江西师范大学
Jiangxi Normal University

Studying two-body χ_{cJ} (J=1,2,3) decays with SU(3) flavor analysis

Bo Lan (兰波)

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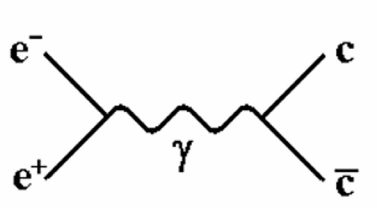
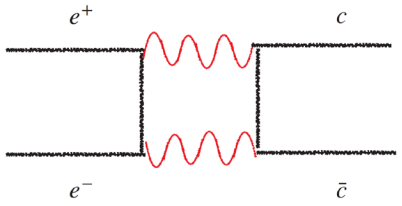
Introduction

BESIII@BEPCII



$e^+e^- \to c\bar{c}$



PRL129, 122001 (2022)

$e^+e^- \to \chi_{c1}$

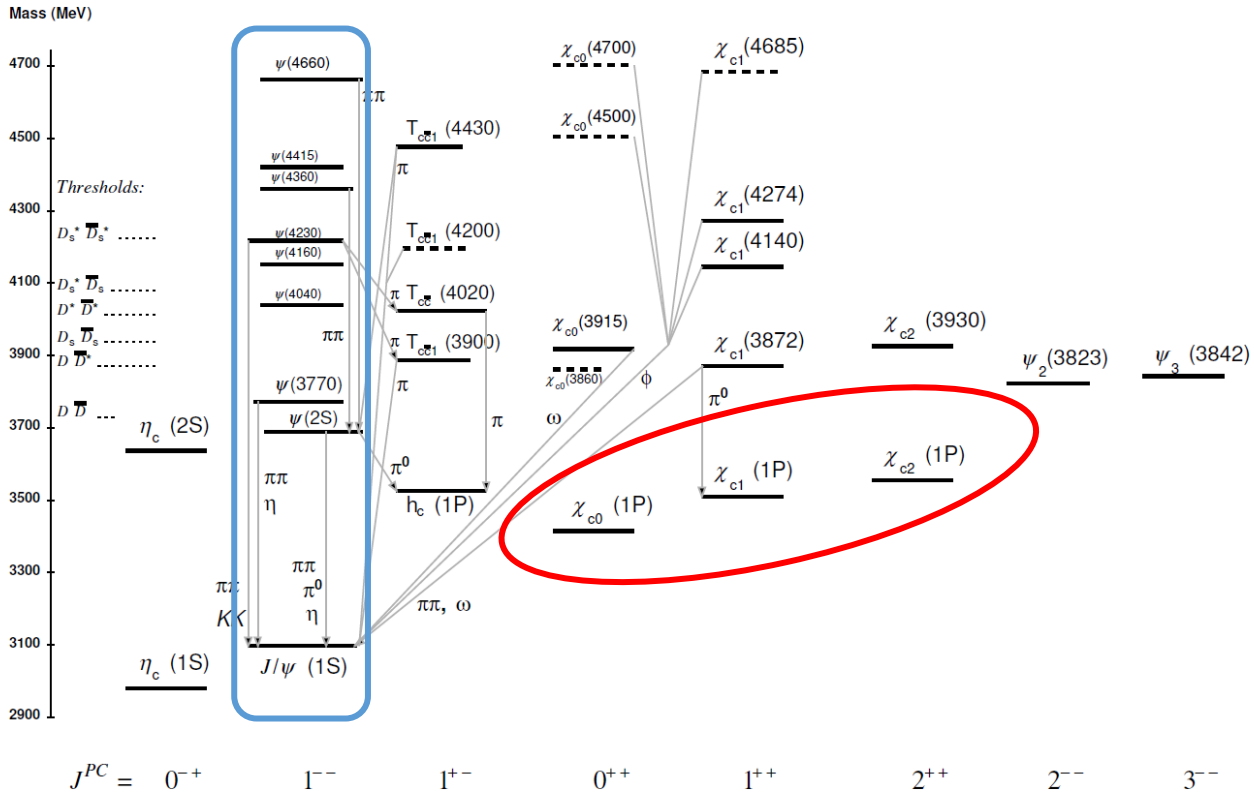
$e^+e^- \to J/\Psi$

more attention

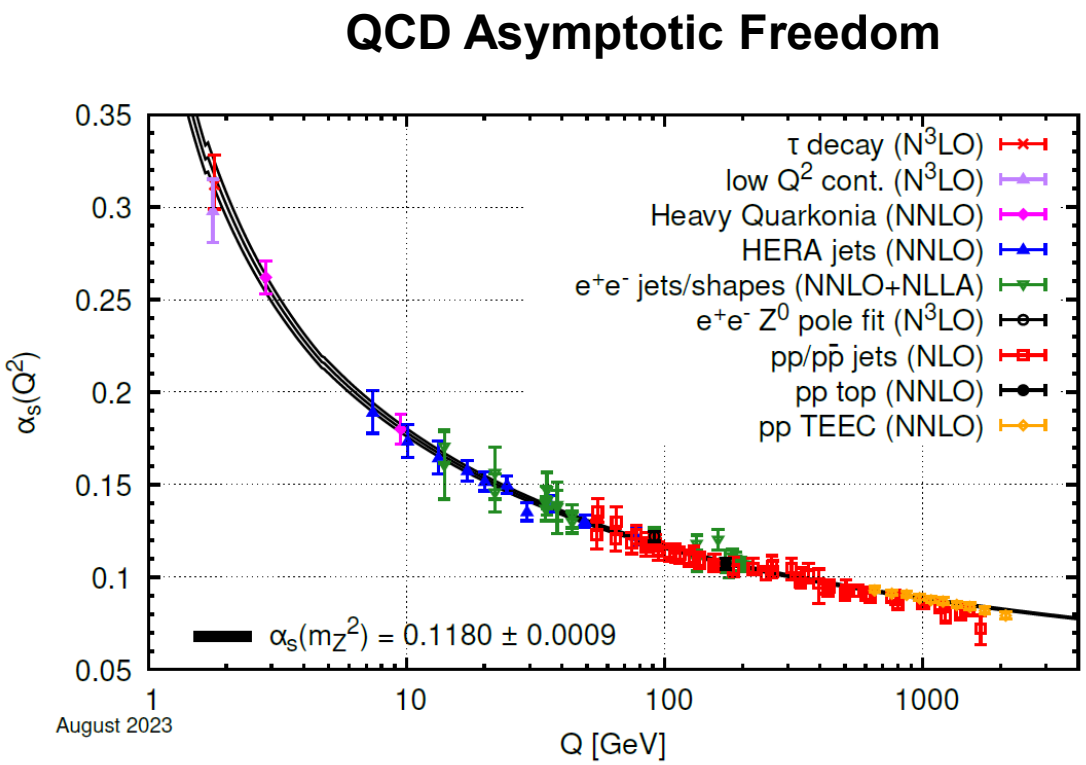
$e^+e^- \to \Psi(2S) \to \gamma\chi_{cJ}$

$(J = 0,1,2)$

less attention



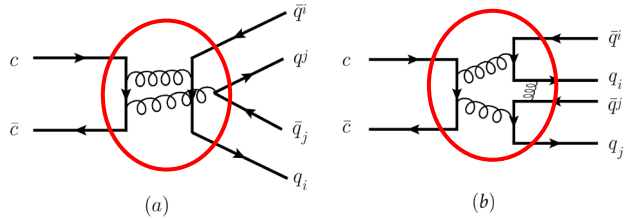
Introduction



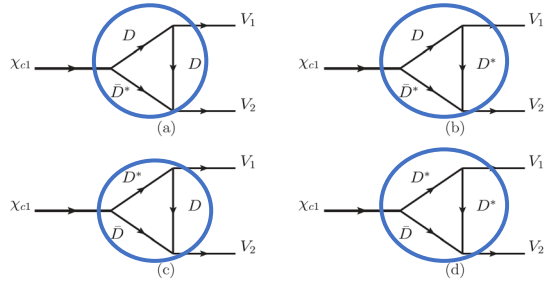
Mass of *charmonium*

pQCD or non-pQCD ?

Short-ranged mechanism:



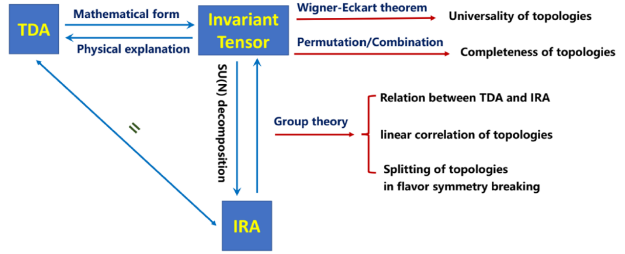
Long-ranged mechanism:



Xiao-Hai Liu, and Qiang Zhao,
PRD 81, 014017 (2010)

SU(3) flavor symmetry

Parametrization
Simplify the complex



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$\chi_{cJ} \to B_8\bar{B}_8, B_{10}\bar{B}_{10}$

$$\mathcal{H}_{\text{eff}} = H_0 + \Delta H$$

symmetry *breaking*

***I*-spin** ***U*-spin**

$$\mathbf{S}_m = \frac{g_m}{3}\begin{pmatrix} 1 & & \\ & 1 & \\ & & -2 \end{pmatrix} \quad \mathbf{S}_e = \frac{g_e}{3}\begin{pmatrix} 2 & & \\ & -1 & \\ & & -1 \end{pmatrix}$$

J/Ψ

$\Psi(2S)$ →

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- octet

$$\mathcal{H}_{\text{eff}} = g_0 \cdot \bar{B}^i_j B^j_i + g_m \cdot ([\bar{B}B]_f)^3_3 + g'_m \cdot ([\bar{B}B]_d)^3_3 + g_e \cdot ([\bar{B}B]_f)^1_1 + g'_e \cdot ([\bar{B}B]_d)^1_1.$$

$$([\bar{B}B]_f)^i_j = \bar{B}^i_k B^k_j - \bar{B}^k_j B^i_k,$$

$$([\bar{B}B]_d)^i_j = \bar{B}^i_k B^k_j + \bar{B}^k_j B^i_k - \frac{2}{3}\delta^i_j \cdot \bar{B}^i_j B^j_i.$$

$$\mathcal{B}_8 = \begin{pmatrix} \frac{\Lambda}{\sqrt{6}} + \frac{\Sigma^0}{\sqrt{2}} & \Sigma^+ & p \\ \Sigma^- & \frac{\Lambda}{\sqrt{6}} - \frac{\Sigma^0}{\sqrt{2}} & n \\ \Xi^- & \Xi^0 & \frac{-2\Lambda}{\sqrt{6}} \end{pmatrix},$$

- decuplet

$$\mathcal{H}_{eff} = g_0 \cdot B_{ijk}B^{ijk} + g_m \cdot H^3_3 + g_e \cdot H^1_1$$

$$H^3_3 = B_{3jk}B^{3jk} - \frac{1}{3}(B_{ijk}B^{ijk})$$

$$H^1_1 = B_{1jk}B^{1jk} - \frac{1}{3}(B_{ijk}B^{ijk})$$

$$\mathcal{B}_{10} = \frac{1}{\sqrt{3}}\left(\begin{pmatrix} \sqrt{3}\Delta^{++} & \Delta^+ & \Sigma^{*+} \\ \Delta^+ & \Delta^0 & \frac{\Sigma^{*0}}{\sqrt{2}} \\ \Sigma^{*+} & \frac{\Sigma^{*0}}{\sqrt{2}} & \Xi^{*0} \end{pmatrix}, \begin{pmatrix} \Delta^+ & \Delta^0 & \frac{\Sigma^{*0}}{\sqrt{2}} \\ \Delta^0 & \sqrt{3}\Delta^- & \Sigma^{*-} \\ \frac{\Sigma^{*0}}{\sqrt{2}} & \Sigma^{*-} & \Xi^{*-} \end{pmatrix}, \begin{pmatrix} \Sigma^{*+} & \frac{\Sigma^{*0}}{\sqrt{2}} & \Xi^{*0} \\ \frac{\Sigma^{*0}}{\sqrt{2}} & \Sigma^{*-} & \Xi^{*-} \\ \Xi^{*0} & \Xi^{*-} & \sqrt{3}\Omega^- \end{pmatrix}\right).$$

$\chi_{cJ} \to B_8\overline{B}_8, B_{10}\overline{B}_{10}$

SU(3) amplitude:

$B_8\overline{B}_8$

TABLE I: Amplitudes for the $\chi_{cJ} \to \mathcal{B}_8\bar{\mathcal{B}}_8$ decays. $A_J = g_0$, $D_J = g'_e/3$, $F_J = -g_e$, $D'_J = -g'_m/3$, and $F'_J = g_m$.

Decay modes	Amplitudes
$\chi_{cJ} \to p\bar{p}$	$A_J + D_J + F_J - D'_J + F'_J$
$\chi_{cJ} \to n\bar{n}$	$A_J - 2D_J - D'_J + F'_J$
$\chi_{cJ} \to \Sigma^+\bar{\Sigma}^-$	$A_J + D_J + F_J + 2D'_J$
$\chi_{cJ} \to \Sigma^-\bar{\Sigma}^+$	$A_J + D_J - F_J + 2D'_J$
$\chi_{cJ} \to \Xi^0\bar{\Xi}^0$	$A_J - 2D_J - D'_J - F'_J$
$\chi_{cJ} \to \Xi^-\bar{\Xi}^+$	$A_J + D_J - F_J - D'_J - F'_J$
$\chi_{cJ} \to \Sigma^0\bar{\Sigma}^0$	$A_J + D_J + 2D'_J$
$\chi_{cJ} \to \Lambda\bar{\Lambda}$	$A_J - D_J - 2D'_J$
$\chi_{cJ} \to \Sigma^0\bar{\Lambda}, \bar{\Sigma}^0\Lambda$	$\sqrt{3}D_J$
$\chi_{cJ} \to \Sigma'^0\bar{\Sigma}'^0$	$A_J + (\cos 2\alpha + \sqrt{3}\sin 2\alpha)D_J + (2\cos 2\alpha)D'_J$
$\chi_{cJ} \to \Lambda'\bar{\Lambda}'$	$A_J - (\sqrt{3}\sin 2\alpha + \cos 2\alpha)D_J - (2\cos 2\alpha)D'_J$
$\chi_{cJ} \to \Sigma'^0\bar{\Lambda}' + \bar{\Sigma}'^0\Lambda'$	$(\sqrt{3}\cos 2\alpha - \sin 2\alpha)D_J - (2\sin 2\alpha)D'_J$

$$\Lambda' = \Lambda\cos\alpha - \Sigma^0\sin\alpha,$$

$$\Sigma'^0 = \Lambda\sin\alpha + \Sigma^0\cos\alpha.$$

The mixing angle $\boldsymbol{\alpha}$: $[0.14,2.0] \times 10^{-2}$

We choose $\boldsymbol{\alpha}$: $(1.5 \pm 0.1) \times 10^{-2}$

The α cite from:

The quark model,
Chiral perturbation theory,
QCD sum rules,
Lattice QCD

$B_{10}\overline{B}_{10}$

TABLE II. Amplitudes for the $\chi_{cJ} \to \mathcal{B}_{10}\bar{\mathcal{B}}_{10}$ decays. $\hat{A}_J = \hat{g}_0$, $\hat{D}_J = \hat{g}_e/3$, and $\hat{D}'_J = \hat{g}_m/3$.

Decay modes	Amplitudes	Decay modes	Amplitudes
$\chi_{cJ} \to \Delta^{++}\bar{\Delta}^{--}$	$\hat{A}_J + 2\hat{D}_J - \hat{D}'_J$	$\chi_{cJ} \to \Sigma^{*0}\bar{\Sigma}^{*0}$	$\hat{A}_J$
$\chi_{cJ} \to \Delta^{+}\bar{\Delta}^{-}$	$\hat{A}_J + \hat{D}_J - \hat{D}'_J$	$\chi_{cJ} \to \Sigma^{*-}\bar{\Sigma}^{*+}$	$\hat{A}_J - \hat{D}_J$
$\chi_{cJ} \to \Delta^{0}\bar{\Delta}^{0}$	$\hat{A}_J - \hat{D}'_J$	$\chi_{cJ} \to \Xi^{*0}\bar{\Xi}^{*0}$	$\hat{A}_J + \hat{D}'_J$
$\chi_{cJ} \to \Delta^{-}\bar{\Delta}^{+}$	$\hat{A}_J - \hat{D}_J - \hat{D}'_J$	$\chi_{cJ} \to \Xi^{*-}\bar{\Xi}^{*+}$	$\hat{A}_J - \hat{D}_J + \hat{D}'_J$
$\chi_{cJ} \to \Sigma^{*+}\bar{\Sigma}^{*-}$	$\hat{A}_J + \hat{D}_J$	$\chi_{cJ} \to \Omega^{-}\bar{\Omega}^{+}$	$\hat{A}_J - \hat{D}_J + 2\hat{D}'_J$

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$\chi_{cJ} \to B_8\overline{B}_8$

Branching ratio: $B_8\overline{B}_8$

Case I: No mixing **Case II: $\Lambda - \Sigma^0$ mixing**

The impact of the $\Lambda - \Sigma^0$ mixed effect is not significant

	Exp. data 2022	case I (1σ)	case I (2σ)	case II (1σ)	case II (2σ)
$Br(\chi_{c0} \to p\bar{p})$	2.21 ± 0.08	$4.41 \pm 1.68^{\sharp}$	2.21 ± 0.16	$4.39 \pm 1.67^{\sharp}$	2.21 ± 0.16
$Br(\chi_{c0} \to n\bar{n})$	$\cdots$	3.31 ± 1.34	3.98 ± 2.00	3.32 ± 1.33	3.97 ± 1.99
$Br(\chi_{c0} \to \Lambda^{(')}\bar{\Lambda}^{(')})$	3.59 ± 0.15	3.59 ± 0.15	3.59 ± 0.30	3.59 ± 0.15	3.59 ± 0.30
$Br(\chi_{c0} \to \Sigma^{(')0}\bar{\Sigma}^{(')0})$	4.68 ± 0.32	4.68 ± 0.32	4.67 ± 0.63	4.68 ± 0.32	4.66 ± 0.62
$Br(\chi_{c0} \to \Sigma^{+}\bar{\Sigma}^{-})$	4.6 ± 0.8	4.60 ± 0.80	3.91 ± 0.91	4.57 ± 0.77	3.94 ± 0.94
$Br(\chi_{c0} \to \Sigma^{-}\bar{\Sigma}^{+})$	5.1 ± 0.5	5.10 ± 0.50	5.10 ± 1.00	5.10 ± 0.50	5.10 ± 1.00
$Br(\chi_{c0} \to \Xi^{0}\bar{\Xi}^{0})$	3.1 ± 0.8	3.10 ± 0.80	4.04 ± 0.66	3.10 ± 0.80	4.05 ± 0.65
$Br(\chi_{c0} \to \Xi^{-}\bar{\Xi}^{+})$	4.8 ± 0.7	4.80 ± 0.70	4.80 ± 1.40	4.80 ± 0.70	4.80 ± 1.40
$Br(\chi_{c0} \to \Lambda^{(')}\bar{\Sigma}^{(')0} + c.c.)$	$\cdots$	0.25 ± 0.25	0.13 ± 0.13	0.24 ± 0.24	0.26 ± 0.26

χ_{c0} $\times 10^{-4}$

	Exp. data 2022	case I (1σ)	case I (2σ)	case II (1σ)	case II (2σ)
$Br(\chi_{c1} \to p\bar{p})$	7.60 ± 0.34	7.56 ± 0.30	7.60 ± 0.68	7.60 ± 0.33	7.60 ± 0.68
$Br(\chi_{c1} \to n\bar{n})$	$\cdots$	5.20 ± 0.55	5.75 ± 1.90	5.13 ± 0.69	5.79 ± 1.81
$Br(\chi_{c1} \to \Lambda^{(')}\bar{\Lambda}^{(')})$	12.7 ± 0.8	13.24 ± 0.26	12.99 ± 1.31	13.19 ± 0.31	12.99 ± 1.31
$Br(\chi_{c1} \to \Sigma^{(')0}\bar{\Sigma}^{(')0})$	4.2 ± 0.6	$2.53 \pm 0.28^{\sharp}$	4.20 ± 1.20	$2.57 \pm 0.35^{\sharp}$	4.18 ± 1.18
$Br(\chi_{c1} \to \Sigma^{+}\bar{\Sigma}^{-})$	3.6 ± 0.7	3.12 ± 0.22	3.71 ± 1.29	3.20 ± 0.30	3.71 ± 1.29
$Br(\chi_{c1} \to \Sigma^{-}\bar{\Sigma}^{+})$	5.7 ± 1.5	$2.13 \pm 0.45^{\sharp}$	4.33 ± 1.63	$2.32 \pm 0.64^{\sharp}$	4.33 ± 1.63
$Br(\chi_{c1} \to \Xi^{0}\bar{\Xi}^{0})$	< 6	3.20 ± 0.28	3.77 ± 1.21	3.27 ± 0.36	3.76 ± 1.12
$Br(\chi_{c1} \to \Xi^{-}\bar{\Xi}^{+})$	8.0 ± 2.1	6.18 ± 0.28	5.19 ± 1.39	6.18 ± 0.28	5.23 ± 1.43
$Br(\chi_{c1} \to \Lambda^{(')}\bar{\Sigma}^{(')0} + c.c.)$	$\cdots$	0.11 ± 0.02	0.06 ± 0.06	0.16 ± 0.03	0.18 ± 0.18

χ_{c1} $\times 10^{-5}$

	Exp. data 2022	case I (1σ)	case I (2σ)	case II (1σ)	case II (2σ)
$Br(\chi_{c2} \to p\bar{p})$	7.33 ± 0.33	7.33 ± 0.33	7.33 ± 0.66	7.33 ± 0.33	7.33 ± 0.66
$Br(\chi_{c2} \to n\bar{n})$	$\cdots$	8.52 ± 3.75	8.26 ± 4.04	8.57 ± 3.78	8.23 ± 3.79
$Br(\chi_{c2} \to \Lambda^{(')}\bar{\Lambda}^{(')})$	18.3 ± 1.6	18.30 ± 1.60	18.32 ± 3.18	18.30 ± 1.60	18.35 ± 3.15
$Br(\chi_{c2} \to \Sigma^{(')0}\bar{\Sigma}^{(')0})$	3.7 ± 0.6	3.70 ± 0.60	3.70 ± 1.20	3.70 ± 0.60	3.69 ± 1.19
$Br(\chi_{c2} \to \Sigma^{+}\bar{\Sigma}^{-})$	3.4 ± 0.7	3.40 ± 0.70	3.51 ± 1.29	3.40 ± 0.70	3.51 ± 1.29
$Br(\chi_{c2} \to \Sigma^{-}\bar{\Sigma}^{+})$	4.4 ± 1.8	3.87 ± 1.27	3.38 ± 1.66	3.86 ± 1.26	3.36 ± 1.67
$Br(\chi_{c2} \to \Xi^{0}\bar{\Xi}^{0})$	< 10	7.43 ± 2.51	7.03 ± 2.97	7.31 ± 2.39	7.11 ± 2.89
$Br(\chi_{c2} \to \Xi^{-}\bar{\Xi}^{+})$	14.2 ± 3.2	$6.84 \pm 3.48^{\sharp}$	9.93 ± 2.13	$6.78 \pm 3.53^{\sharp}$	9.93 ± 2.13
$Br(\chi_{c2} \to \Lambda^{(')}\bar{\Sigma}^{(')0} + c.c)$	$\cdots$	0.10 ± 0.10	0.12 ± 0.12	0.11 ± 0.11	0.27 ± 0.27

χ_{c2} $\times 10^{-5}$

$\chi_{cJ} \to B_8\overline{B}_8$

Branching ratio: $B_8\overline{B}_8$

The latest experimental data provides better constraints !

	Exp. data 2022	Exp. data 2024	$SU(3)$ (1σ)
$Br(\chi_{c0} \to p\bar{p})$	2.21 ± 0.08	2.21 ± 0.14	2.21 ± 0.14
$Br(\chi_{c0} \to n\bar{n})$	$\cdots$	$\cdots$	4.10 ± 0.43
$Br(\chi_{c0} \to \Lambda^{(\prime)}\bar{\Lambda}^{(\prime)})$	3.59 ± 0.15	3.61 ± 0.16	3.61 ± 0.16
$Br(\chi_{c0} \to \Sigma^{(\prime)0}\bar{\Sigma}^{(\prime)0})$	4.68 ± 0.32	4.70 ± 0.32	4.70 ± 0.32
$Br(\chi_{c0} \to \Sigma^{+}\bar{\Sigma}^{-})$	4.6 ± 0.8	4.7 ± 0.8	4.32 ± 0.42
$Br(\chi_{c0} \to \Sigma^{-}\bar{\Sigma}^{+})$	5.1 ± 0.5	5.2 ± 0.5	5.20 ± 0.50
$Br(\chi_{c0} \to \Xi^{0}\bar{\Xi}^{0})$	3.1 ± 0.8	4.5 ± 0.5	4.66 ± 0.34
$Br(\chi_{c0} \to \Xi^{-}\bar{\Xi}^{+})$	4.8 ± 0.7	4.48 ± 0.20	4.48 ± 0.20
$Br(\chi_{c0} \to \Lambda^{(\prime)}\bar{\Sigma}^{(\prime)0} + c.c.)$	$\cdots$	$\cdots$	0.055 ± 0.054

χ_{c0} $\times 10^{-4}$

	Exp. data 2022	Exp. data 2024	$SU(3)$ (1σ)
$Br(\chi_{c1} \to p\bar{p})$	7.60 ± 0.34	7.6 ± 0.4	7.6 ± 0.4
$Br(\chi_{c1} \to n\bar{n})$	$\cdots$	$\cdots$	13.1 ± 4.9
$Br(\chi_{c1} \to \Lambda^{(\prime)}\bar{\Lambda}^{(\prime)})$	12.7 ± 0.8	12.7 ± 0.9	12.7 ± 0.9
$Br(\chi_{c1} \to \Sigma^{(\prime)0}\bar{\Sigma}^{(\prime)0})$	4.2 ± 0.6	4.2 ± 0.6	4.2 ± 0.6
$Br(\chi_{c1} \to \Sigma^{+}\bar{\Sigma}^{-})$	3.6 ± 0.7	3.6 ± 0.7	3.6 ± 0.7
$Br(\chi_{c1} \to \Sigma^{-}\bar{\Sigma}^{+})$	5.7 ± 1.5	5.7 ± 1.5	5.7 ± 1.5
$Br(\chi_{c1} \to \Xi^{0}\bar{\Xi}^{0})$	< 6	7.5 ± 1.3	7.5 ± 1.3
$Br(\chi_{c1} \to \Xi^{-}\bar{\Xi}^{+})$	8.0 ± 2.1	6.0 ± 0.6	6.0 ± 0.6
$Br(\chi_{c1} \to \Lambda^{(\prime)}\bar{\Sigma}^{(\prime)0} + c.c.)$	$\cdots$	$\cdots$	4.83 ± 1.86

χ_{c1} $\times 10^{-5}$

	Exp. data 2022	Exp. data 2024	$SU(3)$ (1σ)
$Br(\chi_{c2} \to p\bar{p})$	7.33 ± 0.33	7.3 ± 0.4	7.3 ± 0.4
$Br(\chi_{c2} \to n\bar{n})$	$\cdots$	$\cdots$	16.4 ± 3.1
$Br(\chi_{c2} \to \Lambda^{(\prime)}\bar{\Lambda}^{(\prime)})$	18.3 ± 1.6	18.6 ± 1.6	18.6 ± 1.6
$Br(\chi_{c2} \to \Sigma^{(\prime)0}\bar{\Sigma}^{(\prime)0})$	3.7 ± 0.6	3.7 ± 0.6	3.8 ± 0.5
$Br(\chi_{c2} \to \Sigma^{+}\bar{\Sigma}^{-})$	3.4 ± 0.7	3.4 ± 0.7	3.16 ± 0.46
$Br(\chi_{c2} \to \Sigma^{-}\bar{\Sigma}^{+})$	4.4 ± 1.8	4.5 ± 1.8	5.16 ± 1.12
$Br(\chi_{c2} \to \Xi^{0}\bar{\Xi}^{0})$	< 10	18.6 ± 2.2	18.6 ± 2.2
$Br(\chi_{c2} \to \Xi^{-}\bar{\Xi}^{+})$	14.2 ± 3.2	14.6 ± 1.2	14.6 ± 1.2
$Br(\chi_{c2} \to \Lambda^{(\prime)}\bar{\Sigma}^{(\prime)0} + c.c.)$	$\cdots$	$\cdots$	0.45 ± 0.35

χ_{c2} $\times 10^{-5}$

$\chi_{cJ} \to B_{10}\overline{B}_{10}$

Branching ratio: $B_{10}\overline{B}_{10}$

Large error!

Need more experimental data to do the analysis

	Exp. data 2022	Predictions (1σ)	Predictions (2σ)
$B(\chi_{c0} \to \Delta^{++}\bar{\Delta}^{--})$	$\cdots$	3.43 ± 2.34	4.34 ± 4.16
$B(\chi_{c0} \to \Delta^{+}\bar{\Delta}^{-})$	$\cdots$	2.87 ± 1.45	3.57 ± 3.20
$B(\chi_{c0} \to \Delta^{0}\bar{\Delta}^{0})$	$\cdots$	3.11 ± 1.59	3.63 ± 3.10
$B(\chi_{c0} \to \Delta^{-}\bar{\Delta}^{+})$	$\cdots$	3.92 ± 1.75	4.59 ± 3.75
$B(\chi_{c0} \to \Sigma^{*+}\bar{\Sigma}^{*-})$	1.60 ± 0.60	1.60 ± 0.60	1.60 ± 1.20
$B(\chi_{c0} \to \Sigma^{*0}\bar{\Sigma}^{*0})$	$\cdots$	1.80 ± 0.78	1.87 ± 1.35
$B(\chi_{c0} \to \Sigma^{*-}\bar{\Sigma}^{*+})$	2.30 ± 0.70	2.30 ± 0.70	2.30 ± 1.40
$B(\chi_{c0} \to \Xi^{*0}\bar{\Xi}^{*0})$	$\cdots$	0.87 ± 0.38	0.92 ± 0.59
$B(\chi_{c0} \to \Xi^{*-}\bar{\Xi}^{*+})$	$\cdots$	1.19 ± 0.30	1.19 ± 0.61
$B(\chi_{c0} \to \Omega^{-}\bar{\Omega}^{+})$	0.351 ± 0.061	0.351 ± 0.061	0.351 ± 0.122

χ_{c0}

$\times 10^{-4}$

	Exp. data 2022	Predictions (1σ)	Predictions (2σ)
$B(\chi_{c1} \to \Delta^{++}\bar{\Delta}^{--})$	$\cdots$	< 20.46	< 20.76
$B(\chi_{c1} \to \Delta^{+}\bar{\Delta}^{-})$	$\cdots$	< 12.85	< 13.04
$B(\chi_{c1} \to \Delta^{0}\bar{\Delta}^{0})$	$\cdots$	< 9.85	< 10.15
$B(\chi_{c1} \to \Delta^{-}\bar{\Delta}^{+})$	$\cdots$	< 7.74	< 8.32
$B(\chi_{c1} \to \Sigma^{*+}\bar{\Sigma}^{*-})$	< 9.00	< 9.00	< 9.00
$B(\chi_{c1} \to \Sigma^{*0}\bar{\Sigma}^{*0})$	$\cdots$	< 6.79	< 6.74
$B(\chi_{c1} \to \Sigma^{*-}\bar{\Sigma}^{*+})$	< 5.00	< 5.00	< 5.00
$B(\chi_{c1} \to \Xi^{*0}\bar{\Xi}^{*0})$	$\cdots$	< 4.68	< 4.92
$B(\chi_{c1} \to \Xi^{*-}\bar{\Xi}^{*+})$	$\cdots$	< 3.34	< 3.53
$B(\chi_{c1} \to \Omega^{-}\bar{\Omega}^{+})$	1.49 ± 0.25	1.49 ± 0.25	1.49 ± 0.50

χ_{c1}

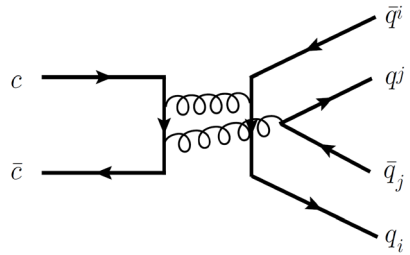
$\times 10^{-5}$

	Exp. data 2022	Predictions (1σ)	Predictions (2σ)
$B(\chi_{c2} \to \Delta^{++}\bar{\Delta}^{--})$	$\cdots$	< 32.01	< 29.79
$B(\chi_{c2} \to \Delta^{+}\bar{\Delta}^{-})$	$\cdots$	< 18.40	< 18.07
$B(\chi_{c2} \to \Delta^{0}\bar{\Delta}^{0})$	$\cdots$	< 13.44	< 13.07
$B(\chi_{c2} \to \Delta^{-}\bar{\Delta}^{+})$	$\cdots$	< 9.32	< 9.58
$B(\chi_{c2} \to \Sigma^{*+}\bar{\Sigma}^{*-})$	< 16.00	< 16.00	< 16.00
$B(\chi_{c2} \to \Sigma^{*0}\bar{\Sigma}^{*0})$	$\cdots$	< 11.59	< 11.23
$B(\chi_{c2} \to \Sigma^{*-}\bar{\Sigma}^{*+})$	< 8.00	< 8.00	< 8.00
$B(\chi_{c2} \to \Xi^{*0}\bar{\Xi}^{*0})$	$\cdots$	< 9.79	< 9.95
$B(\chi_{c2} \to \Xi^{*-}\bar{\Xi}^{*+})$	$\cdots$	< 6.78	< 7.01
$B(\chi_{c2} \to \Omega^{-}\bar{\Omega}^{+})$	4.52 ± 0.30	4.52 ± 0.30	4.52 ± 0.60

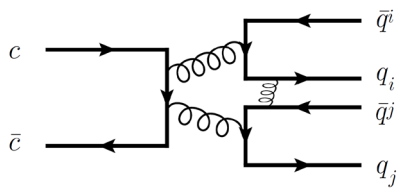
χ_{c2}

$\times 10^{-5}$

$\chi_{cJ} \to PP, VV, PV, PT$



(a)



(b)

singly OZI-suppressed **doubly OZI-suppressed**

$$\mathcal{A}^s(\chi_{cJ} \to M_1 M_2) = a_{1J}^M M_{1\,j}^i M_{2i}^j + a_{2J}^M M_{1i}^i M_{2\,j}^j,$$

$$\mathcal{A}^b(\chi_{cJ} \to M_1 M_2) = b_J^M M_{1\,j}^i W_k^j M_{2i}^k,$$

***I*-spin breaking**

$$\begin{pmatrix} m_u & 0 & 0 \\ 0 & m_d & 0 \\ 0 & 0 & m_s \end{pmatrix} = \frac{1}{3}(m_u + m_d + m_s)I + \frac{1}{2}(m_u - m_d)X + \frac{1}{6}(m_u + m_d - 2m_s)W,$$

$$X = \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 0 \end{pmatrix}, \quad W = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -2 \end{pmatrix}.$$

$$P = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} & \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} & K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta_8}{\sqrt{6}} + \frac{\eta_1}{\sqrt{3}} \end{pmatrix}, \quad \begin{aligned} \eta &= (\eta_8 \cos\theta_P - \eta_1 \sin\theta_P), \\ \eta' &= (\eta_8 \sin\theta_P + \eta_1 \cos\theta_P), \end{aligned}$$

$$V = \begin{pmatrix} \frac{\rho^0}{\sqrt{2}} + \frac{\omega_8}{\sqrt{6}} + \frac{\omega_1}{\sqrt{3}} & \rho^+ & K^{*+} \\ \rho^- & -\frac{\rho^0}{\sqrt{2}} + \frac{\omega_8}{\sqrt{6}} + \frac{\omega_1}{\sqrt{3}} & K^{*0} \\ K^{*-} & \bar{K}^{*0} & -\frac{2\omega_8}{\sqrt{6}} + \frac{\omega_1}{\sqrt{3}}. \end{pmatrix}, \quad \begin{aligned} \phi &= (\omega_8 \cos\theta_V - \omega_1 \sin\theta_V), \\ \omega &= (\omega_8 \sin\theta_V + \omega_1 \cos\theta_V). \end{aligned}$$

$$T = \begin{pmatrix} \frac{a_2^0}{\sqrt{2}} + \frac{f_2^8}{\sqrt{6}} + \frac{f_2^0}{\sqrt{3}} & a_2^+ & K_2^{*+} \\ a_2^- & -\frac{a_2^0}{\sqrt{2}} + \frac{f_2^8}{\sqrt{6}} + \frac{f_2^0}{\sqrt{3}} & K_2^{*0} \\ K_2^{*-} & \bar{K}_2^{*0} & -\frac{2f_2^8}{\sqrt{6}} + \frac{f_2^0}{\sqrt{3}} \end{pmatrix}, \quad \begin{aligned} f_2 &= \left(f_2^8 \cos\theta_T - f_2^0 \sin\theta_T\right), \\ f_2' &= \left(f_2^8 \sin\theta_T + f_2^0 \cos\theta_T\right), \end{aligned}$$

$\theta_P = [-20°, -10°]$

$\theta_V = 36.5°$

$\theta_T = [26°, 28°]$

The θ cite from PDG

$\chi_{cJ} \to PP, VV, PV, PT$

SU(3) amplitude:

PP/VV

Decay modes	Symmetry amplitudes	Breaking amplitudes
$\chi_{cJ} \to \pi^+\pi^-/\rho^+\rho^-$	$2a_{1J}^M$	$2b_J^M$
$\chi_{cJ} \to K^+K^-/K^{*+}K^{*-}$	$2a_{1J}^M$	$-b_J^M$
$\chi_{cJ} \to K^0\bar{K}^0/K^{*0}\bar{K}^{*0}$	$2a_{1J}^M$	$-b_J^M$
$\chi_{cJ} \to \pi^0\pi^0/\rho^0\rho^0$	$\sqrt{2}a_{1J}^M$	$\sqrt{2}b_J^M$
$\chi_{cJ} \to \eta_1\eta_1/\omega_1\omega_1$	$\sqrt{2}(a_{1J}^M + 3a_{2J}^M)$	0
$\chi_{cJ} \to \eta_1\eta_8/\omega_1\omega_8$	0	$2\sqrt{2}b_J^M$
$\chi_{cJ} \to \eta_8\eta_8/\omega_8\omega_8$	$\sqrt{2}a_{1J}^M$	$-\sqrt{2}b_J^M$
$\chi_{cJ} \to \eta\eta/\phi\phi$	$\sqrt{2}a_{1J}^M + \frac{3\sqrt{2}}{2}a_{2J}^M(1-\cos 2\theta_M)$	$-\sqrt{2}b_J^M \cos\theta_M(4\sin\theta_M + \cos\theta_M)$
$\chi_{cJ} \to \eta\eta'/\phi\omega$	$-\frac{3\sqrt{2}}{2}a_{2J}^M \sin 2\theta_M$	$\sqrt{2}b_J^M(2\cos 2\theta_M - \frac{1}{2}\sin 2\theta_M)$
$\chi_{cJ} \to \eta'\eta'/\omega\omega$	$\sqrt{2}a_{1J}^M + \frac{3\sqrt{2}}{2}a_{2J}^M(1+\cos 2\theta_M)$	$\sqrt{2}b_J^M \sin\theta_M(4\cos\theta_M - \sin\theta_M)$

doubly OZI-suppressed

PV/PT

Decay modes	Symmetry amplitudes	Breaking amplitudes
$\chi_{cJ} \to \pi^+\rho^-/\pi^+a_2^-$	$2a_{1J}^{MM}$	$2b_J^{MM}$
$\chi_{cJ} \to \pi^-\rho^+/\pi^-a_2^+$	$2a_{1J}^{MM}$	$2b_J^{MM}$
$\chi_{cJ} \to K^+K^{*-}/K^-K_2^{*+}$	$2a_{1J}^{MM}$	$-b_J^{MM}$
$\chi_{cJ} \to K^-K^{*+}/K^-K_2^{*+}$	$2a_{1J}^{MM}$	$-b_J^{MM}$
$\chi_{cJ} \to K^0\bar{K}^{*0}/K^0\bar{K}_2^{*0}$	$2a_{1J}^{MM}$	$-b_J^{MM}$
$\chi_{cJ} \to \bar{K}^0K^{*0}/\bar{K}^0K_2^{*0}$	$2a_{1J}^{MM}$	$-b_J^{MM}$
$\chi_{cJ} \to \pi^0\rho^0/\pi^0a_2^0$	$2a_{1J}^{MM}$	$2b_J^{MM}$
$\chi_{cJ} \to \eta_1\omega_1/\eta_1f_2^0$	$2a_{1J}^{MM} + 6a_{2J}^{MM}$	0
$\chi_{cJ} \to \eta_1\omega_8/\eta_1f_2^8$	0	$2\sqrt{2}b_J^{MM}$
$\chi_{cJ} \to \eta_8\omega_1/\eta_8f_2^0$	0	$2\sqrt{2}b_J^{MM}$
$\chi_{cJ} \to \eta_8\omega_8/\eta_8f_2^8$	$2a_{1J}^{MM}$	$-2b_J^{MM}$
$\chi_{cJ} \to \eta\phi/\eta f_2'$	$2a_{1J}^{MM}(\cos(\theta_P - \theta_{V/T})) + 6a_{2J}^{MM}\sin\theta_P\sin\theta_{V/T}$	$-2b_J^{MM}(\cos\theta_P\cos\theta_{V/T} + \sqrt{2}\sin(\theta_P + \theta_{V/T}))$
$\chi_{cJ} \to \eta\omega/\eta f_2$	$2a_{1J}^{MM}(-\sin(\theta_P - \theta_{V/T})) - 6a_{2J}^{MM}\sin\theta_P\cos\theta_{V/T}$	$-2b_J^{MM}(\cos\theta_P\sin\theta_{V/T} - \sqrt{2}\cos(\theta_P + \theta_{V/T}))$
$\chi_{cJ} \to \eta'\phi/\eta' f_2'$	$2a_{1J}^{MM}(\sin(\theta_P - \theta_{V/T})) - 6a_{2J}^{MM}\cos\theta_P\sin\theta_{V/T}$	$-2b_J^{MM}(\sin\theta_P\cos\theta_{V/T} - \sqrt{2}\cos(\theta_P + \theta_{V/T}))$
$\chi_{cJ} \to \eta'\omega/\eta' f_2$	$2a_{1J}^{MM}(\cos(\theta_P - \theta_{V/T})) + 6a_{2J}^{MM}\cos\theta_P\cos\theta_{V/T}$	$-2b_J^{MM}(\sin\theta_P\sin\theta_{V/T} - \sqrt{2}\sin(\theta_P + \theta_{V/T}))$

$\chi_{cJ} \to PP$

Branching ratio: PP

	Exp. data 2024	Symmetry	Including breaking
$\mathcal{B}(\chi_{c0} \to \pi^+\pi^-)$	$\cdots$	6.33 ± 0.33	5.67 ± 0.27
$\mathcal{B}(\chi_{c0} \to \pi^0\pi^0)$	$\cdots$	3.17 ± 0.16	2.83 ± 0.13
$\mathcal{B}(\chi_{c0} \to \pi\pi)$	8.50 ± 0.40	$9.50 \pm 0.49^\sharp$	8.50 ± 0.40
$\mathcal{B}(\chi_{c0} \to K^+K^-)$	6.07 ± 0.33	6.08 ± 0.32	6.07 ± 0.33
$\mathcal{B}(\chi_{c0} \to K^0\bar{K}^0)$	$\cdots$	6.08 ± 0.32	6.07 ± 0.33
$\mathcal{B}(\chi_{c0} \to K\bar{K})$	$\cdots$	12.16 ± 0.63	12.14 ± 0.66
$\mathcal{B}(\chi_{c0} \to \eta\eta)$	3.01 ± 0.25	2.91 ± 0.15	2.92 ± 0.16
$\mathcal{B}(\chi_{c0} \to \eta\eta')$	0.09 ± 0.01	0.09 ± 0.01	0.09 ± 0.01
$\mathcal{B}(\chi_{c0} \to \eta'\eta')$	2.17 ± 0.12	2.17 ± 0.12	2.17 ± 0.12

χ_{c0}

$\times 10^{-3}$

$\chi_{c1} \to PP$ are forbidden by spin-parity conservation

	Exp. data 2024	Symmetry	Including breaking
$\mathcal{B}(\chi_{c2} \to \pi^+\pi^-)$	$\cdots$	11.67 ± 0.47	15.13 ± 0.67
$\mathcal{B}(\chi_{c2} \to \pi^0\pi^0)$	$\cdots$	5.83 ± 0.24	7.57 ± 0.33
$\mathcal{B}(\chi_{c2} \to \pi\pi)$	22.70 ± 1.00	$17.50 \pm 0.71^\sharp$	22.70 ± 1.00
$\mathcal{B}(\chi_{c2} \to K^+K^-)$	10.20 ± 1.50	11.24 ± 0.46	10.20 ± 1.50
$\mathcal{B}(\chi_{c2} \to K^0\bar{K}^0)$	$\cdots$	11.24 ± 0.46	10.20 ± 1.50
$\mathcal{B}(\chi_{c2} \to K\bar{K})$	$\cdots$	22.48 ± 0.91	20.39 ± 3.00
$\mathcal{B}(\chi_{c2} \to \eta\eta)$	5.50 ± 0.50	5.22 ± 0.22	5.42 ± 0.42
$\mathcal{B}(\chi_{c2} \to \eta\eta')$	0.22 ± 0.05	0.22 ± 0.05	0.22 ± 0.05
$\mathcal{B}(\chi_{c2} \to \eta'\eta')$	0.46 ± 0.06	0.46 ± 0.06	0.46 ± 0.06

χ_{c2}

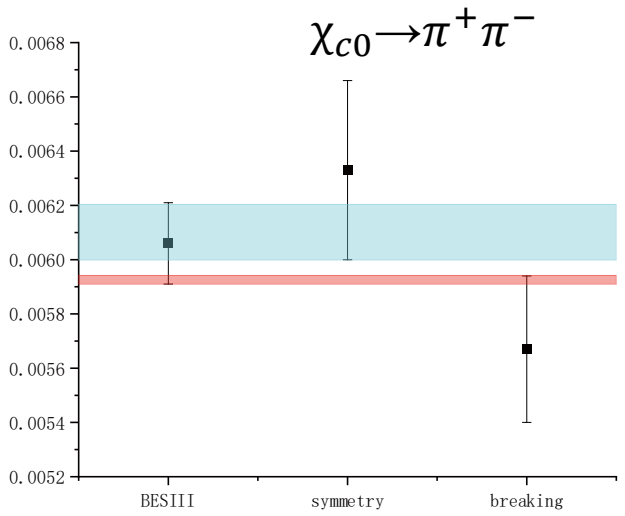
$\times 10^{-4}$

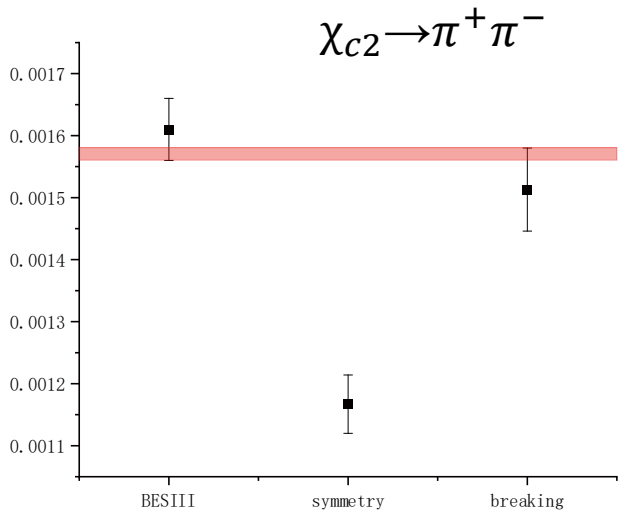
Latest Results from BESIII:

$$\mathrm{Br}(\chi_{c0} \to \pi^+\pi^-) = (6.06 \pm 0.15) \times 10^{-3}$$

$$\mathrm{Br}(\chi_{c2} \to \pi^+\pi^-) = (1.61 \pm 0.04) \times 10^{-3}$$

M. Ablikim et al., (BESIII), arXiv:2502.08929 [hep-ex]





$\chi_{cJ} \to VV$

Branching ratio: VV

SU(3) approach can explain the doubly OZI-suppressed

	Exp. data 2024	Symmetry	Including breaking
$\mathcal{B}(\chi_{c0} \to \rho^+\rho^-)$	$\cdots$	2.04 ± 0.37	2.28 ± 1.56
$\mathcal{B}(\chi_{c0} \to \rho^0\rho^0)$	$\cdots$	1.02 ± 0.18	1.14 ± 0.78
$\mathcal{B}(\chi_{c0} \to \rho\rho)$	$\cdots$	3.06 ± 0.55	3.42 ± 2.33
$\mathcal{B}(\chi_{c0} \to K^{*+}K^{*-})$	$\cdots$	1.95 ± 0.35	1.70 ± 0.60
$\mathcal{B}(\chi_{c0} \to K^{*0}\bar{K}^{*0})$	1.70 ± 0.60	1.95 ± 0.35	1.70 ± 0.60
$\mathcal{B}(\chi_{c0} \to K^*\bar{K}^*)$	$\cdots$	3.90 ± 0.70	3.40 ± 1.20
$\mathcal{B}(\chi_{c0} \to \phi\phi)$	0.848 ± 0.031	0.85 ± 0.03	0.85 ± 0.03
$\mathcal{B}(\chi_{c0} \to \phi\omega)$	0.142 ± 0.013	0.142 ± 0.013	0.142 ± 0.013
$\mathcal{B}(\chi_{c0} \to \omega\omega)$	0.97 ± 0.11	0.97 ± 0.11	0.97 ± 0.11

χ_{c0}
$\times 10^{-3}$

	Exp. data 2024	Symmetry	Including breaking
$\mathcal{B}(\chi_{c1} \to \rho^+\rho^-)$	$\cdots$	12.35 ± 1.92	15.18 ± 4.96
$\mathcal{B}(\chi_{c1} \to \rho^0\rho^0)$	$\cdots$	6.17 ± 0.96	7.59 ± 2.48
$\mathcal{B}(\chi_{c1} \to \rho\rho)$	$\cdots$	18.52 ± 2.87	22.77 ± 7.43
$\mathcal{B}(\chi_{c1} \to K^{*+}K^{*-})$	$\cdots$	11.85 ± 1.84	13.22 ± 3.21
$\mathcal{B}(\chi_{c1} \to K^{*0}\bar{K}^{*0})$	14.00 ± 4.00	11.84 ± 1.84	13.20 ± 3.20
$\mathcal{B}(\chi_{c1} \to K^*\bar{K}^*)$	$\cdots$	23.69 ± 3.68	26.42 ± 6.41
$\mathcal{B}(\chi_{c1} \to \phi\phi)$	4.26 ± 0.21	$5.24 \pm 0.73^\sharp$	4.26 ± 0.21
$\mathcal{B}(\chi_{c1} \to \phi\omega)$	0.27 ± 0.04	0.27 ± 0.04	0.27 ± 0.04
$\mathcal{B}(\chi_{c1} \to \omega\omega)$	5.70 ± 0.70	5.70 ± 0.70	5.70 ± 0.70

χ_{c1}
$\times 10^{-4}$

	Exp. data 2024	Symmetry	Including breaking
$\mathcal{B}(\chi_{c2} \to \rho^+\rho^-)$	$\cdots$	27.08 ± 5.20	18.38 ± 8.04
$\mathcal{B}(\chi_{c2} \to \rho^0\rho^0)$	$\cdots$	13.54 ± 2.60	9.19 ± 4.02
$\mathcal{B}(\chi_{c2} \to \rho\rho)$	$\cdots$	40.62 ± 7.80	27.57 ± 12.06
$\mathcal{B}(\chi_{c2} \to K^{*+}K^{*-})$	$\cdots$	26.04 ± 5.00	20.39 ± 7.37
$\mathcal{B}(\chi_{c2} \to K^{*0}\bar{K}^{*0})$	22.00 ± 9.00	26.00 ± 4.99	20.36 ± 7.36
$\mathcal{B}(\chi_{c2} \to K^*\bar{K}^*)$	$\cdots$	52.03 ± 9.99	40.76 ± 14.74
$\mathcal{B}(\chi_{c2} \to \phi\phi)$	12.30 ± 0.70	12.30 ± 0.70	12.30 ± 0.70
$\mathcal{B}(\chi_{c2} \to \phi\omega)$	0.097 ± 0.028	0.097 ± 0.028	0.097 ± 0.028
$\mathcal{B}(\chi_{c2} \to \omega\omega)$	8.60 ± 1.00	$13.57 \pm 2.36^\sharp$	8.60 ± 1.00

χ_{c2}
$\times 10^{-4}$

$\chi_{cJ} \to PV, PT$

Branching ratio:

PV:

	Exp. data 2024	Symmetry	Including breaking
$\mathcal{B}(\chi_{c1} \to \pi^{\pm}\rho^{\mp})$	$\cdots$	1.12 ± 0.11	2.78 ± 2.65
$\mathcal{B}(\chi_{c1} \to \pi^{0}\rho^{0})$	$\cdots$	0.56 ± 0.05	1.39 ± 1.32
$\mathcal{B}(\chi_{c1} \to K^{\pm}K^{*\mp})$	1.21 ± 0.23	1.08 ± 0.10	1.08 ± 0.10
$\mathcal{B}(\chi_{c1} \to K^{0}\bar{K}^{*0} + c.c.)$	1.03 ± 0.15	1.08 ± 0.10	1.08 ± 0.10
$\mathcal{B}(\chi_{c1} \to \eta\phi)$	$\cdots$	0.21 ± 0.17	0.40 ± 0.40
$\mathcal{B}(\chi_{c1} \to \eta\omega)$	$\cdots$	0.47 ± 0.34	1.16 ± 1.15
$\mathcal{B}(\chi_{c1} \to \eta'\phi)$	$\cdots$	0.64 ± 0.64	0.87 ± 0.87
$\mathcal{B}(\chi_{c1} \to \eta'\omega)$	$\cdots$	0.77 ± 0.77	1.93 ± 1.93

χ_{c1}
$\times 10^{-3}$

	Exp. data 2024	Symmetry	Including breaking
$\mathcal{B}(\chi_{c2} \to \pi^{\pm}\rho^{\mp})$	0.06 ± 0.04	$1.45 \pm 0.15^{\sharp}$	0.06 ± 0.04
$\mathcal{B}(\chi_{c2} \to \pi^{0}\rho^{0})$	$\cdots$	0.73 ± 0.08	0.03 ± 0.02
$\mathcal{B}(\chi_{c2} \to K^{\pm}K^{*\mp})$	1.46 ± 0.21	1.40 ± 0.15	1.40 ± 0.15
$\mathcal{B}(\chi_{c2} \to K^{0}\bar{K}^{*0} + c.c.)$	1.27 ± 0.27	1.40 ± 0.15	1.40 ± 0.15
$\mathcal{B}(\chi_{c2} \to \eta\phi)$	$\cdots$	0.28 ± 0.23	0.91 ± 0.48
$\mathcal{B}(\chi_{c2} \to \eta\omega)$	$\cdots$	0.59 ± 0.43	0.09 ± 0.09
$\mathcal{B}(\chi_{c2} \to \eta'\phi)$	$\cdots$	0.81 ± 0.81	1.55 ± 1.25
$\mathcal{B}(\chi_{c2} \to \eta'\omega)$	$\cdots$	1.01 ± 1.01	0.31 ± 0.31

χ_{c2}
$\times 10^{-4}$

$\chi_{c0} \to PV, PT$ are forbidden by spin-parity conservation

PT:

	Exp. data 2024	Symmetry	Including breaking
$\mathcal{B}(\chi_{c1} \to \pi^{\pm}a_2^{\mp})$	$\cdots$	1.42 ± 0.04	3.03 ± 2.14
$\mathcal{B}(\chi_{c1} \to \pi^{0}a_2^{0})$	$\cdots$	0.71 ± 0.02	1.51 ± 1.07
$\mathcal{B}(\chi_{c1} \to K^{\pm}K_2^{*\mp})$	1.61 ± 0.31	1.34 ± 0.04	1.34 ± 0.04
$\mathcal{B}(\chi_{c1} \to K^{0}\bar{K}_2^{*0} + c.c.)$	1.17 ± 0.20	1.33 ± 0.04	1.33 ± 0.04
$\mathcal{B}(\chi_{c1} \to \eta f_2')$	$\cdots$	0.23 ± 0.03	0.42 ± 0.31
$\mathcal{B}(\chi_{c1} \to \eta f_2)$	0.67 ± 0.11	0.61 ± 0.05	0.67 ± 0.11
$\mathcal{B}(\chi_{c1} \to \eta' f_2')$	$\cdots$	0.60 ± 0.07	0.76 ± 0.76
$\mathcal{B}(\chi_{c1} \to \eta' f_2)$	$\cdots$	0.99 ± 0.21	1.49 ± 1.49

χ_{c1}
$\times 10^{-3}$

	Exp. data 2024	Symmetry	Including breaking
$\mathcal{B}(\chi_{c2} \to \pi^{\pm}a_2^{\mp})$	1.80 ± 0.60	1.50 ± 0.03	2.16 ± 0.24
$\mathcal{B}(\chi_{c2} \to \pi^{0}a_2^{0})$	1.31 ± 0.35	$0.75 \pm 0.02^{\sharp}$	1.08 ± 0.12
$\mathcal{B}(\chi_{c2} \to K^{\pm}K_2^{*\mp})$	1.51 ± 0.13	1.41 ± 0.03	1.41 ± 0.03
$\mathcal{B}(\chi_{c2} \to K^{0}\bar{K}_2^{*0} + c.c.)$	1.27 ± 0.17	1.41 ± 0.03	1.41 ± 0.03
$\mathcal{B}(\chi_{c2} \to \eta f_2')$	$\cdots$	0.37 ± 0.15	0.45 ± 0.36
$\mathcal{B}(\chi_{c2} \to \eta f_2)$	$\cdots$	0.41 ± 0.25	0.77 ± 0.62
$\mathcal{B}(\chi_{c2} \to \eta' f_2')$	$\cdots$	0.37 ± 0.32	0.81 ± 0.81
$\mathcal{B}(\chi_{c2} \to \eta' f_2)$	$\cdots$	0.70 ± 0.70	1.59 ± 1.59

χ_{c2}
$\times 10^{-3}$

Summary

- SU(3) symmetry/breaking approach is applicable to charmonium decays
- The results of SU(3) can describe the OZI-suppressed decays
- More theoretical studies
- More accurate experimental data

Thank you for your attention!

Back up

$$\chi_{cJ} \to B_8 \overline{B}_8$$

	Exp. data 2022	case I (1σ)	case I (2σ)	case II (1σ)	case II (2σ)
$Br(\chi_{c0} \to p\bar{p})$	2.21 ± 0.08	$4.41 \pm 1.68^{\sharp}$	2.21 ± 0.16	$4.39 \pm 1.67^{\sharp}$	2.21 ± 0.16
$Br(\chi_{c0} \to n\bar{n})$	$\cdots$	3.31 ± 1.34	3.98 ± 2.00	3.32 ± 1.33	3.97 ± 1.99
$Br(\chi_{c0} \to \Lambda^{(\prime)}\bar{\Lambda}^{(\prime)})$	3.59 ± 0.15	3.59 ± 0.15	3.59 ± 0.30	3.59 ± 0.15	3.59 ± 0.30
$Br(\chi_{c0} \to \Sigma^{(\prime)0}\bar{\Sigma}^{(\prime)0})$	4.68 ± 0.32	4.68 ± 0.32	4.67 ± 0.63	4.68 ± 0.32	4.66 ± 0.62
$Br(\chi_{c0} \to \Sigma^{+}\bar{\Sigma}^{-})$	4.6 ± 0.8	4.60 ± 0.80	3.91 ± 0.91	4.57 ± 0.77	3.94 ± 0.94
$Br(\chi_{c0} \to \Sigma^{-}\bar{\Sigma}^{+})$	5.1 ± 0.5	5.10 ± 0.50	5.10 ± 1.00	5.10 ± 0.50	5.10 ± 1.00
$Br(\chi_{c0} \to \Xi^{0}\bar{\Xi}^{0})$	3.1 ± 0.8	3.10 ± 0.80	4.04 ± 0.66	3.10 ± 0.80	4.05 ± 0.65
$Br(\chi_{c0} \to \Xi^{-}\bar{\Xi}^{+})$	4.8 ± 0.7	4.80 ± 0.70	4.80 ± 1.40	4.80 ± 0.70	4.80 ± 1.40
$Br(\chi_{c0} \to \Lambda^{(\prime)}\bar{\Sigma}^{(\prime)0} + c.c.)$	$\cdots$	0.25 ± 0.25	0.13 ± 0.13	0.24 ± 0.24	0.26 ± 0.26
A_0 (10^{-2} GeV)	$\cdots$	3.20 ± 0.19	3.17 ± 0.29	3.20 ± 0.19	3.17 ± 0.29
D_0 (10^{-3} GeV)	$\cdots$	0.05 ± 6.60	-0.10 ± 4.74	0.05 ± 6.58	-0.10 ± 4.74
F_0 (10^{-3} GeV)	$\cdots$	0.00 ± 6.63	-0.09 ± 6.69	0.00 ± 6.63	-0.09 ± 6.69
D_0' (10^{-3} GeV)	$\cdots$	0.83 ± 0.93	1.74 ± 1.74	0.83 ± 0.93	1.73 ± 1.75
F_0' (10^{-3} GeV)	$\cdots$	-2.67 ± 3.67	-2.95 ± 3.89	-2.67 ± 3.67	-2.95 ± 3.89
φ_0 (radian)	$\cdots$	$0 \pm \pi$	$0 \pm \pi$	$0 \pm \pi$	$0 \pm \pi$
α (radian)	$\cdots$	0	0	0.015 ± 0.001	0.015 ± 0.002

	Exp. data 2022	case I (1σ)	case I (2σ)	case II (1σ)	case II (2σ)
$Br(\chi_{c1} \to p\bar{p})$	7.60 ± 0.34	7.56 ± 0.30	7.60 ± 0.68	7.60 ± 0.33	7.60 ± 0.68
$Br(\chi_{c1} \to n\bar{n})$	$\cdots$	5.20 ± 0.55	5.75 ± 1.90	5.13 ± 0.69	5.79 ± 1.81
$Br(\chi_{c1} \to \Lambda^{(\prime)}\bar{\Lambda}^{(\prime)})$	12.7 ± 0.8	13.24 ± 0.26	12.99 ± 1.31	13.19 ± 0.31	12.99 ± 1.31
$Br(\chi_{c1} \to \Sigma^{(\prime)0}\bar{\Sigma}^{(\prime)0})$	4.2 ± 0.6	$2.53 \pm 0.28^{\sharp}$	4.20 ± 1.20	$2.57 \pm 0.35^{\sharp}$	4.18 ± 1.18
$Br(\chi_{c1} \to \Sigma^{+}\bar{\Sigma}^{-})$	3.6 ± 0.7	3.12 ± 0.22	3.71 ± 1.29	3.20 ± 0.30	3.71 ± 1.29
$Br(\chi_{c1} \to \Sigma^{-}\bar{\Sigma}^{+})$	5.7 ± 1.5	$2.13 \pm 0.45^{\sharp}$	4.33 ± 1.63	$2.32 \pm 0.64^{\sharp}$	4.33 ± 1.63
$Br(\chi_{c1} \to \Xi^{0}\bar{\Xi}^{0})$	< 6	3.20 ± 0.28	3.77 ± 1.21	3.27 ± 0.36	3.76 ± 1.12
$Br(\chi_{c1} \to \Xi^{-}\bar{\Xi}^{+})$	8.0 ± 2.1	6.18 ± 0.28	5.19 ± 1.39	6.18 ± 0.28	5.23 ± 1.43
$Br(\chi_{c1} \to \Lambda^{(\prime)}\bar{\Sigma}^{(\prime)0} + c.c.)$	$\cdots$	0.11 ± 0.02	0.06 ± 0.06	0.16 ± 0.03	0.18 ± 0.18
A_1 (10^{-3} GeV)	$\cdots$	1.41 ± 0.08	1.17 ± 0.28	1.40 ± 0.09	1.17 ± 0.28
D_1 (10^{-3} GeV)	$\cdots$	0.00 ± 0.29	0.00 ± 0.28	0.00 ± 0.29	0.00 ± 0.28
F_1 (10^{-3} GeV)	$\cdots$	0.00 ± 0.29	0.00 ± 0.28	0.00 ± 0.29	0.00 ± 0.28
D_1' (10^{-3} GeV)	$\cdots$	-1.96 ± 0.08	-2.06 ± 0.25	-1.97 ± 0.09	-2.07 ± 0.25
F_1' (10^{-3} GeV)	$\cdots$	0.16 ± 0.12	0.06 ± 0.22	0.13 ± 0.15	0.06 ± 0.22
φ_1 (radian)	$\cdots$	$0 \pm \pi$	$0 \pm \pi$	$0 \pm \pi$	$0 \pm \pi$
α (radian)	$\cdots$	0	0	0.015 ± 0.001	0.015 ± 0.002

	Exp. data 2022	case I (1σ)	case I (2σ)	case II (1σ)	case II (2σ)
$Br(\chi_{c2} \to p\bar{p})$	7.33 ± 0.33	7.33 ± 0.33	7.33 ± 0.66	7.33 ± 0.33	7.33 ± 0.66
$Br(\chi_{c2} \to n\bar{n})$	$\cdots$	8.52 ± 3.75	8.26 ± 4.04	8.57 ± 3.78	8.23 ± 3.79
$Br(\chi_{c2} \to \Lambda^{(\prime)}\bar{\Lambda}^{(\prime)})$	18.3 ± 1.6	18.30 ± 1.60	18.32 ± 3.18	18.30 ± 1.60	18.35 ± 3.15
$Br(\chi_{c2} \to \Sigma^{(\prime)0}\bar{\Sigma}^{(\prime)0})$	3.7 ± 0.6	3.70 ± 0.60	3.70 ± 1.20	3.70 ± 0.60	3.69 ± 1.19
$Br(\chi_{c2} \to \Sigma^{+}\bar{\Sigma}^{-})$	3.4 ± 0.7	3.40 ± 0.70	3.51 ± 1.29	3.40 ± 0.70	3.51 ± 1.29
$Br(\chi_{c2} \to \Sigma^{-}\bar{\Sigma}^{+})$	4.4 ± 1.8	3.87 ± 1.27	3.38 ± 1.66	3.86 ± 1.26	3.36 ± 1.67
$Br(\chi_{c2} \to \Xi^{0}\bar{\Xi}^{0})$	< 10	7.43 ± 2.51	7.03 ± 2.97	7.31 ± 2.39	7.11 ± 2.89
$Br(\chi_{c2} \to \Xi^{-}\bar{\Xi}^{+})$	14.2 ± 3.2	$6.84 \pm 3.48^{\sharp}$	9.93 ± 2.13	$6.78 \pm 3.53^{\sharp}$	9.93 ± 2.13
$Br(\chi_{c2} \to \Lambda^{(\prime)}\bar{\Sigma}^{(\prime)0} + c.c)$	$\cdots$	0.10 ± 0.10	0.12 ± 0.12	0.11 ± 0.11	0.27 ± 0.27
A_2 (10^{-3} GeV)	$\cdots$	2.46 ± 0.42	2.69 ± 0.64	2.46 ± 0.41	2.69 ± 0.63
D_2 (10^{-3} GeV)	$\cdots$	0.00 ± 0.56	0.00 ± 0.61	0.00 ± 0.56	0.00 ± 0.61
F_2 (10^{-3} GeV)	$\cdots$	0.00 ± 0.55	0.01 ± 0.65	0.00 ± 0.55	0.01 ± 0.64
D_2' (10^{-3} GeV)	$\cdots$	-3.23 ± 0.34	-3.37 ± 0.41	-3.23 ± 0.34	-3.39 ± 0.40
F_2' (10^{-3} GeV)	$\cdots$	-0.13 ± 0.43	-0.21 ± 0.44	-0.13 ± 0.43	-0.21 ± 0.43
φ_2 (radian)	$\cdots$	$0 \pm \pi$	$0 \pm \pi$	$0 \pm \pi$	$0 \pm \pi$
α (radian)	$\cdots$	0	0	0.015 ± 0.001	0.015 ± 0.002

Back up

$$\chi_{cJ} \to B_{10}\overline{B}_{10}$$

	Exp. data 2022	Predictions (1σ)	Predictions (2σ)
$B(\chi_{c1} \to \Delta^{++}\bar{\Delta}^{--})$	$\cdots$	< 20.46	< 20.76
$B(\chi_{c1} \to \Delta^{+}\bar{\Delta}^{-})$	$\cdots$	< 12.85	< 13.04
$B(\chi_{c1} \to \Delta^{0}\bar{\Delta}^{0})$	$\cdots$	< 9.85	< 10.15
$B(\chi_{c1} \to \Delta^{-}\bar{\Delta}^{+})$	$\cdots$	< 7.74	< 8.32
$B(\chi_{c1} \to \Sigma^{*+}\bar{\Sigma}^{*-})$	< 9.00	< 9.00	< 9.00
$B(\chi_{c1} \to \Sigma^{*0}\bar{\Sigma}^{*0})$	$\cdots$	< 6.79	< 6.74
$B(\chi_{c1} \to \Sigma^{*-}\bar{\Sigma}^{*+})$	< 5.00	< 5.00	< 5.00
$B(\chi_{c1} \to \Xi^{*0}\bar{\Xi}^{*0})$	$\cdots$	< 4.68	< 4.92
$B(\chi_{c1} \to \Xi^{*-}\bar{\Xi}^{*+})$	$\cdots$	< 3.34	< 3.53
$B(\chi_{c1} \to \Omega^{-}\bar{\Omega}^{+})$	1.49 ± 0.25	1.49 ± 0.25	1.49 ± 0.50
$\hat{A}_1$ (10^{-3} GeV)	$\cdots$	2.55 ± 1.57	2.51 ± 1.65
$\hat{D}_1$ (10^{-3} GeV)	$\cdots$	0.00 ± 1.95	0.02 ± 1.93
$\hat{D}'_1$(10^{-3} GeV)	$\cdots$	0.21 ± 0.82	0.15 ± 0.94

	Exp. data 2022	Predictions (1σ)	Predictions (2σ)
$B(\chi_{c0} \to \Delta^{++}\bar{\Delta}^{--})$	$\cdots$	3.43 ± 2.34	4.34 ± 4.16
$B(\chi_{c0} \to \Delta^{+}\bar{\Delta}^{-})$	$\cdots$	2.87 ± 1.45	3.57 ± 3.20
$B(\chi_{c0} \to \Delta^{0}\bar{\Delta}^{0})$	$\cdots$	3.11 ± 1.59	3.63 ± 3.10
$B(\chi_{c0} \to \Delta^{-}\bar{\Delta}^{+})$	$\cdots$	3.92 ± 1.75	4.59 ± 3.75
$B(\chi_{c0} \to \Sigma^{*+}\bar{\Sigma}^{*-})$	1.60 ± 0.60	1.60 ± 0.60	1.60 ± 1.20
$B(\chi_{c0} \to \Sigma^{*0}\bar{\Sigma}^{*0})$	$\cdots$	1.80 ± 0.78	1.87 ± 1.35
$B(\chi_{c0} \to \Sigma^{*-}\bar{\Sigma}^{*+})$	2.30 ± 0.70	2.30 ± 0.70	2.30 ± 1.40
$B(\chi_{c0} \to \Xi^{*0}\bar{\Xi}^{*0})$	$\cdots$	0.87 ± 0.38	0.92 ± 0.59
$B(\chi_{c0} \to \Xi^{*-}\bar{\Xi}^{*+})$	$\cdots$	1.19 ± 0.30	1.19 ± 0.61
$B(\chi_{c0} \to \Omega^{-}\bar{\Omega}^{+})$	0.351 ± 0.061	0.351 ± 0.061	0.351 ± 0.122
$\hat{A}_0$ (10^{-2} GeV)	$\cdots$	2.35 ± 0.58	2.29 ± 1.03
$\hat{D}_0$ (10^{-2} GeV)	$\cdots$	0.00 ± 1.37	0.02 ± 1.57
$\hat{D}'_0$ (10^{-2} GeV)	$\cdots$	-0.53 ± 0.37	-0.66 ± 0.86

	Exp. data 2022	Predictions (1σ)	Predictions (2σ)
$B(\chi_{c2} \to \Delta^{++}\bar{\Delta}^{--})$	$\cdots$	< 32.01	< 29.79
$B(\chi_{c2} \to \Delta^{+}\bar{\Delta}^{-})$	$\cdots$	< 18.40	< 18.07
$B(\chi_{c2} \to \Delta^{0}\bar{\Delta}^{0})$	$\cdots$	< 13.44	< 13.07
$B(\chi_{c2} \to \Delta^{-}\bar{\Delta}^{+})$	$\cdots$	< 9.32	< 9.58
$B(\chi_{c2} \to \Sigma^{*+}\bar{\Sigma}^{*-})$	< 16.00	< 16.00	< 16.00
$B(\chi_{c2} \to \Sigma^{*0}\bar{\Sigma}^{*0})$	$\cdots$	< 11.59	< 11.23
$B(\chi_{c2} \to \Sigma^{*-}\bar{\Sigma}^{*+})$	< 8.00	< 8.00	< 8.00
$B(\chi_{c2} \to \Xi^{*0}\bar{\Xi}^{*0})$	$\cdots$	< 9.79	< 9.95
$B(\chi_{c2} \to \Xi^{*-}\bar{\Xi}^{*+})$	$\cdots$	< 6.78	< 7.01
$B(\chi_{c2} \to \Omega^{-}\bar{\Omega}^{+})$	4.52 ± 0.30	4.52 ± 0.30	4.52 ± 0.60
A_2 (10^{-3} GeV)	$\cdots$	5.32 ± 2.83	5.35 ± 2.78
D_2 (10^{-3} GeV)	$\cdots$	0.08 ± 3.87	0.03 ± 3.72
D'_2 (10^{-3} GeV)	$\cdots$	1.22 ± 1.26	1.18 ± 1.32

Back up

$$\chi_{cJ} \to PP$$

	Exp. data [29]	Symmetry	Including breaking
$\mathcal{B}(\chi_{c0} \to \pi^+\pi^-)$	$\cdots$	6.33 ± 0.33	5.67 ± 0.27
$\mathcal{B}(\chi_{c0} \to \pi^0\pi^0)$	$\cdots$	3.17 ± 0.16	2.83 ± 0.13
$\mathcal{B}(\chi_{c0} \to \pi\pi)$	8.50 ± 0.40	$9.50 \pm 0.49^\sharp$	8.50 ± 0.40
$\mathcal{B}(\chi_{c0} \to K^+K^-)$	6.07 ± 0.33	6.08 ± 0.32	6.07 ± 0.33
$\mathcal{B}(\chi_{c0} \to K^0\bar{K}^0)$	$\cdots$	6.08 ± 0.32	6.07 ± 0.33
$\mathcal{B}(\chi_{c0} \to K\bar{K})$	$\cdots$	12.16 ± 0.63	12.14 ± 0.66
$\mathcal{B}(\chi_{c0} \to \eta\eta)$	3.01 ± 0.25	2.91 ± 0.15	2.92 ± 0.16
$\mathcal{B}(\chi_{c0} \to \eta\eta')$	0.09 ± 0.01	0.09 ± 0.01	0.09 ± 0.01
$\mathcal{B}(\chi_{c0} \to \eta'\eta')$	2.17 ± 0.12	2.17 ± 0.12	2.17 ± 0.12
$a_{10}^P\ (10^{-2})$	$\cdots$	5.41 ± 0.28	5.30 ± 0.27
$a_{20}^P\ (10^{-2})$	$\cdots$	1.48 ± 0.56	1.12 ± 1.04
$b_0^P\ (10^{-2})$	$\cdots$	$\cdots$	0.40 ± 0.35
$\lvert\alpha_{0P}\rvert$	$\cdots$	$(119.75 \pm 9.74)^\circ$	$(142.09 \pm 37.82)^\circ$
$\lvert\beta_{0P}\rvert$	$\cdots$	$\cdots$	$(136.36 \pm 43.54)^\circ$

	Exp. data [29]	Symmetry	Including breaking
$\mathcal{B}(\chi_{c2} \to \pi^+\pi^-)$	$\cdots$	11.67 ± 0.47	15.13 ± 0.67
$\mathcal{B}(\chi_{c2} \to \pi^0\pi^0)$	$\cdots$	5.83 ± 0.24	7.57 ± 0.33
$\mathcal{B}(\chi_{c2} \to \pi\pi)$	22.70 ± 1.00	$17.50 \pm 0.71^\sharp$	22.70 ± 1.00
$\mathcal{B}(\chi_{c2} \to K^+K^-)$	10.20 ± 1.50	11.24 ± 0.46	10.20 ± 1.50
$\mathcal{B}(\chi_{c2} \to K^0\bar{K}^0)$	$\cdots$	11.24 ± 0.46	10.20 ± 1.50
$\mathcal{B}(\chi_{c2} \to K\bar{K})$	$\cdots$	22.48 ± 0.91	20.39 ± 3.00
$\mathcal{B}(\chi_{c2} \to \eta\eta)$	5.50 ± 0.50	5.22 ± 0.22	5.42 ± 0.42
$\mathcal{B}(\chi_{c2} \to \eta\eta')$	0.22 ± 0.05	0.22 ± 0.05	0.22 ± 0.05
$\mathcal{B}(\chi_{c2} \to \eta'\eta')$	0.46 ± 0.06	0.46 ± 0.06	0.46 ± 0.06
$a_{12}^P\ (10^{-2})$	$\cdots$	1.02 ± 0.04	1.03 ± 0.08
$a_{22}^P\ (10^{-2})$	$\cdots$	0.35 ± 0.11	0.32 ± 0.16
$b_2^P\ (10^{-2})$	$\cdots$	$\cdots$	0.14 ± 0.08
$\lvert\alpha_{2P}\rvert$	$\cdots$	$(170.17 \pm 9.74)^\circ$	$(165.58 \pm 14.32)^\circ$
$\lvert\beta_{2P}\rvert$	$\cdots$	$\cdots$	$(31.51 \pm 30.94)^\circ$

Back up

$$\chi_{cJ} \to VV$$

	Exp. data [29]	Symmetry	Including breaking
$\mathcal{B}(\chi_{c1} \to \rho^+\rho^-)$	$\cdots$	12.35 ± 1.92	15.18 ± 4.96
$\mathcal{B}(\chi_{c1} \to \rho^0\rho^0)$	$\cdots$	6.17 ± 0.96	7.59 ± 2.48
$\mathcal{B}(\chi_{c1} \to \rho\rho)$	$\cdots$	18.52 ± 2.87	22.77 ± 7.43
$\mathcal{B}(\chi_{c1} \to K^{*+}K^{*-})$	$\cdots$	11.85 ± 1.84	13.22 ± 3.21
$\mathcal{B}(\chi_{c1} \to K^{*0}\bar{K}^{*0})$	14.00 ± 4.00	11.84 ± 1.84	13.20 ± 3.20
$\mathcal{B}(\chi_{c1} \to K^*\bar{K}^*)$	$\cdots$	23.69 ± 3.68	26.42 ± 6.41
$\mathcal{B}(\chi_{c1} \to \phi\phi)$	4.26 ± 0.21	$5.24 \pm 0.73^\sharp$	4.26 ± 0.21
$\mathcal{B}(\chi_{c1} \to \phi\omega)$	0.27 ± 0.04	0.27 ± 0.04	0.27 ± 0.04
$\mathcal{B}(\chi_{c1} \to \omega\omega)$	5.70 ± 0.70	5.70 ± 0.70	5.70 ± 0.70
a_{11}^V (10^{-3})	$\cdots$	6.96 ± 0.54	7.59 ± 1.17
a_{21}^V (10^{-3})	$\cdots$	1.06 ± 0.10	1.05 ± 0.19
b_1^V (10^{-3})	$\cdots$	$\cdots$	1.17 ± 1.14
$\lvert\alpha_{1V}\rvert$	$\cdots$	$(105.42 \pm 28.65)^\circ$	$(134.65 \pm 45.26)^\circ$
$\lvert\beta_{1V}\rvert$	$\cdots$	$\cdots$	$(47.56 \pm 47.56)^\circ$

	Exp. data [29]	Symmetry	Including breaking
$\mathcal{B}(\chi_{c0} \to \rho^+\rho^-)$	$\cdots$	2.04 ± 0.37	2.28 ± 1.56
$\mathcal{B}(\chi_{c0} \to \rho^0\rho^0)$	$\cdots$	1.02 ± 0.18	1.14 ± 0.78
$\mathcal{B}(\chi_{c0} \to \rho\rho)$	$\cdots$	3.06 ± 0.55	3.42 ± 2.33
$\mathcal{B}(\chi_{c0} \to K^{*+}K^{*-})$	$\cdots$	1.95 ± 0.35	1.70 ± 0.60
$\mathcal{B}(\chi_{c0} \to K^{*0}\bar{K}^{*0})$	1.70 ± 0.60	1.95 ± 0.35	1.70 ± 0.60
$\mathcal{B}(\chi_{c0} \to K^*\bar{K}^*)$	$\cdots$	3.90 ± 0.70	3.40 ± 1.20
$\mathcal{B}(\chi_{c0} \to \phi\phi)$	0.848 ± 0.031	0.85 ± 0.03	0.85 ± 0.03
$\mathcal{B}(\chi_{c0} \to \phi\omega)$	0.142 ± 0.013	0.142 ± 0.013	0.142 ± 0.013
$\mathcal{B}(\chi_{c0} \to \omega\omega)$	0.97 ± 0.11	0.97 ± 0.11	0.97 ± 0.11
a_{10}^V (10^{-2})	$\cdots$	3.24 ± 0.37	2.98 ± 0.85
a_{20}^V (10^{-2})	$\cdots$	0.86 ± 0.06	0.85 ± 0.14
b_0^V (10^{-2})	$\cdots$	$\cdots$	0.78 ± 0.78
$\lvert\alpha_{0V}\rvert$	$\cdots$	$(106.57 \pm 16.04)^\circ$	$(120.89 \pm 59.01)^\circ$
$\lvert\beta_{0V}\rvert$	$\cdots$	$\cdots$	$(90.00 \pm 90.00)^\circ$

	Exp. data [29]	Symmetry	Including breaking
$\mathcal{B}(\chi_{c2} \to \rho^+\rho^-)$	$\cdots$	27.08 ± 5.20	18.38 ± 8.04
$\mathcal{B}(\chi_{c2} \to \rho^0\rho^0)$	$\cdots$	13.54 ± 2.60	9.19 ± 4.02
$\mathcal{B}(\chi_{c2} \to \rho\rho)$	$\cdots$	40.62 ± 7.80	27.57 ± 12.06
$\mathcal{B}(\chi_{c2} \to K^{*+}K^{*-})$	$\cdots$	26.04 ± 5.00	20.39 ± 7.37
$\mathcal{B}(\chi_{c2} \to K^{*0}\bar{K}^{*0})$	22.00 ± 9.00	26.00 ± 4.99	20.36 ± 7.36
$\mathcal{B}(\chi_{c2} \to K^*\bar{K}^*)$	$\cdots$	52.03 ± 9.99	40.76 ± 14.74
$\mathcal{B}(\chi_{c2} \to \phi\phi)$	12.30 ± 0.70	12.30 ± 0.70	12.30 ± 0.70
$\mathcal{B}(\chi_{c2} \to \phi\omega)$	0.097 ± 0.028	0.097 ± 0.028	0.097 ± 0.028
$\mathcal{B}(\chi_{c2} \to \omega\omega)$	8.60 ± 1.00	$13.57 \pm 2.36^\sharp$	8.60 ± 1.00
a_{12}^V (10^{-3})	$\cdots$	16.31 ± 1.87	13.73 ± 3.09
a_{22}^V (10^{-3})	$\cdots$	0.98 ± 0.17	0.98 ± 0.48
b_2^V (10^{-3})	$\cdots$	$\cdots$	2.92 ± 2.59
$\lvert\alpha_{2V}\rvert$	$\cdots$	$(90.00 \pm 90.00)^\circ$	$(90.00 \pm 90.00)^\circ$
$\lvert\beta_{2V}\rvert$	$\cdots$	$\cdots$	$(131.21 \pm 48.70)^\circ$

Back up

PV:

	Exp. data [29]	Symmetry	Including breaking
$\mathcal{B}(\chi_{c1} \to \pi^{\pm}\rho^{\mp})$	$\cdots$	1.12 ± 0.11	2.78 ± 2.65
$\mathcal{B}(\chi_{c1} \to \pi^{0}\rho^{0})$	$\cdots$	0.56 ± 0.05	1.39 ± 1.32
$\mathcal{B}(\chi_{c1} \to K^{\pm}K^{*\mp})$	1.21 ± 0.23	1.08 ± 0.10	1.08 ± 0.10
$\mathcal{B}(\chi_{c1} \to K^{0}\bar{K}^{*0} + c.c.)$	1.03 ± 0.15	1.08 ± 0.10	1.08 ± 0.10
$\mathcal{B}(\chi_{c1} \to \eta\phi)$	$\cdots$	0.21 ± 0.17	0.40 ± 0.40
$\mathcal{B}(\chi_{c1} \to \eta\omega)$	$\cdots$	0.47 ± 0.34	1.16 ± 1.15
$\mathcal{B}(\chi_{c1} \to \eta'\phi)$	$\cdots$	0.64 ± 0.64	0.87 ± 0.87
$\mathcal{B}(\chi_{c1} \to \eta'\omega)$	$\cdots$	0.77 ± 0.77	1.93 ± 1.93
a_{11}^{PV} (10^{-3})	$\cdots$	4.68 ± 0.32	5.12 ± 1.65
a_{21}^{PV} (10^{-3})	$\cdots$	1.11 ± 1.11	1.46 ± 1.46
b_{1}^{PV} (10^{-3})	$\cdots$	$\cdots$	1.85 ± 1.85
$\lvert\alpha_{1PV}\rvert$	$\cdots$	$(90.00 \pm 90.00)^{\circ}$	$(90.00 \pm 90.00)^{\circ}$
$\lvert\beta_{1PV}\rvert$	$\cdots$	$\cdots$	$(90.00 \pm 90.00)^{\circ}$

	Exp. data [29]	Symmetry	Including breaking
$\mathcal{B}(\chi_{c2} \to \pi^{\pm}\rho^{\mp})$	0.06 ± 0.04	$1.45 \pm 0.15^{\sharp}$	0.06 ± 0.04
$\mathcal{B}(\chi_{c2} \to \pi^{0}\rho^{0})$	$\cdots$	0.73 ± 0.08	0.03 ± 0.02
$\mathcal{B}(\chi_{c2} \to K^{\pm}K^{*\mp})$	1.46 ± 0.21	1.40 ± 0.15	1.40 ± 0.15
$\mathcal{B}(\chi_{c2} \to K^{0}\bar{K}^{*0} + c.c.)$	1.27 ± 0.27	1.40 ± 0.15	1.40 ± 0.15
$\mathcal{B}(\chi_{c2} \to \eta\phi)$	$\cdots$	0.28 ± 0.23	0.91 ± 0.48
$\mathcal{B}(\chi_{c2} \to \eta\omega)$	$\cdots$	0.59 ± 0.43	0.09 ± 0.09
$\mathcal{B}(\chi_{c2} \to \eta'\phi)$	$\cdots$	0.81 ± 0.81	1.55 ± 1.25
$\mathcal{B}(\chi_{c2} \to \eta'\omega)$	$\cdots$	1.01 ± 1.01	0.31 ± 0.31
a_{12}^{PV} (10^{-3})	$\cdots$	2.60 ± 0.19	1.91 ± 0.18
a_{22}^{PV} (10^{-3})	$\cdots$	0.61 ± 0.61	0.46 ± 0.46
b_{2}^{PV} (10^{-3})	$\cdots$	$\cdots$	1.38 ± 0.21
$\lvert\alpha_{2PV}\rvert$	$\cdots$	$(90.00 \pm 90.00)^{\circ}$	$(90.00 \pm 90.00)^{\circ}$
$\lvert\beta_{2PV}\rvert$	$\cdots$	$\cdots$	$(169.60 \pm 10.31)^{\circ}$

PT:

	Exp. data [29]	Symmetry	Including breaking
$\mathcal{B}(\chi_{c1} \to \pi^{\pm}a_2^{\mp})$	$\cdots$	1.42 ± 0.04	3.03 ± 2.14
$\mathcal{B}(\chi_{c1} \to \pi^{0}a_2^{0})$	$\cdots$	0.71 ± 0.02	1.51 ± 1.07
$\mathcal{B}(\chi_{c1} \to K^{\pm}K_2^{*\mp})$	1.61 ± 0.31	1.34 ± 0.04	1.34 ± 0.04
$\mathcal{B}(\chi_{c1} \to K^{0}\bar{K}_2^{*0} + c.c.)$	1.17 ± 0.20	1.33 ± 0.04	1.33 ± 0.04
$\mathcal{B}(\chi_{c1} \to \eta f_2')$	$\cdots$	0.23 ± 0.03	0.42 ± 0.31
$\mathcal{B}(\chi_{c1} \to \eta f_2)$	0.67 ± 0.11	0.61 ± 0.05	0.67 ± 0.11
$\mathcal{B}(\chi_{c1} \to \eta' f_2')$	$\cdots$	0.60 ± 0.07	0.76 ± 0.76
$\mathcal{B}(\chi_{c1} \to \eta' f_2)$	$\cdots$	0.99 ± 0.21	1.49 ± 1.49
a_{11}^{PT} (10^{-3})	$\cdots$	5.54 ± 0.20	6.09 ± 1.16
a_{21}^{PT} (10^{-3})	$\cdots$	1.18 ± 0.25	1.60 ± 1.60
b_{1}^{PT} (10^{-3})	$\cdots$	$\cdots$	1.92 ± 1.92
$\lvert\alpha_{1PT}\rvert$	$\cdots$	$(25.78 \pm 25.78)^{\circ}$	$(90.00 \pm 90.00)^{\circ}$
$\lvert\beta_{1PT}\rvert$	$\cdots$	$\cdots$	$(90.00 \pm 90.00)^{\circ}$

	Exp. data [29]	Symmetry	Including breaking
$\mathcal{B}(\chi_{c2} \to \pi^{\pm}a_2^{\mp})$	1.80 ± 0.60	1.50 ± 0.03	2.16 ± 0.24
$\mathcal{B}(\chi_{c2} \to \pi^{0}a_2^{0})$	1.31 ± 0.35	$0.75 \pm 0.02^{\sharp}$	1.08 ± 0.12
$\mathcal{B}(\chi_{c2} \to K^{\pm}K_2^{*\mp})$	1.51 ± 0.13	1.41 ± 0.03	1.41 ± 0.03
$\mathcal{B}(\chi_{c2} \to K^{0}\bar{K}_2^{*0} + c.c.)$	1.27 ± 0.17	1.41 ± 0.03	1.41 ± 0.03
$\mathcal{B}(\chi_{c2} \to \eta f_2')$	$\cdots$	0.37 ± 0.15	0.45 ± 0.36
$\mathcal{B}(\chi_{c2} \to \eta f_2)$	$\cdots$	0.41 ± 0.25	0.77 ± 0.62
$\mathcal{B}(\chi_{c2} \to \eta' f_2')$	$\cdots$	0.37 ± 0.32	0.81 ± 0.81
$\mathcal{B}(\chi_{c2} \to \eta' f_2)$	$\cdots$	0.70 ± 0.70	1.59 ± 1.59
a_{12}^{PT} (10^{-3})	$\cdots$	8.77 ± 0.30	9.10 ± 0.70
a_{22}^{PT} (10^{-3})	$\cdots$	1.13 ± 1.13	2.19 ± 2.19
b_{2}^{PT} (10^{-3})	$\cdots$	$\cdots$	2.86 ± 2.14
$\lvert\alpha_{2PT}\rvert$	$\cdots$	$(90.00 \pm 90.00)^{\circ}$	$(90.00 \pm 90.00)^{\circ}$
$\lvert\beta_{2PT}\rvert$	$\cdots$	$\cdots$	$(43.54 \pm 43.54)^{\circ}$