华南師範大學
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Lattice Parton Collaboration

CLQCD

Progress on light hadron lightcone structures for heavy hadron decays: LQCD and global fits

华俊 华南师范大学

On behalf of Lattice Parton Collaboration (LPC)

第22届全国重味物理和CP破坏研讨会

2025 10/27 @北京大学

OUT LINE

Motivation About Meson & Baryon

- Meson — ageless messengers

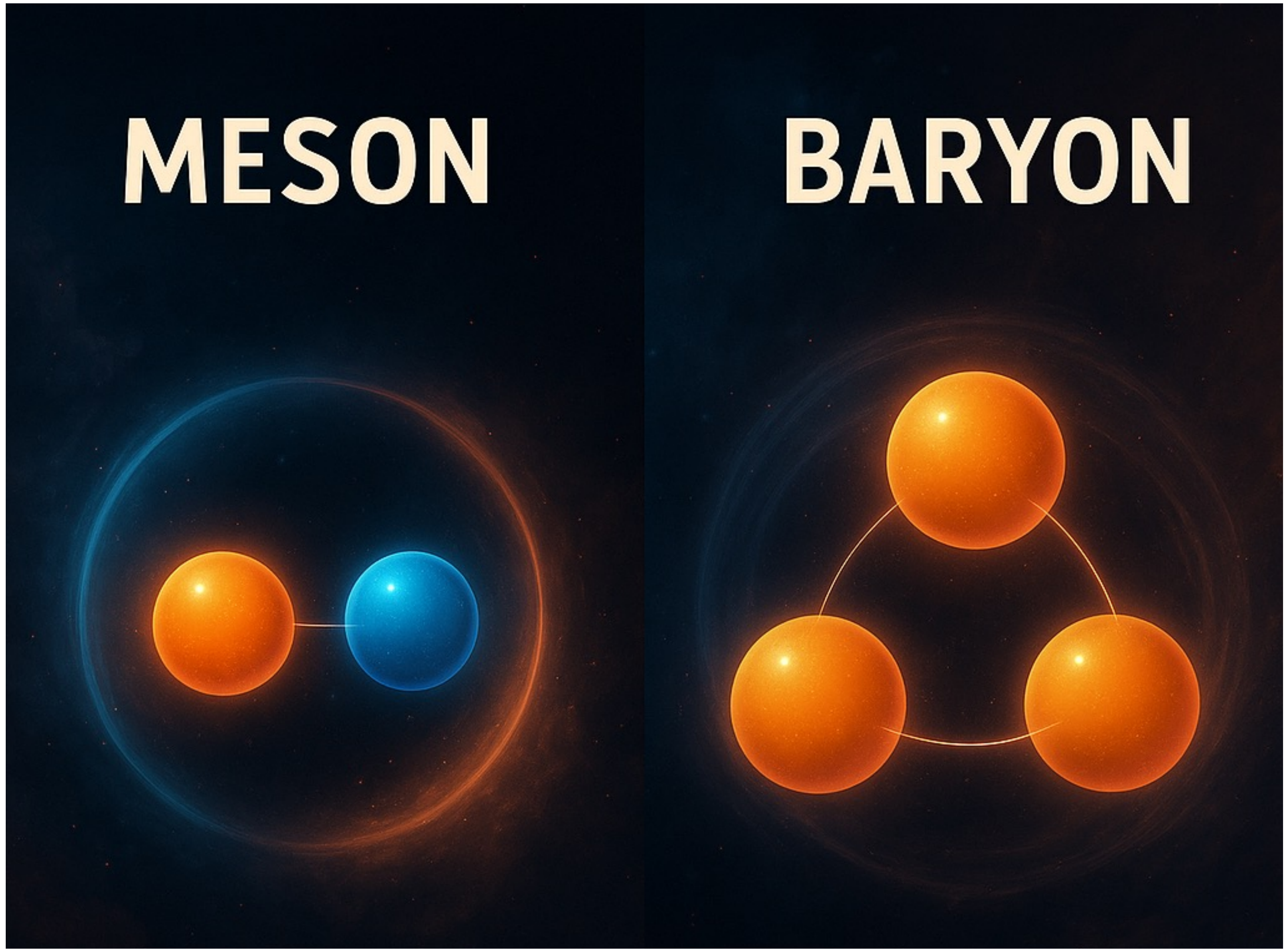
  Light Meson $SU(3)_{flavor}$ $8 \otimes 1$

  Heavy Meson $SU(4)_{flavor}$ …

- Baryon — cornerstone particles

  Octet baryons

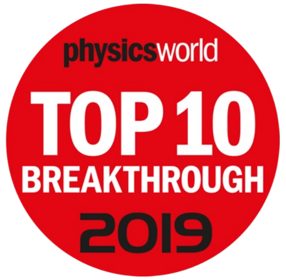
  Decuplet baryons



Motivation About Meson & Baryon

Meson and Baryon in Flavor Physics

- 1956, Parity violation in weak interaction
- 1964, Observation of CP violation in **Kaon meson**
- 1973, Kobayashi-Maskawa mechanism
- 2004, Observation of direct CPV in **B meson**
- 2019, Observation of direct CPV in **D meson**
- 2025, Observation of direct CPV in **Baryon $\Lambda_b^0 \to p$**



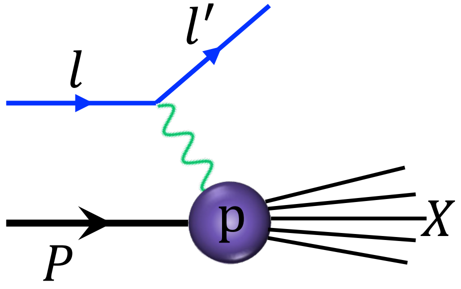
LHCb, Nature 643 8074 (2025)； Theoretical: *J.J.Han, et.al. PRL 134 221801(2025)*

Motivation About LCDA

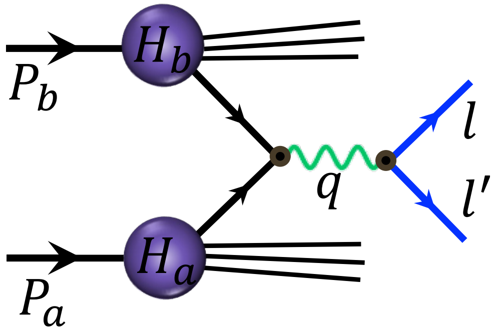
- One try to probe internal structure of nucleons → **Inclusive beam-target collision**

Defined PDFs

DIS



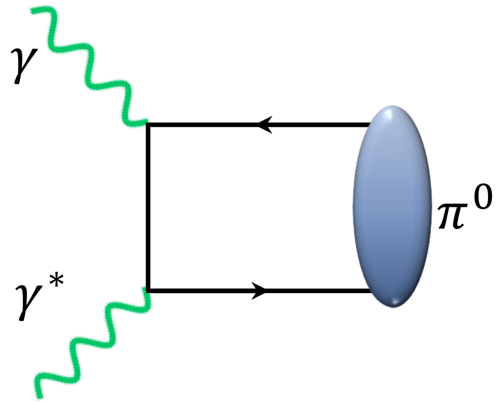
Drell-Yan



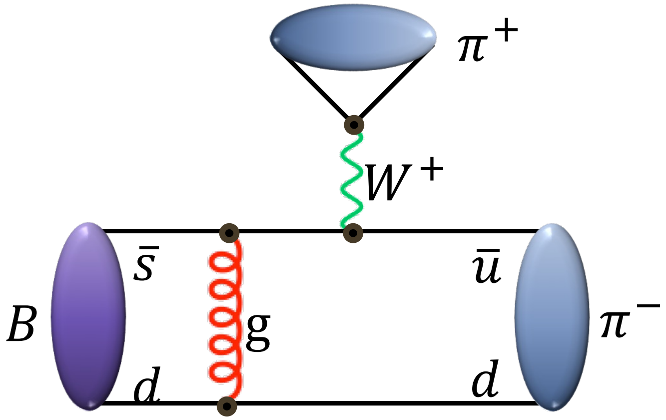
- One try to obtain richer QCD dynamical information → **Exclusive scattering**

Defined LCDAs

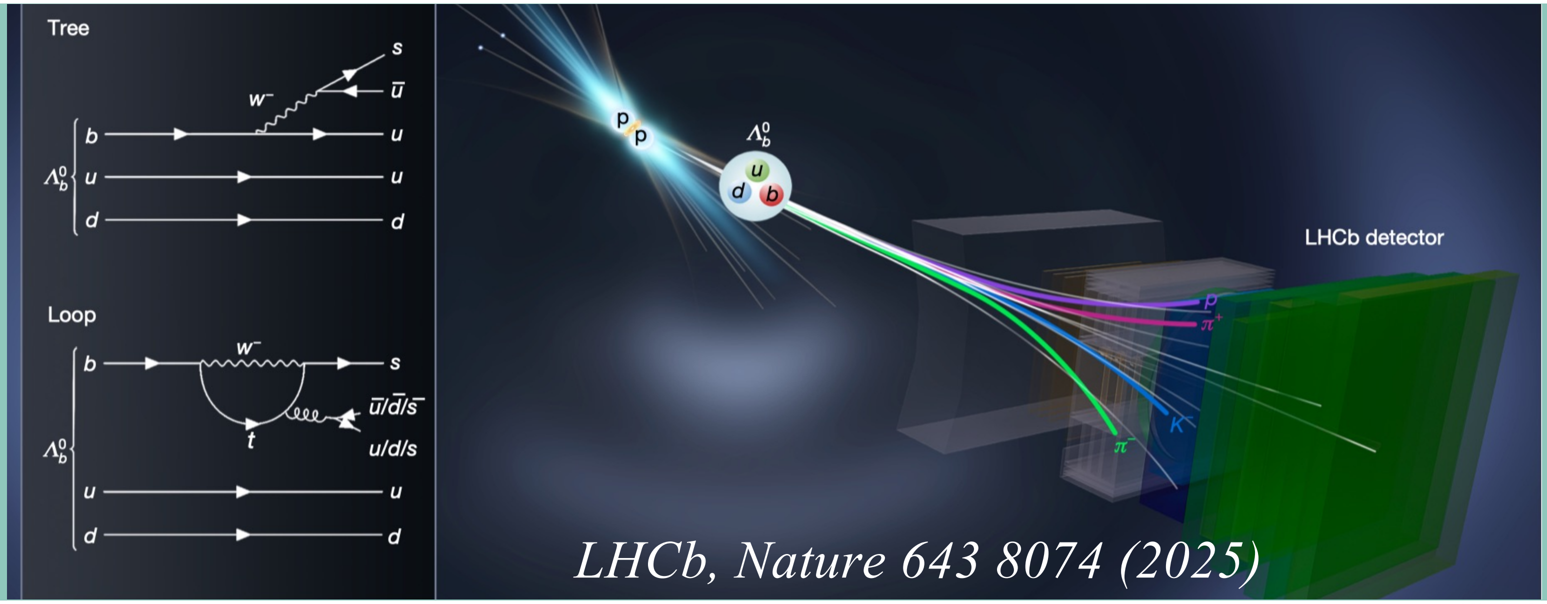
$\gamma\gamma^* \to \pi$



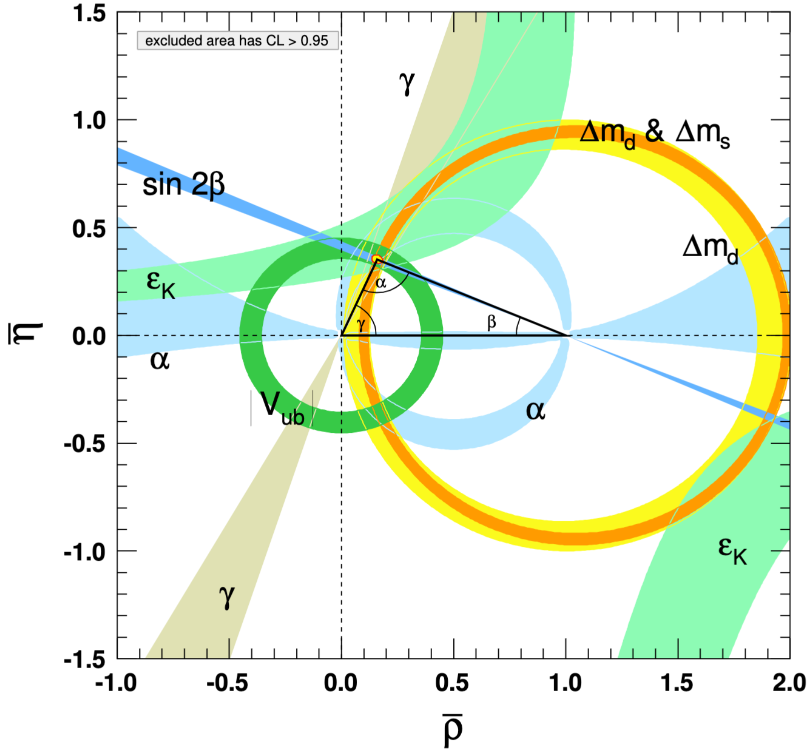
$B \to \pi^+\pi^-$



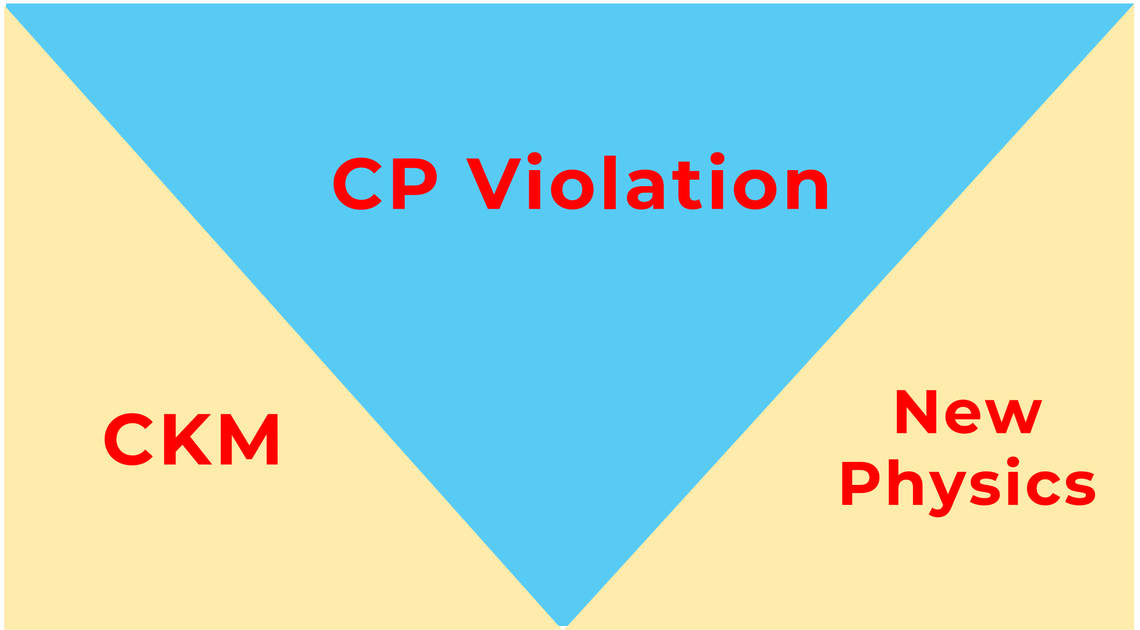
Motivation Exclusive Target



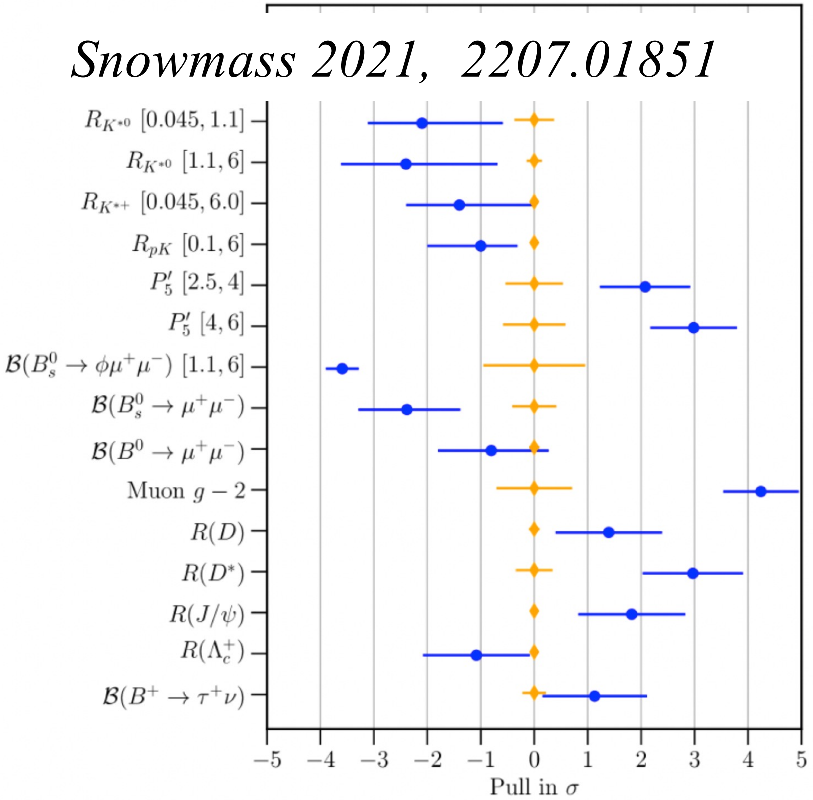
LHCb, Nature 643 8074 (2025)



PDG2024, PRD 110, 030001



Snowmass 2021, 2207.01851



Motivation About LCDA

Light Meson LCDAs: (1977 - now)

- Asymptotic form

  Chernyak, Zhitnitsky, 1977; Lepage, Brodsky, 1979; ……
- QCD Sum rules

  Chernyak, Zhitnitsky, 1982; Braun, Filyanov, 1989; ……
- Dyson-Schwinger Equation

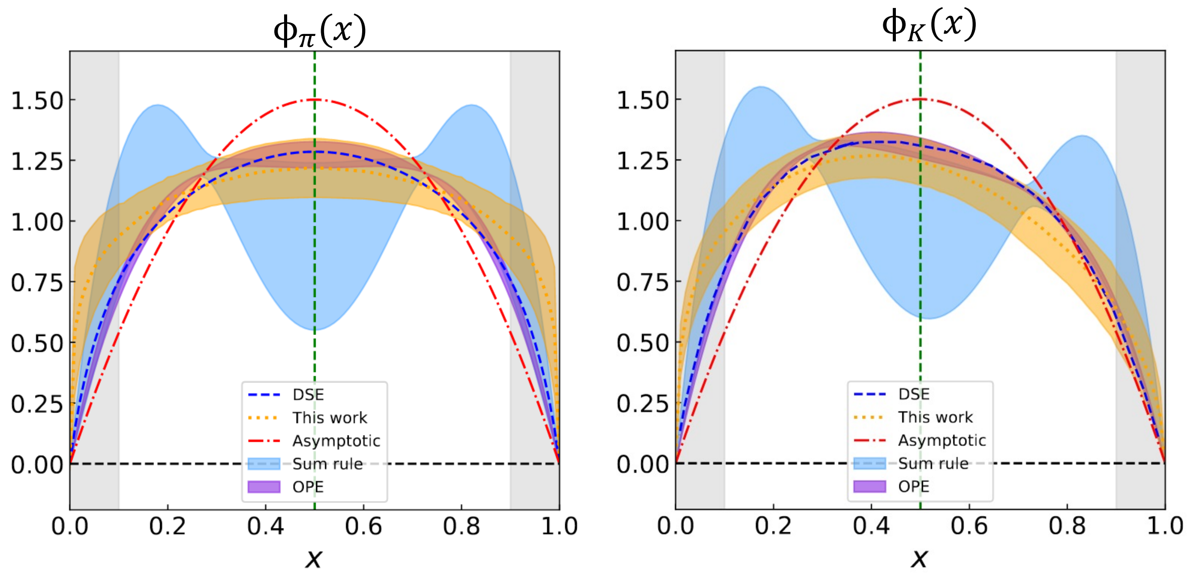
  Chang, Cloet, et.al, 2013; Gao, Chang, et.al, 2014; ……
- Global Fits

  Cheng, et.al, 2020; Hua, Li, Lu, Wang, Xing, 2021; ……
- Models

  Arriola, Broniowski, 2002; Zhong, Zhu, et.al, 2021; ……
- **Lattice QCD with OPE**

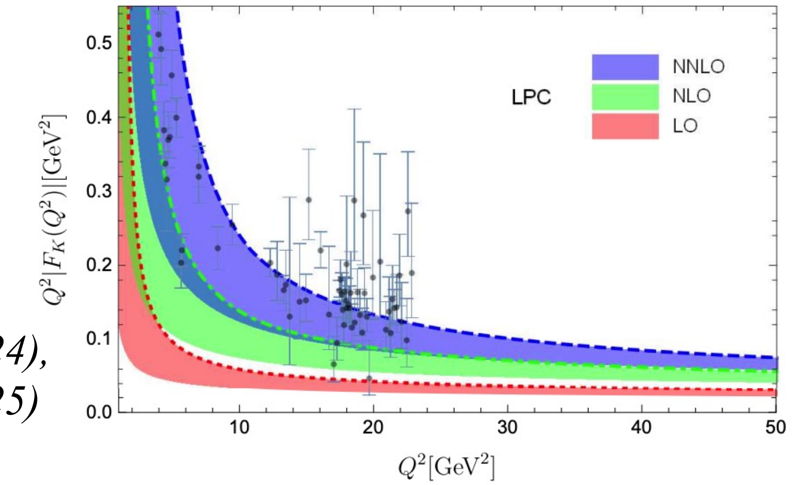
  Braun, et al., 2016; RQCD collaboration, 2019, 2020; ……
- **Lattice QCD with LaMET**

  LP3, 2019; LPC, 2021, 2022; ……



Application:

L.B.Chen et.al
PRL 132 201901(2024),
PLB 870 139886(2025)



02

Global fits Work and Fail

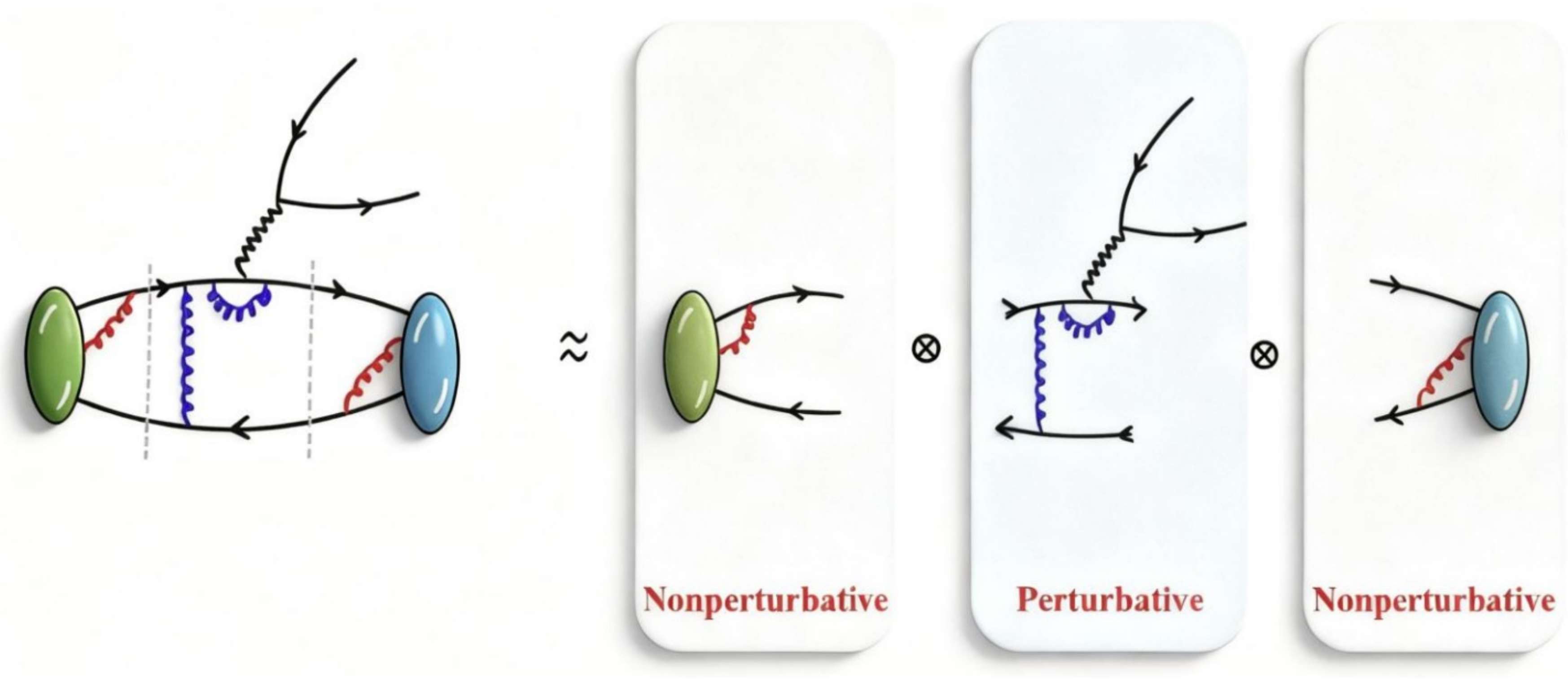
Meson system -> Baryon system

Global fits Work and Fail

How it works ? (in meson system)

JH et.al. PRD 104.016025

I. Nonperturbative LCDA contribute into amplitude by Factorization:



- QCDF
- SCET
- PQCD

$$\langle M_1 M_2 | O_i | \bar{B} \rangle = (\ldots) \otimes \phi_B \otimes \phi_{M_1} \otimes \phi_{M_2}$$

$$\left\{ \int dx\, H(x, \ldots) * 6x(1-x)\left[1 + C_1^{3/2}(2x-1)a_1 + \cdots\right] * \cdots \right\}$$

- a_n can be independent times $M(x)$

Global fits Work and Fail

How it works ? (in meson system)

JH et.al. PRD 104.016025

II. Input experimental data large than fit parameters

	Data		Fit	
Channel	Branching ratio (10^{-6})	A_{CP} (%)	Branching ratio (10^{-6})	A_{CP} (%)
$B^0 \to \bar{K}^0 K^0$	1.21 ± 0.16	-60 ± 70	1.23 ± 0.08	0 ± 0
$B^0 \to \bar{K}^0 \pi^0$	9.90 ± 0.50	0 ± 13	8.98 ± 0.19	-4.02 ± 0.48
$B^0 \to K^- \pi^+$	19.6 ± 0.50	-8.3 ± 0.6	20.3 ± 0.36	-8.34 ± 0.36
$B^0 \to \pi^- \pi^+$	5.12 ± 0.19	32 ± 4	5.24 ± 0.17	23.2 ± 2.1
$B^0 \to \rho^0 \bar{K}^0$	3.40 ± 1.10	4 ± 20	3.06 ± 0.37	2.853 ± 0.068
$B^0 \to \pi^0 \bar{K}^{*0}$	3.30 ± 0.60	-15 ± 13	1.73 ± 0.10	-6.02 ± 0.6
$B^0 \to \pi^- \rho^+ / \pi^+ \rho^-$	23.0 ± 2.30	$13 \pm 6 / -8 \pm 8$	23.33 ± 0.8	$-24.3 \pm 1/8.1 \pm 1.1$
$B^- \to K^0 K^-$	1.31 ± 0.17	4 ± 14	1.47 ± 0.09	22.5 ± 2.7
$B^- \to \pi^0 K^-$	12.9 ± 0.50	3.7 ± 2.1	12.99 ± 0.23	-6.44 ± 0.6
$B^- \to \bar{K}^0 \pi^-$	23.7 ± 0.80	-1.7 ± 1.6	23.15 ± 0.42	-2.84 ± 0.24
$B^- \to \rho^- \pi^0$	10.9 ± 1.40	2 ± 11	8.73 ± 0.25	24.2 ± 2.3
$B^- \to \pi^0 K^{*-}$	6.80 ± 0.90	-39 ± 21	3.51 ± 0.19	-33.5 ± 1.7
$B^- \to K^- K^{*0}$	0.59 ± 0.08	12 ± 10	0.476 ± 0.022	22.5 ± 1.3
$B_s \to K^- K^+$	26.6 ± 2.20	-14 ± 11	24.8 ± 1.50	-8.1 ± 2.3
$B_s \to \pi^- \pi^+$	0.7 ± 0.1	$\cdots$	0.798 ± 0.092	-1.62 ± 0.39
$B_s \to K^0 \bar{K}^0$	20.0 ± 6.00	0 ± 0	26.2 ± 1.60	0 ± 0
$B_s \to \pi^- K^+$	5.80 ± 0.70	22.1 ± 1.5	5.69 ± 0.64	22.1 ± 1.2
$B_s \to K^+ K^{*-} / K^- K^{*+}$	19.0 ± 5.0	$\cdots$	15.28 ± 0.90	$-33.8 \pm 1.3/53.5 \pm 2.4$
$B_s \to K^0 \bar{K}^{*0} / \bar{K}^0 K^{*0}$	20.0 ± 6.00	$\cdots$	15.06 ± 0.96	0 ± 0

$a_2^{\pi}, a_4^{\pi}, a_{P2}^{\pi}, a_{T2}^{\pi}, a_1^{K}, a_2^{K}, a_4^{K}$

$a_2^{\rho||}, a_2^{K^*||}, \qquad \gamma$

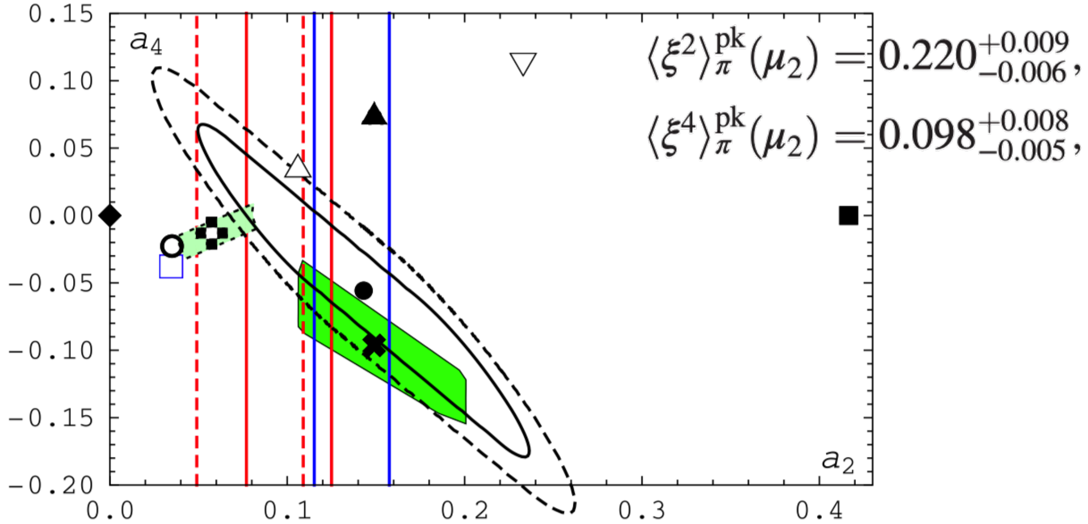
Fit parameters = 10
Experimental Data > 20

I & II → Global fits Works

Global fits Work and Fail

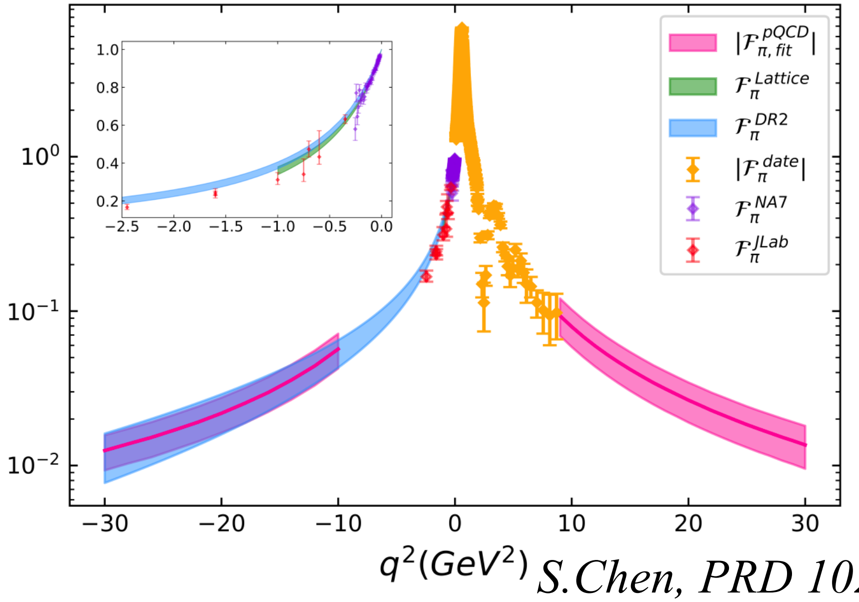
Sometimes, work well

- **Fit from Pion-photon transition form factor**



Stefanis, PRD 102, 034022(2020)

- **Global analysis for pion electromagnetic form factor**



S.Chen, PRD 102, 034022(2020)

- Global analysis for two body decays: C.Shan, N.G.Stefanis… → Meson LCDA
- Global analysis for multibody decays: Y.Li, R.Zhou… → Two meson distribution

Global fits Work and Fail

Why it fails ? (in baryon system)

- Nonperturbative LCDA contribute into amplitude by Factorization ✓
- Input experimental data large than fit parameters ?

I. Global analysis for lots of channels → several LCDAs ?

II. Few precise channels → few specific LCDA ?

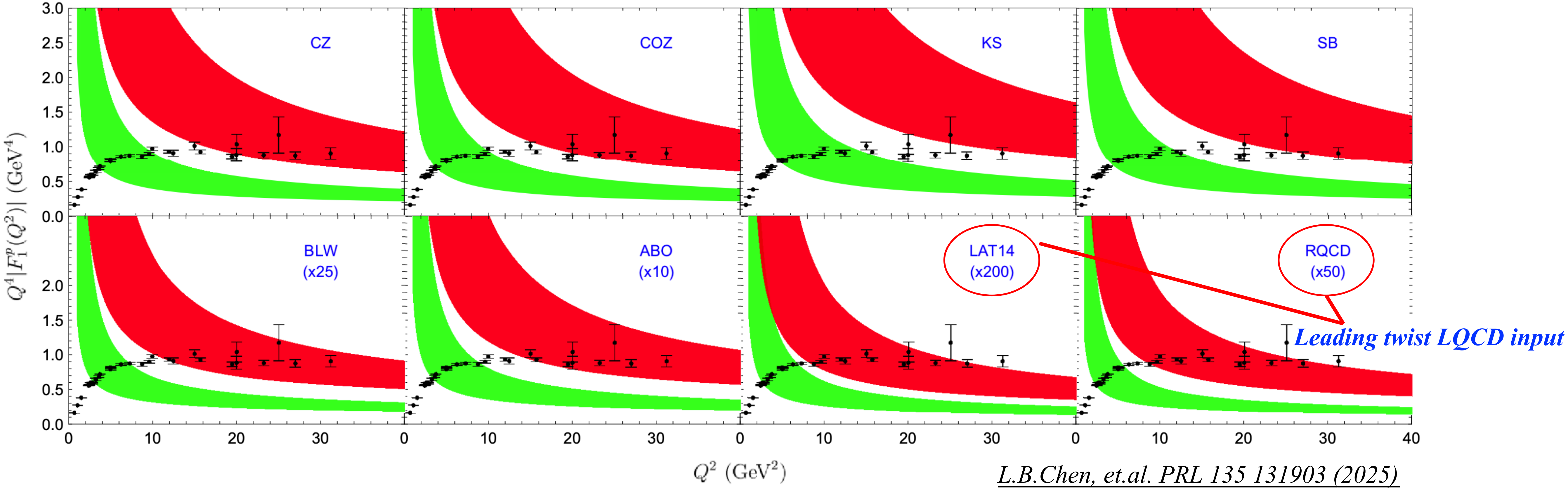
Global fits Work and Fail

Why it fails ? (in baryon system)

I. Global analysis for lots of channels → several LCDAs ?

II. Few precise channels → few specific LCDA ✕

NLO Nucleon Dirac form factors:



L.B.Chen, et.al. PRL 135 131903 (2025)

Global fits Work and Fail

Why it fails ? (in baryon system)

I. Global analysis for lots of channels → several LCDAs ✕

II. Few precise channels → few specific LCDA ✕

LO full calculation with PQCD framework

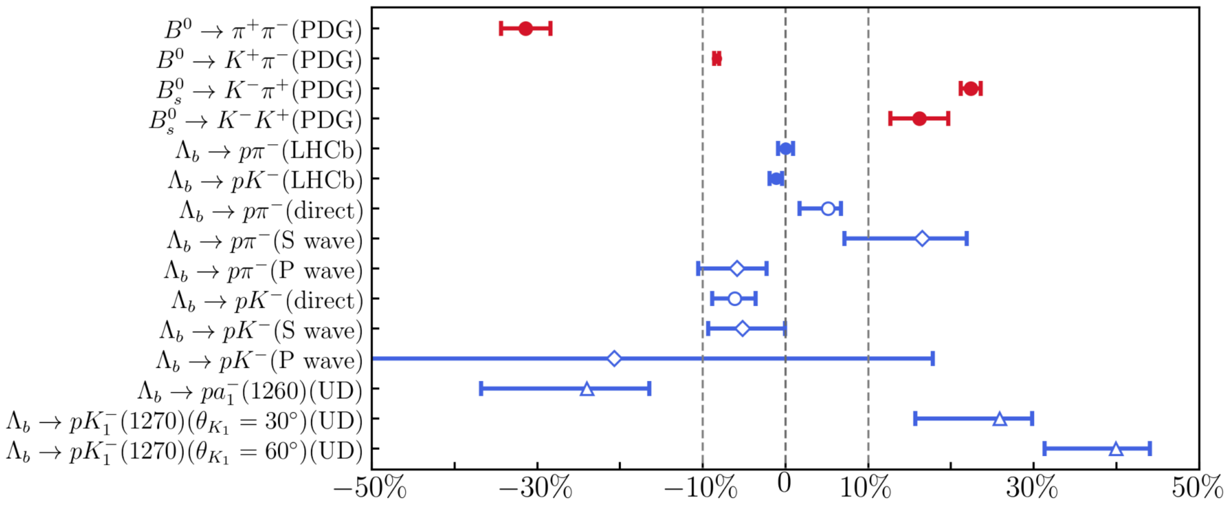


FIG. 2: CPVs measured in B meson and $\Lambda_b \to p\pi^-, pK^-$ decays, and our predictions.

J.J.Han, et.al. PRL 134 221801(2025)

$$
\begin{aligned}
&4\langle 0|\varepsilon^{ijk}u_\alpha^i(a_1z)u_\beta^j(a_2z)d_\gamma^k(a_3z)|P\rangle\\
&= S_1MC_{\alpha\beta}(\gamma_5N^+)_\gamma + S_2MC_{\alpha\beta}(\gamma_5N^-)_\gamma + P_1M(\gamma_5C)_{\alpha\beta}N_\gamma^+ + P_2M(\gamma_5C)_{\alpha\beta}N_\gamma^-\\
&+ V_1(\not{p}C)_{\alpha\beta}(\gamma_5N^+)_\gamma + V_2(\not{p}C)_{\alpha\beta}(\gamma_5N^-)_\gamma + \frac{V_3}{2}M(\gamma_\perp C)_{\alpha\beta}(\gamma^\perp\gamma_5N^+)_\gamma\\
&+ \frac{V_4}{2}M(\gamma_\perp C)_{\alpha\beta}(\gamma^\perp\gamma_5N^-)_\gamma + V_5\frac{M^2}{2pz}(\not{z}C)_{\alpha\beta}(\gamma_5N^+)_\gamma\\
&+ \frac{M^2}{2pz}V_6(\not{z}C)_{\alpha\beta}(\gamma_5N^-)_\gamma + A_1(\not{p}\gamma_5C)_{\alpha\beta}N_\gamma^+ + A_2(\not{p}\gamma_5C)_{\alpha\beta}N_\gamma^-\\
&+ \frac{A_3}{2}M(\gamma_\perp\gamma_5C)_{\alpha\beta}(\gamma^\perp N^+)_\gamma + \frac{A_4}{2}M(\gamma_\perp\gamma_5C)_{\alpha\beta}(\gamma^\perp N^-)_\gamma\\
&+ A_5\frac{M^2}{2pz}(\not{z}\gamma_5C)_{\alpha\beta}N_\gamma^+ + \frac{M^2}{2pz}A_6(\not{z}\gamma_5C)_{\alpha\beta}N_\gamma^-\\
&+ T_1(i\sigma_{\perp p}C)_{\alpha\beta}(\gamma^\perp\gamma_5N^+)_\gamma + T_2(i\sigma_{\perp p}C)_{\alpha\beta}(\gamma^\perp\gamma_5N^-)_\gamma\\
&+ T_3\frac{M}{pz}(i\sigma_{pz}C)_{\alpha\beta}(\gamma_5N^+)_\gamma + T_4\frac{M}{pz}(i\sigma_{zp}C)_{\alpha\beta}(\gamma_5N^-)_\gamma\\
&+ T_5\frac{M^2}{2pz}(i\sigma_{\perp z}C)_{\alpha\beta}(\gamma^\perp\gamma_5N^+)_\gamma + \frac{M^2}{2pz}T_6(i\sigma_{\perp z}C)_{\alpha\beta}(\gamma^\perp\gamma_5N^-)_\gamma\\
&+ M\frac{T_7}{2}(\sigma_{\perp\perp'}C)_{\alpha\beta}(\sigma^{\perp\perp'}\gamma_5N^+)_\gamma + M\frac{T_8}{2}(\sigma_{\perp\perp'}C)_{\alpha\beta}(\sigma^{\perp\perp'}\gamma_5N^-)_\gamma,
\end{aligned}
$$

Baryon LCDA:

- Twist3: 3
- Twist4: 9
- Twist5: 12

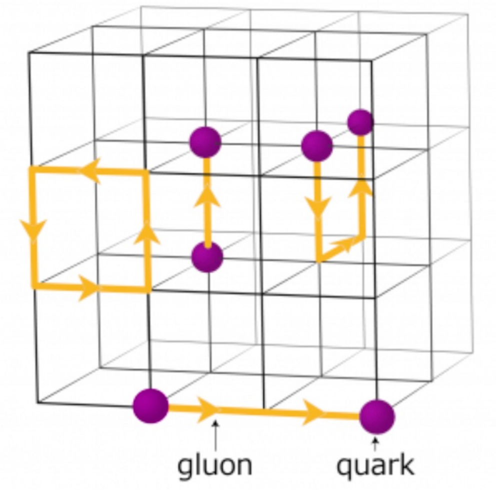
03

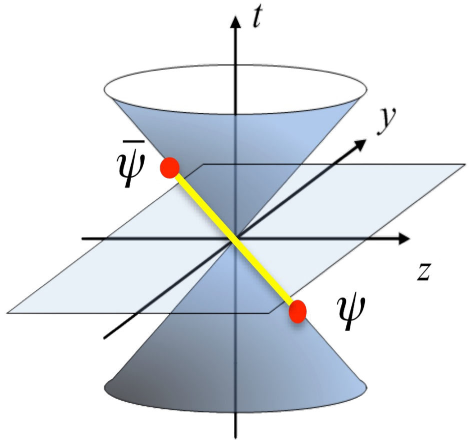
LQCD for LCDAs

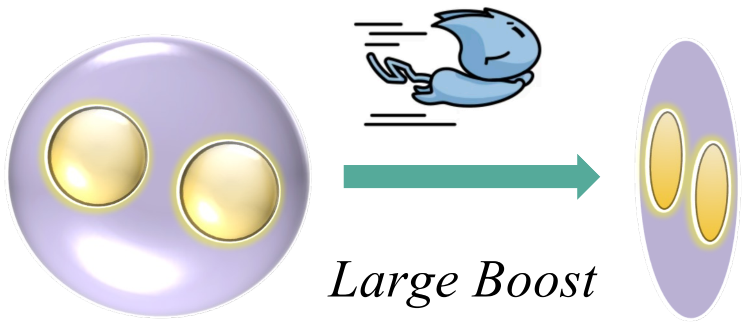
OPE VS LaMET

LQCD for LCDAs OPE VS LaMET

LQCD is formulated as a Feynman path integral on a 4D Euclidean grid, with a Wick rotation from real time to imaginary time.







π LCDA: $\langle 0|\bar{q}_2(\xi^+)\gamma^+\gamma_5 U\left(0,\xi^-\right)q_2\left(\xi^-\right)|\pi\rangle$

- **OPE** ~ in local limit: → moments of DA

$$\langle 0|\bar{q}_2\left(\xi^+\right)\gamma^+\gamma_5 U\left(\xi^+,\xi^-\right)q_2\left(\xi^-\right)|\pi\rangle \overset{\xi\to 0}{\sim} C_m(z)^m \bar{q}_2(0)\left(\gamma^t+\gamma^z\right)\gamma_5(in_z\cdot \overleftrightarrow{D})^m q_1(0)$$

- **LaMET** ~ in large P^z limit: → shape of DA

$$\langle 0|\bar{q}_2\left(\xi^+\right)\gamma^+\gamma_5 U\left(\xi^+,\xi^-\right)q_2\left(\xi^-\right)|\pi\rangle \overset{P^z\to\infty}{\sim} \langle 0|\bar{q}_2\left(z_+\right)(\gamma^t+\gamma^z)\gamma_5 U\left(z_+,z_-\right)q_2\left(z_-\right)|\pi(P^z)\rangle$$

04

Progress on LCDAs from LQCD

Light meson → Light baryon

Progress on LCDAs from LQCD

Moments tension:

How to solve ?

Light Meson LCDA on LQCD

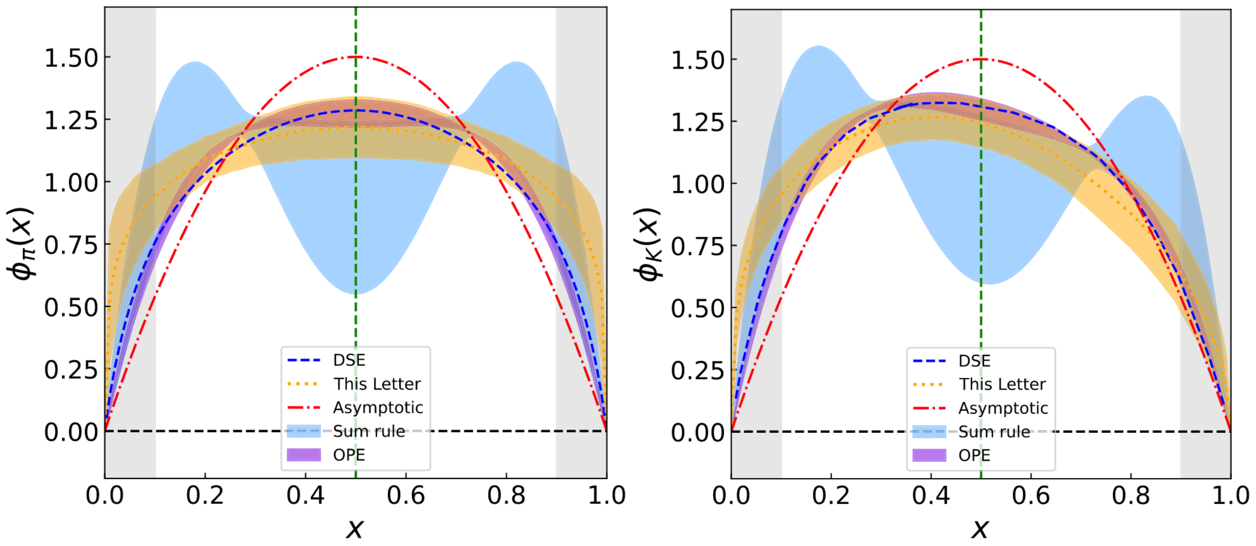
- **Lattice QCD with OPE**

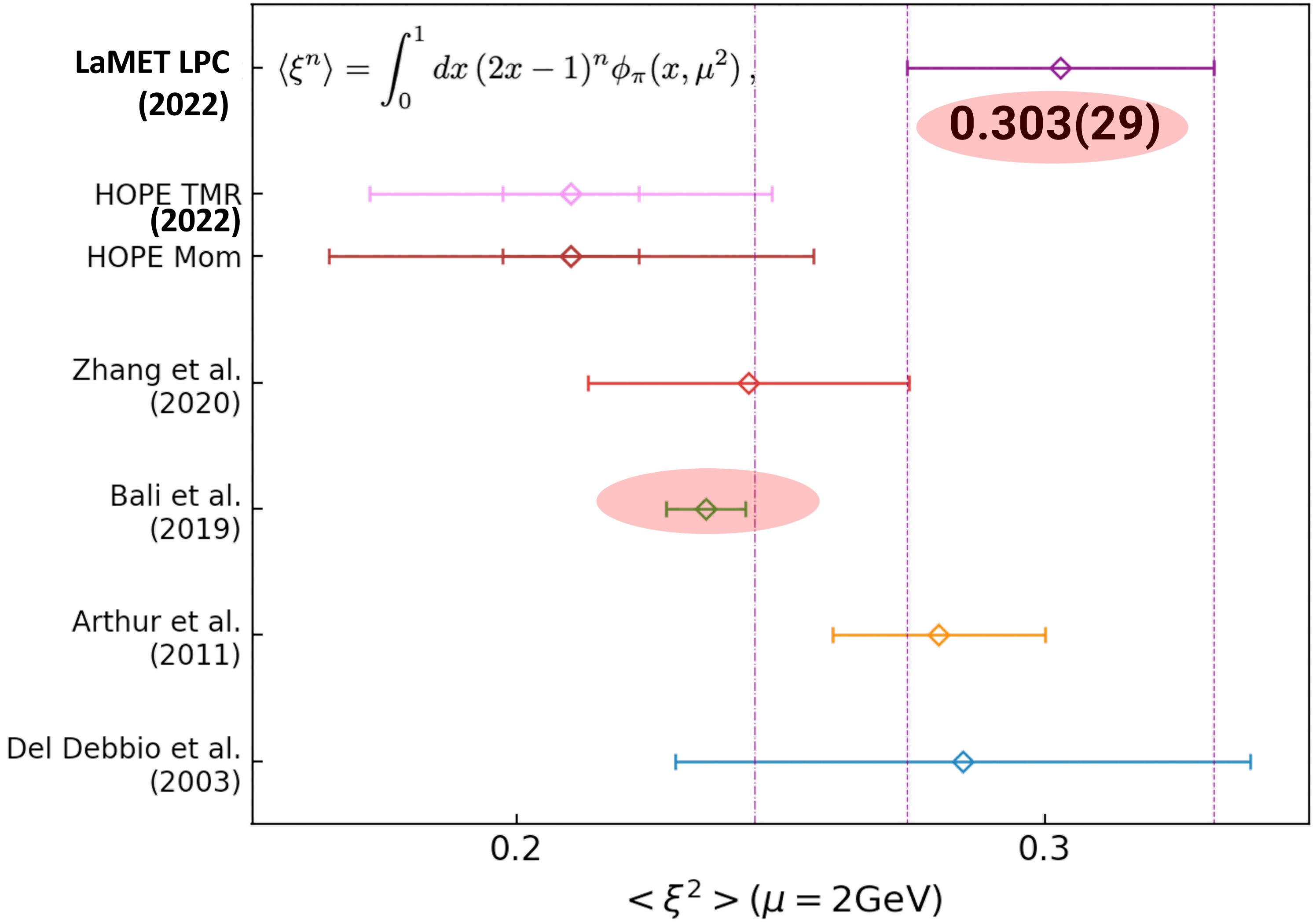
  Braun, et al., 2016; RQCD collaboration, 2019, 2020; ……

- **Lattice QCD with LaMET**

  LP3, 2019; LPC, 2021, 2022; ……

(LPC) *J.Hua et.al., PRL 129, 132001(2022)*





Advances of Framework

Advances: (2021-)

- Hybrid & Self renormalization

  LPC NPB 964 115311, 969 115443(2021)…
- Renormalon resummation (LRR)

  NPB 993 116282(2023)…
- RGR & Threshold resummation

  NPB 993, JHEP 03 045(2025)…
- Two loop matching

  arXiv:2504.09367 …
- Power correction estimation

  PRD 110 094038(2025), 112 016013(2025)…

Linear divergence

Multi-a, extract linear-div

Match with perturbative $\overline{MS}$

Renormalon in perturbative results

Advances of Precision Calculation

Advances: (2021-)

- Kinematically-enhance
  PRD 112 L051502(2025)
- Precision check for meson LCDA
  … JHW et.al.
- Inverse Fourier transform
  arXiv:2505.14619, arXiv:2506.16689…
- CLQCD ensembles & Pyquda
  CLQCD PRD 109(2024) , arXiv: 2411.08461…
- … Gradient flow
  JHEP 06 210 (2024), arXiv:2507.18233…

➢ **Precision**

Signal Improve

Kinematically-enhance:

$$C_2^{\pi}(z,\vec{P},t)=\int d^3y e^{-i\vec{P}\cdot\vec{y}}\langle 0|\underbrace{\bar{\psi}_1(\vec{y},t)\Gamma_1 U(\vec{y},\vec{y}-z\hat{z})\psi_2(\vec{y}-z\hat{z},t)}_{Sink}\underbrace{\bar{\psi}_2(0,0)\Gamma_2\psi_1(0,0)}_{Src}|0\rangle$$

RZ et.al. PRD 112 L051502(2025)

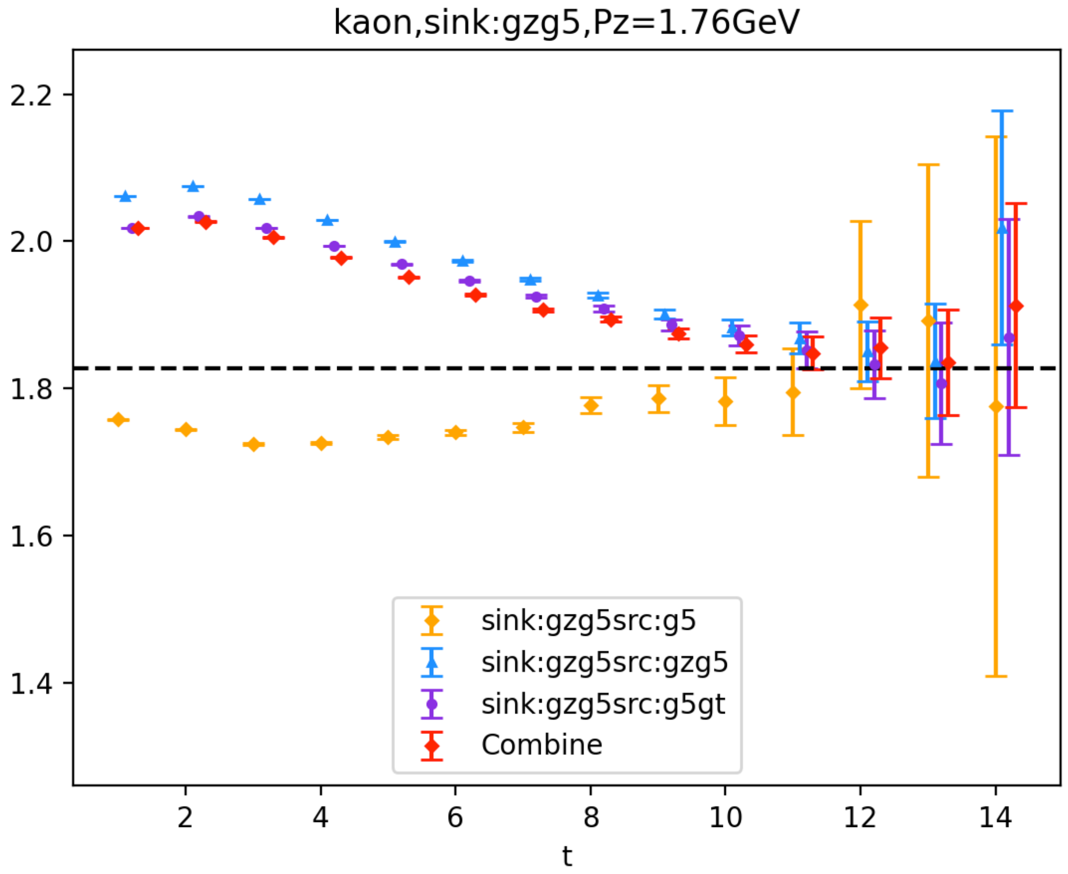
Decomposition in light-cone limit :

PLB 545,345 (2002), EPJC 33, 75(2004)

$$u_+^{\dagger}\gamma_5 d_+ = \sqrt{2}\bar{u}\gamma_+\gamma_5 d$$

$$(\bar{u}\gamma_5 d) = \left(u_+^{\dagger}\gamma_t\gamma_5 d_- + u_-^{\dagger}\gamma_t\gamma_5 d_+\right)/2$$

Can be further extended in baryon case.



1 step towards precision calculation

- **OPE: Local moments**

$$\xi \equiv 2x-1, \qquad \langle \xi^n \rangle_K(\mu) \equiv \int_0^1 dx\,(2x-1)^n\,\phi_K(x,\mu)$$

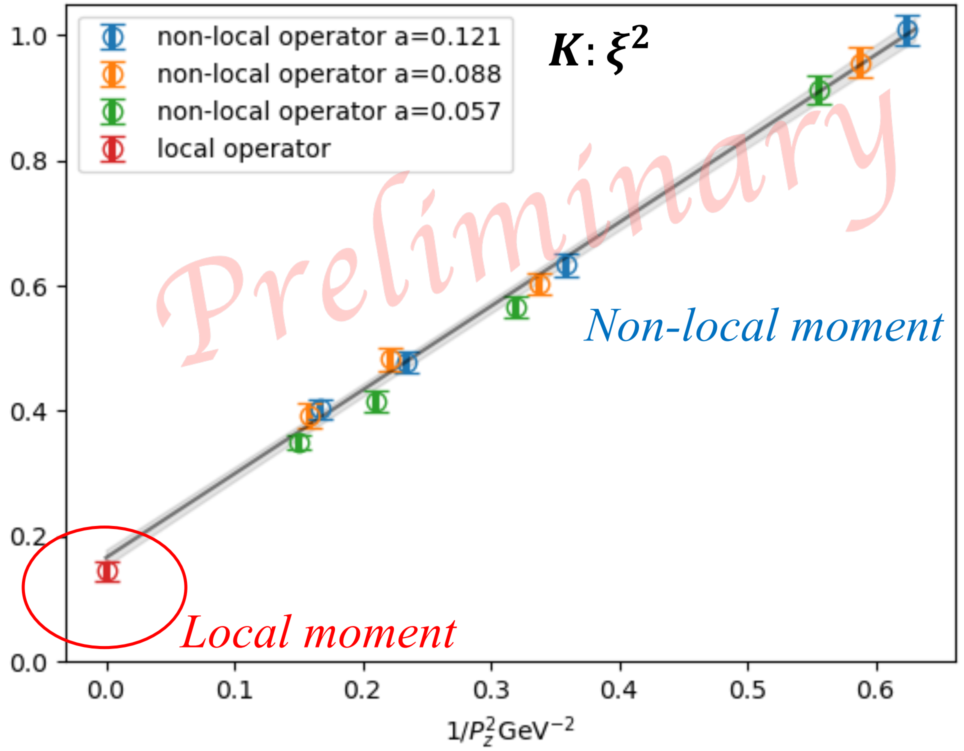
$$\langle 0|\bar{q}(0)\gamma^+\gamma_5(i\overleftrightarrow{D}^+)^n s(0)|K(P)\rangle = i f_K P^{+(n+1)}\langle \xi^n \rangle_K(\mu)$$

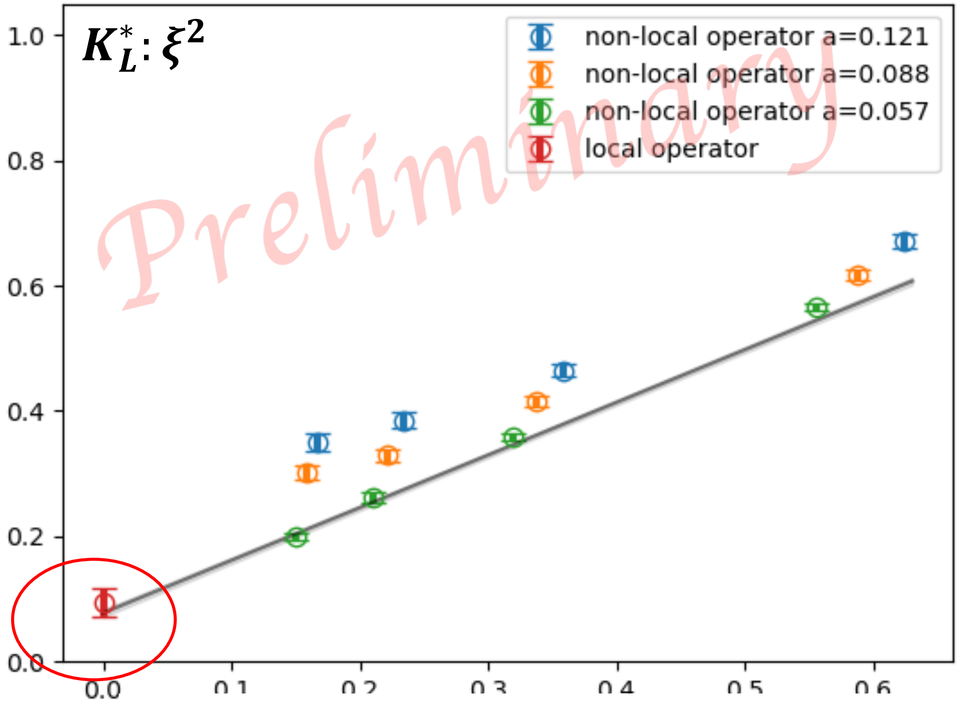
- **Small z expansion: Non-local moments**

$$\tilde{\Phi}(z,P_z) \sim \int_0^1 dx\, e^{i(2x-1)P_z z}\phi_K(x)$$

$$\tilde{\Phi}(z,P_z) = \int_0^1 dx\left[1 + i(2x-1)P_z z - \frac{1}{2}(2x-1)^2 P_z^2 z^2 + \cdots\right]\phi_K(x)$$

Consistent moments from local and non-local

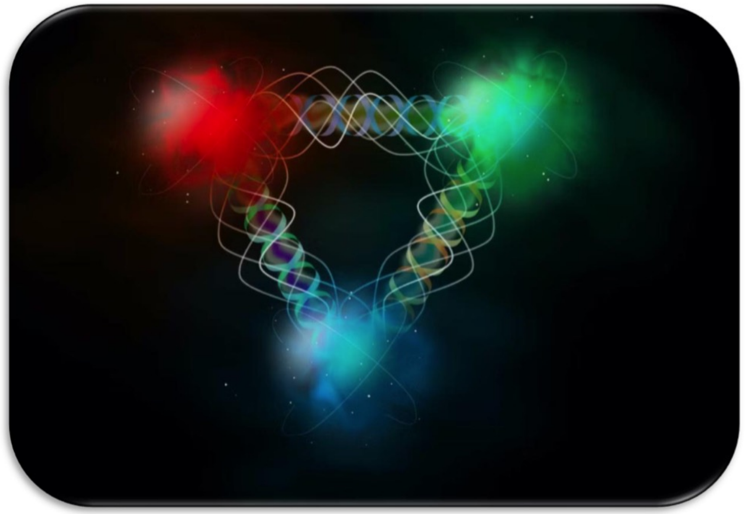


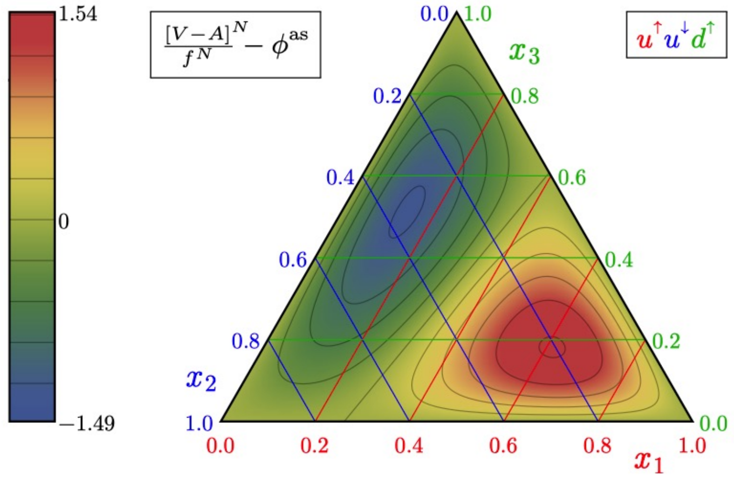


The first precision check in LaMET

Light Baryon LCDAs: (1983 - now)

- Asymptotic form
  Chernyak, Zhitnitsky, 1983 ; ……
- QCD Sum rules
  King, Sachrajda,1987; Stefanis, Bergmann, 1993; Braun, et.al, 2000,2006; Huang, 2009; ……
- Models parametrization
  Bell, Feldmann, Wang, Matthew 2013; ……
- **Lattice QCD with OPE**
  QCDSF, 2008, 2009; RQCD, 2016, 2019(2025) ……
- **Lattice QCD with LaMET**
  LPC, 2024,2025; ……
  PRD 111, 034510, arXiv:2508.08971





Global fit have failed in Baryon,

How about OPE moments on LQCD ?

Operator Product Expansion (OPE) VS Large Momentum Effective theory(LaMET)

Limited for only few moments:

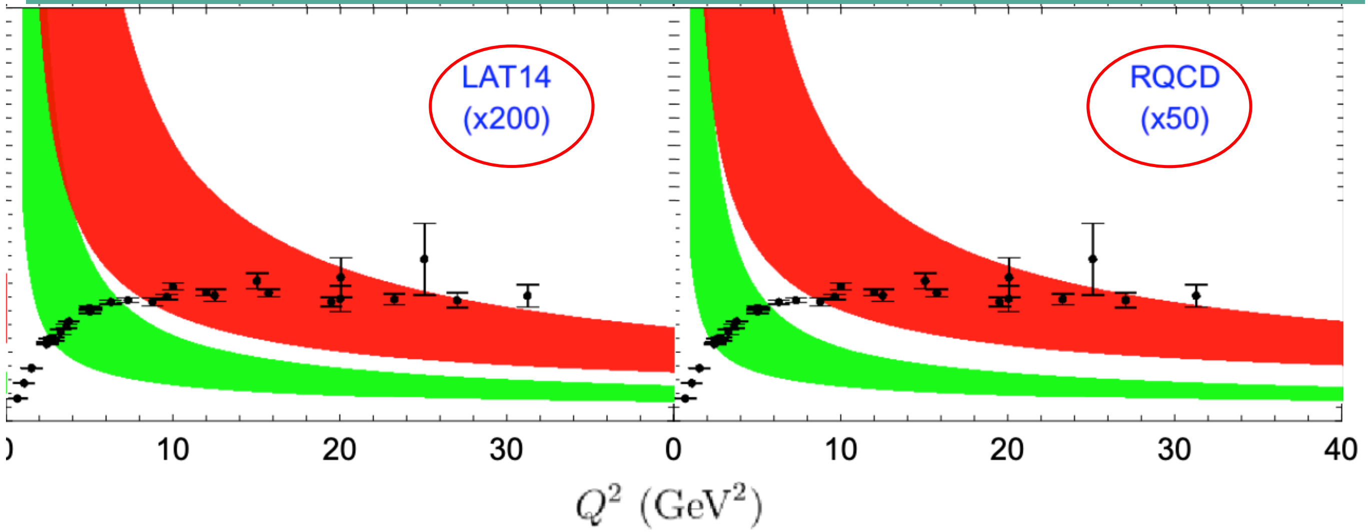
- First two moments for light mesons
- First moments for light baryons
- Inverse problem from moments to LCDA

High twist/moments are more important in Baryon

J.J.Han, et.al. EPJC 82 8 (2022), 686 PRL 134 221801(2025)

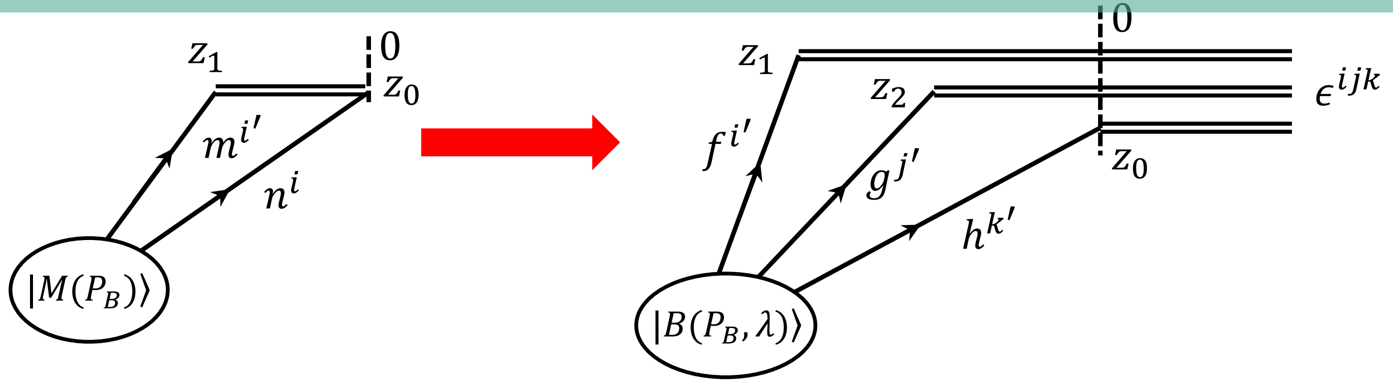
L.B.Chen, et.al. PRL 135 131903 (2025)

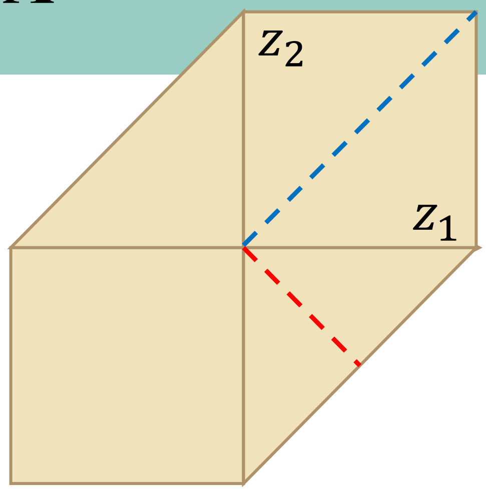
Nucleon Dirac form factors:

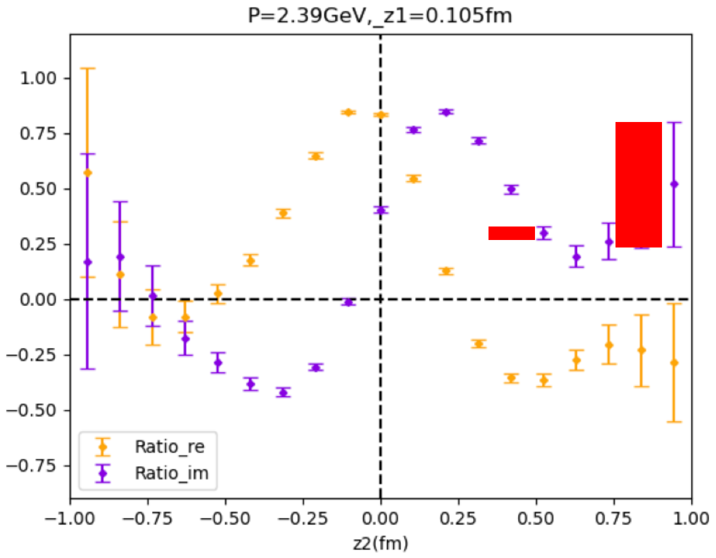


Soft contribution dominates in baryon decay , leading twist far from enough

Challenge from light meson LCDA to light baryon LCDA







- **From effective Wilson length → Signal to Noise Ratio**
  - Signal to Noise Ratio * 8 -> Statistics * 64
  - Totally 64*10*2
- **Dynamic enhance: SNR 200% improve (eff 400%) !**
- **Coding in Pyquda: Computing efficiency 800% !**

arXiv: 2411.08461

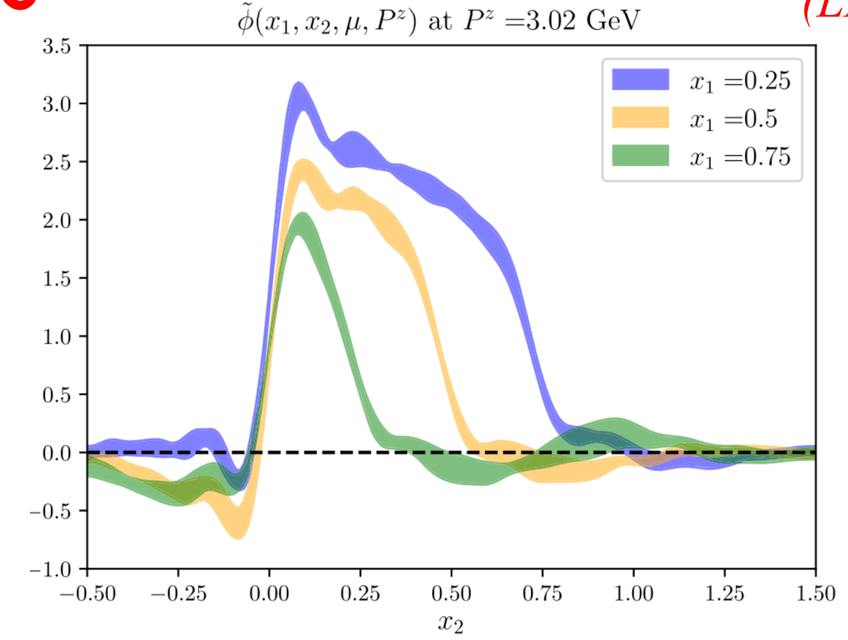
Progress of LCDA on LQCD

Light Meson

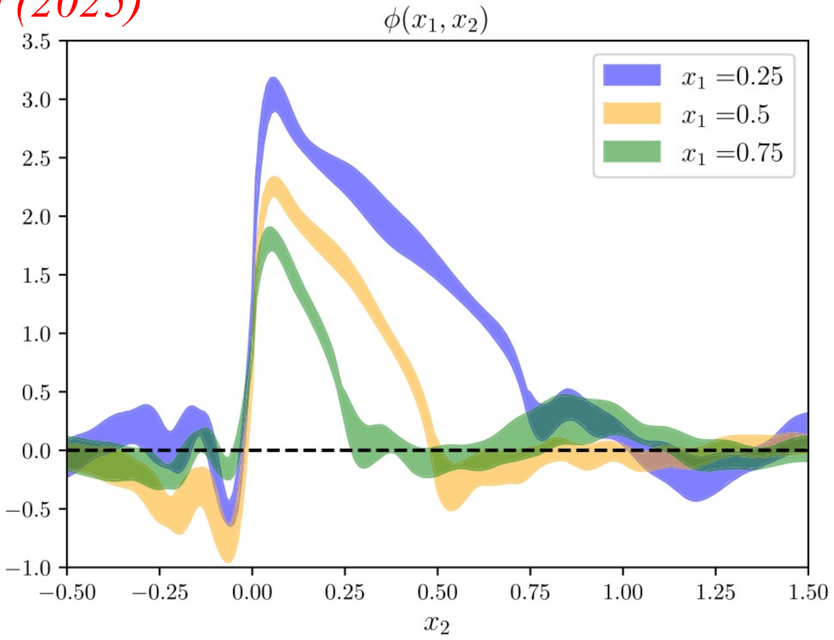
Light Baryon

What we have done ?

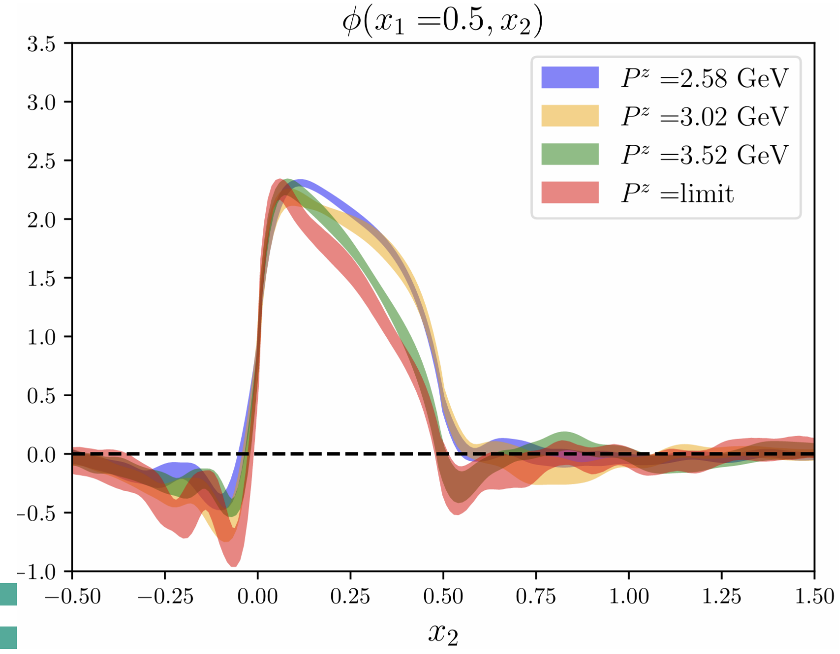
(LPC), PRD 111, 034510 (2025)



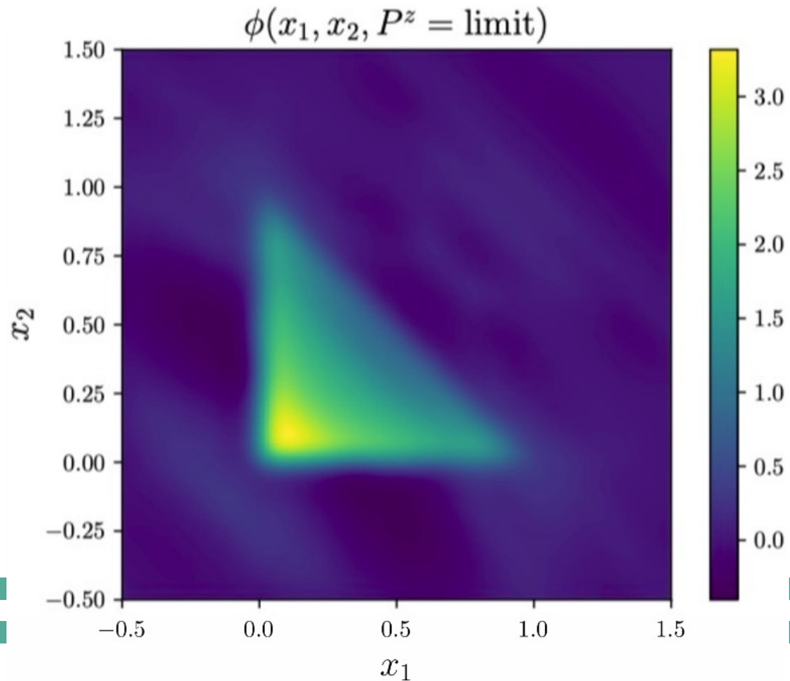
Matching



Large Pz limit

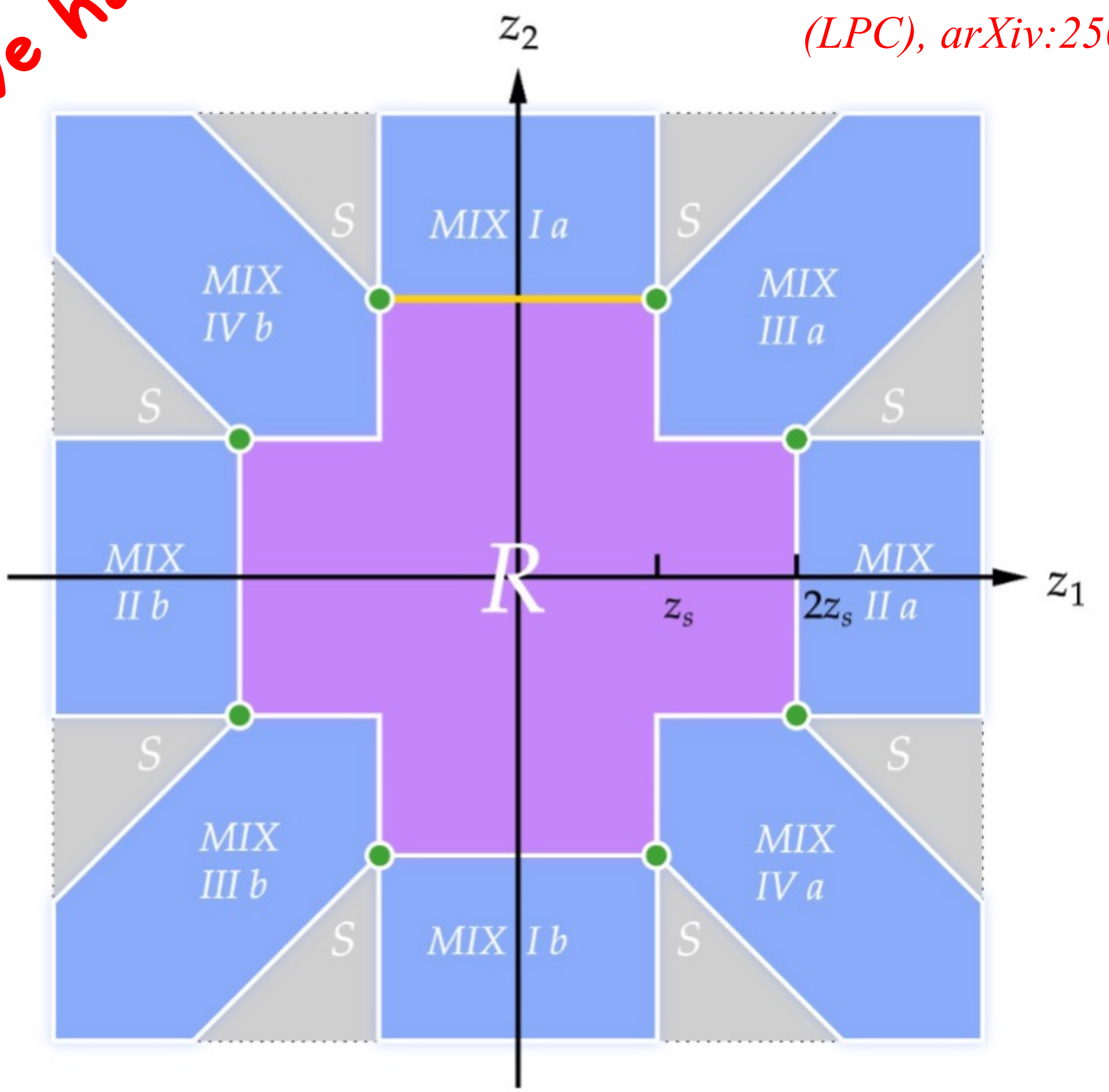


LCDA



What we have done ?

(LPC), arXiv:2508.08971 (2025)



- **Hybrid & Self renormalization**

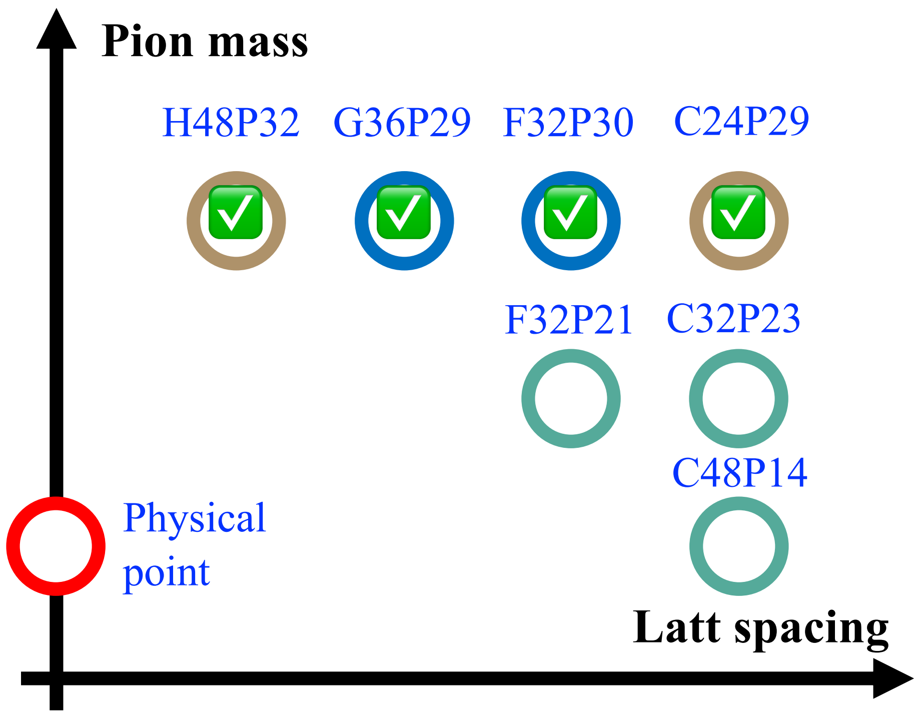
The only renormalization scheme that is self-consistent with the lattice data.

— for LCDA on LQCD

In process …

- ☐ 4 lattice spacings for $\boldsymbol{a} \to \mathbf{0}$ limit & renormalization
- ☐ 4 momentums for $\boldsymbol{P_z} \to \infty$ limit
- ☐ More pion masses to **physical mass** limit

CLQCD Ensembles



Ensemble	Lattice spacing	π mass	measurements
C24P29	0.1052 fm	292 MeV	864 *4 src *9 it0
F32P30	0.0775 fm	300 MeV	777 *4 src *8 it0
H48P32	0.0520 fm	317 MeV	550 *6 src *9 it0
G36P29	0.0689 fm	297 MeV	656 *6 src *8 it0
Other ensembles with different pion masses			

Summary & Outlook

Global fits

- Work under factorization framework
- Input data >> Fit parameters

⬇

- Global analysis for channels
- High precision fit in specific channel

➢ Industry benchmark: CTEQ, NNPDF

➢ Progress in excusive processes

Lattice QCD

- OPE for moments
- LaMET for shape of LCDA

⬇

- Limited in lower moments
- How to give precision prediction

➢ Progress on light meson *(LPC) PRL 2021,2022…*

➢ Progress on light baryon *(LPC)PRD 2025…*

Thanks for Your Attention !