

The radially excited D meson production in B decays

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2025 年 10 月 26 日

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Introduction

- The ground states $D_{(s)}$ and $D_{(s)}^*$ have been well determined in experiments, their productions and decays have been extensively studied.
- While the first radially excited states $D_{(s)}(2S)$ and $D_{(s)}^*(2S)$ have not been well confirmed.
- For example both $D(2550)^0$ and $D_J(2580)^0$ are considered as the candidates of natural $D(2^1S_0)$.
- $D_{s0}(2590)^+$ with $J^P = 0^-$ observed by LHCb is suggested to a candidate of $D_s(2^1S_0)$ state. Some theoretical works suggest that $D_{s0}(2590)^+$ may not be a pure $D_s(2^1S_0)$ state but rather has D^*K component.

Introduction

- $D_1^*(2680)^0$, $D^*(2650)^0$ and $D_1^*(2600)^0$ observed by LHCb are considered as the candidates of neutral $D(2^3S_1)$. The $D^*(2640)^\pm$ discovered by Delphi have masses consistent with the predictions of charged $D(2^3S_1)$, while it has not been confirmed in any other experiments.
- Except for the assignment of $D_s(2^3S_1)$ to $D_{s1}^*(2700)^\pm$, $D_s(1^3D_1)$ and a mixture of $D_s(2^3S_1)$ and $D_s(1^3D_1)$ are also been proposed.
- We will assume the $D_0(2550)$, $D_{s0}(2590)^\pm$ as $D(2^1S_0)$, $D_s(2^1S_0)$, $D_1^*(2600)^0$, $D^*(2640)^\pm$ as $D(2^3S_1)$, the $D_{s1}^*(2700)^\pm$ as $D_s(2^3S_1)$, respectively.

Introduction

At the end of the twentieth century, Jaus put forward the covariant light-front quark model (CLFQM). The CLFQM has some unique advantages:

- The light-front wave functions describing the hadron through quark and gluon degrees of freedom can preserve a Lorentz invariant formalism.
- The final state meson at $q^2 = 0$ is usually relativistic. The CLFQM with relativistic effects involved is suitable to study hadronic transition form factors.

The CLFQM general calculation procedure

The CLFQM general calculation procedure

The Bauer-Stech-Wirble (BSW) form factors for $B \rightarrow D$ and $B \rightarrow D^*$ transitions are defined as follows,

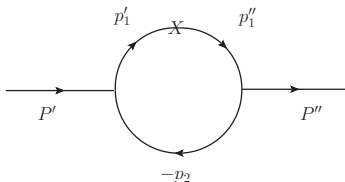
$$\begin{aligned}\langle D(P'') | V_\mu | B(P') \rangle &= \left(P_\mu - \frac{m_B^2 - m_D^2}{q^2} q_\mu \right) F_1^{BD}(q^2) + \frac{m_B^2 - m_D^2}{q^2} q_\mu F_0^{BD}(q^2), \\ \langle D^*(P'', \epsilon'') | V_\mu | B(P') \rangle &= -\frac{1}{m_{D^*} + m_B} \epsilon_{\mu\nu\alpha\beta} \epsilon''^{*\nu} P^\alpha q^\beta V^{BD*}(q^2), \\ \langle D^*(P'', \epsilon'') | A_\mu | B(P') \rangle &= i \left\{ (m_{D^*} + m_B) \epsilon''^{*\mu} A_1^{BD*}(q^2) - \frac{\epsilon''^{*} \cdot P}{m_{D^*} + m_B} P_\mu A_2^{BD*}(q^2) \right. \\ &\quad \left. - 2m_{D^*} \frac{\epsilon''^{*} \cdot P}{q^2} q_\mu [A_3^{BD*}(q^2) - A_0^{BD*}(q^2)] \right\},\end{aligned}$$

where $P = P' + P''$, $q = P' - P''$, ϵ is the polarization vector. The four-momentum of the initial (final) meson is $P' = p_1' + p_2$ ($P'' = p_1'' + p_2$).

The CLFQM general calculation procedure

The decay amplitude in the lowest order for the transitions $B \rightarrow D$ and $B \rightarrow D^*$

$$\begin{aligned}\mathcal{B}_\mu^{BD} &= -i^3 \frac{N_c}{(2\pi)^4} \int d^4 p'_1 \frac{h'_B h''_D}{N'_1 N''_1 N_2} S_\mu^{BD}, \\ \mathcal{B}_\mu^{BD^*} &= -i^3 \frac{N_c}{(2\pi)^4} \int d^4 p'_1 \frac{h'_B (i h''_{D^*})}{N'_1 N''_1 N_2} S_{\mu\nu}^{BD^*} \varepsilon^{\prime\prime * \nu},\end{aligned}\quad (1)$$



- $N_1^{\prime(n)} = p_1^{\prime(n)2} - m_1^{\prime(n)2}$, $N_2 = p_2^2 - m_2^2$ arise from the quark propagators.

The CLFQM general calculation procedure

The traces S_{μ}^{BD} and $S_{\mu\nu}^{BD*}$ can be obtained directly using Lorentz contraction,

$$\begin{aligned}
 S_{\mu}^{BD} &= \text{Tr} \left[\gamma_5 (\not{p}_1'' + m_1'') \gamma_{\mu} (\not{p}_1' + m_1') \gamma_5 (-\not{p}_2 + m_2) \right], \\
 S_{\mu\nu}^{BD*} &= \left(S_V^{BD*} - S_A^{BD*} \right)_{\mu\nu} \\
 &= \text{Tr} \left[\left(\gamma_{\nu} - \frac{1}{W_V''} (p_1'' - p_2)_{\nu} \right) (\not{p}_1'' + m_1'') (\gamma_{\mu} - \gamma_{\mu} \gamma_5) (\not{p}_1' + m_1') \gamma_5 (-\not{p}_2 + m_2) \right], \quad (2)
 \end{aligned}$$

The covariant vertex function h_M'' with $M = D(1S, 2S), D^*(1S, 2S)$ is defined as

$$\begin{aligned}
 h_M'' &= (M''^2 - M_0''^2) \sqrt{\frac{x_1 x_2}{N_c}} \frac{1}{\sqrt{2\tilde{M}_0''}} \varphi, \\
 M_0''^2 &= (e_1'' + e_2)'^2 = \frac{p_{\perp}^2 + m_1''^2}{x_1} + \frac{p_{\perp}^2 + m_2^2}{x_2}, \quad \tilde{M}_0'' = \sqrt{M_0''^2 - (m_1'' - m_2)^2}. \quad (3)
 \end{aligned}$$

The CLFQM general calculation procedure

The phenomenological Gaussian-type wave function φ depicts the light-front momentum distribution amplitude for the S-wave mesons,

$$\begin{aligned}\varphi(1S) &= 4 \left(\frac{\pi}{\beta^2} \right)^{\frac{3}{4}} \sqrt{\frac{dp_z}{dx_2}} \exp \left(-\frac{p_z^2 + p_{\perp}^2}{2\beta^2} \right), \\ \varphi(2S) &= 4 \left(\frac{\pi}{\beta^2} \right)^{\frac{3}{4}} \sqrt{\frac{dp_z}{dx_2}} \exp \left(-\frac{p_z^2 + p_{\perp}^2}{2\beta^2} \right) \left(3 - 2\frac{p_z^2 + p_{\perp}^2}{\beta^2} \right),\end{aligned}$$

- β is a phenomenological parameter and can be fixed by fitting the corresponding decay constant.

Transition Form Factors

Input parameters:

- The constituent quark masses(GeV): $m_c = 1.4$, $m_s = 0.37$, $m_{u,d} = 0.25$;
- The masses of the initial and the final mesons(GeV):

$$m_\pi = 0.140, m_K = 0.494, m_\rho = 0.775, m_{K^*} = 0.892, m_D = 1.86966,$$

$$m_{D(2S)} = 2.549, m_{D_s} = 1.96835, m_{D_s(2S)} = 2.591,$$

$$m_{D_s^{*\pm}} = 2.1122, m_{D^{*0}} = 2.0068, m_{D^{*\pm}} = 2.0102, m_B = 5.279,$$

$$m_{D_s^{*\pm}(2S)} = 2.732, m_{D^{*0}(2S)} = 2.627, m_{D^{*\pm}(2S)} = 2.637, m_{\bar{B}_s^0} = 5.4154.$$

- The CKM matrix elements: $V_{cd} = 0.221 \pm 0.004$, $V_{cs} = 0.975 \pm 0.006$,
 $V_{ud} = 0.97373 \pm 0.00031$;

Transition Form Factors

Input parameters:

- The shape parameters fitted by the decay constants:

$$\beta_{D_s^{*\pm}(1S)} = 0.534_{-0.014}^{+0.014}, \beta_{D^{*0}(1S)} = 0.500_{-0.187}^{+0.140}, \beta_D = 0.466_{-0.021}^{+0.022},$$

$$\beta_{D_s^{*\pm}(2S)} = 0.473_{-0.041}^{+0.041}, \beta_{D^{*0}(2S)} = 0.456_{-0.003}^{+0.004}, \beta_{D_s} = 0.600_{-0.025}^{+0.026},$$

$$\beta_{D^{*\pm}(1S)} = 0.502_{-0.041}^{+0.041}, \beta_{D^{*\pm}(2S)} = 0.453_{-0.003}^{+0.004}, \beta_{\bar{B}_s^0} = 0.626_{-0.045}^{+0.045},$$

$$\beta_{D(2S)} = 0.297_{-0.041}^{+0.041}, \beta_{D_s(2S)} = 0.422_{-0.025}^{+0.026}, \beta_B = 0.555_{-0.060}^{+0.060}.$$

- mean life($10^{-12}s$): $\tau_{B^0} = (1.519 \pm 0.004)$, $\tau_{B_s^0} = (1.520 \pm 0.005)$,
 $\tau_{B^\pm} = (1.638 \pm 0.004)$.

Transition Form Factors

- All the calculations are carried out within the $q^+ = 0$ reference frame, where the form factors can only be obtained at spacelike momentum transfers $q^2 = -q_\perp^2 \leq 0$,
- The parameterized form factors are extrapolated from the space-like region to the time-like region by using

$$F(q^2) = \frac{F(0)}{(1 - q^2/m^2) [1 - a(q^2/m^2) + b(q^2/m^2)^2]}. \quad (4)$$

- $F(q^2)$ denotes different form factors, such as $F_1(q^2)$, $F_0(q^2)$, $V(q^2)$, $A_0(q^2)$, $A_1(q^2)$, $A_2(q^2)$.

Transition Form Factors

- The form factors of the transitions $B_{(s)} \rightarrow D_{(s)}(1S, 2S)$ in the CLFQM. The uncertainties are from the decay constants of $B_{(s)}$ and final state mesons.

	$F(0)$	$F(q_{max}^2)$	a	b
F_1^{BD}	$0.66^{+0.00+0.01}_{-0.01-0.01}$	$0.81^{+0.01+0.01}_{-0.00-0.01}$	$0.80^{+0.01+0.04}_{-0.02-0.04}$	$0.86^{+0.01+0.02}_{-0.01-0.02}$
F_0^{BD}	$0.66^{+0.00+0.01}_{-0.01-0.01}$	$0.70^{+0.02+0.01}_{-0.03-0.01}$	$0.46^{+0.14+0.00}_{-0.12-0.01}$	$0.77^{+0.01+0.05}_{-0.01-0.05}$
$F_1^{BD(2S)}$	$0.26^{+0.01+0.01}_{-0.02-0.02}$	$0.34^{+0.02+0.01}_{-0.03-0.03}$	$0.99^{+0.04+0.16}_{-0.10-0.18}$	$0.66^{+0.07+0.17}_{-0.12-0.15}$
$F_0^{BD(2S)}$	$0.26^{+0.01+0.02}_{-0.01-0.02}$	$0.32^{+0.03+0.02}_{-0.00-0.04}$	$0.65^{+0.03+0.05}_{-0.01-0.04}$	$-0.24^{+0.02+0.01}_{-0.03-0.03}$
$F_1^{B_s D_s}$	$0.67^{+0.00+0.01}_{-0.00-0.01}$	$0.81^{+0.00+0.00}_{-0.01-0.01}$	$0.82^{+0.00+0.02}_{-0.01-0.02}$	$0.96^{+0.01+0.03}_{-0.02-0.03}$
$F_0^{B_s D_s}$	$0.67^{+0.00+0.01}_{-0.00-0.01}$	$0.71^{+0.01+0.01}_{-0.02-0.01}$	$0.48^{+0.00+0.08}_{-0.01-0.08}$	$0.85^{+0.02+0.06}_{-0.02-0.06}$
$F_1^{B_s D_s(2S)}$	$0.26^{+0.02+0.02}_{-0.02-0.02}$	$0.29^{+0.02+0.01}_{-0.02-0.02}$	$0.59^{+0.12+0.05}_{-0.16-0.01}$	$0.35^{+0.11+0.04}_{-0.13-0.02}$
$F_0^{B_s D_s(2S)}$	$0.26^{+0.01+0.01}_{-0.02-0.02}$	$0.25^{+0.02+0.01}_{-0.02-0.02}$	$-0.09^{+0.22+0.22}_{-0.14-0.26}$	$-0.07^{+0.13+0.18}_{-0.21-0.08}$

Transition Form Factors

- The form factors of the transitions $B \rightarrow D^*(1S, 2S)$ in the CLFQM. The uncertainties are from the decay constants of B and final state mesons.

F	F(0)	$F(q_{max}^2)$	a	b
V^{BD^*}	$0.77^{+0.00+0.01}_{-0.00-0.01}$	$0.94^{+0.00+0.01}_{-0.00-0.01}$	$0.78^{+0.00+0.03}_{-0.01-0.03}$	$0.82^{+0.04+0.14}_{-0.08-0.16}$
$A_0^{BD^*}$	$0.75^{+0.00+0.06}_{-0.01-0.02}$	$0.79^{+0.00+0.05}_{-0.02-0.02}$	$0.17^{+0.01+0.01}_{-0.01-0.00}$	$0.12^{+0.07+0.02}_{-0.06-0.03}$
$A_1^{BD^*}$	$0.67^{+0.00+0.01}_{-0.01-0.11}$	$0.76^{+0.00+0.01}_{-0.01-0.01}$	$0.38^{+0.01+0.01}_{-0.01-0.01}$	$0.19^{+0.05+0.09}_{-0.05-0.09}$
$A_2^{BD^*}$	$0.58^{+0.00+0.00}_{-0.01-0.02}$	$0.69^{+0.01+0.01}_{-0.01-0.02}$	$0.68^{+0.02+0.00}_{-0.02-0.00}$	$0.66^{+0.25+0.10}_{-0.12-0.16}$
$V^{BD^*(2S)}$	$0.19^{+0.05+0.01}_{-0.06-0.01}$	$0.19^{+0.01+0.03}_{-0.04-0.01}$	$0.11^{+0.05+0.03}_{-0.06-0.05}$	$0.34^{+0.00+0.03}_{-0.04-0.06}$
$A_0^{BD^*(2S)}$	$0.27^{+0.04+0.01}_{-0.05-0.00}$	$0.28^{+0.00+0.01}_{-0.02-0.00}$	$0.24^{+0.05+0.00}_{-0.06-0.00}$	$-0.07^{+0.15+0.03}_{-0.29-0.06}$
$A_1^{BD^*(2S)}$	$0.16^{+0.04+0.01}_{-0.05-0.00}$	$0.15^{+0.05+0.02}_{-0.04-0.02}$	$-0.23^{+0.04+0.00}_{-0.04-0.00}$	$0.17^{+0.03+0.01}_{-0.01-0.01}$
$A_2^{BD^*(2S)}$	$-0.05^{+0.04+0.01}_{-0.04-0.00}$	$0.01^{+0.00+0.00}_{-0.00-0.00}$	$-1.10^{+0.02+0.01}_{-0.01-0.01}$	$-0.63^{+0.02+0.01}_{-0.01-0.00}$

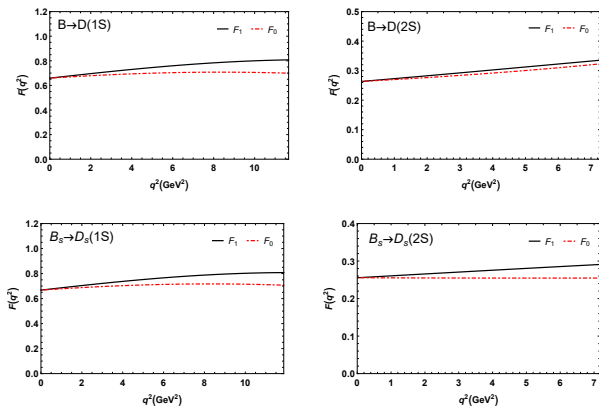
Transition Form Factors

- The form factors of the transitions $B_s \rightarrow D_s^*(1S, 2S)$ in the CLFQM. The uncertainties are from the decay constants of B_s and final state mesons..

F	F(0)	$F(q_{max}^2)$	a	b
$V^{B_s D_s^*}$	$0.78^{+0.01+0.01}_{-0.01-0.01}$	$0.87^{+0.00+0.00}_{-0.00-0.01}$	$0.86^{+0.01+0.04}_{-0.01-0.04}$	$1.11^{+0.01+0.02}_{-0.01-0.02}$
$A_0^{B_s D_s^*}$	$0.74^{+0.01+0.01}_{-0.01-0.01}$	$0.69^{+0.01+0.01}_{-0.01-0.00}$	$0.23^{+0.01+0.01}_{-0.01-0.01}$	$0.21^{+0.00+0.01}_{-0.00-0.01}$
$A_1^{B_s D_s^*}$	$0.66^{+0.01+0.01}_{-0.02-0.02}$	$0.95^{+0.00+0.00}_{-0.00-0.01}$	$0.81^{+0.01+0.02}_{-0.01-0.02}$	$0.93^{+0.01+0.01}_{-0.01-0.01}$
$A_2^{B_s D_s^*}$	$0.57^{+0.00+0.00}_{-0.01-0.00}$	$0.68^{+0.01+0.00}_{-0.01-0.00}$	$0.80^{+0.00+0.03}_{-0.01-0.03}$	$0.96^{+0.01+0.03}_{-0.01-0.02}$
$V^{B_s D_s^*(2S)}$	$0.26^{+0.03+0.04}_{-0.03-0.04}$	$0.28^{+0.00+0.02}_{-0.09-0.10}$	$0.25^{+0.02+0.04}_{-0.01-0.04}$	$0.30^{+0.03+0.04}_{-0.01-0.00}$
$A_0^{B_s D_s^*(2S)}$	$0.31^{+0.02+0.01}_{-0.03-0.02}$	$0.33^{+0.00+0.08}_{-0.01-0.07}$	$0.21^{+0.01+0.03}_{-0.05-0.00}$	$-0.09^{+0.04+0.10}_{-0.02-0.14}$
$A_1^{B_s D_s^*(2S)}$	$0.21^{+0.02+0.03}_{-0.03-0.03}$	$0.20^{+0.00+0.01}_{-0.07-0.06}$	$-0.16^{+0.20+0.04}_{-0.17-0.02}$	$0.12^{+0.00+0.12}_{-0.01-0.07}$
$A_2^{B_s D_s^*(2S)}$	$-0.01^{+0.02+0.06}_{-0.02-0.06}$	$0.01^{+0.00+0.03}_{-0.01-0.01}$	$-4.03^{+0.51+0.75}_{-0.47-1.38}$	$4.31^{+0.00+2.13}_{-0.00-2.44}$

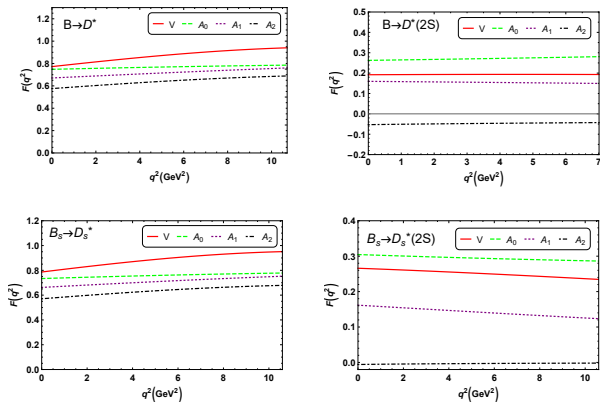
Transition Form Factors

图: The q^2 -dependence of the $B_{(s)} \rightarrow D_{(s)}(1S, 2S)$ transition form factors.



Transition Form Factors

图: The q^2 -dependence of the $B_{(s)} \rightarrow D_{(s)}^*(1S, 2S)$ transition form factors.



Non-leptonic Decays

$$B \rightarrow D^{(*)}(1S, 2S) M$$

Non-leptonic Decays $B \rightarrow D^{(*)}(1S, 2S)M$

The effective Hamiltonian for the decays $B_{(s)} \rightarrow D_{(s)}^{(*)}(1S, 2S)M$ with $M = \pi, K, \rho, K^*$ can be written as

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cb}^* V_{uq} \{C_1 Q_1 + C_2 Q_2\}, \quad (5)$$

The local tree four-quark operators $Q_{1,2}$ are defined by :

$$Q_1 = [\bar{c}_\alpha \gamma_\mu (1 - \gamma_5) b_\beta] [\bar{q}_\beta \gamma^\mu (1 - \gamma_5) u_\alpha], \quad (6)$$

$$Q_2 = [\bar{c}_\alpha \gamma_\mu (1 - \gamma_5) b_\alpha] [\bar{q}_\beta \gamma^\mu (1 - \gamma_5) u_\beta], \quad (7)$$

where α and β are color indices.

Non-leptonic Decays $B \rightarrow D^{(*)}(1S, 2S)M$

Combining with the form factors, we can obtain the partial widths for our considered non-leptonic decays, which are written as

$$\Gamma(B \rightarrow DP) = \frac{G_F^2 (m_B^2 - m_D^2)^2 |\vec{p}_c|}{16\pi m_B^2} |V_{uq}^* V_{cb}|^2 |a_1|^2 f_P^2 F_0^2(m_P^2), \quad (8)$$

$$\Gamma(B \rightarrow D^* P) = \frac{G_F^2 |\vec{p}_c|^3}{4\pi} |V_{uq}^* V_{cb}|^2 |a_1|^2 f_P^2 A_0^2(m_P^2), \quad (9)$$

$$\Gamma(B \rightarrow DV) = \frac{G_F^2 |\vec{p}_c|^3}{4\pi} |V_{uq}^* V_{cb}|^2 |a_1|^2 f_V^2 F_+^2(m_V^2), \quad (10)$$

$$\Gamma(B \rightarrow D^* V) = \frac{G_F^2 |\vec{p}_c|}{16\pi m_B^2} |V_{uq}^* V_{cb}|^2 \left(|H_0|^2 + |H_+|^2 + |H_-|^2 \right), \quad (11)$$

Non-leptonic Decays $B \rightarrow D^{(*)}(1S, 2S)M$

Here $\vec{p}_c$ is the momentum of either of the two final state meson in the B rest frame and $H_{0,\pm}$ are the helicity amplitudes,

$$H_0 = \frac{if_V a_1}{2m_{D^*}} \left[(m_B^2 - m_{D^*}^2 - m_V^2) (m_B + m_{D^*}) A_1^{BD^*} (m_V^2) - \frac{4m_B^2 p_c^2}{m_B + m_{D^*}} A_2^{BD^*} (m_V^2) \right], \quad (12)$$

$$H_{\pm} = if_V m_V a_1 \left[- (m_B + m_{D^*}) A_1^{BD^*} (m_V^2) \mp \frac{2m_B p_c}{m_B + m_{D^*}} V^{BD^*} (m_V^2) \right]. \quad (13)$$

The polarization fractions are defined as $f_{L,\parallel,\perp} = \frac{H_{0,\parallel,\perp}^2}{H_0^2 + H_{\parallel}^2 + H_{\perp}^2}$, where $H_{\parallel}$ and $H_{\perp}$ are parallel and perpendicular amplitudes, respectively, and can be obtained through

$$H_{\parallel,\perp} = \frac{(H_- \pm H_+)}{\sqrt{2}}.$$

Non-leptonic Decays $B \rightarrow D^{(*)}(1S, 2S)M$

- The branching ratios of the decays $B \rightarrow D(1S, 2S)M$ with $M = \pi, K, \rho, K^*$.

Modes	This work	Bethe-Salpeter	QCDF	PQCD	Exp.	Unit
$\bar{B}^0 \rightarrow D^+(1S)\pi^-$	$4.37^{+0.01+0.01+0.01}_{-0.01-0.09-0.01}$	3.24	3.93	2.69	2.52	(10^{-3})
$\bar{B}^0 \rightarrow D^+(1S)K^-$	$0.34^{+0.00+0.00+0.01}_{-0.00-0.01-0.01}$	0.245	0.30	0.243	0.186	
$\bar{B}^0 \rightarrow D^+(1S)\rho^-$	$10.09^{+0.03+0.10+0.25}_{-0.03-0.20-0.28}$	7.91	10.42	6.96	7.6	
$\bar{B}^0 \rightarrow D^+(1S)K^{*-}$	$0.56^{+0.00+0.01+0.01}_{-0.00-0.00-0.01}$	0.431	0.53	0.407	0.45	
$\bar{B}^0 \rightarrow D^+(2S)\pi^-$	$4.76^{+0.01+0.48+0.33}_{-0.01-0.62-0.57}$	0.458	—	—	—	(10^{-4})
$\bar{B}^0 \rightarrow D^+(2S)K^-$	$0.37^{+0.00+0.04+0.04}_{-0.00-0.05-0.03}$	0.034	—	—	—	
$\bar{B}^0 \rightarrow D^+(2S)\rho^-$	$10.54^{+0.03+1.06+0.73}_{-0.03-1.37-1.26}$	1.03	—	—	—	
$\bar{B}^0 \rightarrow D^+(2S)K^{*-}$	$0.58^{+0.00+0.06+0.04}_{-0.00-0.08-0.07}$	0.0557	—	—	—	
$B^- \rightarrow D^0(1S)\pi^-$	$4.71^{+0.01+0.02+0.12}_{-0.01-0.09-0.13}$	3.49	—	5.11	4.68	(10^{-3})
$B^- \rightarrow D^0(1S)K^-$	$0.37^{+0.00+0.00+0.01}_{-0.00-0.01-0.01}$	0.264	—	0.400	0.363	
$B^- \rightarrow D^0(1S)\rho^-$	$10.88^{+0.03+0.00+0.27}_{-0.03-0.20-0.31}$	8.40	—	11.3	13.4	
$B^- \rightarrow D^0(1S)K^{*-}$	$0.60^{+0.00+0.00+0.02}_{-0.00-0.01-0.02}$	0.466	—	0.696	0.53	
$B^- \rightarrow D^0(2S)\pi^-$	$5.14^{+0.01+0.52+0.36}_{-0.01-0.67-0.61}$	0.488	—	—	—	(10^{-4})
$B^- \rightarrow D^0(2S)K^-$	$0.40^{+0.00+0.04+0.03}_{-0.00-0.05-0.05}$	0.0362	—	—	—	
$B^- \rightarrow D^0(2S)\rho^-$	$11.36^{+0.03+1.14+0.79}_{-0.03-1.48-1.35}$	1.09	—	—	—	
$B^- \rightarrow D^0(2S)K^{*-}$	$0.62^{+0.00+0.06+0.04}_{-0.00-0.08-0.07}$	0.0595	—	—	—	

Non-leptonic Decays $B \rightarrow D^{(*)}(1S, 2S)M$

- One can find that the branching ratios of the decays $B \rightarrow D(1S)(\pi, \rho, K^{(*)})$ fall in the range of $10^{-4} \sim 10^{-3}$, which are about one order larger than those of the corresponding decays $B \rightarrow D(2S)(\pi, \rho, K^{(*)})$.
- The small branching ratios for the latter are related to the node structure of the wave function of $D(2S)$ meson.
- Our predictions for the neutral channels $\bar{B}^0 \rightarrow D^+(\pi, K, \rho, K^*)^-$ are consistent with the QCDF results but exceed experimental data. By comparison, the results from the PQCD approach are closer to the experimental data, where except for the factorizable emission diagram amplitude, the nonfactorizable one should also give nonnegligible contribution. Furthermore, these two kinds of amplitudes partially cancel each other.
- It is strange that the difference between theoretical results and experimental data for the charged B decays is not significant.

Non-leptonic Decays $B \rightarrow D^{(*)}(1S, 2S)M$

- Our predictions for the branching ratios of the decays $B \rightarrow D(2S)(\pi, \rho, K^{(*)})$ are about one order larger than those given by the BS equation.
- The branching ratios of the decays $B_s \rightarrow D_s(1S, 2S)M$ with $M = \pi, K, \rho, K^*$.

Modes	This work	BS	RQM	NCQM	QCDSRs	Instant-BS	PQCD	QCDF	Exp.	Unit
$\bar{B}_s^0 \rightarrow D_s^+(1S)\pi^-$	$4.57^{+0.02+0.04+0.07}_{-0.02-0.09-0.09}$	2.92	3.5	5.3	5	2.7	1.7	4.39	2.98 ± 0.14	(10^{-3})
$\bar{B}_s^0 \rightarrow D_s^+(1S)K^-$	$0.36^{+0.00+0.00+0.01}_{-0.00-0.01-0.01}$	0.221	0.28	0.4	0.4	0.21	0.13	0.33	0.225 ± 0.012	
$\bar{B}_s^0 \rightarrow D_s^+(1S)\rho^-$	$10.55^{+0.04+0.09+0.16}_{-0.04-0.22-0.21}$	7.04	9.4	12.6	13	6.4	4.7	—	6.8 ± 1.4	
$\bar{B}_s^0 \rightarrow D_s^+(1S)K^{*-}$	$0.59^{+0.00+0.01+0.01}_{-0.00-0.01-0.01}$	0.392	0.47	0.8	0.6	0.38	0.281	—	—	
$\bar{B}_s^0 \rightarrow D_s^+(2S)\pi^-$	$4.70^{+0.02+0.61+0.50}_{-0.02-0.68-0.59}$	1.13	7	—	—	—	—	—	—	(10^{-4})
$\bar{B}_s^0 \rightarrow D_s^+(2S)K^-$	$0.37^{+0.00+0.05+0.04}_{-0.00-0.05-0.05}$	0.084	0.5	—	—	—	—	—	—	
$\bar{B}_s^0 \rightarrow D_s^+(2S)\rho^-$	$10.43^{+0.04+1.36+1.11}_{-0.04-1.52-1.31}$	2.49	17	—	—	—	—	—	—	
$\bar{B}_s^0 \rightarrow D_s^+(2S)K^{*-}$	$0.57^{+0.00+0.07+0.06}_{-0.00-0.08-0.07}$	0.134	0.8	—	—	—	—	—	—	

- A similar situation occurs in the decays $\bar{B}_s^0 \rightarrow D_s^+(1S)M^-$, whose branching ratios are also larger than the data.
- our predictions are comparable with those given by the BS approach for the decays $\bar{B}_s^0 \rightarrow D_s^+(1S)M^-$, but about 4 times as large as them for the decays $\bar{B}_s^0 \rightarrow D_s^+(2S)M^-$.

Non-leptonic Decays $B \rightarrow D^{(*)}(1S, 2S)M$

- The branching ratios(10^{-3}) of the decays $B \rightarrow D^*(1S, 2S)(\pi, \rho, K^{(*)})$.

Modes	$\bar{B}^0 \rightarrow D^{*+}(1S)\pi^-$	$\bar{B}^0 \rightarrow D^{*+}(1S)K^-$	$\bar{B}^0 \rightarrow D^{*+}(1S)\rho^-$	$\bar{B}^0 \rightarrow D^{*+}(1S)K^{*-}$
This work	$5.25^{+0.01+0.05+0.02}_{-0.01-0.19-0.40}$	$0.40^{+0.00+0.00+0.00}_{-0.00-0.01-0.03}$	$14.32^{+0.04+0.14+0.07}_{-0.04-0.50-1.05}$	$0.83^{+0.00+0.01+0.00}_{-0.00-0.03-0.06}$
BS	3.80	0.281	8.73	0.758
RIQ	—	—	14.24	0.83
PQCD	2.60	0.237	7.94	0.488
Exp.	2.66	0.216	6.8	0.33
Modes	$\bar{B}^0 \rightarrow D^{*+}(2S)\pi^-$	$\bar{B}^0 \rightarrow D^{*+}(2S)K^-$	$\bar{B}^0 \rightarrow D^{*+}(2S)\rho^-$	$\bar{B}^0 \rightarrow D^{*+}(2S)K^{*-}$
This work	$0.45^{+0.00+0.16+0.01}_{-0.00-0.16-0.01}$	$0.03^{+0.00+0.01+0.00}_{-0.00-0.01-0.01}$	$1.09^{+0.00+0.40+0.02}_{-0.00-0.41-0.02}$	$0.06^{+0.00+0.02+0.00}_{-0.00-0.00?2-0.00}$
BS	0.038	0.00274	0.0267	0.00162
RIQ	—	—	1.12	0.07
Modes	$B^- \rightarrow D^{*0}(1S)\pi^-$	$B^- \rightarrow D^{*0}(1S)K^-$	$B^- \rightarrow D^{*0}(1S)\rho^-$	$B^- \rightarrow D^{*0}(1S)K^{*-}$
This work	$5.67^{+0.01+0.05+0.98}_{-0.01-0.19-2.53}$	$0.43^{+0.00+0.00+0.07}_{-0.00-0.01-0.19}$	$15.45^{+0.04+0.13+2.30}_{-0.04-0.53-6.52}$	$0.89^{+0.00+0.01+0.13}_{-0.00-0.03-0.37}$
BS	4.11	0.304	8.73	0.846
RIQ	—	—	15.39	0.90
PQCD	5.04	0.398	11.7	0.682
Exp.	5.17	0.419	9.8	0.81
Modes	$B^- \rightarrow D^{*0}(2S)\pi^-$	$B^- \rightarrow D^{*0}(2S)K^-$	$B^- \rightarrow D^{*0}(2S)\rho^-$	$B^- \rightarrow D^{*0}(2S)K^{*-}$
This work	$0.46^{+0.00+0.16+0.01}_{-0.00-0.17-0.01}$	$0.03^{+0.00+0.01+0.00}_{-0.00-0.01-0.00}$	$1.10^{+0.00+0.04+0.02}_{-0.00-0.04-0.02}$	$0.06^{+0.00+0.02+0.00}_{-0.00-0.02-0.00}$
BS	0.041	0.00295	0.0287	0.00173
RIQ	—	—	1.181	0.070

Non-leptonic Decays $B \rightarrow D^{(*)}(1S, 2S)M$

- The branching ratios (10^{-3}) of the decays $\bar{B}_s \rightarrow D_s^{*+}(1S, 2S)(\pi, \rho, K^{(*)})^-$.

Modes	This work	RQM	QCDSR	3P QCDSR	PQCD	BS	RIQ	Exp.
$\bar{B}_s^0 \rightarrow D_s^{*+}(1S)\pi^-$	$4.04^{+0.02+0.03+0.14}_{-0.02-0.09-0.15}$	2.7	2	2.11	1.89	3.37	—	1.9
$\bar{B}_s^0 \rightarrow D_s^{*+}(1S)K^-$	$0.31^{+0.00+0.00+0.01}_{-0.00-0.00-0.01}$	0.21	0.2	0.159	0.164	0.249	—	0.132
$\bar{B}_s^0 \rightarrow D_s^{*+}(1S)\rho^-$	$11.22^{+0.05+0.08+0.36}_{-0.05-0.25-0.38}$	8.7	13	—	5.23	7.26	11.73	9.5
$\bar{B}_s^0 \rightarrow D_s^{*+}(1S)K^{*-}$	$0.66^{+0.00+0.00+0.02}_{-0.00-0.01-0.02}$	0.48	0.56	0.163	0.322	0.688	0.69	—
$\bar{B}_s^0 \rightarrow D_s^{*+}(2S)\pi^-$	$0.61^{+0.00+0.10+0.06}_{-0.00-0.11-0.09}$	0.8	—	—	—	0.108	—	—
$\bar{B}_s^0 \rightarrow D_s^{*+}(2S)K^-$	$0.05^{+0.00+0.00+0.00}_{-0.00-0.00-0.00}$	0.05	—	—	—	0.00777	—	—
$\bar{B}_s^0 \rightarrow D_s^{*+}(2S)\rho^-$	$1.51^{+0.00+0.27+0.19}_{-0.00-0.29-0.25}$	2.2	—	—	—	0.0475	1.05	—
$\bar{B}_s^0 \rightarrow D_s^{*+}(2S)K^{*-}$	$0.09^{+0.00+0.02+0.01}_{-0.00-0.02-0.01}$	0.08	—	—	—	0.00332	0.06	—

- The similar situation also occurs in the decays $B_{(s)} \rightarrow D_{(s)}^{*}(1S)M$: The branching ratios for some neutral $B_{(s)}$

decays are about two times as large as the data.

- Our predictions for the branching ratios of the decays $B_{(s)} \rightarrow D_{(s)}^{*}(2S)M$ are much larger than the results given by the

BS equation, but agree well with the relativistic quark model (RQM) and the relativistic independent quark (RIQ)

calculations.

Non-leptonic Decays $B \rightarrow D^{(*)}(1S, 2S)M$

- Polarization fractions (10^{-2}) of the decays $B_{(s)} \rightarrow D_{(s)}^*(1S, 2S)(\rho, K^*)$.

Channels	$\bar{B}^0 \rightarrow D^{*+}(1S)\rho^-$	$\bar{B}^0 \rightarrow D^{*+}(1S)K^{*-}$	$\bar{B}^0 \rightarrow D^{*+}(2S)\rho^-$	$\bar{B}^0 \rightarrow D^{*+}(2S)K^{*-}$
f_L [%]	90.59	88.11	93.52	91.56
PDG	88.5 ± 2.0	92^{+38}_{-32}	-	-
$f_{ }$ [%]	8.27	10.47	6.42	8.36
Channel	$B^- \rightarrow D^{*0}(1S)\rho^-$	$B^- \rightarrow D^{*0}(1S)K^{*-}$	$B^- \rightarrow D^{*0}(2S)\rho^-$	$B^- \rightarrow D^{*0}(2S)K^{*-}$
f_L [%]	90.60	88.12	93.25	91.21
PDG	89.2 ± 2.4	86.0 ± 6.7	-	-
$f_{ }$ [%]	8.26	10.46	6.68	8.70
Channels	$\bar{B}_s^0 \rightarrow D_s^{*+}(1S)\rho^-$	$\bar{B}_s^0 \rightarrow D_s^{*+}(1S)K^{*-}$	$\bar{B}_s^0 \rightarrow D_s^{*+}(2S)\rho^-$	$\bar{B}_s^0 \rightarrow D_s^{*+}(2S)K^{*-}$
f_L [%]	90.68	88.22	92.68	90.52
BS+FA	87.40	84.10	—	—
BS+PQCD	85.40	85.70	—	—
PQCD	87	83	—	—
Bell	105^{+6}_{-11}	—	—	—
$f_{ }$ [%]	8.22	10.41	7.32	9.48
BS+FA	10.40	13.30	—	—
BS+PQCD	11.30	10.40	—	—

Non-leptonic Decays $B \rightarrow D^{(*)}(1S, 2S)M$

- Although there exist significant difference between the form factors of the transitions $B_{(s)} \rightarrow D_{(s)}^*(1S)$ and $B_{(s)} \rightarrow D_{(s)}^*(2S)$, the similar polarization behaviors can be observed in these decays $B_{(s)} \rightarrow D_{(s)}^*(1S, 2S)(\rho, K^*)$.
- The longitudinal polarization is dominant, reaching approximately 90%, while the transverse parallel and perpendicular polarization fractions are only a few percent or roughly 10%.

Summary

- The form factors of the transitions $B_{(s)} \rightarrow D_{(s)}(2S), D_{(s)}^*(2S)$ are much smaller than those of the corresponding transitions $B_{(s)} \rightarrow D_{(s)}(1S), D_{(s)}^*(1S)$ because of the different node structures of the wave functions between the ground and radially excited states of $D_{(s)}$ mesons.
- Except for the branching ratios of the neutral decays $B_{(s)}^0 \rightarrow D_{(s)}^{(*)\pm}(1S)(\pi, \rho, K^{(*)})\mp$, which have some excess compared to the experimental data, overall our predictions for the decays $B_{(s)} \rightarrow D_{(s)}^{(*)}(1S)(\pi, \rho, K^{(*)})$ are consistent with the experimental measurements.
- Most the branching ratios of the decays $B_{(s)} \rightarrow D_{(s)}^{(*)}(2S)(\pi, \rho, K^{(*)})$ lie in the range of 10^{-5} to 10^{-4} , which are likely to be detected by the present LHCb and Belle II experiments. Our predictions for these decays are larger than the results given by the BS equation, but agree well with the RQM and RIQ calculations.
- Although there exist obvious difference in the branching ratios between the decays $B_{(s)} \rightarrow D_{(s)}^*(1S)(\rho, K^*)$ and $B_{(s)} \rightarrow D_{(s)}^*(2S)(\rho, K^*)$, the similar polarization behaviour can be observed in these decays $B_{(s)} \rightarrow D_{(s)}^*(1S, 2S)(\rho, K^*)$.
- Study of the decays $B_{(s)} \rightarrow D_{(s)}^{(*)}(2S)(\pi, \rho, K^{(*)})$ are helpful to recognize the radially excited charmed mesons.

Thank you for your attention!