Spectra of Baryonium States

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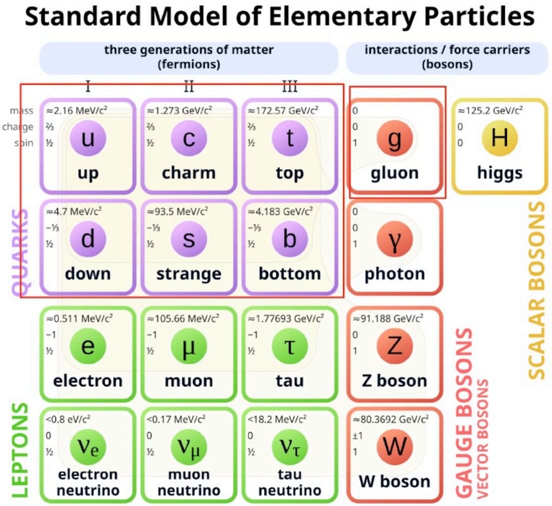
Outline

Section 1

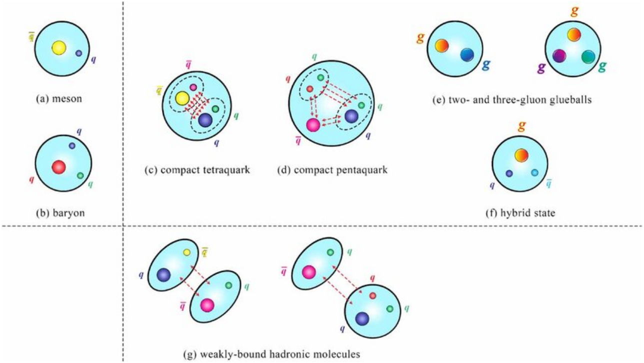
Review

Standard Model and QCD

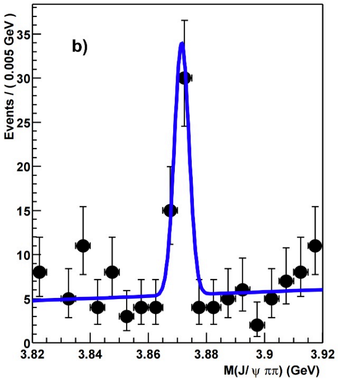


Hadronic States

- Color confinement and Asymptotic freedom – Hadrons – Non-perturbative effects
- Traditional Hadrons in Quark Model:
  - Mesons $(q\bar{q})$;
  - Baryons (qqq).
- QCD allows for hadrons beyond Quark Model – Exotic states:



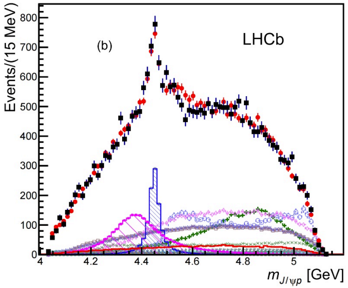
Observation of Exotic States



The Tetra-quark state $X(3872)$

$M_{X(3872)} = (3871.69 \pm 0.17)\text{MeV}$

[Belle.2003]



The Penta-quark state

$P_c(4380)^+,\ \ P_c(4450)^+$

$M_{4380} = (4380 \pm 8 \pm 29)\text{MeV}$

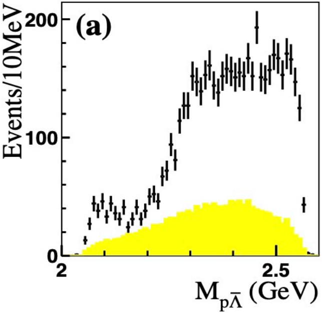
$M_{4450} = (4449.8 \pm 1.7 \pm 2.5)\text{MeV}$

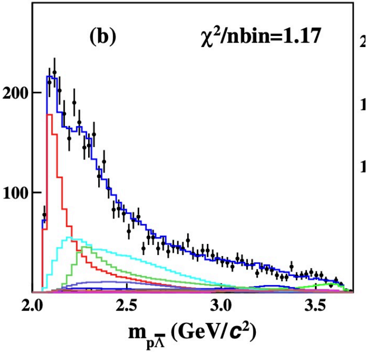
[LHCb.2015]

Hexaquark States

- Experimental candidates: $X(1835)$, $X(1860)$, $X(1880)$, $X(2075)$, $X(2085)$, $Y(4260)$, $Y(4660)$, etc.
- [E.Fermi, C.N.Yang, 1949]: Pions may be $N\bar{N}$ bound states.
- Dibaryon molecular states ($\mathscr{B}\mathscr{B}$) :
  - Deuteron: $J^P = 1^+$, $E_B = 2.225\mathrm{MeV}$ pn dibaryon bound state.
  - Dihyperon states: [R.L.Jaffe. 1977] etc. ;
  - Diproton states: [P.J.Mulders, et.al. 1980];
  - $\Lambda_c\Lambda_c$: [Z.G.Wang et.al. 2021]
- Baryon-antibaryon bound states $\rightarrow$ Baryonium ($\mathscr{B}\bar{\mathscr{B}}$, $\mathscr{B}\bar{\mathscr{B}}'$, $\mathscr{B}$ needless to be color singlet):
  - Hidden-charm: [C.F.Qiao, 2005/2007], [H.X.Chen, et.al. 2006], [Z.G.Wang, et.al. 2021], etc. ;
  - $\Lambda_Q\bar{\Lambda}_Q$: [B.D.Wan, L.Tang and C.F.Qiao. 2019];
  - Light Baryoniums: [B.S.Zou, H.C.Chiang. 2004], [S.L.Zhu, C.S.Gao. 2006], [Z.G.Wang, et.al. 2006], [B.D.Wan, S.Q.Zhang and C.F.Qiao. 2021], etc.
- Compact hexaquark states: [Z.G.Wang. 2022], etc.

$X(2075)$ and $X(2085)$





$M_{X(2075)} = (2075 \pm 12 \pm 5)\text{MeV},\ J = 1$
[BES.2003]

$M_{X(2085)} = (2086 \pm 4 \pm 6)\text{MeV}$
$J^P = 1^+$ [BESIII.2023]

- Theoretical explanation: $p\bar{\Lambda}$ and $p\bar{\Sigma}$ states in constituent quark models. [H.X.Huang, J.L.Ping and F.Wang, 2011].

Further potential structures of these states should be investigated!

Theoretical Methods

- Various methods :
    - Quark Model
    - MIT bag model
    - Chiral effective theories
    - Lattice QCD
    - NRQCD
    - AdS/QCD
    - QCD sum rules (QCDSR)
    - Light-cone sum rules (LCSR)
    - Inverse Problems
    - $\cdots$

[SVZ, 1979]

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QCD AND RESONANCE PHYSICS. THEORETICAL FOUNDATIONS

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Received 24 July 1978

QCD AND RESONANCE PHYSICS. APPLICATIONS

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- Advantages of QCDSR:

1. Based on the first principle of QCD.
2. Gives the analytical results.
3. Sum rules of perturbative and non-perturbative effects.
4. An effective and proven method on researching the properties of hadrons.

QCD Sum Rules for 2-Point Correlation Function

$$\Pi^{\mathrm{QCD}}\left(q^2\right)=\int_{s_{\min}}^{\infty}\mathrm{d}s\,\frac{\rho^{\mathrm{QCD}}(s)}{s-q^2},$$
$$\rho^{\mathrm{QCD}}(s)=\rho^{\mathrm{pert}}(s)+\rho^{\langle\bar{q}q\rangle}(s)+\rho^{\langle g_s^2G^2\rangle}(s)+\cdots,$$

$$L_0(s_0,M_B^2)=\int_{s_{\min}}^{s_0}\mathrm{d}s\;\rho^{\mathrm{OPE}}(s)\mathrm{e}^{-s/M_B^2}+\Pi^{\mathrm{sum}}(M_B^2)$$
$$L_1(s_0,M_B^2)=\frac{\partial}{\partial(M_B^{-2})}L_0(s_0,M_B^2)$$

$$\Pi\left(q^2\right)=\mathrm{i}\int\mathrm{d}^4x\;\mathrm{e}^{\mathrm{i}q\cdot x}\left\langle\Omega\left|T\left\{j(x),j^{\dagger}(0)\right\}\right|\Omega\right\rangle$$

2-Point Correlation Function

OPE

Quark Side

Dispersion Relation

Q-H Duality

$\Pi^{\mathrm{QCD}}(q^2)\approx\Pi^{\mathrm{PHEN}}(q^2)$

$$\lambda^2\mathrm{e}^{-m_h^2/M_B^2}=\int_{s_{\min}}^{s_0}\mathrm{d}s\;\rho^{\mathrm{QCD}}(s)\mathrm{e}^{-s/M_B^2}+\Pi^{\mathrm{sum}}(M_B^2)$$

Borel Transformation

Sum Rule

$$\rho(s)=\frac{1}{\pi}\mathrm{Im}\Pi(s)$$

Hadron Side

extract

$$m_h\left(s_0,M_B^2\right)=\sqrt{-\frac{L_1\left(s_0,M_B^2\right)}{L_0\left(s_0,M_B^2\right)}}$$

$$\lambda_h(s_0,M_B^2)=\sqrt{\mathrm{e}^{m_h^2(s_0,M_B^2)/M_B^2}L_0(s_0,M_B^2)}\,.$$

$$\Pi^{\mathrm{Phen}}\left(q^2\right)=\frac{\lambda_h^2}{m_h^2-q^2}+\int_{s_0}^{+\infty}\mathrm{d}s\,\frac{\rho^H(s)}{s-q^2}$$
$$\rho_h^{\mathrm{Phen}}(s)=\lambda_h^2\delta\left(s-m_h^2\right)+\theta\left(s-s_0\right)\rho^H(s)$$

2-point QCDSR gives the mass and decay constant of the hadrons.

QCDSR for Mesons

- ρ meson: [SVZ, 1979]

$$\left(\frac{4\pi}{g_\rho^2}\right)_{\mathrm{th}} \simeq 0.414\,, \qquad (m_\rho^2)_{\mathrm{th}} \simeq 0.6\ \mathrm{GeV}^2\,,$$

which is our final result for the ρ meson.

$$\left(\frac{g_\rho^2}{4\pi}\right)_{\mathrm{exp}} = 2.36 \pm 0.18\,.$$

$$m_{\rho(770)} = 775.26 \pm 0.23\mathrm{MeV} \quad \text{[PDG, 2024]}$$

- Pion: [SVZ, 1979]

$$f_\pi = m_\rho/2\pi \simeq 125\ \mathrm{MeV}\,,$$

which is to be compared with the experimental value

$$f_\pi|_{\mathrm{exper}} \simeq 133\ \mathrm{MeV}\,.$$

QCDSR for $X(3872)$

$$M_{X(3872)} = (3871.69 \pm 0.17)\mathrm{MeV}$$

Ref	Configuration	QCDSR results	State
W.Chen, et.al 2013	$\xi\bar{c}Gc + \sqrt{1-\xi^2}\bar{D}D^*$	$3.88\mathrm{GeV}$	molecular
Lee, et.al 2009	$D^0\bar{D}^{*0} - \bar{D}^0D^*$	$(3.88 \pm 0.06)\mathrm{GeV}$	molecular
Matheus, et.al 2007	$(q_a c_b)(\bar{q}_d\bar{c}_e)$	$(3925 \pm 127)\mathrm{MeV}$	compact

QCDSR gives accuracy results for the mass of $X(3872)$!

QCDSR for P_c States

$$M_{4380} = (4380 \pm 8 \pm 29)\text{MeV}, \;\; M_{4450} = (4449.8 \pm 1.7 \pm 2.5)\text{MeV}$$

Ref	Configuration	QCDSR results	State
Chen, Zhu, et.al 2015	$\bar{D}^* \Sigma_c$	$4.37^{+0.19}_{-0.12}\text{GeV}$	molecular
Chen, Zhu, et.al 2015	$\sin\theta \bar{D}\Sigma_c^* + \cos\theta \bar{D}^* \Lambda_c$	$4.47^{+0.20}_{-0.13}\text{GeV}$	molecular
Z.G.Wang 2016	$\bar{c}_a(u_j d_k)(u_m c_n)$	$(4.38 \pm 0.13)\text{GeV}$	compact
Z.G.Wang 2016	$\bar{c}_a(u_j d_k)(u_m c_n)$	$(4.44 \pm 0.14)\text{GeV}$	compact

QCDSR gives accuracy results for the mass of P_c states !

QCDSR for Baryoniums

$$M_{X(1835)} = (1833.7 \pm 6.7 \pm 2.1)\mathrm{MeV} \quad \text{[BESIII.2005]}$$

$$M_{X(1880)} = (1882.1 \pm 1.7 \pm 0.7)\mathrm{MeV} \quad \text{[BESIII.2023]}$$

$$M_{\Lambda\bar{\Lambda}\ \text{final state}} = (2356 \pm 7 \pm 17)\mathrm{MeV} \quad \text{[BESIII.2022]}$$

$$M_{Y(4660)} = (4664 \pm 11 \pm 5)\mathrm{MeV} \quad \text{[Belle.2007]}$$

Ref	Configuration	QCDSR results	Possible State
Z.G.Wang, et.al 2006	$p\bar{p}$, $J^{PC} = 0^{-+}$	(1.9 ± 0.1) GeV	$X(1835)$
B.D.Wan, et.al 2021	$p\bar{p}$, $J^{PC} = 0^{-+}/1^{--}$	$(1.81(2) \pm 0.10)$ GeV	$X(1880)$
B.D.Wan, et.al 2021	$\Lambda\bar{\Lambda}$, $J^{PC} = 1^{--}$	(2.34 ± 0.12)GeV	$\Lambda\bar{\Lambda}$ final state
L.Tang, et.al 2019	$\Lambda_c\bar{\Lambda}_c$, $J^{PC} = 0^{+-}$	(4.78 ± 0.23)GeV	$Y(4660)$

Configurations $\mathscr{B}\bar{\mathscr{B}}'$ and compact hexaquark states should be studied!
$\Rightarrow$ $X(2075)$ and $X(2085)$ in $p\bar{\Lambda}$ finial state.

Section 2

Theoretical Framework

Configurations $\mathscr{B}\bar{\mathscr{B}}'$ and Compact Hexaquark States

- $\mathscr{B}\bar{\mathscr{B}}'$ molecular states:

$$[3_c] \otimes [3_c] \otimes [3_c] = [10_c] \oplus [8_c] \oplus [8_c] \oplus [1_c],$$

$[1_c] - [1_c]$ forms the $\mathscr{B}\bar{\mathscr{B}}'$ states.
To explain $X(2075)$ and $X(2085)$, we choose $p\bar{\Lambda}$ and $p\bar{\Sigma}$ states.

- Compact hexaquark states:

$$[\bar{3}_c] \otimes [3_c] \otimes [3_c] = [15_c] \oplus [\bar{6}_c] \oplus [3_c] \oplus [3_c],$$

$qq\bar{q}$ forms a $[3_c]$ state,

$$[3_c] \otimes [\bar{3}_c] = [8_c] \oplus [1_c],$$

$[3_c] - [\bar{3}_c]$ forms the triquark-antitriquark states.
To explain $X(2075)$ and $X(2085)$, we choose structures $qq\bar{q} - \bar{q}\bar{q}\bar{s}$, which have the same strange number and quark components as $p\bar{\Lambda}$ and $p\bar{\Sigma}$.

Interpolating Currents

The chiral limit $m_u = m_d \to 0$ is taken to simplify the currents.

- Baryonic currents

$$\eta_{1\mathscr{B}}(x) = \mathrm{i}\varepsilon_{abc}\left[q_a^{iT}(x)\mathcal{C}q_b^j(x)\right]\gamma_5 q_c^k(x)\,,$$

$$\eta_{2\mathscr{B}}(x) = \mathrm{i}\varepsilon_{abc}\left[q_a^{iT}(x)\mathcal{C}\gamma_5 q_b^j(x)\right] q_c^k(x)\,,$$

where $(i, j, k) = (u, d, u)$ for p, (u, d, s) for Λ, and (u, s, d) for Σ.

- Baryonium currents of the $p\bar{\Lambda}$ and $p\bar{\Sigma}$ states with $J^P = 0^+, 0^-, 1^-, 1^+$

$$j^{0^-}(x) = \mathrm{i}\bar{\eta}_{\mathscr{B}'}(x)\gamma_5\eta_{\mathscr{B}}(x),$$

$$j^{0^+}(x) = \bar{\eta}_{\mathscr{B}'}(x)\eta_{\mathscr{B}}(x),$$

$$j_\mu^{1^-}(x) = \bar{\eta}_{\mathscr{B}'}(x)\gamma_\mu\eta_{\mathscr{B}}(x),$$

$$j_\mu^{1^+}(x) = \bar{\eta}_{\mathscr{B}'}(x)\gamma_\mu\gamma_5\eta_{\mathscr{B}}(x).$$

- $[3_c]$ triquark currents

$$\eta_1^a(x) = \mathrm{i}\left[\bar{q}_b^i(x)q^{a,j}(x)\right]\gamma_5 q^{b,k}(x)\,,$$

$$\eta_2^a(x) = \mathrm{i}\left[\bar{q}_b^i(x)\gamma_5 q^{a,j}(x)\right] q^{b,k}(x)\,,$$

where $(i, j, k) = (u, d, u)$ for $[3_c]_{udu}$, (u, d, s) for $[3_c]_{uds}$, and (u, s, d) for $[3_c]_{usd}$.

- Currents of the $[3_c]_{qqq}$-$[\bar{3}_c]_{qqs}$ and $[3_c]_{qqq}$-$[\bar{3}_c]_{qsq}$ states

$$j_{1(\mu)} = \bar{\eta}_{a;\bar{u}ds}\Gamma_{(\mu)}\eta^a_{\bar{u}du};$$

$$j_{2(\mu)} = \bar{\eta}_{a;\bar{u}sd}\Gamma_{(\mu)}\eta^a_{\bar{u}du};$$

where $\Gamma_{(\mu)} = \mathbb{1}, \mathrm{i}\gamma_5, \gamma_\mu, \gamma_\mu\gamma_5$ correspond to the 4 quantum numbers $0^+, 0^-, 1^-, 1^+$.

Quark Side

- 2-Point correlation functions

$$\text{For } J=0, \quad \Pi(q^2) = \mathrm{i}\int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}q\cdot x} \langle\Omega|\, \mathbb{T}\{j(x), j^\dagger(0)\}\, |\Omega\rangle\,,$$

$$\text{For } J=1, \quad \Pi(q^2) = -\frac{1}{3}\left(g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\right)\Pi_{\mu\nu}(q^2),$$

$$\Pi_{\mu\nu}(q^2) = \mathrm{i}\int \mathrm{d}^4x \mathrm{e}^{\mathrm{i}q\cdot x} \langle\Omega|\, \mathbb{T}\{j_\mu(x), j_\nu^\dagger(0)\}\, |\Omega\rangle\,,$$

Dispersion Relation $\Downarrow$ $\rho(s) = \frac{1}{\pi}\,\mathrm{Im}\,\Pi(s)$

$$\Pi^{\text{QCD}}_{X,J^P}(q^2) = \int_{s_{\min}}^{\infty} \mathrm{d}s\, \frac{\rho^{\text{OPE}}_{X,J^P}(s)}{s-q^2}, \qquad s_{\min} = m_s^2$$

- Do contractions, using full propagators, we obtain the OPE of spectral density, which should be cut off at dimension 13

$$\rho^{\text{OPE}}_{X,J^P} = \rho^{\text{pert}} + \sum_{n=3}^{13} \rho^{\langle \mathcal{O}_n\rangle}.$$

Hadron Side

- Phenomenological framework

$$\rho^{\text{Phen}}_{X,J^P}(s) = \lambda^2_{X,J^P}\,\delta\left(s - m^2_{X,J^P}\right) + \theta(s - s_0)\,\rho_{X,J^P}(s), \quad \lambda_{X,J^P} \equiv \langle\Omega|\,j(x)\,|H_0\rangle\,,$$

$$\Pi^{\text{Phen}}_{X,J^P}(q^2) = \frac{\lambda^2_{X,J^P}}{m^2_{X,J^P} - q^2} + \int_{s_0}^{\infty} \mathrm{d}s\,\frac{\rho_{X,J^P}(s)}{s - q^2},$$

pole of ground state
s_0: continuum threshold

Quark-Hadron duality $\Downarrow$ Borel transformation

$$\lambda^2_{X,J^P}\mathrm{e}^{-m^2_{X,J^P}/M_B^2} = \int_{s_{\min}}^{s_0} \mathrm{d}s\;\rho^{\text{OPE}}_{X,J^P}(s)\mathrm{e}^{-s/M_B^2} + \Pi^{\text{sum}}(M_B^2),$$

which is the sum rule of the correlation function of the ground state hexaquarks.

- Extract the mass and decay constant

$$L_{X,J^P,0}(s_0, M_B^2) = \int_{s_{\min}}^{s_0} \mathrm{d}s\,\rho^{\text{OPE}}(s)\,\mathrm{e}^{-s/M_B^2} + \Pi^{\text{sum}}(M_B^2),$$

$$L_{X,J^P,1}(s_0, M_B^2) = \frac{\partial}{\partial(M_B^{-2})} L_{X,J^P,0}(s_0, M_B^2).$$

$$m_{X,J^P}(s_0, M_B^2) = \sqrt{-\frac{L_{X,J^P,1}(s_0, M_B^2)}{L_{X,J^P,0}(s_0, M_B^2)}},$$

$$\lambda_{X,J^P}(s_0, M_B^2) = \sqrt{\mathrm{e}^{m^2_{X,J^P}(s_0,M_B^2)/M_B^2}\,L_{X,J^P,0}(s_0, M_B^2)}.$$

2 additional parameters s_0 and M_B are inserted!

Criteria of Choosing s_0 and M_B

- Pole Dominate:

$$R^{\mathsf{PC}}_{X,J^P} = \frac{L_{X,J^P,0}(s_0, M_B^2)}{L_{X,J^P,0}(\infty, M_B^2)} \geqslant 15\% \quad \text{for hexaquark states.}$$

- OPE convergence:

$$R^{\mathsf{OPE}}_{X,J^P} = \left| \frac{L^{\langle O_{13}\rangle}_{X,J^P,0}(s_0, M_B^2)}{L_{X,J^P,0}(s_0, M_B^2)} \right| < 5\% \quad \text{in this work,}$$

where

$$L^{\langle \mathcal{O}_{13}\rangle}_{X,J^P,0}(s_0, M_B^2) = \int_{s_{\min}}^{s_0} \mathrm{d}s\ \rho^{\langle \mathcal{O}_{13}\rangle}(s) \mathrm{e}^{-s/M_B^2}.$$

- M_B^2 stability: Observables shouldn't depend on the additional parameters.
  $\Rightarrow$ Find a platform: Borel Window.

Section 3

Numerical Results

Numerical Setup

- Parameters should be chosen at $\mu = 2\,\mathrm{GeV}$:

$$
\begin{aligned}
&\langle \bar{q}q \rangle = -(0.24 \pm 0.01)^3\ \mathrm{GeV}^3, && \langle \bar{s}s \rangle = (1.15 \pm 0.12)\langle \bar{q}q \rangle, \\
&\langle g_s^2 G^2 \rangle = (0.88 \pm 0.25)\ \mathrm{GeV}^4, && \langle g_s^3 G^3 \rangle = (0.045 \pm 0.013)\ \mathrm{GeV}^6, \\
&\langle \bar{q} g_s \sigma \cdot G q \rangle = m_0^2 \langle \bar{q}q \rangle, && \langle \bar{s} g_s \sigma \cdot G s \rangle = m_0^2 \langle \bar{s}s \rangle, \\
&m_0^2 = (0.8 \pm 0.1)\ \mathrm{GeV}^2, && m_s = (95 \pm 5)\ \mathrm{MeV}.
\end{aligned}
$$

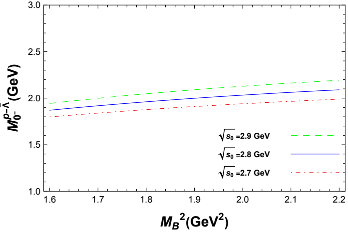
- Reliable region of s_0 should be $\sqrt{s_0} \sim m_X + \delta$, where δ is typically taken as $0.4 - 0.8\,\mathrm{GeV}$. We allow $\sqrt{s_0}$ to vary within a range of $\pm 0.1\mathrm{GeV}$.
- A stable physical state: M_B^2 within the Borel window exceeds $0.5\,\mathrm{GeV}^2$, ensuring the stability of the results.

Results of $p\bar{\Lambda}$ and $p\bar{\Sigma}$

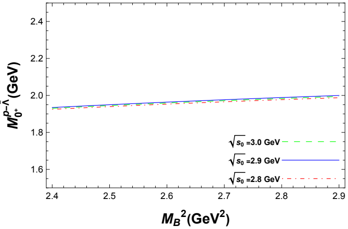
Current	J^P	State	$\sqrt{s_0}$(GeV)	M_B^2(GeV^2)	M_X(GeV)	$\lambda_X(10^{-5}\text{GeV}^8)$	$R^{\text{PC}}(\%)$	$R^{\text{OPE}}(\%)$
Type-1	0^+	$p\bar{\Lambda}$	2.9 ± 0.1	$2.4 - 2.9$	1.96 ± 0.03	3.54 ± 0.10	$18 - 55$	$3.8 - 4.2$
		$p\bar{\Sigma}$	2.9 ± 0.1	$2.4 - 2.9$	1.98 ± 0.04	3.62 ± 0.12	$18 - 54$	$3.8 - 4.3$
Type-2	0^-	$p\bar{\Lambda}$	2.8 ± 0.1	$1.6 - 2.2$	2.00 ± 0.20	3.57 ± 0.89	$18 - 60$	$1.7 - 2.9$
		$p\bar{\Sigma}$	2.8 ± 0.1	$1.6 - 2.1$	1.99 ± 0.18	3.52 ± 0.83	$21 - 60$	$2.2 - 3.5$
	1^-	$p\bar{\Lambda}$	2.8 ± 0.1	$1.7 - 2.2$	2.03 ± 0.17	3.52 ± 0.81	$17 - 52$	$2.5 - 4.2$
		$p\bar{\Sigma}$	2.8 ± 0.1	$1.7 - 2.2$	2.03 ± 0.17	3.52 ± 0.80	$18 - 53$	$1.6 - 2.7$

- For the other states, no suitable Borel window could be identified regardless of the choice of s_0 and M_B^2. Therefore, we conclude that the corresponding current does not couple to such states, which is consistent with our previous work on the light baryonium. [B.D.Wan, et.al, 2021].

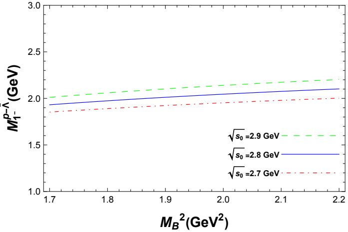
Borel Windows of $p\bar{\Lambda}$



(a) 0^-

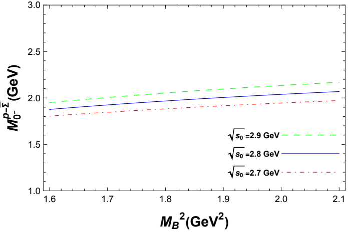


(b) 0^+

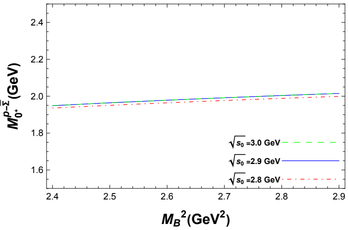


(c) 1^-

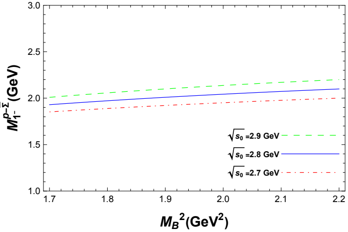
Borel Windows of $p\bar{\Sigma}$



(d) 0^-



(e) 0^+



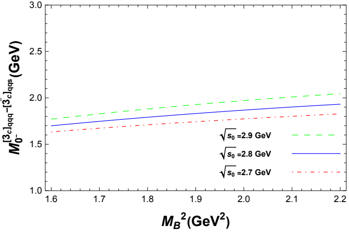
(f) 1^-

Results of the Compact Hexaquark States

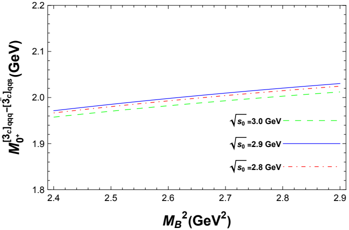
Currents	J^P	Configurations	$\sqrt{s_0}$(GeV)	M_B^2(GeV^2)	m_X(GeV)	$\lambda_X(10^{-5}\text{GeV}^8)$	$R^{\text{PC}}(\%)$	$R^{\text{OPE}}(\%)$
Type-I	0^+	$[3_c]_{qqq}$-$[\bar{3}_c]_{qqs}$	2.9 ± 0.1	$2.4 - 2.9$	1.99 ± 0.02	3.21 ± 0.05	$27 - 81$	$3.6 - 4.1$
	1^+	$[3_c]_{qqq}$-$[\bar{3}_c]_{qqs}$	2.9 ± 0.1	$2.3 - 3.0$	1.97 ± 0.03	2.84 ± 0.05	$18 - 90$	$3.5 - 4.3$
		$[3_c]_{qqq}$-$[\bar{3}_c]_{qsq}$	2.9 ± 0.1	$2.3 - 2.9$	1.97 ± 0.02	2.83 ± 0.04	$23 - 88$	$3.2 - 3.8$
Type-II	0^-	$[3_c]_{qqq}$-$[\bar{3}_c]_{qqs}$	2.8 ± 0.1	$1.6 - 2.2$	1.84 ± 0.21	2.38 ± 0.56	$17 - 62$	$2.2 - 3.3$
	1^-	$[3_c]_{qqq}$-$[\bar{3}_c]_{qqs}$	2.9 ± 0.1	$1.8 - 2.3$	2.01 ± 0.20	2.84 ± 0.70	$16 - 49$	$2.8 - 4.4$
		$[3_c]_{qqq}$-$[\bar{3}_c]_{qsq}$	2.9 ± 0.1	$1.8 - 2.3$	2.02 ± 0.20	2.81 ± 0.69	$16 - 49$	$1.6 - 2.4$

- $J^P = 0^+(0^-)$ states of the configurations $[3_c]_{qqq}$-$[\bar{3}_c]_{qqs}$ and $[3_c]_{qqq}$-$[\bar{3}_c]_{qsq}$ for the Type-I (II) currents are in fact the same state under the consideration of isospin symmetry.
- For the other states, no suitable Borel window could be identified regardless of the choice of s_0 and M_B^2. Therefore, we conclude that the corresponding current does not couple to such states.

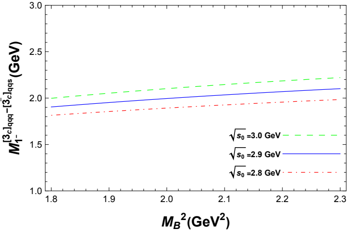
Borel Windows of the $[3_c]_{qqq}$-$[\bar{3}_c]_{qqs}$ Hexaquark states



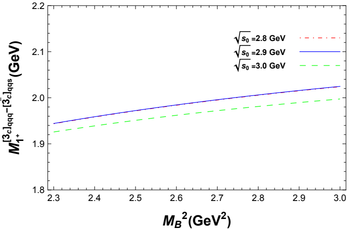
(g) 0^-



(h) 0^+

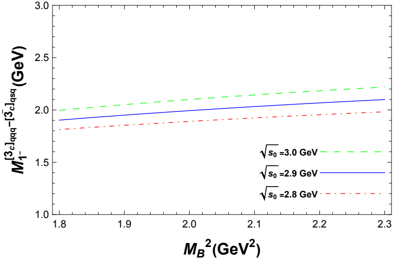


(i) 1^-

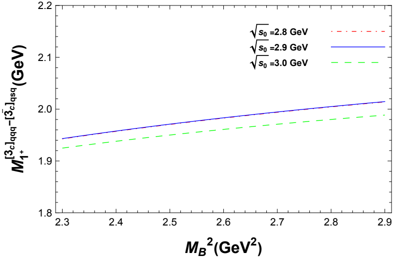


(j) 1^+

Borel Windows of the $[3_c]_{qqq}$-$[\bar{3}_c]_{qsq}$ Hexaquark states



(k) 1^-



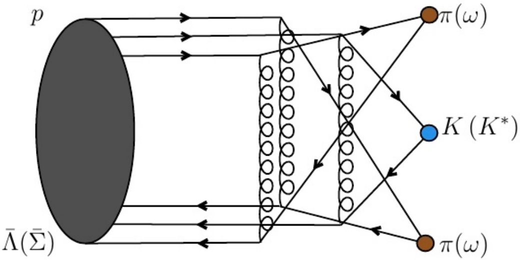
(l) 1^+

Section 4

Discussion and Conclusion

Decay Modes Analysis

- In experiments, the observed hadronic states are usually not pure states of a certain structure, but rather mixtures of several hadrons with nearly degenerate masses and identical quantum number.
    - Masses and Quantum numbers;
    - Rebuild them from the decay modes.
- Strong decay dominant



Decay Modes Analysis

- Possible decay modes

J^P	0^-	0^+	1^-
Modes of $p\bar{\Lambda}$ $(p\bar{\Sigma})$	$\pi\pi K$	$\pi\pi K^*$	$\pi\pi K^*$
	$\omega\omega K$	$\omega\omega K^*$	$\omega\omega K^*$

J^P	0^-	0^+	1^-	1^+
$[3_c]_{qqq}$-$[\bar{3}_c]_{qqs}$	$\pi\pi K$, $\omega\omega K$	$\pi\pi K^*$, $\omega\omega K^*$	$\pi\pi K^*$, $\omega\omega K^*$	$\pi\pi K^*$, $\omega\omega K^*$
		$\pi\rho K$	$\pi\rho K$ $(p\bar{\Lambda}/p\bar{\Sigma})$	$\pi\rho K$
$[3_c]_{qqq}$-$[\bar{3}_c]_{qsq}$	$\pi\pi K$, $\omega\omega K$	$\pi\pi K^*$, $\omega\omega K^*$	$\pi\pi K^*$, $\omega\omega K^*$	$\pi\pi K^*$, $\omega\omega K^*$
		$\pi\rho K$	$\pi\rho K$ $(p\bar{\Lambda}/p\bar{\Sigma})$	$\pi\rho K$

Conclusion

- There are 6 possible $p\bar{\Lambda}$ and $p\bar{\Sigma}$ molecular states with quantum numbers $J^P = 0^-, 0^+, 1^-$.
- There are 6 open-strange compact hexaquark candidates with quantum numbers $J^P = 0^-, 0^+, 1^-, 1^+$.
- The mass of $X(2085)$ does **not** fall into the predicted region for the $J^P = 1^+$ hexaquark states considered here, implying that its internal structure cannot be explained by the present configurations alone.
- The mass of $X(2075)$ *lies within the predicted range* of the $J^P = 1^-$ hexaquark states, and hence possible it possesses large components of these states.

Conclusion

- Moreover, the hexaquark states predicted in this work with quantum numbers $J^P = 0^+$ and $J^P = 0^-$ can serve as candidate for the open strange hadrons near 2 GeV, which are possible to be observed in Belle.II, BESIII and LHCb.
- As such, it is possible that both $X(2075)$ and $X(2085)$ are not pure states of a certain component, but rather mixtures of different hadronic components.
- The molecular and multiquark structures may coexist, which pose a challenge to us in deciphering the exotic states!

Thanks!

The Full Propagator of Light Quarks

- The full propagator

$$
\begin{aligned}
\mathrm{i}\mathcal{S}_q^{jk}(x) =& \mathrm{i}\delta^{jk}\frac{\not{x}}{2\pi^2x^4} - \delta^{jk}m_q\frac{1}{4\pi^2x^2} - \mathrm{i}t_a^{jk}\frac{G_{\alpha\beta}^a}{32\pi^2x^2}\left(\sigma^{\alpha\beta}\not{x}+\not{x}\sigma^{\alpha\beta}\right) - \delta^{jk}\frac{\langle\bar{q}q\rangle}{12} \\
& + \mathrm{i}\delta^{jk}\frac{\not{x}}{48}m_q\langle\bar{q}q\rangle - \delta^{jk}\frac{x^2}{192}\langle g_s\bar{q}\sigma\cdot Gq\rangle + \mathrm{i}\delta^{jk}\frac{x^2\not{x}}{1152}m_q\langle g_s\bar{q}\sigma\cdot Gq\rangle \\
& - t_a^{jk}\frac{\sigma_{\alpha\beta}}{192}\langle g_s\bar{q}\sigma\cdot Gq\rangle - \mathrm{i}t_a^{jk}\frac{1}{768}\left(\sigma_{\alpha\beta}\not{x}+\not{x}\sigma_{\alpha\beta}\right)m_q\langle g_s\bar{q}\sigma\cdot Gq\rangle.
\end{aligned}
$$

- The OPE of spectral density up to dimension 13

$$
\begin{aligned}
\rho^{\mathrm{OPE}}(s) =& \rho^{\mathrm{pert}}(s) + \rho^{\langle\bar{q}q\rangle}(s) + \rho^{\langle G^2\rangle}(s) + \rho^{\langle\bar{q}Gq\rangle}(s) + \rho^{\langle\bar{q}q\rangle^2}(s) + \rho^{\langle G^3\rangle}(s) \\
& + \rho^{\langle\bar{q}q\rangle\langle G^2\rangle}(s) + \rho^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle}(s) + \rho^{\langle G^4\rangle}(s) + \rho^{\langle\bar{q}q\rangle^3}(s) + \rho^{\langle\bar{q}Gq\rangle\langle G^2\rangle}(s) \\
& + \rho^{\langle\bar{q}q\rangle^2\langle G^2\rangle}(s) + \rho^{\langle\bar{q}Gq\rangle^2}(s) + \rho^{\langle\bar{q}q\rangle^2\langle\bar{q}Gq\rangle}(s) + \rho^{\langle\bar{q}q\rangle\langle G^4\rangle}(s) \\
& + \rho^{\langle\bar{q}q\rangle^4}(s) + \rho^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle\langle G^2\rangle}(s) + \rho^{\langle\bar{q}q\rangle\langle\bar{q}Gq\rangle^2}(s) + \rho^{\langle\bar{q}Gq\rangle\langle G^4\rangle}(s).
\end{aligned}
$$

Borel Transformation

- Definition

$$B\left[\Pi\left(q^2\right)\right] = \lim_{\substack{q^2, n \to \infty \\ -q^2/n = M_B^2}} \frac{\left(-q^2\right)^{n+1}}{n!} \left(\frac{\partial}{\partial q^2}\right)^n \Pi\left(q^2\right).$$

- Useful Relations

$$\mathcal{B}\left[\left(q^2\right)^k\right] = 0,$$

$$\mathcal{B}\left[\frac{1}{\left(s-q^2\right)^k}\right] = \frac{1}{(k-1)!}\left(\frac{1}{M_B^2}\right)^{k-1} \mathrm{e}^{-s/M_B^2}.$$

- Borel transformation is used for suppressing the contribution of higher excited states and the continuum spectra.