Study of the two-body nonleptonic $B_c(2S)$ weak decays with the QCD factorization approach

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Outline

1 1. Motivation

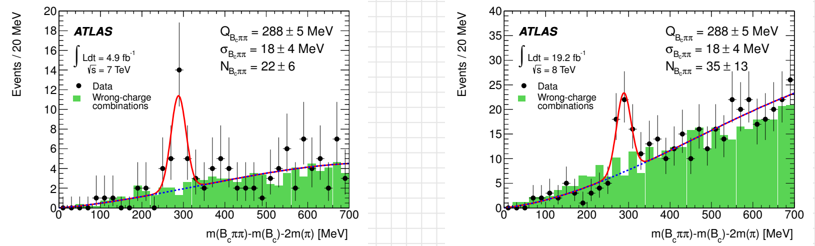
2 2.Theoretical framework

3 3.Numerical results

4 4.Summary

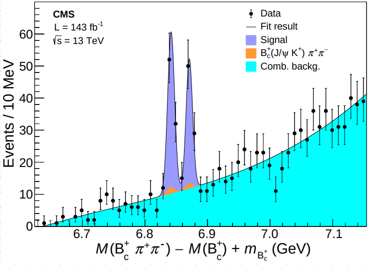
1. Motivation

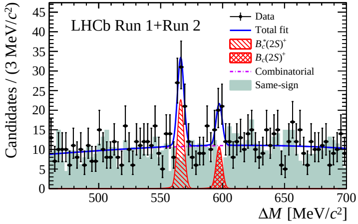
1.1 Observation of $B_c(2S)$



The mass of the observed state is $6842 \pm 4 \pm 5$ MeV. The mass and decay of this state are consistent with expectations for the second S-wave state of the $B_c^{\pm}$ meson, $B_c^{\pm}(2S)$.

Phys.Rev.Lett. 113 (2014) 21, 212004 (ATLAS)

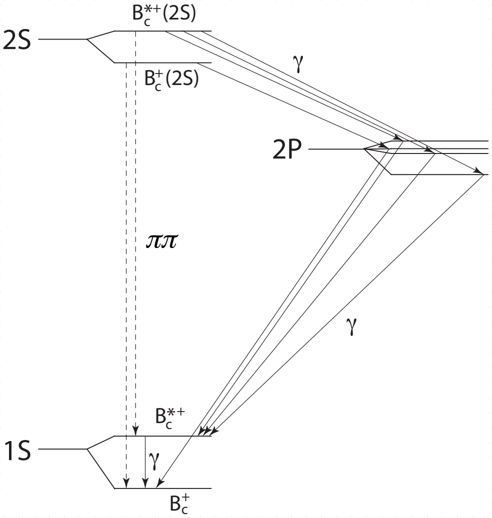




Signals consistent with the $B_c^{\pm}(2S)$ and $B_c^{*\pm}(2S)$ states have been separately observed for the first time by investigating the $B_c^{\pm}\pi^+\pi^-$ invariant mass spectrum measured by CMS. LHCb result confirmed the existence of the two states.

Phys.Rev.Lett. 122 (2019) 13, 132001(CMS)

Phys.Rev.Lett. 122 (2019) 23, 232001 (LHCb)



Transitions between the lightest B_c states, with solid and dashed lines indicating the emission of photons and pion pairs, respectively

1.2 There will be approximately $\mathcal{O}(10^{11})$ $B_c(2S)$ available at HL-LHC experiments

LHC $\sqrt{s} \approx 14$ TeV $\sigma \approx 1\,\mu\mathrm{b}$ $\Longrightarrow$ $\mathcal{O}(10^9)$ $B_c(1S)$ with $1\,\mathrm{fb}^{-1}$

HL-LHC $4000\,\mathrm{fb}^{-1}$ $\Longrightarrow$ $\mathcal{O}(10^{12})$ $B_c(1S)$

CMS group, $R = (3.47{\pm}0.63{\pm}0.33)\%$ $\Longrightarrow$ $\mathcal{O}(10^{11})$ $B_c(2S)$

1.3The $B_c(2S)$ meson is still an immature and up-and-coming particle

- The mass of the $B_c(2S)$ meson is $m = 6871.2 \pm 1.0$ MeV and lies below the open BD threshold.
- The favourite decays, predominantly by hadronic cascades and the electric dipole (E1) or magnetic dipole (M1) transitions, lead to the total width of the $B_c(2S)$ meson are generally less than one hundred keV.
- The weak decays of the $B_c(2S)$ meson are similar to those of the lowest $B_c(1S)$ meson.

2. Theoretical framework

2.1 The effective Hamiltonian

For the $B_c(2S) \to B\,P$, $B\,V$ decays induced by the c quark decays, the effective weak interaction Hamiltonian can be written as

$$\mathcal{H}_{\rm eff}^{c} = \frac{G_F}{\sqrt{2}} \sum_{q_1,q_2} V_{cq_1}^{*} V_{uq_2} \Big[C_1(\mu)\, O_1 + C_2(\mu)\, O_2 \Big] + \text{h.c.}, \tag{1}$$

$$O_1 = [\bar{q}_{1,\alpha}\, \gamma_\mu\, (1-\gamma_5)\, c_\alpha]\, [\bar{u}_\beta\, \gamma^\mu\, (1-\gamma_5)\, q_{2,\beta}], \tag{2}$$

$$O_2 = [\bar{q}_{1,\alpha}\, \gamma_\mu\, (1-\gamma_5)\, c_\beta]\, [\bar{u}_\beta\, \gamma^\mu\, (1-\gamma_5)\, q_{2,\alpha}], \tag{3}$$

both the penguin and annihilation contributions being proportional to the CKM factor $V_{ub}\, V_{cb}^{*} \sim \mathcal{O}(\lambda^5)$ are highly suppressed relative to those from the operator $O_{1,2}$ being proportional to $V_{ud}\, V_{cs}^{*} \sim \mathcal{O}(1)$ or $V_{ud}\, V_{cd}^{*} \sim \mathcal{O}(\lambda)$ or $V_{us}\, V_{cs}^{*} \sim \mathcal{O}(\lambda)$ or $V_{us}\, V_{cd}^{*} \sim \mathcal{O}(\lambda^2)$.

For the $B_c(2S) \to \psi(1S, 2S)\, P$, $\eta_c(1S, 2S)\, P$ decays induced by the $b \to c$ quark transition, the expression of the low-energy effective Hamiltonian is written as

$$\mathcal{H}_{\rm eff}^{b} = \frac{G_F}{\sqrt{2}} \sum_{q=d,s} V_{cb} V_{uq}^{*} \Big[C_1(\mu)\, O_1' + C_2(\mu)\, O_2' \Big] + \text{h.c.}, \tag{4}$$

$$O_1' = [\bar{c}_\alpha\, \gamma_\mu\, (1-\gamma_5)\, b_\alpha]\, [\bar{q}_\beta\, \gamma^\mu\, (1-\gamma_5)\, u_\beta], \tag{5}$$

$$O_2' = [\bar{c}_\alpha\, \gamma_\mu\, (1-\gamma_5)\, b_\beta]\, [\bar{q}_\beta\, \gamma^\mu\, (1-\gamma_5)\, u_\alpha]. \tag{6}$$

Note that because the valence quarks in both final states differ from each other, there is no contributions from penguin and annihilation in Eq.(4).

The decay amplitude can be written as,

$$\mathcal{A} = \langle\, \psi\, \pi\, |\, \mathcal{H}_{\rm eff}\, |\, B_c(2S)\, \rangle = \frac{G_F}{\sqrt{2}}\, V_{cb}\, V_{ud}^{*} \sum_{i=1}^{2} C_i\, \langle\, \psi\, \pi\, |\, O_i'\, |\, B_c(2S)\, \rangle, \tag{7}$$

2.2 Hadronic matrix elements

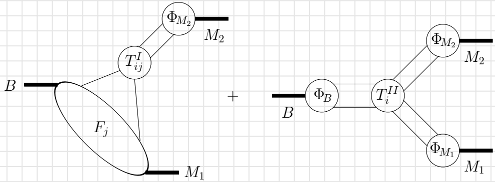


Figure: Graphical representation of the factorization formula. Only one of the two form-factor terms in (8) is shown for simplicity.

$$\begin{aligned}
\langle M_1 M_2|\mathcal{O}_i|\bar{B}\rangle &= \sum_j F_j^{B\to M_1}(m_2^2)\int_0^1 du\, T^I_{ij}(u)\,\Phi_{M_2}(u) \;+\; (M_1\leftrightarrow M_2)\\
&\quad + \int_0^1 d\xi du dv\, T_i^{II}(\xi,u,v)\,\Phi_B(\xi)\,\Phi_{M_1}(v)\,\Phi_{M_2}(u)\\
&\qquad \text{if } M_1 \text{ and } M_2 \text{ are both light,} \qquad (8)\\
\langle M_1 M_2|\mathcal{O}_i|\bar{B}\rangle &= \sum_j F_j^{B\to M_1}(m_2^2)\int_0^1 du\, T^I_{ij}(u)\,\Phi_{M_2}(u)\\
&\qquad \text{if } M_1 \text{ is heavy and } M_2 \text{ is light.}
\end{aligned}$$

Combined the nonfactorizable contributions with the Wilson coefficients, the scale independent effective coefficients at the order α_s can be obtained as follows:

$$a_1 = C_1^{\rm NLO} + \frac{1}{N_c}\, C_2^{\rm NLO} + \frac{\alpha_s}{4\pi}\, \frac{C_F}{N_c}\, C_2^{\rm LO}\, V, \tag{9}$$

$$a_2 = C_2^{\rm NLO} + \frac{1}{N_c}\, C_1^{\rm NLO} + \frac{\alpha_s}{4\pi}\, \frac{C_F}{N_c}\, C_1^{\rm LO}\, V, \tag{10}$$

c quark decay:

$$V = 6\log\left(\frac{m_c^2}{\mu^2}\right) - 18 - \left(\frac{1}{2} + i3\pi\right) a_0^M + \left(\frac{11}{2} - i3\pi\right) a_1^M - \frac{21}{20}\, a_2^M + \cdots, \tag{11}$$

b $\to$ c decay:

$$V' = 3\ln\left(\frac{m_b^2}{\mu^2}\right) + 3\ln\left(\frac{m_c^2}{\mu^2}\right) - 18 - \int \mathbf{d}z\, \phi(z) H_1(z) \tag{12}$$

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Adv.High Energy Phys. 2015 (2015) 104378

2.3 Form factors

$$
\begin{aligned}
&\langle\, X(p_2)\,|\,\bar q_\alpha\,\gamma^\mu\,(1-\gamma_5)\,b_\alpha\,|\,B_c(2S)(p_1)\,\rangle \\
=\quad & \frac{m^2_{B_c(2S)}-m^2_X}{q^2}\,q^\mu\,F_0^{B_c(2S)\to X}(q^2) \\
+\quad & \left(p_1^\mu+p_2^\mu-\frac{m^2_{B_c(2S)}-m^2_X}{q^2}\,q^\mu\right)F_1^{B_c(2S)\to X}(q^2),
\end{aligned}
\tag{13}
$$

$$
\begin{aligned}
&\langle\, \psi(p_2,\epsilon)\,|\,\bar q_\alpha\,\gamma^\mu\,(1-\gamma_5)\,b_\alpha\,|\,B_c(2S)(p_1)\,\rangle \\
=\quad & \epsilon_{\mu\nu\alpha\beta}\,\epsilon_\psi^{*\nu}\,p_1^\alpha\,p_2^\beta\,\frac{2\,V^{B_c(2S)\to\psi}(q^2)}{m_{B_c(2S)}+m_\psi}+i\,\frac{2\,m_\psi\,(\epsilon_\psi^*\cdot q)}{q^2}\,q_\mu\,A_0^{B_c(2S)\to\psi}(q^2) \\
+\quad & i\,\epsilon^*_{\psi,\mu}\,(m_{B_c(2S)}+m_\psi)\,A_1^{B_c(2S)\to\psi}(q^2) \\
-\quad & i\,\frac{(\epsilon_\psi^*\cdot q)}{m_{B_c(2S)}+m_\psi}\,(p_1+p_2)_\mu\,A_2^{B_c(2S)\to\psi}(q^2) \\
-\quad & i\,\frac{2\,m_\psi\,(\epsilon^*_{\psi,\mu}\cdot q)}{q^2}\,q_\mu\,A_3^{B_c(2S)\to\psi}(q^2),
\end{aligned}
\tag{14}
$$

At the large recoil limit, $q^2 = 0$, there is

$$F_0^{B_c\to X}(0) \;=\; F_1^{B_c\to X}(0), \tag{15}$$

$$A_0^{B_c\to \psi}(0) \;=\; A_3^{B_c\to \psi}(0). \tag{16}$$

Currently, there is no theoretical calculation for these form factors. We will use the Wirbel-Stech-Bauer approach to give a rough estimation of the form factor.

$$F_0^{B_c(2S)\to X}(0) \;=\; \int d\vec{k}_\perp \int_0^1 dx\, \Phi_X(\vec{k}_\perp, x, 0, 0)\, \Phi_{B_c(2S)}(\vec{k}_\perp, x, 0, 0), \tag{17}$$

$$A_0^{B_c(2S)\to \psi}(0) \;=\; \int d\vec{k}_\perp \int_0^1 dx\, \Phi_\psi(\vec{k}_\perp, x, 1, 0)\, \sigma_z\, \Phi_{B_c(2S)}(\vec{k}_\perp, x, 0, 0), \tag{18}$$

where σ_z is the Pauli matrix acting on the spin indices of the decaying quark; the variables $\vec{k}_\perp$, x, J, J_z of the wave function $\Phi(\vec{k}_\perp, x, J, J_z)$ denote respectively the transverse momentum and the fraction of the longitudinal momentum carried by the nonspectator quark, the total spin and its z-component.

$$\Phi_{B_c(2S)}(\vec{k}_\perp, x) \;=\; A\left\{\frac{\vec{k}_\perp^2 + \bar{x}\, m_c^2 + x\, m_b^2}{6\,\alpha_1^2\, x\, \bar{x}} - 1\right\} \exp\left\{-\frac{\vec{k}_\perp^2 + \bar{x}\, m_c^2 + x\, m_b^2}{8\,\alpha_1^2\, x\, \bar{x}}\right\}, \tag{19}$$

$$\Phi_{\eta_c(1S)}(\vec{k}_\perp, x) \;=\; \Phi_{\psi(1S)}(\vec{k}_\perp, x) \;=\; B \exp\left\{ - \frac{\vec{k}_\perp^2 + m_c^2}{8\,\alpha_2^2\, x\, \bar{x}}\right\}, \tag{20}$$

$$\Phi_{\eta_c(2S)}(\vec{k}_\perp, x) \;=\; \Phi_{\psi(2S)}(\vec{k}_\perp, x) \;=\; C\left\{\frac{\vec{k}_\perp^2 + m_c^2}{6\,\alpha_2^2\, x\, \bar{x}} - 1\right\} \exp\left\{-\frac{\vec{k}_\perp^2 + m_c^2}{8\,\alpha_2^2\, x\, \bar{x}}\right\}, \tag{21}$$

the mean value of square of transverse momentum is $\alpha_1^2 = \mu\,\omega$ with the reduced mass $\mu = \frac{m_b\, m_c}{m_b + m_c}$ and the characteristic frequency $\omega \approx 0.50 \pm 0.05$ GeV; and $\alpha_2 = m_c\, \alpha_s$ for the charmonium.

$$\Phi_{B_{u,d,s}}(\vec{k}_\perp, x) \;=\; D\, x^2\, \bar{x}^2 \exp\left\{ - \frac{\vec{k}_\perp^2 + x^2\, m_B^2}{2\,\alpha_3^2}\right\},, \tag{22}$$

$\alpha_3 = 0.45 \pm 0.05$ GeV for $B_{u,d}$ meson and 0.55 ± 0.05 GeV for B_s meson

$$F_0^{B_c(2S)\to B_{u,d}}(0) = 0.397, \tag{23}$$

$$F_0^{B_c(2S)\to B_s}(0) = 0.426, \tag{24}$$

$$F_0^{B_c(2S)\to \eta_c(1S)}(0) = A_0^{B_c(2S)\to \psi(1S)}(0) = 0.270, \tag{25}$$

$$F_0^{B_c(2S)\to \eta_c(2S)}(0) = A_0^{B_c(2S)\to \psi(2S)}(0) = 0.260, \tag{26}$$

3.Numerical results

$$\mathcal{B}r \, = \, \frac{p_{\rm cm}}{8\,\pi\, m^2_{B_c(2S)}\,\Gamma_{B_c(2S)}}\,|\mathcal{A}|^2, \tag{27}$$

Table: The partial widths (Γ_i in unit of keV) and branching ratios ($\mathcal{B}r_i$ in unit of %) of the $B_c(2S)$ meson decay.

decay mode	final states	E. Eichten[1]		L. Fulcher[2]		S. Godfrey[3]		I. Asghar[4]		B. Martín-González [5]		X. Li[6]	
		Γ_i	$\mathcal{B}r_i$	Γ_i	$\mathcal{B}r_i$	Γ_i	$\mathcal{B}r_i$	Γ_i	$\mathcal{B}r_i$	Γ_i	$\mathcal{B}r_i$	Γ_i	$\mathcal{B}r_i$
hadronic	$B_c(1S)\,\pi\,\pi$	50	90.4	50	71.9	57	88.1	10.68	51.42	42	54.4	25	76.9
E1	$B_c(1P)\,\gamma$	0.0	0.0	6.4	9.2	1.3	2.0	7.11	34.23	0.0	0.0	2.8	8.6
E1	$B_c(1P')\,\gamma$	5.2	9.4	13.1	18.8	6.1	9.4	2.91	14.01	35	45.3	4.4	13.5
M1	$B_c^*(1S)\,\gamma$	0.1	0.2	0.06	0.1	0.3	0.5	0.07	0.34	0.25	0.3	0.3	1.0
total		55.3	100	69.6	100	64.7	100	20.77	100	77.3	100	33	100

[1] nonrelativistic quarkonium quantum mechanics. PhysRevD.49.5845
[2] potential models. PhysRevD.60.074006
[3] the relativized quark model. PhysRevD.70.054017
[4] non-relativistic quark potential model. PhysRevD.100.096002
[5] non-relativistic constituent quark model. PhysRevD.106.054009
[6] modified Godfrey-Isgur model. EPJC.83.1080

Table: The numerical results on CP-averaged branching ratios for the $B_c(2S) \to BP$, BV decays, where the theoretical uncertainties come from the CKM parameters, the renormalization scale $\mu = (1\pm0.2)\,m_c$, decay constants, and Gegenbauer moments, respectively.

final states	decay amplitude ($\mathcal{A}$)		case	branching ratio ($\mathcal{B}r$)
	coefficient	CKM factor		
$B_s^0\,\pi^+$	a_1	$V_{cs}^*\,V_{ud} \sim \mathcal{O}(1)$	I-a	$(1.93^{+0.00+0.18+0.01}_{-0.00-0.10-0.01}) \times 10^{-9}$
$B_s^0\,\rho^+$	a_1	$V_{cs}^*\,V_{ud} \sim \mathcal{O}(1)$	I-a	$(3.34^{+0.00+0.31+0.10}_{-0.00-0.17-0.10}) \times 10^{-9}$
$B_s^0\,K^+$	a_1	$V_{cs}^*\,V_{us} \sim \mathcal{O}(\lambda)$	I-b	$(1.40^{+0.01+0.13+0.02}_{-0.01-0.07-0.02}) \times 10^{-10}$
$B_s^0\,K^{*+}$	a_1	$V_{cs}^*\,V_{us} \sim \mathcal{O}(\lambda)$	I-b	$(1.53^{+0.01+0.14+0.07}_{-0.01-0.08-0.07}) \times 10^{-10}$
$B_d^0\,\pi^+$	a_1	$V_{cd}^*\,V_{ud} \sim \mathcal{O}(\lambda)$	I-b	$(1.04^{+0.01+0.10+0.01}_{-0.01-0.05-0.01}) \times 10^{-10}$
$B_d^0\,\rho^+$	a_1	$V_{cd}^*\,V_{ud} \sim \mathcal{O}(\lambda)$	I-b	$(1.90^{+0.01+0.18+0.06}_{-0.01-0.10-0.05}) \times 10^{-10}$
$B_d^0\,K^+$	a_1	$V_{cd}^*\,V_{us} \sim \mathcal{O}(\lambda^2)$	I-c	$(7.60^{+0.09+0.72+0.10}_{-0.09-0.40-0.10}) \times 10^{-12}$
$B_d^0\,K^{*+}$	a_1	$V_{cd}^*\,V_{us} \sim \mathcal{O}(\lambda^2)$	I-c	$(8.97^{+0.11+0.84+0.43}_{-0.11-0.47-0.42}) \times 10^{-12}$
$B_u^+\,\overline{K}^0$	a_2	$V_{cs}^*\,V_{ud} \sim \mathcal{O}(1)$	II-a	$(2.85^{+0.00+1.60+0.07}_{-0.00-0.79-0.07}) \times 10^{-10}$
$B_u^+\,\overline{K}^{*0}$	a_2	$V_{cs}^*\,V_{ud} \sim \mathcal{O}(1)$	II-a	$(3.33^{+0.00+1.87+0.19}_{-0.00-0.92-0.18}) \times 10^{-10}$
$B_u^+\,\pi^0$	a_2	$V_{cd}^*\,V_{ud} \sim \mathcal{O}(\lambda)$	II-b	$(5.61^{+0.03+3.15+0.09}_{-0.03-1.55-0.09}) \times 10^{-12}$
$B_u^+\,\rho^0$	a_2	$V_{cd}^*\,V_{ud} \sim \mathcal{O}(\lambda)$	II-b	$(1.02^{+0.01+0.57+0.04}_{-0.01-0.28-0.03}) \times 10^{-11}$
$B_u^+\,\omega$	a_2	$V_{cd}^*\,V_{ud} \sim \mathcal{O}(\lambda)$	II-b	$(7.55^{+0.04+4.24+0.46}_{-0.04-2.09-0.44}) \times 10^{-12}$
$B_u^+\,\eta$	a_2	$V_{cs}^*\,V_{us}$, $V_{cd}^*\,V_{ud}$	II-b	$(2.17^{+0.01+1.22+0.19}_{-0.01-0.60-0.18}) \times 10^{-11}$
$B_u^+\,\eta'$	a_2	$V_{cs}^*\,V_{us}$, $V_{cd}^*\,V_{ud}$	II-b	$(2.79^{+0.02+1.56+0.64}_{-0.02-0.77-0.56}) \times 10^{-12}$
$B_u^+\,\phi$	a_2	$V_{cs}^*\,V_{us} \sim \mathcal{O}(\lambda)$	II-b	$(1.37^{+0.01+0.77+0.00}_{-0.01-0.38-0.00}) \times 10^{-11}$
$B_u^+\,K^0$	a_2	$V_{cd}^*\,V_{us} \sim \mathcal{O}(\lambda^2)$	II-c	$(8.21^{+0.10+4.61+0.21}_{-0.10-2.27-0.21}) \times 10^{-13}$

Table: The numerical results on CP-averaged branching ratios for the $B_c(2S) \to \eta_c(1S,2S)\,P$, $\psi(1S,2S)\,P$ decays, where the theoretical uncertainties come from the CKM parameters, the renormalization scale $\mu = (1{\pm}0.2)\,m_b$, decay constants, and Gegenbauer moments, respectively.

final states	decay amplitude ($\mathcal{A}$) coefficient	CKM factor	case	branching ratio ($\mathcal{B}r$)
$\eta_c(1S)\,\pi^+$	a_1	$V_{cb}\,V_{ud}^{\ast} \sim \mathcal{O}(\lambda^2)$	I-c	$(9.90^{+0.48+0.14+0.03}_{-0.47-0.11-0.03}) \times 10^{-12}$
$\eta_c(2S)\,\pi^+$	a_1	$V_{cb}\,V_{ud}^{\ast} \sim \mathcal{O}(\lambda^2)$	I-c	$(6.40^{+0.31+0.10+0.02}_{-0.30-0.07-0.02}) \times 10^{-12}$
$\psi(1S)\,\pi^+$	a_1	$V_{cb}\,V_{ud}^{\ast} \sim \mathcal{O}(\lambda^2)$	I-c	$(9.36^{+0.46+0.14+0.03}_{-0.44-0.11-0.03}) \times 10^{-12}$
$\psi(2S)\,\pi^+$	a_1	$V_{cb}\,V_{ud}^{\ast} \sim \mathcal{O}(\lambda^2)$	I-c	$(6.19^{+0.30+0.09+0.02}_{-0.29-0.07-0.02}) \times 10^{-12}$
$\eta_c(1S)\,K^+$	a_1	$V_{cb}\,V_{us}^{\ast} \sim \mathcal{O}(\lambda^3)$	I-d	$(7.49^{+0.42+0.11+0.07}_{-0.40-0.09-0.07}) \times 10^{-13}$
$\eta_c(2S)\,K^+$	a_1	$V_{cb}\,V_{us}^{\ast} \sim \mathcal{O}(\lambda^3)$	I-d	$(4.83^{+0.27+0.07+0.04}_{-0.26-0.06-0.04}) \times 10^{-13}$
$\psi(1S)\,K^+$	a_1	$V_{cb}\,V_{us}^{\ast} \sim \mathcal{O}(\lambda^3)$	I-d	$(6.95^{+0.39+0.11+0.06}_{-0.37-0.08-0.06}) \times 10^{-13}$
$\psi(2S)\,K^+$	a_1	$V_{cb}\,V_{us}^{\ast} \sim \mathcal{O}(\lambda^3)$	I-d	$(4.56^{+0.25+0.07+0.04}_{-0.24-0.05-0.04}) \times 10^{-13}$

4.Summary

- It is expected to have hundreds or dozens of the $B_c(2S)$ weak decay events belonging to the I-a, I-b and II-a cases with the potential $\mathcal{O}(10^{11})$ $B_c(2S)$ data in the coming HL-LHC experiment, which might be observed and used to identify the $B_c(2S)$ mesons.
- The $B_c(2S)$ weak decays into final states containing one charmonium are severely suppressed by the CKM factors, and have significantly small branching ratios, $\mathcal{O}(10^{-12})$ and less, which might be outside the future experimental detectability.

感谢各位专家、老师
批评指正！