河南师範大學
HENAN NORMAL UNIVERSITY

Form Factors for $B_c^* \to J/\psi\,(\eta_c) + l\nu_l$ at NLO in QCD

陶伟

河南师范大学

In collaboration with Qin Chang, Ruilin Zhu, Zhen-Jun Xiao, et al.

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Outline

1. Introduction
2. Calculation Procedure
3. Results
4. Summary

B_c^* semileptonic decay and form factors

➢ $c\bar{b}$ meson

- the only meson containing two different heavy flavors
- Bc(1S) discovered in 1998 through $B_c \to J/\psi + l\nu_l$
- Bc*(1S) not yet observed due to low production rate and difficulty in detecting γ of $B_c^* \to B_c + \gamma$
- Bc*(1S) weak decays (e.g. $B_c^* \to J/\psi\,(\eta_c) + l\nu_l$) help search for Bc*(1S)

[CDF, PRL(1998)]
[CMS, PRL(2019)]
[LHCb, PRL(2019)]

➢ Form factors (FFs)

- Building blocks for decay widths and branching fractions
- Lattice QCD data available for $B_c \to J/\psi\,(\eta_c)$ FFs, not for B_c^*
- $B_c^* \to J/\psi\ (\eta_c)$ FFs calculated by light-front quark model (LFQM), Bauer-Stech-Wirbel (BSW) model, QCD sum rules (QCDSR),

[HPQCD, 1611.01987]
[HPQCD, PRD(2020)]

[Q.Chang,L.T.Wang,X.N.Li, JHEP(2019)]
[Q.Chang,et al., AHEP(2020)]
[S.Y.Wang,et al., CPC(2024)]

[Q.Chang,L.L.Chen,S.Xu, JPG(2018)]
[P.R,R.Dhir, 1908.00242]
[Y.Yang,et al., CPC(2023)]

Why NRQCD higher-order calculations

[G.T.Bodwin,E.Braaten,G.P.Lepage,PRD(1995)]

- Non-Relativistic Quantum Chromodynamics (NRQCD)

- Form factor = $\boxed{\text{short-distance coefficient}}$ ✖ $\boxed{\text{wavefunction at origin}}$

  perturbative expansion in α_s nonperturbative

- To test perturbative expansion convergence and renormalization scale dependence in NRQCD
- To obtain more precise theoretical predictions and test the Standard Model
- To study new physics by calculating (axial-)tensor form factors

Review NRQCD calculations for $c\bar{b} \to c\bar{c}$ form factors

- 2007, first one-loop for $B_c \to \eta_c$ vector and tensor FFs
  [G. Bell, 0705.3133]

- 2011-2022, next-to-leading order (NLO) QCD corrections to $B_c \to J/\psi\ (\eta_c)$ (axial-)vector and (axial-)tensor FFs
  [C.F.Qiao,P.Sun, JHEP(2012)]
  [C.F.Qiao,R.Zhu, PRD(2013)]
  [W.Tao,Z.J.Xiao,R.Zhu, PRD(2022)]

- From 2017 onward, relativistic corrections for Bc decaying into S(P)-wave charmonium FFs
  [R.Zhu,et al., PRD(2017)]
  [R.Zhu, NPB(2018)]
  [D.Shen,et al., IJMPA(2021)]

- 2024, $B_c^* \to J/\psi$ (axial-)vector FFs at leading order (LO)
  [Y.Geng,M.Cao,R.Zhu, PRD(2024)]

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Definition for $B_c^* \to J/\psi$ form factors

$$\langle J/\psi(\epsilon',p')|\bar{b}\gamma_\mu c|B_c^*(\epsilon,p)\rangle = -(\epsilon\cdot\epsilon'^*)\left[P_\mu V_1(q^2) - q_\mu V_2(q^2)\right] - (\epsilon\cdot q)\epsilon'^*_\mu V_3(q^2) + (\epsilon'^*\cdot q)\epsilon_\mu V_4(q^2) + (\epsilon\cdot q)(\epsilon'^*\cdot q)\left[\left(\frac{P_\mu}{M^2-M'^2} - \frac{q_\mu}{q^2}\right)V_5(q^2) + \frac{q_\mu}{q^2}V_6(q^2)\right],$$

$$\langle J/\psi(\epsilon',p')|\bar{b}\gamma_\mu\gamma_5 c|B_c^*(\epsilon,p)\rangle = i\varepsilon_{\mu\nu\alpha\beta}\epsilon^\alpha\epsilon'^{*\beta}\left[P^\nu A_1(q^2) - q^\nu A_2(q^2)\right] + \frac{i\varepsilon_{\mu\nu\alpha\beta}P^\alpha q^\beta}{M^2-M'^2}\left[\epsilon'^*\cdot q\epsilon^\nu A_3(q^2) - \epsilon\cdot q\epsilon'^{*\nu}A_4(q^2)\right] - \frac{i\varepsilon_{\rho\nu\alpha\beta}\epsilon^\alpha\epsilon'^{*\beta}P^\nu q^\rho}{M^2-M'^2}\left[P_\mu A_5(q^2) - q_\mu A_6(q^2)\right].$$

Independent FFs:
$V_{1,2,3,4,5,6}$, $A_{1,2,3,4}$,
$T_{2,3,4,6}$, $T'_{2,3,4}$

$$\langle J/\psi(\epsilon',p')|\bar{b}\sigma_{\mu\nu}q^\nu c|B_c^*(\epsilon,p)\rangle = -i(\epsilon\cdot\epsilon'^*)\left[P_\mu T_1(q^2) - q_\mu T_2(q^2)\right](M+M') - i\left[(\epsilon\cdot q)\epsilon'^*_\mu T_3(q^2) - (\epsilon'^*\cdot q)\epsilon_\mu T_4(q^2)\right](M+M') + i\frac{(\epsilon\cdot q)(\epsilon'^*\cdot q)}{M+M'}\left[P_\mu T_5(q^2) + q_\mu T_6(q^2)\right],$$

$$\langle J/\psi(\epsilon',p')|\bar{b}\sigma_{\mu\nu}\gamma_5 q^\nu c|B_c^*(\epsilon,p)\rangle = -\varepsilon_{\mu\nu\alpha\beta}\epsilon^\alpha\epsilon'^{*\beta}\left[P^\nu T'_1(q^2) + q^\nu T'_2(q^2)\right](M+M') + \frac{\varepsilon_{\mu\nu\alpha\beta}P^\alpha q^\beta}{M+M'}\left[\epsilon'^*\cdot q\epsilon^\nu T'_3(q^2) + \epsilon\cdot q\epsilon'^{*\nu}T'_4(q^2)\right] - \frac{\varepsilon_{\rho\nu\alpha\beta}\epsilon^\alpha\epsilon'^{*\beta}P^\nu q^\rho}{M+M'}\left[P_\mu T'_5(q^2) - q_\mu T'_6(q^2)\right].$$

- $P = p + p'$
- $q = p - p'$: transfer momentum

Definition for $B_c^* \to \eta_c$ form factors

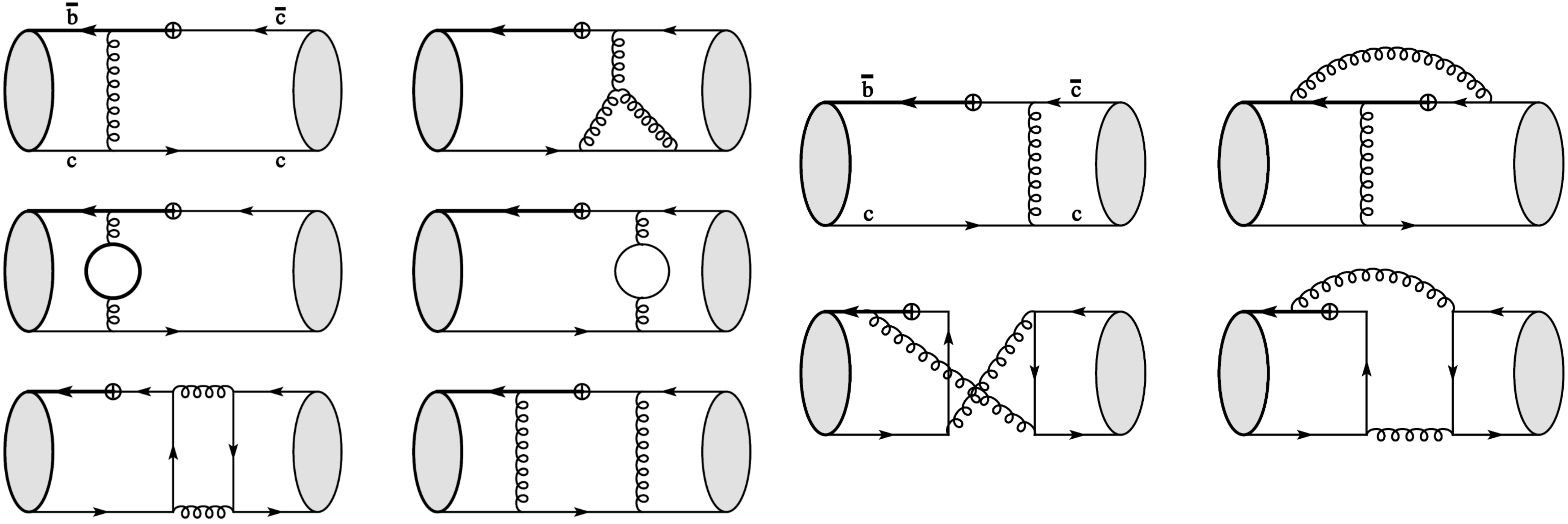
$$\left\langle \eta_c(p') \left| \bar{b}\gamma_\mu c \right| B_c^*(\epsilon, p) \right\rangle = \frac{i\varepsilon_{\mu\nu\alpha\beta}\epsilon^\nu P^\alpha q^\beta}{m_{B_c^*} + m_{\eta_c}} V,$$

$$\begin{aligned} &\left\langle \eta_c(p') \left| \bar{b}\gamma_\mu \gamma_5 c \right| B_c^*(\epsilon, p) \right\rangle \\ &= \frac{2m_{B_c^*}\epsilon \cdot q q_\mu}{q^2} A_0 + \left(\epsilon_\mu - \frac{\epsilon \cdot q}{q^2} q_\mu \right) (m_{B_c^*} + m_{\eta_c}) A_1 \\ &+ \epsilon \cdot q \left(\frac{P_\mu}{m_{B_c^*} + m_{\eta_c}} - \frac{m_{B_c^*} - m_{\eta_c}}{q^2} q_\mu \right) A_2, \end{aligned}$$

$$\left\langle \eta_c(p') \left| \bar{b}\sigma_{\mu\nu} q^\nu c \right| B_c^*(\epsilon, p) \right\rangle = -\varepsilon_{\mu\nu\alpha\beta}\epsilon^\nu P^\alpha q^\beta T,$$

$$\begin{aligned} &\left\langle \eta_c(p') \left| \bar{b}\sigma_{\mu\nu}\gamma_5 q^\nu c \right| B_c^*(\epsilon, p) \right\rangle \\ &= i\left(\epsilon_\mu - \frac{\epsilon \cdot q}{q^2} q_\mu \right) (m_{B_c^*}^2 - m_{\eta_c}^2) T_1' \\ &- i\epsilon \cdot q \left(P_\mu - \frac{m_{B_c^*}^2 - m_{\eta_c}^2}{q^2} q_\mu \right) T_2', \end{aligned}$$

Step 1: generate Feynman diagrams & amplitudes



Step 2: amplitude simplification

- Dirac matrix simplification, index contraction, color algebra simplification, and trace calculation
- γ_5 scheme for the trace of a fermion chain containing γ_5

➢ Naïve γ_5 scheme when containing 0/2 γ_5 [V.Shtabovenko,R.Mertig,F.Orellana, CPC(2025)]

$$\gamma_5\gamma_\mu + \gamma_\mu\gamma_5 = 0, \gamma_5^2 = \mathbf{1} \text{ and cyclicity}$$

➢ Fixed reading point γ_5 scheme when containing 1/3 γ_5

- the fermion chain contains current vertex $\Gamma = \gamma_\mu\gamma_5$ or $\sigma_{\mu\nu}\gamma_5$

$$\mathrm{Trace}\left(a\cdot\Gamma\cdot b\right) \to \mathrm{Trace}\left(\frac{\Gamma\cdot b\cdot a + b\cdot a\cdot\Gamma}{2}\right)$$

- otherwise

[J.G.Korner,D.Kreimer,K.Schilcher, ZPC(1992)]
[S.A.Larin, PLB(1993)]
[S.Moch,J.A.M.Vermaseren,A.Vogt, PLB(2015)]

$$\mathrm{Trace}\left(a\cdot\gamma_5\cdot b\right) \to \mathrm{Trace}\left(b\cdot a\cdot\gamma_5\right)$$

2.5 Step 3: Express amplitudes as $A_0, B_0, C_{0,1}, D_0$ & calculate them

amplitudes

Consisting of scalar products of momenta

FeynCalc TID

$A_0, B_0, C_{0,1}, D_0, E_0$

IBP reduction

Package-X

[K.G.Chetyrkin,F.V.Tkachov, NPB(1981)]
[H.H.Patel, CPC(2017)]

Li_2, Log, Sqrt

Series: expanding in small m_c and taking the leading-order terms

hierarchical heavy quark limit

Step 4: Renormalization

- One-loop diagrams
- Tree diagrams inserted with one $\mathcal{O}(\alpha_s^1)$ counterterm vertex

- QCD coupling $\overline{\mathbf{MS}}$ renormalization constant
- QCD heavy quark field (mass) **OS** renormalization constant
- QCD heavy flavor-changing current **OS** renormalization constant Z_J^{OS}

$$Z_v^{\mathrm{OS}} = Z_a^{\mathrm{OS}} = 1$$

[W.Tao,Z.J.Xiao, JHEP(2023)]
[W.Tao,Z.J.Xiao, JHEP(2024)]

$$Z_t^{\mathrm{OS}} = Z_{t5}^{\mathrm{OS}} = 1 + \frac{\alpha_s C_F}{4\pi}\left(\frac{1}{\epsilon} - \frac{2x\log x}{1+x} + 2\log y + \mathcal{O}(\epsilon)\right) + \mathcal{O}(\alpha_s^2)$$

$$x = \frac{m_c}{m_b}, \qquad y = \frac{\mu}{m_b}, \qquad s = \frac{1}{1 - \frac{q^2}{m_b^2}}$$

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Analytical results for $B_c^* \to J/\psi$ form factors

- LO

$$V_1 = \frac{16\sqrt{2}\pi\alpha_s C_F s^2(1+x)^{\frac{5}{2}}\Psi_{B_c^*}(0)\Psi_{J/\psi}(0)}{m_b^3 x^{\frac{3}{2}}(1+s(x-2)x)^2},$$

$$V_2 = A_2 = T_2 = \frac{1-x}{1+x}V_1 = \frac{2(1+x)}{1+3x}T_6,$$

$$T_3 = \frac{1+x}{2x}T_4 = \frac{-1+s(4+10x+3x^2)}{2s(1+x)(1+3x)}V_1,$$

$$V_1 = \frac{V_3}{2} = A_1 = \frac{1+x}{4x}V_4 = \frac{s(1+x)(1+3x)}{1+4sx+3sx^2}T_2',$$

$$T_4' = \frac{1+x}{2}T_3' = \frac{1+3x}{2(1+x)}V_1,$$

$$V_5 = V_6 = A_3 = A_4 = 0,$$

$\Psi_{B_c^*(J/\psi)}(0)$: $B_c^*(J/\psi)$ wavefunction at origin

$n_f = n_b + n_c + n_l$

- Asymptotic NLO

$$\frac{V_1^{\mathrm{NLO}}}{V_1^{\mathrm{LO}}} = 1 + \frac{\alpha_s}{4\pi}\Bigg\{\left(\frac{11C_A}{3} - \frac{2}{3}n_f\right)\ln\frac{2sy^2}{x} - \frac{10}{9}n_f + \left(\frac{2\ln s}{3} - \frac{2\ln x}{3} + \frac{10}{9} + \frac{2\ln 2}{3}\right)n_b$$
$$- C_A\left[\frac{\ln^2 x}{2} + \left(\ln s + 2\ln 2 + \frac{3}{2}\right)\ln x + \frac{1}{2}\ln^2 s + \left(\frac{3}{2} + 2\ln 2\right)\ln s + 2\ln^2 2 + \frac{3\ln 2}{2}\right.$$
$$\left. - \frac{1}{9}(67 - 3\pi^2)\right] + C_F\Big[2\mathrm{Li}_2(1-s) + \ln^2 x + (2\ln s + 10\ln 2 - 5)\ln x + 2\ln^2 s$$
$$+ (10\ln 2 - 2)\ln s + 7\ln^2 2 + 9\ln 2 + \frac{1}{3}(\pi^2 - 51)\Big]\Bigg\},$$

Analytical results for $B_c^* \to \eta_c$ form factors

$\Psi_{B_c^*(\eta_c)}(0)$: $B_c^*(\eta_c)$ wavefunction at origin

- LO

$$V=\frac{16\sqrt{2}\pi\alpha_s C_F s^2(1+3x)(1+x)^{\frac{3}{2}}\Psi_{B_c^*}(0)\Psi_{\eta_c}(0)}{m_b^3 x^{\frac{3}{2}}(1+s(x-2)x)^2},\quad V=\frac{4s^2(1+3x)^2(x^2-1)T_1'}{1-s(6+4x-8x^2)-s^2x(12+10x-8x^2-15x^3)},$$

$$V=\frac{1+3x}{1+x}A_0=\frac{2s(1+3x)^2}{3+sx(6+7x)}A_1=2A_2,\qquad T_2'=T=\frac{1+s(4+6x+5x^2)}{4s(1+x)^2}A_0,$$

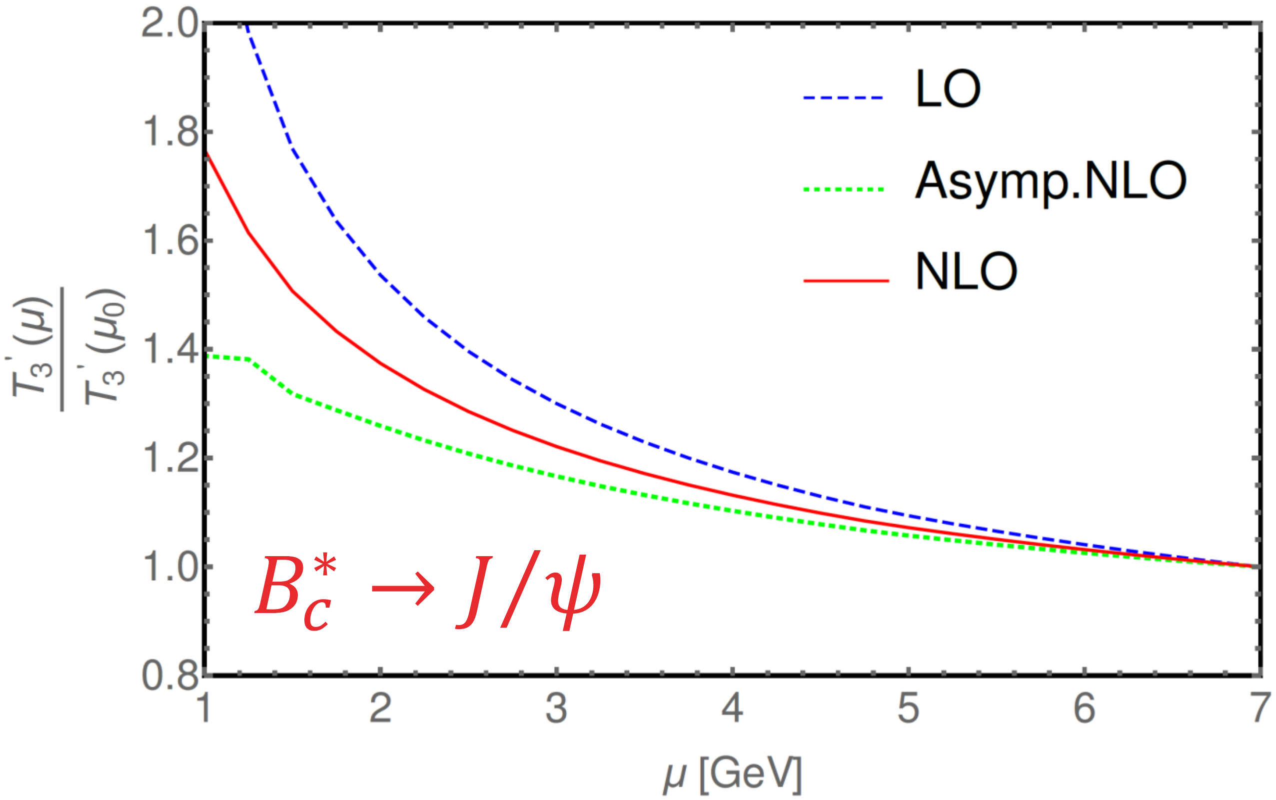
- Asymptotic NLO

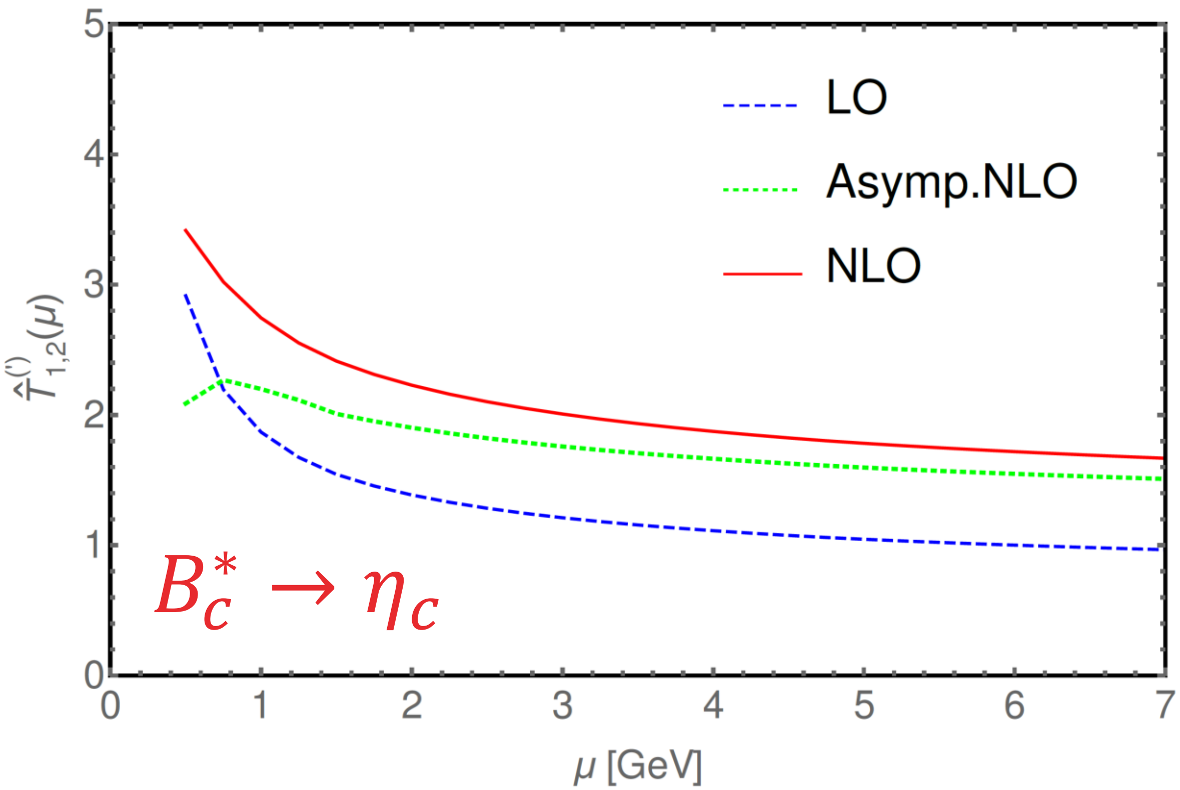
$$x=\frac{m_c}{m_b},\quad y=\frac{\mu}{m_b},\quad s=\frac{1}{1-\frac{q^2}{m_b^2}}$$

$$\begin{aligned}
\frac{T_1'^{\mathrm{NLO}}}{T_1'^{\mathrm{LO}}}=&1+\frac{\alpha_s}{4\pi}\Bigg\{\left(\frac{11C_A}{3}-\frac{2n_f}{3}\right)\ln\frac{2sy^2}{x}-\frac{10n_f}{9}+\left(-\frac{2\ln x}{3}+\frac{2\ln s}{3}+\frac{2\ln 2}{3}+\frac{10}{9}\right)n_b+\frac{2s\ln x}{1-6s}\\
&+\frac{2s\ln s}{1-6s}+\frac{(4s+2)\ln 2}{1-6s}+\frac{\pi^2}{18s-3}+C_A\Bigg[\frac{2(s-1)(4s-1)\mathrm{Li}_2(1-2s)}{6s-1}+\frac{(4s^2-6s+2)\,\mathrm{Li}_2(1-s)}{1-6s}\\
&+\frac{s\ln^2 x}{1-6s}+\left(\frac{2s}{1-6s}+\frac{2s\ln s}{1-6s}+\frac{(2s+2)\ln 2}{1-6s}\right)\ln x+\frac{s(2s-3)\ln^2 s}{6s-1}+\left(\frac{2s}{1-6s}+\frac{4(2s-3)s\ln 2}{6s-1}\right)\ln s\\
&+\frac{2s(2s-3)\ln^2 2}{6s-1}+\frac{6s\ln 2}{1-6s}+\frac{438s+3\pi^2(s(2s-5)+1)-85}{54s-9}\Bigg]+C_F\Bigg[-\frac{4(s-1)(4s-1)\mathrm{Li}_2(1-2s)}{6s-1}\\
&+\frac{(8s^2+2)\,\mathrm{Li}_2(1-s)}{6s-1}+\frac{(3s+1)\ln^2 x}{6s-1}+\left(\frac{6-35s}{6s-1}+\left(\frac{3}{6s-1}+1\right)\ln s+\frac{(20s+2)\ln 2}{6s-1}\right)\ln x\\
&+\frac{(13-4s)s\ln^2 s}{6s-1}+\left(\frac{(27-34s)s-7}{4s(3s-2)+1}+\frac{(8(5-2s)s-2)\ln 2}{6s-1}\right)\ln s+\frac{(4s(2s-5)+1)\ln^2 2}{1-6s}\\
&+\frac{(16s(2s-1)-2)\ln 2}{4s(3s-2)+1}+\frac{-309s+\pi^2((19-4s)s+2)+51}{18s-3}\Bigg]\Bigg\},
\end{aligned}$$

3.3 Renormalization scale dependence of form factors

- At maximum recoil point ($q^2 = 0$)
- $\mu_0 = 7$ GeV
- $m_b = 4.75$ GeV
- $m_c = 1.5$ GeV

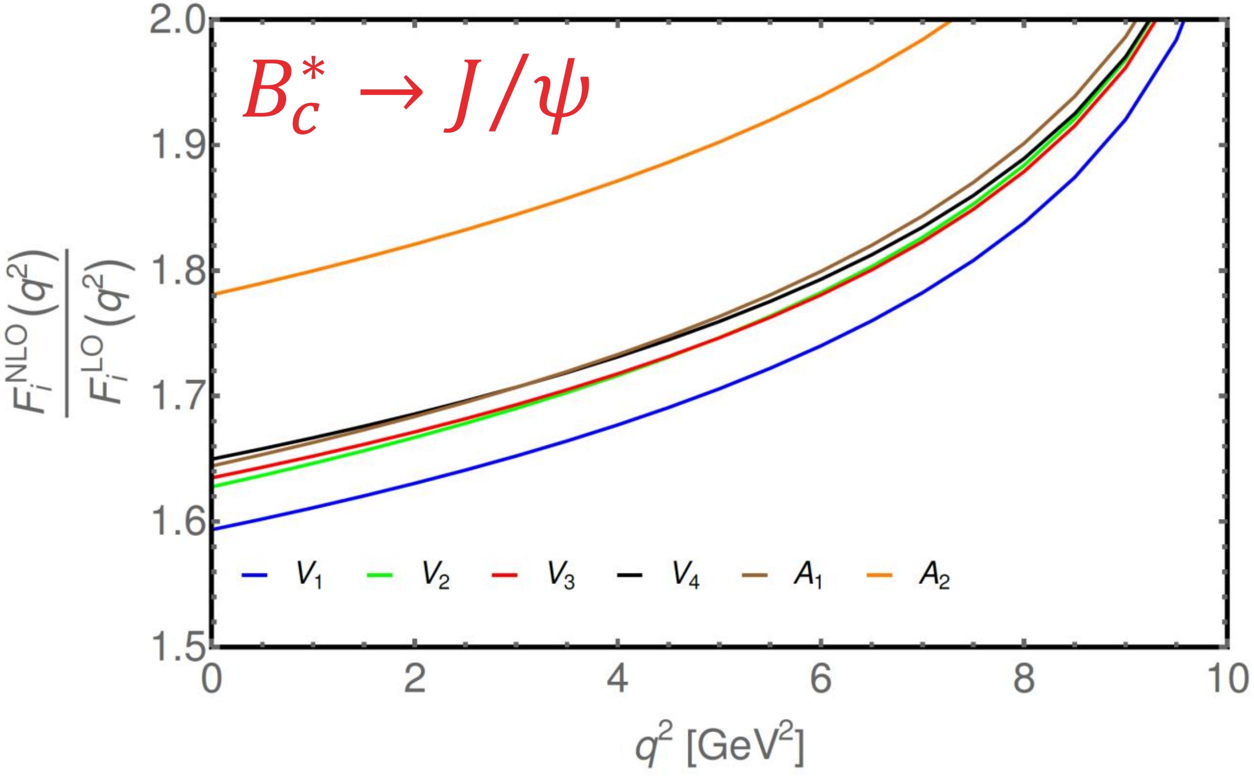


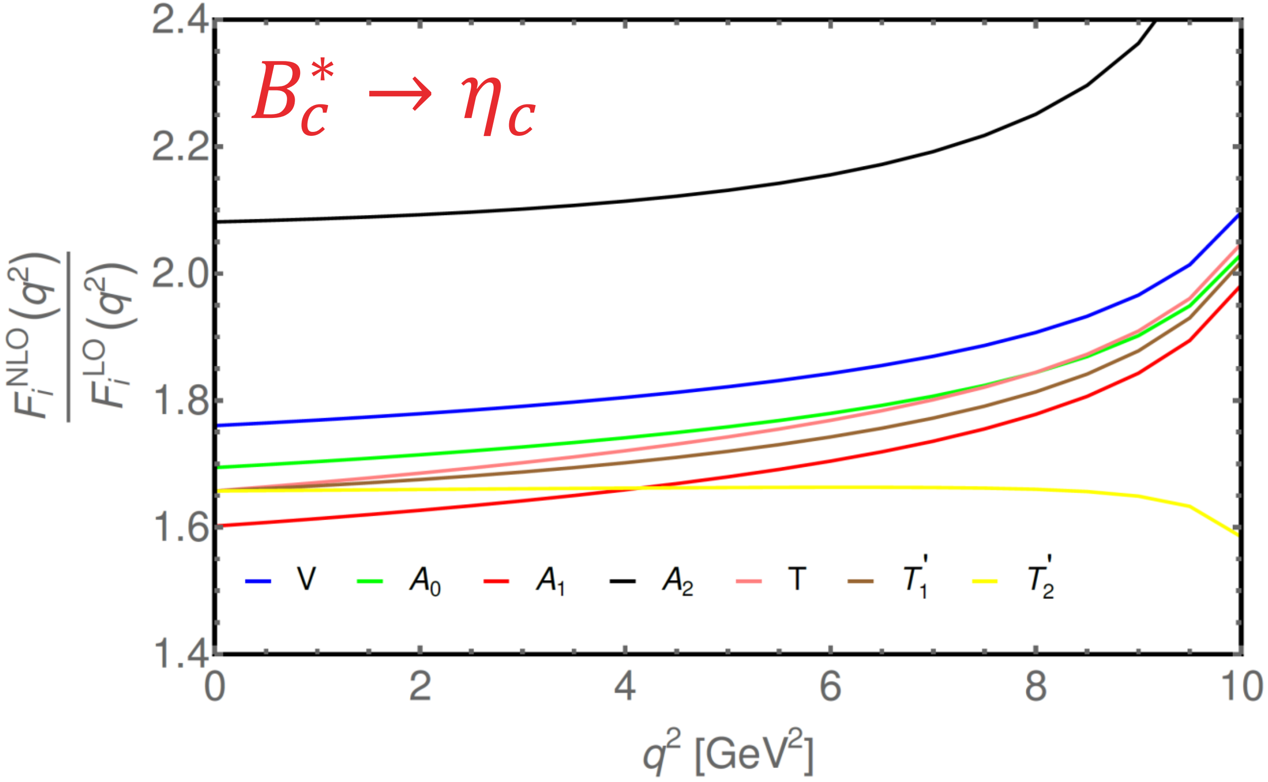


NLO corrections reduce the renormalization scale dependence

3.4 q^2 dependence of NLO to LO form factor ratios

$\mu = 3$ GeV





- NLO corrections are sizable but convergent in low q^2 region
- The convergence breaks down at high q^2

3.5

NRQCD+Lattice predictions for $B_c^* \to J/\psi$ form factors at $q^2 = 0$

$$F_{i,\mathrm{NRQCD+Lattice}}^{B_c^*\to J/\psi}(q^2) = \frac{1}{4}\sum_{j=1}^{4} \frac{F_{i,\mathrm{NRQCD}}^{B_c^*\to J/\psi}(q^2)}{F_{j,\mathrm{NRQCD}}^{B_c\to J/\psi}(q^2)} F_{j,\mathrm{Lattice}}^{B_c\to J/\psi}(q^2)$$

$$\Psi_{B_c^*}(0) \approx \Psi_{B_c}(0)$$

$$F_j^{B_c\to J/\psi} \in \{V, A_{0,1,2}\}^{B_c\to J/\psi}$$

The second uncertainties from lattice data dominate over the first uncertainties from $\mu = 3^{+4}_{-1.5}$ GeV

[HPQCD, PRD(2020)]
[Q.Chang,L.T.Wang,X.N.Li, JHEP(2019)]
[Q.Chang,et al., AHEP(2020)]

	NRQCD+Lattice	LFQM [7, 8]
V_1	$0.4320^{+0.0030}_{-0.0048} \pm 0.0448$	$0.56^{+0.01+0.17}_{-0.01-0.17}$
V_2	$0.2295^{+0.0003}_{-0.0004} \pm 0.0238$	$0.33^{+0.01+0.05}_{-0.01-0.04}$
V_3	$0.8865^{+0.0001}_{+0.0003} \pm 0.0919$	$1.17^{+0.02+0.23}_{-0.02-0.29}$
V_4	$0.4294^{-0.0009}_{+0.0018} \pm 0.0445$	$0.65^{+0.01+0.20}_{-0.01-0.19}$
V_5	$0.1303^{-0.0338}_{+0.0569} \pm 0.0135$	$0.20^{+0.00+0.02}_{-0.00-0.02}$
V_6	$0.1303^{-0.0338}_{+0.0569} \pm 0.0135$	$0.20^{+0.00+0.02}_{-0.00-0.02}$
A_1	$0.4458^{-0.0006}_{+0.0013} \pm 0.0462$	$0.54^{+0.01+0.16}_{-0.01-0.17}$
A_2	$0.2510^{-0.0053}_{+0.0090} \pm 0.0260$	$0.35^{+0.00}_{-0.00}$
A_3	$0.0942^{-0.0244}_{+0.0411} \pm 0.0098$	$0.13^{+0.00+0.03}_{-0.00-0.02}$
A_4	$0.1092^{-0.0284}_{+0.0477} \pm 0.0113$	$0.14^{+0.00+0.02}_{-0.00-0.02}$
T_2	$0.2352^{-0.0012}_{+0.0021} \pm 0.0244$	–
T_3	$0.5724^{-0.0035}_{+0.0062} \pm 0.0593$	–
T_4	$0.2790^{-0.0028}_{+0.0049} \pm 0.0289$	–
T_6	$0.2211^{-0.0131}_{+0.0221} \pm 0.0229$	–
T_2'	$0.4387^{+0.0012}_{-0.0018} \pm 0.0455$	–
T_3'	$0.4546^{+0.0115}_{-0.0190} \pm 0.0471$	–
T_4'	$0.3805^{-0.0136}_{+0.0230} \pm 0.0394$	–

NRQCD+Lattice predictions for $B_c^* \to \eta_c$ form factors at $q^2 = 0$

[HPQCD, 1611.01987]
[Q.Chang,L.L.Chen,S.Xu, JPG(2018)]
[P.R,R.Dhir, 1908.00242]
[Y.Yang,et al., CPC(2023)]
[Q.Chang,L.T.Wang,X.N.Li, JHEP(2019)]
[S.Y.Wang,et al., CPC(2024)]
[Z.G.Wang, CTP(2014)]

$$F_{i,\mathrm{NRQCD+lattice}}^{B_c^*\to\eta_c}(q^2) \approx \frac{1}{2}\sum_{j=1}^{2}\frac{F_{i,\mathrm{NRQCD}}^{B_c^*\to\eta_c}(q^2)}{F_{j,\mathrm{NRQCD}}^{B_c\to\eta_c}(q^2)} F_{j,\mathrm{lattice}}^{B_c\to\eta_c}(q^2), \qquad F_j^{B_c\to\eta_c} \in \{f_+, f_0\}^{B_c\to\eta_c}$$

	NRQCD+Lattice	LFQM		BSW	QCDSR
		[56]	[4]	[6, 7]	[8]
$V(0)$	$0.8062^{-0.0082}_{+0.0121} \pm 0.0682$	$0.91^{+0.02+0.18}_{-0.02-0.22}$	$0.88^{+0.01+0.04}_{-0.01-0.06}$	$0.753^{+0.107}_{-0.129}$	0.71 ± 0.14
$A_0(0)$	$0.5243^{-0.0008}_{+0.0012} \pm 0.0443$	$0.66^{+0.01+0.14}_{-0.01-0.17}$	$0.56^{+0.01+0.01}_{-0.02-0.02}$	0.526	0.47 ± 0.09
$A_1(0)$	$0.5410^{+0.0064}_{-0.0095} \pm 0.0457$	$0.69^{+0.01+0.17}_{-0.01-0.19}$	$0.59^{+0.01+0.02}_{-0.02-0.03}$	$0.561^{+0.080}_{-0.096}$	0.43 ± 0.08
$A_2(0)$	$0.4766^{-0.0211}_{+0.0314} \pm 0.0403$	$0.59^{+0.02+0.12}_{-0.02-0.13}$	$0.45^{+0.02+0.01}_{-0.02-0.02}$	$0.495^{+0.035}_{-0.062}$	0.57 ± 0.11
$T_{1,2}^{(\prime)}(0)$	$0.5475^{+0.0020}_{-0.0030} \pm 0.0463$	–	–	–	–

3.7 NRQCD+Lattice+Z-series predictions in full q^2 range

m_R: masses of low-lying $c\bar{b}$ resonances

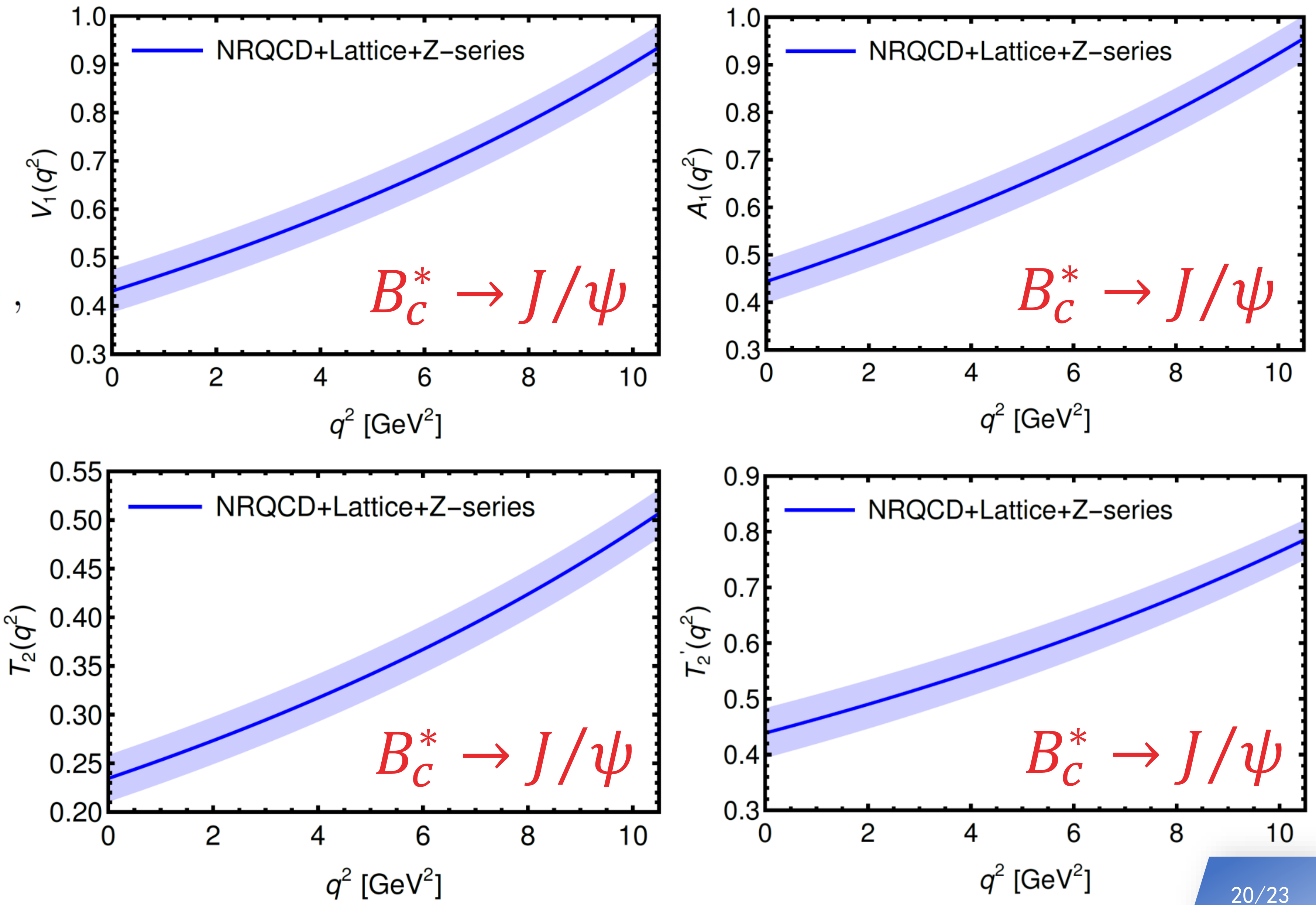
$$F_i(q^2) = \frac{1}{1 - \frac{q^2}{m_R^2}} \sum_{n=0}^{N} \alpha_{i,n}\, z^n\,(q^2)\,,$$

$$z(q^2) = \frac{\sqrt{t_+ - q^2} - \sqrt{t_+ - t_0}}{\sqrt{t_+ - q^2} + \sqrt{t_+ - t_0}},$$

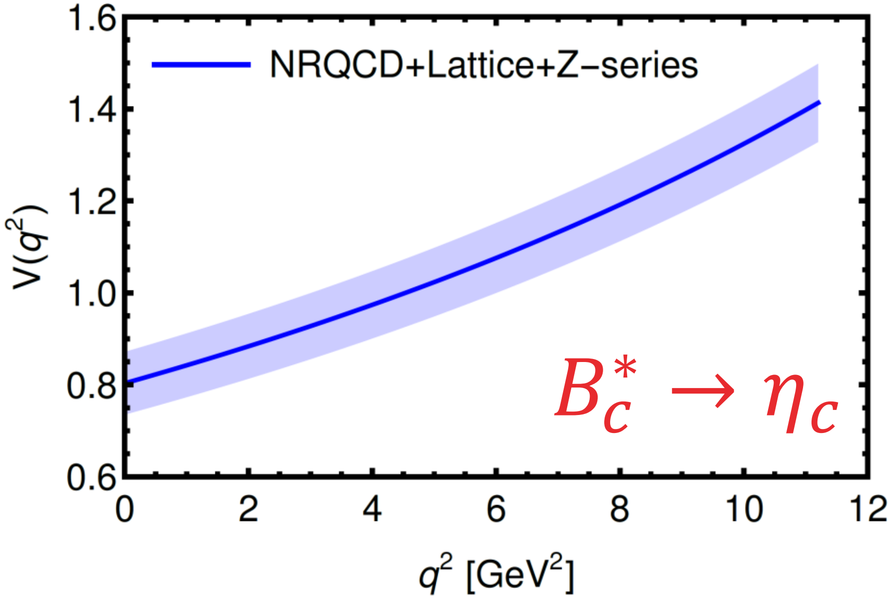
$$t_0 = t_+ \left(1 - \sqrt{1 - \frac{t_-}{t_+}}\right),$$

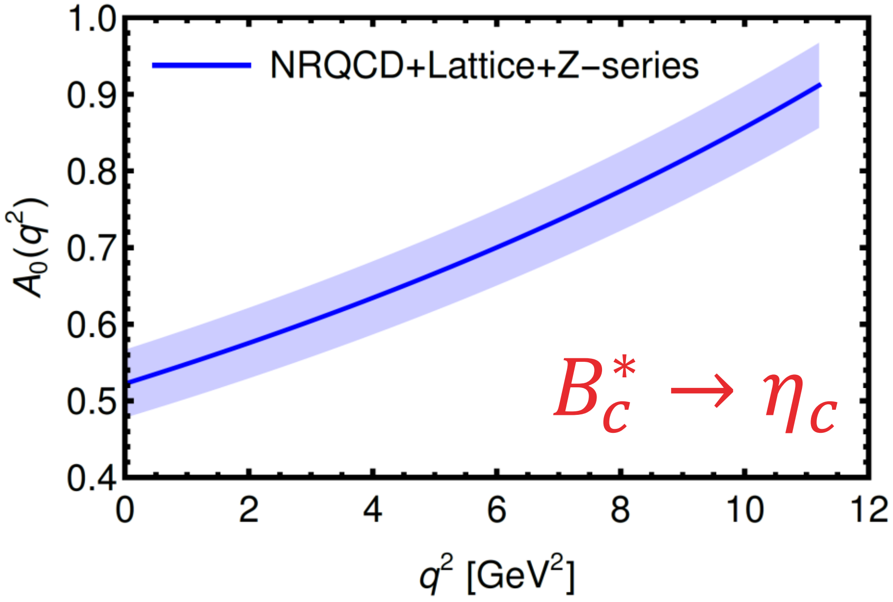
$$t_\pm = \left(m_{B_c^*} \pm m_{J/\psi(\eta_c)}\right)^2,$$

[C.G.Boyd,B.Grinstein,R.F.Lebed, PRD(1997)]
[D.Leljak,B.Melic,M.Patra, JHEP(2019)]

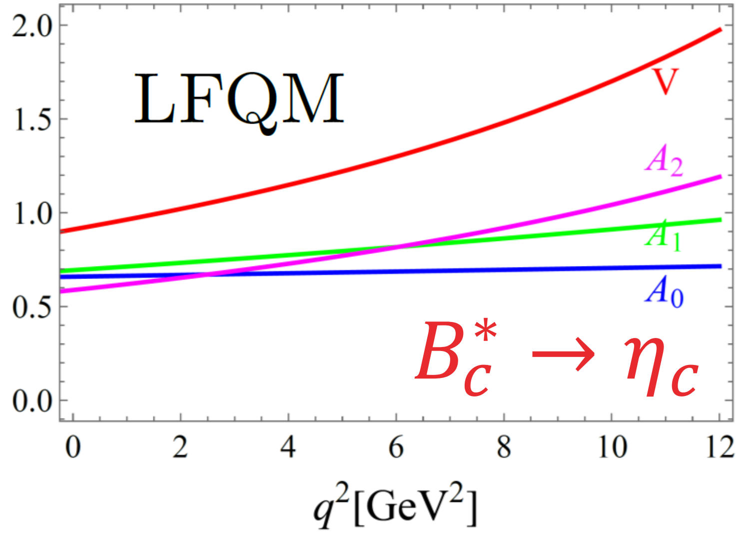


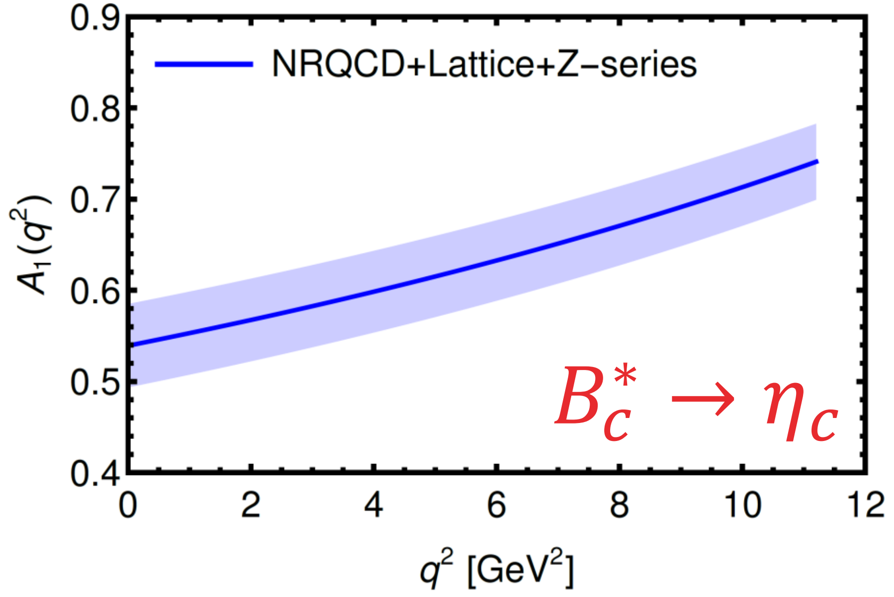
NRQCD+Lattice+Z-series predictions in full q^2 range

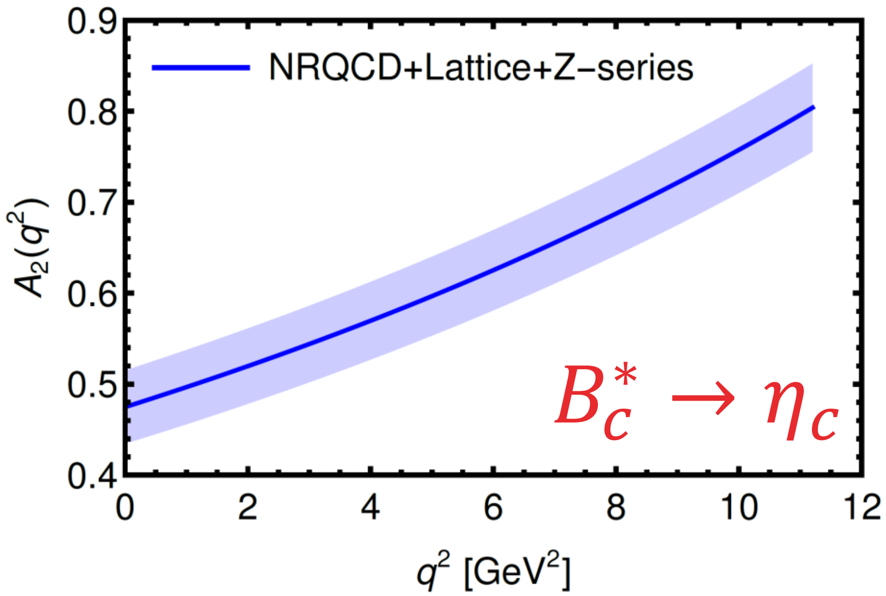




[Y.Yang,et al., CPC(2023)]
[Q.Chang,L.T.Wang,X.N.Li, JHEP(2019)]







Decay Widths & Branching Fractions for $B_c^* \to \eta_c + l\nu_l$

$$\frac{d\Gamma_L}{dq^2} = \left(\frac{q^2-m_l^2}{q^2}\right)^2 \frac{\sqrt{\lambda(q^2)}G_F^2|V_{cb}|^2}{384 m_{B_c^*}^3 \pi^3 q^2}\left\{3m_l^2\lambda(q^2)A_0^2(q^2)\right.$$
$$+\left|(m_{B_c^*}^2-m_{\eta_c}^2-q^2)(m_{B_c^*}+m_{\eta_c})A_1(q^2)\right.$$
$$\left.\left.-\frac{\lambda(q^2)}{m_{B_c^*}+m_{\eta_c}}A_2(q^2)\right|^2\frac{m_l^2+2q^2}{4m_{\eta_c}^2}\right\},$$

$$\frac{d\Gamma_\pm}{dq^2} = \left(\frac{q^2-m_l^2}{q^2}\right)^2 \frac{\sqrt{\lambda(q^2)}G_F^2|V_{cb}|^2}{384 m_{B_c^*}^3 \pi^3}\left\{(m_l^2+2q^2)\lambda(q^2)\right.$$
$$\left.\times\left|\frac{V(q^2)}{m_{B_c^*}+m_{\eta_c}} \mp \frac{(m_{B_c^*}+m_{\eta_c})A_1(q^2)}{\sqrt{\lambda(q^2)}}\right|^2\right\},$$

$$\frac{d\Gamma_T}{dq^2} = \frac{d\Gamma_+}{dq^2}+\frac{d\Gamma_-}{dq^2},\quad \frac{d\Gamma}{dq^2} = \frac{d\Gamma_L}{dq^2}+\frac{d\Gamma_T}{dq^2},$$

$$\Gamma(B_c^* \to \eta_c + l\nu_l) = \int_{m_l^2}^{(m_{B_c^*}-m_{\eta_c})^2}\frac{d\Gamma}{dq^2}dq^2.$$

	This work	LFQM [56]	LFQM [4]	BS [21]	QCDSR [8]
$\frac{10^{14}}{\text{GeV}}\Gamma(B_c^* \to \eta_c + e\nu_e)$	$2.541^{+0.494}_{-0.446}$	$4.20^{+1.63}_{-1.79}$	–	$0.966^{+0.094}_{-0.084}$	$0.686^{+0.225}_{-0.195}$
$\frac{10^{14}}{\text{GeV}}\Gamma(B_c^* \to \eta_c + \mu\nu_\mu)$	$2.529^{+0.489}_{-0.442}$	$4.18^{+1.73}_{-1.78}$	–	$0.963^{+0.094}_{-0.084}$	$0.684^{+0.224}_{-0.195}$
$\frac{10^{14}}{\text{GeV}}\Gamma(B_c^* \to \eta_c + \tau\nu_\tau)$	$0.688^{+0.104}_{-0.096}$	$1.09^{+0.35}_{-0.44}$	–	$0.290^{+0.029}_{-0.026}$	$0.215^{+0.075}_{-0.065}$
$10^7\mathcal{B}(B_c^* \to \eta_c + e\nu_e)$	$4.235^{+0.824}_{-0.743}$	$7.00^{+2.71}_{-2.98}$	$4.48^{+1.14}_{-0.76}$	$4.20^{+0.41}_{-0.37}$	–
$10^7\mathcal{B}(B_c^* \to \eta_c + \mu\nu_\mu)$	$4.214^{+0.816}_{-0.736}$	$6.96^{+2.89}_{-2.96}$	$4.45^{+1.14}_{-0.75}$	$4.19^{+0.41}_{-0.37}$	–
$10^7\mathcal{B}(B_c^* \to \eta_c + \tau\nu_\tau)$	$1.147^{+0.173}_{-0.161}$	$1.82^{+0.58}_{-0.73}$	$1.03^{+0.26}_{-0.17}$	$1.26^{+0.13}_{-0.11}$	–
$\mathcal{R}_{\eta_c}$	$0.271^{+0.107}_{-0.076}$	$0.26^{-0.01}_{+0.01}$	$0.229^{+0.059}_{-0.059}$	0.300	–

[S.Y.Wang,et al., CPC(2024)]
[T.Wang,et al., JPG(2018)]
[Z.G.Wang, CTP(2014)]
[Y.Yang,et al., CPC(2023)]
[Q.Chang,L.T.Wang,X.N.Li, JHEP(2019)]
[T.D.Cohen,H.Lamm,R.F.Lebed, PRD(2018)]

4 Summary

- Obtain complete and asymptotic analytical results for NLO QCD corrections to $B_c^* \to J/\psi\,(\eta_c)$ (axial-)vector, (axial-)tensor form factors
- Find NLO corrections reduce renormalization scale dependence, and are sizable but convergent in low q^2 region
- Provide NRQCD+Lattice+Z-series predictions for $B_c^* \to J/\psi\,(\eta_c)$ form factors across full physical q^2 range
- Calculate decay widths and branching fractions for $B_c^* \to \eta_c + l\nu_l$

Thank you!