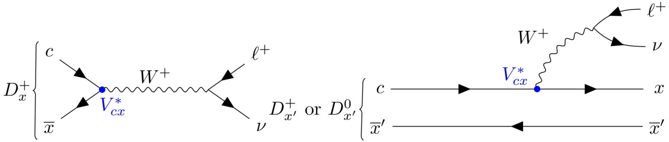
Heavy to light form factors from LQCD

孟 雨（郑州大学）

第22届全国重味物理与CP破坏研讨会，2025年10月24-28日，北京

"November Revolution"— The discovery of J/ψ particle in 1974, greatly facilitated the establishment of the Standard Model.

- Precise test for Standard Model and search for new physics
  — High-precison measurements(BESIII,etc) and theoretical calculations
    — CKM matrix element $V_{cs(d)}$



- Test the interplay of pert and non-pert QCD
  — intermediate energy scale

Charmed high-precision frontier

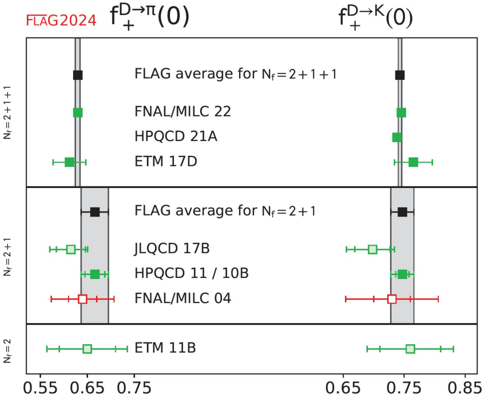
- FLAG2024:

$$
\begin{aligned}
f_+^{D\to\pi}(0) &= 0.6296(50) \\
f_+^{D\to K}(0) &= 0.7430(27)
\end{aligned}
$$

- HFLAV2023

$$|V_{cs}| = 0.9639 \pm 0.0044_{\text{EXP}} \pm 0.0032_{\text{LQCD}}$$

$$|V_{cd}| = 0.2265 \pm 0.0029_{\text{EXP}} \pm 0.0018_{\text{LQCD}}$$



- QED correction needs to be investigated
  @杨一玻 talk on QED correction

$P \to V$ semileptonic decay

- For $P' \to P$:

$$\frac{d\Gamma(P' \to P e \nu_e)}{dq^2 d\cos\theta_l} = \frac{G_F^2}{32\pi^3} |\vec{p}|^3 |V_{xy}|^2 \sin^2\theta_l |f_+(q^2)|^2$$

  for example, $D \to \pi$ and $D \to K$

- For $P \to V$:

$$\frac{\mathrm{d}\Gamma(P \to V e \nu_e)}{\mathrm{d}q^2 \mathrm{d}\chi \mathrm{d}\cos\theta_\ell \mathrm{d}\cos\theta_V} = \frac{G_F^2 |V_{xy}|^2 |\vec{p}| q^2}{12(2\pi)^4 M^2} W\left(q^2, \theta_V, \theta_\ell, \chi\right)$$

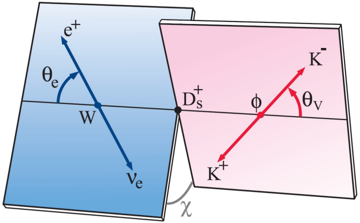
  - Independent determination of the CKM matrix element
  - Stronger test with abundant polarization information

$P \to V$ decay amplitude

$$
\begin{aligned}
W\left(\theta_K,\theta_\ell,\chi\right) = {} & \frac{9}{32}\left(1+\cos^2\theta_\ell\right)\sin^2\theta_K\left(|H_+|^2+|H_-|^2\right)+\frac{9}{8}\sin^2\theta_\ell\cos^2\theta_K|H_0|^2 \\
& +\frac{9}{16}\cos\theta_\ell\sin^2\theta_K\left(|H_+|^2-|H_-|^2\right)-\frac{9}{16}\sin^2\theta_\ell\sin^2\theta_K\cos 2\chi\left(H_+H_-\right) \\
& +\frac{9}{16}\sin\theta_\ell\sin 2\theta_K\cos\chi\left(H_+H_0-H_-H_0\right) \\
& +\frac{9}{32}\sin 2\theta_\ell\sin 2\theta_K\cos\chi\left(H_+H_0+H_-H_0\right) \\
& +\frac{m_\ell^2}{2q^2}\left[\frac{9}{4}\cos^2\theta_K|H_t|^2-\frac{9}{2}\cos\theta_\ell\cos^2\theta_K\left(H_0H_t\right)+\frac{9}{4}\cos^2\theta_\ell\cos^2\theta_K|H_0|^2\right. \\
& +\frac{9}{16}\sin^2\theta_\ell\sin^2\theta_K\left(|H_+|^2+|H_-|^2\right)+\frac{9}{8}\sin^2\theta_\ell\sin^2\theta_K\cos 2\chi\left(H_+H_-\right) \\
& +\frac{9}{8}\sin\theta_\ell\sin 2\theta_K\cos\chi\left(H_+H_t+H_-H_t\right) \\
& \left.-\frac{9}{16}\sin 2\theta_\ell\sin 2\theta_K\cos\chi\left(H_+H_0+H_-H_0\right)\right]
\end{aligned}
$$

and the helicity amplitudes are

$$
\begin{aligned}
H_\pm\left(q^2\right) &= \left(M+m\right)A_1\left(q^2\right)\mp\frac{2M|\vec{p}|}{M+m}V\left(q^2\right) \\
H_0\left(q^2\right) &= \frac{1}{2M\sqrt{q^2}}\times\left[\left(M^2-m^2-q^2\right)\left(M+m\right)A_1\left(q^2\right)-4\frac{M^2|\vec{p}|^2}{M+m}A_2\left(q^2\right)\right] \\
H_t\left(q^2\right) &= \frac{2M|\vec{p}|}{\sqrt{q^2}}A_0\left(q^2\right)
\end{aligned}
$$



$P \to V$ matrix element

- For $P' \to P$:

$$\langle P(p)|V_\mu|H(p')\rangle = f_+(q^2)\left[(p'+p)_\mu - \frac{M^2-m^2}{q^2}q_\mu\right] + f_0(q^2)\frac{M^2-m^2}{q^2}q_\mu$$

- For $P \to V$:

$$\begin{aligned}\langle h_V(p)|V_\mu - A_\mu|H(p')\rangle &= \varepsilon^*_\nu \epsilon_{\mu\nu\alpha\beta}p'_\alpha p_\beta \frac{2V}{m+M} + (M+m)\,\varepsilon^*_\mu A_1 \\ &+ \frac{\varepsilon^*\cdot q}{M+m}\left(p+p'\right)_\mu A_2 - 2m\frac{\varepsilon^*\cdot q}{q^2}q_\mu\,(A_0 - A_3)\end{aligned}$$

  - Two key physical quantities:

$$r_V \equiv V(0)/A_1(0), \qquad r_2 \equiv A_2(0)/A_1(0)$$

Charmed $P \to V$ decay

- Experiments:

Channels	r_V	r_2	Refs
$D^0 \to K^*(892)$			
(μ)	$1.37 \pm 0.09 \pm 0.03$	$0.76 \pm 0.06 \pm 0.02$	PRL134,011803(2025)
(μ)	$1.46 \pm 0.11 \pm 0.04$	$0.71 \pm 0.08 \pm 0.03$	PRL135,111803(2025)
(e)	$1.48 \pm 0.05 \pm 0.02$	$0.70 \pm 0.04 \pm 0.02$	JHEP03,197(2025)
Average	$1.456 \pm 0.040 \pm 0.016$	$0.715 \pm 0.031 \pm 0.014$	
$D_s \to K^{*,0} e \nu_e$	$1.67 \pm 0.34 \pm 0.16$	$0.77 \pm 0.28 \pm 0.07$	PRL122,061801(2019)
$D_s \to \phi$			
(e)	$1.807 \pm 0.046 \pm 0.065$	$0.816 \pm 0.036 \pm 0.030$	PRD78,051101(2008)
(μ)	$1.58 \pm 0.17 \pm 0.02$	$0.71 \pm 0.14 \pm 0.02$	JHEP12,072(2023)
$D^0 \to \rho^- e^+ \nu_e$	$1.695 \pm 0.083 \pm 0.051$	$0.845 \pm 0.056 \pm 0.039$	PRL122,062001(2019)

- We target on $D \to K^*$ and $D_s \to \phi$ using lattice QCD
  - Both experimental precision $\sim 3\%$

$D_s \to \phi$ & $D \to K^*(892)$

$K^*(892)$ $\quad I(J^P) = 1/2(1^-)$

$K^*(892)$ T-Matrix Pole $\sqrt{s}$ $\quad$ (890+-14) - i (26+-6) MeV

$K^*(892)$ **DECAY MODES**

	Mode	Fraction (Γ_i/Γ)	Scale Factor/ Conf. Level	P(MeV/c)
Γ_1	$K\pi$	~ 100 %		289

$\phi(1020)$ $\quad I^G(J^{PC}) = 0^-(1^{--})$

Expand/Collapse All

$\phi(1020)$ MASS	1019.460 ± 0.016 MeV
$\phi(1020)$ WIDTH	4.249 ± 0.013 MeV (S = 1.1)

- Γ_{K^*} is not so small, $K^* \to K\pi$ may have visible effect
- Γ_ϕ is only ~ 4 MeV, a reasonable approximation to regard ϕ as stable

This work: $D_s \to \phi l \nu_l$

- Previous lattice calculation: HPQCD,PRD90,074506(2014)
  - Three ensembles with two coarse lattice spacings, ~ 0.09 fm and ~ 0.12 fm
  - Extrapolated to physical limit by z-expansion

$$F(z) = \sum_{n=0}^{3} B_n^F \left[1 + C_n^F a^2 + D_n^F a^4 + E_n^F (m_l/m_{s,\text{phys}})\right] z^n$$

leading to $r_V = 1.72(21)$, $r_2 = 0.74(12)$

- Experiment:

	r_V	r_2	Refs
BaBar(e)	$1.807 \pm 0.046 \pm 0.065$	$0.816 \pm 0.036 \pm 0.030$	PRD78,051101(2008)
BESIII(μ)	$1.58 \pm 0.17 \pm 0.02$	$0.71 \pm 0.14 \pm 0.02$	JHEP12,072(2023)

- A more precise and systematic lattice study is essential

Extraction of the form factor: scalar function method

- Matrix element parameterization

$$\langle h_\nu/J_\nu^{\mathrm{em}}(p)|J_\mu^{\mathrm{em}/W}(0)|H(p')\rangle = \sum_j \mathcal{Q}^j_{\mu\nu}(p,p')F_j(q^2)$$

  - $P \to 2\gamma$: $\pi_0/\eta/\eta'/\eta_c \to 2\gamma$
  - $V \to \gamma P$: $J/\psi \to \gamma\eta_c, D_s^* \to \gamma D_s, D^* \to \gamma D$
  - $P \to P'l\nu_l$, $P \to V l\nu_l$
  - $P \to \gamma l\nu_l$, $P \to l'\bar{l}'l\nu_l$

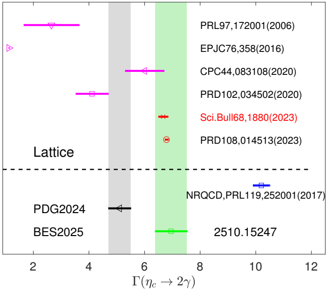
- Constructing scalar functions

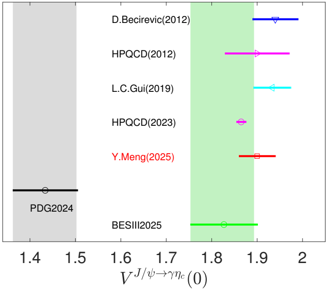
$$\begin{aligned}\tilde{I}_i(q^2) &\equiv \mathcal{Q}^i_{\mu\nu}\cdot\langle h_\nu(p)|J_\mu^{\mathrm{em}/W}(0)|H(p')\rangle \\ &= \sum_i \omega_{ij}(p,p')F_j(q^2), \quad \omega_{ij}\equiv\mathcal{Q}^i_{\mu\nu}\cdot\mathcal{Q}^j_{\mu\nu}\end{aligned}$$

- Solving the linear equation

$$F = \omega^{-1}\cdot\tilde{I}$$

Scalar function method: applications





- $\Gamma(\eta_c \to 2\gamma) = 6.67(16)(6)$ keV, $\Gamma(J/\psi \to \gamma\eta_c) = 2.30(10)$ keV

  Y.M et al,Sci Bull68,1880(2023) **Y.M** et al,PRD111,014508(2025)
- A first step toward resolving the long-standing charmonium-decay puzzle
  - We first find 2.9σ tension and PDG-fit is misleading, arXiv:2109.09381
  - H-P.Wang and C-Z.Yuan,New puzzle in charmonium decays, arXiv:2112.08584
  - HPQCD(2023) and BESIII(2025)

Scalar function method: applications

- The most precise value $\Gamma(D_s^* \to \gamma D_s) = 0.0549(54)$ keV
  - Precision $\sim 10\%$, much imporved than $0.066(26)$ keV by HPQCD PRL112,212002(2014)
  - Combined with experiment: $f_{D_s^*}|V_{cs}| = (190.5^{+55.1}_{-41.7}{}_{\text{stat.}} \pm 9.1_{\text{syst.exp}} \pm 8.7_{\text{syst.latt}})$ MeV

Y.M et al, PRD109,074511(2024)

- First lattice study on J/ψ semileptonic decay

channels	Upper limit/Br	Refs
$J/\psi \to D_s e \nu_e$	4.9×10^{-5}	PLB639,418(2006)
$J/\psi \to D_s e \nu_e$	1.3×10^{-6}	PRD90,112014(2014)
$J/\psi \to D e \nu_e$	7.1×10^{-8}	JHEP06,157(2021)
$J/\psi \to D \mu \nu_\mu$	5.6×10^{-7}	JHEP01,126(2024)
$J/\psi \to D_s e \nu_e$	$1.90(8) \times 10^{-10}$	this work
$J/\psi \to D_s \mu \nu_\mu$	$1.84(8) \times 10^{-10}$	this work
$J/\psi \to D e \nu_e$	$1.21(11) \times 10^{-11}$	this work
$J/\psi \to D \mu \nu_\mu$	$1.18(11) \times 10^{-11}$	this work

Y.M et al,PRD110,074510(2024)

Scalar function method: $D_s \to \phi l \nu_l$

- $\langle \phi_\sigma(\vec{p}) | J^W_\mu(0) | D_s(p') \rangle = \frac{F_0(q^2)}{Mm} \epsilon_{\mu\sigma\alpha\beta} p'_\alpha p_\beta + F_1\left(q^2\right) \delta_{\mu\sigma} + \frac{F_2(q^2)}{Mm} p_\mu p'_\sigma + \frac{F_3(q^2)}{M^2} p'_\mu p'_\sigma$
- Constructing various scalar functions $\tilde{I}_i(q^2)$

$$
\begin{aligned}
F_0 &= \frac{m}{2}\tilde{\mathcal{I}}_0, \qquad F_1 = \frac{1}{2}\tilde{\mathcal{I}}_1 + \frac{1}{2}\tilde{\mathcal{I}}_2 - \frac{m^2}{2M^2}\tilde{\mathcal{I}}_3 \\
F_2 &= \frac{mE}{2\left(E^2 - m^2\right)}\tilde{\mathcal{I}}_1 + \frac{mE^2 + 2m^3}{2\left(E^3 - Em^2\right)}\tilde{\mathcal{I}}_2 - \frac{3Em^3}{2M^2\left(E^2 - m^2\right)}\tilde{\mathcal{I}}_3 \\
F_3 &= -\frac{m^2}{2\left(E^2 - m^2\right)}\tilde{\mathcal{I}}_1 - \frac{3m^2}{2\left(E^2 - m^2\right)}\tilde{\mathcal{I}}_2 + \frac{3m^4}{2M^2\left(E^2 - m^2\right)}\tilde{\mathcal{I}}_3
\end{aligned}
$$

- Matching to V, A_0, A_1, A_2

$$
\begin{aligned}
V &= \frac{(m+M)}{2mM}F_0, \quad A_0 = \frac{F_1}{2m} + \frac{m^2 - M^2 + Q^2}{4m^2 M}F_2 + \frac{m^2 - M^2 - Q^2}{4mM^2}F_3 \\
A_1 &= \frac{F_1}{M+m}, \qquad A_2 = \frac{M+m}{2mM^2}\left(MF_2 + mF_3\right)
\end{aligned}
$$

Lattice setup

Configuration	Volume	a/fm	m_π/MeV	$N_{\text{conf}} \times N_t \times N_s$
C24P29	$24^3 \times 72$	0.10524(05)	292.3(1.0)	$450 \times 72 \times 2$
C32P29	$32^3 \times 64$	0.10524(05)	293.1(0.8)	$397 \times 64 \times 2$
C32P23	$32^3 \times 64$	0.10524(05)	227.9(1.2)	$333 \times 64 \times 3$
F32P30	$32^3 \times 96$	0.07753(03)	300.4(1.2)	$360 \times 96 \times 2$
F48P21	$48^3 \times 96$	0.07753(03)	207.5(1.1)	$241 \times 48 \times 4$
G36P29	$36^3 \times 108$	0.06887(12)	297.2(0.9)	$300 \times 54 \times 2$
F48P32	$48^3 \times 144$	0.05199(08)	316.6(1.0)	$300 \times 72 \times 2$

- Four lattice spacings $[0.052 - 0.105]$ fm → continuum limit
- $m_\pi = 208 - 317$ MeV → physical pion msss
- Same lattice spacings and pion mass: C24P29 vs C32P29 → finite-volume effect

Mass and decay constant

- CLQCD,PRD111,054504(2025)

$$m_{D_s} = 1966.7(1.5)\ \text{MeV}$$
$$f_{D_s} = 247.0(2.1)\ \text{MeV}$$

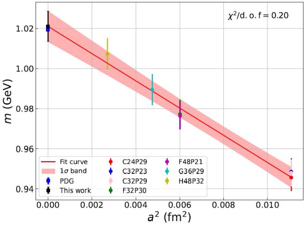
- This work 2510.14478

$$m_{\phi} = 1021.1(7.6)\ \text{MeV}$$
$$f_{\phi} = 246.2(4.1)\ \text{MeV}$$

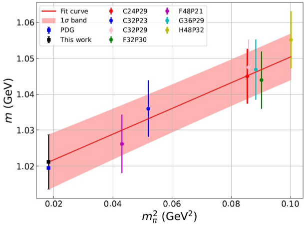
much improved than

$$m_{\phi} = 1032(16)\ \text{MeV}$$
$$f_{\phi} = 241(18)\ \text{MeV}$$

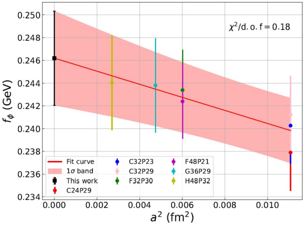
HPQCD,PRD90,074506(2014)



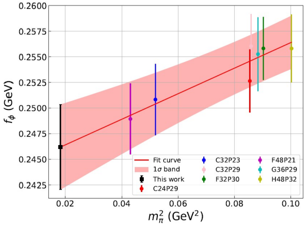
(i) ϕ mass at $m_\pi = 0.135$ GeV.



(ii) ϕ mass at $a = 0.0$ fm.



(iii) ϕ decay constant at $m_\pi = 0.135$ GeV.



(iv) ϕ decay constant at $a = 0.0$ fm.

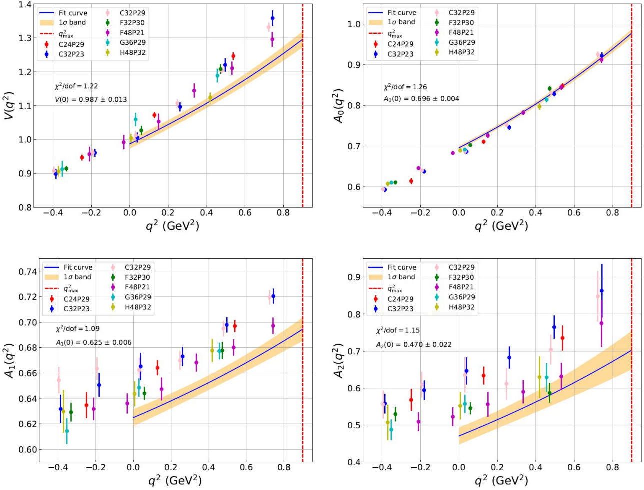
$D_s \to \phi$ form factors

- z-expansion:

$$V/A(q^2) = \frac{1}{1-q^2/m^2_{V/A}} \sum_{i=0}^{2} \left(c_i + d_i a^2\right) \times \left[1 + f_i \left(m_\pi^2 - m_{\pi,\mathrm{phys}}^2\right)\right] z^i$$

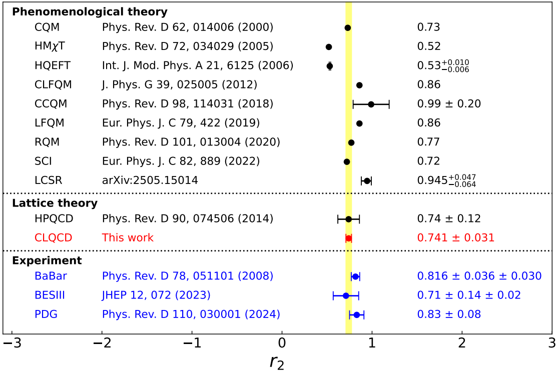
- Test other schemes
  - Single pole
  - Modified pole
  - Phase moment form

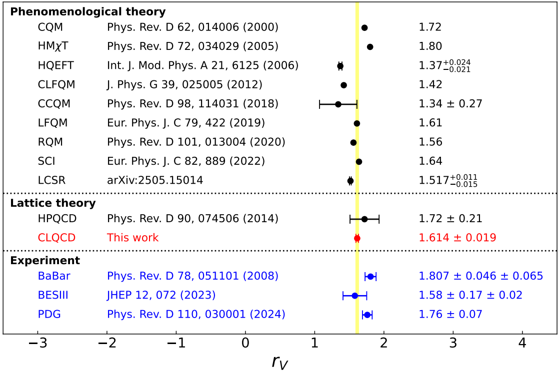
D.L.Yao etc,PRD101,034014(2020)



$D_s \to \phi$ form factors

$$r_2 \equiv A_2(0)/A_1(0), \quad r_V \equiv V(0)/A_1(0)$$

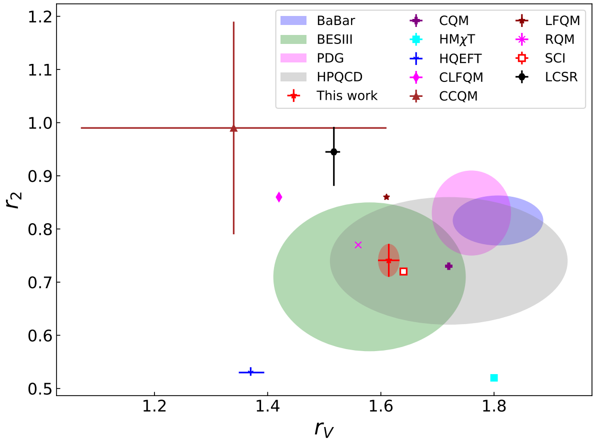




G.-F.Fan,**Y.M**,C.Liu,Z.-F Liu,T.-X Wang,T.-H Shen,K.-Z Zhang, and L.Zhang, arXiv:2510.14478

- Precision improved by up to an order of magnitude

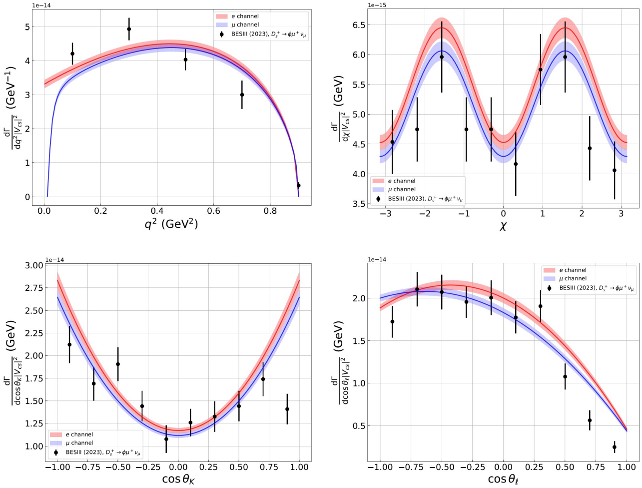
$D_s \to \phi$ form factors



- The PDG may be misleading

Differential decay width

$\mathcal{B}(D_s \to \phi \ell \nu_\ell) \times 10^{-2}$	μ channel	e channel
This work	2.351(67)	2.493(73)
BaBar [7]	–	2.61(17)
CLEO [8]	–	2.14(19)
BESIII (2018) [9]	1.94(54)	2.26(46)
BESIII (2023) [10]	2.25(11)	–
PDG [39]	2.24(11)	2.34(12)

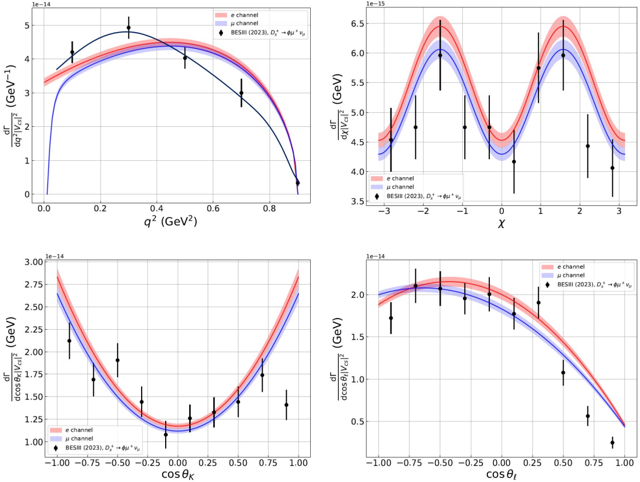


- Discrepancy in q^2-dsitribution compared to BESIII measurement on $D_s \to \phi\mu\nu_\mu$

Differential decay width

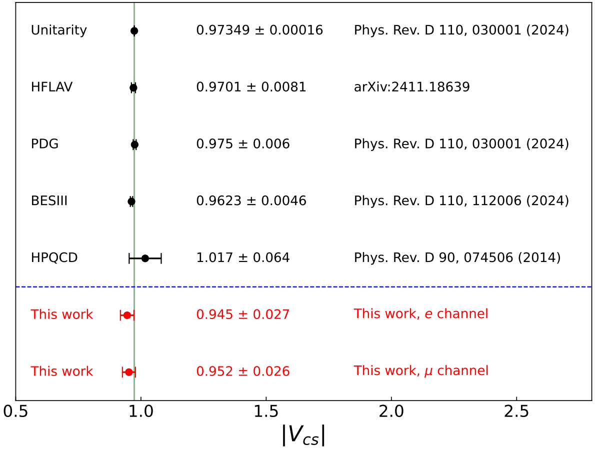
$\mathcal{B}(D_s \to \phi \ell \nu_\ell) \times 10^{-2}$	μ channel	e channel
This work	2.351(67)	2.493(73)
BaBar [7]	–	2.61(17)
CLEO [8]	–	2.14(19)
BESIII (2018) [9]	1.94(54)	2.26(46)
BESIII (2023) [10]	2.25(11)	–
PDG [39]	2.24(11)	2.34(12)

- $D_s \to \phi e \nu_e$ is needed



- Discrepancy in q^2-dsitribution compared to BESIII measurement on $D_s \to \phi\mu\nu_\mu$

Extraction of $|V_{cs}|$



Conclusion and outlook

- Conclusion
  - Scalar function method for $P \to V$ semileptonic decay
  - Systematic study on $D_s \to \phi l \nu_l$ using seven CLQCD ensembles
  - Most precise form factors at physical point after continuum and chiral limit
- Outlook
  - Unstable decay of final vector particle, $P \to V(\to P_1 P_2) l \nu_l$
  - Charmed meson semileptonic decay: $D \to K^*$, $D \to \rho$, etc

Thank you for attention!