QED correction of the hadron masses with heavy quark

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For CLQCD collaboration

CLQCD

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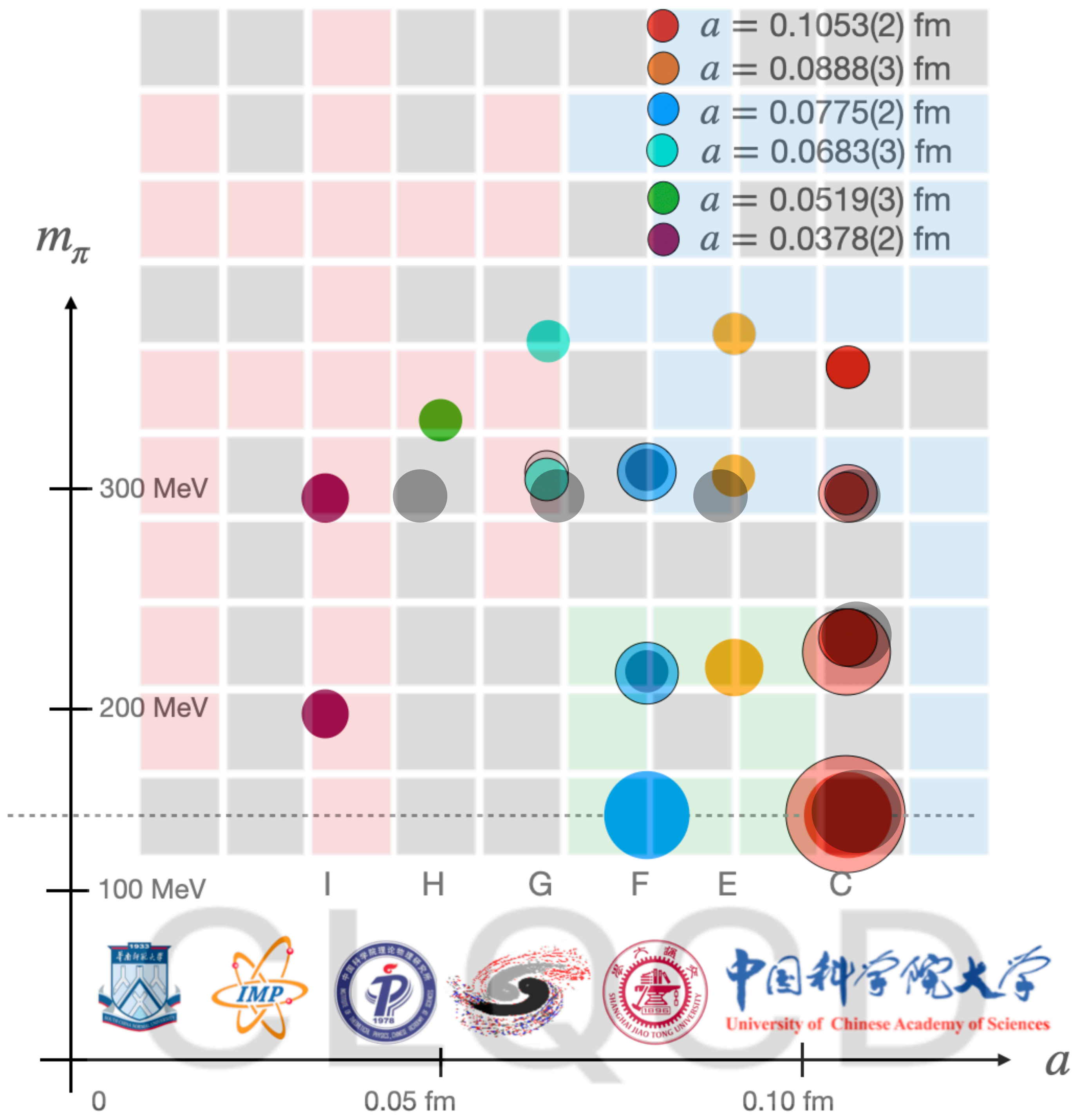
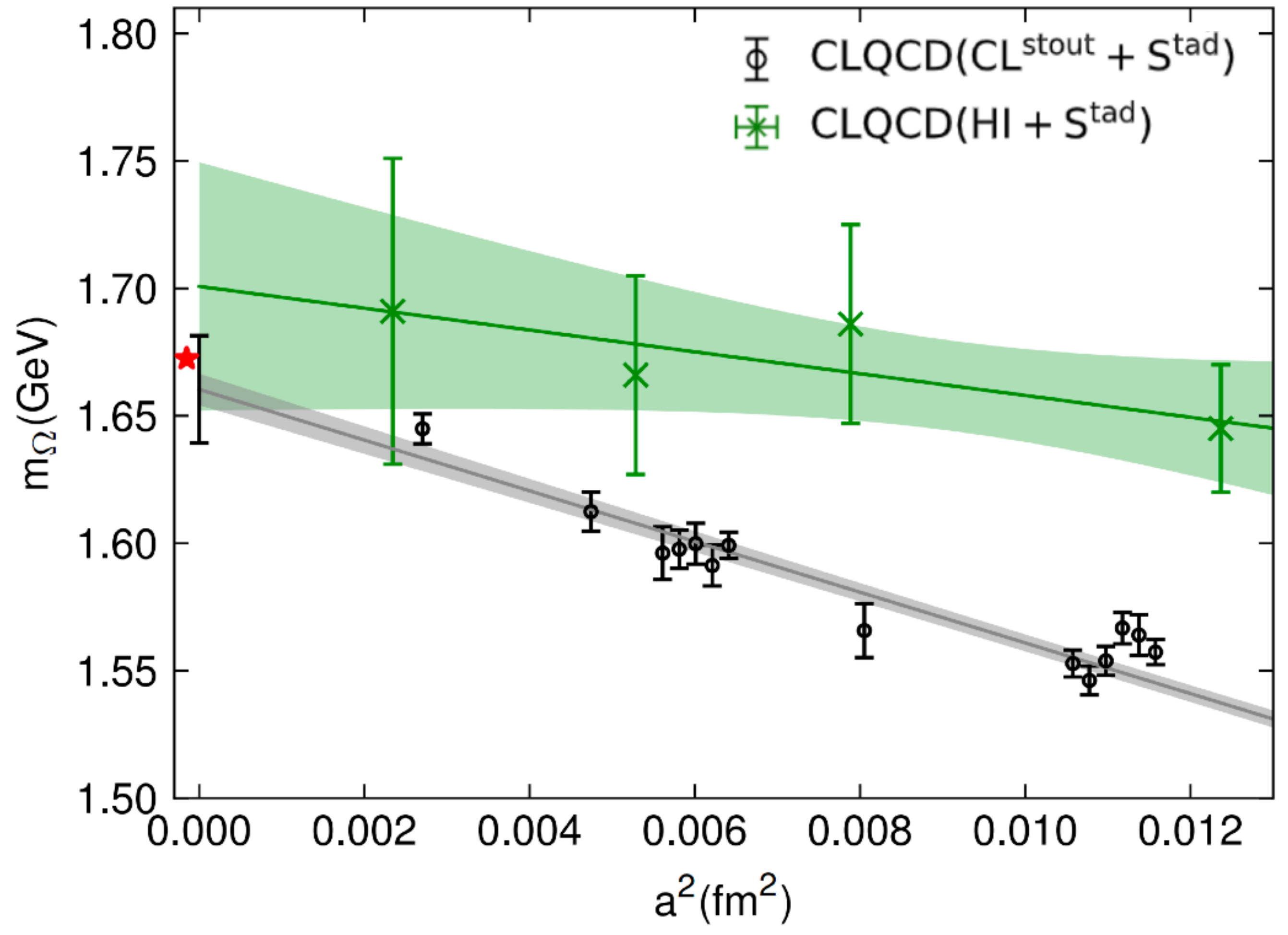
CLQCD ensembles

	Country/ Region	Smallest lattice spacing	No. of physical point ensembles	Largest spacial size	No. of fermion discretization
MILC	US	0.03 fm	5	5.8 fm	1
RBC	US	0.06 fm	3	5.5 fm	1
BMW	EN	0.05 fm	15	10 fm	2
CLS	EN	0.04 fm	2	5.5 fm	1
ETM	EN	0.05 fm	5	6.3 fm	1
PACS	JP	0.06 fm	3	10 fm	1
CLQCD	**CN**	**0.04 fm**	**3**	**6.7 fm**	**2**

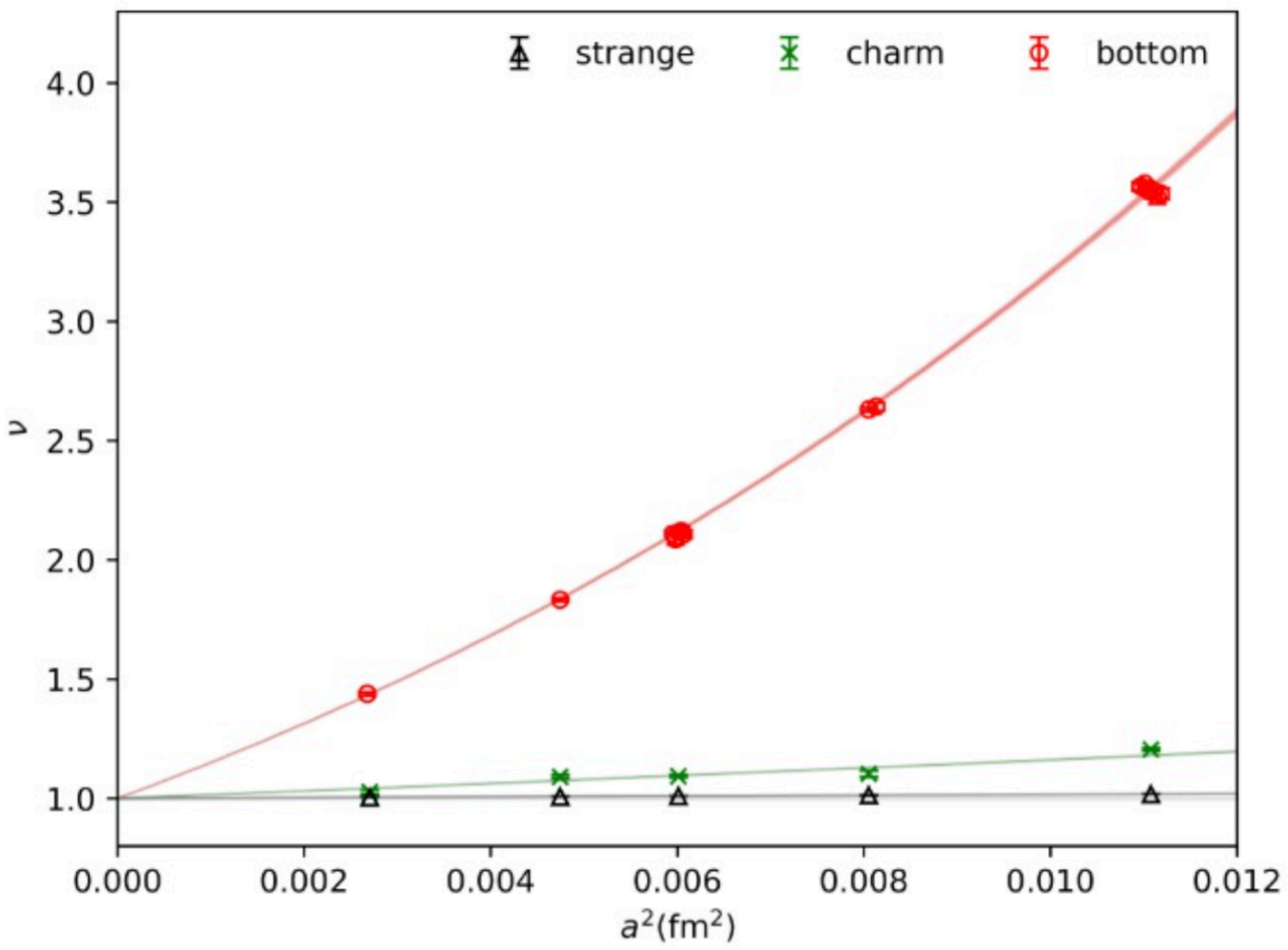


- The first ensemble set from China which can control most of the systematic uncertainties；
- Unique advantage on finite volume studies.

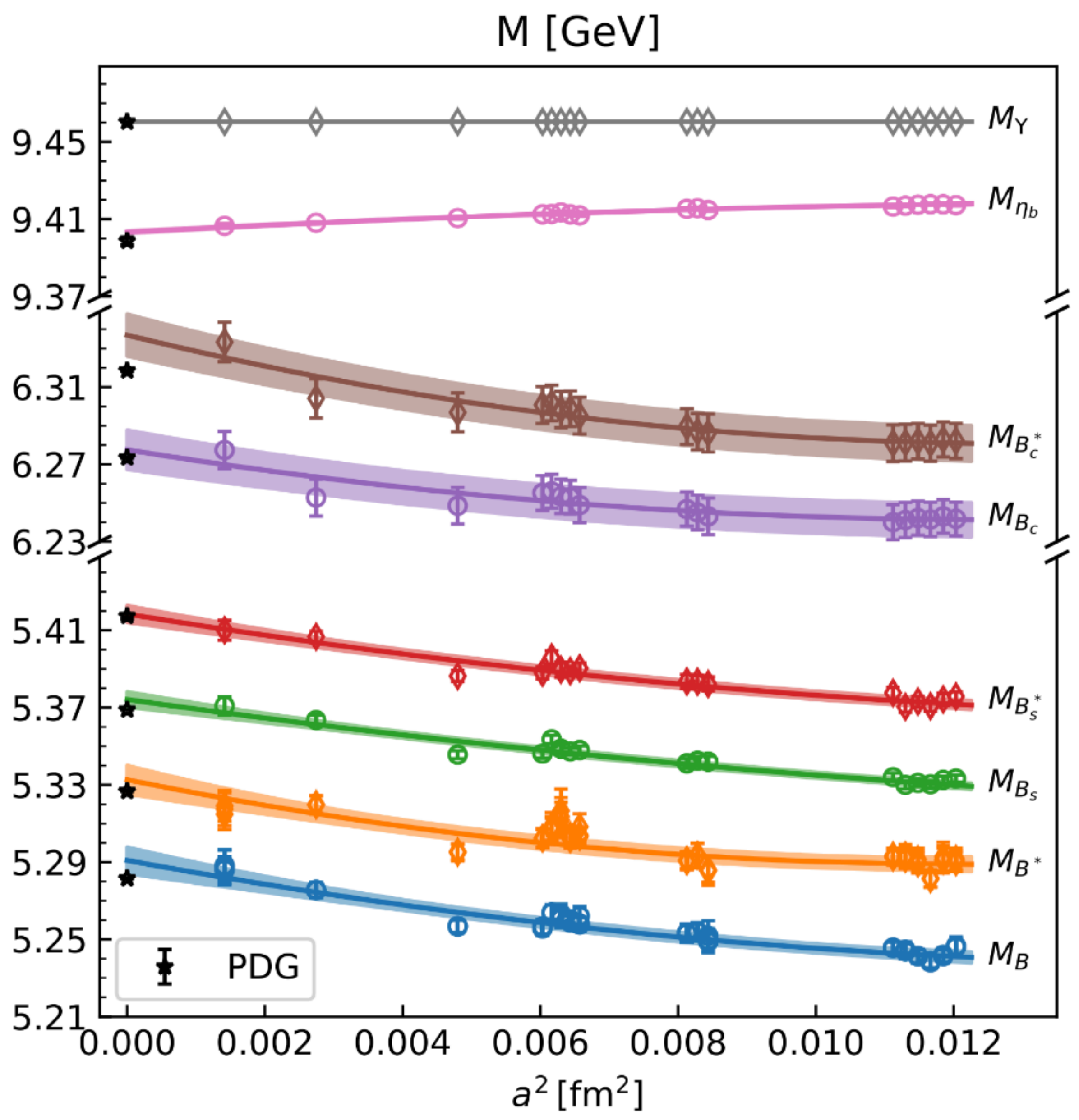


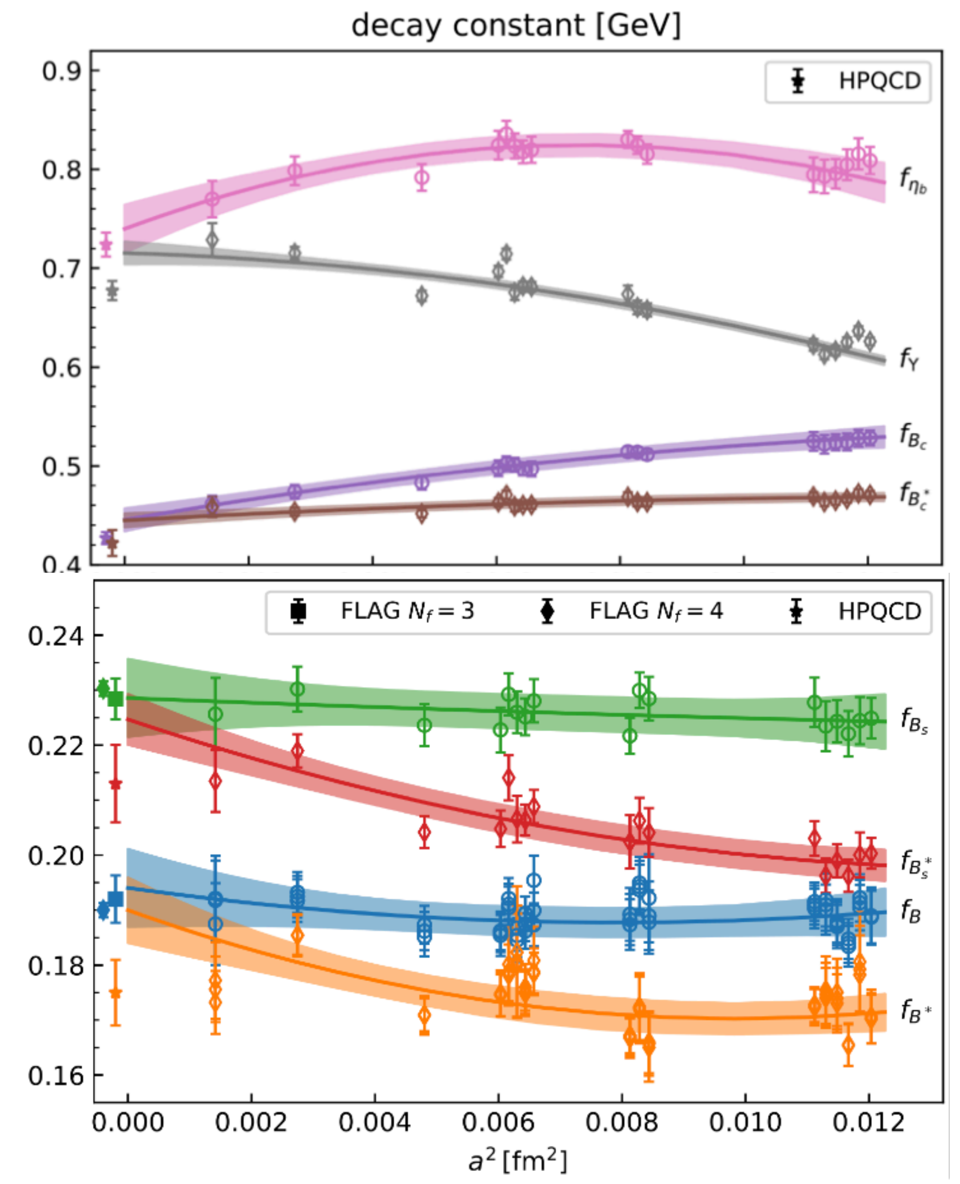
- New ensembles $(\mathrm{HI} + \mathrm{S}^{\mathrm{tad}})$ with 2+1+1 flavor HISQ fermion can provide proper estimate of the charm sea effects;
- Compared to the current 2+1 flavor Clover fermion ensembles $(\mathrm{CL}^{\mathrm{stout}} + \mathrm{S}^{\mathrm{tad}})$, the discretization errors are also suppressed in kinds of the cases.

CLQCD ensembles



- Absorb most of the discretization errors of the bottom quark into an rescale factor along the temporal direction;
- And developed the corresponding non-perturbative renormalization.

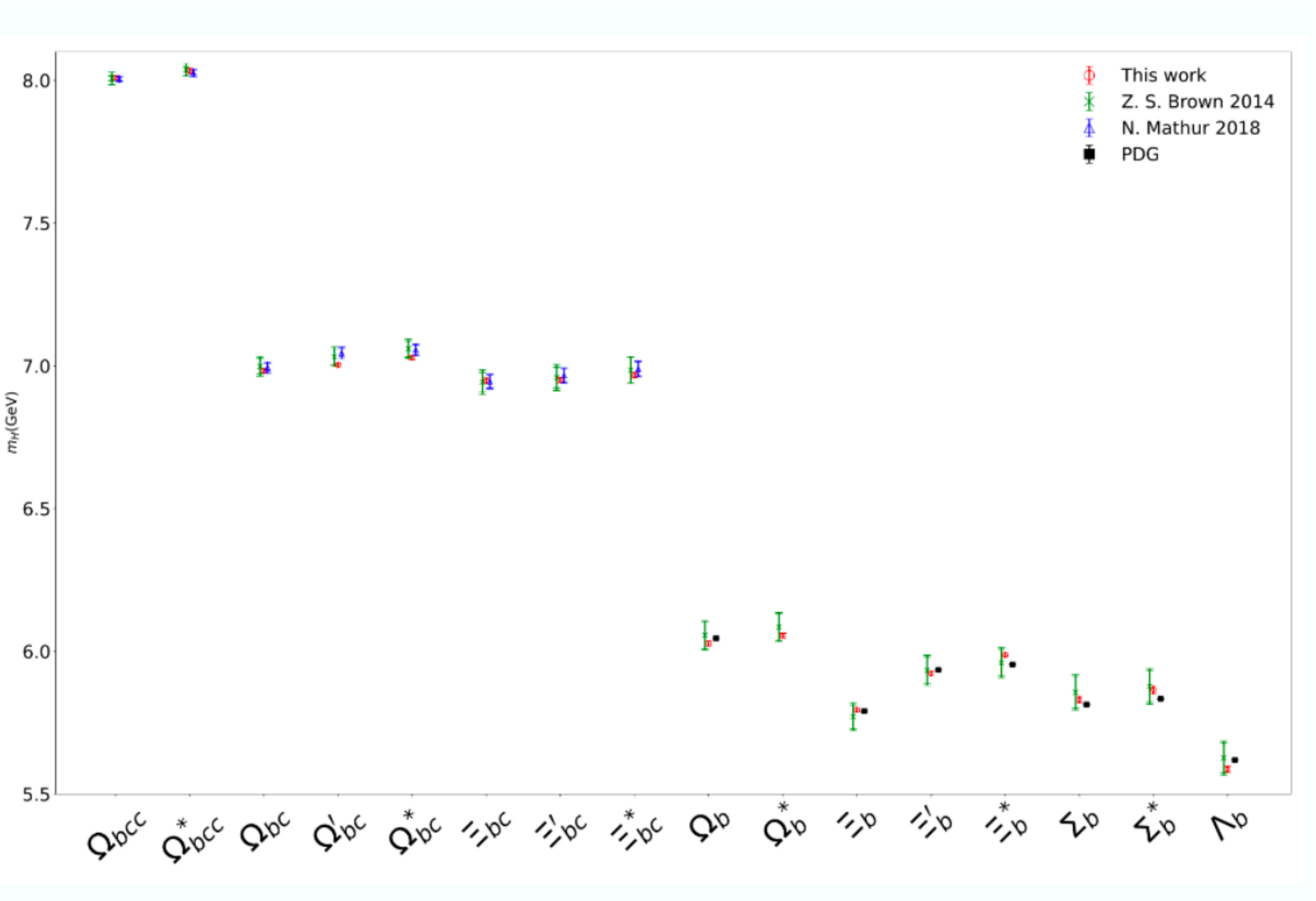


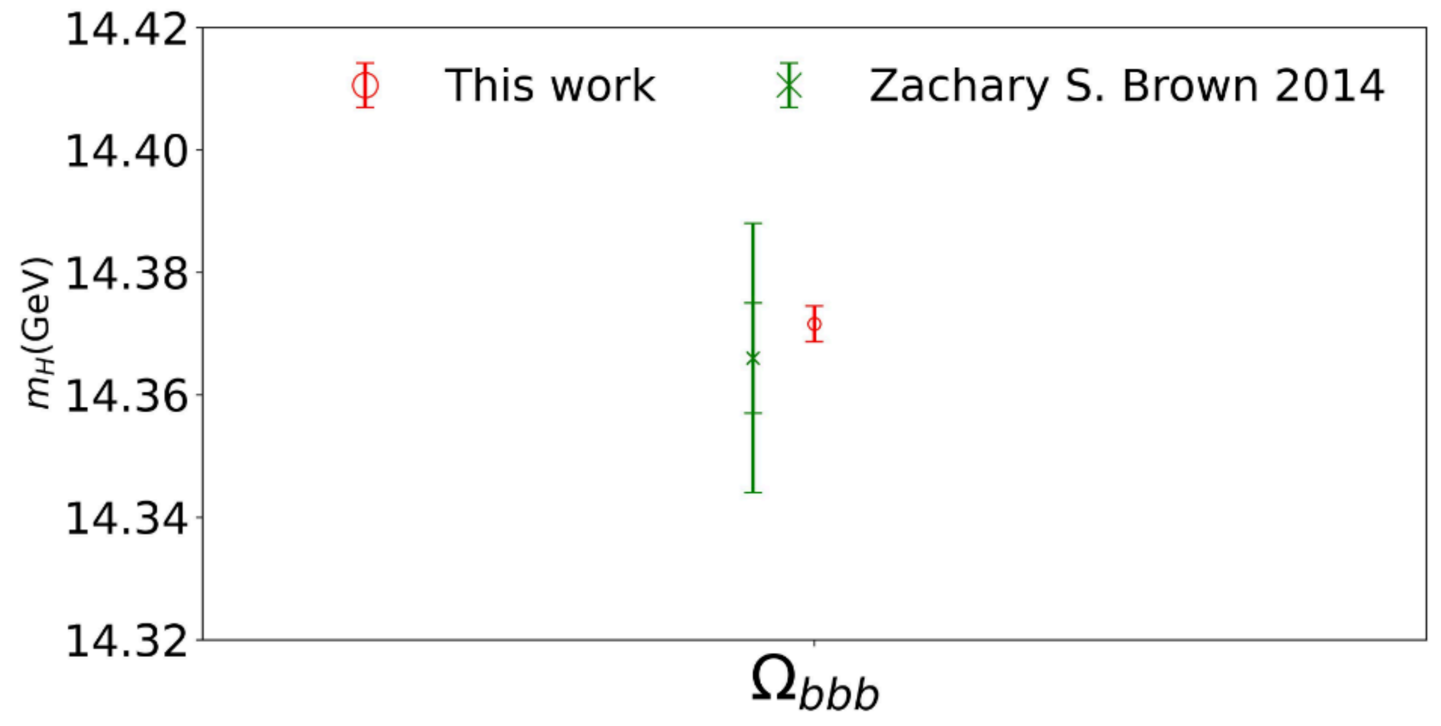


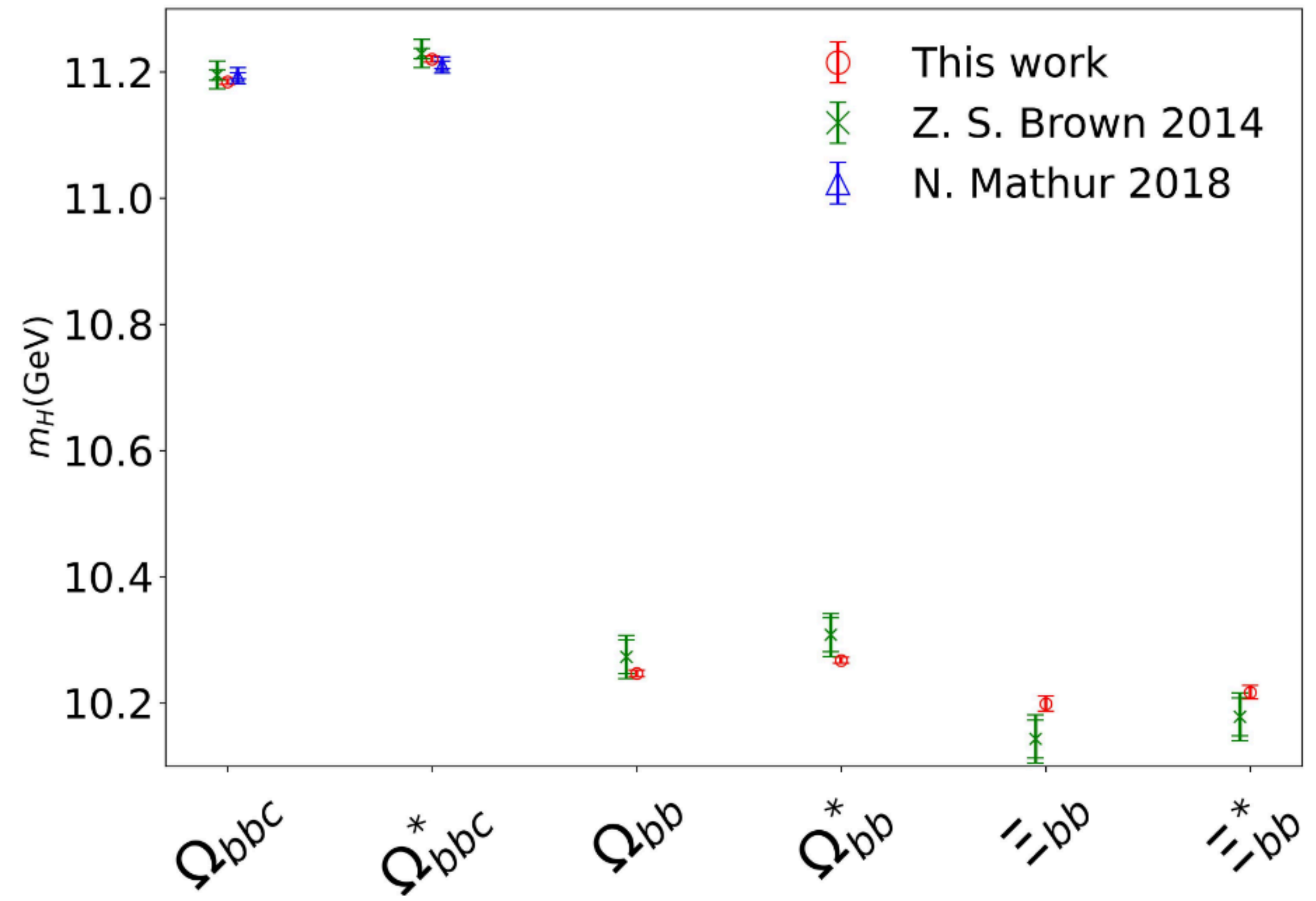
Z.-H. Hu et al. [CLQCD], PRD109(2024)054507
H.Y. Du et al. [CLQCD], PRD111(2025)054504
M.C. Cai et al. [CLQCD], in preparation

CLQCD ensembles

Bottom baryons





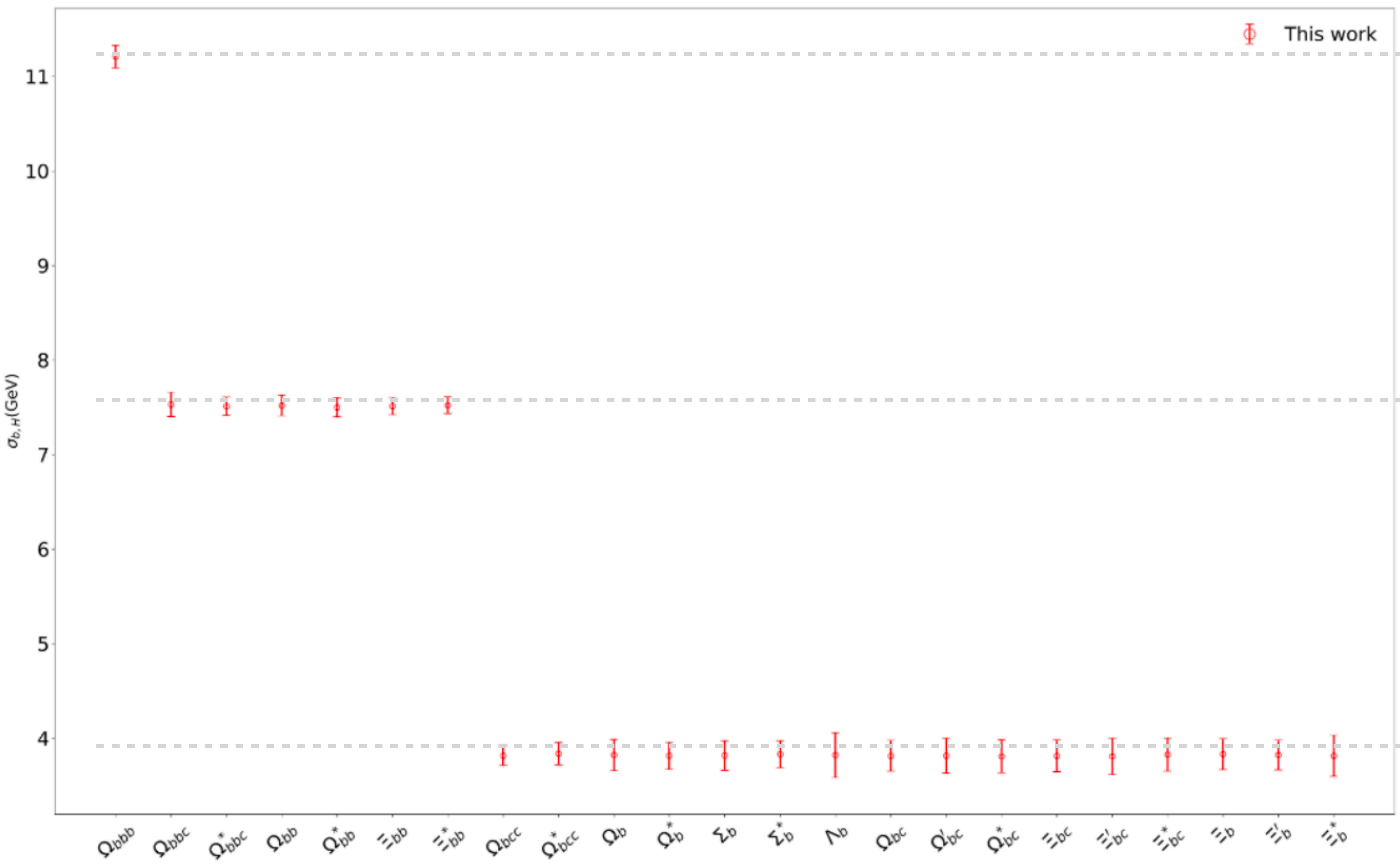


H.Y. Du, et. al., CLQCD, in preparation

- Our prediction of the bottomed baryon masses avoid the dependences on additional nonrelativistic QCD parameters;
- And much more precise the previous lattice calculations.

CLQCD ensembles

Bottom sigma term



$$\sigma_{b,\Omega_{bbb}} = 3 * 3.74(4)\ \mathrm{GeV}$$

$$\sigma_{b,H_{bb}} = 2 * 3.75(5)\ \mathrm{GeV}$$

$$\sigma_{b,H_b} = 1 * 3.82(7)\ \mathrm{GeV}$$

H.Y. Du, et. al., CLQCD, in preparation

- The $\sigma_{b,H} = m_b\langle\bar{b}b\rangle_H$ is the scale and scheme independent bottom quark mass contribution to the baryon mass;
- “Intrinsic scale” to make $\langle\bar{b}b\rangle = 1$ would be around $\overline{\mathrm{MS}}$ 7.6 GeV.

QCD+QED

Decomposition scheme

- The hadron mass and matrix elements in the real world require the full QCD+QED calculation. But since the lattice calculation can only reach an $\mathcal{O}(1\%)$ ($\mathcal{O}(0.1\%)$ in the heavy quark case) precision, one can expand the prediction in term of the polynomial of α and also $\delta_{\rm ISB} \equiv (m_d - m_u)/\Lambda_{\rm QCD}$:

$$\mathcal{M}^{\rm QCD+QED} = \mathcal{M}^{\rm isoQCD} + \alpha \mathcal{M}^{(0,1)} + \delta_{\rm ISB}\mathcal{M}^{(1,0)} + \mathcal{O}(\alpha^2, \alpha\delta_{\rm ISB}, \delta^2_{\rm ISB}).$$

- Naive power counting suggests that both ISB and QED corrections are 1%;

- There are kinds of known results for the ISB and QED corrections:

$$m_n - m_p = 2.52_{\rm ISB}(29)\ {\rm MeV} - 1.00_{\rm QED}(16)\ {\rm MeV},$$

$$m_{D^+} - m_{D^0} = 2.54_{\rm ISB}(13)\ {\rm MeV} + 2.14_{\rm QED}(13)\ {\rm MeV}, \qquad m_{B^+} - m_{B^0} = -1.88_{\rm ISB}(60)\ {\rm MeV} + 1.58_{\rm QED}(24)\ {\rm MeV}\,.$$

BMWc, Science 347(2015)1452

M. Rowe, R. Zwicky, JHEP(2023)089

- ISB effect is only 0.1% for the charmed hadron, how about the QED effect?

- How to understand the sizable QED correction for the charged pion mass in the chiral limit?

$$m_{\pi^+}|_{m_q\to 0} = 0_{m_q\bar{q}q} + 0_{\mathcal{O}(\alpha_s)G^2} + 32_{\mathcal{O}(\alpha)}\ {\rm MeV} \qquad \text{given } (m^2_{\pi^+} - m^2_{\pi^0})|_{m_q\to 0} \simeq\ 1000\ {\rm MeV}^2$$

QED corrections QED_L

- The QCD+QED calculation can be done under the quenched QED approximation using QED_L for the valence fermion:

$$U_\mu^{\mathrm{QCD+QED}} = U_\mu^{\mathrm{QCD}} e^{-\mathrm{i}ee_qA_\mu},\ A_\mu(x) = \int \frac{\mathrm{d}^4p}{(2\pi)^4} e^{-\mathrm{i}p\cdot x}A_\mu(p), P_{A_\mu(p)}|_{\vec{p}\neq 0} \propto e^{-\frac{1}{2N_V}\hat{p}^2A_\mu^2(p)}.$$

- The QED finite volume correction (FVC) of hadron masses using QED_L is independent of the hadron structure until $\mathcal{O}(1/(m_HL)^3)$:

$$\delta_{\mathrm{QED-FVC}}m_H = e_H^2\frac{c_1}{L}(1+\frac{2}{m_HL}+\mathcal{O}(\frac{1}{(m_HL)^2})).$$

BMWc, Science 347(2015)1452

Thus the QED-FVC of neutral particles using QED_L are highly suppressed, likes that using QED_∞.

- One can further improve QED_L by enlarging the weights of the near-zero momentum modes:

$$\delta_{\mathrm{QED-FVC}}^{\mathrm{inf.imp.}}m_H = \mathcal{O}(\frac{e_H^2}{(m_HL)^2}),\ P_{A_\mu(p)}^{\mathrm{inf.imp.}}|_{\vec{p}\neq 0} \propto e^{-\frac{1+1.4856\delta(p^2-\frac{4\pi^2}{L^2})}{2N_V}\hat{p}^2A_\mu^2(p)}.$$

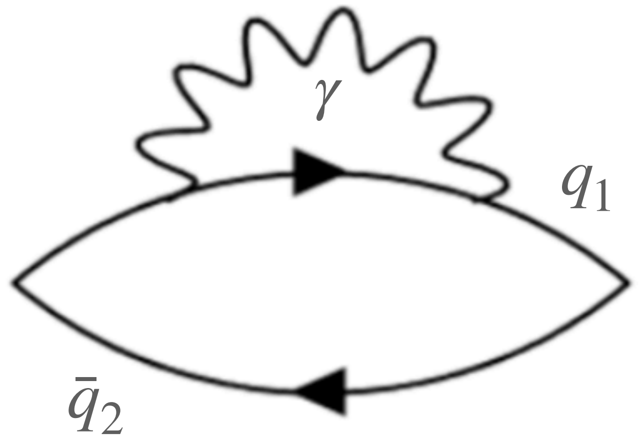
Z. Davoudi et.al., PRD99(2019)034510

QED corrections

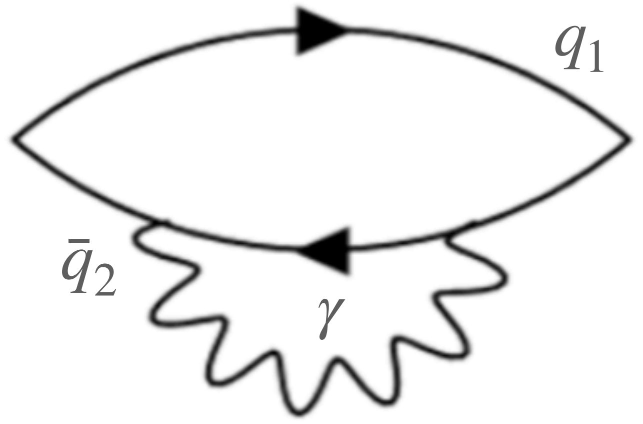
NPU Scheme

- One can define the neutral pion uncorrected (NPU) scheme by tuning the bare mass $m_q^b(e_q)$ of the quark with a QED charge e_q, to ensure the neutral iso-vector pseudoscalar meson mass to be the same as that using the QED-neutral quark with bare mass $m_q^b(0)$:

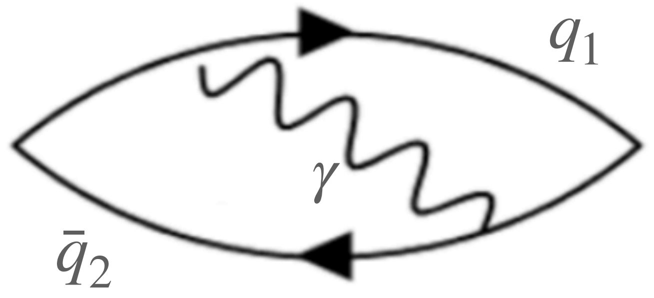
$$m_{\eta_q}^{\rm QCD+QED}(m_q^b(e_q)) = m_{\eta_q}^{\rm QCD}(m_q^b(0)).$$



$e_{q_1}^2\delta^{\rm self}m$



$e_{q_2}^2\delta^{\rm self}m$



$-2e_{q_1}e_{q_2}\delta^{\rm int}m$

- Using the QED quark diagram decomposition, we have ($\delta^{\rm self}m = m^{\rm QCD+QED} - m^{\rm QCD}$):

$$\delta m_{\pi^0} = \frac{5}{18}\delta^{\rm self}m_{\pi^0} - \frac{5}{18}\delta^{\rm int}m_{\pi^0} + \mathcal{O}(\alpha\alpha_s^2, \alpha^2),$$

$$\delta m_{\pi^+} = \frac{5}{18}\delta^{\rm self}m_{\pi^+} + \frac{4}{18}\delta^{\rm int}m_{\pi^+} + \mathcal{O}(\alpha\alpha_s^2, \alpha^2),$$

$$\delta m_{\eta_c} = \frac{4}{9}\delta^{\rm self}m_{\eta_c} - \frac{4}{9}\delta^{\rm int}m_{\eta_c} + \mathcal{O}(\alpha\alpha_s^2, \alpha^2)$$

$$\delta m_{\eta_b} = \frac{1}{9}\delta^{\rm self}m_{\eta_b} - \frac{1}{9}\delta^{\rm int}m_{\eta_b} + \mathcal{O}(\alpha\alpha_s^2, \alpha^2)$$

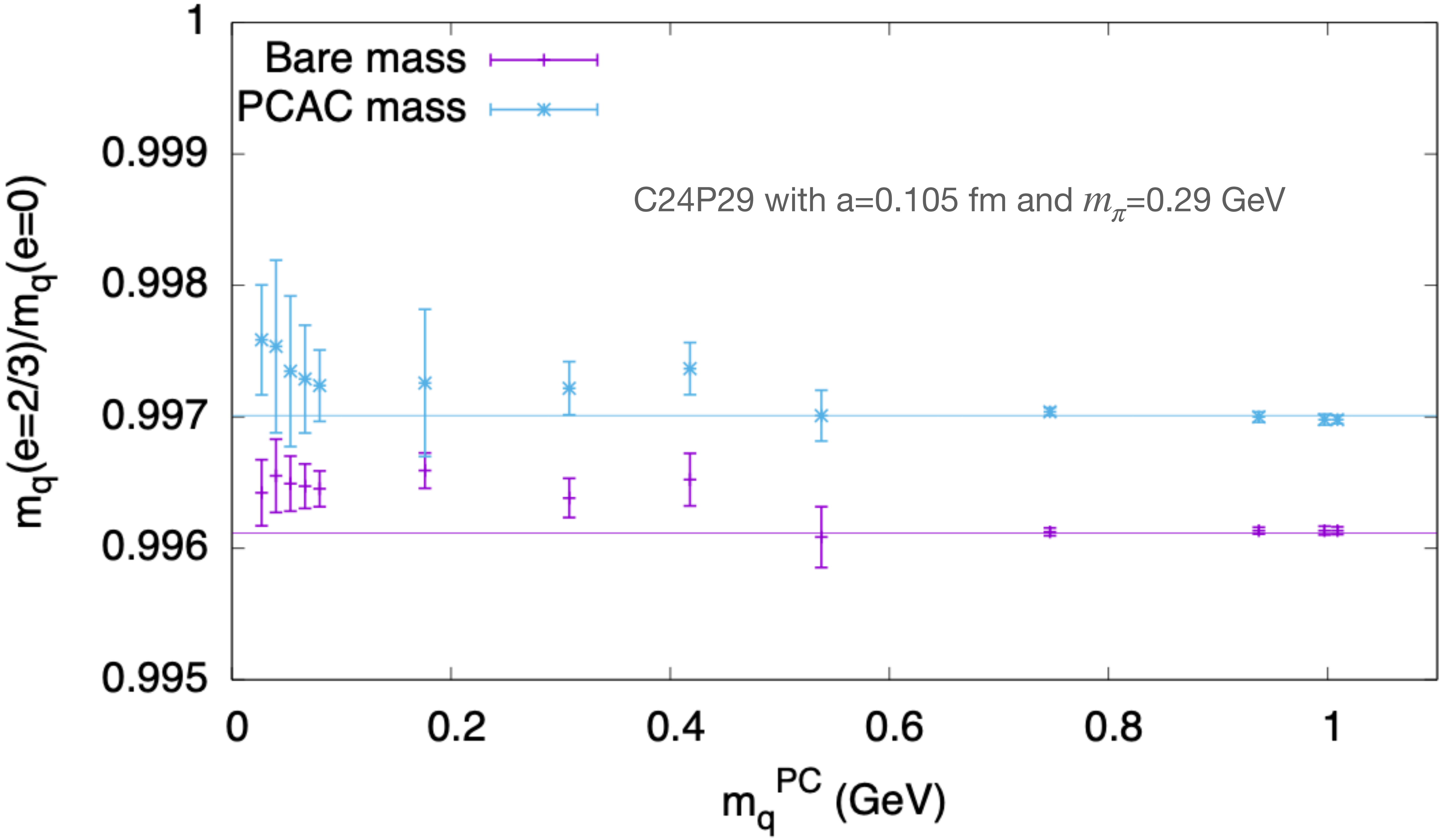
- $\delta^{\rm self}m_{\eta_q}$ requires from the QED UV renormalization and a matching condition.

- The NPU scheme defines $\delta^{\rm self}m_{\eta_q} = \delta^{\rm int}m_{\eta_q}$, and then

$$\delta m_{\pi^+} = \frac{1}{2}\delta^{\rm int}m_{\pi^+}. \\ +\mathcal{O}(\alpha\alpha_s^2, \alpha^2)$$

QED corrections

Quark mass renormalization



- For the u-type quarks:

1. PCAC quark mass is changed by 0.30(5)%,
2. Bare quark mass $m_q^b - m_q^{\rm crti}$ is changed by 0.39(5)%,

using the NPU scheme, with their difference coming from the additive chiral symmetry breaking of the clover fermion;

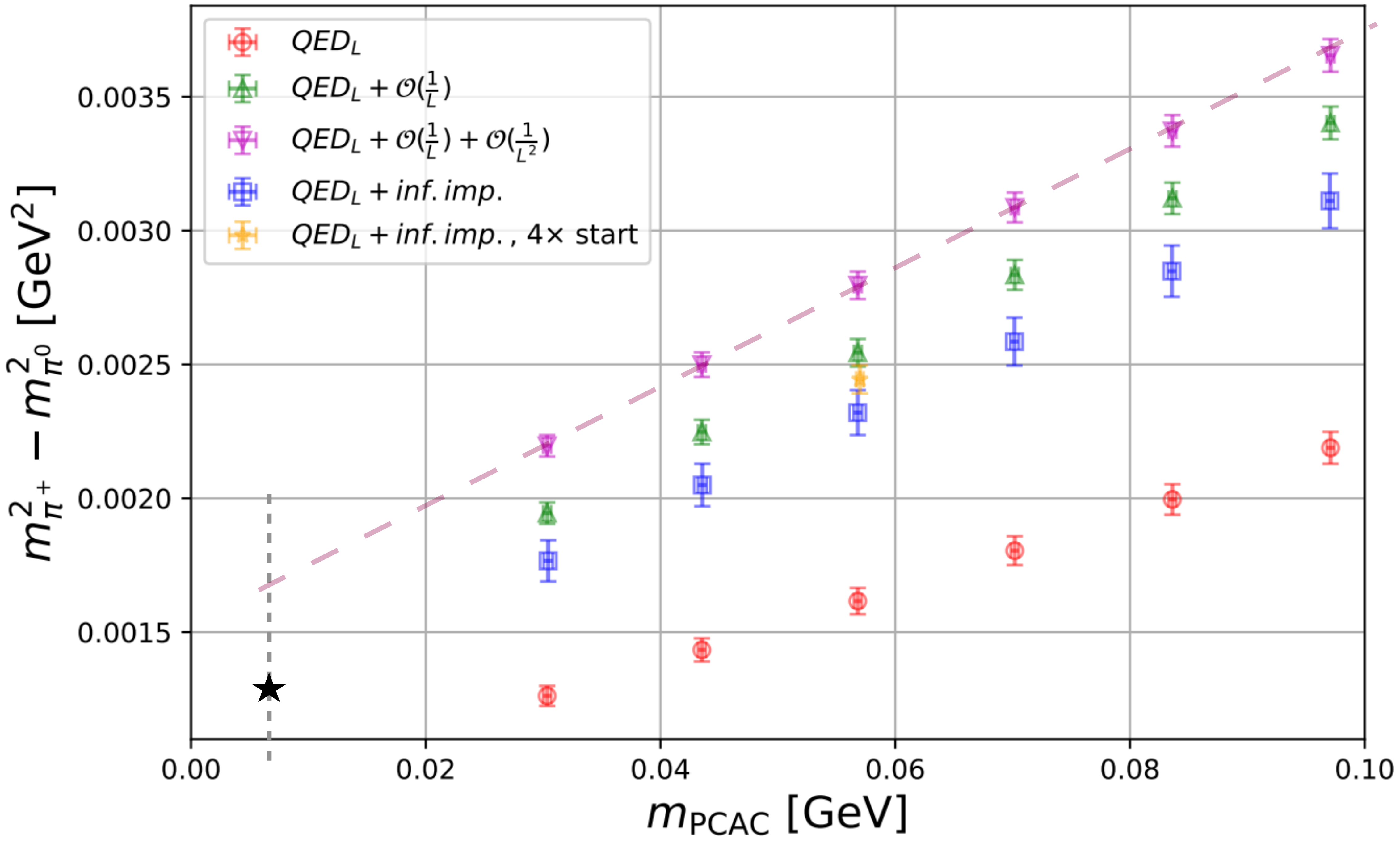
- The correction would be quark mass independent and more statistics is necessary to verify it.
- The perturbative calculation shows that the QED UV scale dependence is 0.12% from a=0.105 fm to a=0.052 fm.
- That of the d-type quarks will be suppressed by a factor of 4.

Y.Y. Liu et al. [CLQCD], in preparation

QED corrections

m_π^+ correction

$$\delta_{\mathrm{QED-FVE}} m_H^2 = e_H^2 m_H \underset{\substack{\uparrow \\ \mathcal{O}(\frac{1}{L})}}{\frac{2c_1}{L}} \Big(1 + \underset{\substack{\uparrow \\ \mathcal{O}(\frac{1}{L^2})}}{\frac{2}{m_H L}} + \mathcal{O}\Big(\frac{1}{(m_H L)^2}\Big)\Big)$$



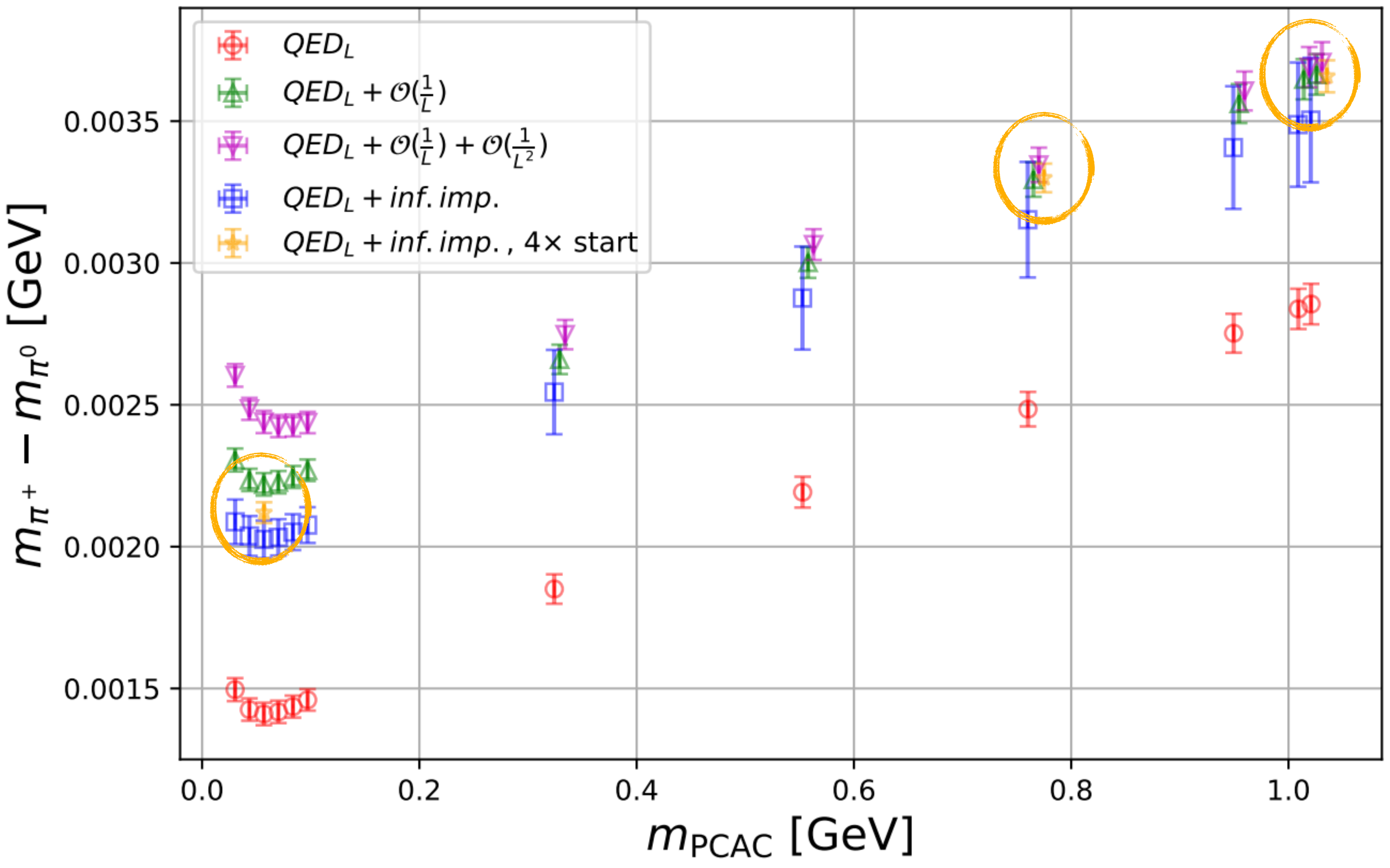
C24P29 with a=0.105 fm and m_π=0.29 GeV

- Ignoring the iso-spin breaking (ISB) correction, the mass difference between $m_{\pi^+}[\bar{q}(e_q)\gamma_5 q(e_q)]$ and $m_{\pi^0}[\bar{q}(-e_q)\gamma_5 q(e_q)]$ is a pure QED correction;
- With the NNLO QED-FVC, we have $m_{\pi^+}^2 - m_{\pi^0}^2 = 1.5 \times 10^3\ \mathrm{MeV}^2$ after the chiral extrapolation of the valence quark mass, at a=0.105 fm and m_π^{sea}=0.29 GeV, which is not far away from the **physical value.**
- The inferred improved QED_L result can include the $\mathcal{O}(1/L)$ FVC automatically and closes to the NLO QED_L result.

Y.Y. Liu et al. [CLQCD], in preparation

QED corrections

Inferred improved QED_L



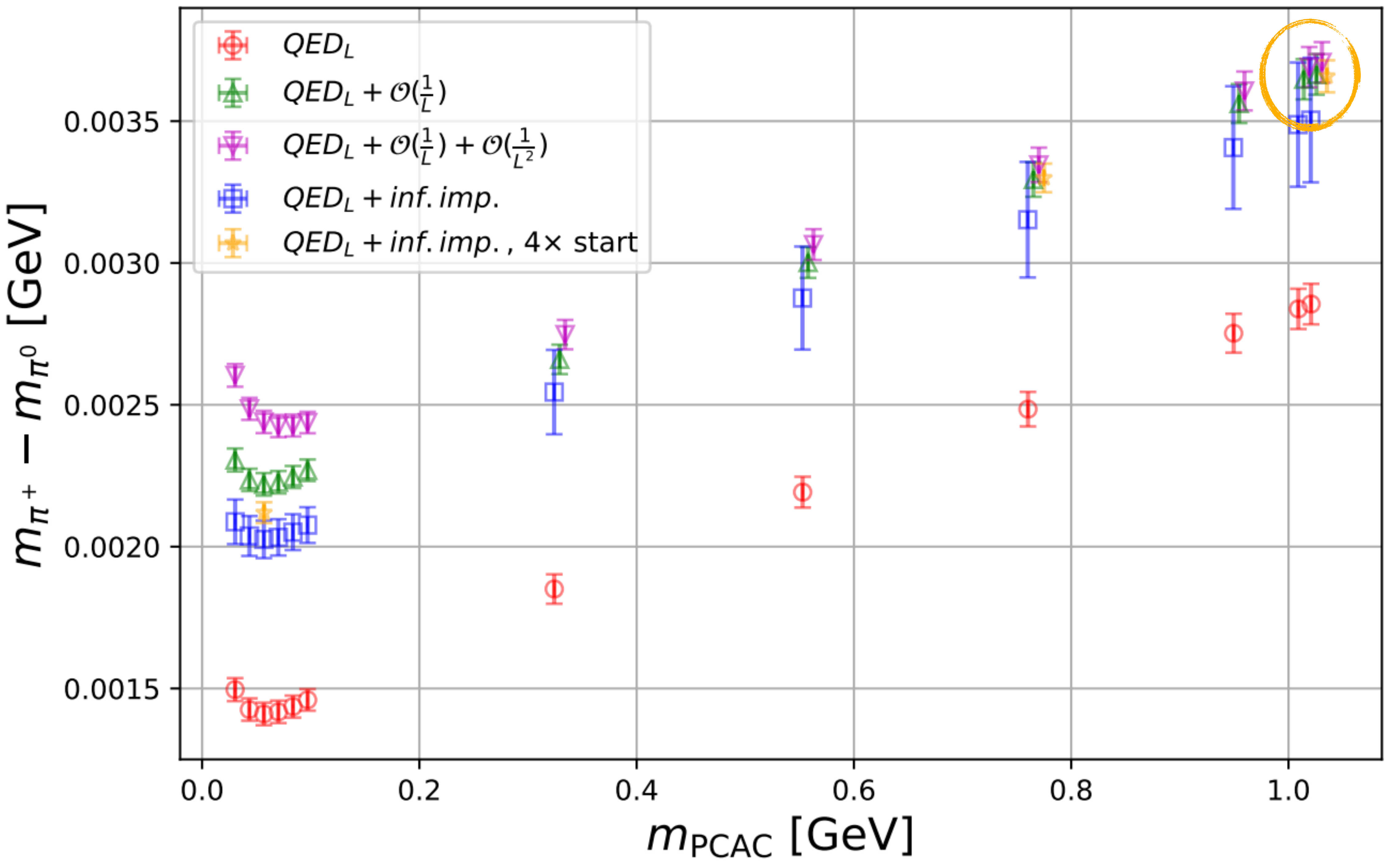
Y.Y. Liu et al. [CLQCD], in preparation

C24P29 with a=0.105 fm and m_π=0.29 GeV

- With heavier quark mass, the NNLO QED-FVC becomes negligible and then only the NLO QED-FVE matters;
- The inferred improved QED_L result becomes closer to the NLO QED-FVE result with heavier quark mass, while the statistical uncertainty is larger;
- The agreement becomes even better after the statistics of the inferred improved QED_L result is improved.

QED corrections

Impact on the charm physics



Y.Y. Liu et al. [CLQCD], in preparation

C24P29 with a=0.105 fm and m_π=0.29 GeV

- For the charm quark:

1. QED interaction correction $-3.7 * 2 \times (2/3)^2 \simeq -3.2(1)$ MeV of m_{η_c} is also similar with -3.0(1) MeV obtained in 2009.07667.
2. QED self energy correction will be $3.7 \times (2/3)^2 \simeq 1.6(1)$ MeV which is 0.15% of the charm quark mass;
3. Combining the QED interaction correction $-4.7(5) e_c e_l$ MeV of m_{D^0} obtained in 2009.07667 and ignoring the light quark self energy correction, we have $\delta_{\mathrm{QED}} m_{D^0} = -0.5(2)$ MeV and $\delta_{\mathrm{QED}} m_{D^{+(s)}} = 2.6(2)$ MeV which also agree with those from literature.

QCD+QED prediction

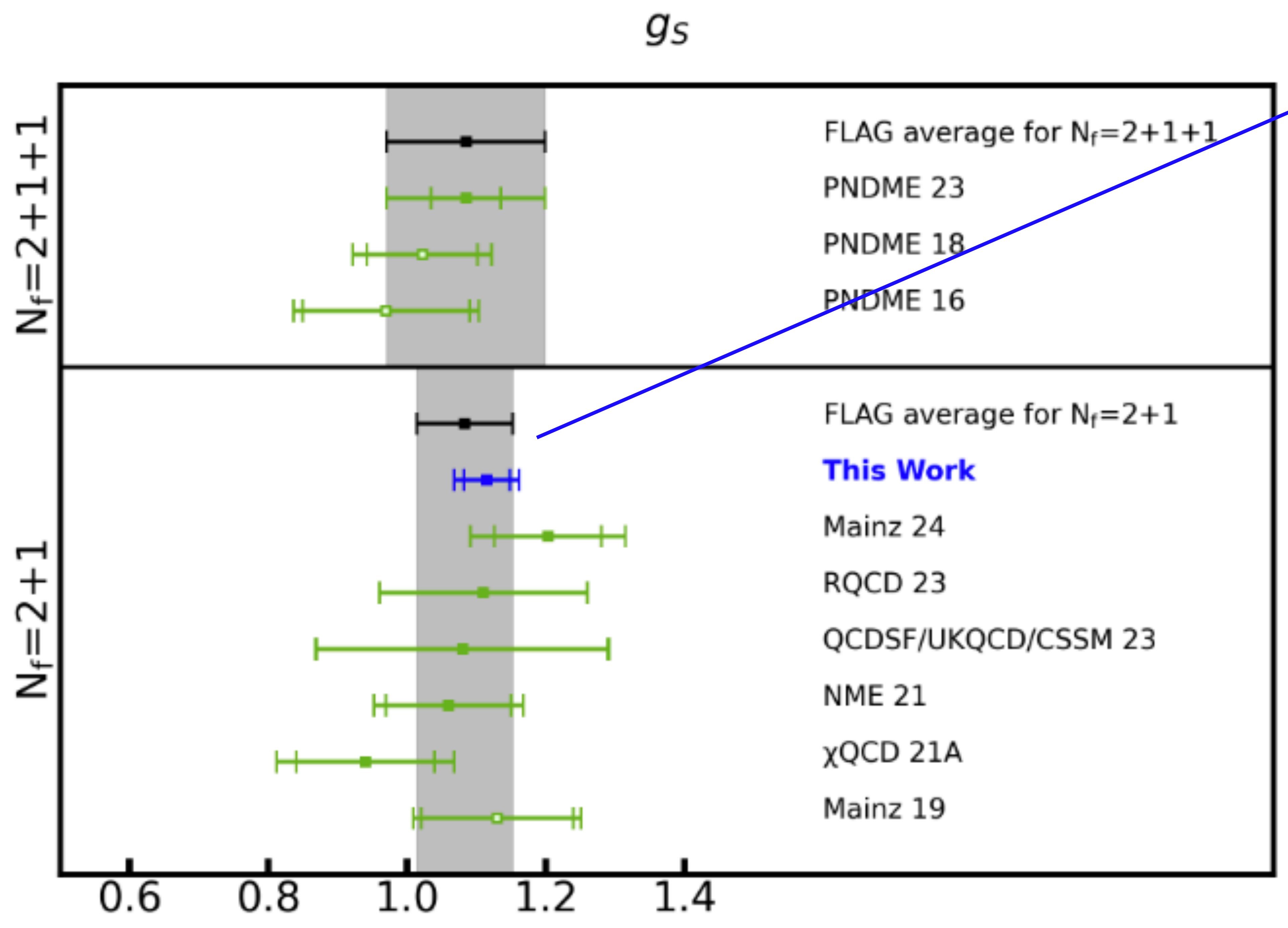
Current status

- Proton-neutron mass difference:

$$m_n - m_p = m_u(\frac{\partial m_n}{\partial m_u} - \frac{\partial m_p}{\partial m_u}) + m_d(\frac{\partial m_n}{\partial m_d} - \frac{\partial m_p}{\partial m_d}) + \delta^{\text{QED}} m_p^{\text{isoQCD}} = (m_d - m_u) g_S^{u-d} + \delta^{\text{QED}} m_p^{\text{isoQCD}}$$

$$= (2.35(12)\ \text{MeV})_{m_d-m_u} * 1.12(5)_{g_s} - 0.85(2)(?)\ \text{MeV}_{\text{QED}}$$

$$= 1.78(17)(?)\ \text{MeV}\,.$$



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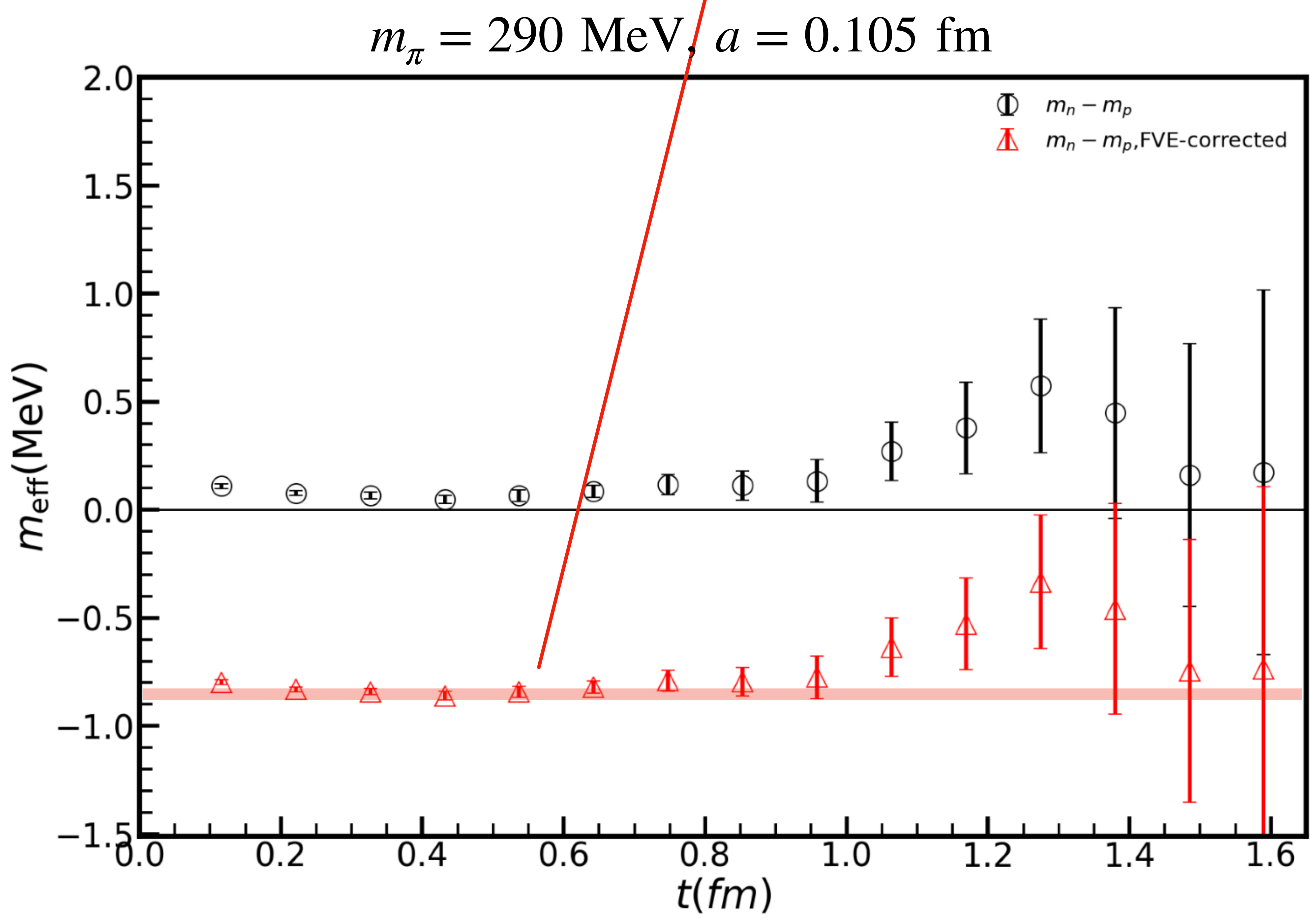
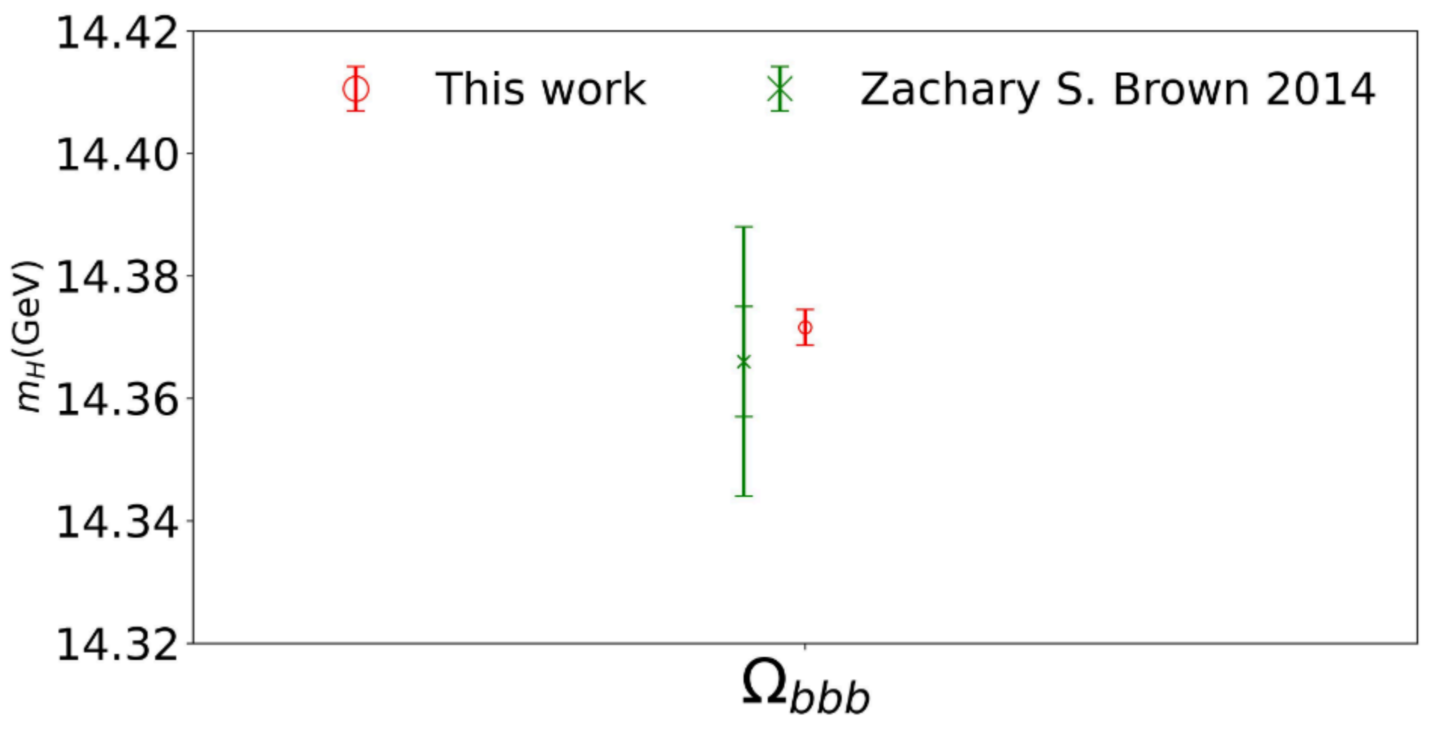
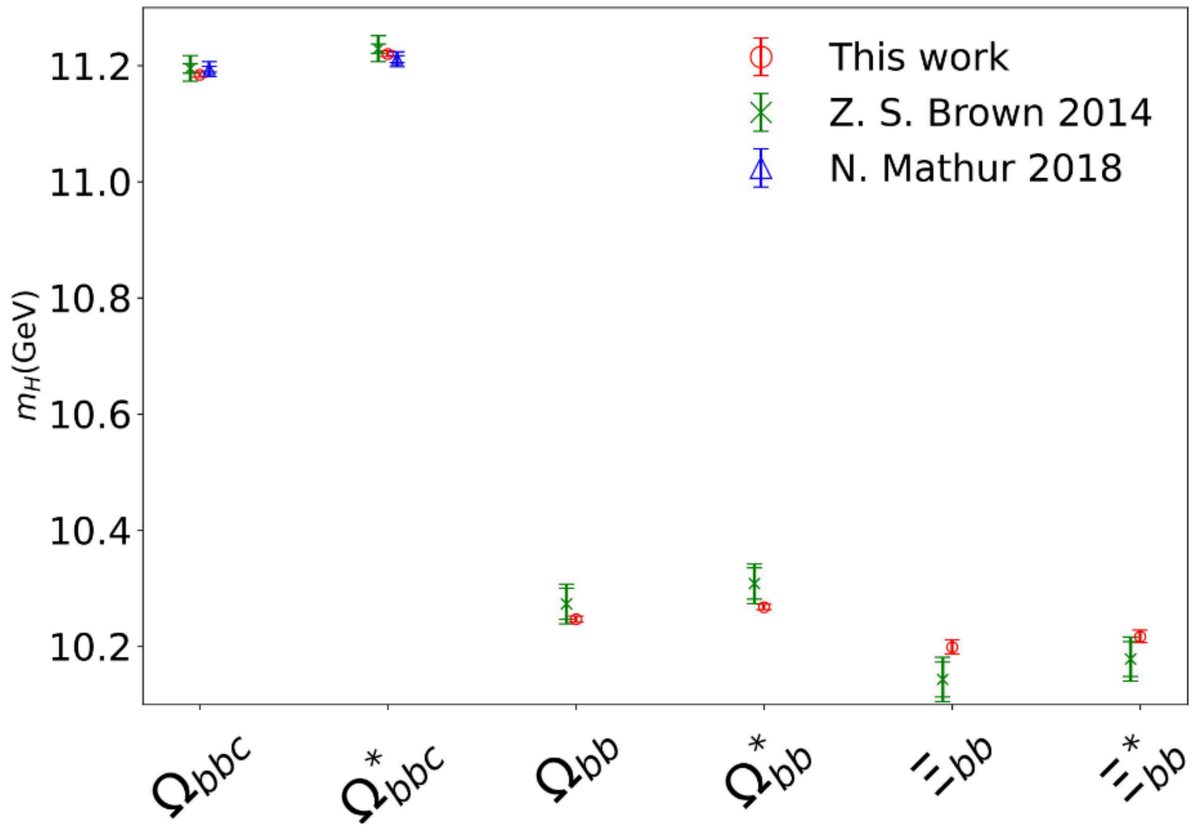


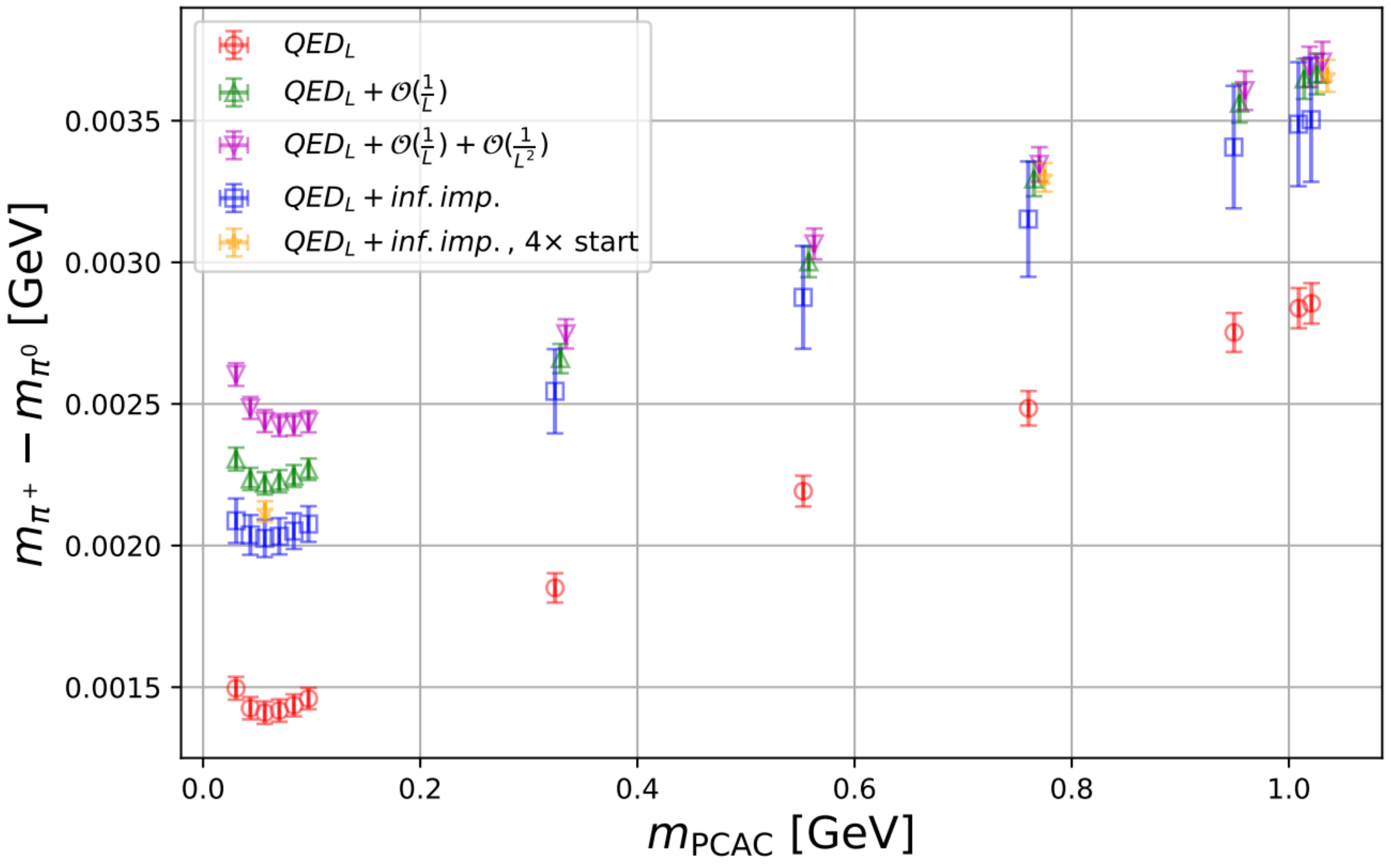
Z.C. Hu et al. [CLQCD], in preparation

Summary

- CLQCD ensembles can now provide high precision hadron matrix elements with heavy quark;
- And also control all the systematic uncertainties of the iso-symmetric QCD.







- The QCD+QED simulation on the CLQCD ensemble C24P29 (a=0.105 fm and m_π=0.29 GeV) agrees with that in the literature reasonably;
- The QED corrections of the charm and bottom quark masses would be around 3-4 MeV.
- Full QCD+QED prediction of baryon masses from CLQCD are in progress.