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北京航空航天大學 BEIHANG UNIVERSITY

重味介子光锥分布振幅的格点QCD精确计算

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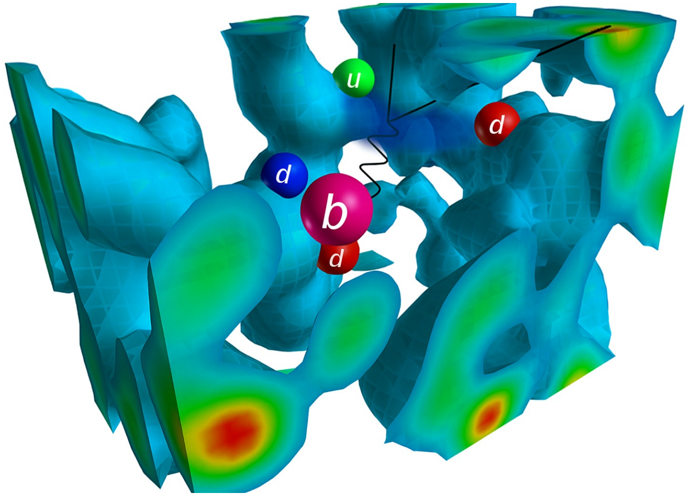
Collaborated with LPC Lattice Parton Collaboration

10/27/2025 @ HFCPV2025, 北京

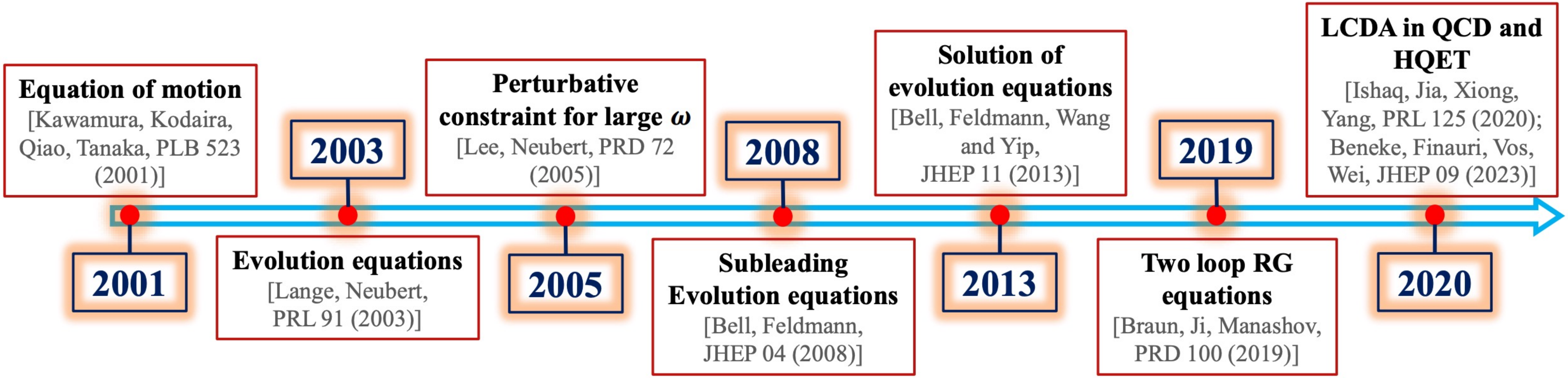


Outline

- Our Knowledge of Heavy Meson LCDA
- Recipe for the *B* meson LCDAs
- Towards High-Precision LQCD Calculations
- QCD LCDA of *D* Meson from LQCD
- Prediction for HQET LCDA
- Summary and Outlook

Our Knowledge of Heavy Meson LCDA

- **Theoretical Researches:** *See Ji Xu's review talk*



- Existing Studies: Perturbative, Evolution

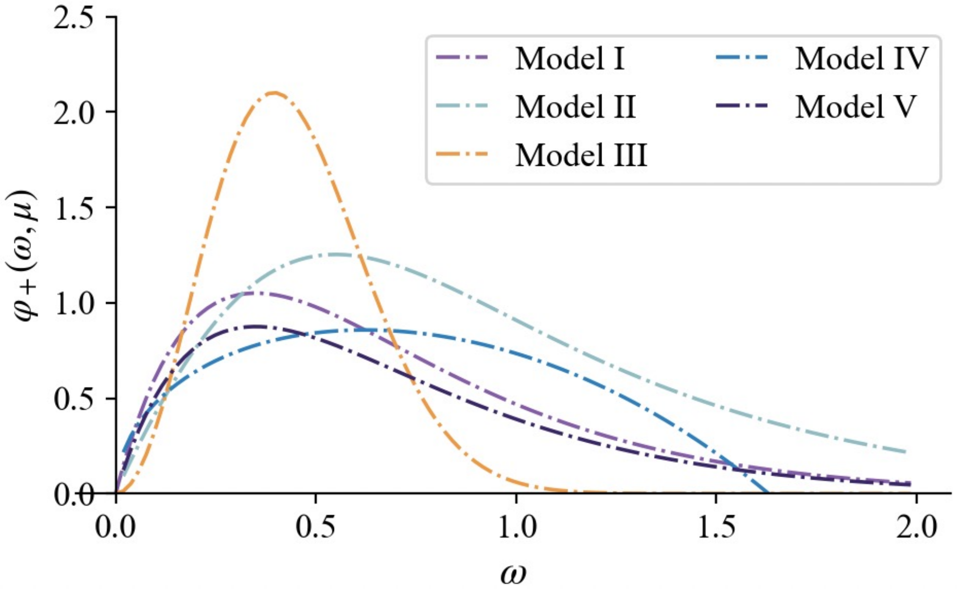
Our Knowledge of Heavy Meson LCDA

- **Limited understanding** of the nonperturbative heavy meson LCDA itself:

  Full Distribution: Only Models

- **Uncertainties from B meson LCDA** **dominate** the errors in theoretical prediction.

  - *e.g.*: $B \to \pi,\ K^*$ form factors from LCSRs:



$$\mathcal{V}_{B\to K^*}(0) = 0.359\,{}^{+0.141}_{-0.085}\Big|_{\lambda_B}\,{}^{+0.019}_{-0.019}\Big|_{\sigma_1}\,{}^{+0.001}_{-0.062}\Big|_{\mu}\,{}^{+0.010}_{-0.004}\Big|_{M^2}\,{}^{+0.016}_{-0.017}\Big|_{s_0}\,{}^{+0.153}_{-0.079}\Big|_{\varphi_\pm(\omega)},$$

$$f^+_{B\to\pi}(0) = 0.122 \times \Big[1 \pm 0.07\Big|_{S_0^\pi} \pm 0.11\Big|_{\Lambda_q} \pm 0.02\Big|_{\lambda_E^2/\lambda_H^2}\,{}^{+0.05}_{-0.06}\Big|_{M^2} \pm 0.05\Big|_{2\lambda_E^2+\lambda_H^2}$$
$$\qquad {}^{+0.06}_{-0.10}\Big|_{\mu_h} \pm 0.04\Big|_{\mu}\,{}^{+1.36}_{-0.56}\Big|_{\lambda_B}\,{}^{+0.25}_{-0.43}\Big|_{\sigma_1,\sigma_2}\Big].$$

Gao, Lu, Shen, Wang, Wei, PRD 101 (2020) 074035

Cui, Huang, Shen, Wang, JHEP 03 (2023) 140

Our Knowledge of Heavy Meson LCDA

- Lattice QCD Calculations ?

Could you list the lattice QCD calculations of B meson LCDAs from lattice QCD before 2020?

Short answer up front — there are **no published direct lattice-QCD calculations of the B-meson light-cone distribution amplitude (ϕ_B(ω)) completed before 2020**.

Direct lattice QCD calculations of B-meson LCDA before 2020

- **None found.**
  — I searched the literature and conference material up to 2019 and did not find any completed lattice determination of the B-meson light-cone distribution amplitude ϕ_B(ω) (point-by-point reconstruction or its full quasi-DA on the lattice) published before 2020. The first concrete lattice-implementation proposals/algorithms for B-meson LCDA appear around 2020 (see Wang 2020), and practical lattice results begin to appear in the literature and conference talks only after that.

Recipe for the B meson LCDAs

- **A proposal from amplitude, e.g. $B\to\gamma l\nu$:**

…… there has been no further progress.

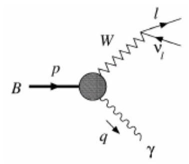
Light-cone distribution amplitude

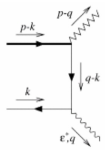
- Light-like separation… possible?

Shoji Hashimoto (KEK)
@ INT Workshop "Effective Field Theory, QCD, and Heavy Hadrons," U. of Washington, Seattle; April 28, 2005.

What to calculate?

- Any amplitude instead of the LCDA itself. (Not necessarily be a physical one.)





- The simplest process: $B\to l\nu\gamma^{(*)}$

$$\frac{1}{\sqrt{4\pi\alpha}}\langle\gamma(\varepsilon^*,q)|\bar{u}\gamma_\mu(1-\gamma_5)b|\bar{B}(p)\rangle$$
$$= \epsilon_{\mu\nu\rho\sigma}\varepsilon^{*\nu}v^\rho q^\sigma F_V(E_\gamma) + i[\varepsilon^*_\mu(v\cdot q)-q_\mu(v\cdot\varepsilon^*)]F_A(E_\gamma)$$

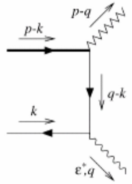
- At the leading order, it is given as

$$F_V = F_A = \frac{f_B m_B Q_u}{2\sqrt{2}E_\gamma}\int_0^\infty dk_+\frac{\Phi_+^B(k_+)}{k_+}$$

proportional to the inverse moment $1/\lambda_B$.

Boost!

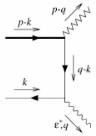
- Need a large momentum insertion from the photon.
- Then, the internal light quark becomes energetic.



- Boost back to the "rest frame" of the photon. Or, the soft photon limit (red shift).
- Not possible: it requires v=1 for the massless photon.
- Consider a virtual (space-like) photon instead. Q^2 must be large enough to keep $v < 0.8$.

$$F \sim \int_0^\infty dk_+\frac{\Phi_+^B(k_+)}{Q^2+(E_\gamma+\sqrt{E_\gamma^2+Q^2})k_+}$$

Recipe



1. Calculate the 3-point function with a space-like four-momentum (q^2=-Q^2) inserted. Boost the b quark so that the photon becomes soft.
2. Fit the data with the expected functional form to obtain the form factor.
3. Extrapolate to the $Q^2\to0$ limit.
4. Compare with the perturbative calculation. Then, $1/\lambda_B$ is obtained at the leading order; higher orders can also be included if the data are precise enough.

Recipe for the B meson LCDAs

- A proposal from lattice HQET fields:

  ✓ Equal-time matrix elements

$$\langle H_Q(P_{H_Q}) \,|\, \bar{q}(z)\not{n}_z\gamma_5\, W_c(z,0) h_v(0) \,|\, 0\rangle$$

Wang, Wang, Xu, Zhao, PRD 102, 011502 (2020);
Xu, Zhang, PRD 106, 114019 (2022);
Hu, Xu, Zhao, EPJC 84, 502 (2024); ……

? Boosted HQET field on the lattice — Mandula, Ogilvie PRD 45, 2183-2187 (1992)

- HQET:

$$\mathcal{L}_v = \bar{h}_v(x) i\, v \cdot D\, h_v(x)$$

The original field is recovered by

$$\psi(x) = e^{imv\cdot x}\frac{1+v\cdot\gamma}{2}h_v(x)$$

- The lattice version:

$$\bar{h}_v(x)[-v_0 D_0 + i\vec{v}\cdot\vec{D}]h_v(x)$$

Evolution equation:

$$\tilde{S}(x+\hat{t}\,,y) = U(x, x+\hat{t}) * \left[\tilde{S}(x,y) - i\sum_{\mu=1}^{3}\frac{v_\mu}{v_0}\frac{U(x,x+\hat{\mu})\tilde{S}(x+\hat{\mu},y) - U(x,x-\hat{\mu})\tilde{S}(x-\hat{\mu},y)}{2a} + \frac{1}{a^3 v_0}\delta(t)\delta_{\vec{x},\vec{y}}\right]$$

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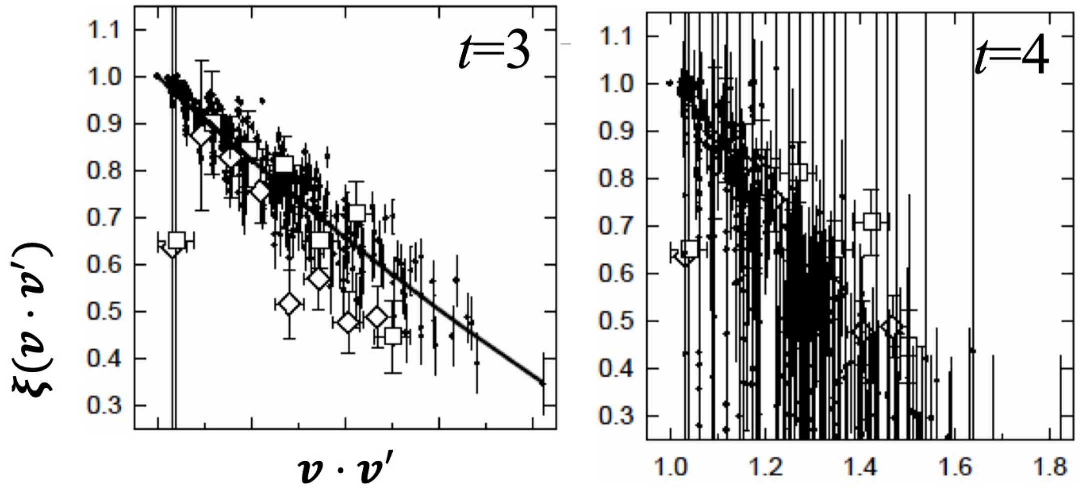


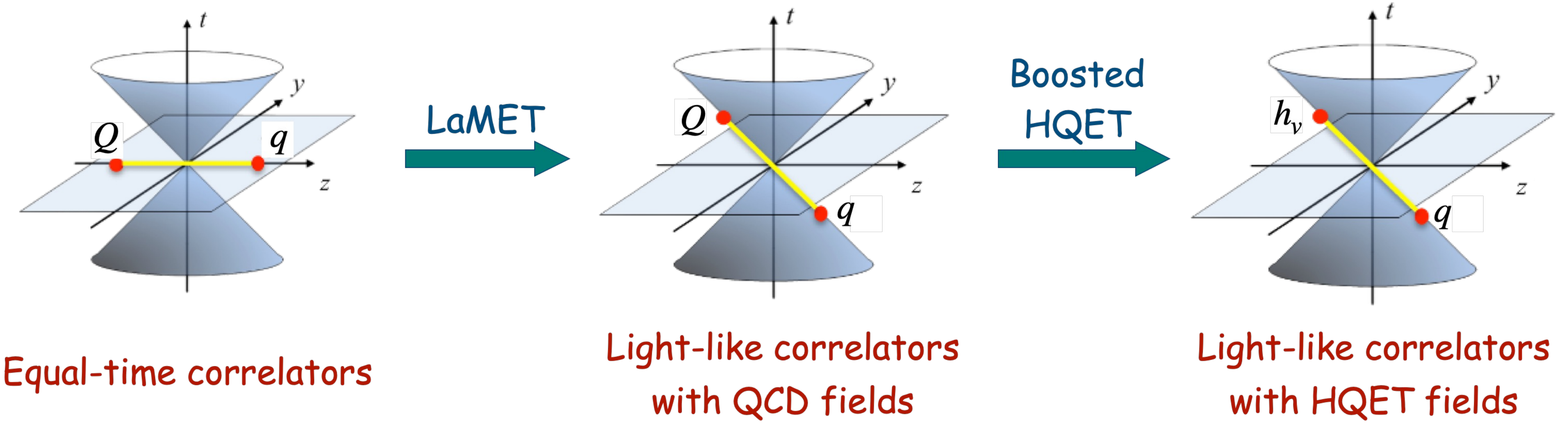
Very noisy……

…… there has been no further progress.

Recipe for the B meson LCDAs

- A sequential effective theory combining LaMET and bHQET:

Liu, Wang, Xu, **QAZ**, Zhao, PRD 99, 094036 (2019)
Han, **QAZ**, *et.al.*, PRD111, 034503, (2025)



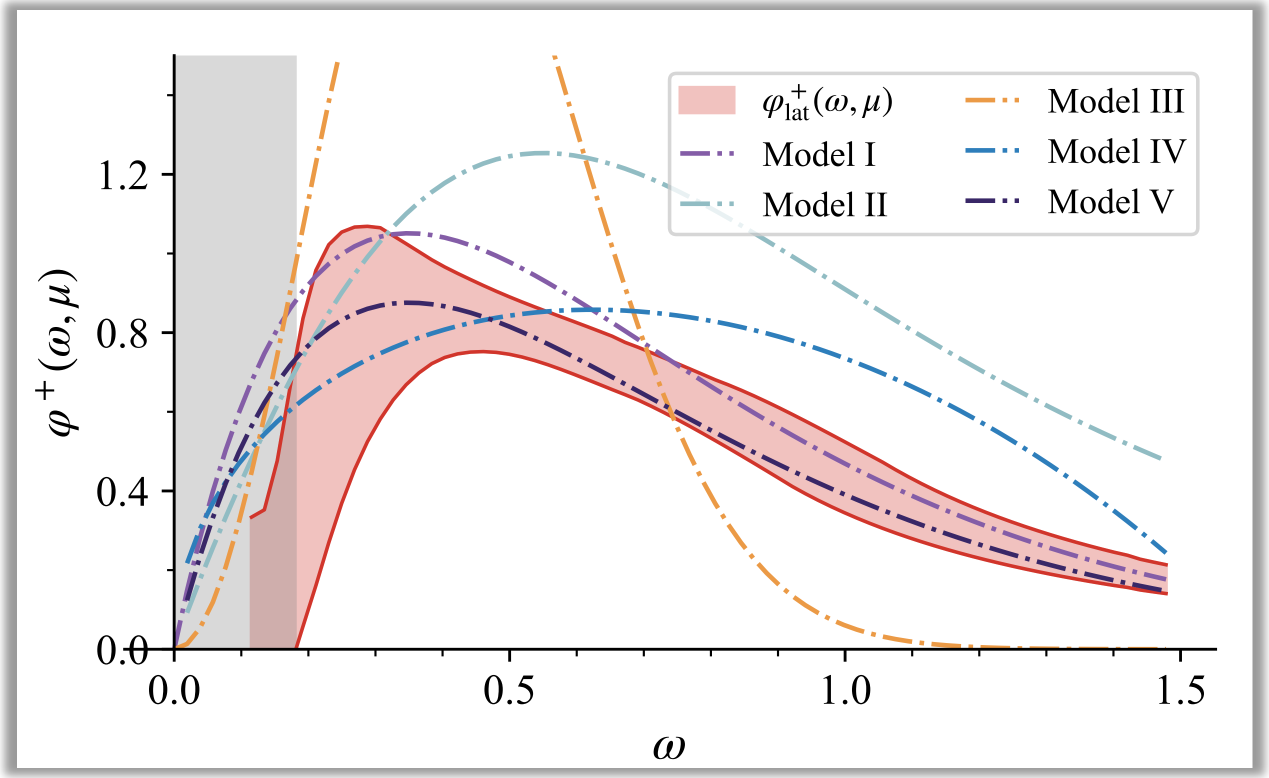
✓ Fully distributions from lattice QCD!

✓ Avoid cusp divergence!

Recipe for the B meson LCDAs

- A sequential effective theory combining LaMET and bHQET:

Liu, Wang, Xu, **QAZ**, Zhao, PRD 99, 094036 (2019)
Han, **QAZ**, *et.al.*, PRD111, 034503, (2025)



✓ The first lattice QCD prediction for heavy meson LCDA!

A more precision calculation?

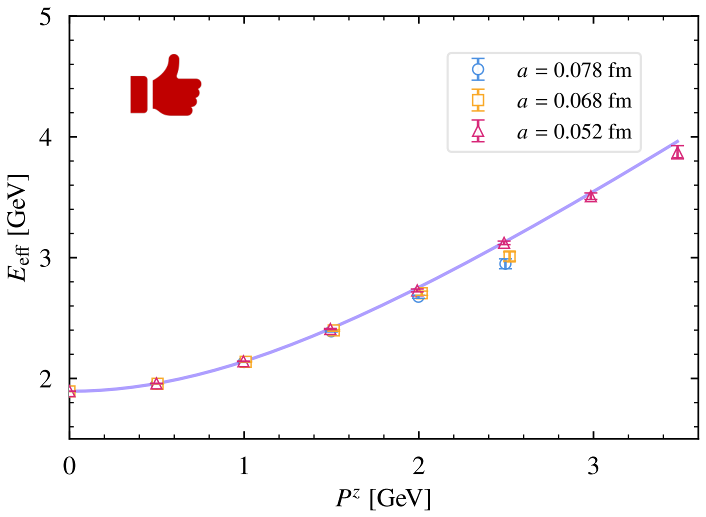
- Statistic fluctuations
- Discretization effects
- Power corrections

Towards High-Precision Calculation

Large statistics calculations on CLQCD ensembles

Ensemble	$L^3 \times T$	a (fm)	m_π (GeV)	P^z (GeV)	N_{stats}
F32P30	$32^3 \times 96$	0.07750	303.2(1.3)	Up to 2.5	183600
F32P21	$32^3 \times 64$	0.07750	210.9(2.2)	Up to 2.5	29376
G36P29	$36^3 \times 108$	0.06826	295.1(1.2)	Up to 2.5	112832
H48P32	$48^3 \times 144$	0.05187	317.2(0.9)	Up to 3.5	108000

- ✓ Continuum extrapolation.
- ✓ Chiral extrapolation.
- ✓ Infinite momentum extrapolation.
- ✓ Large statistic.
- ✓ Discretization effects well controlled.

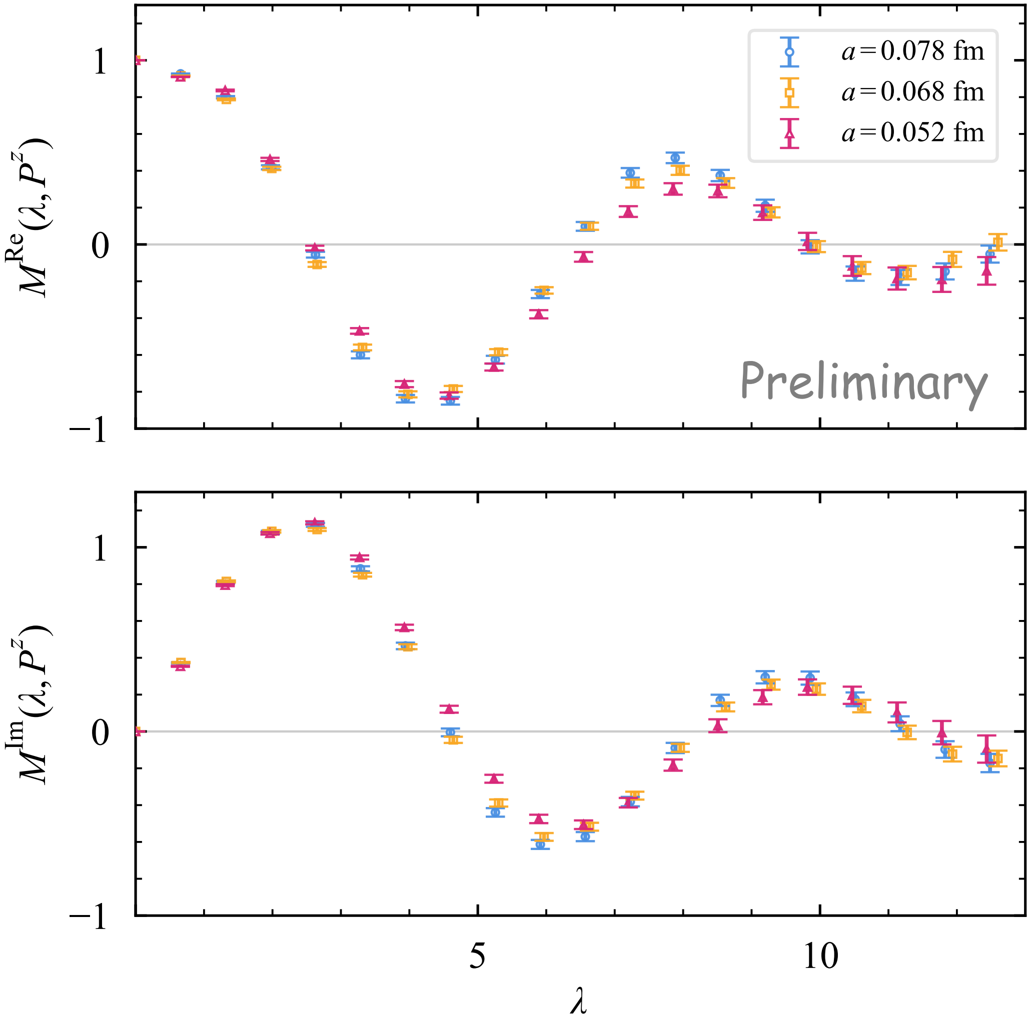


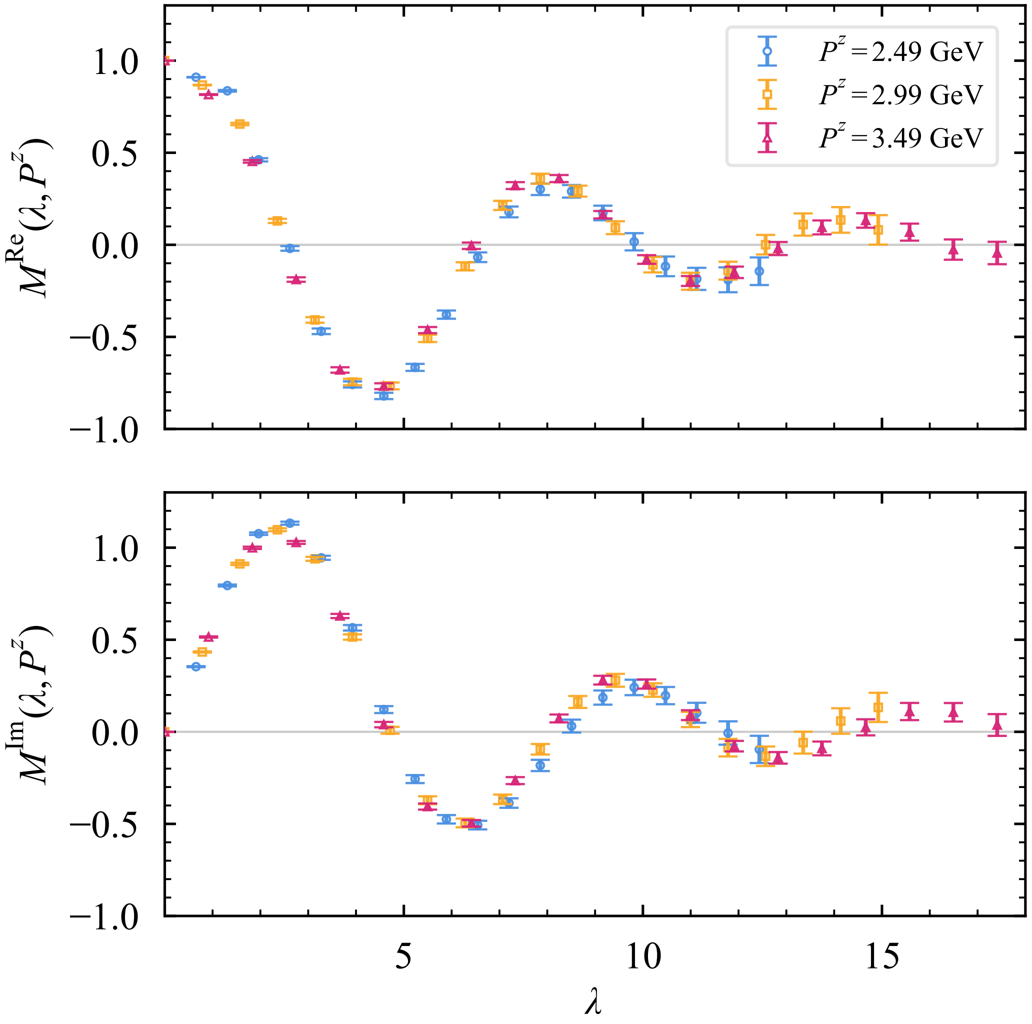
- Multi-source enhancement;
- HYP smearing to improve signal-to-noise ratio;
- State-of-the-art hybrid renormalization scheme;
- Long-range extrapolation and Fourier transformation;
- ……

Details are important, … but skipped

Renormalized equal-time distributions from LQCD

- a-dependence, $P^z = 2.5\text{GeV}$
- P^z-dependence, $a = 0.052\text{fm}$





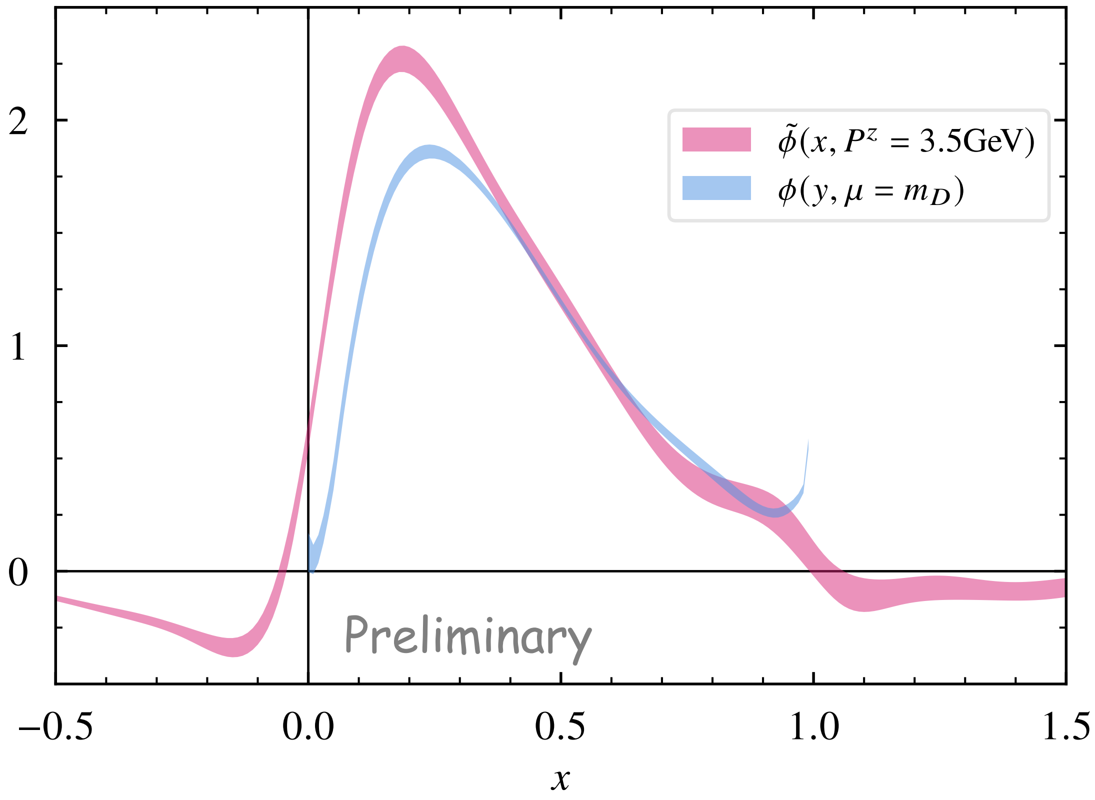
LaMET: from equal-time correlator to QCD distributions

Matching from quasi-DA to QCD LCDA:

$$\phi(y,\mu) = \int_{-\infty}^{+\infty} \frac{dx}{|x|}\, \mathcal{C}\left(\frac{y}{x}, \frac{\mu}{xP^z}\right) \tilde{\phi}(x, P^z) + \mathcal{O}\left(\frac{m_H^2}{(P^z)^2}, \frac{\Lambda_{\rm QCD}^2}{(yP^z,(1-y)P^z)^2}\right).$$

Liu, Wang, Xu, **QAZ**, Zhao, PRD 99, 094036 (2019)

Han, **QAZ**, *et.al.*, PRD111, 034503, (2025)



- Power corrections:

$\frac{\Lambda_{\rm QCD}^2}{(yP^z,(1-y)P^z)^2}$: Contribute to end-point region.

$\frac{m_H^2}{(P^z)^2}$: All region, can be benchmarked by predictions without LaMET.

Moments of Heavy Meson

- Gegenbauer moments from LaMET:

$$\phi(x,\mu^2) \;=\; 6x(1-x)\left[1+\sum_{n=1}^{\infty} a_n(\mu^2)\, C_n^{3/2}(2x-1)\right],$$

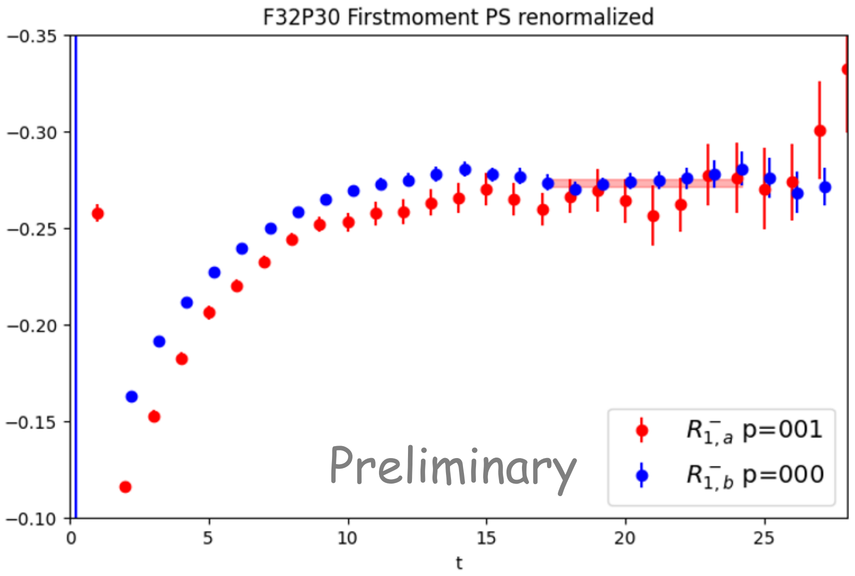
Fitting from data excluding the end-point region:

n	a_1	a_2	a_3	a_4
2	-0.397(18)	0.118(10)		
4	-0.412(20)	0.134(16)	-0.016(11)	0.005(8)
6	-0.391(22)	0.111(18)	0.026(20)	-0.019(15)
8	-0.376(24)	0.109(19)	0.041(22)	-0.034(18)

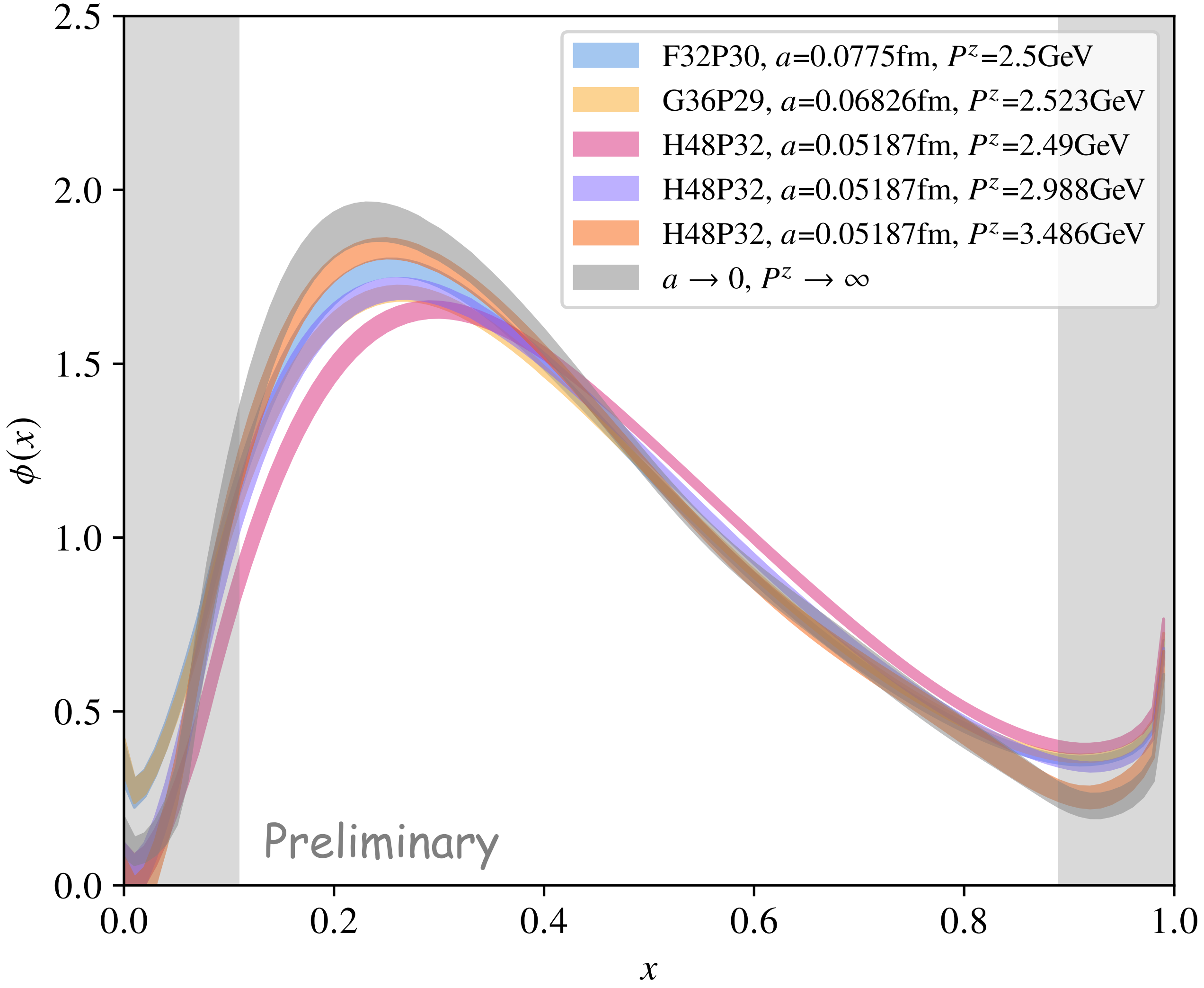
- Moments from local operators:

$$\bar{q}(z_2 n)\not{n}\gamma_5 W_c(z_2 n, z_1 n)\, q'(z_1 n)$$
$$= \sum_{k,l=0}^{\infty} \frac{z_2^k z_1^l}{k!\,l!}\, n^\rho n^{\mu_1} \ldots n^{k+l} \mathcal{M}^{(k,l)}_{\rho\mu_1\ldots\mu_{k+l}} + \cdots,$$

$$\mathcal{M}^{(k,l)}_{\rho\mu_1\ldots\mu_{k+l}} \;=\; \bar{q}(0)\overleftarrow{D}_{(\mu_1}\ldots\overleftarrow{D}_{\mu_k}\overrightarrow{D}_{\mu_{k+1}}\ldots\overrightarrow{D}_{\mu_{k+l}}\gamma_{\rho)}\gamma_5 q'(0).$$



QCD LCDA of D Meson from LQCD



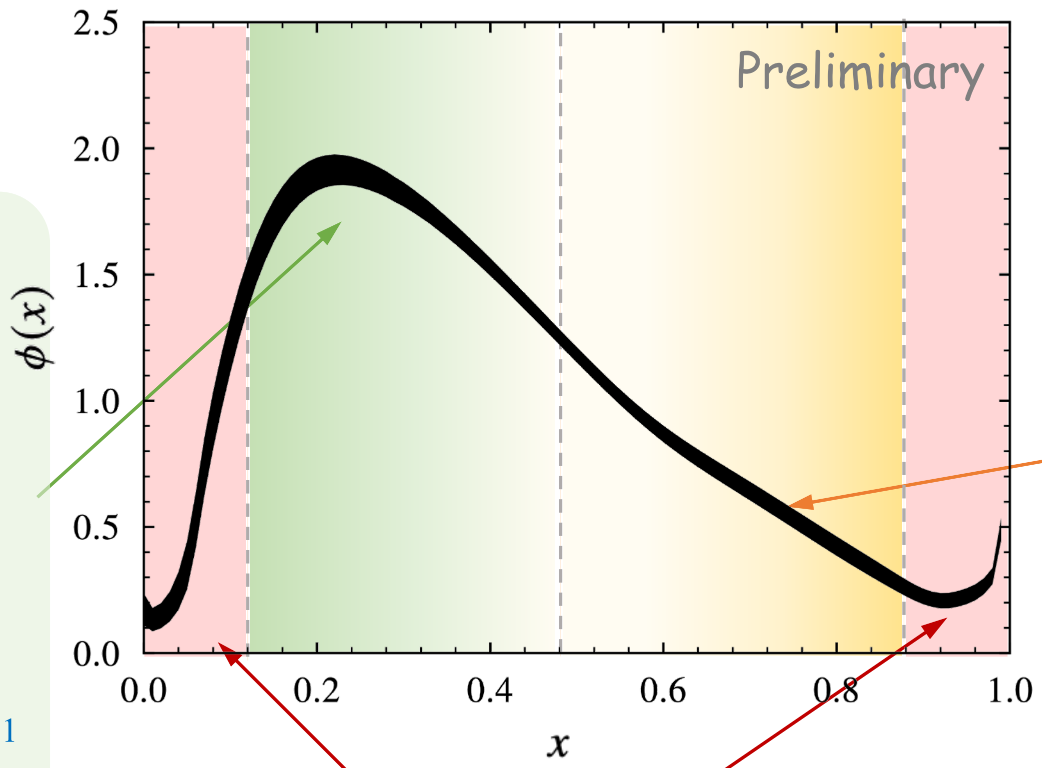
- ✓ Continuum extrapolation
- ✓ Infinite momentum extrapolation
- Chiral extrapolation: in progress

QCD LCDA of D Meson from LQCD

Peak region: $y \sim \frac{\Lambda_{\rm QCD}}{m_H}$

- Light quark carries small momentum fraction;
- Related to the HQET LCDA.

Ishaq, Jia, Xiong, Yang, PRL125(2020)132001
Beneke, Finauri, Vos, Wei, JHEP 09, 066 (2023)



Tail region: $y \sim 1$

- Contain only hard-collinear physics;
- Suppressed in LCDA.

End-point region:

- LaMET matching kernel suffer large power corrections.
- Lattice QCD predictions fail

Prediction for HQET LCDA

- Leading twist HQET LCDA:

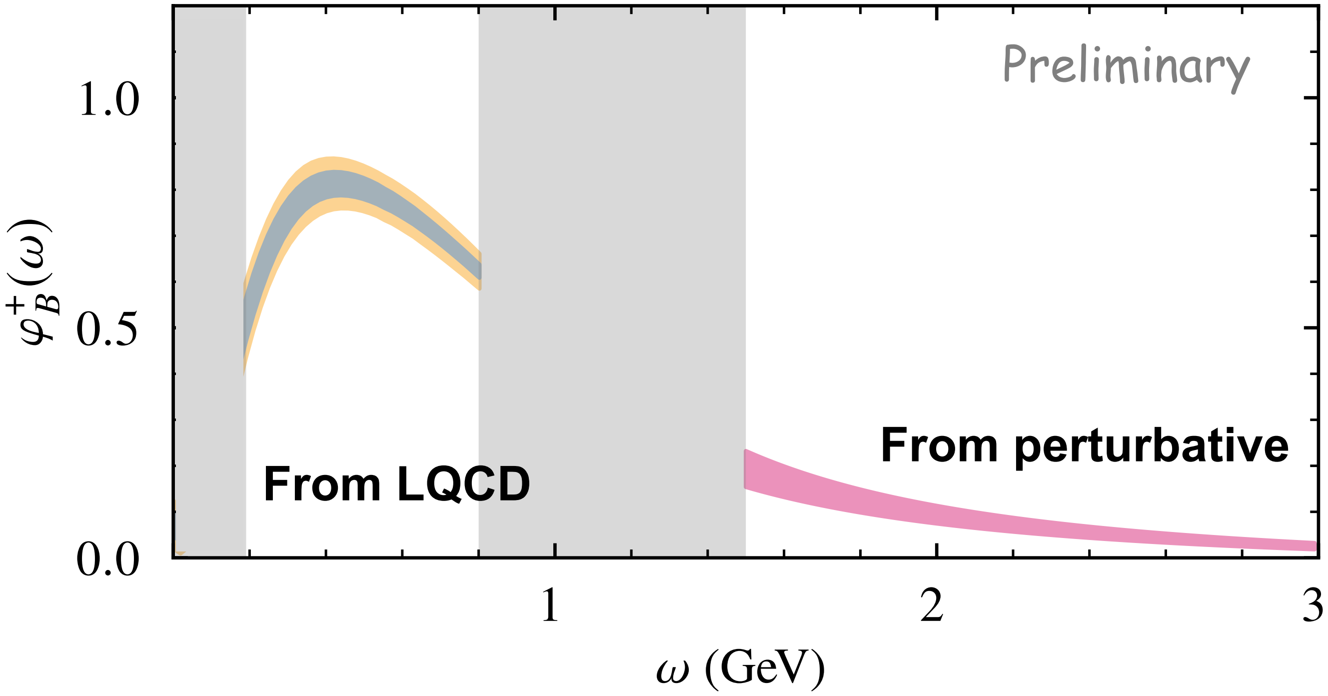
QCD LCDA, nonperturbative

$$\varphi^+(\omega,\mu) = \begin{cases} \frac{1}{m_H}\left[1-\frac{\alpha_s C_F}{4\pi}\left(\frac{1}{2}\ln^2\frac{\mu^2}{m_H^2}+\frac{1}{2}\ln\frac{\mu^2}{m_H^2}+\frac{3}{2}\ln\frac{\mu^2}{m_Q^2}+\frac{\pi^2}{12}+4\right)+\mathcal{O}\left(\alpha_s^2\right)\right]\phi\left(\frac{\omega}{m_H},\mu\right), & \omega\sim\Lambda_{\rm QCD} \\ \frac{\alpha_s C_F}{\pi\omega}\left[\left(\frac{1}{2}-\ln\frac{\omega}{\mu}\right)+\frac{4\bar{\Lambda}}{3\omega}\left(2-\ln\frac{\omega}{\mu}\right)\right]+\mathcal{O}\left(\alpha_s^2\right), & \omega\sim m_H \end{cases}$$

Beneke, Finauri, Vos, Wei, JHEP 09, 066 (2023)

Lee, Neubert, PRD 72, 094028 (2005)

Prediction for HQET LCDA from first-principle:

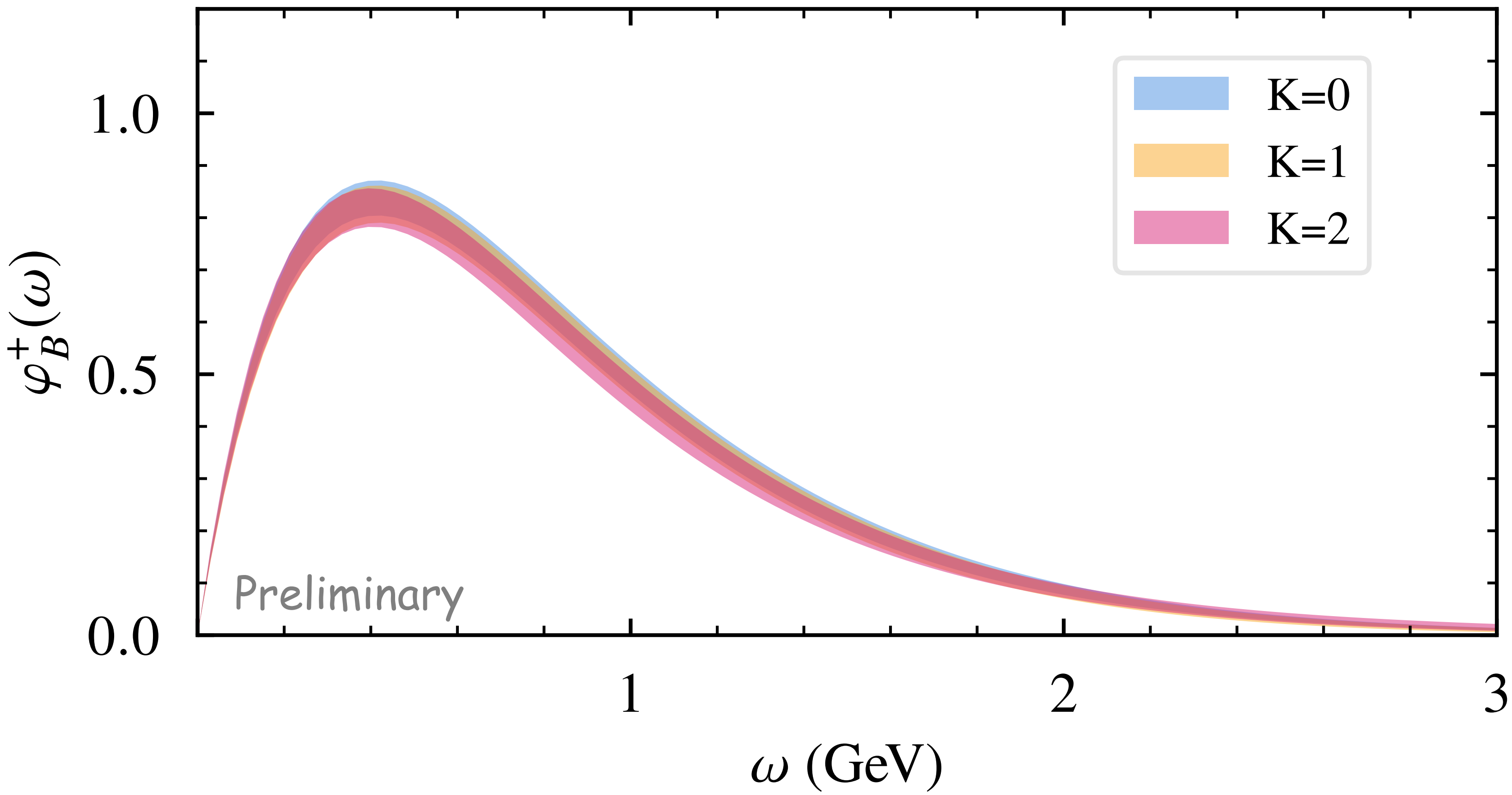


Prediction for HQET LCDA

- Rebuild the full distribution of the HQET LCDA from a **model-independent parametrization:**

Feldmann, Lughausen, Dyk, JHEP10, 162 (2020)

$$\varphi^+(\omega,\mu) = \frac{\omega e^{-\omega/\omega_0}}{\omega_0^2} \sum_{k=0}^{K} \frac{a_k(\mu)}{1+k} L_k^{(1)}(2\omega/\omega_0),$$



Inverse and inverse-logarithmic moments

Predictions of inverse and inverse-logarithmic moments from the full distribution of HEQT LCDA:

$$\lambda_B^{-1}(\mu) = \int_0^\infty \frac{d\omega}{\omega} \varphi^+(\omega, \mu),$$

$$\sigma_B^{(n)}(\mu) = \lambda_B(\mu) \int_0^\infty \frac{d\omega}{\omega} \ln\left(\frac{\mu}{\omega}\right)^{(n)} \varphi^+(\omega, \mu).$$

	λ_B (GeV)	$\sigma_B^{(1)}$
Our work	0.414 (16)	1.451 (19)
PRD98 (2018)	> 0.24	
JHEP (2020)	0.383 (153)	
PRD72 (2005)	0.48 (11)	1.6 (2)
PRD69 (2004)	0.46 (11)	1.4 (4)
PRD55 (1997)	0.35 (15)	
PRD101 (2020)	$0.343^{+0.064}_{-0.079}$	
PLB848 (2024)	0.338 (68)	

Preliminary

Summary and Outlook

✓ We present a first-principles precision determination of the QCD and HQET LCDA for heavy mesons at leading power.

✓ Systematic uncertainties from non-perturbative renormalization, scale conversion, continuum and infinite-momentum extrapolation etc., are comprehensively addressed.

• Power corrections ($\propto m_H/P^z$, $\Lambda_{\rm QCD}/m_Q$) are the most important goal for next stage.

Thanks for your attention