FCCC Semileptonic Ξ_b decays in PQCD

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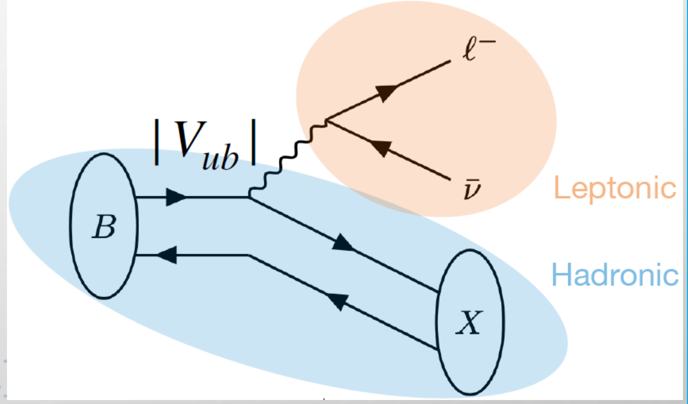
Contents

- Current status of FCCC transitions in B meson semileptonic decays
  - ✓ $|V_{ub}|$ & $|V_{cb}|$ puzzles;
  - ✓ $R(D)$— $R(D^*)$ anomalies.

- What can we learn from the FCCC semileptonic b-baryon (Ξ_b) decays ?

Why FCCC Semileptonic b-hadron decays?

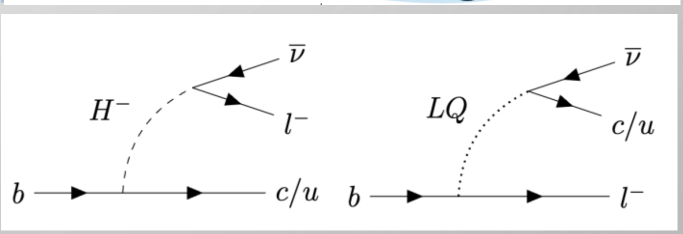
- Initiated by a b quark which is heavy (various final states accessed) and long lived (rel. easy to detect).
- Large samples of b-hadron species ($B, B_s, B_c, \Lambda_b, \Xi_b, \Omega_b$ ...).
- $b \to c,u$ tree-level transitions in SM (large BR, $|V_{cb}|$ & $|V_{ub}|$).
- Abundant observables : BR, angular and ratio observables, ⋯ (both differential and integrated values, rich phenomenology).



- Theoretically calculable: leptonic and hadronic currents factorize
- Semitauonic b decays, a path towards NP. e.g. Charged Higgs or Leptoquarks.

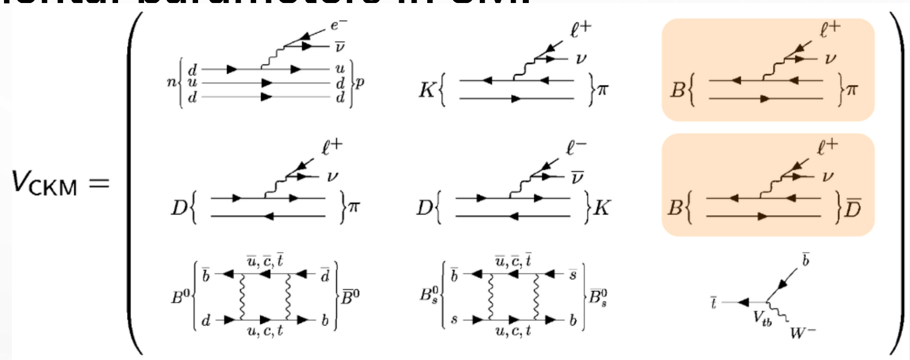
[PRL 116, 081801, PRD 94, 115021, 2304.01694...]

- Good probes for testing SM.
  - ✓ CKM matrix elements measurements
  - ✓ Lepton Flavour Universality (LFU) tests



$|V_{ub}|$ & $|V_{cb}|$ puzzles

- CKM matrix elements are fundamental parameters in SM.



- $|V_{ub}|$ and $|V_{cb}|$ are important to constrain CKM unitarity triangle.

- All FCCC processes depend on the CKM parameters.

- Measured FCCC processes is more than 4 independent parameters.

⇒ Over-constraining the CKM parameters.

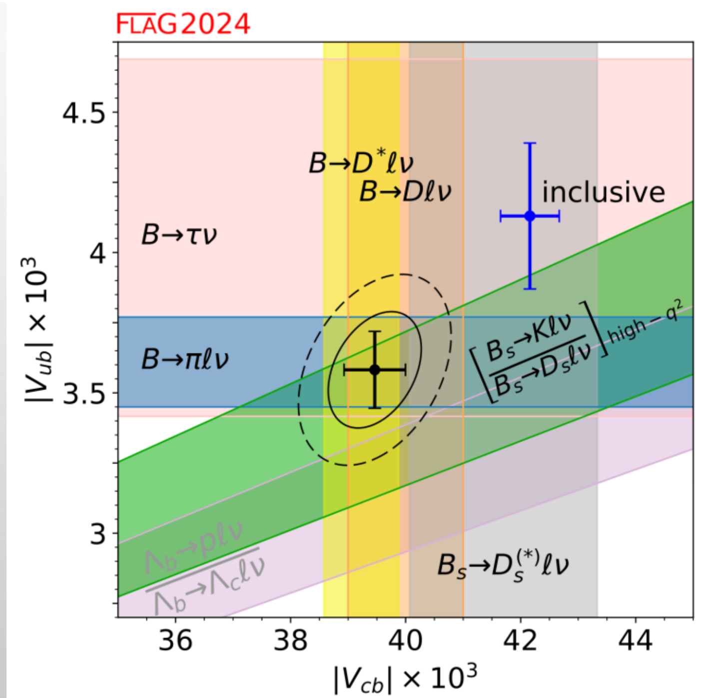
- Long standing tension for $|V_{ub}|$ and $|V_{cb}|$ (inclusive versus exclusive puzzle) 10^{-3}

[FLAG Review 2024 [arXiv:2411.04268]]

$$(|V_{cb}|, |V_{ub}|)_{\rm incl} = (42.16 \pm 0.51, 4.13 \pm 0.26)$$

Tension: $\sim 3\sigma$

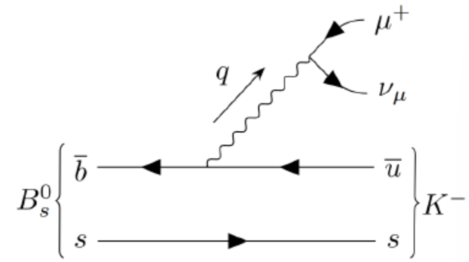
$$(|V_{cb}|, |V_{ub}|)_{\rm excl} = (39.46 \pm 0.53, 3.60 \pm 0.14)$$



Measurement of $|V_{ub}/V_{cb}|$ [PhysRevLett.126.081804]

- **Analysis strategy**

$$\frac{|V_{ub}|^2}{|V_{cb}|^2} \times \underbrace{\boxed{\frac{\mathrm{FF}_{B_s^0\to K^-}}{\mathrm{FF}_{B_s^0\to D_s^-}}}}_{\text{Theory Input}} = \underbrace{\frac{\mathcal{B}(B_s^0 \to K^-\mu^+\nu_\mu)}{\mathcal{B}(B_s^0 \to D_s^-\mu^+\nu_\mu)}}_{\text{Experiment}}$$



- **Extraction of $|V_{ub}|^2/|V_{cb}|^2$ in two bins of q^2 (LHCb,2021):**

$$q^2 < 7\mathrm{GeV}^2: \frac{\mathcal{B}(B_s^0 \to K^-\mu^+\nu_\mu)}{\mathcal{B}(B_s^0 \to D_s^-\mu^+\nu_\mu)} = 1.66 \pm 0.08(\mathrm{stat}) \pm 0.07(\mathrm{syst}) \pm 0.05(D_s) \times 10^{-3}$$

$$q^2 > 7\mathrm{GeV}^2: \frac{\mathcal{B}(B_s^0 \to K^-\mu^+\nu_\mu)}{\mathcal{B}(B_s^0 \to D_s^-\mu^+\nu_\mu)} = 3.25 \pm 0.21(\mathrm{stat})^{+0.16}_{-0.17}(\mathrm{syst}) \pm 0.09(D_s) \times 10^{-3}$$

- **Theory input on signal form-factors (complementary approaches)**

  $q^2 < 7\ GeV^2$ from LCSR [JHEP 2017, 112 (2017)]

  $q^2 > 7\ GeV^2$ from Lattice QCD [Phys. Rev. D 100, 034051 (2019)]

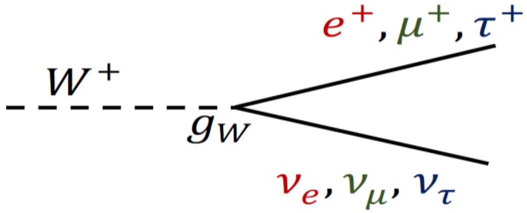
$|V_{ub}/V_{cb}|$(low) =0.0607$\pm 0.0015(stat) \pm 0.0013(syst) \pm 0.0008(D_s) \pm 0.0030(FF)$

$|V_{ub}/V_{cb}|$(high)=0.0946 $\pm 0.0030(stat)^{+0.0024}_{-0.0025}(syst) \pm 0.0013(D_s) \pm 0.0068(FF)$

- **Tension is driven by differences in FF calculations.**

Lepton Flavour Universality Tests

- SM couplings of charged leptons to gauge bosons are identical: $g_e = g_\mu = g_\tau$.



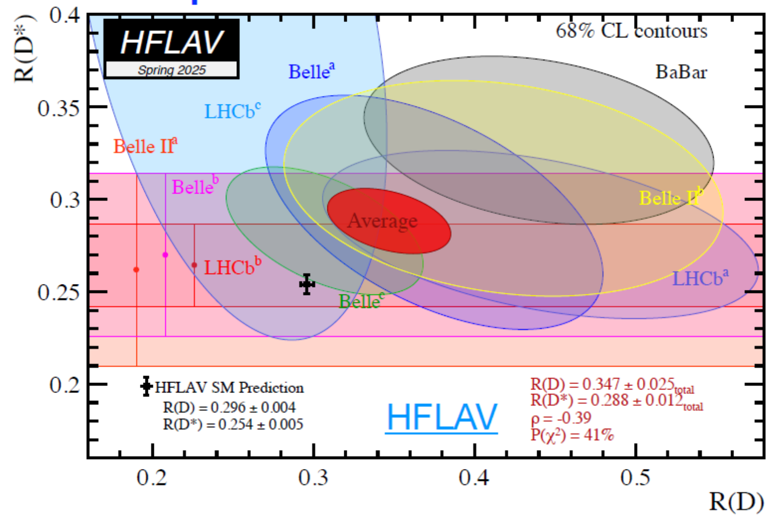
- **LFU Test in $b \to c$ transition.**

$$R(H_c) = \frac{\mathcal{B}(H_b \to H_c \tau^+ \nu_\tau)}{\mathcal{B}(H_b \to H_c \mu^+ \nu_\mu)},\ H_b = B^0, B^+_{(c)}, \Lambda_b^0, B_s^0, \ldots, H_c = D^{(*)\pm}, D^0, D_s, \Lambda_c^+, J/\psi, \ldots$$

  - ✓ Large BR (few %)
  - ✓ Theoretically clean: hadronic uncertainties and V_{cb} cancel out (% level).

- The average of the combined $R(D) - R(D^*)$ measurements is 3.8σ from the SM prediction.
- New physics at tree-level?
- Similar deviations may appear in suppressed $b \to u$ transition.
- More measurements are needed!
- LHCb has unique access to b-baryons including $\Lambda_b, \Xi_b, \Omega_b$.

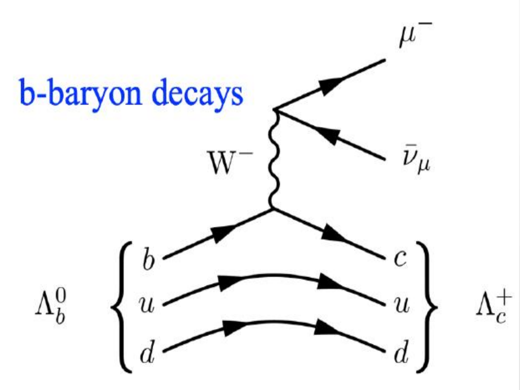
LFU Experimental ~3σ tension with SM



Cross-checks with b-baryon decays

- $|V_{ub}/V_{cb}|$ in $\Lambda_b \to p\mu\nu_\mu$ and $\Lambda_b \to \Lambda_c\mu\nu_\mu$ decays.[Nature Physics 11 (2015)]

  $$\left|\frac{V_{ub}}{V_{cb}}\right| = \mathbf{0.083 \pm 0.004(expt) \pm 0.004(lattice)}$$



- First measurement of $|V_{ub}|$ using baryonic decay channels.

  $$|V_{ub}| = \mathbf{(3.27 \pm 0.15 \pm 0.16 \pm 0.06) \times 10^{-3}}$$

- Agreement with the exclusively measured world average, but disagrees with the inclusive measurement at 3.5σ.

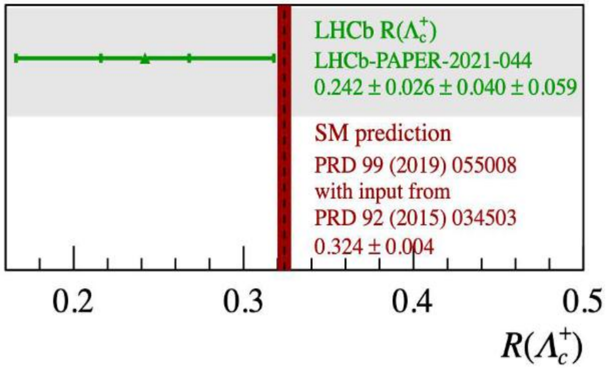
- Combined analyses have been performed within LQCD, the RQM, and various NP scenarios.
  PRD 92, 034503 (2015), PRD 94, 073008 (2016), PRD 93, 054003 (2016)

- A discrepancy exceeding 3σ remains between the LQCD and RQM.

- First observation of the mode $\Lambda_b \to \Lambda_c \tau \nu_\tau$ with a 6σ significance and first LFU test in a baryonic $b \to c\ell\nu$ decay.
  [PRL128, 191803 (2022)]

$$R(\Lambda_c) = 0.242 \pm 0.026 \pm 0.040 \pm 0.059$$

- Agreement with SM within 1σ but points towards a downward shift from the SM expectation.
  [PhysRevD.99.055008]



- Approximate sum rule relating $R(D^{(*)})$ and $R(\Lambda_c)$: $\frac{R_{\Lambda_c}}{R_{\Lambda_c}^{SM}} \simeq 0.280\,\frac{R_D}{R_D^{SM}} + 0.720\,\frac{R_{D^*}}{R_{D^*}^{SM}}$
  [1811.09603, 1905.08253, 2410.21384, 2405.06062]

- Enhancement of $R(D^{(*)})$ implies: $R(\Lambda_c) > 0.324(4)$.

- Sum rule relation seems to be challenged.

- Continued experimental and theoretical efforts are motivated.

- $\Xi_b^0(bus)$ and $\Xi_b^-(bds)$ are two antitriplet partners of $\Lambda_b(bud)$, the corresponding FCCC processes are:
  $\Xi_b^0 \to \Sigma^+ l\nu_l$ VS $\Xi_b^0 \to \Xi_c^+ l\nu_l$ and $\Xi_b^- \to \Lambda l\nu_l$ VS $\Xi_b^- \to \Xi_c^0 l\nu_l$.

- A valuable supplement to the well studied B decays, and an alternative avenue for determining $|V_{ub}/V_{cb}|$, once the relevant hadronic form factors are established.

- Theoretical processes on $\Xi_b \to \Sigma, \Lambda, \Xi_c$ form factors:
  - ✓ QM, H.Y. Cheng (1997); RQM, Ivanov(1997), Faustov (2018); Bethe-Salpeter approach, Ivanov(1999); CQM, Cardarelli (1999), Albertus (2005); QCDSR, Azizi (2012); LF approach, Zhen-Xing Zhao (2018);NP scenarios, Dutta (2018), J.Zhang (2019).

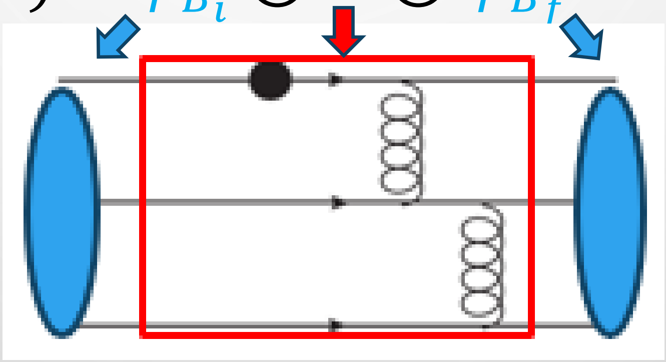
- We now focus on these decays and perform a first PQCD calculations of the baryonic transitions form factors.

- Vector and axial-vector transitions form factors: $\boldsymbol{q = p - p'}$

$$\langle \mathcal{B}_f | \bar{u}(1-\gamma_5)\gamma_\mu b | \mathcal{B}_i \rangle = \bar{u}_{\mathcal{B}_f}(p',\lambda') \left[\gamma_\mu f_1(q^2) - i\sigma_{\mu\nu} \frac{q^\nu}{M} f_2(q^2) + \frac{q_\mu}{M} f_3(q^2) \right] u_{\mathcal{B}_i}(p,\lambda)$$
$$+ \bar{u}_{\mathcal{B}_f}(p',\lambda') \left[\gamma_\mu g_1(q^2) - i\sigma_{\mu\nu} \frac{q^\nu}{M} g_2(q^2) + \frac{q_\mu}{M} g_3(q^2) \right] \gamma_5 u_{\mathcal{B}_i}(p,\lambda),$$

- PQCD factorization formula:

$$F(G) \propto \psi_{B_i} \otimes H \otimes \psi_{B_f}$$



14 diagrams

- H starts at α_s^2, LCDAs are the necessary inputs.

- **Helicity amplitudes (capture the impact of hadronic effects):**

$$H_{\lambda'\lambda_W} = H^V_{\lambda'\lambda_W} - H^A_{\lambda'\lambda_W}, \qquad Q_\pm = M \pm m$$ [JHEP 08 (2017) 131.]

$$H^V_{\frac{1}{2}t} = \sqrt{\frac{Q_+^2 - q^2}{q^2}}\left[Q_- f_1 + \frac{q^2}{M} f_3\right], H^A_{\frac{1}{2}t} = \sqrt{\frac{Q_-^2 - q^2}{q^2}}\left[Q_+ g_1 - \frac{q^2}{M} g_3\right],$$

$$H^V_{\frac{1}{2}0} = \sqrt{\frac{Q_-^2 - q^2}{q^2}}\left[Q_+ f_1 + \frac{q^2}{M} f_2\right], H^A_{\frac{1}{2}0} = \sqrt{\frac{Q_+^2 - q^2}{q^2}}\left[Q_- g_1 - \frac{q^2}{M} g_2\right],$$

$$H^V_{\frac{1}{2}1} = \sqrt{2(Q_-^2 - q^2)}\left[-f_1 - \frac{Q_+}{M} f_2\right], H^A_{\frac{1}{2}1} = \sqrt{2(Q_+^2 - q^2)}\left[-g_1 + \frac{Q_-}{M} g_2\right]$$

- **Angular distribution:** [PRD 94, 073008 (2016)].

$$\frac{d^2\Gamma_\ell}{dq^2 d\cos\theta} = \frac{G_F^2 V^2 q^2 |P|}{192 M^2 \pi^3}(1 - 2\delta_\ell)^2 W(\theta), \qquad \delta_\ell = \frac{m_\ell^2}{2q^2}$$

θ : the polar angle of the charged lepton.

$$W(\theta) = \frac{3}{8}(1 + \cos^2\theta)\mathcal{H}_T - \frac{3}{4}\cos\theta\,\mathcal{H}_{TP} + \frac{3}{4}\sin^2\theta\mathcal{H}_L$$
$$+\delta_\ell\left\{\frac{3}{2}\mathcal{H}_S + \frac{3}{4}\sin^2\theta\mathcal{H}_T + \frac{3}{2}\cos^2\theta\mathcal{H}_L - 3\cos\theta\mathcal{H}_{SL}\right\}$$

- **Branching ratio:** $\mathcal{B}_\ell = \tau_{\Xi_b}\int \frac{d^2\Gamma_\ell}{dq^2 d\cos\theta} dq^2 d\cos\theta\,, \qquad \mathcal{R} = \frac{\mathcal{B}_\tau}{\mathcal{B}_e}$

Angular observables:

- **Leptonic forward-backward asymmetries :**

$$A_{FB}^{\ell}(q^2) = -\frac{3}{4H}(H_{TP} + 4\delta_\ell H_{SL}),$$

$$H = \int d\cos\theta\, W(\theta) = H_T + H_L + \frac{m_\ell^2}{2q^2}(H_T + H_L + 3H_S)$$

- **Convexity parameter:**

$$C_F^{\ell}(q^2) = \frac{3}{4}(1 - 2\delta_\ell)\frac{H_T - 2H_L}{H},$$

- **Lepton polarization asymmetries :**

$$H_z^{\ell}(q^2) = \frac{H_{TP} + H_{LP} + \delta_\ell(H_{TP} + H_{LP} + 3H_{SP})}{H},$$

$$H_x^{\ell}(q^2) = -\frac{3\pi}{4\sqrt{2}}\frac{H_{LT} - 2\delta_\ell H_{STP}}{H},$$

- **Hadron polarization asymmetries :**

$$P_z^{\ell}(q^2) = -\frac{H_T + H_L - \delta_\ell(H_T + H_L + 3H_S)}{H},$$

$$P_x^{\ell}(q^2) = -\frac{3\pi}{4\sqrt{2}}\sqrt{\delta_\ell}\,\frac{H_{TP} - 2H_{SL}}{H}.$$

- **The integrated observables are obtained by integrating over q^2 separately before taking the ratio.**

Numerical results

1. Baryonic Form factors

- PQCD calculations for the form factors are only reliable in the low q^2 region (take ten q^2 values from 0 to m_τ^2).

- Fit the form factors by z-series parametrization to extrapolate to the whole physical region.

$$F(q^2) = \frac{a_0 + a_1 z(q^2)}{1 - q^2/M_{\text{pole}}^2}$$

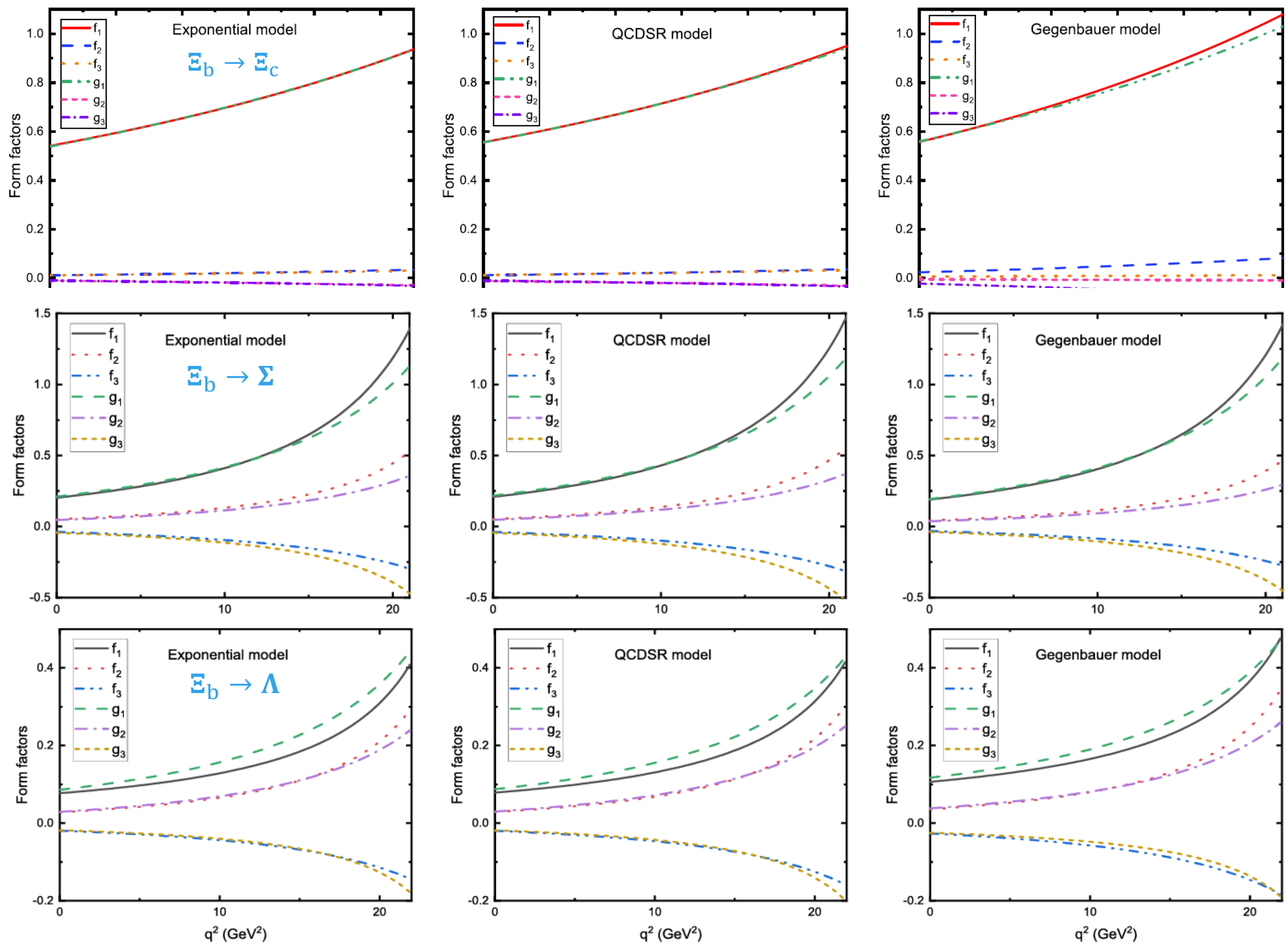
$$z(q^2) = \frac{\sqrt{Q_+^2 - q^2} - \sqrt{Q_+^2 - Q_-^2}}{\sqrt{Q_+^2 - q^2} + \sqrt{Q_+^2 - Q_-^2}}$$

- The fitted parameters for the $\Xi_b \to \Xi_c$ transition form factors with different models of baryonic LCDAs.

Model	Parameter	f_1	f_2	f_3	g_1	g_2	g_3
Exponential	a_0	0.611	0.018	0.017	0.624	−0.017	−0.017
	a_1	−3.159	−0.312	−0.266	−3.873	0.265	0.289
QCDSR	a_0	0.623	0.019	0.018	0.635	−0.018	−0.018
	a_1	−3.051	−0.345	−0.277	−3.641	0.276	0.316
Gegenbauer	a_0	0.671	0.042	0.007	0.671	−0.007	−0.040
	a_1	−5.152	−0.816	−0.074	−5.050	0.052	0.790

- Insensitive to the different models of baryonic LCDAs.

1. Baryonic Form factors



➢ **Following with the expectations in the heavy-quark limit:**

$$f_1(q^2) \approx g_1(q^2), \quad f_{2,3}(q^2) \approx g_{2,3}(q^2) \approx 0$$

2. Decay branching ratios and the LFU ratios:

$\Xi_b \to \Xi_c$	$\mathcal{B}_\tau$	$\mathcal{B}_e$	$\mathcal{R}_{\Xi_c}$
Exponential model	$(2.1^{+2.6+0.5+0.1}_{-1.0-0.2-0.2}) \times 10^{-2}$	$(7.3^{+9.3+1.7+0.2}_{-3.7-1.2-0.9}) \times 10^{-2}$	$0.295^{+0.000+0.000+0.000}_{-0.008-0.008-0.002}$
QCDSR model	$(2.2^{+0.4+0.2+0.2}_{-0.1-0.1-0.2}) \times 10^{-2}$	$(7.6^{+1.0+0.9+1.4}_{-0.4-0.5-0.8}) \times 10^{-2}$	$0.293^{+0.001+0.003+0.003}_{-0.000-0.000-0.000}$
Gegenbauer model	$(2.5^{+0.3+0.5+0.4}_{-0.0-0.0-0.3}) \times 10^{-2}$	$(8.4^{+0.9+1.1+0.9}_{-0.0-0.0-1.1}) \times 10^{-2}$	$0.304^{+0.008+0.011+0.004}_{-0.005-0.002-0.000}$
Other predictions	$(2.0 \sim 2.8) \times 10^{-2}$	$(6.1 \sim 9.2) \times 10^{-2}$	$0.25 \sim 0.34$

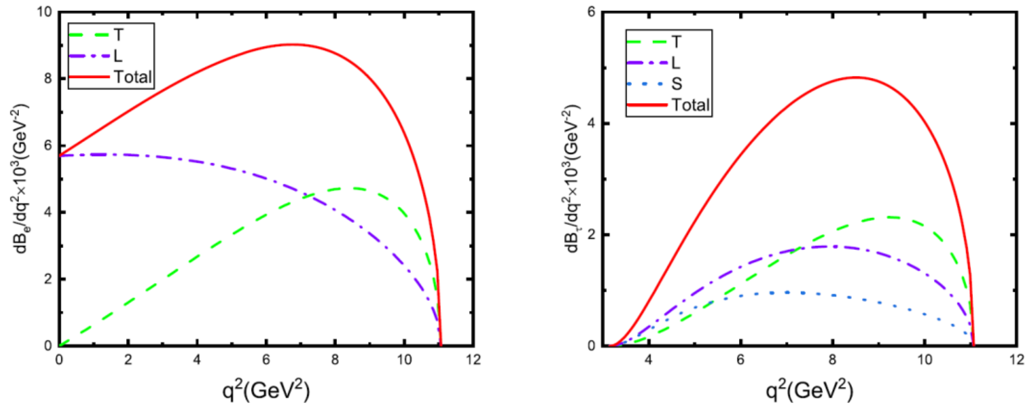
$\Xi_b \to \Sigma$	$\mathcal{B}_\tau$	$\mathcal{B}_e$	$\mathcal{R}_{\Sigma}$
Exponential model	$(4.3^{+4.3}_{-1.5}) \times 10^{-4}$	$(6.8^{+6.8}_{-2.4}) \times 10^{-4}$	$0.645^{+0.011}_{-0.002}$
QCDSR model	$(4.8^{+1.3}_{-0.8}) \times 10^{-4}$	$(7.4^{+1.9}_{-1.3}) \times 10^{-4}$	$0.648^{+0.005}_{-0.006}$
Gegenbauer model	$(4.6^{+0.9}_{-0.4}) \times 10^{-4}$	$(7.0^{+1.6}_{-0.8}) \times 10^{-4}$	$0.660^{+0.014}_{-0.008}$
LF	...	2.83×10^{-4}	...

$\Xi_b \to \Lambda$	$\mathcal{B}_\tau$	$\mathcal{B}_e$	$\mathcal{R}_{\Lambda}$
Exponential model	$(5.3^{+6.1}_{-2.8}) \times 10^{-5}$	$(8.5^{+9.6}_{-4.3}) \times 10^{-5}$	$0.617^{+0.002}_{-0.004}$
QCDSR model	$(5.1^{+2.6}_{-1.9}) \times 10^{-5}$	$(8.4^{+4.0}_{-3.0}) \times 10^{-5}$	$0.611^{+0.011}_{-0.009}$
Gegenbauer model	$(7.5^{+1.8}_{-2.3}) \times 10^{-5}$	$(12.8^{+2.8}_{-3.2}) \times 10^{-5}$	$0.588^{+0.003}_{-0.008}$
LF,RQM	18×10^{-5} (RQM)	$(5.42 \sim 26) \times 10^{-5}$	0.717 ± 0.021 (RQM)

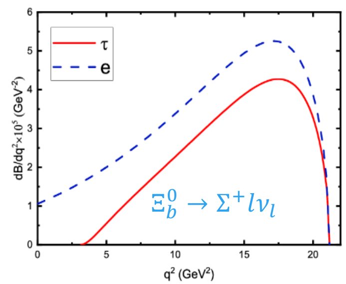
- Exponential model shows greater sensitivity to the model parameters.
- Results from different LCDA models are consistent within uncertainties.
- Large BRs, $b \to c$ (10^{-2}) ,$b \to u$ (10^{-4}), accessible to measurements.

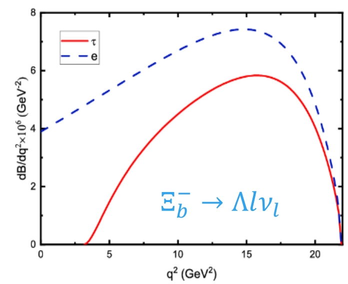
- ➢ Differential branching ratios.

$\Xi_b^0 \to \Xi_c^+ l \nu_l$



- L .VS. T.
- Scalar is suppressed by the lepton mass.
- The tau modes are kinematically suppressed.





- ➢ LFU ratios for $b \to c$ transition with spin ½ and 1 approximately equal in PQCD:
  $\mathcal{R}_{\Xi_c} \approx \mathcal{R}_{\Lambda_c} \approx \mathcal{R}_{\mathrm{D}^*} \approx \mathcal{R}_{\mathrm{D}_s^*} \approx \mathcal{R}_{\mathrm{J}/\psi} \approx 0.3.$
  [1401.0571 ,2509.02257, 2509.12566]
- LFU ratios for $b \to$ u transition are larger： $\mathcal{R}_{\Sigma} \sim 0.65$, $\mathcal{R}_{\Lambda} \sim 0.60$.
  $\mathcal{R}_{\pi} = 0.641 \pm 0.016$, $\mathcal{R}_{p} = 0.688 \pm 0.064$ [RMP94, 015003 (2022)]
- Substantial contribution of the spin-flip component in the $b \to$ u transition.

3. Combined analysis of b $\rightarrow u\ell\nu$ and b $\rightarrow c\ell\nu$ transitions

- The ratio of decay rates:

$$\frac{\Gamma(b\to u\ell\nu_\ell)}{\Gamma(b\to c\ell\nu_\ell)}=\frac{|V_{ub}|^2}{|V_{cb}|^2}\mathcal{R}_\ell\,,\qquad \mathcal{R}_\ell=\frac{\int d\,q^2q^2|P|(1-\delta_\ell)^2H(b\to u\ell\nu_\ell)}{\int d\,q^2q^2|P|(1-\delta_\ell)^2H(b\to c\ell\nu_\ell)}$$

- The integration interval depends on the selected $\boldsymbol{q^2}$ regions in forthcoming measurements.
- Integrated over the whole kinematical region: $q^2\in\left[m_\ell^2,(M-m)^2\right]$:

$\mathcal{R}_\ell$	Exponential	QCDSR	Gegenbauer
$\mathcal{R}_\tau(\Lambda/\Xi_c)$	$0.23^{+0.10}_{-0.08}$	$0.22^{+0.09}_{-0.07}$	$0.28^{+0.05}_{-0.05}$
$\mathcal{R}_e(\Lambda/\Xi_c)$	$0.113^{+0.045}_{-0.034}$	$0.106^{+0.042}_{-0.032}$	$0.147^{+0.025}_{-0.031}$
$\mathcal{R}_\tau(\Sigma/\Xi_c)$	2.1 ± 0.2	2.2 ± 0.2	$1.8^{+0.1}_{-0.2}$
$\mathcal{R}_e(\Sigma/\Xi_c)$	0.94 ± 0.10	0.99 ± 0.07	$0.85^{+0.06}_{-0.08}$

- A total uncertainty for Σ mode is around 10%, more suitable for extracting $|\boldsymbol{V_{ub}/V_{cb}}|$.
- Using the latest average of $|\boldsymbol{V_{ub}/V_{cb}}|$ = 0.089 $\pm$ 0.005 from PDG

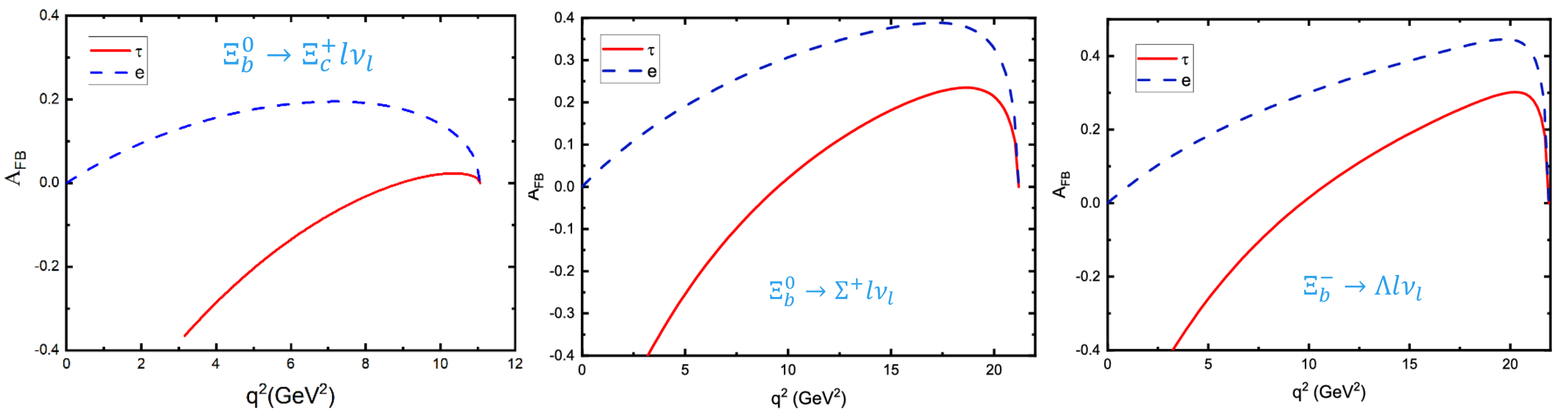
$$\frac{\Gamma(\Xi_b^-\to\Lambda\tau\nu_\tau)}{\Gamma(\Xi_b^-\to\Xi_c^0\tau\nu_\tau)}=(2.2\pm0.4\pm0.2)\times10^{-3},\qquad \frac{\Gamma(\Xi_b^-\to\Lambda e\nu_e)}{\Gamma(\Xi_b^-\to\Xi_c^0 e\nu_e)}=(1.2\pm0.2\pm0.1)\times10^{-3},$$

$$\frac{\Gamma(\Xi_b^0\to\Sigma^+\tau\nu_\tau)}{\Gamma(\Xi_b^0\to\Xi_c^+\tau\nu_\tau)}=(14\pm0.1\pm0.2)\times10^{-2},\qquad \frac{\Gamma(\Xi_b^0\to\Sigma^+ e\nu_e)}{\Gamma(\Xi_b^0\to\Xi_c^+ e\nu_e)}=(6.7^{+0.5}_{-0.6}\pm0.8)\times10^{-3}.$$

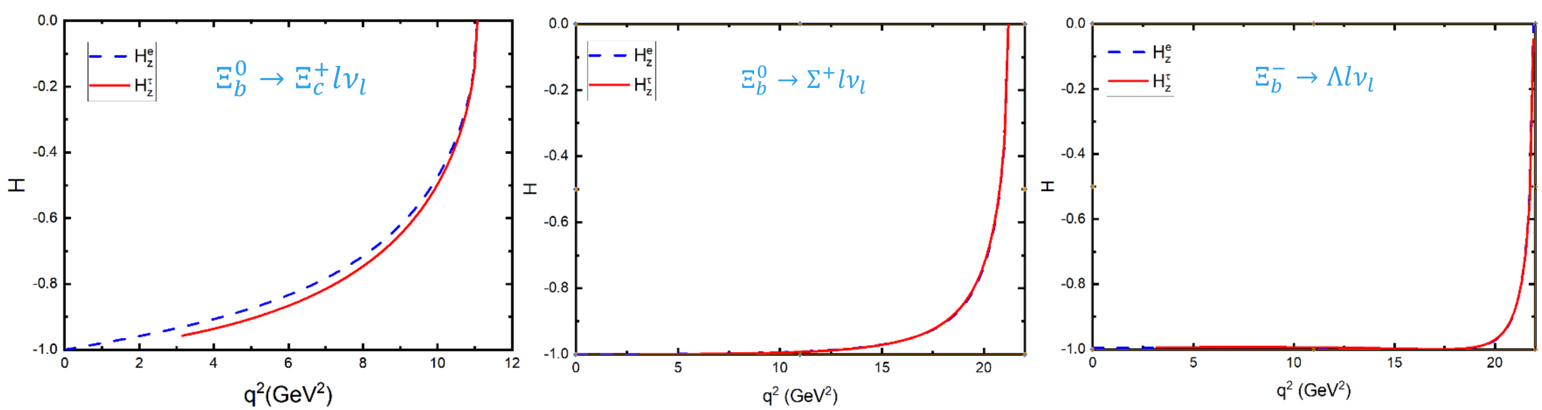
4. Differential angular observables

- Not sensitive to the models of baryonic LCDAs (Gegenbauer model)

- Leptonic forward-backward asymmetry, large lepton mass effect.



- Longitudinal hadronic polarization, not sensitive to lepton flavor.



5. Integrated angular observables

Observable	$\Xi_b^0 \to \Xi_c^+ l \nu_l$	$\Xi_b^0 \to \Sigma^+ l \nu_l$	$\Xi_b^- \to \Lambda l \nu_l$
$\langle A_{\mathrm{FB}}^{\tau} \rangle$	$-0.135^{+0.003}_{-0.001}$	$0.063^{+0.005}_{-0.003}$	$0.025^{+0.004}_{-0.003}$
$\langle A_{\mathrm{FB}}^{e} \rangle$	$0.148^{+0.003}_{-0.003}$	$0.313^{+0.008}_{-0.004}$	$0.302^{+0.001}_{-0.006}$
$\langle C_{\mathrm{F}}^{\tau} \rangle$	$-0.099^{+0.002}_{-0.002}$	$-0.187^{+0.004}_{-0.006}$	$-0.200^{+0.005}_{-0.003}$
$\langle C_{\mathrm{F}}^{\mathrm{e}} \rangle$	$-0.618^{+0.020}_{-0.002}$	$-0.466^{+0.011}_{-0.013}$	$-0.578^{+0.007}_{-0.009}$
$\langle H_z^{\tau} \rangle$	$-0.762^{+0.018}_{-0.004}$	$-0.947^{+0.008}_{-0.015}$	$-0.989^{+0.007}_{-0.005}$
$\langle H_z^{e} \rangle$	$-0.764^{+0.022}_{-0.004}$	$-0.943^{+0.009}_{-0.018}$	$-0.990^{+0.014}_{-0.004}$
$\langle H_x^{\tau} \rangle$	$-0.159^{+0.007}_{-0.021}$	$-0.115^{+0.031}_{-0.025}$	$-0.015^{+0.014}_{-0.052}$
$\langle H_x^{e} \rangle$	$-0.463^{+0.005}_{-0.031}$	$-0.195^{+0.057}_{-0.028}$	$-0.004^{+0.020}_{-0.052}$
$\langle P_z^{\tau} \rangle$	$-0.177^{+0.003}_{-0.019}$	$-0.438^{+0.018}_{-0.011}$	$-0.377^{+0.009}_{-0.011}$
$\langle P_z^{\mathrm{e}} \rangle$	-1	-1	-1
$\langle P_x^{\tau} \rangle$	$0.599^{+0.003}_{-0.014}$	$0.652^{+0.015}_{-0.007}$	$0.695^{+0.007}_{-0.010}$
$\langle P_x^{\mathrm{e}} \rangle$	0	0	0

- Theoretical uncertainties are largely reduced.
- Sizable lepton mass effect in the lepton-side observables (spin flip contributions).
- Electrons absolutely polarized opposite to its momentum direction (negative helicity).
- Comparable with RQM's results for Ξ_c and Λ modes.
- First predictions for the Σ mode, pending for comparison.

SUMMARY

- Many tensions in $b \to c,u$ transitions, but current attention focuses primarily on the B and Λ_b decays.

- We have investigated FCCC semileptonic Ξ_b decays in PQCD. Branching ratios and angular observables and LFU ratios are obtained and compared with other predictions.

- Combined analysis of $\Xi_b^0 \to \Sigma^+ l\nu_l$ and $\Xi_b^0 \to \Xi_c^+ l\nu_l$ enabes an independent determination of $|V_{ub}/V_{cb}|$.

- The obtained LFU ratios are consistent with their counterparts in B decays.

- FCNC semileptonic $b \to s,d$ decays are in progress.

Thanks for your attention!

Backup Slides

$$f_1 = F_1 + \frac{1}{2}(F_2+F_3)(1+r), \quad f_2 = -\frac{1}{2}(F_2+F_3), \quad f_3 = \frac{1}{2}(F_2-F_3),$$
$$g_1 = G_1 - \frac{1}{2}(G_2+G_3)(1-r), \quad g_2 = -\frac{1}{2}(G_2+G_3), \quad g_3 = \frac{1}{2}(G_2-G_3).$$

z-series parametrization

$$z(q^2) = \frac{\sqrt{Q_+^2-q^2}-\sqrt{Q_+^2-Q_0^2}}{\sqrt{Q_+^2-q^2}+\sqrt{Q_+^2-Q_0^2}},$$

$$F(q^2) = \frac{a_0 + a_1 z(q^2)}{1-q^2/M_{\text{pole}}^2}$$

$$M_{\text{pole}} = \begin{cases} 6.333\ \text{GeV}, & \text{for } f_{1,2} \\ 6.743\ \text{GeV}, & \text{for } g_{1,2} \\ 6.699\ \text{GeV}, & \text{for } f_3 \\ 6.275\ \text{GeV}, & \text{for } g_3, \end{cases}$$

$$M_{\text{pole}} = \begin{cases} 5.325\ \text{GeV}, & \text{for } f_{1,2} \\ 5.723\ \text{GeV}, & \text{for } g_{1,2} \\ 5.749\ \text{GeV}, & \text{for } f_3 \\ 5.280\ \text{GeV}, & \text{for } g_3. \end{cases}$$

(i) The exponential model [64],

$$\phi_2^b(\omega,u) = \frac{\omega^2 u(1-u)}{\omega_0^4} e^{-\frac{\omega}{\omega_0}},$$
$$\phi_{3s}^b(\omega,u) = \frac{\omega}{2\omega_0^3} e^{-\frac{\omega}{\omega_0}},$$
$$\phi_{3a}^b(\omega,u) = \frac{\omega(2u-1)}{2\omega_0^3} e^{-\frac{\omega}{\omega_0}},$$
$$\phi_4^b(\omega,u) = \frac{1}{\omega_0^2} e^{-\frac{\omega}{\omega_0}},$$

(ii) The QCD sum rule (QCDSR) model [59],

$$\phi_2^b(\omega,u) = \frac{15}{2\mathcal{N}}\omega^2 u(1-u)\int_{\frac{\omega}{2}}^{s_0} ds e^{-s/\tau}\left(s-\frac{\omega}{2}\right),$$
$$\phi_{3s}^b(\omega,u) = \frac{15}{4\mathcal{N}}\omega\int_{\frac{\omega}{2}}^{s_0} ds e^{-s/\tau}\left(s-\frac{\omega}{2}\right)^2,$$
$$\phi_{3a}^b(\omega,u) = \frac{15}{4\mathcal{N}}\omega(2u-1)\int_{\frac{\omega}{2}}^{s_0} ds e^{-s/\tau}\left(s-\frac{\omega}{2}\right)^2,$$
$$\phi_4^b(\omega,u) = \frac{5}{\mathcal{N}}\int_{\frac{\omega}{2}}^{s_0} ds e^{-s/\tau}\left(s-\frac{\omega}{2}\right)^3, \qquad (8)$$

(iii) The Gegenbauer model [61],

$$\phi_2^b(\omega,u) = \omega^2 u(1-u)\sum_{l=0}^{2}\frac{a_l}{\epsilon_l^4}\frac{c_l^{3/2}(2u-1)}{|c_l^{3/2}|^2} e^{-\frac{\omega}{\epsilon_l}},$$
$$\phi_{3s,3a}^b(\omega,u) = \frac{\omega}{2}\sum_{l=0}^{2}\frac{a_l}{\epsilon_l^3}\frac{c_l^{1/2}(2u-1)}{|c_l^{1/2}|^2} e^{-\frac{\omega}{\epsilon_l}},$$
$$\phi_4^b(\omega,u) = \sum_{l=0}^{2}\frac{a_l}{\epsilon_l^2}\frac{c_l^{1/2}(2u-1)}{|c_l^{1/2}|^2} e^{-\frac{\omega}{\epsilon_l}}, \qquad (9)$$

$$F_i/G_i = \frac{4 f_{\mathcal{B}_i} \pi^2 G_F}{27\sqrt{2}} \sum_{\xi} \int \mathcal{D}x \mathcal{D}b \alpha_s^2(t_\xi) e^{-S_{\mathcal{B}_i} - S_{\mathcal{B}_f}} \Omega_\xi(b_i, b_i') H_\xi^{F_i/G_i}(x_i, x_i')$$

Summation ξ extends over all possible diagrams.

various parity conserving and violating helicity structures

$$\begin{aligned}
H_T(q^2) &= |H_{\frac{1}{2}1}|^2 + |H_{-\frac{1}{2}-1}|^2, \\
H_L(q^2) &= |H_{\frac{1}{2}0}|^2 + |H_{-\frac{1}{2}0}|^2, \\
H_S(q^2) &= |H_{\frac{1}{2}t}|^2 + |H_{-\frac{1}{2}t}|^2, \\
H_{SL}(q^2) &= Re[H_{\frac{1}{2}0} H^\dagger_{\frac{1}{2}t} + H_{-\frac{1}{2}0} H^\dagger_{-\frac{1}{2}t}], \\
H_{ST}(q^2) &= Re[H_{\frac{1}{2}1} H^\dagger_{-\frac{1}{2}t} + H_{-\frac{1}{2}-1} H^\dagger_{\frac{1}{2}t}], \\
H_{LT}(q^2) &= Re[H_{\frac{1}{2}1} H^\dagger_{-\frac{1}{2}0} + H_{-\frac{1}{2}-1} H^\dagger_{\frac{1}{2}0}], \\
H_{TP}(q^2) &= |H_{\frac{1}{2}1}|^2 - |H_{-\frac{1}{2}-1}|^2, \\
H_{LP}(q^2) &= |H_{\frac{1}{2}0}|^2 - |H_{-\frac{1}{2}0}|^2, \\
H_{SP}(q^2) &= |H_{\frac{1}{2}t}|^2 - |H_{-\frac{1}{2}t}|^2, \\
H_{SLP}(q^2) &= Re[H_{\frac{1}{2}0} H^\dagger_{\frac{1}{2}t} - H_{-\frac{1}{2}0} H^\dagger_{-\frac{1}{2}t}], \\
H_{STP}(q^2) &= Re[H_{\frac{1}{2}1} H^\dagger_{-\frac{1}{2}t} - H_{-\frac{1}{2}-1} H^\dagger_{\frac{1}{2}t}], \\
H_{LTP}(q^2) &= Re[H_{\frac{1}{2}1} H^\dagger_{-\frac{1}{2}0} - H_{-\frac{1}{2}-1} H^\dagger_{\frac{1}{2}0}].
\end{aligned}$$

$$\begin{pmatrix} \mathcal{S}_1 \\ 2p' \cdot z\mathcal{S}_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 1 & -1 \end{pmatrix} \begin{pmatrix} S_1 \\ S_2 \end{pmatrix},$$

$$\begin{pmatrix} \mathcal{P}_1 \\ 2p' \cdot z\mathcal{P}_2 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ -1 & 1 \end{pmatrix} \begin{pmatrix} P_1 \\ P_2 \end{pmatrix},$$

$$\begin{pmatrix} \mathcal{V}_1 \\ 2p' \cdot z\mathcal{V}_2 \\ 2\mathcal{V}_3 \\ 4p' \cdot z\mathcal{V}_4 \\ 4p' \cdot z\mathcal{V}_5 \\ (2p' \cdot z)^2\mathcal{V}_6 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & -1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -2 & 0 & 1 & 1 & 2 & 0 \\ 0 & 0 & -1 & 1 & 0 & 0 \\ -1 & 1 & 1 & 1 & 1 & -1 \end{pmatrix} \begin{pmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{pmatrix},$$

$$\begin{pmatrix} \mathcal{A}_1 \\ 2p' \cdot z\mathcal{A}_2 \\ 2\mathcal{A}_3 \\ 4p' \cdot z\mathcal{A}_4 \\ 4p' \cdot z\mathcal{A}_5 \\ (2p' \cdot z)^2\mathcal{A}_6 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 1 & -1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ -2 & 0 & -1 & -1 & 2 & 0 \\ 0 & 0 & 1 & -1 & 0 & 0 \\ 1 & -1 & 1 & 1 & -1 & 1 \end{pmatrix} \begin{pmatrix} A_1 \\ A_2 \\ A_3 \\ A_4 \\ A_5 \\ A_6 \end{pmatrix},$$

$$\begin{pmatrix} \mathcal{T}_1 \\ 2p' \cdot z\mathcal{T}_2 \\ 2\mathcal{T}_3 \\ 2p' \cdot z\mathcal{T}_4 \\ 2p' \cdot z\mathcal{T}_5 \\ (2p' \cdot z)^2\mathcal{T}_6 \\ 4p' \cdot z\mathcal{T}_7 \\ (2p' \cdot z)^2\mathcal{T}_8 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & -2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & -1 & 0 & 0 & 0 & 0 & -2 & 0 \\ -1 & 0 & 0 & 0 & 1 & 0 & 0 & 2 \\ 0 & 2 & -2 & -2 & 2 & 0 & 2 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & -1 \\ -1 & 1 & 0 & 0 & 1 & -1 & 2 & 2 \end{pmatrix} \begin{pmatrix} T_1 \\ T_2 \\ T_3 \\ T_4 \\ T_5 \\ T_6 \\ T_7 \\ T_8 \end{pmatrix}.$$

Parameter	Central value	Variation	Unit
A	0.5	± 0.2	$\cdots$
f_Ξ	9.9	± 0.4	$10^{-3}\mathrm{GeV}^2$
$\lambda_1(\Xi)$	-2.8	± 0.1	$10^{-2}\mathrm{GeV}^2$
$\lambda_2(\Xi)$	5.2	± 0.2	$10^{-2}\mathrm{GeV}^2$
$\lambda_3(\Xi)$	1.7	± 0.1	$10^{-2}\mathrm{GeV}^2$
f_Σ	9.4	± 0.4	$10^{-3}\mathrm{GeV}^2$
$\lambda_1(\Sigma)$	-2.5	± 0.1	$10^{-2}\mathrm{GeV}^2$
$\lambda_2(\Sigma)$	4.4	± 0.1	$10^{-2}\mathrm{GeV}^2$
$\lambda_3(\Sigma)$	2.0	± 0.1	$10^{-2}\mathrm{GeV}^2$
f_Λ	6.0	± 0.3	$10^{-3}\mathrm{GeV}^2$
$\lambda_1(\Lambda)$	1.0	± 0.3	$10^{-2}\mathrm{GeV}^2$
$\lambda_2(\Lambda)^a$	0.83	± 0.05	$10^{-2}\mathrm{GeV}^2$
$\lambda_3(\Lambda)^a$	0.83	± 0.05	$10^{-2}\mathrm{GeV}^2$

TABLE II. Comparisons of the form factors $f_i(q^2)$ and $g_i(q^2)$ at $q^2 = 0$ and $q^2 = q^2_{\max}$ from three different LCDA models obtained in this work and previous studies in different literature.

Model	Form factor	$i = 1$	$i = 2$	$i = 3$
This work (exponential model)	$f_i(q^2 = 0)$	$0.545^{+0.292+0.066+0.012}_{-0.160-0.046-0.037}$	$0.011^{+0.005+0.001+0.001}_{-0.003-0.000-0.002}$	$0.011^{+0.004+0.001+0.001}_{-0.002-0.001-0.001}$
	$g_i(q^2 = 0)$	$0.542^{+0.310+0.073+0.011}_{-0.162-0.040-0.031}$	$-0.011^{+0.003+0.001+0.001}_{-0.004-0.001-0.001}$	$-0.010^{+0.002+0.000+0.001}_{-0.005-0.002-0.002}$
	$f_i(q^2 = q^2_{\max})$	$0.940^{+0.501+0.124+0.007}_{-0.283-0.049-0.042}$	$0.034^{+0.017+0.003+0.004}_{-0.009-0.001-0.002}$	$0.030^{+0.011+0.004+0.001}_{-0.005-0.002-0.002}$
	$g_i(q^2 = q^2_{\max})$	$0.938^{+0.414+0.075+0.005}_{-0.279-0.084-0.063}$	$-0.030^{+0.006+0.002+0.003}_{-0.010-0.003-0.003}$	$-0.032^{+0.007+0.002+0.002}_{-0.015-0.002-0.004}$
This work (QCDSR model)	$f_i(q^2 = 0)$	$0.554^{+0.038+0.040+0.006}_{-0.012-0.010-0.037}$	$0.012^{+0.000+0.001+0.001}_{-0.001-0.001-0.001}$	$0.012^{+0.001+0.000+0.002}_{-0.000-0.000-0.001}$
	$g_i(q^2 = 0)$	$0.552^{+0.041+0.035+0.019}_{-0.012-0.018-0.034}$	$-0.012^{+0.000+0.001+0.001}_{-0.001-0.001-0.001}$	$-0.011^{+0.001+0.000+0.001}_{-0.001-0.001-0.001}$
	$f_i(q^2 = q^2_{\max})$	$0.954^{+0.066+0.033+0.067}_{-0.024-0.034-0.037}$	$0.037^{+0.0018+0.000+0.003}_{-0.001-0.001-0.003}$	$0.032^{+0.003+0.002+0.004}_{-0.000-0.000-0.002}$
	$g_i(q^2 = q^2_{\max})$	$0.946^{+0.054+0.041+0.022}_{-0.020-0.012-0.041}$	$-0.032^{+0.001+0.000+0.003}_{-0.001-0.000-0.004}$	$-0.034^{+0.001+0.000+0.003}_{-0.001-0.000-0.003}$
This work (Gegenbauer model)	$f_i(q^2 = 0)$	$0.560^{+0.039+0.041+0.028}_{-0.000-0.000-0.038}$	$0.024^{+0.001+0.001+0.003}_{-0.001-0.000-0.002}$	$0.006^{+0.002+0.002+0.000}_{-0.002-0.001-0.001}$
	$g_i(q^2 = 0)$	$0.567^{+0.034+0.029+0.010}_{-0.000-0.006-0.039}$	$-0.005^{+0.002+0.001+0.000}_{-0.002-0.002-0.001}$	$-0.023^{+0.000+0.000+0.001}_{-0.002-0.001-0.003}$
	$f_i(q^2 = q^2_{\max})$	$1.084^{+0.109+0.088+0.071}_{-0.000-0.024-0.073}$	$0.082^{+0.008+0.007+0.011}_{-0.002-0.001-0.007}$	$0.012^{+0.005+0.006+0.001}_{-0.007-0.007-0.000}$
	$g_i(q^2 = q^2_{\max})$	$1.034^{+0.067+0.107+0.079}_{-0.000-0.000-0.060}$	$-0.010^{+0.005+0.005+0.001}_{-0.005-0.006-0.001}$	$-0.080^{+0.002+0.000+0.008}_{-0.006-0.005-0.010}$
NRQM [37]	$f_i(q^2 = 0)$	0.533	0.124	−0.018
	$g_i(q^2 = 0)$	0.580	0.019	−0.135
LFQM [93]	$f_i(q^2 = 0)$	0.481	0.127	−0.046
	$g_i(q^2 = 0)$	0.471	0.026	−0.154
	$f_i(q^2 = q^2_{\max})$	1.015	0.312	−0.097
	$g_i(q^2 = q^2_{\max})$	0.978	0.068	−0.377
LFQM [47]	$f_i(q^2 = 0)$	0.467	0.185	...
	$g_i(q^2 = 0)$	0.448	0.052	...
LF [95]	$f_i(q^2 = 0)$	0.654	0.143	...
	$g_i(q^2 = 0)$	0.640	0.015	...
LFQM [94]	$f_i(q^2 = 0)$	0.437	0.123	0.057
	$g_i(q^2 = 0)$	0.429	−0.059	−0.145
	$f_i(q^2 = q^2_{\max})$	0.714	0.206	0.102
	$g_i(q^2 = q^2_{\max})$	0.693	−0.099	−0.214
RQM [90]	$f_i(q^2 = 0)$	0.474	0.150	0.081
	$g_i(q^2 = 0)$	0.449	−0.030	−0.285
	$f_i(q^2 = q^2_{\max})$	0.945	0.426	0.161
	$g_i(q^2 = q^2_{\max})$	0.962	−0.104	−0.752
QCDSR [96]	$f_i(q^2 = 0)$	0.500	0.141	0.016
	$g_i(q^2 = 0)$	0.500	−0.035	−0.191
	$f_i(q^2 = q^2_{\max})$	0.971	0.346	0.070
	$g_i(q^2 = q^2_{\max})$	1.107	−0.125	−0.519
HQET [97]	$f_i(q^2 = q^2_{\max})$	1.029	0.271	−0.050
	$g_i(q^2 = q^2_{\max})$	1.029	0.050	−0.271

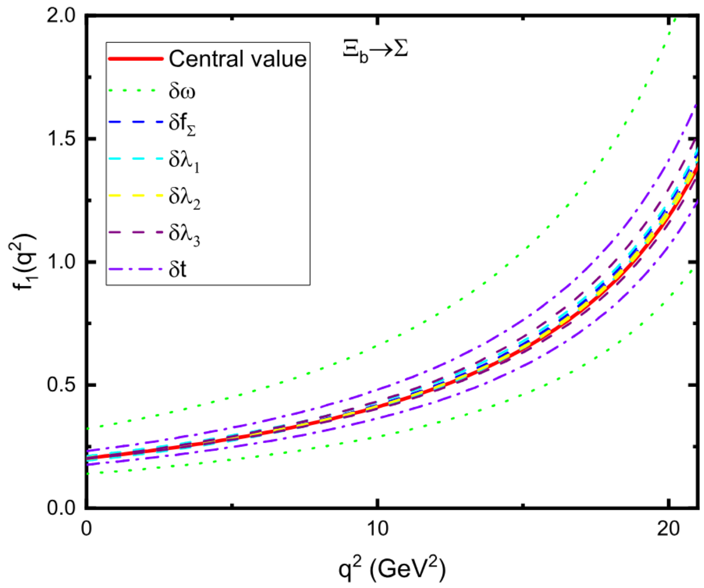
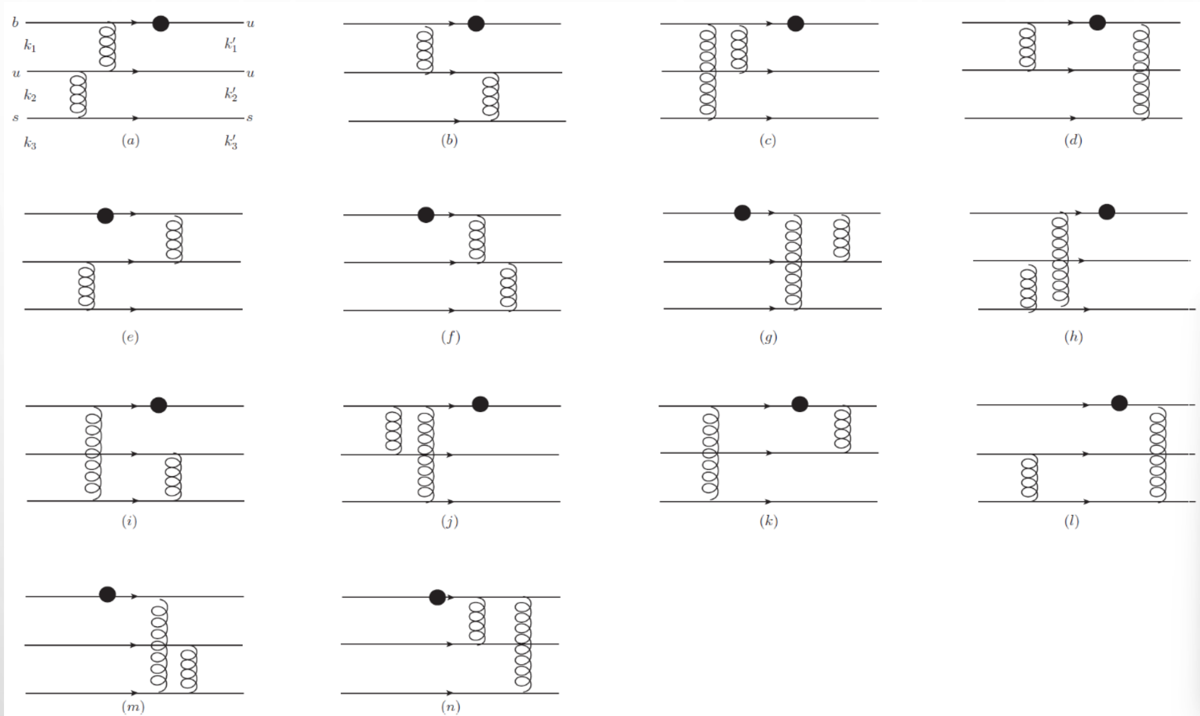


FIG. 2: The theoretical uncertainties in S1 scheme for the form factor $f_1(q^2)$ of $\Xi_b \to \Sigma$ transition . The solid red line shows the q^2 shape at central values of model parameters. The dotted green curves show the effect of the variation of $\omega_0 = (0.4 \pm 0.1)$GeV from the Ξ_b baryon LCDAs. The dashed blue, cyan, yellow, and purple curves show the variation of the parameters f_Σ, λ_1, λ_2, and λ_3 of Σ baryon LCDAs, respectively. The dot-dashed violet curves show the scale t variation between $0.8t$ and $1.2t$.

- **Feynman diagrams:**



- **Kinematics:**

$$p = \frac{M}{\sqrt{2}}(1{,}1,\mathbf{0}_T), p' = \frac{M}{\sqrt{2}}(f^+, f^-, \mathbf{0}_T), f^{\pm} = \left(f \pm \sqrt{f^2-1}\right)\frac{\mathrm{m}}{\mathrm{M}}, f = \mathrm{v}\cdot\mathrm{v}'$$

$$k_{2,3} = \left(0, \frac{M}{\sqrt{2}}x_{2,3}, \mathrm{k}_{\mathrm{iT}}\right), k_1 = p - k_2 - k_3,\ k_i' = \left(\frac{M}{\sqrt{2}}f^+x_i', 0, \mathrm{k}_{iT}'\right).$$

- **The conservation laws:** $x_1 + x_2 + x_3 = 1, \mathbf{k}_{1T} + \mathbf{k}_{2T} + \mathbf{k}_{3T} = 0.$

Baryonic LCDAs:

- Heavy baryons LCDAs can be simplified in the heavy-quark symmetry limit.
  - *P.Ball, V.M.Braun, E.Gardi (2008);*
  - *A. Ali, C. Hambrock, and A. Y. Parkhomenko (2012);*
  - *G.Bell, T.Feldmann, Yu-Ming Wang, M.W.Y.Yip (2013);*
  - *A. Ali, C. Hambrock, A. Y. Parkhomenko, and W. Wang (2013);*
  - *V. M. Braun, S. E. Derkachov, and A. N. Manashov (2014);*
  - *Yu-Ming Wang, Yue-Long Shen (2016).*

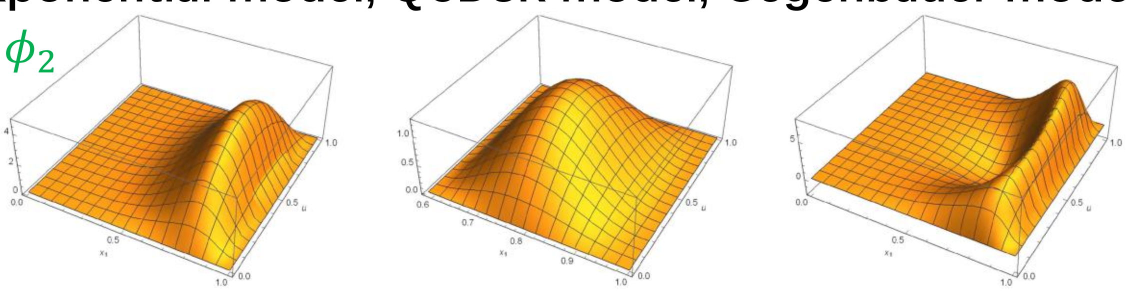
$$\begin{aligned}&\left(\Psi_{\mathcal{B}_i}\right)_{\alpha\beta\gamma}(\omega,u)\\ &=\frac{1}{8N_c}\left\{\frac{f^{(1)}}{\sqrt{2}}\left[(\not{n}\,\gamma_5 C)_{\alpha\beta}\phi_2(\omega,u)+(\not{v}\gamma_5 C)_{\alpha\beta}\phi_4(\omega,u)\right]\left(u_{\mathcal{B}_i}\right)_\gamma\right.\\ &\left.+f^{(2)}\left[(\gamma_5 C)_{\alpha\beta}\phi_{3s}(\omega,u)-\frac{i}{2}(\sigma_{nv}\gamma_5 C)_{\alpha\beta}\phi_{3a}(\omega,u)\right]\left(u_{\mathcal{B}_i}\right)_\gamma\right\}\end{aligned}$$

up to twist 4

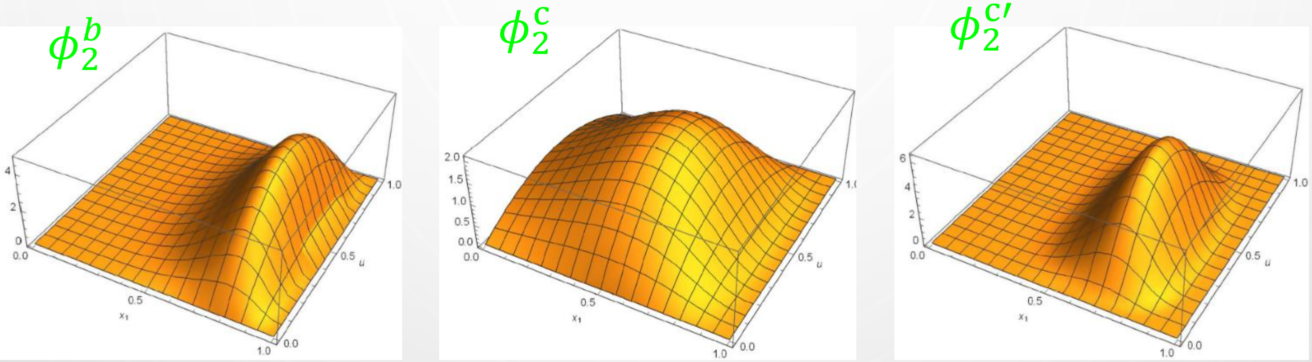
$$\omega=(x_2+x_3)M$$
$$u=x_2/(x_2+x_3)$$

- Three popular models exist in the literature (Ξ_b):
  Exponential model, QCDSR model, Gegenbauer model.

ϕ_2



- The Ξ_c LCDAs were not available at the moment.
- In heavy-quark symmetry: $\phi_i^c(\omega, u) = \phi_i^b(\omega, u)|_{M\to m}$,
- Twist-two LCDAs of Exponential model:



- Based on the light-front formalism for baryons:

$$\phi_i^c(\omega, u) \to N_i \phi_i^c(\omega, u) e^{-\frac{m^2}{2\beta^2 x_1} - \frac{m_q^2}{2\beta^2 x_2} - \frac{m_q^2}{2\beta^2 x_3}},$$

- Constituent quark masses: $m_u = m_d = 0.22$ GeV, $m_s = 0.419$ GeV.
  [Semirelativistic potential model PRD 104, 013005 (2021)]
- Confinement scale parameter $\boldsymbol{\beta} = 1.0 \pm 0.2$GeV.
  [PRD 61,114002 (2000)].
- Peak around $x_1 \sim 0.7$, the on-shell charm quark is generally recovered.

- Outgoing light baryonic LCDAs up to twist six in the momentum-space. [Braun, NPB 553, 355 (1999)]

$$
\begin{aligned}
(\bar{\Psi}_{\mathcal{B}_f})_{\alpha\beta\gamma}(x_i') = & -\frac{1}{8\sqrt{2}N_c}[\mathcal{S}_1 m(\bar{u}_{\mathcal{B}_f}\gamma_5)_\gamma C_{\beta\alpha} + \mathcal{S}_2 m^2(\bar{u}_{\mathcal{B}_f}\gamma_5 \not{z})_\gamma C_{\beta\alpha} + \mathcal{P}_1 m(\bar{u}_{\mathcal{B}_f})_\gamma (C\gamma_5)_{\beta\alpha} \\
& + \mathcal{P}_2 m^2(\bar{u}_{\mathcal{B}_f}\not{z})_\gamma (C\gamma_5)_{\beta\alpha} + \mathcal{V}_1(\bar{u}_{\mathcal{B}_f}\gamma_5)_\gamma (C\not{p}')_{\beta\alpha} + \mathcal{V}_2 m(\bar{u}_{\mathcal{B}_f}\gamma_5\not{z})_\gamma (C\not{p}')_{\beta\alpha} \\
& + \mathcal{V}_3 m(\bar{u}_{\mathcal{B}_f}\gamma_5\gamma_\mu)_\gamma (C\gamma^\mu)_{\beta\alpha} + \mathcal{V}_4 m^2(\bar{u}_{\mathcal{B}_f}\gamma_5)_\gamma (C\not{z})_{\beta\alpha} - i\mathcal{V}_5 m^2(\bar{u}_{\mathcal{B}_f}\gamma_5\sigma^{\mu\nu}z_\nu)_\gamma (C\gamma_\mu)_{\beta\alpha} \\
& + \mathcal{V}_6 m^3(\bar{u}_{\mathcal{B}_f}\gamma_5\not{z})_\gamma (C\not{z})_{\beta\alpha} + \mathcal{A}_1(\bar{u}_{\mathcal{B}_f})_\gamma (C\gamma_5\not{p}')_{\beta\alpha} + \mathcal{A}_2 m(\bar{u}_{\mathcal{B}_f}\not{z})_\gamma (C\gamma_5\not{p}')_{\beta\alpha} \\
& + \mathcal{A}_3 m(\bar{u}_{\mathcal{B}_f}\gamma_\mu)_\gamma (C\gamma_5\gamma^\mu)_{\beta\alpha} + \mathcal{A}_4 m^2(\bar{u}_{\mathcal{B}_f})_\gamma (C\gamma_5\not{z})_{\beta\alpha} - i\mathcal{A}_5 m^2(\bar{u}_{\mathcal{B}_f}\sigma^{\mu\nu}z_\nu)_\gamma (C\gamma_5\gamma_\mu)_{\beta\alpha} \\
& + \mathcal{A}_6 m^3(\bar{u}_{\mathcal{B}_f}\not{z})_\gamma (C\gamma_5\not{z})_{\beta\alpha} - i\mathcal{T}_1(\bar{u}_{\mathcal{B}_f}\gamma_5\gamma^\mu)_\gamma (C\sigma_{\mu\nu}p'^\nu)_{\beta\alpha} - i\mathcal{T}_2 m(\bar{u}_{\mathcal{B}_f}\gamma_5)_\gamma (C\sigma_{\mu\nu}p'^\mu z^\nu)_{\beta\alpha} \\
& + \mathcal{T}_3 m(\bar{u}_{\mathcal{B}_f}\gamma_5\sigma^{\mu\nu})_\gamma (C\sigma_{\mu\nu})_{\beta\alpha} + \mathcal{T}_4 m(\bar{u}_{\mathcal{B}_f}\gamma_5\sigma^{\mu\rho}z_\rho)_\gamma (C\sigma_{\mu\nu}p'^\nu)_{\beta\alpha} \\
& - i\mathcal{T}_5 m^2(\bar{u}_{\mathcal{B}_f}\gamma_5\gamma^\mu)_\gamma (C\sigma_{\mu\nu}z^\nu)_{\beta\alpha} - i\mathcal{T}_6 m^2(\bar{u}_{\mathcal{B}_f}\gamma_5\not{z})_\gamma (C\sigma_{\mu\nu}z^\mu p'^\nu)_{\beta\alpha} \\
& + \mathcal{T}_7 m^2(\bar{u}_{\mathcal{B}_f}\gamma_5\not{z}\sigma^{\mu\rho})_\gamma (C\sigma_{\mu\nu})_{\beta\alpha} + \mathcal{T}_8 m^3(\bar{u}_{\mathcal{B}_f}\gamma_5\sigma^{\mu\rho}z_\rho)_\gamma (C\sigma_{\mu\nu}z^\nu)_{\beta\alpha}],
\end{aligned}
$$

- $\mathcal{S}, \mathcal{P}, \mathcal{V}, \mathcal{A}, \mathcal{T}$ can be expanded in terms of 24 LCDAs with definite twist and symmetry.

- 4 independent parameters f, λ_1, λ_2, λ_3, at LO conformal spin accuracy. [Y. L. Liu, NPA 821, 80 (2009)]