# Revisiting semileptonic and leptonic $B$ decays via $U(2)^5$ flavor symmetry:

# SMEFT meets SM

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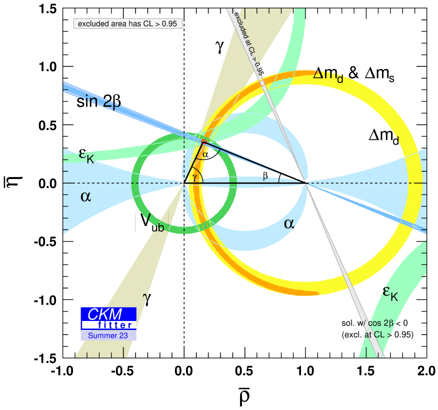
# Outlines

- Motivation
- $U(2)^5$ flavor symmetry
- Matching with CKM in SM
- Flavor structures in SMEFT
- Numerical discussions and conclusions

# $U(2)^5$ flavor symmetry




CKMfitter 2023

$$V_{\text{CKM}} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4)$$

"the success of the CKM picture be due to the existence of a suitable flavour symmetry appropriately broken in some definite direction, thus allowing a scale of new flavour physics phenomena sufficiently near to the Fermi scale to leave room for relatively small but nevertheless observable deviations from the SM in the flavour sector."

$U(2)_q \otimes U(2)_u \otimes U(2)_d$: act on the first two generations of quarks

$U(2)_\ell \otimes U(2)_e$: act the first two generations of leptons

A. Pomarol and D. Tommasini, hep-ph/9507462

R. Barbieri, G. R. Dvali and L. J. Hall, hep-ph/9512388

$$V_{\text{CKM}} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & s_u s e^{-i\delta} \\ -\lambda & 1-\lambda^2/2 & c_u s \\ -s_d s e^{i(\delta+\alpha_u-\alpha_d)} & -s c_d & 1 \end{pmatrix}$$

R. Barbieri, G. Isidori, J. Jones-Perez, P. Lodone and D. M. Straub, 1105.2296

M. Bordone, C. Cornella, J. Fuentes-Martín and G. Isidori, 1805.09328

# Plenty of literature on $U(2)^5$

**New physics in the third generation. A comprehensive SMEFT analysis and future prospects**

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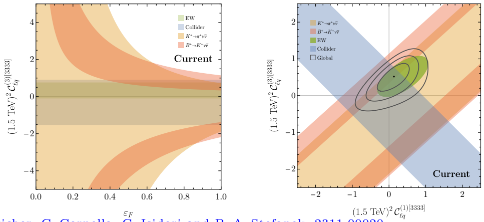
Abstract: We present a comprehensive analysis of electroweak, flavor, and collider bounds on the complete set of dimension-six SMEFT operators in the $U(2)^5$-symmetric limit. This




L. Allwicher, C. Cornella, G. Isidori and B. A. Stefanek, 2311.00020

**Correlating $\epsilon'/\epsilon$ with hadronic $B$ decays via $U(2)^3$ flavor symmetry**

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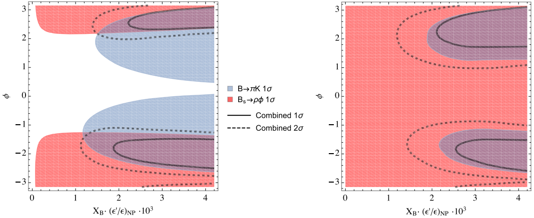
There are strong similarities between charge-parity ($CP$) violating observables in hadronic $B$ decays (in particular $\Delta A_{CP}^{-}$ in $B\to K\pi$) and direct $CP$ violation in kaon decays ($\epsilon'$): All these observables are very sensitive to new physics (NP) which is at the same time $CP$ and isospin violating (i.e., NP with complex couplings which are different for up quarks and down quarks). Intriguingly, both the measurements of $\epsilon'$




A. Crivellin, C. Gross, S. Pokorski and L. Vernazza, 1909.02101

# Some issues need further research!

$$L_d \approx \begin{pmatrix} c_d & -s_d\, e^{i\alpha_d} & 0 \\ s_d\, e^{-i\alpha_d} & c_d & s_b \\ -s_d\, s_b\, e^{-i(\alpha_d+\phi_q)} & -c_d\, s_b\, e^{-i\phi_q} & e^{-i\phi_q} \end{pmatrix},$$

$$R_d \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{m_s}{m_b}\, s_b \\ 0 & -\frac{m_s}{m_b}\, s_b\, e^{-i\phi_q} & e^{-i\phi_q} \end{pmatrix},$$

$$R_u \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{m_c}{m_t}\, s_t \\ 0 & -\frac{m_c}{m_t}\, s_t\, e^{-i\phi_q} & e^{-i\phi_q} \end{pmatrix},$$

$$L_e \approx \begin{pmatrix} c_e & -s_e & 0 \\ s_e & c_e & s_\tau \\ -s_e s_\tau & -c_e s_\tau & 1 \end{pmatrix},$$

$$R_e \approx \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{m_\mu}{m_\tau}\, s_\tau \\ 0 & -\frac{m_\mu}{m_\tau}\, s_\tau & 1 \end{pmatrix},$$

with $V_{\mathrm{CKM}} = L_u^\dagger L_d$.

$$\mathcal{O}_{ledq} = (\bar{l}_\alpha e_\beta)(\bar{d}_i q_j)$$

$$\Lambda_S^{[ij\alpha\beta]} = (\Gamma_L^\dagger)^{\alpha j} \times \Gamma_R^{i\beta}\,,$$

where, in the interaction basis,

$$\Gamma_L^{i\alpha} = \begin{pmatrix} x_{q\ell} V_q^i (V_\ell^\alpha)^* & x_q V_q^i \\ x_\ell (V_\ell^\alpha)^* & 1 \end{pmatrix}, \quad \Gamma_R = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}.$$

$$\hat{\Gamma}_L = e^{i\phi_q} \begin{pmatrix} \Delta_{q\ell}^{de} & \Delta_{q\ell}^{d\mu} & \lambda_q^d \\ \Delta_{q\ell}^{se} & \Delta_{q\ell}^{s\mu} & \lambda_q^s \\ \lambda_\ell^e & \lambda_\ell^\mu & x_{q\ell}^{b\tau} \end{pmatrix} \approx e^{i\phi_q} \begin{pmatrix} 0 & 0 & \lambda_q^d \\ 0 & \Delta_{q\ell}^{s\mu} & \lambda_q^s \\ \lambda_\ell^e & \lambda_\ell^\mu & 1 \end{pmatrix},$$

$$\hat{\Gamma}_R \approx e^{i\phi_q} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & -\frac{m_s}{m_b}\, s_b \\ 0 & -\frac{m_\mu}{m_\tau}\, s_\tau & 1 \end{pmatrix}.$$

$$\lambda_q^s = O(|V_q|)\,, \qquad \lambda_\ell^\mu = O(|V_\ell|)\,,$$

$$x_{q\ell}^{b\tau} = O(1)\,, \qquad \Delta_{q\ell}^{s\mu} = O(\lambda_q^s \lambda_\ell^\mu)\,,$$

$$\frac{\lambda_q^d}{\lambda_q^s} = \frac{\Delta_{q\ell}^{d\alpha}}{\Delta_{q\ell}^{s\alpha}} = \frac{V_{td}^*}{V_{ts}^*}\,, \qquad \frac{\lambda_\ell^e}{\lambda_\ell^\mu} = \frac{\Delta_{q\ell}^{ie}}{\Delta_{q\ell}^{i\mu}} = s_e\,.$$

# Theoretical framework

- $U(2)^5$ symmetry:

$$U(2)^5 = U(2)_q \otimes U(2)_u \otimes U(2)_d \otimes U(2)_\ell \otimes U(2)_e.$$

- Quantum number $(q, u, d, \ell, e)$:

$$q = \begin{pmatrix} q_{12} \sim (2,1,1,1,1) \\ q_3 \sim (1,1,1,1,1) \end{pmatrix}, \quad u = \begin{pmatrix} u_{12} \sim (1,2,1,1,1) \\ u_3 \sim (1,1,1,1,1) \end{pmatrix}, \quad d = \begin{pmatrix} d_{12} \sim (1,1,2,1,1) \\ d_3 \sim (1,1,1,1,1) \end{pmatrix}$$
$$\ell = \begin{pmatrix} \ell_{12} \sim (1,1,1,2,1) \\ \ell_3 \sim (1,1,1,1,1) \end{pmatrix}, \quad e = \begin{pmatrix} e_{12} \sim (1,1,1,1,2) \\ e_3 \sim (1,1,1,1,1) \end{pmatrix},$$

- Minimal *spurions*:

$$\Delta_e \sim (1,1,1,2,\bar{2}) \ , \qquad \Delta_u \sim (2,\bar{2},1,1,1) \ , \qquad \Delta_d \sim (2,1,\bar{2},1,1) \ .$$
$$V_\ell \sim (1,1,1,2,1) \ , \qquad V_q \sim (2,1,1,1,1)$$

# Yukawa interaction

$$(\bar{Q}_q, \bar{Q}_3) \begin{pmatrix} Y_d^{2\times 2} & Y_d^{2\times 1} \\ Y_d^{1\times 2} & Y_d^{33} \end{pmatrix} \begin{pmatrix} d_q \\ d_3 \end{pmatrix} H$$

$$\begin{pmatrix} Y_d^{2\times 2} & Y_d^{2\times 1} \\ Y_d^{1\times 2} & Y_d^{33} \end{pmatrix} \sim \begin{pmatrix} 2 \otimes \bar{2} & 2 \otimes \bar{1} \\ 1 \otimes \bar{2} & 1 \otimes \bar{1} \end{pmatrix} \qquad (q \otimes d)$$

- Yukawa matrices:

$$Y_u = |y_t| \begin{pmatrix} U_q^\dagger O_u^\intercal \hat{\Delta}_u & |V_q|\,|x_t|\, e^{i\phi_q}\, \vec{n} \\ 0 & 1 \end{pmatrix}, \qquad Y_d = |y_b| \begin{pmatrix} U_q^\dagger \hat{\Delta}_d & |V_q|\,|x_b|\, e^{i\phi_q}\, \vec{n} \\ 0 & 1 \end{pmatrix},$$

$$Y_e = |y_\tau| \begin{pmatrix} O_e^\intercal \hat{\Delta}_e & |V_\ell|\,|x_\tau|\, \vec{n} \\ 0 & 1 \end{pmatrix}$$

  - Diagonal positive $2 \times 2$ matrices: $\hat{\Delta}_{u,d,e}$, $\quad \vec{n} = (0,1)^T$, $\quad \hat{\Delta}_d^{1,2} = \epsilon_{d,s}$, $\quad |V_q||x_b| = \epsilon_b$
  - Unitary $U_q$ and Orthogonal $O_{u,e}$ matrices:

$$U_q = \begin{pmatrix} c_d & s_d\, e^{i\alpha_d} \\ -s_d\, e^{-i\alpha_d} & c_d \end{pmatrix}, \qquad O_u = \begin{pmatrix} c_u & s_u \\ -s_u & c_u \end{pmatrix}, \qquad O_e = \begin{pmatrix} c_e & s_e \\ -s_e & c_e \end{pmatrix}$$

# Diagonalization

- Unitary transformations:

$$L_f^\dagger Y_f R_f = \mathrm{diag}(Y_f) = \hat{Y}_f, \quad \text{with} f = u, d, e,$$

- Arbitrary phase:

$$P_u = \begin{pmatrix} -e^{i\phi_x} & 0 & 0 \\ 0 & -e^{i\phi_y} & 0 \\ 0 & 0 & e^{i\phi_z} \end{pmatrix}, \qquad P_d = \begin{pmatrix} -e^{i\phi_a} & 0 & 0 \\ 0 & -e^{i\phi_b} & 0 \\ 0 & 0 & e^{i\phi_c} \end{pmatrix},$$

$$P_f^\dagger L_f^\dagger Y_f R_f P_f = P_f^\dagger \hat{Y}_f P_f = \hat{Y}_f$$

  - $L'_f = L_f P_f, \qquad R'_f = R_f P_f$

# Results

- General CKM:

$$V_{\text{CKM}} = \begin{pmatrix} c_u e^{i\alpha_d - i\theta + i\phi_a - i\phi_x} & s_u e^{-i\theta + i\phi_b - i\phi_x} & x_{bt}\lambda e^{i\phi_c - i\phi_x} \\ -s_u e^{i\alpha_d - i\delta + i\phi_a - i\phi_y} & c_u e^{-i\delta + i\phi_b - i\phi_y} & x_{bt}\chi e^{i\phi_c - i\phi_y} \\ -x_{bt} s_d e^{i\phi_a - i\phi_z} & -x_{bt} c_d e^{i\phi_b - i\phi_z} & e^{i\phi_c - i\phi_z} \end{pmatrix} + \mathcal{O}(\epsilon^2)$$

  - $\epsilon = m_d/m_s$, $\epsilon_s \sim \epsilon_b \approx \mathcal{O}(\epsilon)$, $\lambda e^{i\theta} = (c_d s_u + s_d c_u e^{i\alpha_d})$, $\chi e^{i\delta} = (c_d c_u - s_d s_u e^{i\alpha_d})$

- Matching with SM Wolfstein parameterization:
  - $\phi_z = \phi_b = \phi_c$, $\phi_x = \phi_b - \theta$, $\phi_y = \phi_b - \delta$, $\phi_a = \phi_b - \alpha_d$

$$V_{\text{CKM}} = \begin{pmatrix} c_u & s_u & x_{bt}(c_u s_d e^{i\alpha_d} + s_u c_d) \\ -s_u & c_u & x_{bt}(c_u c_d - s_u s_d e^{i\alpha_d}) \\ -x_{bt} s_d e^{-i\alpha_d} & -x_{bt} c_d & 1 \end{pmatrix} + \mathcal{O}(\epsilon^2)$$

where $x_{bt} = \epsilon_b - \epsilon_t$

# Matching with SM Wolfstein parameterization

$$V_{\rm CKM} = \begin{pmatrix} 1-\lambda^2/2 & \lambda & A\lambda^3(\rho - i\eta) \\ -\lambda & 1-\lambda^2/2 & A\lambda^2 \\ A\lambda^3(1-\rho-i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4),$$

$$V_{\rm CKM}^{\rm U(2)} = \begin{pmatrix} c_u & s_u & x_{bt}(c_u s_d e^{i\alpha_d} + s_u c_d) \\ -s_u & c_u & x_{bt}(c_u c_d - s_u s_d e^{i\alpha_d}) \\ -x_{bt} s_d e^{-i\alpha_d} & -x_{bt} c_d & 1 \end{pmatrix} + \mathcal{O}(\epsilon^2)$$

We set

- $s_d \approx \mathcal{O}(\lambda)$, then $x_{bt} \approx \mathcal{O}(|V_{\rm CKM}^{ts}|) \approx \mathcal{O}(\lambda^2)$
- $\lambda = s_u, \quad A = \frac{x_{bt} c_d}{s_u^2}, \quad \rho = 1 + \frac{x_{bt} s_d \cos\alpha_d}{x_{bt} c_d s_u}, \quad \eta = -\frac{x_{bt} s_d \sin\alpha_d}{x_{bt} c_d s_u}$

We can get

- $-x_{bt} s_d e^{-i\alpha_d} = A\lambda^3(1-\rho-i\eta) = V_{\rm CKM}^{td}$
- $|s_d| = |V_{\rm CKM}^{td} c_d / A\lambda^2|, \quad e^{-i\alpha_d} = (V_{\rm CKM}^{td}/V_{\rm CKM}^{ts})(c_d/s_d)$

# Matching with SM Wolfstein parameterization

Use power counting for $\lambda$:

$$V_{\text{CKM}} = \begin{pmatrix} c_u & s_u & x_{bt}(c_u s_d e^{i\alpha_d} + s_u c_d) \\ -s_u & c_u & x_{bt}(c_u c_d - s_u s_d e^{i\alpha_d}) \\ -x_{bt} s_d e^{-i\alpha_d} & -x_{bt} c_d & 1 \end{pmatrix} + \mathcal{O}(\epsilon^2)$$

$$= \begin{pmatrix} 1 - \lambda^2/2 & \lambda & V_{\text{CKM}}^{ub} \\ -\lambda & 1 - \lambda^2/2 & V_{\text{CKM}}^{cb} \\ A\lambda^3(1 - \rho - i\eta) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4),$$

$$\begin{aligned} V_{\text{CKM}}^{ub} =& (A\lambda^3 + x_{bt} s_d \cos\alpha_d) + i x_{bt} s_d \sin\alpha_d + \mathcal{O}(\lambda^4) \\ =& A\lambda^3(\rho - i\eta) + \mathcal{O}(\lambda^4) \\ V_{\text{CKM}}^{cb} =& A\lambda^2 + \mathcal{O}(\lambda^4) \end{aligned}$$

# Comparison with the literature ($\mathcal{O}(\epsilon)$)

$$L_d = \begin{pmatrix} c_d & -s_d\, e^{i\alpha_d} & 0 \\ s_d\, e^{-i\alpha_d} & c_d & \epsilon_b \epsilon \\ -s_d \epsilon_b \epsilon e^{-i(\phi_q+\alpha_d)} & -c_d \epsilon_b \epsilon e^{-i\phi_q} & e^{-i\phi_q} \end{pmatrix}$$

$$R_d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \color{red}{0} \\ 0 & \color{red}{0} & e^{-i\phi_q} \end{pmatrix}$$

$$R_u = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \color{red}{0} \\ 0 & \color{red}{0} & e^{-i\phi_q} \end{pmatrix}$$

$$L_d = \begin{pmatrix} c_d & -s_d\, e^{i\alpha_d} & 0 \\ s_d\, e^{-i\alpha_d} & c_d & s_b \\ -s_d\, s_b\, e^{-i(\alpha_d+\phi_q)} & -c_d\, s_b\, e^{-i\phi_q} & e^{-i\phi_q} \end{pmatrix}$$

$$R_d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \color{red}{\frac{m_s}{m_b}\, s_b} \\ 0 & \color{red}{-\frac{m_s}{m_b}\, s_b\, e^{-i\phi_q}} & e^{-i\phi_q} \end{pmatrix}$$

$$R_u = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \color{red}{\frac{m_c}{m_t}\, s_t} \\ 0 & \color{red}{-\frac{m_c}{m_t}\, s_t\, e^{-i\phi_q}} & e^{-i\phi_q} \end{pmatrix}$$

**D. A. Faroughy, G. Isidori, F. Wilsch and K. Yamamoto, 2005.05366**

# Comparison with the literature ($\mathcal{O}(\epsilon)$)

$$L_d = \begin{pmatrix} c_d & -s_d\, e^{i\alpha_d} & 0 \\ s_d\, e^{-i\alpha_d} & c_d & \epsilon_b\epsilon \\ -s_d\epsilon_b\epsilon e^{-i(\phi_q+\alpha_d)} & -c_d\epsilon_b\epsilon e^{-i\phi_q} & e^{-i\phi_q} \end{pmatrix}$$

$$R_d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \epsilon_s\epsilon_b c_d \\ 0 & -\epsilon_s\epsilon_b c_d e^{-i\phi_q} & e^{-i\phi_q} \end{pmatrix}$$

$$R_u = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \chi\epsilon_c\epsilon_t\epsilon^2 e^{i\delta} \\ 0 & -\chi\epsilon_c\epsilon_t\epsilon^2 e^{-i(\phi_q+\delta)} & e^{-i\phi_q} \end{pmatrix}$$

$$L_d = \begin{pmatrix} c_d & -s_d\, e^{i\alpha_d} & 0 \\ s_d\, e^{-i\alpha_d} & c_d & s_b \\ -s_d\, s_b\, e^{-i(\alpha_d+\phi_q)} & -c_d\, s_b\, e^{-i\phi_q} & e^{-i\phi_q} \end{pmatrix}$$

$$R_d = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{m_s}{m_b}\, s_b \\ 0 & -\frac{m_s}{m_b}\, s_b\, e^{-i\phi_q} & e^{-i\phi_q} \end{pmatrix}$$

$$R_u = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & \frac{m_c}{m_t}\, s_t \\ 0 & -\frac{m_c}{m_t}\, s_t\, e^{-i\phi_q} & e^{-i\phi_q} \end{pmatrix}$$

**D. A. Faroughy, G. Isidori, F. Wilsch and K. Yamamoto, 2005.05366**

# SMEFT

- Semileptonic and leptonic decays at tree-level:

$$
\begin{aligned}
\mathcal{Q}_{lq}^{(1)} &= (\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t), & \mathcal{Q}_{lq}^{(3)} &= (\bar{l}_p\gamma_\mu\tau^I l_r)(\bar{q}_s\gamma^\mu\tau^I q_t),\\
\mathcal{Q}_{ld} &= (\bar{l}_p\gamma_\mu l_r)(\bar{d}_s\gamma^\mu d_t), & \mathcal{Q}_{qe} &= (\bar{q}_p\gamma^\mu q_r)(\bar{e}_s\gamma^\mu e_t),\\
\mathcal{Q}_{ed} &= (\bar{e}_p\gamma_\mu e_r)(\bar{d}_s\gamma^\mu d_t), & \mathcal{Q}_{ledq} &= (\bar{l}_p^j e_r)(\bar{d}_s q_t^j),\\
\mathcal{Q}_{lequ}^{(1)} &= (\bar{l}_p^j e_r)\varepsilon_{ik}(\bar{q}_s^k u_t), & \mathcal{Q}_{lequ}^{(3)} &= (\bar{l}_p^j\sigma_{\mu\nu} e_r)\varepsilon_{ik}(\bar{q}_s^k\sigma^{\mu\nu} u_t),
\end{aligned}
$$

- Quark and lepton bilinear:

$$
\begin{array}{lllll}
\Gamma_{qq}\bar{q}\gamma^\mu q, & \Gamma_{dd}\bar{d}\gamma^\mu d, & \Gamma_{dq}\bar{d}q, & \Gamma_{qu}\bar{q}u, & \Gamma_{qu}\bar{q}\sigma^\mu u,\\
\Gamma_{ll}\bar{l}\gamma_\mu l, & \Gamma_{ee}\bar{e}\gamma_\mu e, & \Gamma_{le}\bar{l}e, & \Gamma_{le}\bar{l}\sigma_{\mu\nu}e, &
\end{array}
$$

# Power-counting of *spurions*

$$\Delta_e \sim (1,1,1,2,\bar{2}) \ , \qquad \Delta_u \sim (2,\bar{2},1,1,1) \ , \qquad \Delta_d \sim (2,1,\bar{2},1,1) \ ,$$
$$V_\ell \sim (1,1,1,2,1) \ , \qquad V_q \sim (2,1,1,1,1)$$

$$\mathcal{L}_{\rm eff} = C_S(\bar{l}_p^j \bar{d}_s)\Lambda_{pstr}(q_t^j e_r) + \cdots$$

$$\begin{aligned}\Lambda_{pstr} =& A\{V_q^{\dagger n_q'}, V_q^{n_q}, V_l^{\dagger n_l'}, V_l^{n_l}, \Delta_d^{\dagger n_d'}, \Delta_d^{n_d}, \Delta_u^{\dagger n_u'}, \Delta_u^{n_u}, \Delta_e^{\dagger n_e'}, \Delta_e^{n_e}\} \\ & \sim (f(n_q + n_d + n_u - n_q' - n_d' - n_u'), f(n_u' - n_u), f(n_d' - n_d), \\ & f(n_l + n_e - n_l' - n_e'), f(n_e' - n_e))\end{aligned}$$

- quantum-number function: $f(-1) = \bar{2}$, $f(0) = 1$, and $f(1) = 2$.
- fermion-generation variables: $Q$, $D$, $L$, $E = 1$ (first two generations) or 0 (the 3rd one).

$$n_q + n_d - n_q' - n_d' = -Q, \qquad n_l + n_e - n_l' - n_e' = L,$$
$$n_d' - n_d = D, \qquad n_u' - n_u = 0 \qquad n_e' - n_e = -E.$$

# Wilson coefficients

$$[\Lambda^{(1)}_{lq}]_{prst} \sim [\Lambda^{(3)}_{\ell q}]_{prst} = \begin{pmatrix} a_l & x_l V_l \\ x_l^* V_l^\dagger & a_l' \end{pmatrix}_{pr} \times \begin{pmatrix} a_q & x_q V_q \\ x_q^* V_q^\dagger & a_q' \end{pmatrix}_{st},$$

$$[\Lambda_{ld}]_{prst} = \begin{pmatrix} a_l & x_l V_l \\ x_l^* V_l^\dagger & a_l' \end{pmatrix}_{pr} \times \begin{pmatrix} a_d & \Delta_d^\dagger x_{dq} V_q \\ V_q^\dagger x_{dq}^\dagger \Delta_d & a_d' \end{pmatrix}_{st},$$

$$[\Lambda_{qe}]_{prst} = \begin{pmatrix} a_q & x_q V_q \\ x_q^* V_q^\dagger & a_q' \end{pmatrix}_{pr} \times \begin{pmatrix} a_e & \Delta_e^\dagger x_{el} V_l \\ V_l^\dagger x_{el}^\dagger \Delta_e & a_e' \end{pmatrix}_{st},$$

$$[\Lambda_{ed}]_{prst} = \begin{pmatrix} a_e & \Delta_e^\dagger x_{el} V_l \\ V_l^\dagger x_{el}^\dagger \Delta_e & a_e' \end{pmatrix}_{pr} \times \begin{pmatrix} a_d & \Delta_d^\dagger x_{dq} V_q \\ V_q^\dagger x_{dq}^\dagger \Delta_d & a_d' \end{pmatrix}_{st},$$

$$[\Lambda_{ledq}]_{prst} = \begin{pmatrix} x_e \Delta_e & x_l V_l \\ V_l^\dagger x_{el}^\dagger \Delta_e & x_e' \end{pmatrix}_{pr} \times \begin{pmatrix} x_d \Delta_d^\dagger & \Delta_d^\dagger x_{dq} V_q \\ x_q V_q^\dagger & x_d' \end{pmatrix}_{st},$$

$$[\Lambda^{(1)}_{lequ}]_{prst} \sim [\Lambda^{(3)}_{lequ}]_{prst} = \begin{pmatrix} x_e \Delta_e & x_l V_l \\ V_l^\dagger x_{el}^\dagger \Delta_e & x_e' \end{pmatrix}_{pr} \times \begin{pmatrix} x_u \Delta_u & x_q V_q \\ V_q^\dagger x_{uq}^\dagger \Delta_u & x_u' \end{pmatrix}_{st}.$$

# Rotating to the mass basis

$\mathcal{Q}_{lq}^{(1)} = (\bar{l}_p\gamma_\mu l_r)(\bar{q}_s\gamma^\mu q_t)$:

$$\hat{\Gamma}^{ee} = \begin{pmatrix} a_l - s_e^2 k'_{l\tau}\epsilon_\tau^2 & -c_e s_e k'_{l\tau}\epsilon_\tau^2 & s_e k_{l\tau}\epsilon_\tau \\ -c_e s_e k'_{l\tau}\epsilon_\tau^2 & a_l - c_e^2 k'_{l\tau}\epsilon_\tau^2 & c_e k_{l\tau}\epsilon_\tau \\ s_e k_{l\tau}\epsilon_\tau & c_e k_{l\tau}\epsilon_\tau & a'_l + k'_{l\tau}\epsilon_\tau^2 \end{pmatrix},$$

$$\hat{\Gamma}^{dd} = \begin{pmatrix} a_q - s_d^2 k'_{qb}\epsilon_b^2 & -c_d s_d k'_{qb} K_{ds}^*\epsilon_b^2 & s_d k_{qb} K_{ds}^*\epsilon_b \\ -c_d s_d k'_{qb} K_{ds}\epsilon_b^2 & a_q - c_d^2 k'_{qb}\epsilon_b^2 & c_d k_{qb}\epsilon_b \\ s_d k_{qb} K_{ds}\epsilon_b & c_d k_{qb}\epsilon_b & a'_q + k'_{qb}\epsilon_b^2 \end{pmatrix},$$

$$\hat{\Gamma}^{\nu\nu} = \begin{pmatrix} a_l & 0 & 0 \\ 0 & a_l & \frac{x_l}{x_\tau}\epsilon_\tau \\ 0 & \frac{x_l}{x_\tau}\epsilon_\tau & a'_l \end{pmatrix},$$

$$\hat{\Gamma}^{uu} = \begin{pmatrix} a_q - \lambda^2\epsilon_t(\frac{2x_q\epsilon_b}{x_b} + x_{qq}\epsilon_t) & -\lambda\chi e^{-i(\delta-\theta)}\epsilon_t(\frac{2x_q\epsilon_b}{x_b} + x_{qq}\epsilon_t) & \lambda e^{i\theta}(\frac{x_q\epsilon_b}{x_b} + x_{qq}\epsilon_t) \\ -\lambda\chi e^{i(\delta-\theta)}\epsilon_t(\frac{2x_q\epsilon_b}{x_b} + x_{qq}\epsilon_t) & a_q - \chi^2\epsilon_t(\frac{2x_q\epsilon_b}{x_b} + x_{qq}\epsilon_t) & \chi e^{i\delta}(\frac{x_q\epsilon_b}{x_b} + x_{qq}\epsilon_t) \\ \lambda e^{-i\theta}(\frac{x_q\epsilon_b}{x_b} + x_{qq}\epsilon_t) & \chi e^{-i\delta}(\frac{x_q\epsilon_b}{x_b} + x_{qq}\epsilon_t) & a'_q + \epsilon_t(\frac{2x_q\epsilon_b}{x_b} + x_{qq}\epsilon_t) \end{pmatrix},$$

where $k_{l\tau} = a_l - a'_l + \frac{x_l}{x_\tau}$, $k'_{l\tau} = a_l - a'_l + \frac{2x_l}{x_\tau}$, $k_{qb} = a_q - a'_q + \frac{x_q}{x_b}$, $k'_{qb} = a_q - a'_q + \frac{2x_q}{x_b}$, $x_{qq} = a_q - a'_q$.

# Rotating to the mass basis

$\mathcal{Q}_{ledq} = (\bar{l}_p^j e_r)(\bar{d}_s q_t^j)$:

$$\hat{\Gamma}^{e_L e} = \begin{pmatrix} x_e\epsilon_e & 0 & -k_{\mu\tau}s_e\epsilon_\tau \\ 0 & x_e\epsilon_\mu & -k_{\mu\tau}c_e\epsilon_\tau \\ 0 & (x_e - k'_{\mu\tau})c_e\epsilon_\mu\epsilon_\tau & x'_e + (-k_{\mu\tau} + \frac{x'_e}{2})\epsilon_\tau^2 \end{pmatrix},$$

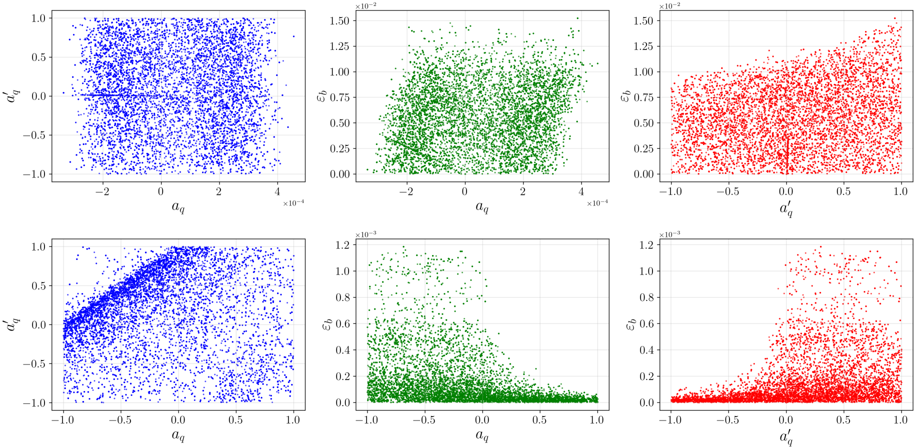
$$(\hat{\Gamma}^{dd_L})^\dagger = \begin{pmatrix} x_d\epsilon_d & 0 & -k_{sb}K_{ds}^* c_d\epsilon_b \\ 0 & x_d\epsilon_s & -k_{sb}c_d\epsilon_b \\ 0 & (x_d - k'_{sb})c_d\epsilon_s\epsilon_b & x'_d + (-k_{sb} + \frac{x'_d}{2})\epsilon_b^2 \end{pmatrix},$$

$$\hat{\Gamma}^{\nu e} = \begin{pmatrix} x_e c_e\epsilon_e & -x_e s_e\epsilon_\mu & 0 \\ x_e s_e\epsilon_e & x_e c_e\epsilon_\mu & \frac{x_l}{x_\tau}\epsilon_\tau \\ 0 & -k'_{\mu\tau}c_e\epsilon_\mu\epsilon_\tau & x'_e \end{pmatrix},$$

$$(\hat{\Gamma}^{du_L})^\dagger = \begin{pmatrix} x_d c_u\epsilon_d & x_d s_u\epsilon_s & c_d(s_u + c_u K_{ds}^*)(\frac{x_q}{x_b}\epsilon_b - x'_d\epsilon_t) \\ -x_d s_u\epsilon_d & x_d c_u\epsilon_s & c_d(c_u - s_u K_{ds}^*)(\frac{x_q}{x_b}\epsilon_b - x'_d\epsilon_t) \\ 0 & -c_d k'_{sb}\epsilon_b\epsilon_s + x_d c_d\epsilon_t\epsilon_s & x'_d + \frac{x_q}{x_b}\epsilon_b\epsilon_s - \frac{x'_d}{2}\epsilon_t^2 \end{pmatrix},$$

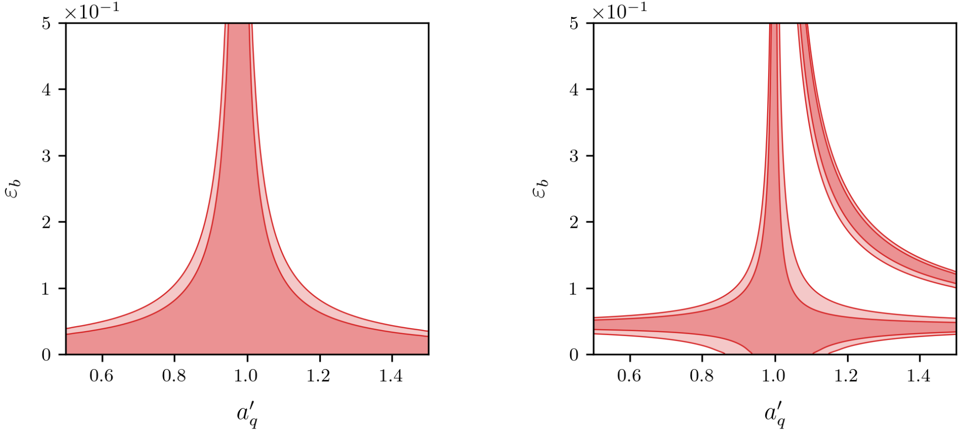
where $K_{ds} = V_{td}/V_{ts}$, $k_{\mu\tau} = x'_e - \frac{x_l}{x_\tau}$, $k_{sb} = x'_d - \frac{x_q}{x_b}$, $k'_{\mu\tau} = x'_e - \frac{x_{el}}{x_\tau}$, $k'_{sb} = x'_d - \frac{x_{dq}}{x_b}$.

# Numerical discussions




Parameters of $C_{lq}^{(1)}$ constrained by the leptonic processes of $b \to d\ell^+\ell^-$ (upper) and the $b \to s\ell^+\ell^-$ (lower).

# Numerical discussions




Parameters of $C_{ledq}$ constrained by the leptonic processes of $b \to s\tau^+\tau^-$ process (left) and $b \to u\tau\nu$ (right).

# Conclusions

In the $U(2)^5$ flavor symmetry framework, we obtain the following analytical results with strict power counting:

- the CKM of $U(2)^5$ matching with Standard model.
- the flavour structures in Wilson coefficients of semileptonic and leptonic $B$ decays.

Then some numerical calculations are also performed.

Thank you !!