重介子四体半轻衰变理论进展

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Overview

I: 重介子半轻衰变的核心问题

II: 重介子四体半轻衰变的初步探索

i: $B \to \pi\pi l\nu$ 和 $|V_{ub}|$ 测量

ii: $D_s \to [\pi\pi]_{\mathrm{S}}\, e^+\nu_e$ 和 $f_0, [\pi\pi]_{\mathrm{S}}$ 结构初探

III: 总结和展望

重介子半轻衰变的核心问题

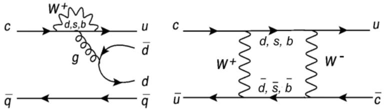
重介子半轻衰变

- fundamental parameters, such as the CKM matrix elements
  $|V_{ub}| = (3.82 \pm 0.20) \times 10^{-3}$, $|V_{cb}| = (4.11 \pm 0.12) \times 10^{-2}$,
  $|V_{cs}| = 0.975 \pm 0.006$ [PDG 2024]

- new physical mechanism via the FCNC

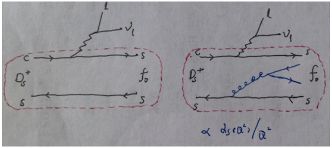
- ◦ anomalous measured in $B \to K^* \mu^+ \mu^-$, 3.6σ derivation of $d\mathcal{B}/dq^2$ in $q^2 \in [1, 6]$ GeV^2
- ◦ a plausible effect in up-type FCNC process $c \to ull$ [Bharucha, Boito and Méaux, JHEP 04 (2021) 158] SM $\mathcal{B}(D \to \pi l^+ l^-) \sim \mathcal{O}(10^{-9})$, current best-world limit $\mathcal{O}(10^{-8})$
- ◦ LCSRs prediction $\mathcal{B}(D \to \pi\mu\mu) = 1.33^{+0.17}_{-0.24} \times 10^{-8}$, at the same order of LHCb limit 6.7×10^{-8} [A. Bansal, A. Khodjamirian and T. Mannel, JHEP 08 (2025) 026]

- ◦ first measurement of $D^0 \to \pi^+\pi^- e^+ e^-$ [LHCb, PRD 111(2025) L091101]
  $(4.53 \pm 1.38) \times 10^{-7}$ in ρ/ω and
  $(3.84 \pm 0.96) \times 10^{-7}$ in ϕ
- ◦ LD contribution from ϕ, $D_s \to \pi, \rho(\phi \to)e^+ e^- \sim 10^{-5}$ [BESIII, PRL 133(2024)121801]



重介子半轻衰变 理解标量介子结构的干净平台

$$|f_0(980)\rangle, \quad |[\pi\pi]_{\mathrm{S}}\rangle = \psi_{q\bar{q}}|q\bar{q}\rangle + \psi_{q\bar{q}g}|q\bar{q}g\rangle + \psi_{q\bar{q}q\bar{q}}|q\bar{q}q\bar{q}\rangle + \cdots$$



- ⋆ in semileptonic $B_{(s)}$ decays
- ⋆ tetraquark contribution is suppressed doubly by strong coupling and power
- ⋆ FSI is weak too

- **how about the energetic $q\bar{q}$ picture $f_0(980)$ in D_s decays ?**

† the produced $q\bar{q}$ is nonrelativistic, likely proceeds through the creation of additional g or dynamical $q\bar{q}$

† hadron spectroscopy analysis provides evidence of the $q\bar{q}q\bar{q}$ states (compact tetraquarks or molecules)

† the cascade decay analyses of $D_s \to (f_0 \to \pi\pi)e\nu$ under $q\bar{q}$ ansatz consists with data, with $D_s \to f_0$ FFs and Flatté model of f_0 resonant

⋆ physical observables are usually written in a QCD convolution

$$\frac{d\sigma}{d\Omega} = \sum_t \int_0^1 dx_i \mathcal{H}^t(x_i, Q)\psi^t(x_i, \mu)$$

⋆ ψ^t is universal, however H^t is process dependent, hence different observables might highlike the contributions from different components

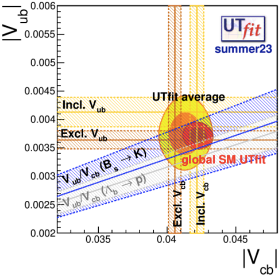
$|V_{ub}|$ 疑难

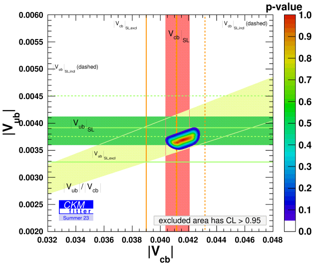
- $|V_{ub}|$ tension $\quad |V_{ub}| = (3.82 \pm 0.20) \times 10^{-3}$ [PDG 2024]

‡ $\sim 2.5\sigma$ tension between $(4.13 \pm 0.25) \times 10^{-3}$ and $(3.70 \pm 0.16) \times 10^{-3}$ measured via the $B \to X_u l^- \bar{\nu}$ and $B \to \pi l^- \bar{\nu}$ processes, respectively.

- $|V_{cb}|$ tension $\quad |V_{cb}| = (41.1 \pm 1.2) \times 10^{-3}$ [PDG 2024]

‡ $\sim 2.5\sigma$ tension between $(42.2 \pm 0.5) \times 10^{-3}$ and $(39.8 \pm 0.6) \times 10^{-3}$ measured via the $B \to X_c l^- \bar{\nu}$ and $B \to D^{(*)} l^- \bar{\nu}$ processes, respectively.

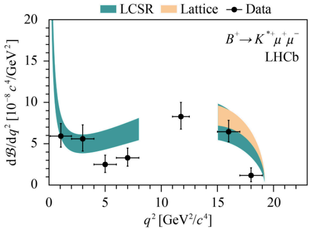


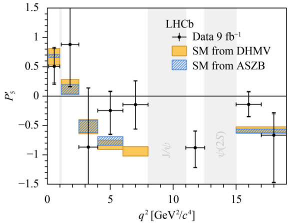


味物理反常现象

- LFU in $b \to c l^- \bar{\nu}$ processes $R_{D^{(*)}} = \mathcal{B}(B \to D^{(*)} \tau^- \bar{\nu}) / \mathcal{B}(B \to D^{(*)} \mu^- \bar{\nu})$
  - $R_D = 0.407 \pm 0.046, R_{D^*} = 0.306 \pm 0.015$ Average with [Belle PRL124, 161803 (2020)] new result from Belle II, [PRD 110, 072020(2024)], see Qi-dong's talk
  - $R_D = 0.441 \pm 0.089, R_{D^*} = 0.281 \pm 0.030$ [LHCb PRL131,111802 (2023)] new result using muonic τ decays, [PRL134, 061801(2025)], see Ji-ke's talk
  - SM: $R_D = 0.298 \pm 0.004, R_{D^*} = 0.254 \pm 0.005, \quad 2.1\sigma, 3.0\sigma$ [HFLAV]
  - make the CKM measurements more complicated

- Anomalies in FCNC processes $B \to K^* \mu^+ \mu^-$
  - 3.6σ derivation from SM of $d\mathcal{B}(B \to K^* \mu^+ \mu^-)/dq^2$ in $q^2 \in [1, 6]$ GeV^2
  - 1.9σ derivation from SM of $P_5' = S_5 / \sqrt{F_L(1 - F_L)}$ in $q^2 \in [4, 8]$ GeV^2



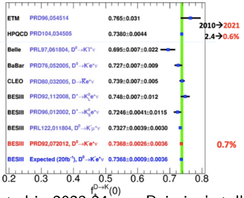


[Heavy Flavour Physics and CP Violation at LHCb: a Ten-Year Review, Front. Phys.18,44601(2023)]

解决方案　在传统过程继续奋斗

- 更精确的测量和格点计算

$|V_{cs}|$ issue　$|V_{cs}| = 0.975 \pm 0.006$ [PDG 2022, 24]



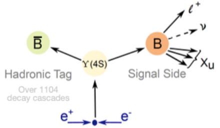
updated in 2023,24, see Bai-qian's talk

* 0.972 ± 0.007 and 0.984 ± 0.012 measured via the $D \to K l \nu$ and $D_s \to \mu^+ \nu_\mu$ processes　$\sim 3\sigma \to \sim 1.5\sigma$

理论与实验联合研讨的成功典范

- 更全面更系统的物理分析方法

$|V_{ub}|$ result from Belle collaboration with Simultaneous Determination in excl. and incl. processes [Belle PRL131, 211801 (2023)]



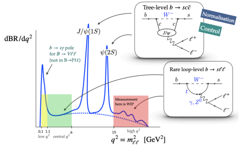
$$\mathcal{B}(B \to \pi^0 \ell \nu) + \mathcal{B}(B \to \pi^+ \ell \nu) + \mathcal{B}(B \to X_u^{\text{other}} \ell \nu) = \mathcal{B}(B \to X_u \ell \nu)$$

* $(3.78 \pm 0.23 \pm 0.16 \pm 0.14) \times 10^{-3}$ and $(3.88 \pm 0.20 \pm 0.31 \pm 0.09) \times 10^{-3}$

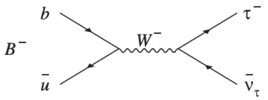
- 精细结构、丰富的 QCD 效应

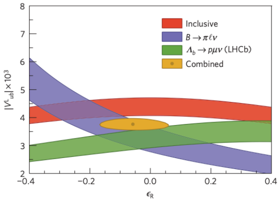
high order QCD corrections, more structures

[AK, TM, YMW, JHEP 02 (2013) 010]
[AK, TM, AAP, YMW, JHEP 09 (2010) 089]



解决方案 寻找新的增长极





‡ $|V_{ub}| f_B$ in **pure leptonic decay**

* 0.72 ± 0.09 MeV from Belle, 1.01 ± 0.14 MeV from BABAR, 0.77 ± 0.12 MeV average [FLAG2021]

‡ $|V_{ub}|$ in **baryon decay**

[LHCb Nature Physics 11, 743-747 (2015)]

$$\frac{|V_{ub}|^2}{|V_{cb}|^2} = \frac{\mathcal{B}(\Lambda_b^0 \to p\mu^-\bar{\nu})}{\mathcal{B}(\Lambda_b^0 \to \Lambda_c^+\mu^-\bar{\nu})} R_{\rm FF} = 0.68 \pm 0.07 \Downarrow$$

$$\frac{|V_{ub}|}{|V_{cb}|} = 0.083 \pm 0.06 \xrightarrow{|V_{cb}|} |V_{ub}| = (3.72 \pm 0.23) \times 10^{-3}$$

* consistent with determinations in exclusive $B \to \pi l \bar{\nu}$ decay
  confirms the existing incompatibility with the inclusive sample

‡ $|V_{ub}|/|V_{cb}|$ in $\mathcal{B}(B_s \to K^-\mu^+\nu)/\mathcal{B}(B_s \to D_s^-\mu^+\nu)$ [LHCb PRL126, 081804 (2021)]

‡ $|V_{cb}|$ in $B_s \to D_s\mu^+\nu$, $\frac{d\mathcal{B}(B\to K^*\mu^+\mu^-)}{dq^2}$ and p_5' in $\Lambda_b^0 \to \Lambda\mu^+\mu^-$

解决方案 寻找新的增长极

- 以上过程都只涉及到基态粒子
- 含有激发态粒子的过程也可以提供独立的测量
- $|V_{ub}|$ in the $B \to \rho l \nu$ channel   the same $b \to u l \nu$ transition as in the golden channel
- simultaneous measurements of the $\frac{d\mathcal{B}}{dq^2}$ for $B \to \pi^- l^+ \nu_l$ and $B \to \rho^0 l^+ \nu_l$
  [Belle-II PRD 111. 112009(2025)] [HFLAV 2023] [P. Bharucha and et.al., JHEP 08(2016)098]

$$|V_{ub}|_{B\to\pi l\nu} = \left(3.73 \pm 0.10 \pm 0.16|_{\mathrm{LQCD+LCSRs}}\right) \times 10^{-3},$$
$$|V_{ub}|_{B\to\rho l\nu} = \left(3.19 \pm 0.22 \pm 0.26|_{\mathrm{LCSRs}}\right) \times 10^{-3}.$$

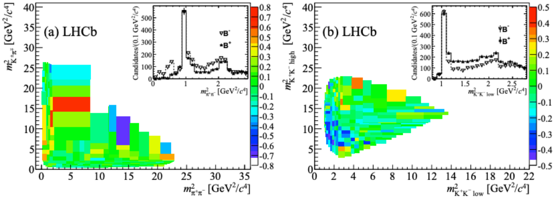
- $B \to V$ FFs updated via B-meson LCSRs [Gao, et.al., PRD 101.074035(2020)]

$$|V_{ub}|_{B\to\rho l\nu} = \left(3.05^{+1.34}_{-1.30}|_{\mathrm{theo}}\,{}^{+0.19}_{-0.20}|_{\mathrm{data}}\right) \times 10^{-3},$$
$$|V_{ub}|_{B\to\omega l\nu} = \left(2.54^{+1.09}_{-1.05}|_{\mathrm{theo}}\,{}^{+0.18}_{-0.19}|_{\mathrm{data}}\right) \times 10^{-3}$$

- a notably smaller value is obtained in $B \to \rho$ transition
- uniformity of $|V_{ub}|$ determinations across different exclusive channels ?
- $|V_{ub}|_{B_s\to K l\nu} = 3.58(9) \times 10^{-3}$, consists with the golden channel [PRD104.114041 (2021)]

$B \to \pi\pi l\nu$ 和 $|V_{ub}|$ 测量

- ρ is an unstable resonance that decays into $\pi\pi$ via the strong interaction
  - ∗ the signal channel in a $B \to \rho l\nu$-type decay is $B \to \pi\pi l\nu$ (B_{l4})
  - ∗ the $\pi\pi$ spectra in $[0.554, 0.996]$GeV serves as the candidate region for ρ
    [X.-W. Kang, B. Kubis, C. Hanhart, and U.-G. Meißner, PRD 89. 053015 (2014)]
    [S. Faller, T. Feldmann, A. Khodjamirian, T. Mannel and D. van. Dyk PRD 89. 014015 (2014)]
  - ∗ the QCD studies usually treat ρ as a stable single particle
- how to address the finite width of the ρ mesons, nonresonant QCD backgrounds, and the effects of different partial waves in the calculation of $B \to \pi\pi$ form factor?



∗ small ($< 10\%$) in three-body D decays, large/dominate in the penguin dominated three-body B decays ($B \to K\pi\pi$, KKK) [PRL 111.101801(2013) LHCb], theoretically unresolved

重介子四体半轻衰变的初步探索

i: $B \to \pi\pi l\nu$ 和 $|V_{ub}|$ 测量

ii: $D_s \to [\pi\pi]_{\rm S}\, e^+\nu_e$ 和 $f_0, [\pi\pi]_{\rm S}$ 结构初探

$B \to \pi\pi$ 形状因子

$$
\begin{aligned}
i\langle \pi^+(k_1)\pi^-(k_2)|\bar{u}\gamma_\nu(1-\gamma_5)b|\bar{B}^0(p)\rangle = {}& F_\perp(q^2,k^2,\zeta)\,\frac{2}{\sqrt{k^2}\sqrt{\lambda_B}}\, i\epsilon_{\nu\alpha\beta\gamma}\, q^\alpha\, k^\beta\, \bar{k}^\gamma \\
&+F_t(q^2,k^2,\zeta)\,\frac{q_\nu}{\sqrt{q^2}} + F_0(q^2,k^2,\zeta)\,\frac{2\sqrt{q^2}}{\sqrt{\lambda_B}}\left(k_\nu - \frac{k\cdot q}{q^2}q_\nu\right) \\
&+F_\parallel(q^2,k^2,\zeta)\,\frac{1}{\sqrt{k^2}}\left(\bar{k}_\nu - \frac{4(q\cdot k)(q\cdot\bar{k})}{\lambda_B}\,k_\nu + \frac{4k^2(q\cdot\bar{k})}{\lambda_B}\,q_\nu\right)
\end{aligned}
$$

† $\lambda = \lambda(m_B^2, k^2, q^2)$ is the Källén function, $q\cdot k = (m_B^2 - q^2 - k^2)/2$,
$q\cdot\bar{k} = \sqrt{\lambda}\beta_\pi(k^2)\cos\theta_\pi/2 = \sqrt{\lambda}\,(2\zeta - 1)$

† $\beta_\pi(k^2) = \sqrt{1 - 4m_\pi^2/k^2}$, θ_π is the angle between the 3-momenta of the neutral pion and the B-meson in the dipion rest frame

- **QCDF** (QCD factorization) in the large dipion mass
  - [P. Böer, T. Feldmann and D. van Dyk, JHEP02, 133(2017)] $T_I \propto F_{Bto\pi} \otimes \phi_\pi$,
- **SU(3) flavor symmetry/breaking** with the intermediate resonance
  - [R.M. Wang, Y.G. Xu, J.H. Sheng, X.D. Cheng and et.al., 2301.00090, EPJC84(2024)1110, PRD112(2025)033002]

$B \to \pi\pi$ 形状因子

- **LQCD** (Lattice QCD) in the ρ resonance region with a simple BW model
  - [L. Leskovec and et.al, PRL 134.161901 (2025), **Editors' Suggestion**]
- **HChPT** (Heavy-meson Chiral Perturbative Theory) in the large q^2 by taking dispersive methods in terms of Omnés functions
  - [X.-W. Kang, B. Kubis, C. Hanhart, and U.-G. Meißner, PRD 89. 053015 (2014)]

  in the full phase-space by a novel parameterization with unitarity
  - [F. Herren, B. Kubis and R. van Tonder, PRD 112, 014037 (2025), **Editors' Suggestion**]
- **LCSRs** (Light-cone sum rules) in the small and intermediate q^2
  - [**SC**, A. Khodjamirian and J. Virto, JHEP 05(2017)157] *B*-meson LCSRs , [S. Descotes-Genon, A. Khodjamirian, J. Virto and K.K. Vos, JHEP 12(2019)083, 06(2023)034] $B \to K\pi$
  - [C. Hambrock and A. Khodjamirian, NPB 905(2016)379-390] 2πDAs LCSRS of $F_{\parallel,\perp}$
  - [**SC**, A. Khodjamirian and J. Virto, PRD(R) 96 (2017)051901] timelike-helicity FF F_t and F_0
  - [**SC**, PRD 99 (2019) 053005] 2πDAs updates and $B \to [\pi\pi]_{\mathrm{S,P}}$ FFs
  - [**SC** and J.M Shen, EPJC(2020)6:554, **SC** and S.L Zhang, EPJC (2024)84:379] Pheno
  - [**SC**, arXiv: 2502.07333] first study of twist-three 2πDAs and $|V_{ub}|$ extraction
  - [**SC**, L.Y. Dai, J.M. Shen and S.L. Zhang, arXiv:2509.15659] $D_s \to [\pi\pi]_{\mathrm{S}}\, e\nu$, minor $q\bar{q}$ contribution

$B \to \pi\pi$ 形状因子

- $B \to \pi\pi$ form factors from the B-meson LCSRs

$$F_{\mu\nu}(k,q) = i\int d^4x\, e^{ik\cdot x}\langle 0|\mathrm{T}\{j_\mu(x), j_\nu^{V-A}(0)\}|\bar{B}^0(q+k)\rangle$$
$$\equiv \quad \varepsilon_{\mu\nu\rho\sigma} q^\rho k^\sigma F_{(\varepsilon)}(k^2,q^2) + i g_{\mu\nu} F_{(g)}(k^2,q^2) + i q_\mu k_\nu F_{(qk)}(k^2,q^2) + \cdots$$

- $B \to \pi\pi$ form factors from the 2πDAs LCSRs

$$F_\mu(k_1,k_2,q) = i\int d^4x e^{iq\cdot x}\langle \pi^+(k_1)\pi^-(k_2)|\mathrm{T}\{j_\mu^{V-A}(x), j_5(0)\}|0\rangle$$
$$\equiv \quad \varepsilon_{\mu\nu\rho\sigma} q^\nu k_1^\rho k_1^\sigma F^V + q_\mu F^{(A,q)} + k_\mu F^{(A,k)} + \bar{k}_\mu F^{(A,\bar{k})}$$

- Advantages of the 2πDAs LCSRs

  * the QCD calculation does not rely on any resonant model, however, encounter a inverse problem in B-meson LCSRs

$$2\,\mathrm{Im}F_{\mu\nu}(k,q) = \int d\tau_{2\pi}\,\langle 0|\bar{d}\gamma_\mu u\,|\pi^+\pi^-\rangle\langle \pi^+\pi^-|\bar{u}\gamma_\nu(1-\gamma_5)b|\bar{B}^0\rangle + \cdots$$

  * provides a unifies framework to predict contributions from different partial-waves
  * theoretical uncertainties are better controlled, higher-power corrections are suppressed by $\mathcal{O}(1/m_b)$ when k^2 is not too large

2πDAs

- Chiral-even LC expansion with gauge factor $[x, 0]$ [Polyakov 1999, Diehl 1998]

$$\langle \pi^a(k_1)\pi^b(k_2)|\bar{q}_f(zn)\gamma_\mu \tau q_{f'}(0)|0\rangle = \kappa_{ab}\, k_\mu \int dx\, e^{iuz(k\cdot n)}\, \Phi_{\parallel}^{ab,ff'}(u,\zeta,k^2)$$

‡ Three independent kinematic variables

- 2πDAs is decomposed in terms of $C_n^{3/2}(2z-1)$ and $C_\ell^{1/2}(2\zeta-1)$

$$\Phi^{I=1}(z,\zeta,k^2,\mu) = 6z(1-z) \sum_{n=0,\mathrm{even}}^{\infty} \sum_{l=1,\mathrm{odd}}^{n+1} B_{n\ell}^{I=1}(k^2,\mu) C_n^{3/2}(2z-1) C_\ell^{1/2}(2\zeta-1)$$

$$\Phi^{I=0}(z,\zeta,k^2,\mu) = 6z(1-z) \sum_{n=1,\mathrm{odd}}^{\infty} \sum_{l=0,\mathrm{even}}^{n+1} B_{n\ell}^{I=0}(k^2,\mu) C_n^{3/2}(2z-1) C_\ell^{1/2}(2\zeta-1)$$

- How to describe **the evolution from $4m_\pi^2$ to large invariant mass k^2** ?

‡ Watson theorem of π-π scattering amplitudes

$$B_{n\ell}^{I}(k^2) = B_{n\ell}^{I}(0)\, \mathrm{Exp}\left[\sum_{m=1}^{N-1} \frac{k^{2m}}{m!} \frac{d^m}{dk^{2m}} \ln B_{n\ell}^{I}(0) + \frac{k^{2N}}{\pi} \int_{4m_\pi^2}^{\infty} ds\, \frac{\delta_\ell^I(s)}{s^N(s-k^2-i0)}\right]$$

△ **2πDAs in a wide range of energies is given by δ_ℓ^I and a few subtraction constants**

2πDAs

- The subtraction constants of $B_{n\ell}(s)$ at low s (around the threshold)

(nl)	$B^{\parallel}_{n\ell}(0)$	$c_1^{\parallel,(nl)}$	$\frac{d}{dk^2}\ \ln B^{\parallel}_{n\ell}(0)$	$B^{\perp}_{n\ell}(0)$	$c_1^{\perp,(nl)}$	$\frac{d}{dk^2}\ \ln B^{\perp}_{n\ell}(0)$
(01)	1	0	1.46 $\rightarrow$ 1.80	1	0	0.68 $\rightarrow$ 0.60
(21)	-0.113 $\rightarrow$ 0.218	-0.340	0.481	0.113 $\rightarrow$ 0.185	-0.538	-0.153
(23)	0.147 $\rightarrow$ -0.038	0	0.368	0.113 $\rightarrow$ 0.185	0	0.153
(10)	-0.556	-	0.413	-	-	-
(12)	0.556	-	0.413	-	-	-

△ firstly studied in the effective low-energy theory based on instanton vacuum [Polyakov 1999]

△ updated with the kinematical constraints and the new a_2^{π}, a_2^{ρ} [**SC** 2019, 2023]

- All the above discussions are at leading twist

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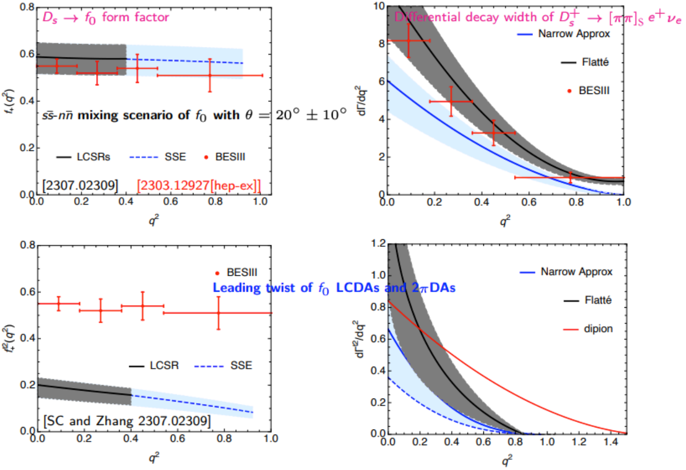
pion EMFF widely used in the three-body B decays studied from pQCD and QCDF are the asymptotic formula of 2πDAs

[J. Chai, **SC** and A.J Ma PRD 105 (2022) 033003]

normalized to unit as $\Gamma^{I=1}_{M_1M_2}(0) = 1$. When the invariant mass of dimeson system is small, the higher $\mathcal{O}(s)$ terms in the expansion of coefficient $B_{nl}(s,\mu)$ around the resonance pole can be safely neglected due to the large suppression $\mathcal{O}(s/m_b^2)$ in contrast to the energetic dimeson system in B decay, so the relation $B_{n1}(s,\mu) \rightarrow a_n(\mu)\Gamma^{I=1}_{M_1M_2}(s)$ can be obtained in the lowest partial wave approximation. This argument induces the basic assumption in PQCD that the energetic dimeson DAs can be deduced from the DAs of resonant meson by replacing the decay constant by the timelike form factor.

- my talk in the HFCPV2023 at FDU, Dec 16th, 2023

DiPion LCDAs and D_{l4} decays



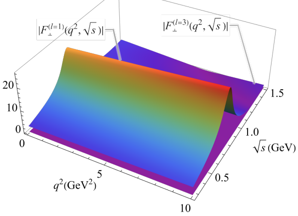
- Twist-3 LCDAs give dominate contribution in $D_s \to f_0, [\pi\pi]_S$ transitions
- further measurements would help us to understand the DiPion system

- twist three 2πDAs are studied recently [S. Cheng, arXiv:2502.07333]

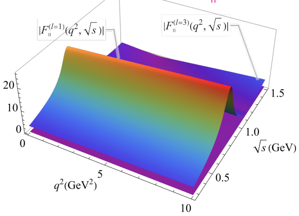
$B \to \pi\pi$ 形状因子

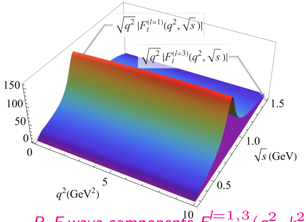
[S Cheng, 2502.07333]

P, F-wave components $F_{\perp}^{l=1,3}(q^2, k^2)$

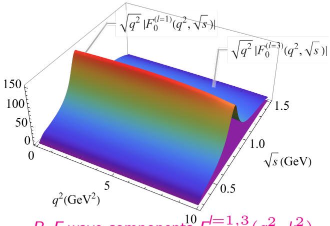


P, F-wave components $F_{\parallel}^{l=1,3}(q^2, k^2)$





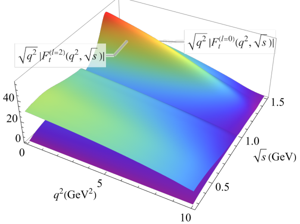
P, F-wave components $F_t^{l=1,3}(q^2, k^2)$



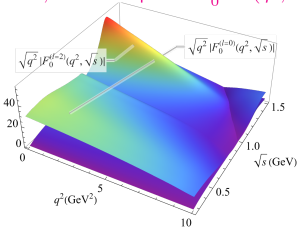
P, F-wave components $F_0^{l=1,3}(q^2, k^2)$

$B \to \pi\pi$ 形状因子

S, *D*-wave components $F_t^{l=0,2}(q^2, k^2)$



S, *D*-wave components $F_0^{l=0,2}(q^2, k^2)$



- twist-three contribution vanish in $F_{\perp,\parallel}^{(l=1,3,\cdots)}$
- persist in $F_{t,0}^{(l=0,1,\cdots)}$, give a significant correction $\sim 40\%$ to leading twist result
- ρ resonance is dominate in $F_{\perp,\parallel,t,0}^{(l=1)}(q^2,k^2)$, *F*-wave contributions are negligible
- *S*-wave component is dominate in the small k^2, *D*-wave component is comparable in the large k^2 and small q^2

通过 B_{l4} 过程测量 $|V_{ub}|$

- 2D partial differential decay width

$$\frac{d^2\Gamma}{dq^2\,dk^2} = G_F^2|V_{ub}|^2 \frac{\beta_\pi\sqrt{\lambda}q^2}{3\,(4\pi)^5\,m_B^3}\left[\left(|F_0^{(S)}|^2 + |F_0^{(P)}|^2\right) + \beta_\pi^2\left(|F_\parallel^{(P)}|^2 + |F_\perp^{(P)}|^2\right) + \cdots\right]$$

- Partial branching fractions $\triangle\mathcal{B}^i$ (in unit of 10^{-5}) in different bins

take the PDG average value of $|V_{ub}| = (3.82 \pm 0.20) \times 10^{-3}$

bins	$\sqrt{s}$	q^2	$\triangle\mathcal{B}^i$	$\triangle\mathcal{B}^i$ [Belle 2021]
1	$[4m_\pi^2, 0.6]$	$[0, 8]$	$0.27 \pm 0.03 \pm 0.06$	$0.84^{+0.39}_{-0.32} \pm 0.18$
2	$(0.6, 0.9]$	$[0, 4]$	$1.91 \pm 0.21 \pm 0.38$	$2.39^{+0.53}_{-0.47} \pm 0.32$
3	$(0.6, 0.9]$	$(4, 8]$	$1.54 \pm 0.17 \pm 0.27$	$2.16^{+0.47}_{-0.42} \pm 0.23$
4	$(0.9, 1.2]$	$[0, 4]$	$0.65 \pm 0.07 \pm 0.12$	$0.70^{+0.32}_{-0.25} \pm 0.20$
5	$(0.9, 1.2]$	$(4, 8]$	$0.41 \pm 0.04 \pm 0.08$	$0.64^{+0.28}_{-0.22} \pm 0.11$
6	$(1.2, 1.5]$	$[0, 4]$	$0.57 \pm 0.05 \pm 0.10$	$0.91^{+0.35}_{-0.28} \pm 0.12$
7	$(1.2, 1.5]$	$(4, 8]$	$0.16 \pm 0.02 \pm 0.02$	$0.64^{+0.32}_{-0.26} \pm 0.08$

- $|V_{ub}|$ extraction in the regions of ρ and f_0 resonances

$$\boxed{|V_{ub}|_{B^+\to[\rho^0\to]\pi^+\pi^-l^+\nu_l} = (4.27 \pm 0.49|_{\rm Data} \pm 0.55|_{\rm LCSRs}) \times 10^{-3}}$$

$$\boxed{|V_{ub}|_{B^+\to[f_0\to]\pi^+\pi^-l^+\nu_l} = (3.96 \pm 0.47|_{\rm Data} \pm 0.52|_{\rm LCSRs}) \times 10^{-3}}$$

$D_s \to [\pi\pi]_S\, e^+\nu_e$ 和 $f_0, [\pi\pi]_S$ 结构初探

- Semileptonic $D_{(s)}$ decays provide a clean environment to study scalar mesons
  - $D_s \to f_0 e^+\nu$ [CLEO '09], $D_{(s)} \to a_0 e^+\nu$ [BESIII '18, '21], $D^+ \to f_0/\sigma e^+\nu$[BESIII '19]
  - $D_s \to f_0(\to \pi^0\pi^0, K_sK_s)e^+\nu$ [BESIII 22], $D_s \to f_0(\to \pi^+\pi^-)e^+\nu$[BESIII 23]

$$\mathcal{B}(D_s \to f_0(\to \pi^0\pi^0)e^+\nu) = (7.9 \pm 1.4 \pm 0.3) \times 10^{-4}$$
$$\mathcal{B}(D_s \to f_0(\to \pi^+\pi^-)e^+\nu) = (17.2 \pm 1.3 \pm 1.0) \times 10^{-4}$$
$$f_+^{f_0}(0)|V_{cs}| = 0.504 \pm 0.017 \pm 0.035$$

- single particle (narrow width limit) $D_s \to f_0 e^+\nu$

$$\frac{d\Gamma(D_s^+ \to f_0 l^+\nu)}{dq^2} = \frac{G_F^2|V_{cs}|^2\lambda^{3/2}(m_{D_s}^2, m_{f_0}^2, q^2)}{192\pi^3 m_{D_s}^3}|f_+(q^2)|^2, \; D_s \to f_0 \text{ FF}$$

- improvement with the width effect by resonant model $D_s \to [f_0 \to]\, \pi\pi e^+\nu$

$$\frac{d\Gamma(D_s^+ \to [\pi\pi]_S\, l^+\nu)}{dsdq^2} = \frac{1}{\pi}\frac{G_F^2|V_{cs}|^2}{192\pi^3 m_{D_s}^3}|f_+(q^2)|^2 \frac{\lambda^{3/2}(m_{D_s}^2, s, q^2)\, g_1\beta_\pi(s)}{|m_S^2 - s + i(g_1\beta_\pi(s)) + g_2\beta_K(s))\,|^2}, \text{ BESIII}$$

- calculate directly the signal channel $D_s \to [\pi\pi]_S\, e^+\nu$

$$\frac{d^2\Gamma(D_s^+ \to [\pi\pi]_S\, l^+\nu)}{dk^2 dq^2} = \frac{G_F^2|V_{cs}|^2\, \beta_{\pi\pi}(k^2)\sqrt{\lambda_{D_s}}q^2}{3(4\pi)^5 m_{D_s}^3}\sum_{\ell=0}^{\infty}|F_0^{(\ell)}(q^2, k^2)|^2, \; D_s \to \pi\pi \text{ FF}$$

$D_s \to f_0$ 形状因子和 $D_s \to (f_0 \to)\,\pi\pi e^+\nu$ 衰变

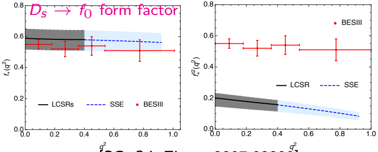
- $\{M^2, s_0\} = \{5.0 \pm 0.5, 6.0 \pm 0.5\}\mathrm{GeV}^2$

this work	3pSRs(07)	LFQM(09)	CLFD/DR(08)	LCSRs(10)
0.63 ± 0.04	0.96	0.87	$0.86/0.90$	0.30 ± 0.03

- the BESIII result in the $\pi^+\pi^-$ system $f_+(0) = 0.518 \pm 0.018 \pm 0.036$ [BESIII 23]

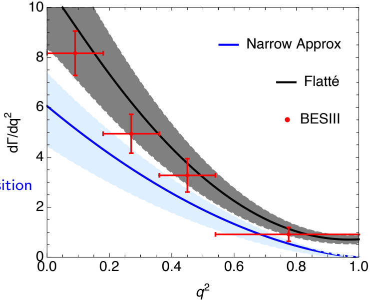
different input of the decay constant $\tilde{f}_{f_0} = 335$ MeV, much larger than 180 MeV in LCSRs(10)
we add the first gegenbauer expansion terms in the LCDAs, up-to-date parameters

$s\bar{s}$-$n\bar{n}$ mixing scenario of f_0 with $\theta = 20^\circ \pm 10^\circ$



[**SC**, S.L Zhang 2307.02309]

- Twist-3 LCDAs give dominate contribution in $D_s \to f_0$ transition
  - the uncertainty estimation is conservative
  - without NLO correction
  - we need a model independent calculation
  - not only for the QCD understanding
  - but also for the future partial-wave measurement

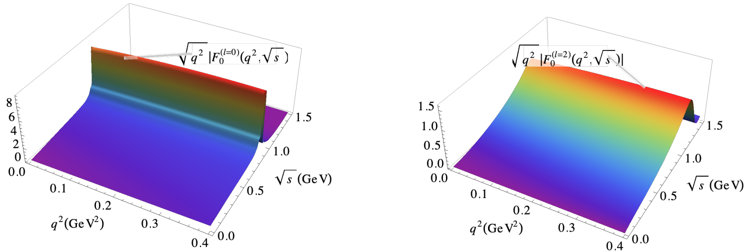


Differential decay width of $D_s^+ \to (f_0, [\pi\pi]_S) e^+ \nu_e$

$D_s \to [\pi\pi]_{\rm S}$ 形状因子和 $D_s \to [\pi\pi]_{\rm S}\, e^+\nu$ 衰变

- The LCSRs ℓ'-wave $D_s \to [\pi\pi]_{\rm S}$ form factors ($\ell' = \text{even}$ & $\ell' \leqslant n+1$)

$$\sqrt{q^2}\,F_0^{(\ell')}(q^2,k^2) = \frac{m_c(m_c+m_s)\sqrt{q^2}\sqrt{\lambda_{D_s}}}{m_{D_s}^2 f_{D_s}} \sum_{n=1,\text{odd}}^{\infty} \frac{\beta_\pi(k^2)}{\sqrt{2\ell'+1}} J_n^0(q^2,k^2,M^2,s_0)\, B_{n\ell,\parallel}^{l=0}(k^2)\, I_{\ell\ell'}$$

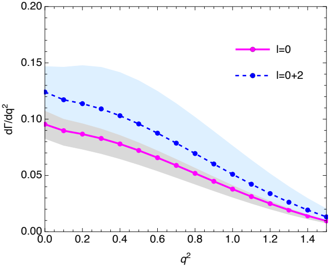


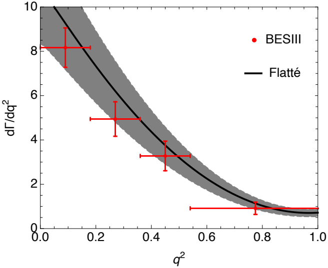
[**SC**, L.Y Dai, J.M Shen and S.L Zhang, 2509.15659]

- S- and D-wave FFs: $\sqrt{q^2}|F_0^{(l=0)}(\sqrt{s},q^2)|$ and $\sqrt{q^2}|F_0^{(l=2)}(\sqrt{s},q^2)|$

$D_s \to [\pi\pi]_S$ 形状因子和 $D_s \to [\pi\pi]_S\, e^+\nu$ 衰变

- The differential decay width on momentum transfers





- Differential widths $d\Gamma/dq^2$ is two-order in magnitude smaller than the data
- the FFs at the two-particle level are one-order lower than the required
- the conventional $q\bar{q}$ is not the dominate component in the charm decays
- we have to go further to multi-particle DiPion LCDAs in CHARM ($q\bar{q}g$, $q\bar{q}q\bar{q}$)
- much different in B decays   leading twist dominated   [**SC**, arXiv: 2502.07333]

总结和展望

总结和展望

- 2πDAs 提供了 $\pi\pi$ 质量谱的一般描述

  † 准确厘清不同分波和同一个分波中不同共振态的贡献， † 有效分离、清晰描述非共振态背景的贡献， † 基于交换对称性来理解 SPD/GPD

- 2πDAs 是重介子四体半轻衰变 (H_{l4}) 研究的关键输入

  † 为 $|V_{ub}|$ 疑难和 FCNC 反常研究提供了新动能， † 为重味强子分布振幅的研究提供了并行检测

- 首次在三扭度水平研究了 2πDAs，实现了在 B_{l4} 过程抽取 $|V_{ub}|$
  [**SC**, 2502.07333]
- 理解重介子衰变中的同位旋标量介子 (对) 的部分子结构和多粒子图像
  [**SC**, L.Y Dai, J.M Shen and S.L Zhang 2509.15659]
- 在 $D \to \pi\pi l^+ l^-$ 衰变中深入理解 FCNC 过程可能的反常行为 wishlists
- 在 e^+e^- 湮灭 $\gamma^* \to \pi\pi\gamma, \gamma^*\gamma \to \pi\pi$ 过程中进一步理解 2πDAs

Thank you for your patience.

Colliders: Pion DAs in the light-cone dominated processes

Conformal spin and collinear twist definition

[Braun, Korchemsky, Müller 2003]

- A convenient tool to study DAs is provided by conformal expansion
- the underlying idea of *conformal expansion of LCDAs* is similar to *partial-wave expansion of wave function in quantum mechanism*
- *invariance of massless QCD under conformal trans.* *VS* *rotation symmetry*
- *longitudinal* ⊗ *transversal* dofs *VS* *angular* ⊗ *radial* dofs for spherically symmetry potential
- the transversal-momentum dependence (scale dependence of the relevant operators) is governed by the RGE
- the longitudinal-momentum dependence (orthogonal polynomials) is described in terms of irreducible representations of the corresponding symmetry group

  collinear subgroup of conformal group $SL(2,R) \cong SU(1,1) \cong SO(2,1)$