CP violation in *b*-baryon two-body decays

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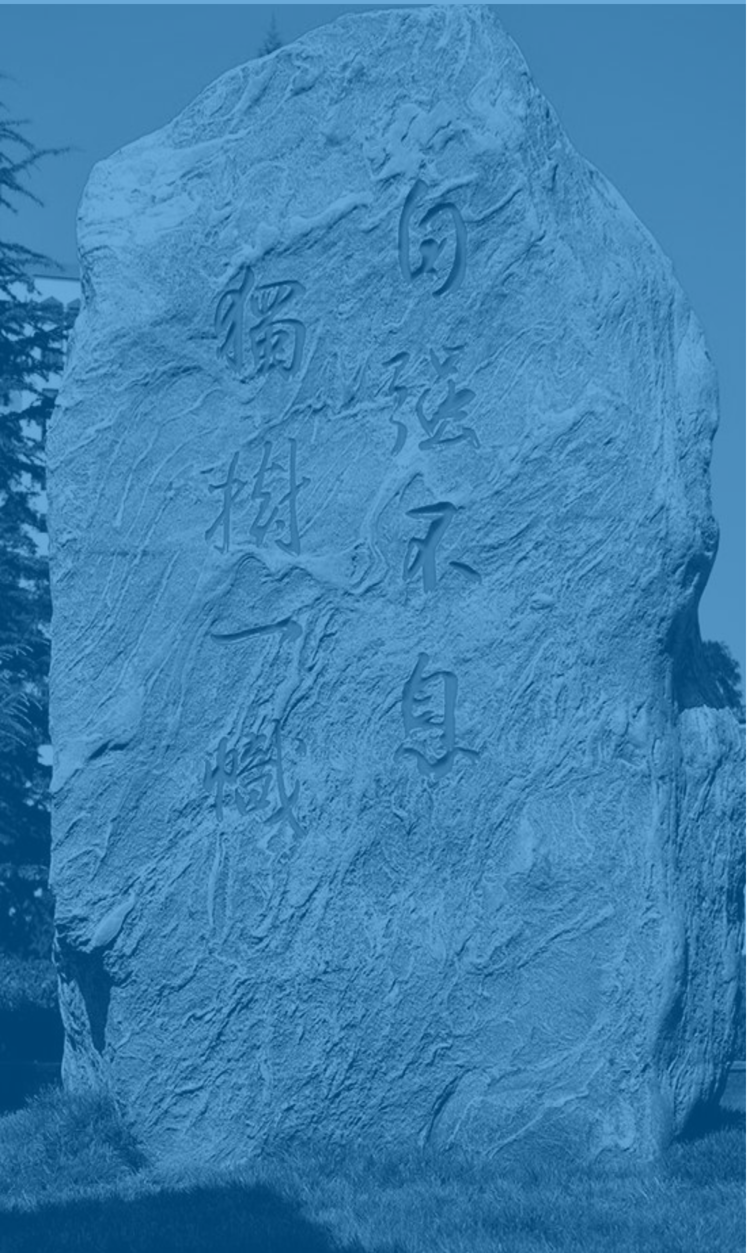
2025·10 HFCPV2025



Why baryon CPVs?

Establish CPVs

$\Lambda_b \to p\pi^-, pK^-$

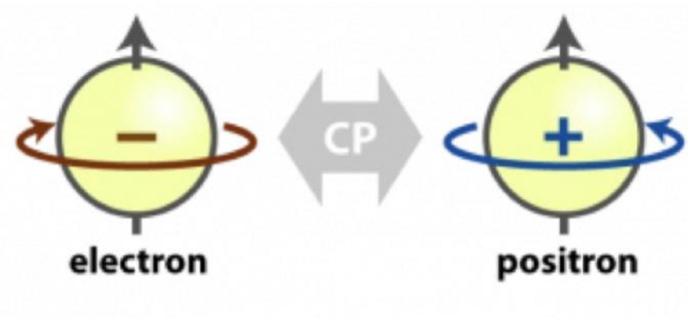
Predict CPVs

$\Lambda_b \to p\rho^-, pK^{*-}, pa_1^-, pK_1^-$

Summary & Outlook

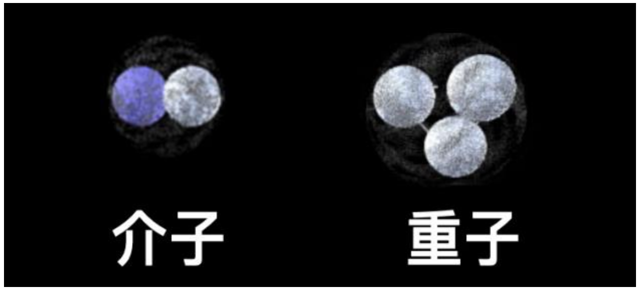
CP violations

- Matter-antimatter asymmetry in the Universe
  - ➢ Science 109 (2005) 5731
- Charge-Parity violation is necessary
  - ➢ Sakharov JETP Lett.(1967)



CP: 电荷共轭+宇称变换

- **Study of baryon CP violation is crucial for understanding the matter-antimatter asymmetry in Universe.**



Baryon CPVs

- Hyperon sector:
  - SM: $\mathcal{O}(10^{-5} \sim 10^{-4})$ [Donoghue, X.G.He, Pakvasa, 1986]
  - BESIII [Nature, 2022] $A_{CP}^{\alpha}(\Lambda \to p\pi^-) = (2.5 \pm 4.8) \times 10^{-3}$

- Charm sector:
  - SM: $\mathcal{O}(10^{-3} \sim 10^{-4})$ [X.G.He, C.W.Liu, 2024] [C.P.Jia, H.Y.Jiang, J.P.Wang, F.S.Yu, 2024]
  - LHCb [JHEP, 2018] $A_{CP}(\Lambda_c \to pK^+K^-/p\pi^+\pi^-) = (3.0 \pm 9.1 \pm 6.1) \times 10^{-3}$

- Beauty sector: SM estimates $\sim 10\%$ due to large weak phase difference

- PDG

$$A_{CP}(B^0 \to K^+\pi^-) = (-8.34 \pm 0.32)\% \quad A_{CP}(B_s^0 \to K^-\pi^+) = (22.4 \pm 1.2)\%$$

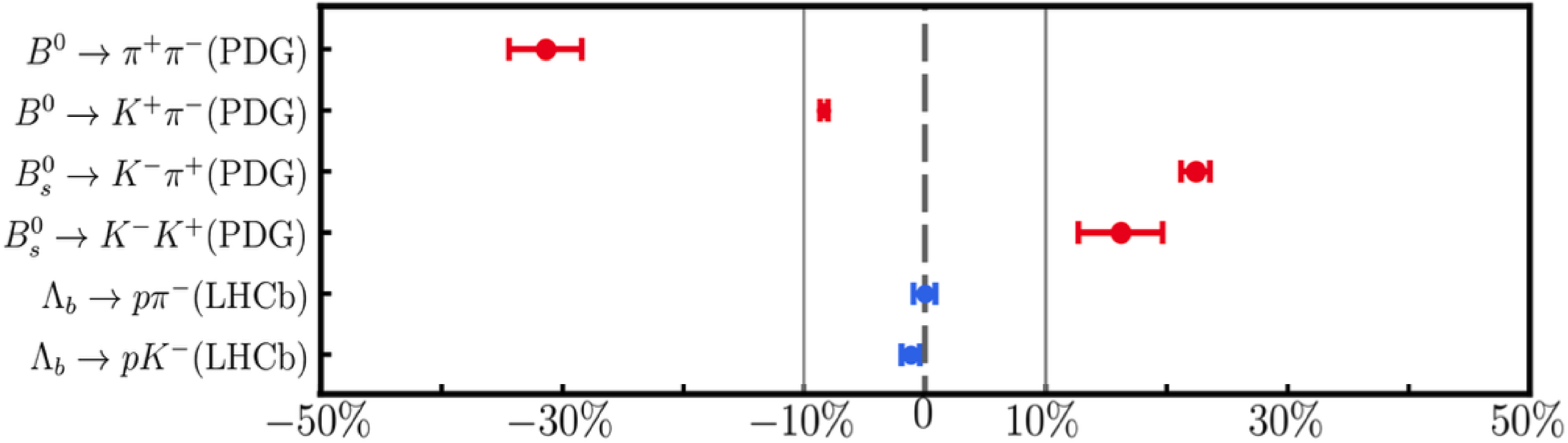
$$\Lambda_b^0 \to pK^-\pi^+\pi^-$$
$$A_{CP} = (2.45 \pm 0.46 \pm 0.10)\%$$

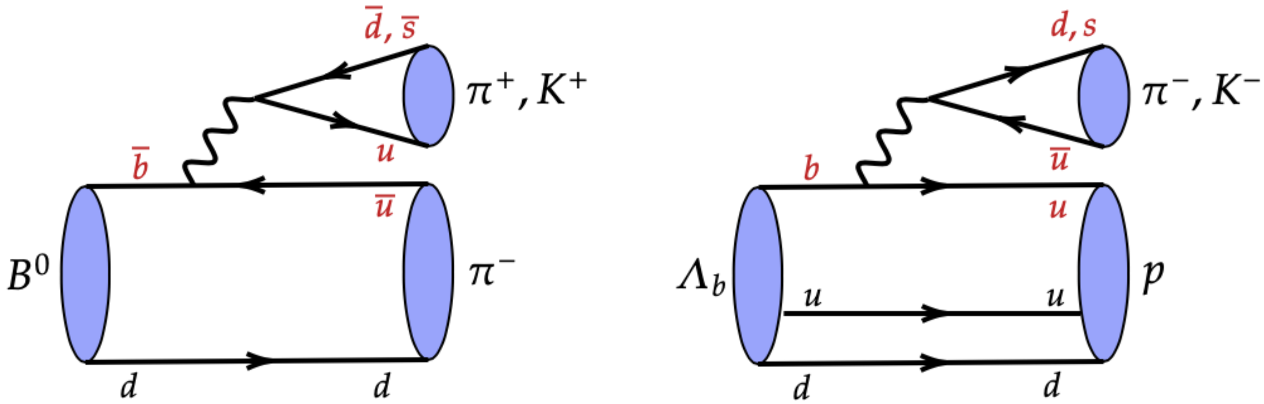
- **LHCb Collaboration**, Nature 643(2025)8074

Puzzle of b-baryon CPVs

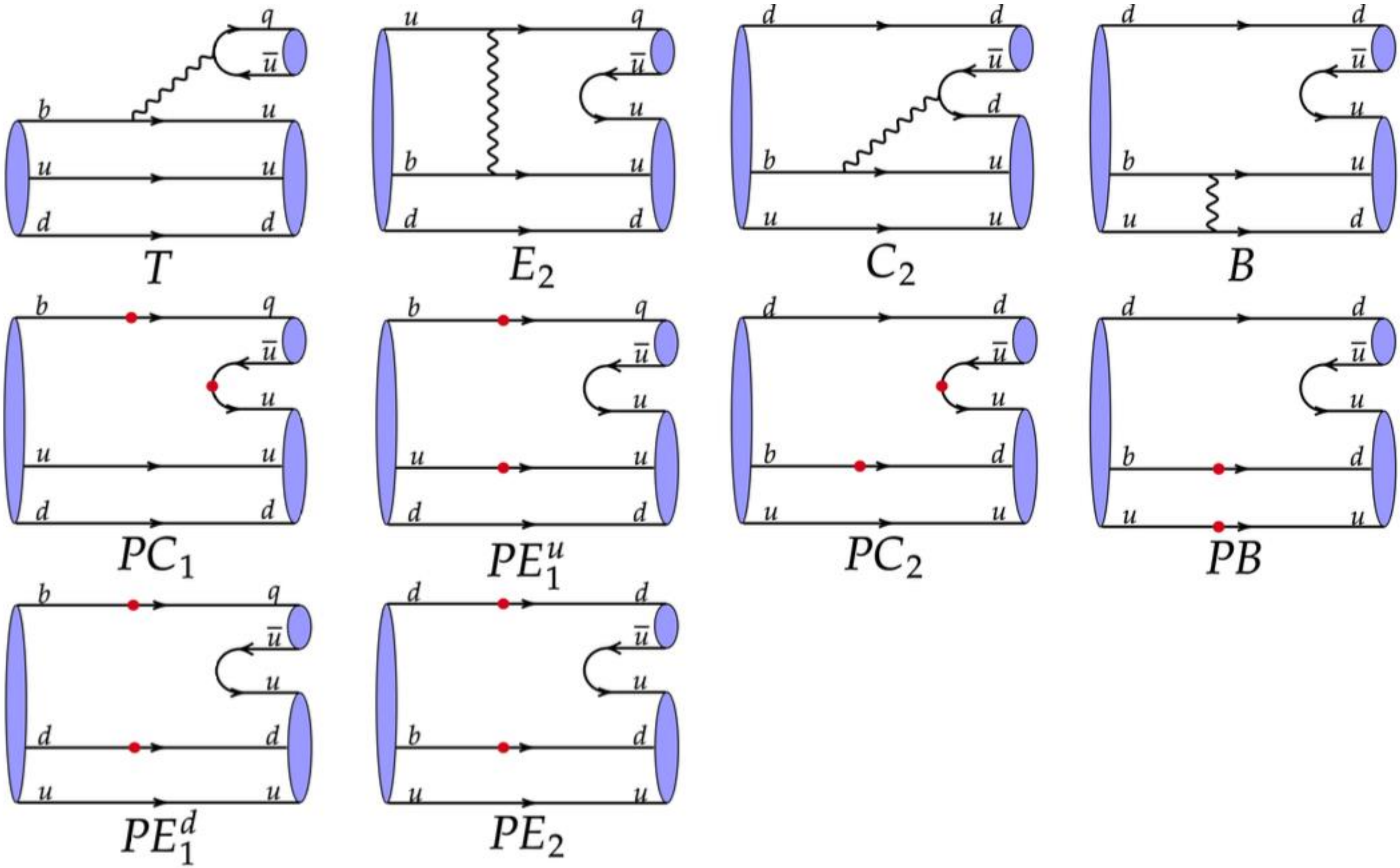
$$A_{CP}(\Lambda_b \to p\pi^-) = (0.20 \pm 0.83 \pm 0.37)\%$$
$$A_{CP}(\Lambda_b \to pK^-) = (-1.14 \pm 0.67 \pm 0.36)\%$$

- LHCb 2024





Topological diagrams of $\Lambda_b \to p\pi^-, pK^-$



$\Lambda_b \to p\pi^-, pK^-$

- Baryons have half-integer spin, and thus two partial wave amplitudes.

$$\mathcal{A}(\Lambda_b \to ph) = i\bar{u}_p(S + P\gamma_5)u_{\Lambda_b}$$

S wave $\quad S = \boxed{\lambda_{\mathcal{T}}|S_{\mathcal{T}}|e^{i\delta^S_{\mathcal{T}}}} + \boxed{\lambda_{\mathcal{P}}|S_{\mathcal{P}}|e^{i\delta^S_{\mathcal{P}}}} \qquad \Delta\delta_S = \delta^S_{\mathcal{P}} - \delta^S_{\mathcal{T}}$

P wave $\quad P = \boxed{\lambda_{\mathcal{T}}|P_{\mathcal{T}}|e^{i\delta^P_{\mathcal{T}}}} + \boxed{\lambda_{\mathcal{P}}|P_{\mathcal{P}}|e^{i\delta^P_{\mathcal{P}}}} \qquad \Delta\delta_P = \delta^P_{\mathcal{P}} - \delta^P_{\mathcal{T}}$

Tree **Penguin** **strong phase difference**

$\Lambda_b \to p\pi^-, pK^-$

- Baryons have half-integer spin, and thus two partial wave amplitudes.

$$\mathcal{A}(\Lambda_b \to ph) = i\bar{u}_p(S + P\gamma_5)u_{\Lambda_b}$$

$$A_{CP}^{dir} \equiv \frac{\Gamma(\Lambda_b\to ph^-)-\overline{\Gamma}(\overline{\Lambda}_b\to\bar{p}h^+)}{\Gamma(\Lambda_b\to ph^-)+\overline{\Gamma}(\overline{\Lambda}_b\to\bar{p}h^+)}$$

$$= \frac{M_+^2(|S|^2-|\bar{S}|^2)+M_-^2|P|^2-|\bar{P}|^2}{M_+^2(|S|^2+|\bar{S}|^2)+M_-^2|P|^2+|\bar{P}|^2}$$

$$= \frac{|S|^2}{|S|^2+\frac{M_-^2}{M_+^2}\frac{1+A_{CP}^S}{1+A_{CP}^P}|P|^2}A_{CP}^S + \frac{\frac{M_-^2}{M_+^2}|P|^2}{\frac{1+A_{CP}^P}{1+A_{CP}^S}|S|^2+\frac{M_-^2}{M_+^2}|P|^2}A_{CP}^P$$

$$\approx \frac{|S|^2}{|S|^2+|P|^2}A_{CP}^S + \frac{|P|^2}{|S|^2+|P|^2}A_{CP}^P$$

$$A_{CP}^S \equiv \frac{|S|^2-|\bar{S}|^2}{|S|^2+|\bar{S}|^2} = \frac{-2r_S sin\Delta\phi sin\Delta\delta_S}{1+r_S^2+2r_S cos\Delta\phi cos\Delta\delta_S},$$

$$A_{CP}^P \equiv \frac{|P|^2-|\bar{P}|^2}{|P|^2+|\bar{P}|^2} = \frac{-2r_P sin\Delta\phi sin\Delta\delta_P}{1+r_P^2+2r_P cos\Delta\phi cos\Delta\delta_P}$$

partial-wave CPVs

$$\Delta\delta_S = \delta_{\mathcal{P}}^S - \delta_{\mathcal{T}}^S$$

$$\Delta\delta_P = \delta_{\mathcal{P}}^P - \delta_{\mathcal{T}}^P$$

strong phase difference

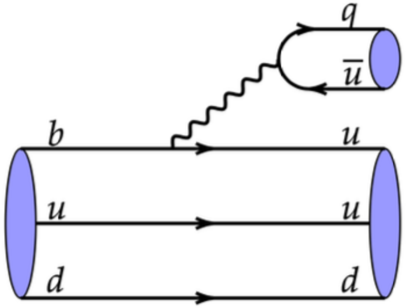
Explain CPVs of $\Lambda_b \to p\pi^-, pK^-$ in PQCD

- Baryons have half-integer spin, and thus two partial wave amplitudes.

$$\mathcal{A}(\Lambda_b \to ph) = i\bar{u}_p(S + P\gamma_5)u_{\Lambda_b}$$

$$S\ wave\quad S = \underbrace{\boxed{\lambda_{\mathcal{T}}|S_{\mathcal{T}}|e^{i\delta^S_{\mathcal{T}}}}}_{} + \underbrace{\boxed{\lambda_{\mathcal{P}}|S_{\mathcal{P}}|e^{i\delta^S_{\mathcal{P}}}}}_{}$$

$$P\ wave\quad P = \underbrace{\boxed{\lambda_{\mathcal{T}}|P_{\mathcal{T}}|e^{i\delta^P_{\mathcal{T}}}}}_{\textbf{Tree}} + \underbrace{\boxed{\lambda_{\mathcal{P}}|P_{\mathcal{P}}|e^{i\delta^P_{\mathcal{P}}}}}_{\textbf{Penguin}}$$



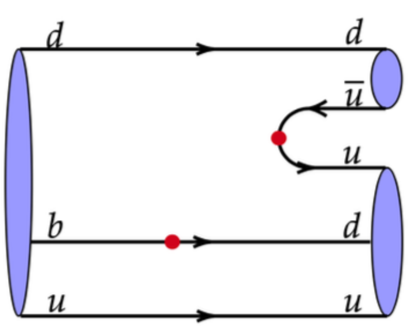
T

$$\sim q^\mu \bar{u}_p\gamma_\mu(1-\gamma_5)u_{\Lambda_b} \sim \bar{u}_p(1+\gamma_5)u_{\Lambda_b}$$

$$\Delta\delta_{S-wave} = \delta^{S-wave}_{PC_2} - \delta^{S-wave}_{T}$$

$$\Delta\delta_{P-wave} = \delta^{P-wave}_{PC_2} - \delta^{P-wave}_{T}$$

different by π



PC_2

$$\sim \bar{u}_p(1+\gamma_5)(\gamma_5\not{p}_\pi)(\not{p}_{\Lambda_b}\gamma_5)\not{p}_p(1-\gamma_5)u_{\Lambda_b} \sim \bar{u}_p(1-\gamma_5)u_{\Lambda_b}$$

CPVs in PQCD

- Strong phases need to be determined by QCD calculations

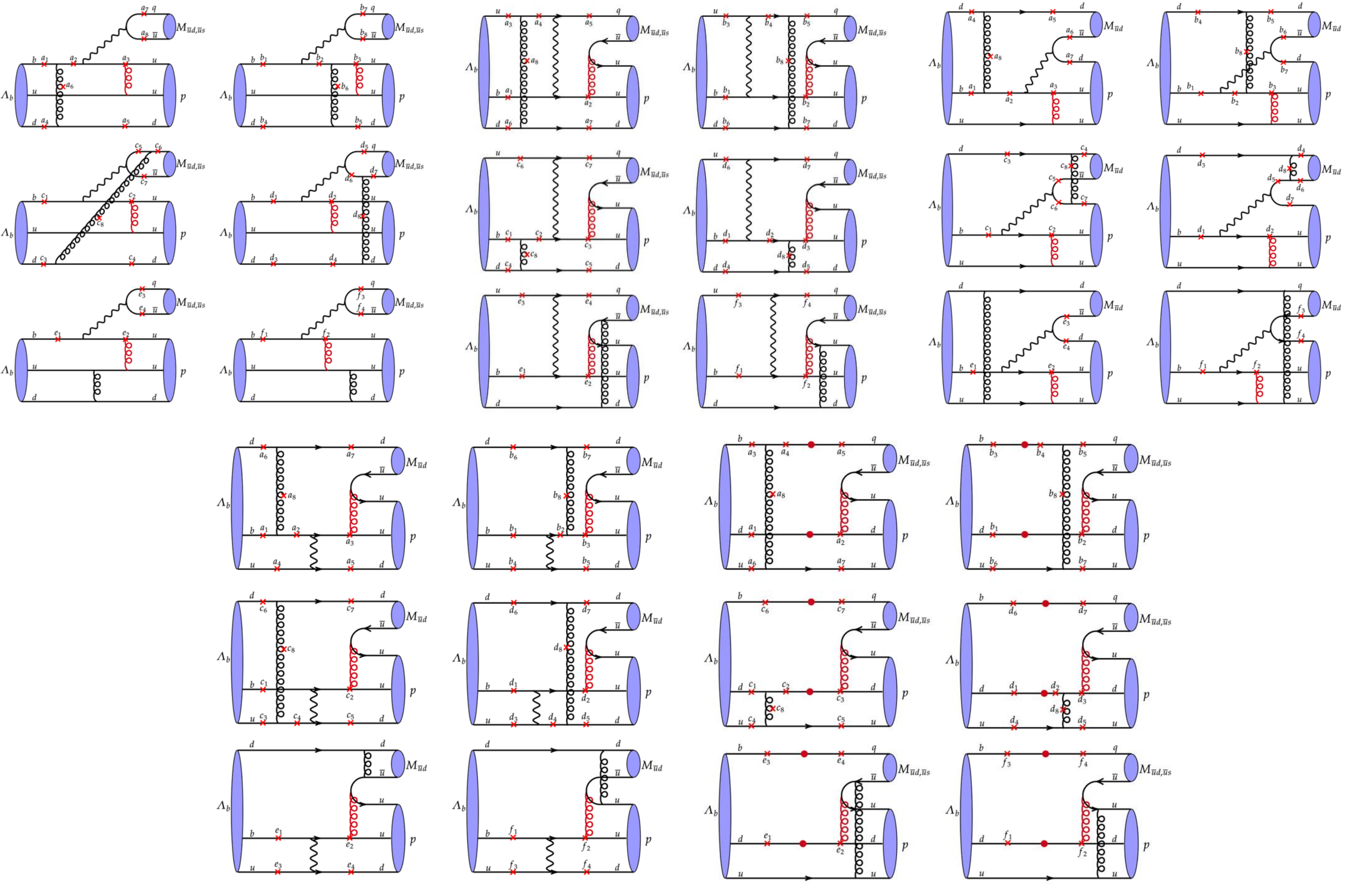
- Based on k_T factorization, PQCD approach has successfully predicted B meson CPV

$C_{\pi\pi}(B \to \pi^+\pi^-)\%$	$A_{CP}(B \to K^+\pi^-)\%$
~ -40 [Lü,Ukai,Yang,2000]	~ -18 [Keum,Li,Sanda,2000]
$-30 \pm 25 \pm 4$ [BaBar,2002]	$-19 \pm 10 \pm 3$ [BaBar,2001]
$-12.8^{+3.48}_{-3.29}$ [Chai,Cheng,Ju,Yan, Lü,Xiao,2022]	$-5.43^{+2.25}_{-2.34}$ [Chai,Cheng,Ju,Yan, Lü,Xiao,2022]
-31.4 ± 3 [PDG]	-8.31 ± 0.31 [PDG]

- Amplitudes are expressed as convolution of hard kernels and LCDAs

$$\begin{aligned}
\mathcal{M} &= \langle pM|H_{eff}|\Lambda_b\rangle \\
&\sim \int [d^4k_p][d^4k_M][d^4k_{\Lambda_b}]\Psi_p([k_p],\mu)\Psi_M([k_M],\mu)\Psi_{\Lambda_b}([k_{\Lambda_b}],\mu)\cdot C_i(\mu)H([k_p],[k_M],[k_{\Lambda_b}],\mu) \\
&\sim \int_0^1 [dx_p]\,[dx_M][dx_{\Lambda_b}] \int [d^2k_p^T]\,[d^2k_M^T][d^2k_{\Lambda_b}^T]\phi_p([x_p],\mu)\phi_M([x_M],\mu)\phi_{\Lambda_b}([x_{\Lambda_b}],\mu) \\
&\cdot C_i(\mu)H([x_p,k_p^T],[x_M,k_M^T],[x_{\Lambda_b},k_{\Lambda_b}^T],\mu)
\end{aligned}$$

Feynman diagrams



Partial-wave amplitudes of $\Lambda_b \to p\pi^-$ in PQCD

- without the CKM matrix elements, $\times 10^{-9}$

$\Lambda_b \to p\pi^-$	$\lvert S\rvert$	$\delta^S(^\circ)$	Real(S)	Imag(S)	$\lvert P\rvert$	$\delta^P(^\circ)$	Real(P)	Imag(P)
T_f	707.17	0.00	707.17	0.00	1004.44	0.00	1004.44	0.00
T_{nf}	51.72	-96.64	-5.98	-51.38	267.72	-97.92	-36.90	-265.17
$T_f + T_{nf}$	703.07	-4.19	701.19	-51.38	1003.22	-15.33	967.54	-265.17
C_2	29.37	154.96	-26.61	12.43	41.51	179.80	-41.51	0.14
E_2	66.94	-145.26	-55.01	-38.14	72.58	119.94	-36.23	62.89
B	10.40	112.64	-4.00	9.60	23.65	-122.56	-12.73	-19.93
Tree	619.26	-6.26	615.57	-67.49	904.75	-14.21	877.08	-222.06
$P_f^{C_1}$	58.44	0.00	58.44	0.00	2.90	0.00	2.90	0.00
$P_{nf}^{C_1}$	1.24	-115.38	-0.53	-1.12	11.16	-95.25	-1.02	-11.11
$P_f^{C_1} + P_{nf}^{C_1}$	57.91	-1.11	57.90	-1.12	11.27	-80.38	1.88	-11.11
P^{C_2}	13.36	-116.10	-5.88	-12.00	14.93	71.96	4.62	14.20
$P^{E_1^u}$	9.48	-87.62	0.39	-9.47	8.83	114.44	-3.65	8.04
P^B	1.36	-51.30	0.85	-1.06	1.55	-159.86	-1.46	-0.53
$P^{E_1^d} + P^{E_2}$	3.87	-98.18	-0.55	-3.83	1.41	-12.55	1.37	-0.31
Penguin	59.45	-27.54	52.71	-27.49	10.65	74.93	2.77	10.28

Observables of $\Lambda_b \to p\pi^-, pK^-$

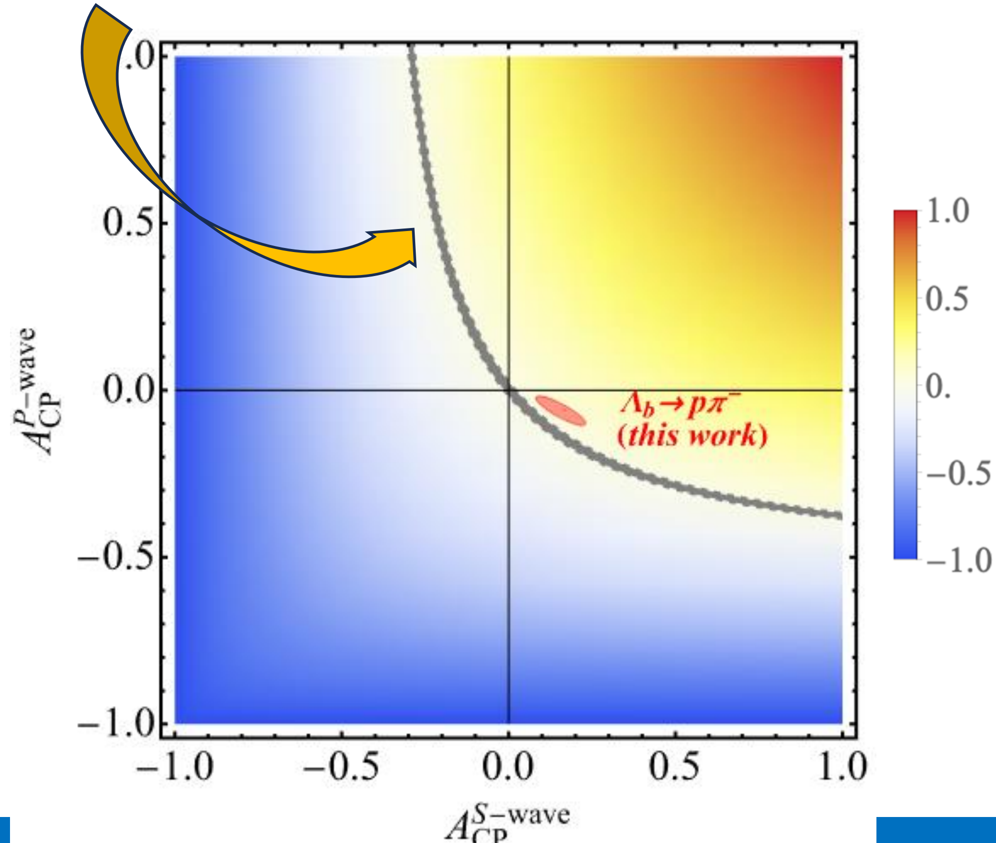
$$A_{CP}^{dir} = \frac{|S|^2}{|S|^2 + \frac{M_-^2}{M_+^2}\frac{1+A_{CP}^{S\text{-wave}}}{1+A_{CP}^{P\text{-wave}}}|P|^2} A_{CP}^{S\text{-wave}} + \frac{\frac{M_-^2}{M_+^2}|P|^2}{\frac{1+A_{CP}^{P\text{-wave}}}{1+A_{CP}^{S\text{-wave}}}|S|^2 + \frac{M_-^2}{M_+^2}|P|^2} A_{CP}^{P\text{-wave}}$$

	$Br(\times 10^{-6})$		
$\Lambda_b \to p\pi^-$	$3.34^{+2.53+1.33+0.10+0.47}_{-1.30-1.10-0.11-0.14}$		
$\Lambda_b \to pK^-$	$2.83^{+2.17+1.17+0.49+2.19}_{-1.05-0.92-0.37-0.66}$		
	A_{CP}^{dir}	$A_{CP}^{S}(\kappa_S)$	$A_{CP}^{P}(\kappa_P)$
$\Lambda_b \to p\pi^-$	$0.05^{+0.00+0.00+0.00+0.02}_{-0.02-0.01-0.02-0.01}$	$\boxed{0.17}^{+0.01+0.01+0.03+0.04}_{-0.04-0.04-0.07-0.04}(49\%)$	$\boxed{-0.06}^{+0.01+0.03+0.02+0.00}_{-0.02-0.03-0.03-0.01}(51\%)$
$\Lambda_b \to pK^-$	$-0.06^{+0.01+0.01+0.02+0.00}_{-0.01-0.01-0.01-0.00}$	$-0.05^{+0.02+0.02+0.04+0.00}_{-0.02-0.01-0.03-0.00}\boxed{(94\%)}$	$-0.21^{+0.07+0.23+0.29+0.04}_{-0.15-0.33-0.27-0.01}\boxed{(6\%)}$
	α	A_{CP}^{α}	$\langle\alpha\rangle$
$\Lambda_b \to p\pi^-$	$-0.94^{+0.00+0.02+0.01+0.03}_{-0.02-0.02-0.02-0.02}$	$0.02^{+0.00+0.01+0.00+0.01}_{-0.01-0.01-0.01-0.01}$	$-0.96^{+0.00+0.01+0.01+0.02}_{-0.00-0.01-0.01-0.01}$
$\Lambda_b \to pK^-$	$0.23^{+0.04+0.02+0.10+0.15}_{-0.03-0.05-0.12-0.07}$	$0.04^{+0.02+0.02+0.01+0.01}_{-0.02-0.03-0.01-0.01}$	$0.20^{+0.02+0.01+0.11+0.14}_{-0.02-0.02-0.12-0.06}$
	β	A_{CP}^{β}	$\langle\beta\rangle$
$\Lambda_b \to p\pi^-$	$0.34^{+0.00+0.05+0.01+0.07}_{-0.06-0.06-0.06-0.05}$	$0.22^{+0.00+0.00+0.03+0.07}_{-0.01-0.01-0.04-0.03}$	$0.12^{+0.00+0.05+0.03+0.00}_{-0.05-0.05-0.04-0.02}$
$\Lambda_b \to pK^-$	$-0.39^{+0.03+0.08+0.08+0.12}_{-0.01-0.04-0.07-0.01}$	$-0.44^{+0.01+0.01+0.02+0.08}_{-0.00-0.00-0.01-0.04}$	$0.05^{+0.03+0.08+0.07+0.04}_{-0.01-0.05-0.07-0.02}$
	γ	A_{CP}^{γ}	$\langle\gamma\rangle$
$\Lambda_b \to p\pi^-$	$0.09^{+0.02+0.04+0.04+0.04}_{-0.04-0.06-0.06-0.01}$	$0.11^{+0.01+0.02+0.03+0.03}_{-0.02-0.03-0.04-0.02}$	$-0.02^{+0.01+0.02+0.01+0.01}_{-0.02-0.04-0.01-0.00}$
$\Lambda_b \to pK^-$	$0.89^{+0.02+0.04+0.04+0.00}_{-0.01-0.02-0.05-0.01}$	$0.02^{+0.02+0.05+0.04+0.00}_{-0.01-0.03-0.04-0.00}$	$0.87^{+0.00+0.01+0.02+0.00}_{-0.00-0.01-0.02-0.01}$

$$A_{CP}^{dir} = \frac{|S|^2}{|S|^2 + \frac{M_-^2}{M_+^2}\frac{1+A_{CP}^{S\text{-wave}}}{1+A_{CP}^{P\text{-wave}}}|P|^2} A_{CP}^{S\text{-wave}} + \frac{\frac{M_-^2}{M_+^2}|P|^2}{\frac{1+A_{CP}^{P\text{-wave}}}{1+A_{CP}^{S\text{-wave}}}|S|^2 + \frac{M_-^2}{M_+^2}|P|^2} A_{CP}^{P\text{-wave}}$$

$\Lambda_b \to p\pi^-$ is tree-dominant, so taking $\frac{|S|^2}{|P|^2} = \frac{|619|^2}{|905|^2}$

$A_{CP}(\Lambda_b \to p\pi^-) = (0.20 \pm 0.83 \pm 0.37)\%$ [LHCb,2024]



Results of $\Lambda_b \to pa_1, pK_1$

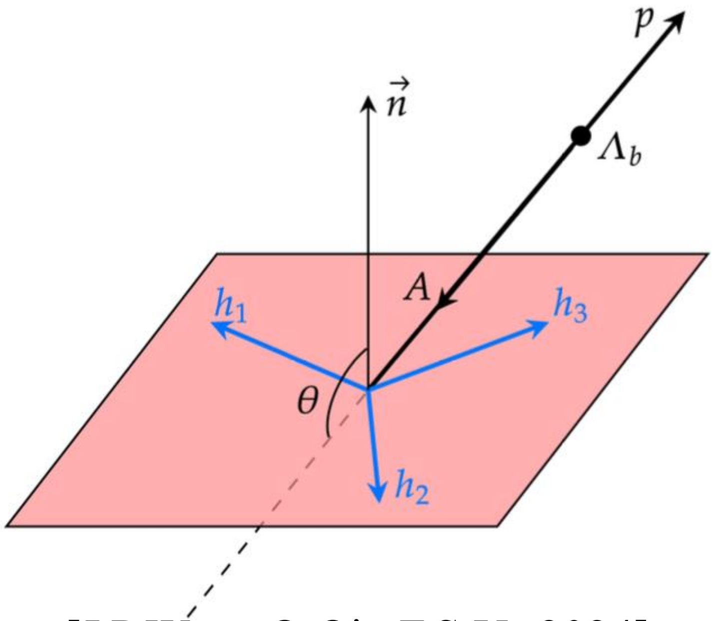
- The angle distribution for $\Lambda_b \to pA \to ph_1h_2h_3$:

$$\frac{d\Gamma}{d\cos\theta} \supset R\,\mathcal{R}e(S^T P_2^*)\ \cos\theta$$

- up-down asymmetry :

$$A_{UD} \equiv \frac{\Gamma(\cos\theta>0)-\Gamma(\cos\theta<0)}{\Gamma(\cos\theta>0)+\Gamma(\cos\theta<0)} = R\,\mathcal{R}e(S^T P_2^*)$$

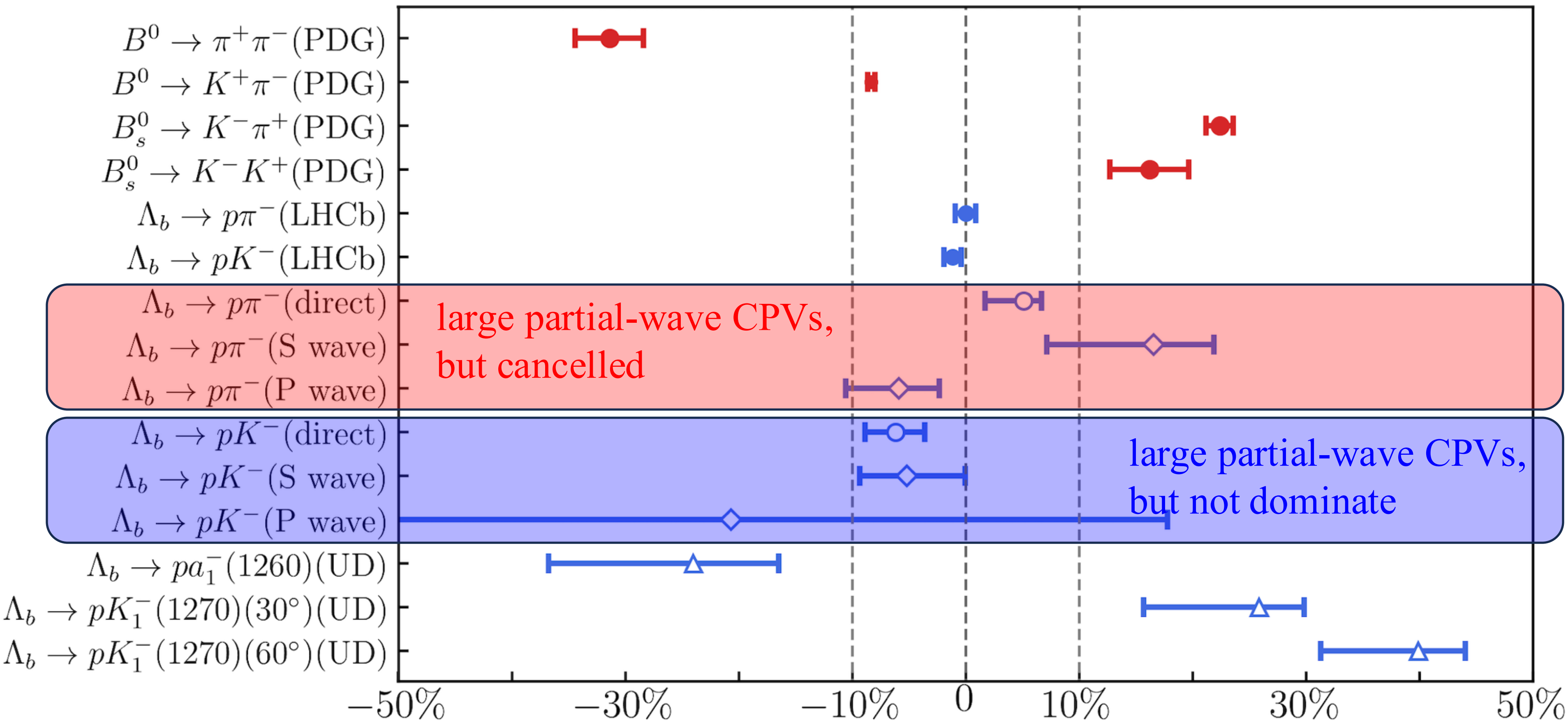
$$A_{CP}^{UD} = \frac{A_{UD}+\bar{A}_{UD}}{A_{UD}-\bar{A}_{UD}}$$



[J.P.Wang,Q.Qin,F.S.Yu,2024]

	a_{UD}	A_{CP}^{UD}
$\Lambda_b \to pa_1^-(1260)$	$-0.09^{+0.00+0.01+0.02+0.00}_{-0.01-0.01-0.01-0.01}$	$-0.24^{+0.03+0.05+0.05+0.03}_{-0.03-0.09-0.06-0.06}$
$\Lambda_b \to pK_1^-(1270)(30°)$	$-0.19^{+0.03+0.02+0.01+0.01}_{-0.02-0.02-0.01-0.02}$	$0.26^{+0.02+0.03+0.01+0.00}_{-0.03-0.08-0.04-0.04}$
$\Lambda_b \to pK_1^-(1400)(30°)$	$-0.38^{+0.06+0.10+0.05+0.00}_{-0.06-0.09-0.07-0.03}$	$0.72^{+0.05+0.13+0.07+0.05}_{-0.05-0.23-0.03-0.12}$
$\Lambda_b \to pK_1^-(1270)(60°)$	$-0.24^{+0.04+0.04+0.01+0.00}_{-0.02-0.03-0.02-0.03}$	$0.40^{+0.02+0.03+0.02+0.01}_{-0.01-0.04-0.03-0.07}$
$\Lambda_b \to pK_1^-(1400)(60°)$	$-0.04^{+0.02+0.02+0.01+0.02}_{-0.01-0.05-0.03-0.01}$	$-0.19^{+0.12+0.14+0.00+0.06}_{-0.18-0.19-0.20-0.00}$

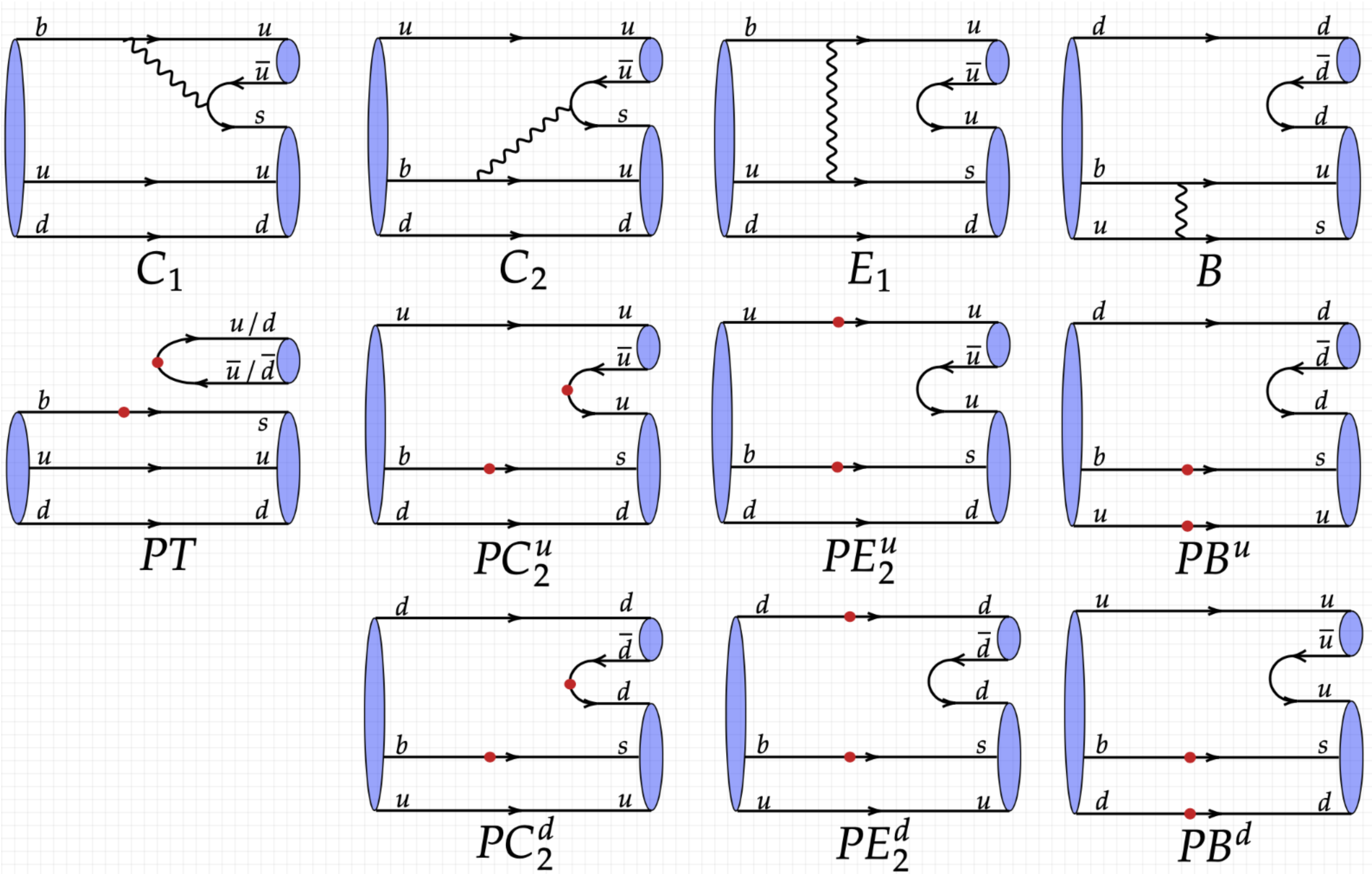
Summary



- **Jia-Jie Han**, Ji-Xin Yu, Ya Li, Hsiang-nan Li, Jian-Peng Wang, Zhen-Jun Xiao, Fu-Sheng Yu, **Physical Review Letters 134 (2025) 221801**.
- **Jia-Jie Han**, Ji-Xin Yu, Ya Li, Hsiang-nan Li, Jian-Peng Wang, Zhen-Jun Xiao, Fu-Sheng Yu, **Physical Review D 112 (2025) 053007.**

$\Lambda_b^0 \to \Lambda^0 h^0$

- Lei Yang, Jia-Jie Han, Qin Chang, Fu-Sheng Yu, in progress



Conclusion

First full QCD analysis of b-baryon decays

Find cancellation of partial wave CPVs

Half-integer spin of baryon, different partial wave amplitudes, different dynamics

Small direct CPVs of $\Lambda_b \to p\pi, pK$ are well explained

Our PQCD calculation have No conflict with known measurements

Large CPV observables are proposed and predicted

Backup

Opportunities and puzzle

- LHCb is a baryon factory !

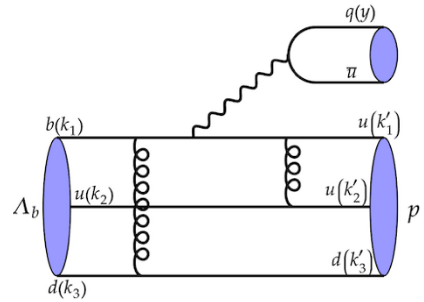
$$\frac{f_{\Lambda_b}}{f_{u,d}} \sim 0.5 \quad \Longrightarrow \quad \frac{N_{\Lambda_b}}{N_B^{0,-}} \sim 0.5 \text{ [LHCb, 2012]}$$

- Precision of b-baryon CPV measurement reached the order of 1%

$$A_{CP}(\Lambda_b \to p\pi^-) = (0.20 \pm 0.83 \pm 0.37)\%$$
$$A_{CP}(\Lambda_b \to pK^-) = (-1.14 \pm 0.67 \pm 0.36)\% \text{ [LHCb,2024]}$$

- Why CPVs of $\Lambda_b \to p\pi, pK$ are small ? What difference of dynamics?

- Baryons are very different from mesons!
  - non-zero spin/polarization, more information from polarizations and partial waves
  - three valence quarks, need at least two hard gluons



  - SCET: power counting of baryon is different from meson
    - heavy-to-light form factor is factorizable at leading power and no end-point singularity!

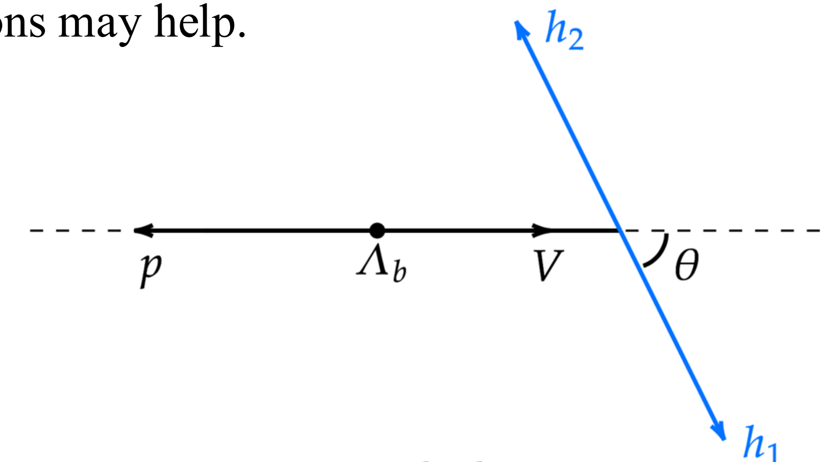
$$\xi_{\Lambda_b\to\Lambda} = f_{\Lambda_b}\Phi_{\Lambda_b}(x_i) \otimes J(x_i, y_i) \otimes \mathrm{f}_{\Lambda}\Phi_{\Lambda}(\mathrm{y}_{\mathrm{i}})$$

    - leading power: $\xi_{\Lambda_b\to\Lambda}(q^2 = 0) = -0.012$ [W.Wang, 2011]
    - Total form factors: $\xi_{\Lambda_b\to\Lambda}(q^2 = 0) = 0.18$ [Y.L.Shen, Y.M.Wang, 2016]

$$A_{CP}^{dir} \approx \kappa_{S^T} A_{CP}^{S^T} + \kappa_{P_2} A_{CP}^{P_2} + \kappa_{D+S^L} A_{CP}^{D+S^L} + \kappa_{P_1} A_{CP}^{P_1}$$

	$Br(\times 10^{-6})$	A_{CP}^{dir}	$A_{CP}^{S^T}(\kappa_{S^T})$
$\Lambda_b \to p\rho^-$	$9.66^{+6.23+3.23+0.21+1.89}_{-3.50-3.03-1.20-0.75}$	$0.03^{+0.02+0.01+0.00+0.02}_{-0.02-0.03-0.03-0.02}$	$0.01^{+0.00+0.00+0.00+0.00}_{-0.01-0.02-0.02-0.02}(7\%)$
$\Lambda_b \to pK^{*-}$	$2.83^{+1.77+0.46+0.37+0.63}_{-1.29-1.23-0.53-0.66}$	$-0.05^{+0.04+0.07+0.01+0.05}_{-0.11-0.07-0.06-0.08}$	$-0.15^{+0.06+0.09+0.02+0.05}_{-0.00-0.04-0.05-0.00}(6\%)$
	$A_{CP}^{S^L+D}(\kappa_{S^L+D})$	$A_{CP}^{P_1}(\kappa_{P_1})$	$A_{CP}^{P_2}(\kappa_{P_2})$
$\Lambda_b \to p\rho^-$	$0.02^{+0.03+0.04+0.02+0.05}_{-0.02-0.02-0.00-0.00}(44\%)$	$0.03^{+0.04+0.00+0.00+0.00}_{-0.05-0.04-0.10-0.05}(45\%)$	$0.17^{+0.00+0.00+0.01+0.03}_{-0.02-0.03-0.03-0.04}(4\%)$
$\Lambda_b \to pK^{*-}$	$0.27^{+0.02+0.06+0.05+0.03}_{-0.17-0.11-0.02-0.18}(33\%)$	$-0.23^{+0.05+0.07+0.02+0.05}_{-0.11-0.11-0.09-0.03}(55\%)$	$-0.14^{+0.01+0.00+0.02+0.01}_{-0.04-0.09-0.02-0.03}(6\%)$
	α	A_{CP}^{α}	$\langle\alpha\rangle$
$\Lambda_b \to p\rho^-$	$-0.83^{+0.02+0.01+0.00+0.00}_{-0.02-0.05-0.04-0.01}$	$-0.01^{+0.01+0.01+0.01+0.00}_{-0.00-0.00-0.01-0.00}$	$-0.83^{+0.01+0.01+0.01+0.00}_{-0.02-0.05-0.04-0.01}$
$\Lambda_b \to pK^{*-}$	$-1.00^{+0.01+0.01+0.00+0.01}_{-0.00-0.00-0.00-0.00}$	$-0.00^{+0.00+0.00+0.00+0.00}_{-0.00-0.00-0.00-0.00}$	$-1.00^{+0.00+0.01+0.00+0.00}_{-0.00-0.00-0.00-0.00}$
	β	A_{CP}^{β}	$\langle\beta\rangle$
$\Lambda_b \to p\rho^-$	$-0.98^{+0.05+0.07+0.05+0.06}_{-0.00-0.00-0.00-0.00}$	$0.00^{+0.01+0.02+0.01+0.02}_{-0.00-0.00-0.00-0.00}$	$-0.99^{+0.04+0.05+0.04+0.04}_{-0.00-0.00-0.00-0.00}$
$\Lambda_b \to pK^{*-}$	$-0.90^{+0.07+0.17+0.11+0.00}_{-0.03-0.03-0.00-0.03}$	$-0.02^{+0.04+0.06+0.04+0.01}_{-0.00-0.04-0.00-0.00}$	$-0.88^{+0.06+0.11+0.08+0.00}_{-0.03-0.06-0.00-0.04}$
	γ	A_{CP}^{γ}	$\langle\gamma\rangle$
$\Lambda_b \to p\rho^-$	$-0.11^{+0.01+0.01+0.01+0.01}_{-0.01-0.01-0.02-0.00}$	$-0.01^{+0.00+0.00+0.00+0.00}_{-0.00-0.00-0.00-0.00}$	$-0.10^{+0.01+0.01+0.01+0.00}_{-0.01-0.01-0.02-0.00}$
$\Lambda_b \to pK^{*-}$	$-0.12^{+0.01+0.00+0.02+0.00}_{-0.06-0.05-0.03-0.05}$	$0.02^{+0.01+0.03+0.01+0.01}_{-0.02-0.02-0.01-0.01}$	$-0.14^{+0.01+0.01+0.02+0.00}_{-0.04-0.07-0.04-0.04}$
	Λ	A_{CP}^{Λ}	$\langle\Lambda\rangle$
$\Lambda_b \to p\rho^-$	$-0.96^{+0.05+0.06+0.04+0.05}_{-0.00-0.00-0.00-0.00}$	$0.00^{+0.01+0.02+0.01+0.02}_{-0.00-0.00-0.00-0.00}$	$-0.97^{+0.04+0.04+0.03+0.04}_{-0.00-0.00-0.00-0.00}$
$\Lambda_b \to pK^{*-}$	$-0.91^{+0.06+0.15+0.09+0.00}_{-0.02-0.02-0.00-0.03}$	$-0.01^{+0.03+0.06+0.03+0.01}_{-0.00-0.03-0.00-0.00}$	$-0.90^{+0.05+0.09+0.07+0.00}_{-0.03-0.05-0.01-0.03}$
	$\mathcal{J}$	$A_{CP}^{\mathcal{J}}$	$\langle\mathcal{J}\rangle$
$\Lambda_b \to p\rho^-$	$1.66^{+0.04+0.04+0.02+0.02}_{-0.03-0.03-0.05-0.00}$	$-0.01^{+0.01+0.01+0.01+0.00}_{-0.01-0.01-0.01-0.00}$	$1.67^{+0.03+0.04+0.02+0.02}_{-0.05-0.03-0.05-0.00}$
$\Lambda_b \to pK^{*-}$	$1.67^{+0.02+0.00+0.04+0.00}_{-0.14-0.12-0.08-0.12}$	$0.04^{+0.02+0.05+0.02+0.01}_{-0.06-0.04-0.02-0.03}$	$1.63^{+0.01+0.03+0.04+0.00}_{-0.08-0.15-0.09-0.09}$

- How to measure the large partial-wave CPV?
- The angular distributions may help.



- The angle distribution for $\Lambda_b \to pV \to ph_1h_2$:

$$\frac{d\Gamma}{d\theta} \propto |H_{\frac{1}{2},0}|^2 + |H_{-\frac{1}{2},0}|^2 + |H_{-\frac{1}{2},-1}|^2 + |H_{\frac{1}{2},1}|^2 + (2|H_{\frac{1}{2},0}|^2 + 2|H_{-\frac{1}{2},0}|^2 - |H_{-\frac{1}{2},-1}|^2 - |H_{\frac{1}{2},1}|^2)P_2$$
$$\propto 1 + \frac{2|H_{\frac{1}{2},0}|^2 + 2|H_{-\frac{1}{2},0}|^2 - |H_{-\frac{1}{2},-1}|^2 - |H_{\frac{1}{2},1}|^2}{|H_{\frac{1}{2},0}|^2 + |H_{-\frac{1}{2},0}|^2 + |H_{-\frac{1}{2},-1}|^2 + |H_{\frac{1}{2},1}|^2} P_2$$
$$\propto 1 + \mathcal{J} \cdot P_2$$

- The CP asymmetry and average for $\mathcal{J}$:

$$A_{CP}^{\mathcal{J}} = \frac{\mathcal{J} - \bar{\mathcal{J}}}{2}, \qquad \langle \mathcal{J} \rangle = \frac{\mathcal{J} + \bar{\mathcal{J}}}{2}$$

[J.P.Wang,Q.Qin,F.S.Yu,2024]

Predict CPVs of $\Lambda_b \to pa_1, pK_1(1270), pK_1(1400)$

$$A_{CP}^{dir} \approx \kappa_{S^T} A_{CP}^{S^T} + \kappa_{P_2} A_{CP}^{P_2} + \kappa_{D+S^L} A_{CP}^{D+S^L} + \kappa_{P_1} A_{CP}^{P_1}$$

$$\begin{pmatrix} |K_1(1270)\rangle \\ |K_1(1400)\rangle \end{pmatrix} = \begin{pmatrix} sin\theta_{K_1} & cos\theta_{K_1} \\ cos\theta_{K_1} & -sin\theta_{K_1} \end{pmatrix} \begin{pmatrix} |K_{1A}\rangle \\ |K_{1B}\rangle \end{pmatrix}$$

$$\theta_K \sim 30^\circ/60^\circ$$

	$Br(\times 10^{-6})$	A_{CP}^{dir}	$A_{CP}^{S^T}(\kappa_{S^T})$
$\Lambda_b \to pa_1^-(1260)$	$11.06^{+8.21+3.88+0.91+1.73}_{-4.30-3.32-0.46-0.06}$	$-0.01^{+0.01+0.03+0.02+0.03}_{-0.00-0.01-0.02-0.00}$	$-0.22^{+0.04+0.07+0.05+0.04}_{-0.03-0.07-0.07-0.01}(6\%)$
$\Lambda_b \to pK_1^-(1270)(30^\circ)$	$5.48^{+3.63+1.94+0.27+2.49}_{-1.87-1.55-0.31-1.11}$	$0.09^{+0.03+0.07+0.03+0.01}_{-0.04-0.02-0.02-0.00}$	$0.34^{+0.00+0.01+0.01+0.00}_{-0.02-0.03-0.01-0.05}(8\%)$
$\Lambda_b \to pK_1^-(1400)(30^\circ)$	$1.25^{+0.59+0.33+0.13+0.64}_{-0.39-0.19-0.19-0.31}$	$0.06^{+0.03+0.05+0.03+0.00}_{-0.03-0.09-0.04-0.01}$	$0.71^{+0.05+0.06+0.03+0.03}_{-0.02-0.16-0.04-0.13}(13\%)$
$\Lambda_b \to pK_1^-(1270)(60^\circ)$	$6.28^{+3.97+1.93+0.18+2.79}_{-2.13-1.51-0.41-1.32}$	$0.07^{+0.01+0.03+0.03+0.01}_{-0.04-0.04-0.03-0.00}$	$0.46^{+0.00+0.00+0.02+0.01}_{-0.02-0.04-0.02-0.07}(9\%)$
$\Lambda_b \to pK_1^-(1400)(60^\circ)$	$0.53^{+0.33+0.38+0.09+0.36}_{-0.16-0.19-0.22-0.13}$	$0.08^{+0.11+0.09+0.12+0.00}_{-0.08-0.11-0.04-0.03}$	$0.07^{+0.00+0.41+0.08+0.22}_{-0.12-0.09-0.15-0.10}(3\%)$
	$A_{CP}^{S^L+D}(\kappa_{S^L+D})$	$A_{CP}^{P_1}(\kappa_{P_1})$	$A_{CP}^{P_2}(\kappa_{P_2})$
$\Lambda_b \to pa_1^-(1260)$	$-0.11^{+0.02+0.01+0.02+0.02}_{-0.00-0.01-0.07-0.03}(46\%)$	$0.18^{+0.03+0.02+0.04+0.09}_{-0.03-0.02-0.03-0.04}(40\%)$	$-0.24^{+0.01+0.05+0.04+0.03}_{-0.02-0.09-0.06-0.06}(8\%)$
$\Lambda_b \to pK_1^-(1270)(30^\circ)$	$-0.11^{+0.01+0.08+0.08+0.03}_{-0.04-0.06-0.03-0.00}(42\%)$	$0.19^{+0.10+0.13+0.05+0.02}_{-0.06-0.09-0.11-0.01}(42\%)$	$0.33^{+0.00+0.04+0.02+0.00}_{-0.02-0.03-0.02-0.03}(8\%)$
$\Lambda_b \to pK_1^-(1400)(30^\circ)$	$0.81^{+0.09+0.17+0.07+0.04}_{-0.12-0.14-0.11-0.00}(17\%)$	$-0.41^{+0.04+0.05+0.08+0.03}_{-0.07-0.05-0.11-0.04}(60\%)$	$0.78^{+0.04+0.11+0.09+0.05}_{-0.06-0.20-0.04-0.10}(10\%)$
$\Lambda_b \to pK_1^-(1270)(60^\circ)$	$0.06^{+0.01+0.08+0.07+0.03}_{-0.03-0.07-0.04-0.00}(37\%)$	$-0.07^{+0.05+0.06+0.04+0.01}_{-0.06-0.05-0.05-0.02}(45\%)$	$0.46^{+0.00+0.04+0.04+0.02}_{-0.01-0.03-0.02-0.06}(9\%)$
$\Lambda_b \to pK_1^-(1400)(60^\circ)$	$-0.82^{+0.14+0.19+0.12+0.21}_{-0.07-0.09-0.07-0.02}(30\%)$	$0.52^{+0.06+0.12+0.37+0.00}_{-0.01-0.14-0.03-0.07}(64\%)$	$-0.28^{+0.27+0.04+0.03+0.03}_{-0.07-0.36-0.25-0.16}(3\%)$

Predict CPVs of $\Lambda_b \to p\rho^-, pK^{*-}$

Invariant amplitudes

$$\mathcal{M}^L\left[B_i(1/2^+) \to B_f(1/2^+) + V\right] = \bar{u}_f(p_f)\epsilon_L^{*\mu}\left[A_1^L\gamma_\mu\gamma_5 + A_2^L\frac{(p_f)_\mu}{m_i}\gamma_5 + B_1^L\gamma_\mu + B_2^L\frac{(p_f)_\mu}{m_i}\right]u_i(p_i),$$

$$\mathcal{M}^T\left[B_i(1/2^+) \to B_f(1/2^+) + V\right] = \bar{u}_f(p_f)\epsilon_T^{*\mu}\left[A_1^T\gamma_\mu\gamma_5 + B_1^T\gamma_\mu\right]u_i(p_i).$$

Partial wave amplitudes

$$S^T = -A_1^T,$$

$$S^L = -A_1^L,$$

$$P_1 = -\frac{p_c}{E_V}\left(\frac{m_i + m_f}{E_f + m_f}B_1^L + B_2^L\right),$$

$$P_2 = \frac{p_c}{E_f + m_f}B_1^T,$$

$$D = -\frac{p_c^2}{E_V(E_f + m_f)}\left(A_1^L - A_2^L\right).$$

$$\Gamma(1/2^+ \to 1/2^+ + V) = \frac{p_c}{4\pi}\frac{E_f + m_f}{m_i}\left\{2(|S|^2 + |P_2|^2) + \frac{E_V^2}{m_V^2}(|S + D|^2 + |P_1|^2)\right\}$$

Helicity amplitudes

$$H_{1/2,1} = -M_+A_1^T - M_-B_1^T,$$

$$H_{-1/2,-1} = M_+A_1^T - M_-B_1^T,$$

$$H_{1/2,0} = \frac{1}{\sqrt{2}m_V}\left[M_+(m_i - m_f)A_1^L - M_-p_cA_2^L + M_-(m_i + m_f)B_1^L + M_+p_cB_2^L\right],$$

$$H_{-1/2,0} = \frac{1}{\sqrt{2}m_V}\left[-M_+(m_i - m_f)A_1^L + M_-p_cA_2^L + M_-(m_i + m_f)B_1^L + M_+p_cB_2^L\right].$$

$$\mathcal{B} = \frac{p_c\tau_{\Lambda_b}}{8\pi m_i^2}(|H_{1/2,1}|^2 + |H_{-1/2,-1}|^2 + |H_{1/2,0}|^2 + |H_{-1/2,0}|^2).$$

[Koener,Kramer,1992]
[Cheng,1996]

Partial wave amplitudes of $\Lambda_b \to pK^-$ in PQCD

- without the CKM matrix elements, $\times 10^{-9}$

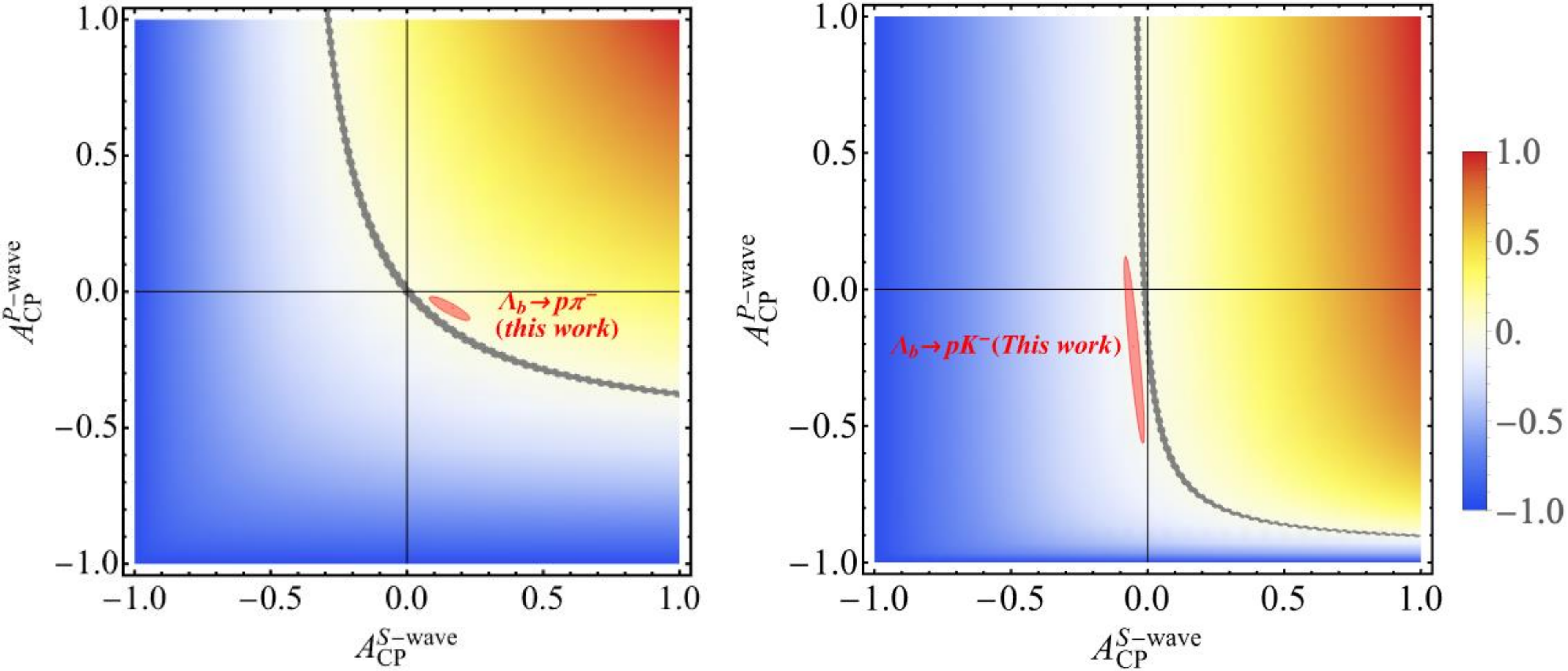
$\Lambda_b \to pK^-$	$\lvert S\rvert$	$\delta^S(°)$	Real(S)	Imag(S)	$\lvert P\rvert$	$\delta^P(°)$	Real(P)	Imag(P)
T^f	865.44	0.00	865.44	0.00	1230.64	0.00	1230.64	0.00
T^{nf}	53.41	-102.81	-11.84	-52.08	343.23	-96.76	-40.43	-340.84
$T^f + T^{nf}$	855.18	-3.49	853.60	-52.08	1238.05	-15.98	1190.21	-340.84
E_2	89.06	-138.10	-66.28	-59.48	94.13	122.31	-50.31	79.56
Tree	795.18	-8.06	787.31	-111.55	1169.46	-12.91	1139.90	-261.28
PC_1^f	76.43	0.00	76.43	0.00	3.30	180.00	-3.30	0.00
PC_1^{nf}	1.14	-134.10	-0.79	-0.82	13.85	-94.36	-1.05	-13.81
$PC_1^f + PC_1^{nf}$	75.64	-0.62	75.64	-0.82	14.48	-107.50	-4.35	-13.81
PE_1^u	11.80	-89.53	0.10	-11.80	11.02	115.62	-4.76	9.93
PE_1^d	7.53	-101.53	-1.50	-7.38	2.67	51.53	1.66	2.09
Penguin	76.88	-15.08	74.23	-20.00	7.66	-166.53	-7.45	-1.79

- SU(3) breaking effect

$$\frac{|T^f(pK)|}{|T^f(p\pi)|} = 1.22, \quad \frac{|T^{nf}(pK)|}{|T^{nf}(p\pi)|} = 1.03, \quad \frac{|E_2(pK)|}{|E_2(p\pi)|} = 1.33 \text{ (S wave)},$$
$$\frac{|T^f(pK)|}{|T^f(p\pi)|} = 1.23, \quad \frac{|T^{nf}(pK)|}{|T^{nf}(p\pi)|} = 1.28, \quad \frac{|E_2(pK)|}{|E_2(p\pi)|} = 1.29 \text{ (P wave)}.$$

$$A_{CP}(\Lambda_b \to p\pi^-) = (0.20 \pm 0.83 \pm 0.37)\%$$
$$A_{CP}(\Lambda_b \to pK^-) = (-1.14 \pm 0.67 \pm 0.36)\%$$ [LHCb,2024]

$$\begin{aligned}
A_{CP}^{dir} &\equiv \frac{\Gamma - \bar{\Gamma}}{\Gamma + \bar{\Gamma}} \\
&= \frac{M_+^2(|S|^2 - |\bar{S}|^2) + M_-^2(|P|^2 - |\bar{P}|^2)}{M_+^2(|S|^2 + |\bar{S}|^2) + M_-^2(|P|^2 + |\bar{P}|^2)} \\
&= \frac{|S|^2}{|S|^2 + \frac{M_-^2}{M_+^2}\frac{1+A_{CP}^{S\text{-wave}}}{1+A_{CP}^{P\text{-wave}}}|P|^2} A_{CP}^{S\text{-wave}} + \frac{\frac{M_-^2}{M_+^2}|P|^2}{\frac{1+A_{CP}^{P\text{-wave}}}{1+A_{CP}^{S\text{-wave}}}|S|^2 + \frac{M_-^2}{M_+^2}|P|^2} A_{CP}^{P\text{-wave}} \\
&= \kappa_S A_{CP}^{S\text{-wave}} + \kappa_P A_{CP}^{P\text{-wave}},
\end{aligned}$$



$\Lambda_b \to p\pi^-$	$\lvert S\rvert$	$\delta^S(^\circ)$	Real(S)	Imag(S)	$\lvert P\rvert$	$\delta^P(^\circ)$	Real(P)	Imag(P)
T_f	707.17	0.00	707.17	0.00	1004.44	0.00	1004.44	0.00
T_{nf}	51.72	-96.64	-5.98	-51.38	267.72	-97.92	-36.90	-265.17
$T_f + T_{nf}$	703.07	-4.19	701.19	-51.38	1003.22	-15.33	967.54	-265.17
C_2	29.37	154.96	-26.61	12.43	41.51	179.80	-41.51	0.14
E_2	66.94	-145.26	-55.01	-38.14	72.58	119.94	-36.23	62.89
B	10.40	112.64	-4.00	9.60	23.65	-122.56	-12.73	-19.93
Tree	619.26	-6.26	615.57	-67.49	904.75	-14.21	877.08	-222.06
$P_f^{C_1}$	58.44	0.00	58.44	0.00	2.90	0.00	2.90	0.00
$P_{nf}^{C_1}$	1.24	-115.38	-0.53	-1.12	11.16	-95.25	-1.02	-11.11
$P_f^{C_1} + P_{nf}^{C_1}$	57.91	-1.11	57.90	-1.12	11.27	-80.38	1.88	-11.11
P^{C_2}	13.36	-116.10	-5.88	-12.00	14.93	71.96	4.62	14.20
$P^{E_1^u}$	9.48	-87.62	0.39	-9.47	8.83	114.44	-3.65	8.04
P^B	1.36	-51.30	0.85	-1.06	1.55	-159.86	-1.46	-0.53
$P^{E_1^d} + P^{E_2}$	3.87	-98.18	-0.55	-3.83	1.41	-12.55	1.37	-0.31
Penguin	59.45	-27.54	52.71	-27.49	10.65	74.93	2.77	10.28

$$S(P_f^{C_1}) = -\frac{G_F}{\sqrt{2}} f_h V_{tb} V_{td}^* \left(\frac{C_3}{3} + C_4 + \frac{C_9}{3} + C_{10} \boxed{+ R_1^\pi} \left(\frac{C_5}{3} + C_6 + \frac{C_7}{3} + C_8\right) \right)$$
$$\left[F_1(m_h^2)(M_{\Lambda_b} - M_p) + F_3(m_h^2) m_h^2\right]$$

chiral factors $R_1 \approx R_2$

$$P(P_f^{C_1}) = -\frac{G_F}{\sqrt{2}} f_h V_{tb} V_{td}^* \left(\frac{C_3}{3} + C_4 + \frac{C_9}{3} + C_{10} \boxed{- R_2^\pi} \left(\frac{C_5}{3} + C_6 + \frac{C_7}{3} + C_8\right) \right)$$
$$\left[G_1(m_h^2)(M_{\Lambda_b} + M_p) - G_3(m_h^2) m_h^2\right]$$

LCDAs of hadrons

Λ_b

$$(Y_{\Lambda_b})_{\alpha\beta\gamma}(x_i,\mu) \equiv \frac{1}{2\sqrt{2}N_c}\int\prod_{l=2}^{3}\frac{dz_l^- d\boldsymbol{z}_l}{(2\pi)^3}e^{ik_l\cdot z_l}\epsilon^{ijk}\langle 0|T[b_\alpha^i(0)u_\beta^j(z_2)d_\gamma^k(z_3)]|\Lambda_b\rangle$$
$$= \frac{1}{8\sqrt{2}N_c}\left\{f_{\Lambda_b}^{(1)}(\mu)[M_1(x_2,x_3)\gamma_5 C^T]_{\gamma\beta} + f_{\Lambda_b}^{(2)}(\mu)[M_2(x_2,x_3)\gamma_5 C^T]_{\gamma\beta}\right\}[\Lambda_b]_\alpha,$$

[P.Ball, V.M.Braun, E.Gardi, 2008]
[G.Bell, T.Feldmann, Y.M.Wang, M.W.Y.Yip, 2013]
[Y.M.Wang, Y.L.Shen, 2016]

$$M_1(x_2,x_3) = \frac{\not{\bar{n}}\not{n}}{4}\psi_3^{+-}(x_2,x_3) + \frac{\not{n}\not{\bar{n}}}{4}\psi_3^{-+}(x_2,x_3),$$

$$M_2(x_2,x_3) = \frac{\not{n}}{\sqrt{2}}\psi_2(x_2,x_3) + \frac{\not{\bar{n}}}{\sqrt{2}}\psi_4(x_2,x_3),$$

$$\psi_2(x_2,x_3) = \frac{x_2 x_3}{\omega_0^4}m_{\Lambda_b}^4 e^{-(x_2+x_3)m_{\Lambda_b}/\omega_0},$$

$$\psi_3^{+-}(x_2,x_3) = \frac{2x_2}{\omega_0^3}m_{\Lambda_b}^3 e^{-(x_2+x_3)m_{\Lambda_b}/\omega_0},$$

$$\psi_3^{-+}(x_2,x_3) = \frac{2x_3}{\omega_0^3}m_{\Lambda_b}^3 e^{-(x_2+x_3)m_{\Lambda_b}/\omega_0},$$

$$\psi_4(x_2,x_3) = \frac{1}{\omega_0^2}m_{\Lambda_b}^2 e^{-(x_2+x_3)m_{\Lambda_b}/\omega_0},$$

$$(Y_{\Lambda_b})_{\alpha\beta\gamma}(x_i,\mu) = \frac{1}{8\sqrt{2}N_c}\left\{f^{(1)}_{\Lambda_b}(\mu)[M_1(x_2,x_3)\gamma_5 C^T]_{\gamma\beta} + f^{(2)}_{\Lambda_b}(\mu)[M_2(x_2,x_3)\gamma_5 C^T]_{\gamma\beta}\right\}[\Lambda_b(p)]_\alpha$$

$$M_1(x_2,x_3) = \frac{\not{\bar n}\not{n}}{4}\psi_3^{+-}(x_2,x_3) + \frac{\not{n}\not{\bar n}}{4}\psi_3^{-+}(x_2,x_3),$$

$$M_2(x_2,x_3) = \frac{\not{n}}{\sqrt{2}}\psi_2(x_2,x_3) + \frac{\not{\bar n}}{\sqrt{2}}\psi_4(x_2,x_3),$$

$$\psi_2(x_2,x_3) = \frac{x_2x_3}{\omega_0^4}m^4_{\Lambda_b}e^{-(x_2+x_3)m_{\Lambda_b}/\omega_0},$$

$$\psi_3^{+-}(x_2,x_3) = \frac{2x_2}{\omega_0^3}m^3_{\Lambda_b}e^{-(x_2+x_3)m_{\Lambda_b}/\omega_0},$$

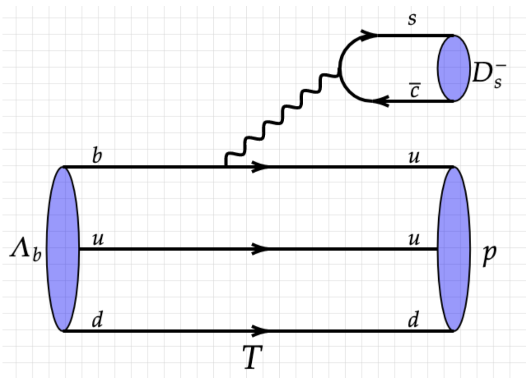
$$\psi_3^{-+}(x_2,x_3) = \frac{2x_3}{\omega_0^3}m^3_{\Lambda_b}e^{-(x_2+x_3)m_{\Lambda_b}/\omega_0},$$

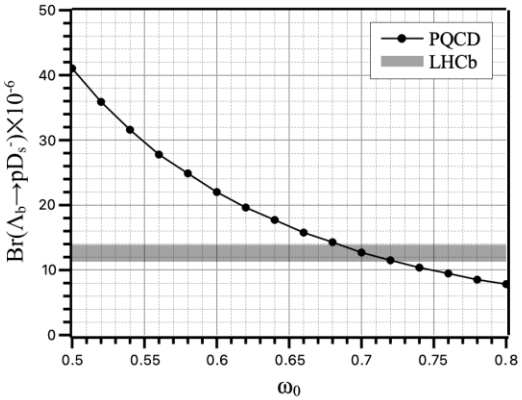
$$\psi_4(x_2,x_3) = \frac{1}{\omega_0^2}m^2_{\Lambda_b}e^{-(x_2+x_3)m_{\Lambda_b}/\omega_0},$$

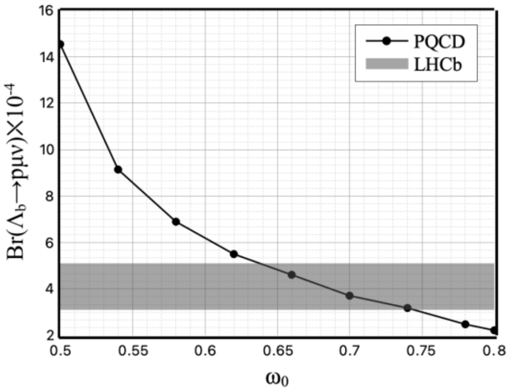
- (LHCb,2212.12574) recently measured the branching fraction:

$$Br(\Lambda_b \to pD_s^-) = (12.6 \pm 1.3)\%$$

- This mode has only W-external emission diagram, used to determine the parameter $\omega_0 = 0.7 \pm 0.1 GeV$







LCDAs of hadrons

proton

$$
\begin{aligned}
(\overline{Y}_P)_{\alpha\beta\gamma}(x_i', \mu) \equiv \frac{1}{2\sqrt{2}N_c} \int \prod_{l=2}^{3} \frac{dz_l^- dz_l}{(2\pi)^3} e^{ik_l\cdot z_l} \epsilon^{i'j'k'} \langle p(p')| T[\bar{u}_\alpha^{i'}(0)\bar{u}_\beta^{j'}(z_2)\bar{d}_\gamma^{k'}(z_3)]|0\rangle \\
= \frac{-1}{8\sqrt{2}N_c} \Big\{ S_1 m_p C_{\beta\alpha}(\bar{N}^+\gamma_5)_\gamma + S_2 m_p C_{\beta\alpha}(\bar{N}^-\gamma_5)_\gamma + P_1 m_p (C\gamma_5)_{\beta\alpha}\bar{N}_\gamma^+ \\
+ P_2 m_p (C\gamma_5)_{\beta\alpha}\bar{N}_\gamma^- + V_1(C\not{P})_{\beta\alpha}(\bar{N}^+\gamma_5)_\gamma + V_2(C\not{P})_{\beta\alpha}(\bar{N}^-\gamma_5)_\gamma \\
+ V_3 \frac{m_p}{2}(C\gamma_\perp)_{\beta\alpha}(\bar{N}^+\gamma_5\gamma^\perp)_\gamma + V_4\frac{m_p}{2}(C\gamma_\perp)_{\beta\alpha}(\bar{N}^-\gamma_5\gamma^\perp)_\gamma + V_5\frac{m_p^2}{2Pz}(C\not{z})_{\beta\alpha}(\bar{N}^+\gamma_5)_\gamma \\
+ V_6\frac{m_p^2}{2Pz}(C\not{z})_{\beta\alpha}(\bar{N}^-\gamma_5)_\gamma + A_1(C\gamma_5\not{P})_{\beta\alpha}(\bar{N}^+)_\gamma + A_2(C\gamma_5\not{P})_{\beta\alpha}(\bar{N}^-)_\gamma \\
+ A_3\frac{m_p}{2}(C\gamma_5\gamma_\perp)_{\beta\alpha}(\bar{N}^+\gamma^\perp)_\gamma + A_4\frac{m_p}{2}(C\gamma_5\gamma_\perp)_{\beta\alpha}(\bar{N}^-\gamma^\perp)_\gamma + A_5\frac{m_p^2}{2Pz}(C\gamma_5\not{z})_{\beta\alpha}(\bar{N}^+)_\gamma \\
+ A_6\frac{m_p^2}{2Pz}(C\gamma_5\not{z})_{\beta\alpha}(\bar{N}^-)_\gamma - T_1(iC\sigma_{\perp P})_{\beta\alpha}(\bar{N}^+\gamma_5\gamma^\perp)_\gamma - T_2(iC\sigma_{\perp P})_{\beta\alpha}(\bar{N}^-\gamma_5\gamma^\perp)_\gamma \\
- T_3\frac{m_p}{Pz}(iC\sigma_{Pz})_{\beta\alpha}(\bar{N}^+\gamma_5)_\gamma - T_4\frac{m_p}{Pz}(iC\sigma_{zP})_{\beta\alpha}(\bar{N}^-\gamma_5)_\gamma - T_5\frac{m_p^2}{2Pz}(iC\sigma_{\perp z})_{\beta\alpha}(\bar{N}^+\gamma_5\gamma^\perp)_\gamma \\
- T_6\frac{m_p^2}{2Pz}(iC\sigma_{\perp z})_{\beta\alpha}(\bar{N}^-\gamma_5\gamma^\perp)_\gamma + T_7\frac{m_p}{2}(C\sigma_{\perp\perp'})_{\beta\alpha}(\bar{N}^+\gamma_5\sigma^{\perp\perp'})_\gamma \\
+ T_8\frac{m_p}{2}(C\sigma_{\perp\perp'})_{\beta\alpha}(\bar{N}^-\gamma_5\sigma^{\perp\perp'})_\gamma \Big\},
\end{aligned}
$$

[V.Braun, R.J.Fries, N.Mahnke, E.Stein, 2000]

Twist classification of the distribution amplitudes in Eq. (2.9)

	twist-3	twist-4	twist-5	twist-6
Vector	V_1	V_2, V_3	V_4, V_5	V_6
Pseudo-vector	A_1	A_2, A_3	A_4, A_5	A_6
Tensor	T_1	T_2, T_3, T_7	T_4, T_5, T_8	T_6
Scalar		S_1	S_2	
Pseudo-scalar		P_1	P_2	

LCDA-proton

$$
\begin{aligned}
(\overline{Y}_P)_{\alpha\beta\gamma}(x_i',\mu) = \frac{-1}{8\sqrt{2}N_c}\Big\{ & S_1 m_p C_{\beta\alpha}(\bar{N}^+\gamma_5)_\gamma + S_2 m_p C_{\beta\alpha}(\bar{N}^-\gamma_5)_\gamma + P_1 m_p (C\gamma_5)_{\beta\alpha}\bar{N}^+_\gamma \\
& + P_2 m_p (C\gamma_5)_{\beta\alpha}\bar{N}^-_\gamma + V_1(C\not{P})_{\beta\alpha}(\bar{N}^+\gamma_5)_\gamma + V_2(C\not{P})_{\beta\alpha}(\bar{N}^-\gamma_5)_\gamma \\
& + V_3\frac{m_p}{2}(C\gamma_\perp)_{\beta\alpha}(\bar{N}^+\gamma_5\gamma^\perp)_\gamma + V_4\frac{m_p}{2}(C\gamma_\perp)_{\beta\alpha}(\bar{N}^-\gamma_5\gamma^\perp)_\gamma + V_5\frac{m_p^2}{2Pz}(C\not{z})_{\beta\alpha}(\bar{N}^+\gamma_5)_\gamma \\
& + V_6\frac{m_p^2}{2Pz}(C\not{z})_{\beta\alpha}(\bar{N}^-\gamma_5)_\gamma + A_1(C\gamma_5\not{P})_{\beta\alpha}(\bar{N}^+)_\gamma + A_2(C\gamma_5\not{P})_{\beta\alpha}(\bar{N}^-)_\gamma \\
& + A_3\frac{m_p}{2}(C\gamma_5\gamma_\perp)_{\beta\alpha}(\bar{N}^+\gamma^\perp)_\gamma + A_4\frac{m_p}{2}(C\gamma_5\gamma_\perp)_{\beta\alpha}(\bar{N}^-\gamma^\perp)_\gamma + A_5\frac{m_p^2}{2Pz}(C\gamma_5\not{z})_{\beta\alpha}(\bar{N}^+)_\gamma \\
& + A_6\frac{m_p^2}{2Pz}(C\gamma_5\not{z})_{\beta\alpha}(\bar{N}^-)_\gamma - T_1(iC\sigma_{\perp P})_{\beta\alpha}(\bar{N}^+\gamma_5\gamma^\perp)_\gamma - T_2(iC\sigma_{\perp P})_{\beta\alpha}(\bar{N}^-\gamma_5\gamma^\perp)_\gamma \\
& - T_3\frac{m_p}{Pz}(iC\sigma_{Pz})_{\beta\alpha}(\bar{N}^+\gamma_5)_\gamma - T_4\frac{m_p}{Pz}(iC\sigma_{zP})_{\beta\alpha}(\bar{N}^-\gamma_5)_\gamma - T_5\frac{m_p^2}{2Pz}(iC\sigma_{\perp z})_{\beta\alpha}(\bar{N}^+\gamma_5\gamma^\perp)_\gamma \\
& - T_6\frac{m_p^2}{2Pz}(iC\sigma_{\perp z})_{\beta\alpha}(\bar{N}^-\gamma_5\gamma^\perp)_\gamma + T_7\frac{m_p}{2}(C\sigma_{\perp\perp'})_{\beta\alpha}(\bar{N}^+\gamma_5\sigma^{\perp\perp'})_\gamma \\
& + T_8\frac{m_p}{2}(C\sigma_{\perp\perp'})_{\beta\alpha}(\bar{N}^-\gamma_5\sigma^{\perp\perp'})_\gamma\Big\}, \qquad (16)
\end{aligned}
$$

	$f_N(GeV^2)$	$\lambda_1(GeV^2)$	$\lambda_2(GeV^2)$	V_1^d
QCDSR(2001)[58]	$(5.3 \pm 0.5) \times 10^{-3}$	$-(2.7 \pm 0.9) \times 10^{-2}$	$(5.1 \pm 1.9) \times 10^{-2}$	0.23 ± 0.03
QCDSR(2006)[59]	$(5.0 \pm 0.5) \times 10^{-3}$	$-(2.7 \pm 0.9) \times 10^{-2}$	$(5.4 \pm 1.9) \times 10^{-2}$	0.23 ± 0.03
LQCD(2019)[51]	$(3.54 \pm 0.06) \times 10^{-3}$	$-(4.49 \pm 0.42) \times 10^{-2}$	$(9.34 \pm 0.48) \times 10^{-2}$	0.19 ± 0.22
	A_1^u	f_1^d	f_2^d	f_1^u
QCDSR(2001)[58]	0.38 ± 0.15	0.6 ± 0.2	0.15 ± 0.06	0.22 ± 0.15
QCDSR(2006)[59]	0.38 ± 0.15	0.4 ± 0.05	0.22 ± 0.05	0.07 ± 0.05
LQCD(2019)[51]	0.30 ± 0.32	...	...	...

LCDA-mesons

- π/K

$$\Phi_{\pi(K)}(q,y)=\frac{i}{\sqrt{2N_C}}\left[\gamma_5 \not{q}\phi^A_{\pi(K)}(y)+m_0^{\pi(K)}\gamma_5\phi^P_{\pi(K)}(y)+m_0^{\pi(K)}\gamma_5(\not{v}\not{n}-1)\phi^T_{\pi(K)}(y)\right]$$

$$\phi^A_{\pi(K)}(y)=\frac{f_{\pi(K)}}{2\sqrt{2N_c}}6y(1-y)\left[1+a_1^{\pi(K)}C_1^{3/2}(2y-1)+a_2^{\pi(K)}C_2^{3/2}(2y-1)+a_4^{\pi(K)}C_4^{3/2}(2y-1)\right],$$

$$\phi^P_{\pi(K)}(y)=\frac{f_{\pi(K)}}{2\sqrt{2N_c}}\left[1+\left(0.45-\frac{5}{2}\rho^2_{\pi(K)}\right)C_2^{1/2}(2y-1)-3\left(-0.045+\frac{9}{20}\rho^2_{\pi(K)}(1+6a_2^{\pi(K)})\right)C_4^{1/2}(2y-1)\right]$$

$$\phi^T_{\pi(K)}(y)=\frac{f_{\pi(K)}}{2\sqrt{2N_c}}(1-2y)\left[1+6\left(0.0975-\frac{7}{20}\rho^2_{\pi(K)}-\frac{3}{5}\rho^2_{\pi(K)}a_2^{\pi(K)}\right)(1-10y+10y^2)\right], \quad (24)$$

- ρ/K^*

$$\langle P|(\bar{q}q')_{V\mp A}|0\rangle=\pm i f_P p_\mu,$$
$$\langle P|(\bar{q}q')_{S\mp P}|0\rangle=\pm i f_P m_{0P},$$

$$\Phi^L_V(q,\epsilon^*_L,y)=\frac{-1}{\sqrt{2N_c}}\left[m_V\not{\epsilon}^*_L\phi_V(y)+\not{\epsilon}^*_L\not{q}\phi^t_V(y)+m_V\phi^s_V(y)\right]_{\alpha\beta},$$

$$\Phi^T_V(q,\epsilon^*_T,y)=\frac{-1}{\sqrt{2N_c}}\left[m_V\not{\epsilon}^*_T\phi^v_V(y)+\not{\epsilon}^*_T\not{q}\phi^T_V(y)+m_V i\epsilon_{\mu\nu\rho\sigma}\gamma_5\gamma^\mu\epsilon^{*\nu}_T v^\rho n^\sigma\phi^a_V(y)\right]_{\alpha\beta}, \quad (27)$$

$$\phi_V(y)=\frac{3f_V}{\sqrt{2N_c}}y(1-y)\left[1+a_1^{\parallel}C_1^{3/2}(2y-1)+a_2^{\parallel}C_2^{3/2}(2y-1)\right], \quad (28)$$

$$\phi^T_V(y)=\frac{3f^T_V}{\sqrt{2N_c}}y(1-y)\left[1+a_1^{\perp}C_1^{3/2}(2y-1)+a_2^{\perp}C_2^{3/2}(2y-1)\right], \quad (29)$$

$$\phi^t_V(y)=\frac{3f^T_V}{2\sqrt{2N_c}}(2y-1)^2, \quad (30)$$

$$\phi^s_V(y)=\frac{3f^T_V}{2\sqrt{2N_c}}(1-2y), \quad (31)$$

$$\phi^v_V(y)=\frac{3f_V}{8\sqrt{2N_c}}(1+(2y-1)^2), \quad (32)$$

$$\phi^a_V(y)=\frac{3f_V}{4\sqrt{2N_c}}(1-2y), \quad (33)$$

$$\langle V|(\bar{q}q')_{V\mp A}|0\rangle=f_V m_V\epsilon^*_\mu,$$
$$\langle V|(\bar{q}q')_{S\mp P}|0\rangle=0,$$

LCDA-mesons

- a_1/K_1

$$\Phi_A^L(q,\epsilon_L^*,y) = \frac{-i}{\sqrt{2N_c}}\left[m_A\gamma_5 \not\epsilon_L^* \phi_A(y) + \not\epsilon_L^* \not q \gamma_5 \phi_A^t(y) + m_A\gamma_5\phi_A^s(y)\right]_{\alpha\beta}, \quad (34)$$

$$\Phi_A^T(q,\epsilon_T^*,y) = \frac{-i}{\sqrt{2N_c}}\left[m_A\gamma_5 \not\epsilon_T^* \phi_A^v(y) + \not\epsilon_T^* \not q \gamma_5 \phi_A^T(y) + m_A i\epsilon_{\mu\nu\rho\sigma}\gamma^\mu \epsilon_T^{*\nu} v^\rho n^\sigma \phi_A^a(y)\right]_{\alpha\beta}, \quad (35)$$

$$\phi_A(y) = \frac{3f_A}{\sqrt{2N_c}} y(1-y)\left[a_{0A}^{\parallel} + 3a_{1A}^{\parallel}(2y-1) + \frac{3}{2}a_{2A}^{\parallel}(5(2y-1)^2-1)\right], \quad (36)$$

$$\phi_A^T(y) = \frac{3f_A}{\sqrt{2N_c}} y(1-y)\left[a_{0A}^{\perp} + 3a_{1A}^{\perp}(2y-1) + \frac{3}{2}a_{2A}^{\perp}(5(2y-1)^2-1)\right]. \quad (37)$$

$$\phi_A^t(y) = \frac{f_A}{2\sqrt{2N_c}}\left[3a_{0A}^{\perp}(2y-1)^2 + \frac{3}{2}a_{1A}^{\perp}(2y-1)(3(2y-1)^2-1)\right], \quad (38)$$

$$\phi_A^s(y) = \frac{3f_A}{2\sqrt{2N_c}}(a_{0A}^{\perp} - a_{1A}^{\perp} - 2a_{0A}^{\perp}y + 6a_{1A}^{\perp}y - 6a_{1A}^{\perp}y^2), \quad (39)$$

$$\phi_A^v(y) = \frac{f_A}{2\sqrt{2N_c}}\left[\frac{3}{4}a_{0A}^{\parallel}(1+(2y-1)^2) + \frac{3}{2}a_{1A}^{\parallel}(2y-1)^3\right], \quad (40)$$

$$\phi_A^a(y) = \frac{3f_A}{4\sqrt{2N_c}}(a_{0A}^{\parallel} - a_{1A}^{\parallel} - 2a_{0A}^{\parallel}y + 6a_{1A}^{\parallel}y - 6a_{1A}^{\parallel}y^2).$$

$$\langle A|(\bar{q}q')_{V\mp A}|0\rangle = \mp i f_A m_A \epsilon_\mu^*,$$

$$\langle A|(\bar{q}q')_{S\mp P}|0\rangle = 0,$$

- mixing angle $\theta_{K_1} \sim 30^\circ/60^\circ$

$$\begin{pmatrix} |K_1(1270)\rangle \\ |K_1(1400)\rangle \end{pmatrix} = \begin{pmatrix} sin\theta_{K_1} & cos\theta_{K_1} \\ cos\theta_{K_1} & -sin\theta_{K_1} \end{pmatrix} \begin{pmatrix} |K_{1A}\rangle \\ |K_{1B}\rangle \end{pmatrix}$$

Λ_b two-body decays

$\Lambda_b \to pM_{\overline{u}q}(q=d,s)$

$$\mathcal{H}_{eff}=\frac{G_F}{\sqrt{2}}\left\{V_{ub}V_{uq}^*\left[C_1(\mu)O_1^u(\mu)+C_2(\mu)O_2^u(\mu)\right]-V_{tb}V_{tq}^*\left[\sum_{i=3}^{10}C_i(\mu)O_i(\mu)\right]\right\}$$

$$O_1^u=(\bar{u}_\alpha b_\beta)_{V-A}(\bar{q}_\beta u_\alpha)_{V-A}, \qquad O_2^u=(\bar{u}_\alpha b_\alpha)_{V-A}(\bar{q}_\beta u_\beta)_{V-A},$$

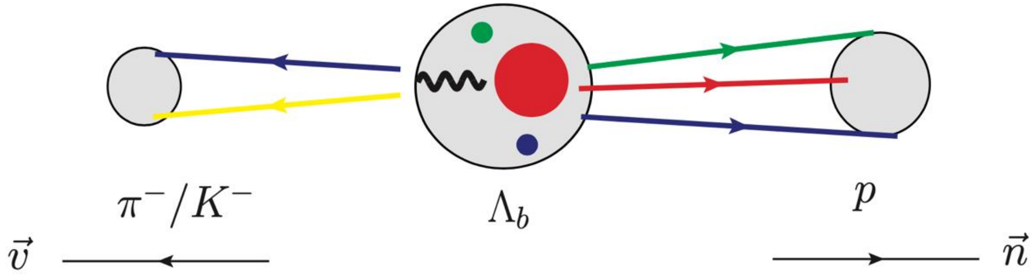
$$O_3=(\bar{q}_\alpha b_\alpha)_{V-A}\sum_{q'}(\bar{q}'_\beta q'_\beta)_{V-A}, \qquad O_4=(\bar{q}_\beta b_\alpha)_{V-A}\sum_{q'}(\bar{q}'_\alpha q'_\beta)_{V-A},$$

$$O_5=(\bar{q}_\alpha b_\alpha)_{V-A}\sum_{q'}(\bar{q}'_\beta q'_\beta)_{V+A}, \qquad O_6=(\bar{q}_\beta b_\alpha)_{V-A}\sum_{q'}(\bar{q}'_\alpha q'_\beta)_{V+A},$$

$$O_7=\frac{3}{2}(\bar{q}_\alpha b_\alpha)_{V-A}\sum_{q'}e_{q'}(\bar{q}'_\beta q'_\beta)_{V+A}, \qquad O_8=\frac{3}{2}(\bar{q}_\beta b_\alpha)_{V-A}\sum_{q'}e_{q'}(\bar{q}'_\alpha q'_\beta)_{V+A},$$

$$O_9=\frac{3}{2}(\bar{q}_\alpha b_\alpha)_{V-A}\sum_{q'}e_{q'}(\bar{q}'_\beta q'_\beta)_{V-A}, \qquad O_{10}=\frac{3}{2}(\bar{q}_\beta b_\alpha)_{V-A}\sum_{q'}e_{q'}(\bar{q}'_\alpha q'_\beta)_{V-A},$$

Kinematics



$$p_i = \frac{m_i}{\sqrt{2}}(1, 1, \mathbf{0}_T),$$
$$p_f = \frac{m_i}{\sqrt{2}}(\eta^+, \eta^-, \mathbf{0}_T),$$
$$q = p_i - p_f = \frac{m_i}{\sqrt{2}}(1-\eta^+, 1-\eta^-, \mathbf{0}_T),$$

$$k_1 = (\frac{m_i}{\sqrt{2}}, \frac{m_i}{\sqrt{2}}x_1, \boldsymbol{k}_{1T}), \quad k_2 = (0, \frac{m_i}{\sqrt{2}}x_2, \boldsymbol{k}_{2T}), \quad k_3 = (0, \frac{m_i}{\sqrt{2}}x_3, \boldsymbol{k}_{3T}),$$
$$k_1' = (\frac{m_i}{\sqrt{2}}\eta^+ x_1', 0, \boldsymbol{k}_{1T}'), \quad k_2' = (\frac{m_i}{\sqrt{2}}\eta^+ x_2', 0, \boldsymbol{k}_{2T}'), \quad k_3' = (\frac{m_i}{\sqrt{2}}\eta^+ x_3', 0, \boldsymbol{k}_{3T}'),$$
$$q_1 = (0, \frac{m_i}{\sqrt{2}}y(1-\eta^-), \boldsymbol{q}_T), \quad q_2 = (0, \frac{m_i}{\sqrt{2}}(1-y)(1-\eta^-), -\boldsymbol{q}_T),$$

for T, E and P diagrams

$$k_1 = (\frac{m_i}{\sqrt{2}}(1-x3), \frac{m_i}{\sqrt{2}}(1-x2), \boldsymbol{k}_{1T}), \quad k_2 = (0, \frac{m_i}{\sqrt{2}}x_2, \boldsymbol{k}_{2T}), \quad k_3 = (\frac{m_i}{\sqrt{2}}x_3, 0, \boldsymbol{k}_{3T})$$

for B and C' diagrams