Matching heavy meson three-body HQET hadronic parameters onto QCD

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Preliminary results

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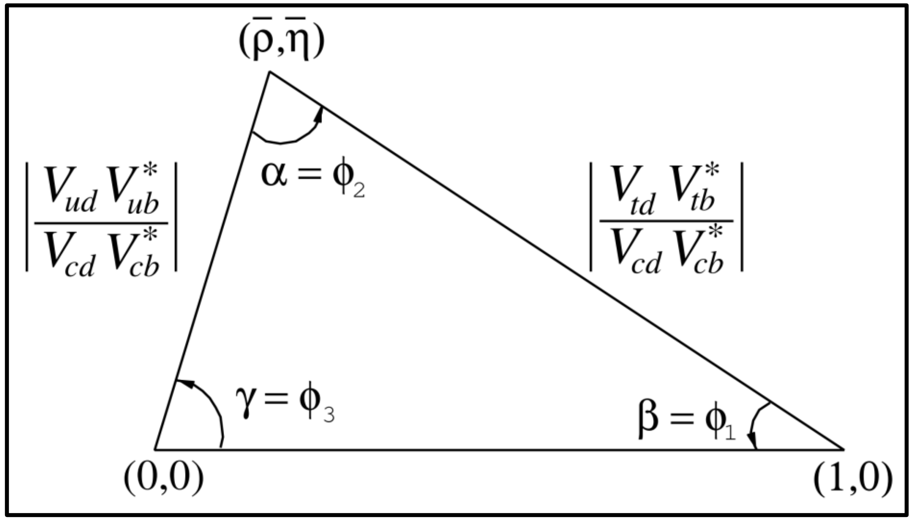
B-meson physics

- B mesons provide phenomenology to study the Standard Model (SM) of particle physics and investigate potential new physics effects.



$b \to c\ell\bar{\nu}_\ell$ $\quad$ $b \to u\ell\bar{\nu}_\ell \; (\ell = e, \mu)$

$\bar{B} \to X\ell\bar{\nu}_\ell$ $\quad$ $X = D,\ D^*,\ \pi,\ \rho$ etc.

$$|V_{ub}|/|V_{cb}| = 0.098 \pm 0.006 \quad \text{(inclusive)},$$
$$|V_{ub}|/|V_{cb}| = 0.093 \pm 0.004 \quad \text{(exclusive)}.$$

B mesons decay constant in HQET

- Decay constants of B mesons is an important measurement for B meson decay

$$\langle 0|\bar{q}\gamma^{\mu}\gamma_5 Q(0)\,|P(p)\rangle = -i f_P\, p^{\mu}.$$

$$\langle 0|\bar{q}\,\gamma^{\mu}\,Q(0)\,|P^*(p,\epsilon)\rangle = f_{P*}\,\epsilon^{\mu}$$

$f_B = 0.195 \pm 0.013\ \mathrm{GeV},$ by LQCD. (HPQCD Collaboration 2009)

$f_B = 0.195 \pm 0.0011 GeV$ by LQCD (Fermilab/MILC collaboration 2009)

$f_B\big|_{(\text{Belle+Babar avg.})} = 233 \pm 37\ \mathrm{MeV}.$ $\mathscr{B}(B^+ \to \tau^+ \nu)$ (Belle and Babar collaborations 2009)

$(1\sigma\ higher\ than\ LQCD\ results)$

- The decay constant represents the quantitative content of the quark-antiquark valence component inside the B meson

B mesons decay constants in HQET

- In heavy quark limit $\Lambda_{\rm QCD}/m_B \ll 1, \ \ \Lambda_{\rm QCD}/m_B \ll 1,$
  $\bar{q}\,\Gamma^\mu Q(0) = \bar{q}\,\Gamma^\mu Q_v(0)$ at tree level

- Decay constant in HQET: $\langle 0|\overline{q}\gamma_\rho\gamma_5 h_v|\bar{B}(v)\rangle = iF(\mu)v_\rho.$
  at tree level: $f_B\sqrt{m_B} \ = \ F(\mu)$

- At one-loop level

$$
\begin{aligned}
f_B\sqrt{m_B} \ &= \ F(\mu)\left[1+\frac{C_F\alpha_s}{4\pi}\left(3\ln\frac{m_b}{\mu}-2\right)+\dots\right] \\
&+ \ O(1/m_b),
\end{aligned}
$$

- To probe more details of B-meson, more operators are introduced.

B mesons decay constants in HQET

- More precise description of B meson requires three-body parameters

$$\langle 0|\bar{q}\boldsymbol{\alpha}\cdot g\boldsymbol{E}\gamma_5 h_v|\bar{B}(v)\rangle = F(\mu)\lambda_E^2(\mu), \quad E_i = G_{0i}, \quad \alpha_i = \gamma^0\gamma^i \quad G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu$$

$$\langle 0|\bar{q}\boldsymbol{\sigma}\cdot g\boldsymbol{H}\gamma_5 h_v|\bar{B}(v)\rangle = iF(\mu)\lambda_H^2(\mu), \quad H_i = -\tfrac{1}{2}\epsilon_{ijk}G_{jk}, \quad \sigma^i = i\epsilon^{ijk}\gamma_j\gamma_k\frac{1}{2} \quad \hat{G}_{\mu\nu} = G^a_{\mu\nu}T^a$$

- High Fock component contribution $|b\bar{q}g\rangle$ contained in $|B>$ hadronic state.
- $\lambda_{E,H}$ are related to the chromo-electric and chromo-magnetic fields in the B meson rest frame.
- They could affect the amplitudes for the exclusive B-meson decays at the leading power.

Effects of three body parameters

- OPE for B-meson LCDA

$$\tilde{\phi}_+(t,\mu) = 1 - \frac{\alpha_s C_F}{4\pi}\left(2L^2 + 2L + \frac{5\pi^2}{12}\right) - it\frac{4\bar{\Lambda}}{3}\left[1 - \frac{\alpha_s C_F}{4\pi}\left(2L^2 + 4L - \frac{9}{4} + \frac{5\pi^2}{12}\right)\right]$$

$$-t^2\bar{\Lambda}^2\left[1 - \frac{\alpha_s C_F}{4\pi}\left(2L^2 + \frac{16}{3}L - \frac{35}{9} + \frac{5\pi^2}{12}\right)\right] - \frac{t^2\boxed{\lambda_E^2(\mu)}}{3}\left[1 - \frac{\alpha_s C_F}{4\pi}\left(2L^2 + 2L - \frac{2}{3} + \frac{5\pi^2}{12}\right)\right.$$

$$\left. + \frac{\alpha_s C_G}{4\pi}\left(\frac{3}{4}L - \frac{1}{2}\right)\right] - \frac{t^2\boxed{\lambda_H^2(\mu)}}{6}\left[1 - \frac{\alpha_s C_F}{4\pi}\left(2L^2 + \frac{2}{3} + \frac{5\pi^2}{12}\right) - \frac{\alpha_s C_G}{8\pi}(L-1)\right],$$

$$\bar{\Lambda} = m_B - m_b$$

Kawamura, Tanaka, 2009

- LCDA received contributions from multi-body HQET parameters that associated with long-distance gluon fields inside the heavy meson
- High precision study on heavy meson structure requires the knowledge of λ_E and λ_H

Effects of three body parameters

- $\lambda_{E/H}$ affects the $B \to \pi$ form factor

$$f^0_{B\to\pi}(0) = 0.122 \times \Big[1 \pm 0.07|_{S_0^\pi} \pm 0.11|_{\Lambda_q} \boxed{\pm 0.02|_{\lambda_E^2/\lambda_H^2}} {}^{+0.05}_{-0.06}|_{M^2} \boxed{\pm 0.05|_{2\lambda_E^2+\lambda_H^2}} {}^{+0.06}_{-0.10}|_{\mu_h} \pm 0.04|_{\mu} {}^{+1.36}_{-0.56}|_{\lambda_B} {}^{+0.25}_{-0.43}|_{\sigma_1,\sigma_2}\Big].$$

J. Gao,C. D. Lv, Y. L. Shen, Y.M. Wang, Y.B. Wei, PRD 2020

B.Y. Cui, Y.K. Huang, Y.L.Shen, C. Wang, Y.M. Wang, JHEP 2023

LPC, PRD 2025

- Could be related to the moment of the subleading distribution amplitude

$$\langle 0|(\bar{q}_s S_n)(\tau_1 n)(S_n^\dagger S_{\bar{n}})(0)(S_{\bar{n}}^\dagger g_s G_{\mu\nu} S_{\bar{n}})(\tau_2 \bar{n})\bar{n}^\nu \not{n} \gamma_\perp^\mu \gamma_5 (S_{\bar{n}}^\dagger h_v)(0)|\bar{B}_v\rangle$$
$$= 2\tilde{f}_B(\mu) m_B \int_{-\infty}^{+\infty} d\omega_1 \int_{-\infty}^{+\infty} d\omega_2 \exp[-i(\omega_1\tau_1 + \omega_2\tau_2)]\Phi_G(\omega_1,\omega_2,\mu),$$

Kawamura, Kodaira, Qiao, Tanaka, PLB 523,(2001) 111

Qin, Shen, Wang, Wang, 2023

Knowledge about three body parameters

- Perturbative aspects: RGE for λ_E and λ_H

$$\mu\frac{d}{d\mu}\begin{pmatrix}\lambda_E^2(\mu)\\ \lambda_H^2(\mu)\end{pmatrix} = -\frac{\alpha_s}{4\pi}\begin{pmatrix}\frac{8}{3}C_F+\frac{3}{2}C_G & \frac{4}{3}C_F-\frac{3}{2}C_G\\ \frac{4}{3}C_F-\frac{3}{2}C_G & \frac{8}{3}C_F+\frac{5}{2}C_G\end{pmatrix}\begin{pmatrix}\lambda_E^2(\mu)\\ \lambda_H^2(\mu)\end{pmatrix}$$

Grozin, Neubert, 1997; Kawamura, Tanaka, 2009

- Nonperturbative aspects:

  Estimation from sum rules;

  Models for LCDAs

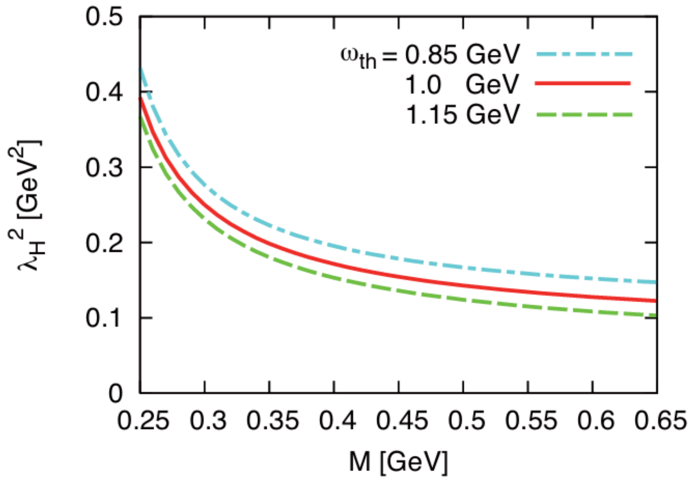
Knowledge about three body parameters

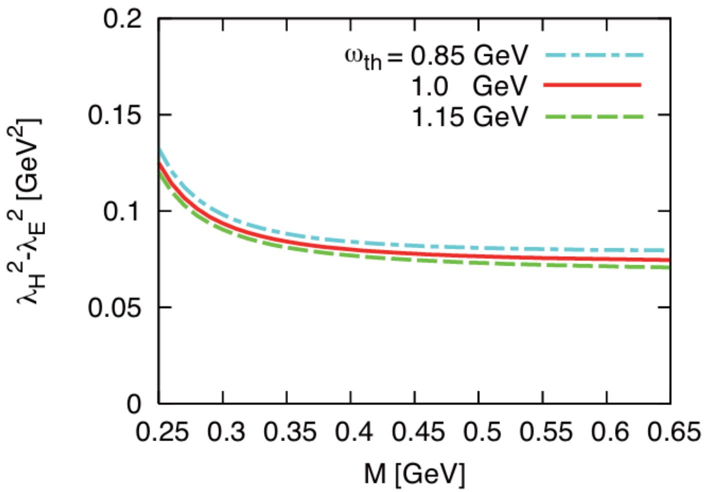
- From QCD sum rules

$$\lambda_E^2(\mu) = 0.11 \pm 0.06 \text{ GeV}^2,$$
$$\lambda_H^2(\mu) = 0.18 \pm 0.07 \text{ GeV}^2,$$

at $\mu = 1$ GeV

A. G. Grozin and M. Neubert 1997





$$\lambda_E^2(1 \text{ GeV}) = 0.03 \pm 0.02 \text{ GeV}^2,$$
$$\lambda_H^2(1 \text{ GeV}) = 0.06 \pm 0.03 \text{ GeV}^2.$$

at $\mu = 1$ GeV

Nishikawa, Tanaka 2011

Parameters	Ref. [3]	Ref. [26]	**this work**
$\mathcal{R}(1 \text{ GeV})$	(0.6 ± 0.4)	(0.5 ± 0.4)	(0.1 ± 0.1)
$\lambda_H^2(1 \text{ GeV})$	(0.18 ± 0.07) GeV^2	(0.06 ± 0.03) GeV^2	(0.15 ± 0.05) GeV^2
$\lambda_E^2(1 \text{ GeV})$	(0.11 ± 0.06) GeV^2	(0.03 ± 0.02) GeV^2	(0.01 ± 0.01) GeV^2

M.Rahimi and M.Wald 2012

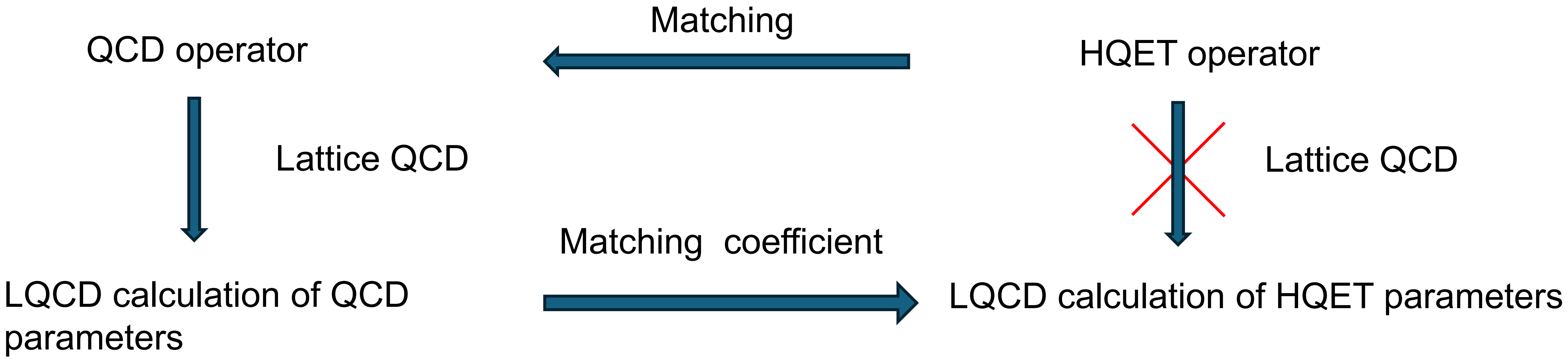
Three body parameters from QCD

- Lack of exact values for $\lambda^2_{E,H}$
- Needs first-principle calculations
- Lattice QCD simulations could be helpful
- Large noise-to-signal ratio when simulating HQET directly on the lattice

HQET operator $\cancel{\xrightarrow{\text{Lattice QCD}}}$ LQCD results of HQET parameters

An approach for three body parameters on the lattice

- Matching HQET parameters onto QCD



- Has been adopted in two-step matching for LCDAs

See the talks by Ji Xu, Yan-Bing Wei, Qi-An Zhang, …

Matching

- There is a matching relation between QCD and HQET three body (dimension 5) operators

$$\boxed{O_{i,\mathrm{QCD}}(m,\mu) = \sum_j C_{ij}(m/\mu) O_{j,\mathrm{HQET}}(\mu) + \mathcal{O}(\Lambda_{\mathrm{QCD}}/m).}$$

- Low scale physics (below m) is encoded in HQET operator; while the high scale physics (above m) is contained in the Wilson coefficients C_{ij}
- One can extract Wilson coefficient from the matching of matrix elements of the operators.

$$\boxed{|B\rangle \to |b(p_2)\bar{q}(p_1)g(l)\rangle}$$

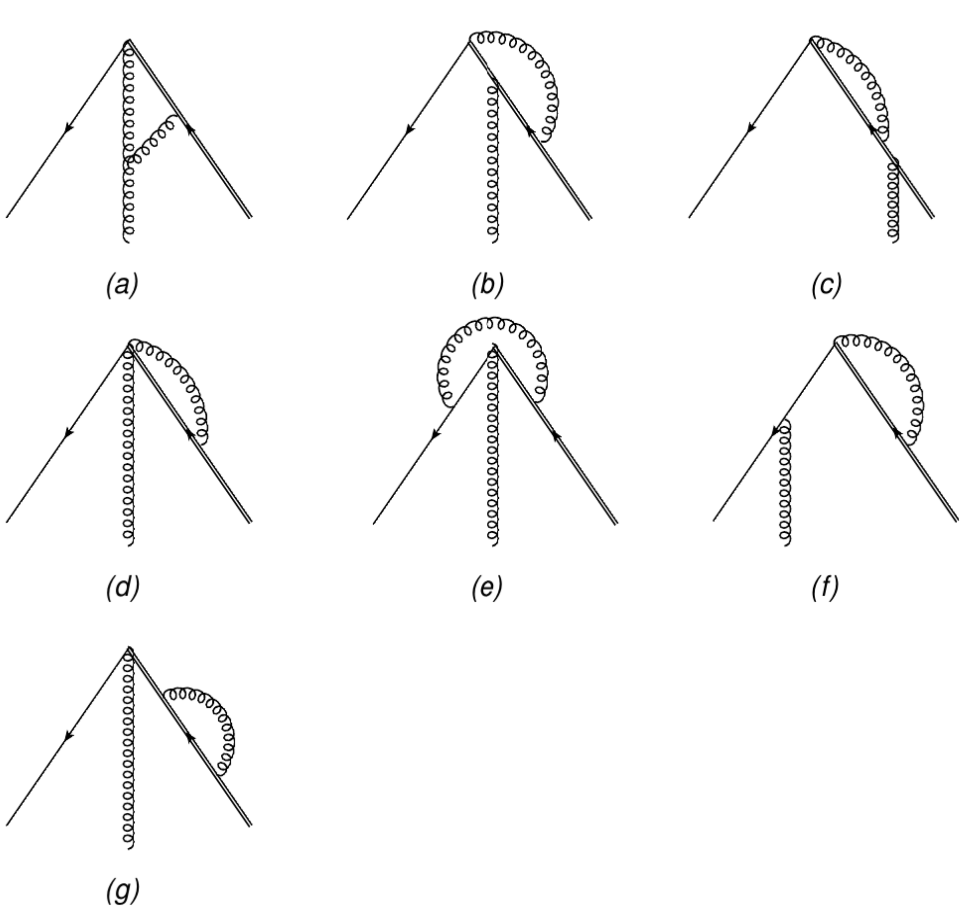
$$\begin{array}{c} \vec{\alpha}\cdot g\vec{E}\gamma_5 h \to \vec{\alpha}\cdot g\vec{E}\gamma_5 h_v \\ \bar{q}\vec{\sigma}\cdot g\vec{H}\gamma_5 h \to \bar{q}\vec{\sigma}\cdot g\vec{H}\gamma_5 h_v \end{array} \xrightarrow[\text{matrix element}]{\text{from operator to}} \begin{array}{c} <0|\bar{q}\vec{\alpha}\cdot g\vec{E}\gamma_5 h|\bar{B}> \\ <0|\bar{q}\vec{\sigma}\cdot g\vec{H}\gamma_5 h|\bar{B}> \end{array} \longrightarrow \begin{array}{c} <0|\bar{q}\vec{\alpha}\cdot g\vec{E}\gamma_5 h_v|\bar{B}> \\ <0|\bar{q}\vec{\sigma}\cdot g\vec{H}\gamma_5 h_v|\bar{B}> \end{array}$$

$$\begin{array}{c} <0|\bar{q}\vec{\alpha}\cdot g\vec{E}\gamma_5 h|\bar{B}> \\ <0|\bar{q}\vec{\sigma}\cdot g\vec{H}\gamma_5 h|\bar{B}> \end{array} \xrightarrow[\text{to Fock state}]{\text{from hadron state}} \begin{array}{c} <0|\bar{q}\vec{\alpha}\cdot g\vec{E}\gamma_5 h|\bar{b}qg> \\ <0|\bar{q}\vec{\sigma}\cdot g\vec{H}\gamma_5 h|\bar{b}qg> \end{array}$$

- The matching at tree-level is trivial:

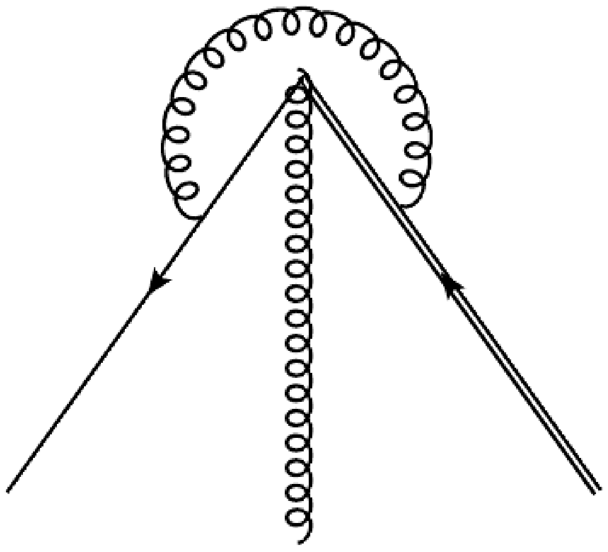
$$C_{ij} \approx \delta_{ij}$$

Matching at one-loop



- Dimensional regularization (DR) and $\overline{\mathrm{MS}}$ renormalization. On-shell states.
- IR divergences are regularized in DR
- IR divergences only exist in (a), (e), (g)
- Power (UV) divergences exist. Operator mixing
- We studied the operator $\bar{q} g_s \sigma^{\alpha\beta} G_{\alpha\beta} \gamma^5 h$. $\lambda_{E,H}$ will be similar.

Matching at one-loop



QCD

$$\frac{g_s^2}{16\pi^2}\frac{1}{2N_c}\left(\boxed{\frac{1}{\epsilon_{IR}}}+2+\ln\frac{\mu_{IR}^2}{m^2}\right)\boxed{\bar{v}(p_1)g_s\epsilon_c^{\rho}t^c\sigma^{\alpha\beta}\gamma_5 u(p_2)(-i)(l_\alpha g_{\beta\rho}-l_\beta g_{\alpha\rho})}$$

Tree-level structure

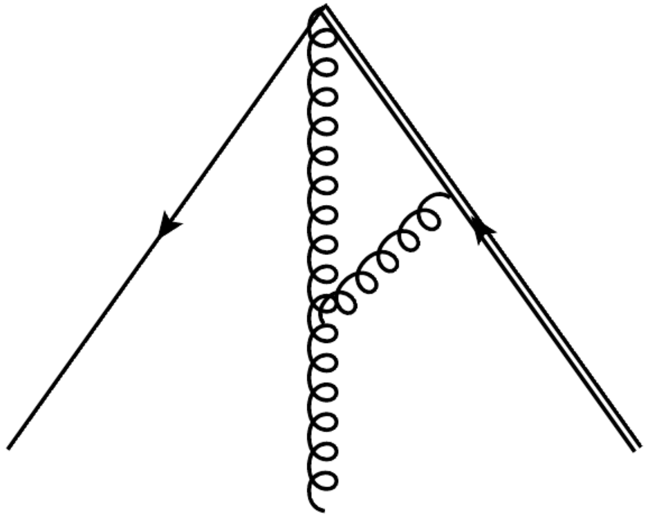
HQET

$$-\frac{g_s^2}{16\pi^2}\frac{1}{2N_c}\left(\frac{1}{\epsilon}-\boxed{\frac{1}{\epsilon_{IR}}}+\ln\frac{\mu^2}{\mu_{IR}^2}\right)\boxed{\bar{v}(p_1)t^c g_s\epsilon_c^{\rho}\sigma^{\alpha\beta}\gamma_5 u_v(-i)(l_\alpha g_{\beta\rho}-l_\beta g_{\alpha\rho})}$$

Tree-level structure

No operator mixing

Matching at one-loop



(a)

QCD

$$-\frac{1}{2}g_s^3 N_c \bar{v}(p_1) t_a \not{l} \not{\epsilon}_a \gamma^5 u(p_2) \frac{1}{\pi^2}\Bigg[-\frac{1}{2\epsilon}+\boxed{\frac{1}{8\epsilon_{IR}^2}}-\boxed{\frac{5+2\ln\frac{\mu^2}{4v\cdot l^2}}{16\epsilon_{IR}}}$$
$$+\frac{1}{96}\left(-42+5\pi^2-48\ln\frac{\mu^2}{4v\cdot l^2}+30\ln\frac{\mu_{IR}^2}{4v\cdot l^2}+6\ln^2\frac{\mu_{IR}^2}{4v\cdot l^2}-42\ln\frac{4v\cdot l^2}{m^2}\right)\Bigg]$$
$$+\ other\ operator\ structures$$

HQET

$$-\frac{1}{2}g_s^3 N_c \bar{v}(p_1) t^a \not{l} \not{\epsilon}^a \gamma^5 u_v \frac{1}{\pi^2}\Bigg[-\frac{1}{16\epsilon}+\boxed{\frac{1}{8\epsilon_{IR}^2}}+\boxed{\frac{1}{16\epsilon_{IR}}\left(5-2\ln\frac{4l\cdot v^2}{\mu_{IR}^2}\right)}$$
$$+\frac{1}{96}\left(48+5\pi^2+2\ln\frac{4l\cdot v^2}{\mu^2}+6\left(-5+\ln\frac{4l\cdot v^2}{\mu_{IR}^2}\right)\ln\frac{4l\cdot v^2}{\mu_{IR}^2}\right)\Bigg]$$
$$+\ other\ operator\ structures$$

Observations:

- Double poles in HQET and QCD results
- Double poles and logs of $l\cdot v$ cancel between QCD and HQET
- Leads to IR finite matching coefficient

$$\boxed{\delta C^{(1)}=-\frac{g_s^2 N_c}{64\pi^2}\left(15+7\ln\frac{\mu^2}{m^2}\right)}$$

Matching

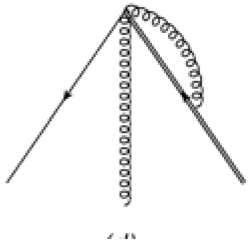
- Power (UV) divergences arise in the calculation, associated with other operator structures.
- The operator mixing should be taken into account in lattice simulations.
- When the external state is off-shell, operators that vanish with equation of motion get involved, which makes lattice simulation complicated.
- More exploration is in progress.

Summary and outlook

- The three-body hadronic parameter in HQET is crucial for high precision study of heavy flavor physics
- We examined the factorization formula linking QCD and HQET three-body parameters and evaluated the Wilson coefficients.
- Could provide insights into the first principle calculation of heavy meson 3-body parameters.
- Operator mixing should be taken into consideration. In progress.

Thank you

Back up



$$=ig^3 f_{abc} T^a T^c \epsilon_0^b(k)\bar{v}(p)\gamma^5 \frac{i}{2(4\pi)^2}\gamma^0\gamma^i g_{i\mu}(m_b^2\mu^2/\pi)^\epsilon\Gamma(\epsilon)[-\frac{m_b}{2-2\epsilon}\not{v}\gamma^\mu + \frac{m_b}{1-2\epsilon}2v^\mu + \frac{m_b}{1-2\epsilon}2m_b\gamma^\mu]u(l)$$

$$+ig^3 f_{abc} T^a T^b \epsilon_i^c(k)\bar{v}(p)\gamma^5 \frac{i}{2(4\pi)^2}\gamma^0\gamma^i g_{0\mu}(m_b^2\mu^2/\pi)^\epsilon\Gamma(\epsilon)[-\frac{m_b}{2-2\epsilon}\not{v}\gamma^\mu + \frac{m_b}{1-2\epsilon}2v^\mu + \frac{m_b}{1-2\epsilon}2m_b\gamma^\mu]u(l)$$

$$= +g^3 f_{abc} T^a T^c \epsilon_0^b(k)\bar{v}(p)\gamma^5 \frac{1}{2(4\pi)^2}[+\frac{1}{2\epsilon} + \frac{1}{2}(\log(m_b^2\mu^2) - \gamma + 1 - \log(\pi))]\gamma^0\gamma^i g_{i\mu} m_b[\not{v}\gamma^\mu]u(l)$$

$$-g^3 f_{abc} T^a T^c \epsilon_0^b(k)\bar{v}(p)\gamma^5 \frac{1}{2(4\pi)^2}[\frac{1}{\epsilon} + (\log(m_b^2\mu^2) - \gamma + 2 - \log(\pi))+]\gamma^0\gamma^i g_{i\mu} m_b[2v^\mu + 2\gamma^\mu]u(l)$$

$$+g^3 f_{abc} T^a T^c \epsilon_i^b(k)\bar{v}(p)\gamma^5 \frac{1}{2(4\pi)^2}[+\frac{1}{2\epsilon} + \frac{1}{2}(\log(m_b^2\mu^2) - \gamma + 1 - \log(\pi))]\gamma^0\gamma^i g_{0\mu} m_b[\not{v}\gamma^\mu]u(l)$$

$$-g^3 f_{abc} T^a T^c \epsilon_i^b(k)\bar{v}(p)\gamma^5 \frac{1}{2(4\pi)^2}[\frac{1}{\epsilon} + (\log(m_b^2\mu^2) - \gamma + 2 - \log(\pi))+]\gamma^0\gamma^i g_{0\mu} m_b[2v^\mu + 2\gamma^\mu]u(l)$$