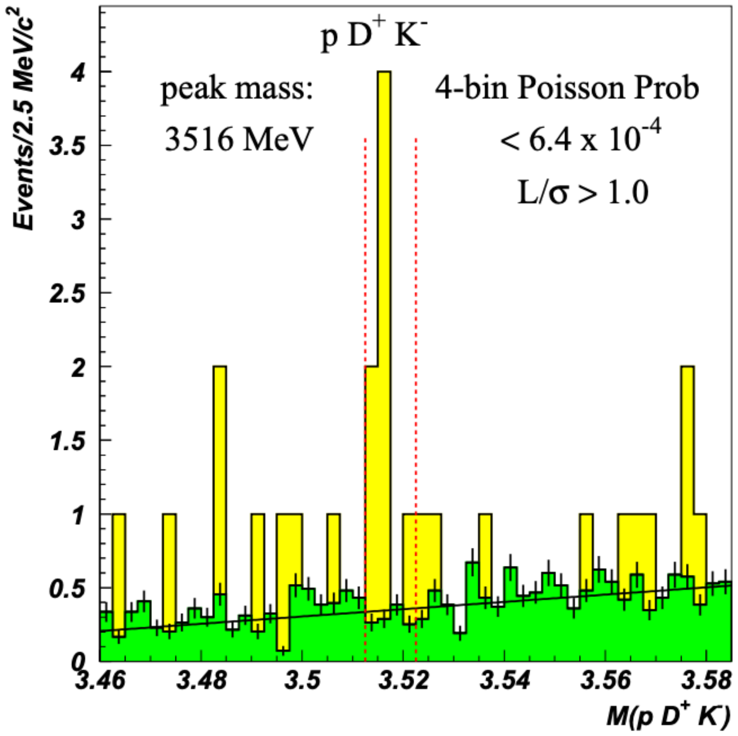
Ξ_{bc}	Ω_{bc}
Ξ_{bb}	Ω_{bb}

Experimental Progress

- Discovery of Doubly Charmed Baryons$\Xi_{cc}^{+}(3520)$



First observed by SELEX Collaboration (2002)



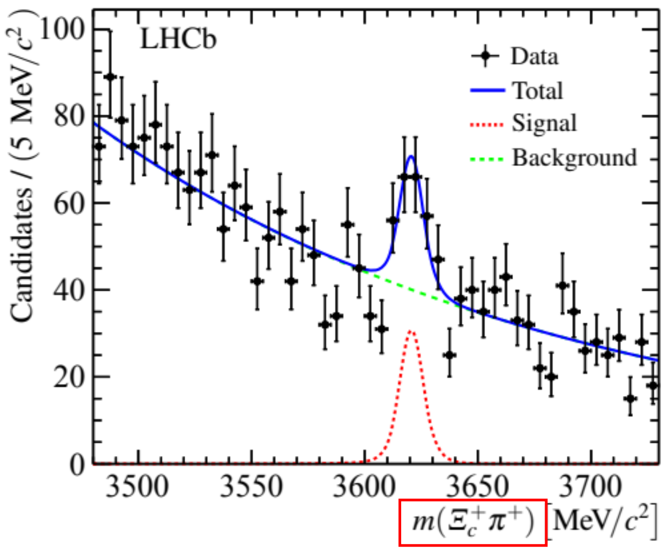
Confirmation by SELEX Collaboration (2004)

No confirmation by other collaboration

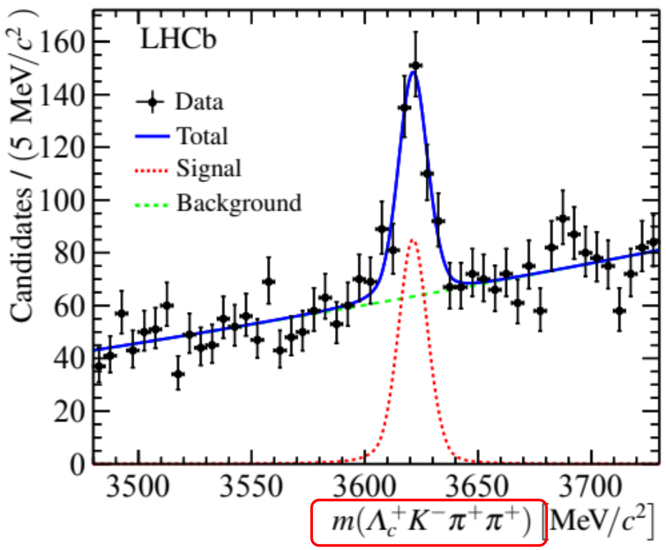
- FOCUS Collaboration （2003） Nucl. Phys. B Proc. Suppl. 115 (2003), 33-36
- BaBar Collaboration（2006） Phys. Rev. D 74 (2006), 011103
- LHCb Collaboration（2013） JHEP 12 (2013), 090
- Belle Collaboration （2014） Phys. Rev. D 89 (2014) no.5, 052003

Discovery of Doubly Charmed Baryons:

F. S. Yu et al., Discovery potentials of doubly charmed baryons, arXiv:1703.09086 *Chin. Phys. C* 42 (2018) 5, 051001



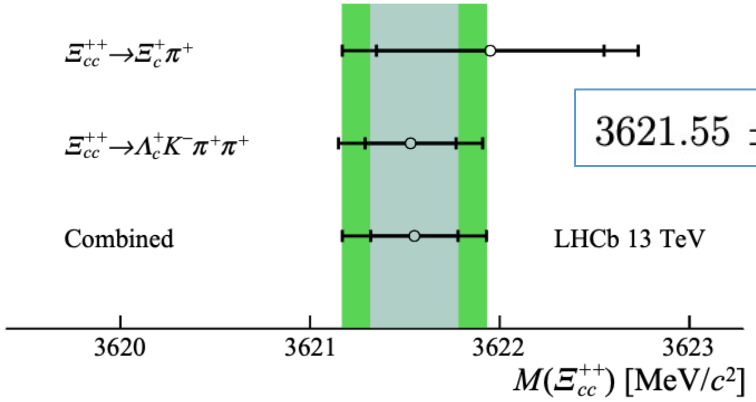
Phys. Rev. Lett. 121 (2018) 16, 162002



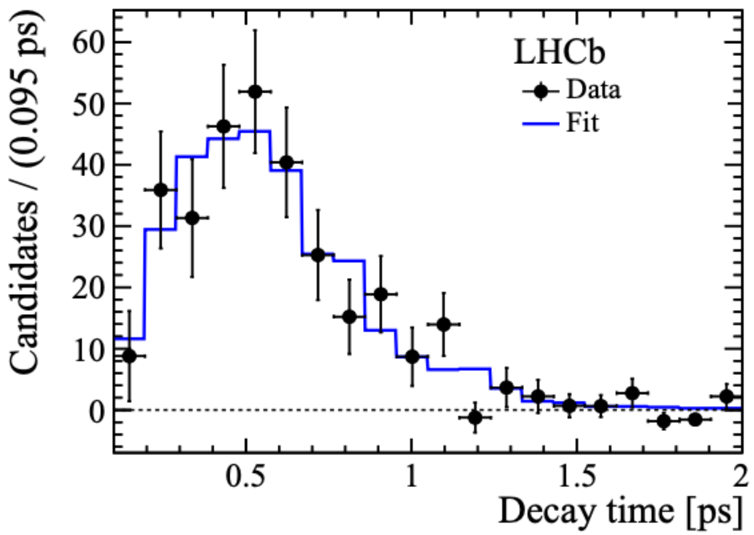
PRL 119, 112001 (2017)

In their Supplemental Material:
The same structure is also observed in the. $\Lambda_c^+K^-\pi^+\pi^+$ spectrum in a p p data sample collected by LHCb at 8 TeV.

Precision measurement of the mass *JHEP* 02 (2020) 049

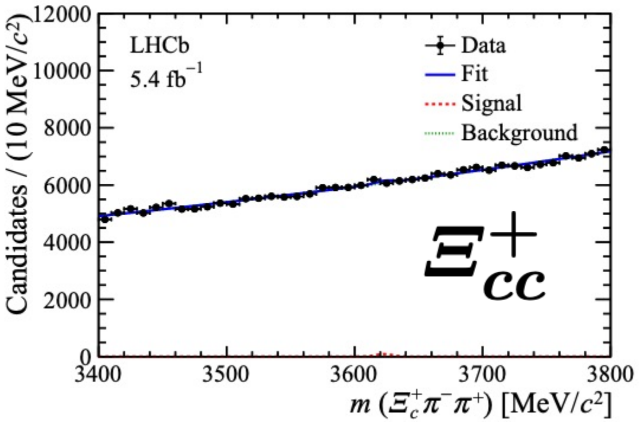


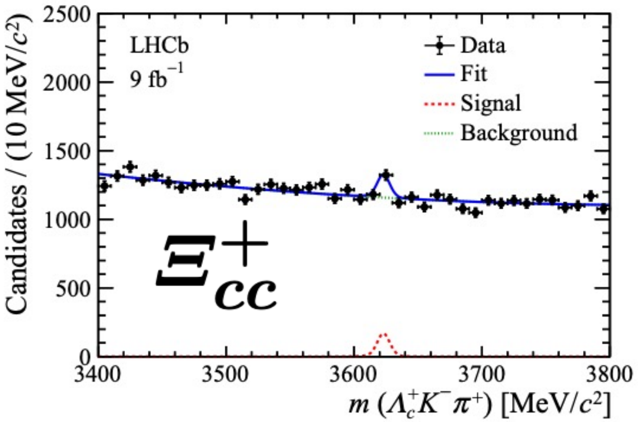
$$3621.55 \pm 0.23\,(\text{stat}) \pm 0.30\,(\text{syst})\ \text{MeV}/c^2.$$

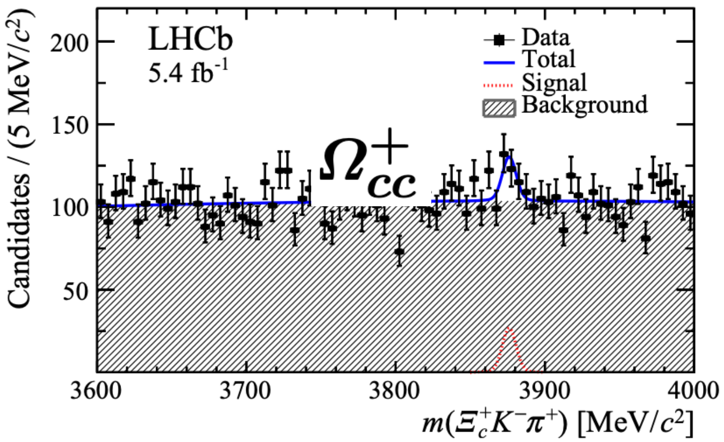


$$\tau\left(\Xi_{cc}^{++}\right) = 0.256\,^{+0.024}_{-0.022}\,(\text{stat}) \pm 0.014\,(\text{syst})\ \text{ps}.$$

The search of other doubly charmed baryon



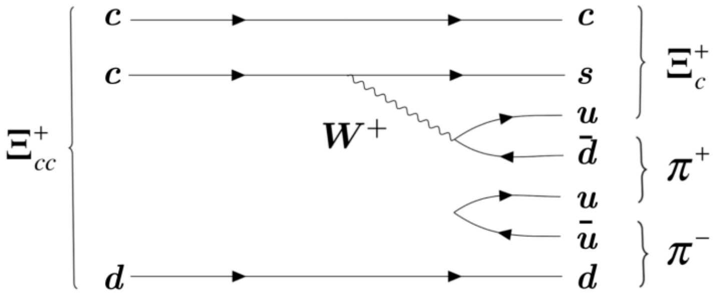




JHEP 12 (2021) 107

Sci. China Phys. Mech. Astron. 63 (2020) 2, 221062

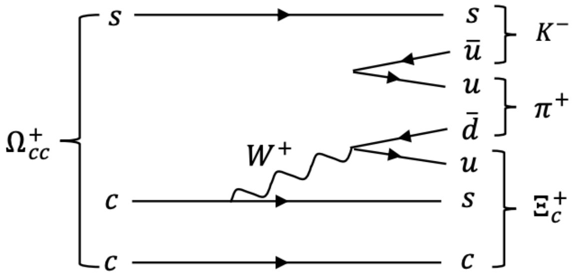
Sci. China Phys. Mech. Astron. 64 (2021) 10, 101062



- SELEX Collaboration (2002)

$$\Xi_{cc}^{+} \to \Lambda_c^+ K^- \pi^+$$

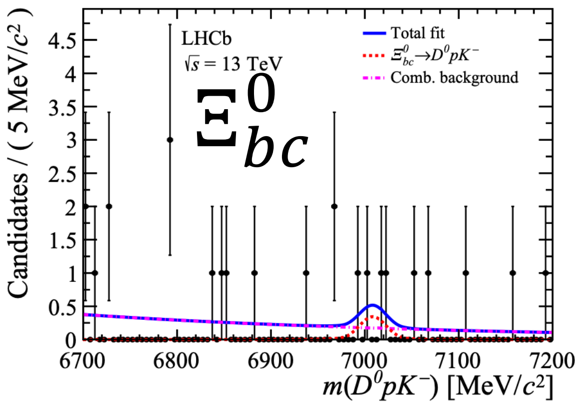
The observed mass of this state is (3519 ± 1) MeV/c^2 .



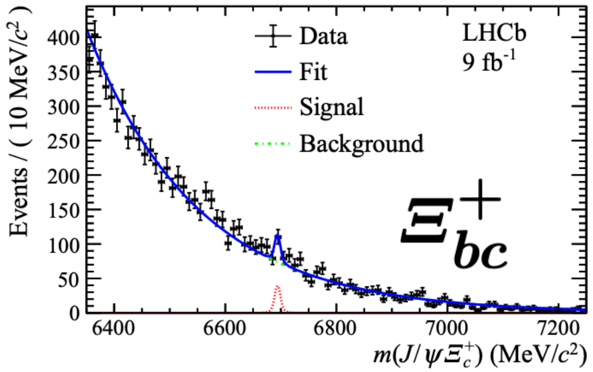
No significant signal is observed in the mass range from 3.4 to3.8GeV/c^2.

No significant signal is observed within the invariant mass range of 3.6 to 4.0 GeV/c^2.

The search of bottom-charmed baryon



JHEP 11 (2020) 095



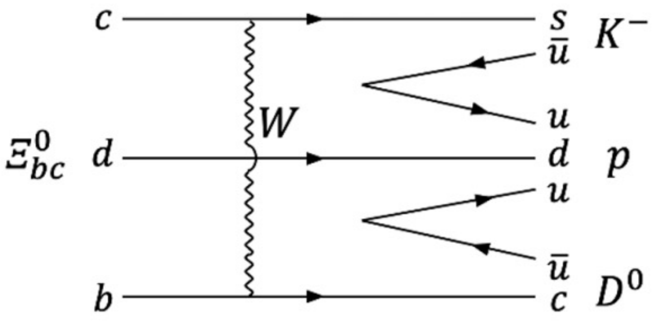
Chin. Phys. C 47 (2023) 9, 093001

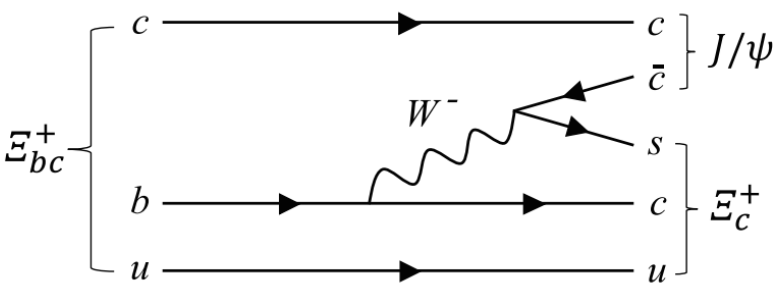
No significant excess is observed across the searched mass range.

Chin. Phys. C 45 (2021) 9, 093002

$$\Omega_{bc}^{0}\to\Lambda_c^+\pi^-\ (\Xi_{bc}^{0}\to\Lambda_c^+\pi^-)$$

$$\Omega_{bc}^{0}\to\Xi_c^+\pi^-\ (\Xi_{bc}^{0}\to\Xi_c^+\pi^-)$$





The LHCb collaboration observed no significant signals in the invariant mass range of 6.7 ~ 7.3 GeV.

So far, the doubly heavy baryon containing a bottom quark has not yet been discovered experimentally.

Theoretical Advances

Baryon	Quark content	S_h^π	J^P	Baryon	Quark content	S_h^π	J^P
Ξ_{cc}	$\{cc\}q$	1^+	$1/2^+$	Ξ_{bb}	$\{bb\}q$	1^+	$1/2^+$
Ξ_{cc}^*	$\{cc\}q$	1^+	$3/2^+$	Ξ_{bb}^*	$\{bb\}q$	1^+	$3/2^+$
Ω_{cc}	$\{cc\}s$	1^+	$1/2^+$	Ω_{bb}	$\{bb\}s$	1^+	$1/2^+$
Ω_{cc}^*	$\{cc\}s$	1^+	$3/2^+$	Ω_{bb}^*	$\{bb\}s$	1^+	$3/2^+$
Ξ_{bc}'	$[bc]q$	0^+	$1/2^+$	Ω_{bc}'	$[bc]s$	0^+	$1/2^+$
Ξ_{bc}	$\{bc\}q$	1^+	$1/2^+$	Ω_{bc}	$\{bc\}s$	1^+	$1/2^+$
Ξ_{bc}^*	$\{bc\}q$	1^+	$3/2^+$	Ω_{bc}^*	$\{bc\}s$	1^+	$3/2^+$

● **Mass Spectrum:**

Lattice QCD, Phys. Rev. D,90,094507,

QCD sum rule, J. Phys. Conf. ser. 556,01 2016

Quark Model, Phys. Rev. D 66,014008 .....

Baryon	BSE[1]	QM[2]	FHT[3]	ECM[4-5]	PM[6]	Three QM[7]	QCDSR[8]	LaQCD[9]	RQM[10]
Ξ_{bc}	6.930	6.990	6.959	6.914	6.933	6.919	6.904	6.943	6.950
Ξ_{bc}'	6.942	7.040	6.943	6.933	6.963	6.948	6.920	6.959	7.000
Ω_{bc}	7.032	7.060	7.032	7.013	7.088	6.986	7.136	6.998	7.050
Ω_{bc}'	7.045	7.090	6.998	7.025	7.116	7.009	7.165	7.032	7.090

a mass in the range of 6700 –7029 MeV/c^2

Theoretical Advances

Lifetimes: $\tau[fs]$
operator product expansion:
Phys. Rev. D 98 (2018), 113004;
the inverse heavy quark mass expansion :
Eur. Phys. J. C 16 (2000), 461-469
heavy quark expansion:
Phys. Rev. D **99**, 073006
A factorization approach:
Phys. Rev. D 90(2014),094007
....

baryon	OPE[1]	IHQME[2]	FA[3]	HQE[4]	…
Ξ_{bc}^{+}	0.24	0.33	0.244	0.409-0.607	
Ξ_{bc}^{0}	0.22	0.28	0.093	0.093-0.118	
Ω_{bc}^{0}	0.18			0.168-0.370	

Its lifetime is predicted to be [240-607]fs

- **Strong Couplings:**

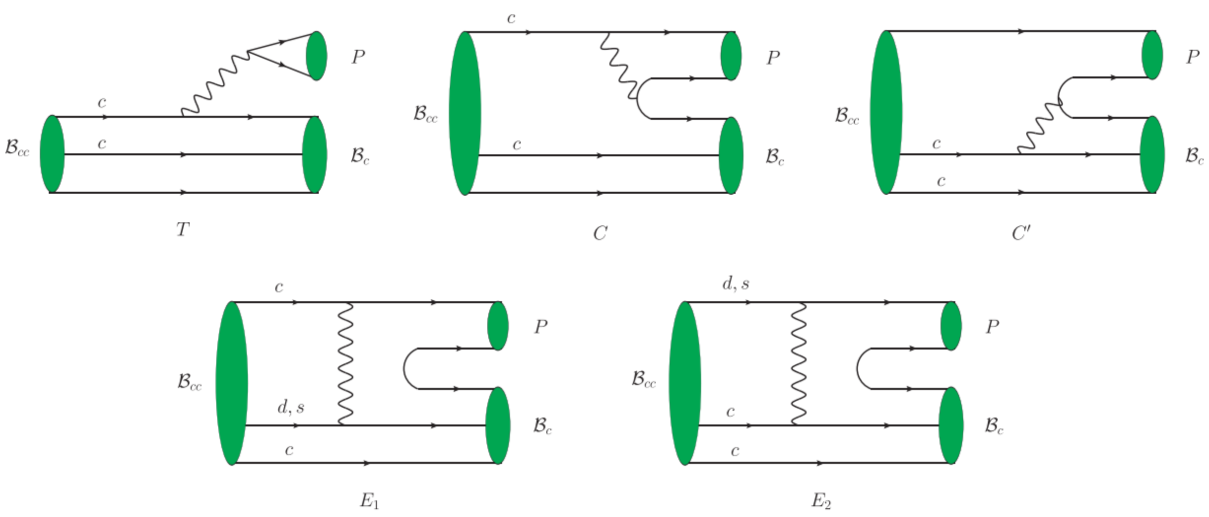
T. M. Aliev, K. Simsek, Eur. Phys. J. C (2020) 80:976.....

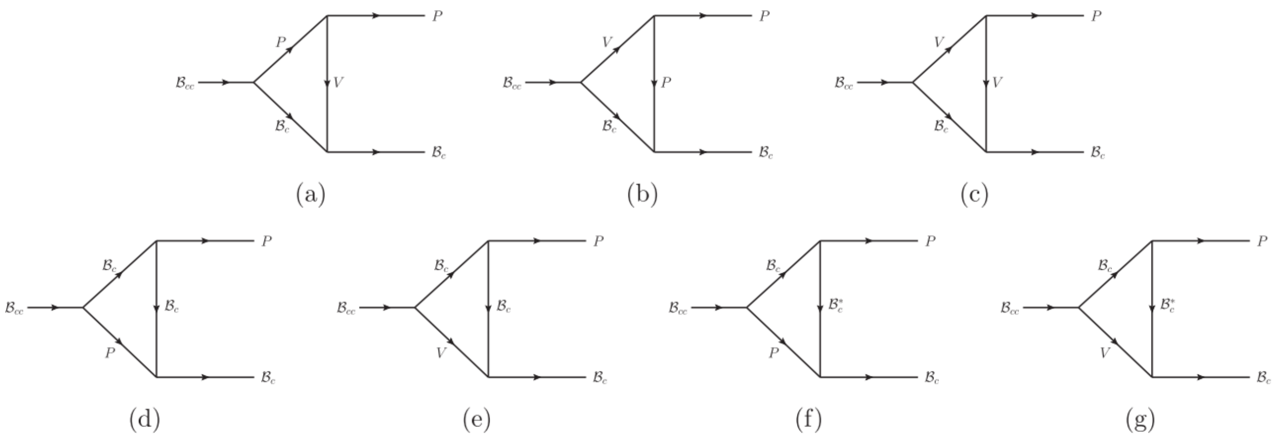
Challenges in calculating transitions between doubly and singly heavy baryons

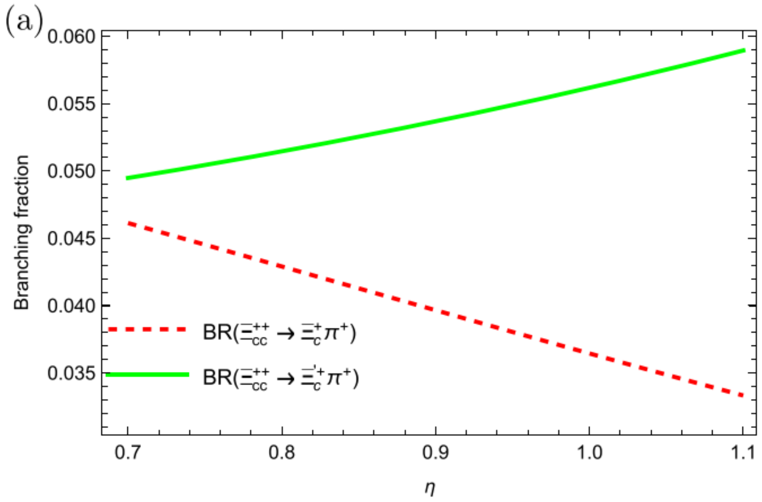
- **Nonleptonic weak decays:**

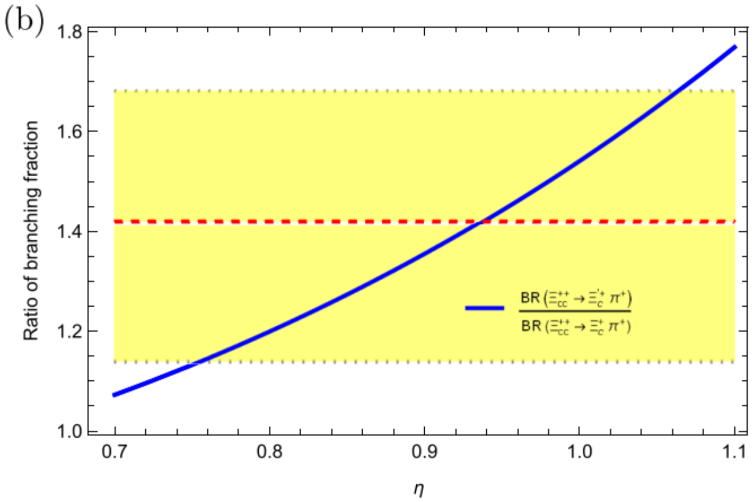
Experiment: Phys. Rev. Lett. 121 (2018) 16, 162002; JHEP 05 (2022) 0381.
Three Loop Calculation: *Phys. Rev. D* 99 (2019) 5, 056013;
Factorization and Pole Model: Phys. Rev. D 96, 113006 (2017).
Final States Interactions: Eur. Phys. J. C 78, 961 (2018).
Light Front Quark Model: Eur. Phys. J. C 77, 781 (2017).
Optimal Theory and Scattering Mechanism: Chin. Phys. C 42, 051001 (2018)....

Nonleptonic Weak Decay









$$\frac{\mathcal{B}(\Xi_{cc}^{++} \to \Xi_c^{\prime +}\pi^+)}{\mathcal{B}(\Xi_{cc}^{++} \to \Xi_c^{+}\pi^+)} = 1.41 \pm 0.17 \pm 0.10.$$

$$\eta = 0.9 \pm 0.2$$

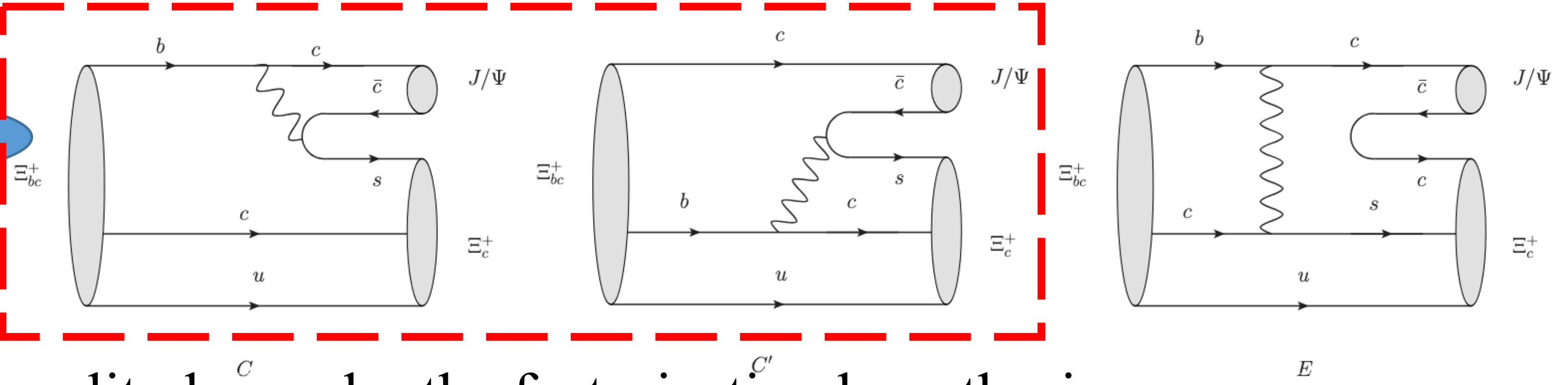
Phys. Rev. D 111(7), 076002 (2025)

Branching ratio and asymmetery parameters

Channels	CKM	$\mathcal{B}_{\rm SD}[10^{-3}]$	$\mathcal{B}_{\rm LD}[10^{-3}]$	$\mathcal{B}_{\rm tot}[10^{-3}]$	α	β	γ
$\Xi_{cc}^{++} \to \Xi_c^{+}\pi^{+}$	CF	$60.70^{+1.12}_{-1.11}$	$2.67^{+1.30+2.37}_{-0.82-1.50}$	$39.70^{+2.97+6.47}_{-2.73-6.33}$	$-0.43^{+0.03+0.04}_{-0.02-0.03}$	$0.13^{+0.06+0.07}_{-0.04-0.05}$	$-0.89^{+0.01+0.01}_{-0.00-0.01}$
$\Xi_{cc}^{++} \to \Xi_c^{\prime +}\pi^{+}$	CF	$42.30^{+0.78}_{-0.78}$	$2.13^{+2.40+2.09}_{-0.87-1.24}$	$53.70^{+5.86+5.26}_{-3.58-4.20}$	$-0.97^{+0.00+0.00}_{-0.00-0.00}$	$0.19^{+0.04+0.05}_{-0.04-0.06}$	$0.14^{+0.04+0.05}_{-0.07-0.06}$
$\Xi_{cc}^{++} \to \Sigma_c^{+}\pi^{+}$	SCS	$2.23^{+0.04}_{-0.04}$	$0.27^{+0.17+0.21}_{-0.08-0.14}$	$1.39^{+0.09+0.18}_{-0.04-0.12}$	$-0.87^{+0.07+0.12}_{-0.04-0.07}$	$0.50^{+0.06+0.14}_{-0.11-0.16}$	$0.00^{+0.10+0.12}_{-0.17-0.18}$
$\Xi_{cc}^{++} \to \Lambda_c^{+}\pi^{+}$	SCS	$3.44^{+0.06}_{-0.06}$	$0.18^{+0.11+0.17}_{-0.06-0.11}$	$4.08^{+0.17+0.32}_{-0.13-0.25}$	$-0.26^{+0.06+0.11}_{-0.05-0.10}$	$0.19^{+0.04+0.06}_{-0.04-0.06}$	$-0.95^{+0.01+0.02}_{-0.00-0.01}$
$\Xi_{cc}^{++} \to \Xi_c^{\prime +}K^{+}$	SCS	$2.74^{+0.11}_{-0.10}$	$0.03^{+0.02+0.02}_{-0.01-0.01}$	$3.18^{+0.19+0.18}_{-0.15-0.16}$	$-0.96^{+0.00+0.01}_{-0.00-0.01}$	$-0.08^{+0.01+0.02}_{-0.00-0.01}$	$0.26^{+0.01+0.02}_{-0.01-0.02}$
$\Xi_{cc}^{++} \to \Xi_c^{+}K+$	SCS	$4.82^{+0.19}_{-0.18}$	$0.15^{+0.07+0.13}_{-0.04-0.08}$	$4.02^{+0.13+0.16}_{-0.04-0.08}$	$-0.31^{+0.03+0.06}_{-0.03-0.05}$	$-0.08^{+0.02+0.03}_{-0.02-0.04}$	$-0.95^{+0.01+0.02}_{-0.01-0.01}$
$\Xi_{cc}^{++} \to \Lambda_c^{+}K^{+}$	DCS	$0.29^{+0.01}_{-0.01}$	$0.01^{+0.00+0.01}_{-0.00-0.00}$	$0.21^{+0.00+0.02}_{-0.00-0.02}$	$-0.44^{+0.00+-0.00}_{-0.01-0.01}$	$-0.09^{+0.01+0.04}_{-0.02-0.06}$	$-0.89^{+0.00+0.01}_{0.00-0.00}$
$\Xi_{cc}^{++} \to \Sigma_c^{+}K^{+}$	DCS	$0.16^{+0.01}_{-0.01}$	$0.01^{+0.00+0.01}_{-0.00-0.01}$	$0.17^{+0.01+0.01}_{-0.01-0.00}$	$-0.79^{+0.05+0.12}_{-0.04-0.09}$	$-0.21^{+0.02+0.08}_{-0.00-0.09}$	$0.58^{+0.06+0.11}_{-0.06-0.12}$
$\Xi_{cc}^{+} \to \Xi_c^{0}\pi^{+}$	CF	$10.40^{+0.19}_{-0.19}$	$0.68^{+0.34+0.63}_{-0.21-0.39}$	$15.60^{+1.38+2.26}_{-1.15-1.90}$	$-0.58^{+0.01+0.02}_{-0.01-0.02}$	$-0.11^{+0.02+0.04}_{-0.02-0.04}$	$-0.81^{+0.01+0.02}_{-0.01-0.02}$
$\Xi_{cc}^{+} \to \Xi_c^{\prime 0}\pi^{+}$	CF	$7.26^{+0.13}_{-0.13}$	$0.10^{+0.06+0.10}_{-0.03-0.06}$	$6.15^{+0.13+0.33}_{-0.14-0.32}$	$-0.95^{+0.01+0.01}_{-0.00-0.00}$	$-0.07^{+0.02+0.02}_{-0.02-0.03}$	$0.31^{+0.01+0.01}_{-0.01-0.01}$
$\Xi_{cc}^{+} \to \Sigma_c^{0}\pi^{+}$	SCS	$0.76^{+0.01}_{-0.01}$	$0.14^{+0.06+0.12}_{-0.04-0.08}$	$0.28^{+0.06+0.13}_{-0.05-0.10}$	$-0.89^{+0.06+0.10}_{-0.02-0.04}$	$0.22^{+0.08+0.11}_{-0.05-0.09}$	$0.40^{+0.05+0.12}_{-0.03-0.06}$
$\Xi_{cc}^{+} \to \Xi_c^{\prime 0}K^{+}$	SCS	$0.47^{+0.02}_{-0.02}$	$0.05^{+0.03+0.06}_{-0.02-0.03}$	$0.41^{+0.00+0.01}_{-0.00-0.01}$	$-0.69^{+0.12+0.23}_{-0.08-0.16}$	$-0.34^{+0.07+0.15}_{-0.07-0.18}$	$0.64^{+0.06+0.08}_{-0.08-0.14}$
$\Xi_{cc}^{+} \to \Xi_c^{0}K^{+}$	SCS	$0.82^{+0.03}_{-0.03}$	$0.01^{+0.00+0.01}_{-0.00-0.00}$	$0.85^{+0.01+0.00}_{-0.01-0.00}$	$-0.55^{+0.01+0.03}_{-0.01-0.03}$	$-0.00^{+0.00+0.01}_{-0.00-0.02}$	$-0.83^{+0.01+0.02}_{-0.01-0.02}$
$\Xi_{cc}^{+} \to \Sigma_c^{0}K^{+}$	DCS	$1.00^{+0.04}_{-0.04}$	$\cdots$	$1.00^{+0.04}_{-0.04}$	-0.98	$\cdots$	0.18
$\Omega_{cc}^{+} \to \Omega_c^{0}\pi^{+}$	CF	$68.10^{+1.26}_{-1.25}$	$\cdots$	$68.10^{+1.26}_{-1.25}$	-0.96	$\cdots$	0.28
$\Omega_{cc}^{+} \to \Xi_c^{0}\pi^{+}$	SCS	$2.46^{+0.05}_{-0.05}$	$0.09^{+0.06+0.09}_{-0.03-0.05}$	$2.99^{+0.21+0.27}_{-0.15-0.21}$	$-0.34^{+0.03+0.06}_{-0.03-0.06}$	$0.05^{+0.00+-0.00}_{-0.01-0.01}$	$-0.94^{+0.01+0.02}_{-0.01-0.02}$
$\Omega_{cc}^{+} \to \Xi_c^{\prime 0}\pi^{+}$	SCS	$1.75^{+0.03}_{-0.03}$	$0.34^{+0.17+0.30}_{-0.10-0.19}$	$0.96^{+0.07+0.18}_{-0.00-0.10}$	$-0.74^{+0.17+0.33}_{-0.09-0.17}$	$0.66^{+0.07+0.18}_{-0.13-0.24}$	$-0.15^{+0.11+0.19}_{-0.15-0.21}$
$\Omega_{cc}^{+} \to \Omega_c^{0}K^{+}$	SCS	$4.43^{+0.17}_{-0.17}$	$0.03^{+0.02+0.02}_{-0.01-0.02}$	$5.03^{+0.24+0.25}_{-0.20-0.21}$	$-0.98^{+0.00+-0.00}_{-0.00-0.00}$	$-0.08^{+0.02+0.02}_{-0.03-0.01}$	$0.17^{+0.01+0.00}_{-0.00-0.01}$
$\Omega_{cc}^{+} \to \Xi_c^{0}K^{+}$	DCS	$0.20^{+0.01}_{-0.01}$	$0.00^{+0.00+0.00}_{-0.00-0.00}$	$0.17^{+0.00+0.01}_{-0.00-0.01}$	$-0.50^{+0.01+0.01}_{-0.01-0.01}$	$-0.13^{+0.03+0.05}_{-0.04-0.06}$	$-0.86^{+0.01+0.02}_{-0.01-0.01}$
$\Omega_{cc}^{+} \to \Xi_c^{\prime 0}K^{+}$	DCS	$0.12^{+0.00}_{-0.00}$	$0.01^{+0.00+0.01}_{-0.00-0.00}$	$0.13^{+0.00+0.01}_{-0.00-0.00}$	$-0.85^{+0.05+0.09}_{-0.04-0.07}$	$-0.28^{+0.05+0.10}_{-0.06-0.11}$	$0.45^{+0.04+0.08}_{-0.05-0.08}$

The branching ratio of $\Xi_{bc}^{+} \to J/\Psi \Xi_{c}^{+}$

$1/2^{+} \to 1/2^{+}$



Short-distance amplitudes under the factorization hypothesis:

$$\langle \Xi_c J/\Psi | \mathcal{H}_{eff} | \Xi_{bc} \rangle_{SD}^{C} = \frac{G_F}{\sqrt{2}} V_{cs}^{*} V_{bc} a_2(\mu) \langle J/\Psi | \bar{c}\gamma^{\mu}(1-\gamma_5)c|0\rangle \langle \Xi_c | \bar{s}\gamma_{\mu}(1-\gamma_5) b | \Xi_{bc}\rangle$$

$$\langle J/\Psi(q)|\bar{c}\gamma^{\mu}(1-\gamma_5)c|0\rangle = m_{J/\Psi} f_{J/\Psi} \epsilon^{*\mu}(q),$$

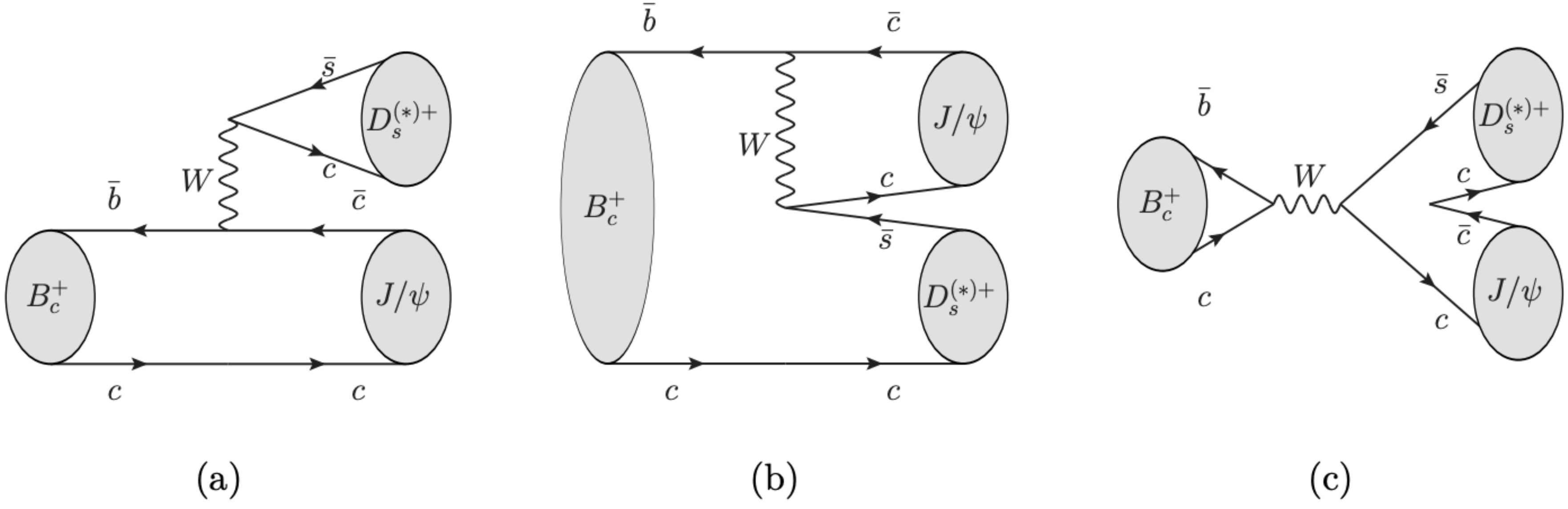
$$\mathcal{B}(J/\psi \to \mu^+\mu^-) = (5.961 \pm 0.033)\%,$$

$$\begin{aligned}\langle \Xi_c(p_f)|(V-A)_{\mu}|\Xi_{bc}(p_i)\rangle = \bar{u}(p_f,s_f)\left[\gamma_{\mu} f_1(q^2) + i\sigma_{\mu\nu}\frac{q^{\nu}}{m_i} f_2(q^2) + \frac{q_{\mu}}{m_i} f_3(q^2)\right] u(p_i,s_i) \\ -\bar{u}(p_f,s_f)\left[\gamma_{\mu} g_1(q^2) + i\sigma_{\mu\nu}\frac{q^{\nu}}{m_i} g_2(q^2) + \frac{q_{\mu}}{m_i} g_3(q^2)\right]\gamma_5 u(p_i,s_i),\end{aligned}$$

$\sim 2\times 10^{-7}$

color surpressed $Br(\Xi_{bc}^{+} \to J/\Psi \Xi_{c}^{+})$/ $Br(B_c^{+} \to J/\Psi D_s^{+})$~(1/3)^2~1/10

The branching ratio of $B_c^+ \to J/\Psi D_s^+$



$$R_{D_s^{(*)+}/\pi^+} = \frac{\mathcal{B}(B_c^+ \to J/\psi D_s^{(*)+})}{\mathcal{B}(B_c^+ \to J/\psi \pi^+)}$$

$$R_{D_s^+/\pi^+} = 2.76 \pm 0.33 \pm 0.29 \pm 0.16$$

JHEP 08(2022), 087

The ATLAS collaboration

$$R_{D_s^+/\pi^+} = 2.90 \pm 0.57 \pm 0.24$$

Phys. Rev. D 87, 112012 (2013)

LHCb collaboration

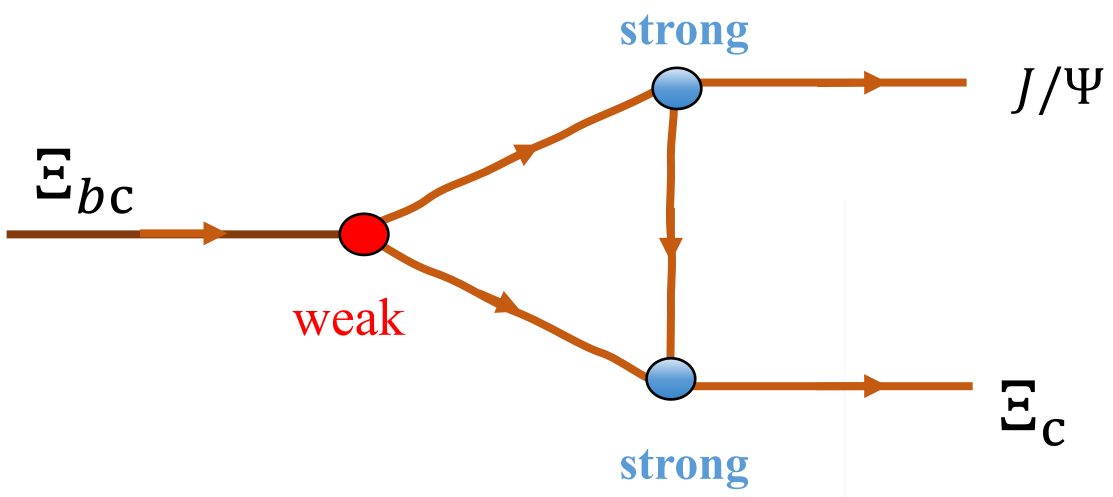
$Br(B_c^+ \to J/\Psi\pi^+)$=1.97×$10^{-3}$

Eur. Phys. J. C83(6), 477 (2023)

$$Br(B_c^+ \to J/\Psi D_s^+) = \begin{cases} 5.44\times10^{-3} (ATLAS) \\ 5.71\times10^{-3} (LHCb) \end{cases}$$

Long distance contribution

The triangle diagram



H. Y. Cheng, C. K. Chua and A. Soni,
Final state interactions in hadronic B decays,
Phys. Rev. D 71, 014030 (2005)

The hadronic strong interaction Lagrangians

$$\mathcal{L}_{P\mathcal{B}_i\mathcal{B}_f} = g_{P\mathcal{B}_i\mathcal{B}_f} Tr[\overline{\mathcal{B}}_f i\gamma_5 P \mathcal{B}_i],$$

$$\mathcal{L}_{V\mathcal{B}_i\mathcal{B}_f} = f_{1V\mathcal{B}_i\mathcal{B}_f} Tr[\overline{\mathcal{B}}_f \gamma_\mu V^\mu \mathcal{B}_i] + \frac{f_{2V\mathcal{B}_i\mathcal{B}_f}}{m_{\mathcal{B}_i}+m_{\mathcal{B}_f}} Tr[\overline{\mathcal{B}}_f \sigma_{\mu\nu}\partial^\mu V^\nu \mathcal{B}_i],$$

$$\mathcal{L}_{DDJ/\psi} = i g_{DDJ/\psi}\psi^\alpha\left(\partial_\alpha \bar{D} D - D\partial_\alpha \bar{D}\right),$$

$$\mathcal{L}_{DD^*J/\psi} = -g_{D^*DJ/\psi}\epsilon^{\alpha\beta\rho\tau}\partial_\alpha\psi_\beta\left(\partial_\rho D^*_\tau \bar{D} + D\partial_\rho \bar{D}^*_\tau\right),$$

$$\mathcal{L}_{D^*D^*J/\psi} = i g_{D^*D^*J/\psi}\left[\psi^\alpha\left(\partial_\alpha D^{*\beta}\bar{D}^*_\beta - D^{*\beta}\partial_\alpha \bar{D}^*_\beta\right)\right.$$
$$\left. + \left(\partial_\alpha\psi^\beta D^*_\beta - \psi^\beta\partial_\alpha D^*_\beta\right)\bar{D}^{*\alpha} + D^{*\alpha}\left(\psi^\beta\partial_\alpha\bar{D}^*_\beta - \partial_\alpha\psi^\beta\bar{D}^*_\beta\right)\right].$$

[Yan, et al, PRD46,1148(1992)]
[Casalbuoni, et al, Phys. Rept.281,145(1997)]
[Meissner, Phys. Rept.161,213(1988)]
Q. Wu, et al, Phys. Rev. D 104, 7, 074011 (2021)
C. J. Xiao, etal, Phys. Rev. D 100, no.1, 014022 (2019)

Input parameters

Masses

- Bottom charmed baryon Ξ_{bc}^{+} — Theoretical prediction
- douly charmed baryon Ξ_{cc}^{++} — PDG
- singly charmd baryon Ξ_{c}^{+} — PDG
- D and D^* meson — PDG

Transition form factor

Theoretical clalculation:

- $\boxed{\text{light front quark model}}$
- light cone sum rules
- QCD sum rules

$\Xi_{bc} \to \Xi_c,\ \Xi_{bc} \to \Xi_{cc}$

	F	$F(0)$	$m_{\rm fit}$	δ	F	$F(0)$	$m_{\rm fit}$	δ
scalar (0+) diquarks	$f_1^{\Xi_{bc}^+\to\Xi_{cc}^{++}}$	0.550	4.45	0.43	$f_2^{\Xi_{bc}^+\to\Xi_{cc}^{++}}$	-0.230	4.07	0.47
	$g_1^{\Xi_{bc}^+\to\Xi_{cc}^{++}}$	0.530	4.57	0.44	$g_2^{\Xi_{bc}^+\to\Xi_{cc}^{++}}$	-0.043	3.90	0.48
axial vector (1+) diquarks.	$f_1^{\Xi_{bc}^+\to\Xi_{cc}^{++}}$	0.527	3.78	0.55	$f_2^{\Xi_{bc}^+\to\Xi_{cc}^{++}}$	0.525	3.91	0.48
	$g_1^{\Xi_{bc}^+\to\Xi_{cc}^{++}}$	-0.146	4.76	0.38	$g_2^{\Xi_{bc}^+\to\Xi_{cc}^{++}}$	-0.060	4.50	0.51

W. Wang, F. S. Yu and Z. X. Zhao, Eur. Phys. J. C 77(11), 781 (2017)

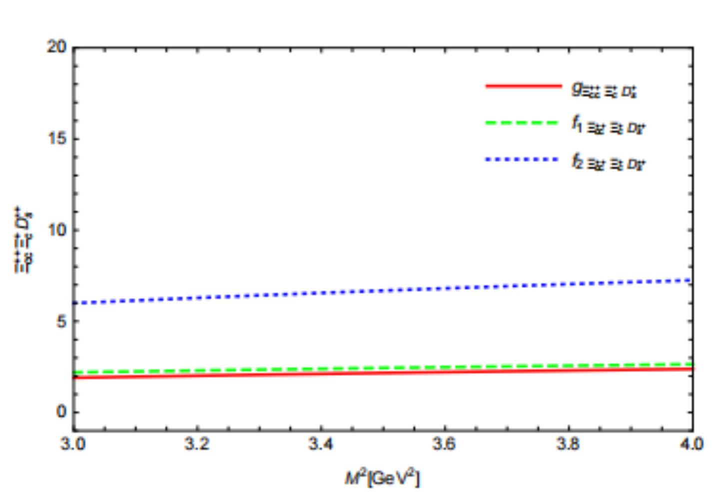
X. H. Hu, R. H. Li and Z. P. Xing, Eur. Phys. J. C 80(4), 320 (2020)

Decay constants and Strong couplings

Therotical clalculation

f_{D_s}	$f_{D_s^*}$	$f_{J/\Psi}$	
249.9 ± 0.5 [20]	314^{+19}_{-14} [90]	407 ± 5 [91]	
$g_{D_sD_sJ/\Psi}$	$g_{D_s^*D_sJ/\Psi}$	$g_{D_s^*D_s^*J/\Psi}$	
$6.24^{+0.41}_{-0.36}$ [92]	$4.39^{+1.44}_{-1.30}$ [93]	4.96 ± 1.42 [94]	
$g_{\Xi_{cc}^{++}\Xi_c^+D_s^+}$	$g_{\Xi_{cc}^{++}\Xi_c^+D_s^{*+}}$	$g_{\Lambda_c^+\Lambda^0D_s^+}$	$g_{\Lambda_c^+\Lambda^0D_s^{*+}}$
2.17 ± 0.68 [71]	$\{2.44 \pm 0.83, 6.68 \pm 2.38\}$ [71]	$-8.74^{+3.51}_{-4.32}$ [95]	$\{4.47^{+2.04}_{-1.71}, -2.94^{+1.47}_{2.37}\}$ [95]

Strong couplings(LCSRs)



$$g_{P\mathcal{B}_Q\mathcal{B}_{QQ'}} = \frac{ie^{m_P^2/M_2^2+m_{\mathcal{B}_{QQ'}}^2/M_1^2}}{f_P m_P^2 f_{\mathcal{B}_{QQ'}}} \frac{1}{\pi^2}\int_{s_{\min}}^{s_{\rm th}} ds\, e^{-s/M_1^2}\int_{t_1(s)}^{t_2(s)} ds'\, e^{-s'/M_2^2}\mathrm{Im}_s\,\mathrm{Im}_{s'}C_{\not{q}\gamma_5}(s,s'),$$

$$f_{1V\mathcal{B}_Q\mathcal{B}_{QQ'}} = \frac{-ie^{m_V^2/M_2^2+m_{\mathcal{B}_{QQ'}}^2/M_1^2}}{2m_{\mathcal{B}_Q} f_V m_V f_{\mathcal{B}_{QQ'}}} \frac{1}{\pi^2}\int_{s_{\min}}^{s_{\rm th}} ds\, e^{-s/M_1^2}\int_{t_1(s)}^{t_2(s)} ds'\, e^{-s'/M_2^2}\mathrm{Im}_s\,\mathrm{Im}_{s'}C_{v_\mu},$$

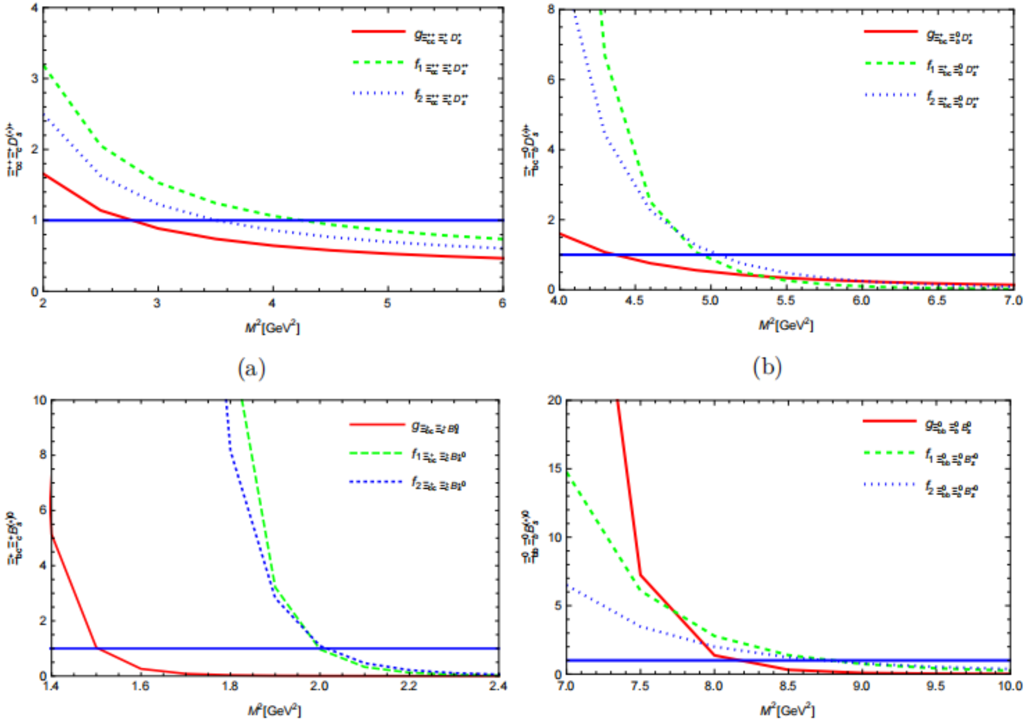
$$f_{2V\mathcal{B}_Q\mathcal{B}_{QQ'}} = \frac{-ie^{m_V^2/M_2^2+m_{\mathcal{B}_{QQ'}}^2/M_1^2}(m_{\mathcal{B}_Q}+m_{\mathcal{B}_{QQ'}})}{2m_{\mathcal{B}_Q} f_V m_V f_{\mathcal{B}_{QQ'}}} \times\frac{1}{\pi^2}\int_{s_{\min}}^{s_{\rm th}} ds\, e^{-s/M_1^2}\int_{t_1(s)}^{t_2(s)} ds'\, e^{-s'/M_2^2}\mathrm{Im}_s\,\mathrm{Im}_{s'}C_{v_\mu\not{q}}.$$

Suppress the contributions from the continuum spectrum：

$$\boxed{\mathrm{Pole/Conti} = \frac{\int_{s_{\min}}^{s_{\rm th}} ds \int_{t_1(s)}^{t_2(s)} ds'\ e^{-s^2/M_1^2}e^{-s'^2/M_2^2}\{\mathrm{Im}_s\,\mathrm{Im}_{s'}C(s,s')\}}{\int_{s_{\rm th}}^{\infty} ds \int_{t_1(s)}^{t_2(s)} ds'\ e^{-s^2/M_1^2}e^{-s'^2/M_2^2}\{\mathrm{Im}_s\,\mathrm{Im}_{s'}C(s,s')\}} > 1.}$$

Simplify the two Borel parameters into one ：

$$\frac{M_1^2}{M_2^2} = \frac{m_{\mathcal{B}_{QQ'}}^2-(m_Q+m_{Q'})^2}{m_{P_{Q'}}^2-m_{Q'}^2}.$$



(a) (b)

(c) (d)

Results

Threshold： $(m_{\mathcal{B}_{QQ}}+0.3\mathrm{GeV})^2 < s_{\mathrm{th}} < (m_{\mathcal{B}_{QQ}}+0.7\mathrm{GeV})^2,$

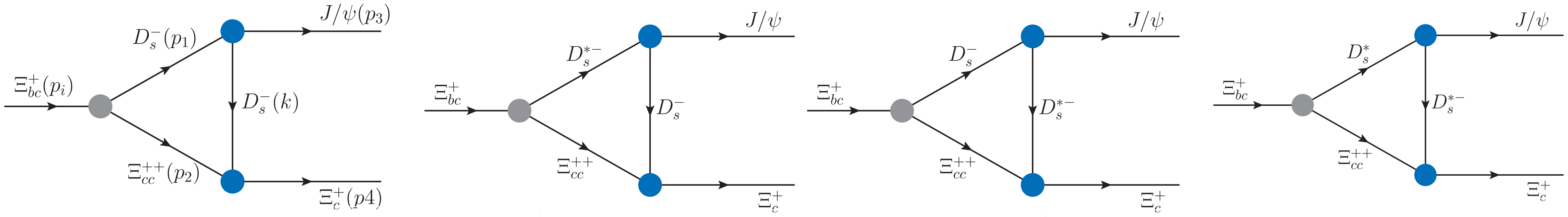
Coupling	s_{th} (GeV^2)	M^2 (GeV^2)
$\mathcal{B}_{cc}\mathcal{B}_c D$	$15\sim19$	$3\sim4$
$\mathcal{B}_{bc}\mathcal{B}_b D$	$51\sim57$	$4.5\sim5$
$\mathcal{B}_{bc}\mathcal{B}_c B$	$51\sim57$	$1.6\sim2.0$
$\mathcal{B}_{bb}\mathcal{B}_b B$	$107\sim117$	$8.0\sim8.5$

Coupling	SU(3)	g	Coupling	SU(3)	f_1	f_2
$\Xi_{cc}^{++}\Lambda_c^+D^+$	$a_1^{\bar{3}}$	$5.34\pm0.86\pm0.54\pm0.58$	$\Xi_{cc}^{++}\Lambda_c^+D^{*+}$	$b_1^{\bar{3}}$	$6.73\pm1.09\pm0.69\pm0.49$	$19.60\pm3.15\pm2.23\pm1.19$
$\Xi_{cc}^{++}\Xi_c^+D_s^+$	$a_1^{\bar{3}}$	$2.17\pm0.35\pm0.22\pm0.11$	$\Xi_{cc}^{++}\Xi_c^+D_s^{*+}$	$b_1^{\bar{3}}$	$2.44\pm0.39\pm0.20\pm0.24$	$6.68\pm1.08\pm0.57\pm0.73$
$\Xi_{cc}^{+}\Lambda_c^+D^0$	$-a_1^{\bar{3}}$	$-5.30\pm0.86\pm0.54\pm0.58$	$\Xi_{cc}^{+}\Lambda_c^+D^{*0}$	$-b_1^{\bar{3}}$	$-6.70\pm1.08\pm0.69\pm0.49$	$-19.50\pm3.14\pm2.24\pm1.19$
$\Xi_{cc}^{+}\Xi_c^0D_s^+$	$a_1^{\bar{3}}$	$2.17\pm0.35\pm0.22\pm0.11$	$\Xi_{cc}^{+}\to\Xi_c^0D_s^{*+}$	$b_1^{\bar{3}}$	$2.44\pm0.39\pm0.20\pm0.24$	$6.68\pm1.08\pm0.57\pm0.73$
$\Omega_{cc}^{+}\Xi_c^+D^0$	$-a_1^{\bar{3}}$	$-1.82\pm0.29\pm0.23\pm0.14$	$\Omega_{cc}^{+}\to\Xi_c^+D^{*0}$	$-b_1^{\bar{3}}$	$-2.20\pm0.31\pm0.23\pm0.23$	$-6.13\pm0.78\pm0.67\pm0.70$
$\Omega_{cc}^{+}\Xi_c^0D^+$	$-a_1^{\bar{3}}$	$-1.83\pm0.30\pm0.23\pm0.13$	$\Omega_{cc}^{+}\to\Xi_c^0D^{*+}$	$-b_1^{\bar{3}}$	$-2.21\pm0.31\pm0.23\pm0.23$	$-6.15\pm0.78\pm0.66\pm0.70$
$\Xi_{bc}^{+}\Lambda_b^0D^+$	$a_3^{\bar{3}}$	$5.39\pm0.89\pm0.48\pm0.14$	$\Xi_{bc}^{+}\to\Lambda_b^0D^{*+}$	$b_3^{\bar{3}}$	$16.30\pm2.68\pm1.48\pm0.42$	$43.30\pm6.75\pm3.94\pm1.04$
$\Xi_{bc}^{+}\Xi_b^0D_s^+$	$a_3^{\bar{3}}$	$6.62\pm0.90\pm0.59\pm0.44$	$\Xi_{bc}^{+}\to\Xi_b^0D_s^{*+}$	$b_3^{\bar{3}}$	$10.70\pm1.46\pm0.95\pm0.27$	$34.50\pm4.26\pm3.07\pm1.44$
$\Xi_{bc}^{0}\Lambda_b^0D^0$	$-a_3^{\bar{3}}$	$-5.39\pm0.89\pm0.49\pm0.14$	$\Xi_{bc}^{0}\to\Lambda_b^0D^{*0}$	$-b_3^{\bar{3}}$	$-16.20\pm2.68\pm1.48\pm0.42$	$-43.20\pm6.74\pm3.93\pm1.04$
$\Xi_{bc}^{0}\Xi_b^-D_s^+$	$a_3^{\bar{3}}$	$12.50\pm1.74\pm1.10\pm0.59$	$\Xi_{bc}^{0}\to\Xi_b^-D_s^{*+}$	$-b_3^{\bar{3}}$	$20.40\pm2.67\pm1.80\pm0.58$	$68.00\pm8.31\pm6.01\pm2.50$
$\Omega_{bc}^{0}\Xi_b^0D^0$	$-a_3^{\bar{3}}$	$-7.02\pm1.03\pm0.62\pm0.49$	$\Omega_{bc}^{0}\to\Xi_b^0D^{*0}$	$-b_3^{\bar{3}}$	$-11.20\pm1.64\pm1.01\pm0.34$	$-37.10\pm5.03\pm3.30\pm1.71$
$\Omega_{bc}^{0}\Xi_b^-D^+$	$-a_3^{\bar{3}}$	$-13.10\pm1.95\pm1.16\pm0.63$	$\Omega_{bc}^{0}\to\Xi_b^-D^{*+}$	$-b_3^{\bar{3}}$	$-21.80\pm3.10\pm1.93\pm0.67$	$-73.30\pm9.92\pm6.49\pm2.83$
$\Xi_{bc}^{+}\Lambda_c^+\bar{B}^0$	$a_2^{\bar{3}}$	$10.80\pm1.69\pm3.01\pm1.08$	$\Xi_{bc}^{+}\to\Lambda_c^+\bar{B}^{*0}$	$b_2^{\bar{3}}$	$87.80\pm13.50\pm17.80\pm8.73$	$126.00\pm18.20\pm25.60\pm12.60$
$\Xi_{bc}^{+}\Xi_c^+\bar{B}_s^0$	$a_2^{\bar{3}}$	$5.28\pm0.65\pm1.01\pm0.37$	$\Xi_{bc}^{+}\to\Xi_c^+\bar{B}_s^{*0}$	$b_2^{\bar{3}}$	$40.40\pm5.01\pm8.30\pm2.50$	$59.60\pm6.71\pm10.70\pm3.56$
$\Xi_{bc}^{0}\Lambda_c^+B^-$	$-a_2^{\bar{3}}$	$-10.80\pm1.68\pm3.00\pm1.07$	$\Xi_{bc}^{0}\to\Lambda_c^+B^{*-}$	$-b_2^{\bar{3}}$	$-87.80\pm13.50\pm17.80\pm8.73$	$-126.00\pm18.20\pm25.60\pm12.60$
$\Xi_{bc}^{0}\Xi_c^+\bar{B}_s^0$	$a_2^{\bar{3}}$	$5.28\pm0.65\pm1.01\pm0.37$	$\Xi_{bc}^{0}\to\Xi_c^+\bar{B}_s^{*0}$	$b_2^{\bar{3}}$	$40.40\pm5.01\pm8.30\pm2.50$	$59.60\pm6.71\pm10.70\pm3.56$
$\Omega_{bc}^{0}\Xi_c^+B^-$	$-a_2^{\bar{3}}$	$-1.37\pm0.19\pm4.59\pm0.98$	$\Omega_{bc}^{0}\to\Xi_c^+B^{*-}$	$-b_2^{\bar{3}}$	$-9.86\pm1.34\pm3.53\pm0.56$	$-14.70\pm1.85\pm4.85\pm0.90$
$\Omega_{bc}^{0}\Xi_c^0\bar{B}^0$	$-a_2^{\bar{3}}$	$-1.38\pm0.19\pm4.60\pm0.99$	$\Omega_{bc}^{0}\to\Xi_c^0\bar{B}^{*0}$	$-b_2^{\bar{3}}$	$-9.86\pm1.34\pm3.53\pm0.56$	$-14.70\pm1.85\pm4.85\pm0.85$
$\Xi_{bb}^{0}\Lambda_b^0\bar{B}^0$	$a_4^{\bar{3}}$	$2.76\pm0.50\pm1.83\pm0.28$	$\Xi_{bb}^{0}\to\Lambda_b^0\bar{B}^{*0}$	$b_4^{\bar{3}}$	$16.90\pm3.05\pm1.11\pm1.68$	$31.80\pm5.59\pm2.10\pm3.17$
$\Xi_{bb}^{0}\Xi_b^0\bar{B}_s^0$	$a_4^{\bar{3}}$	$3.65\pm0.59\pm2.38\pm0.89$	$\Xi_{bb}^{0}\to\Xi_b^0\bar{B}_s^{*0}$	$b_4^{\bar{3}}$	$18.70\pm3.12\pm1.23\pm1.93$	$36.60\pm5.81\pm2.39\pm3.83$
$\Xi_{bb}^{-}\Lambda_b^0B^-$	$-a_4^{\bar{3}}$	$-2.76\pm0.50\pm1.83\pm0.28$	$\Xi_{bb}^{-}\to\Lambda_b^0B^{*-}$	$-b_4^{\bar{3}}$	$-16.90\pm3.05\pm1.11\pm1.68$	$-31.80\pm5.59\pm2.10\pm3.17$
$\Xi_{bb}^{-}\Xi_b^-\bar{B}_s^0$	$a_4^{\bar{3}}$	$1.55\pm0.28\pm1.04\pm0.73$	$\Xi_{bb}^{-}\to\Xi_b^-\bar{B}_s^{*0}$	$b_4^{\bar{3}}$	$10.40\pm1.91\pm1.69\pm1.37$	$18.90\pm3.33\pm1.27\pm1.75$
$\Omega_{bb}^{-}\Xi_b^0B^-$	$-a_4^{\bar{3}}$	$-2.97\pm0.48\pm1.95\pm0.54$	$\Omega_{bb}^{-}\to\Xi_b^0B^{*-}$	$-b_4^{\bar{3}}$	$-15.10\pm2.52\pm1.99\pm1.75$	$-29.90\pm4.76\pm1.96\pm2.50$
$\Omega_{bb}^{-}\Xi_b^-\bar{B}^0$	$-a_4^{\bar{3}}$	$-1.21\pm0.21\pm0.82\pm0.58$	$\Omega_{bb}^{-}\to\Xi_b^-\bar{B}^{*0}$	$-b_4^{\bar{3}}$	$-8.12\pm1.46\pm5.51\pm0.83$	$-14.90\pm2.57\pm1.01\pm1.61$

Eur. Phys. J. C 85 (2025) 6, 625

Long-distance contributions

$$\mathcal{A}(\Xi_{bc}\to\Xi_c J/\Psi)=\mathcal{M}_{SD}(\Xi_{bc}\to\Xi_c J/\Psi)+\mathcal{M}[D_s^-,\Xi_{cc}^{++};D_s^-]+\mathcal{M}[D_s^{*-},\Xi_{cc}^{++};D_s^-]$$
$$+\mathcal{M}[D_s^-,\Xi_{cc}^{++};D_s^{*-}]+\mathcal{M}[D_s^{*-},\Xi_{cc}^{++};D_s^{*-}],$$



$$M[D_s^-,\Xi_{cc}^{++};D_s^-]=\int\frac{d^4p_1}{(2\pi)^4}(2\pi)^4\delta^4(p_3+p_4-p_1)g^D_{\psi DD}g_{\Xi_{cc}\Xi_c D_s}\mathcal{PF}$$
$$\times\,\epsilon^\alpha(p_3)(2p_{1\alpha}-p_{3\alpha})\bar{u}_{\Xi_c}\gamma_5(\not{p}_2+m_2)\left[A_{LD}+B_{LD}\gamma_5\right]u_{\Xi_{bc}},$$

$$M[D_s^{*-},\Xi_{cc}^{++};D_s^-]=\int\frac{d^4p_1}{(2\pi)^4}(2\pi)^4\delta^4(p_3+p_4-p_1)g^D_{D^*DJ/\psi}g_{\Xi_{cc}\Xi_c D_s}\mathcal{PF}$$
$$\times\,\epsilon^{\alpha\beta\rho\tau}\epsilon_\alpha(p_3)\epsilon_\beta(p_1)p_{1\rho}p_{3\tau}\bar{u}_{\Xi_c}\gamma_5(\not{p}_2+m_2)$$
$$\times\,\epsilon^{*\mu}(p_1)\left[A^1_{LD}\gamma_\mu\gamma_5+A^2_{LD}\frac{p_{2\mu}}{m_i}\gamma_5+B^1_{LD}\gamma_\mu+B^2_{LD}\frac{p_{2\mu}}{m_i}\right]u_{\Xi_{bc}},$$

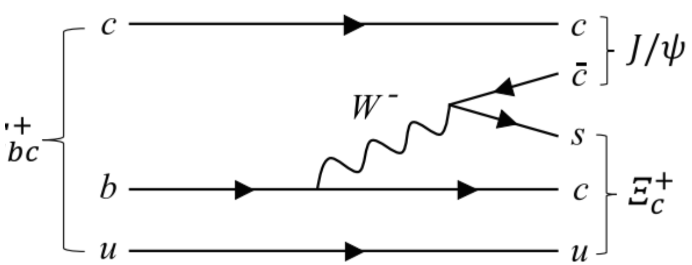
$$\boxed{\mathcal{PF}\;=\;\frac{1}{(p_1^2-m_1^2+i\epsilon)(p_2^2-m_2^2+i\epsilon)(p_k^2-m_k^2+i\epsilon)}\left(\frac{\Lambda_k^2-m_k^2}{\Lambda_k^2-p_k^2}\right)}$$

$$\Lambda_k=m_k+\eta\Lambda_{\rm QCD}.$$

Amplitudes

$$\langle \Xi_c J/\Psi | \mathcal{H}_{eff} | \Xi_{bc} \rangle_{SD}^{C} = \frac{G_F}{\sqrt{2}} V_{cs}^{*} V_{bc} a_2(\mu) \langle J/\Psi | \bar{c}\gamma^{\mu}(1-\gamma_5)c|0\rangle \langle \Xi_c | \bar{s}\gamma_{\mu}(1-\gamma_5) b | \Xi_{bc}\rangle,$$

$$\mathcal{A}(\Xi_{bc} \to \Xi_c J/\Psi) = \epsilon^{*\mu} \bar{u}_{\Xi_c} \left[A_{SD}^{1}\gamma_{\mu}\gamma_5 + A_{SD}^{2}\frac{p_{f\mu}}{m_i}\gamma_5 + B_{SD}^{1}\gamma_{\mu} + B_{SD}^{2}\frac{p_{f\mu}}{m_i} \right] u_{\Xi_{bc}}.$$

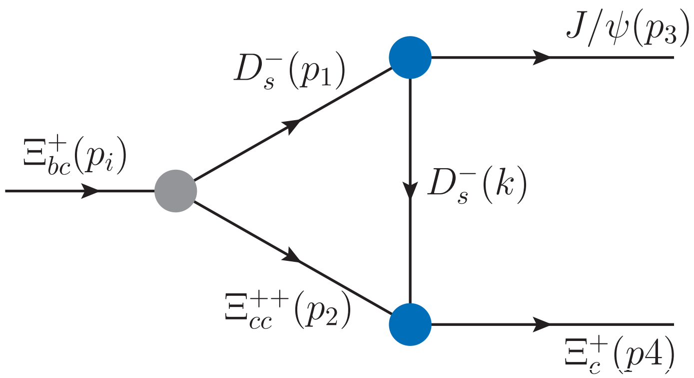


$$A_{SD}^{1} = -\lambda f_{J/\Psi} m_{J/\Psi}\left(g_1(q^2) + g_2(q^2)\frac{m_i - m_f}{m_i}\right), \qquad A_{SD}^{2} = -2\lambda f_{J/\Psi} m_{J/\Psi} g_2(q^2),$$

$$B_{SD}^{1} = \lambda f_{J/\Psi} m_{J/\Psi}\left(f_1(q^2) - f_2(q^2)\frac{m_i + m_f}{m_i}\right), \qquad B_{SD}^{2} = 2\lambda f_{J/\Psi} m_{J/\Psi} f_2(q^2),$$

$$\langle \Xi_{cc} D_s | \mathcal{H}_{eff} | \Xi_{bc}\rangle^{T} = i\bar{u}_{\Xi_{cc}}\left[A_{LD} + B_{LD}\gamma_5\right] u_{\Xi_{bc}}.$$

$$\langle \Xi_{cc} D_s^{*} | \mathcal{H}_{eff} | \Xi_{bc}\rangle^{T} = \epsilon^{*\mu}(p_1)\bar{u}_{\Xi_{cc}}\left[A_{LD}^{1}\gamma_{\mu}\gamma_5 + A_{LD}^{2}\frac{p_{2\mu}}{m_i}\gamma_5 + B_{LD}^{1}\gamma_{\mu} + B_{LD}^{2}\frac{p_{2\mu}}{m_i}\right] u_{\Xi_{bc}}.$$



$$A_{LD} = \lambda f_{D_s}(m_i - m_2) f_1(q^2), \qquad B_{LD} = \lambda f_{D_s}(m_i + m_2) g_1(q^2),$$

$$A_{LD}^{1} = -\lambda f_{D_s^*} m_{D_s^*}\left(g_1(q^2) + g_2(q^2)\frac{m_i - m_2}{m_i}\right), \qquad A_{LD}^{2} = -2\lambda f_{D_s^*} m_{D_s^*} g_2(q^2),$$

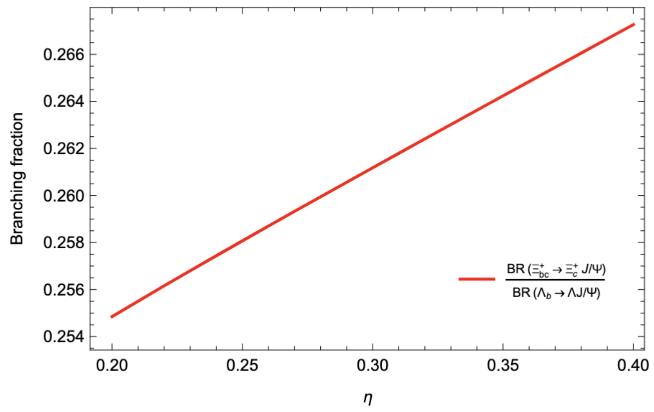
$$B_{LD}^{1} = \lambda f_{D_s^*} m_{D_s^*}\left(f_1(q^2) - f_2(q^2)\frac{m_i + m_2}{m_i}\right), \qquad B_{LD}^{2} = 2\lambda f_{D_s^*} m_{D_s^*} f_2(q^2),$$

Obsevations

$$\Gamma(\Xi_{bc} \to \Xi_c J/\Psi) = \frac{|\vec{p}_c|(E_f + m_f)}{4\pi m_i}\left[2\left(|\mathcal{S}|^2 + |\mathcal{P}_2|^2\right) + \frac{E_{J/\Psi}^2}{m_{J/\Psi}^2}\left(|\mathcal{S}+\mathcal{D}|^2 + |\mathcal{P}_1|^2\right)\right]$$

$$= \frac{|\vec{p}_c|}{8\pi m_i^2}\frac{1}{2}\sum_{\lambda_i,\lambda_f,\lambda_V}|H^{\lambda_i}_{\lambda_f,\lambda_V}(\Xi_{bc}\to\Xi_c J/\Psi)|^2,$$

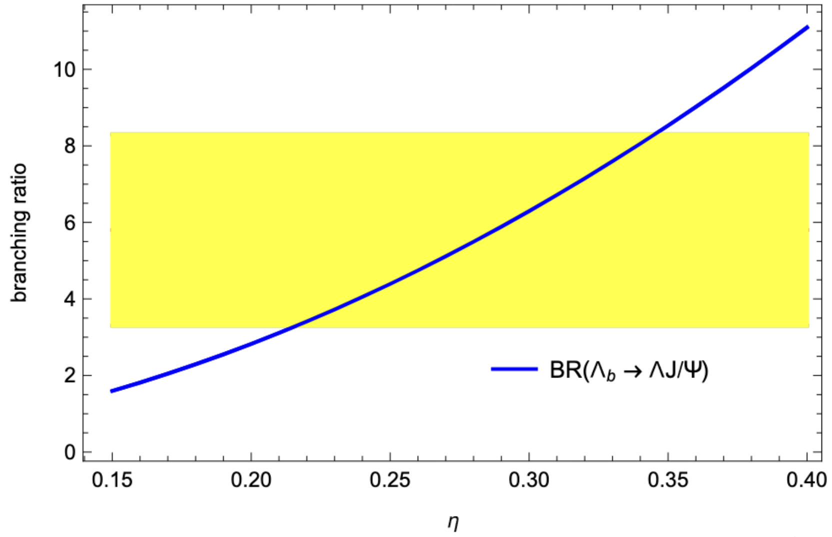
$$\mathcal{BR}_i = \Gamma_i \cdot \tau.$$

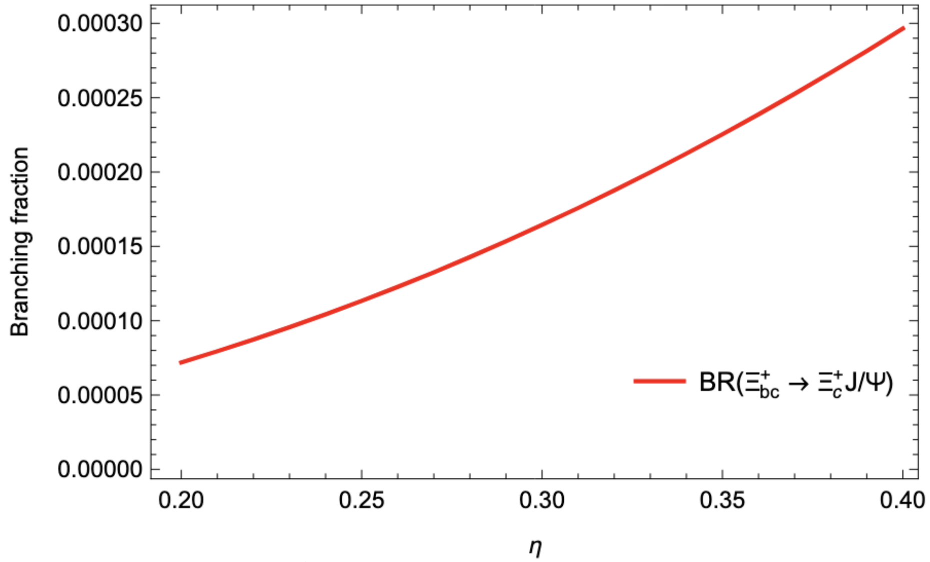


$f(b \to \Lambda_b) = (7 - 17.5)\%$

$$f(b \to \Lambda_b) \times \mathcal{B}(\Lambda_b \to \Lambda J/\psi) = (5.8 \pm 0.8) \times 10^{-5},$$

$$Br(\Lambda_b \to \Lambda J/\Psi) = (3.3 \sim 8.3) \times 10^{-4}$$





$$\eta = [0.2, 0.35]$$

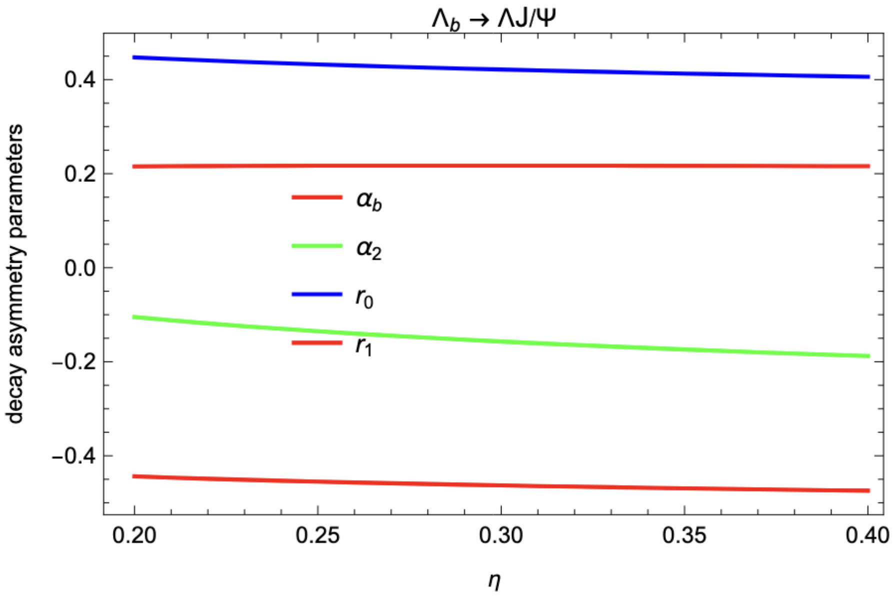
$Br(\Xi_{bc}^+ \to \Xi_c^+ J/\Psi)$=(0.5~2.5)×$10^{-4}$

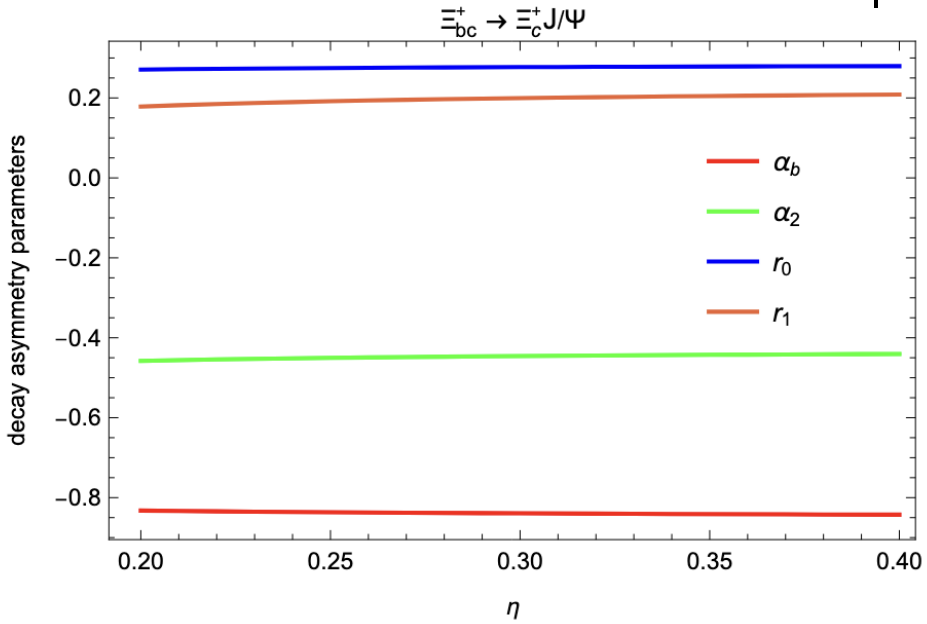
Decay asymmetry parameters

$$
\begin{aligned}
HV^{\frac{1}{2}}_{-\frac{1}{2},0} &= HV^{-\frac{1}{2}}_{\frac{1}{2},0} = -i\sqrt{\frac{Q_-}{q^2}}\left((m_i+m_f)f_1 - \frac{q^2}{m_i}f_2\right), \\
HV^{\frac{1}{2}}_{\frac{1}{2},1} &= HV^{-\frac{1}{2}}_{-\frac{1}{2},-1} = i\sqrt{2Q_-}\left(-f_1 + \frac{m_i+m_f}{m_i}f_2\right), \\
HA^{\frac{1}{2}}_{-\frac{1}{2},0} &= -HA^{-\frac{1}{2}}_{\frac{1}{2},0} = -i\sqrt{\frac{Q_+}{q^2}}\left((m_i-m_f)g_1 + \frac{q^2}{m_i}g_2\right), \\
HA^{\frac{1}{2}}_{\frac{1}{2},1} &= -HA^{-\frac{1}{2}}_{-\frac{1}{2},-1} = i\sqrt{2Q_+}\left(-g_1 - \frac{m_i-m_f}{m_i}g_2\right).
\end{aligned}
$$

$$
\begin{aligned}
\alpha_b &= -|\hat{H}^{\frac{1}{2}}_{\frac{1}{2},1}|^2 + |\hat{H}^{-\frac{1}{2}}_{-\frac{1}{2},-1}|^2 + |\hat{H}^{-\frac{1}{2}}_{\frac{1}{2},0}|^2 - |\hat{H}^{\frac{1}{2}}_{-\frac{1}{2},0}|^2, \\
\alpha_2 &= |\hat{H}^{\frac{1}{2}}_{\frac{1}{2},1}|^2 - |\hat{H}^{-\frac{1}{2}}_{-\frac{1}{2},-1}|^2 + |\hat{H}^{-\frac{1}{2}}_{\frac{1}{2},0}|^2 - |\hat{H}^{\frac{1}{2}}_{-\frac{1}{2},0}|^2, \\
r_0 &= |\hat{H}^{\frac{1}{2}}_{\frac{1}{2},1}|^2 + |\hat{H}^{\frac{1}{2}}_{-\frac{1}{2},0}|^2, \\
r_1 &= |\hat{H}^{\frac{1}{2}}_{\frac{1}{2},1}|^2 - |\hat{H}^{\frac{1}{2}}_{-\frac{1}{2},0}|^2,
\end{aligned}
$$

These asymmetry parameters have a small dependence on the parameter η.





Branching ratio and asymmetery parameters

Mode	$\mathcal{B}$	α_b	α_2	r_0	r_1
$\Lambda_b \to \Lambda J/\psi$					
This work	$6.30^{+4.80}_{-3.47}$	$-0.46^{+0.02}_{-0.01}$	$-0.16^{+0.05}_{-0.03}$	$0.42^{+0.03}_{-0.02}$	$0.22^{+0.01}_{-0.01}$
pQCD [108]	$7.75^{+4.08+1.39}_{-3.17-0.96}$	$0.042^{+0.104+0.027}_{-0.069-0.001}$	$-0.84^{+0.05+0.01}_{-0.01-0.00}$	$0.47^{+0.04+0.00}_{-0.05-0.02}$	$-0.40^{+0.07+0.01}_{-0.04-0.00}$
CCQM [39]	8.0	-0.069	-0.995	0.533	-0.532
LHCb [21]	$\cdots$	$0.05 \pm 0.17 \pm 0.17$	$\cdots$	$0.58 \pm 0.02 \pm 0.01$	$-0.56 \pm 0.10 \pm 0.05$
ATLAS [22]	$\cdots$	$0.30 \pm 0.16 \pm 0.16$	$\cdots$	$\cdots$	$\cdots$
CMS [23]	$\cdots$	$-0.14 \pm 0.14 \pm 0.10$	$-1.11 \pm 0.04 \pm 0.05$	$\cdots$	$\cdots$
$\Xi_{bc} \to \Xi_c J/\Psi$					
This work	$1.64^{+1.32}_{-0.09}$	$-0.84^{+0.01}_{-0.01}$	$-0.45^{+0.01}_{-0.00}$	$0.28^{+0.01}_{-0.00-0.02}$	$0.21^{+0.01}_{-0.01}$

The obtained branching ratios for the two modes are both of order 10^{-4}.

There is no experimental data on the absolute branching ratios since the fragmentation fraction of the b quark to Λ_b baryon are not well determined yet.

Summary

- Theoretical studies on the weak decays of bottom-charmed baryon contribute to experimental detection.
- **Applications of the LCSR method:**

Determined the strong couplings between double-heavy baryons, single-heavy baryons, and heavy-flavor mesons.

- **Analysis of weak decays in double-heavy baryons using the rescattering mechanism of final-state interactions:**
- Calculated the non-leptonic weak decay branching ratios of $\mathcal{B}_{cc} \to \mathcal{B}_c P$.
- Calculated the non-leptonic weak decay branching ratios of $\Xi_{bc}^{+} \to J/\Psi \Xi_c^{+}$.
- **Significance:** Our calculations provide critical guidance for experimental searches of doubly charmed baryons and bottom-charmed baryons.