# The complicated QED corrections in the simplest B decays

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**Based on:**

**Complete analysis on QED corrections to $B \to \tau^+ \tau^-$** *JHEP* 10 (2023) 073

in collaboration with **Y.K. Huang , Y.L. Shen and X.C. Zhao**

**Impact of structure-dependent QED effects on $|V_{ub}|$ extraction**

in preparation with **Y.L. Shen, C. Wang and Y.B. Wei**

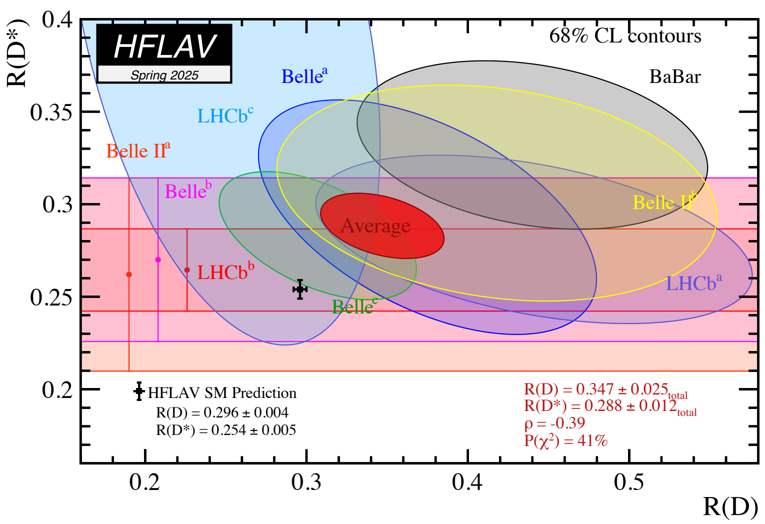
# Why QED corrections in flavor physics ?

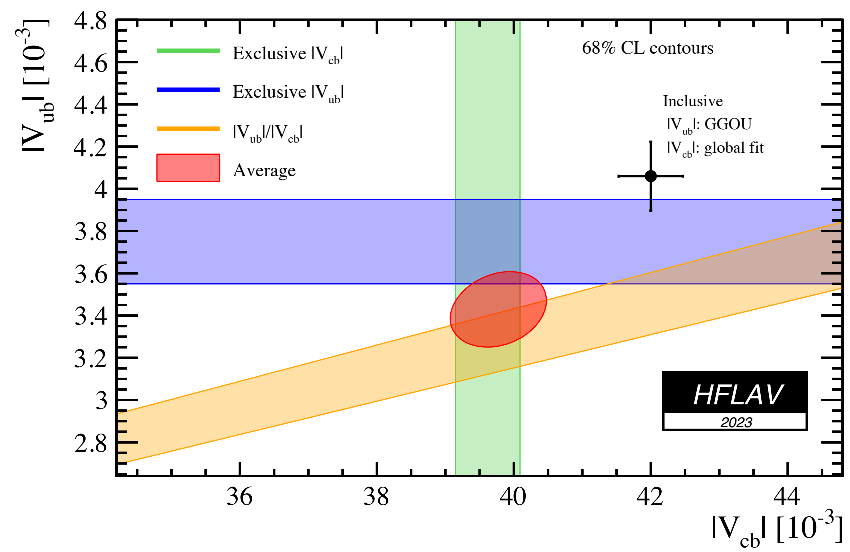
- Flavour physics in high precision in experiments and theoretic aspects

  - ▶ experimental data from $e^+e^-$ "B-factories" and hadron colliders, Belle II, BESIII and LHC.

  - ▶ QCD corrections in $\alpha_s$ and $1/m_b$ and controllable hadronic input parameters

- "Flavour Anomalies" : deviations between theory and experiment for some flavor observables in recent years e.g. $R(D^{(*)}), |V_{ub}|, |V_{cb}|, \ldots$




- QED corrections are important in high precision process

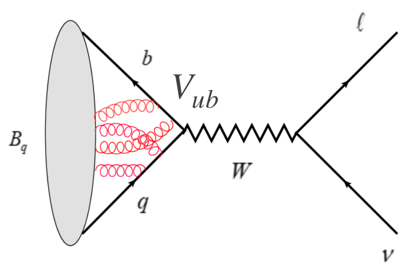
$$\alpha_{\text{em}} \sim \alpha_s^3 \sim 0.01$$

# Why QED corrections in Leptonic B decay ? $B_{d,s} \to \tau^+\tau^-, B_u \to \tau\nu$

- B decay modes receive large uncertainties from the B meson light-cone distribution amplitudes (LCDA)

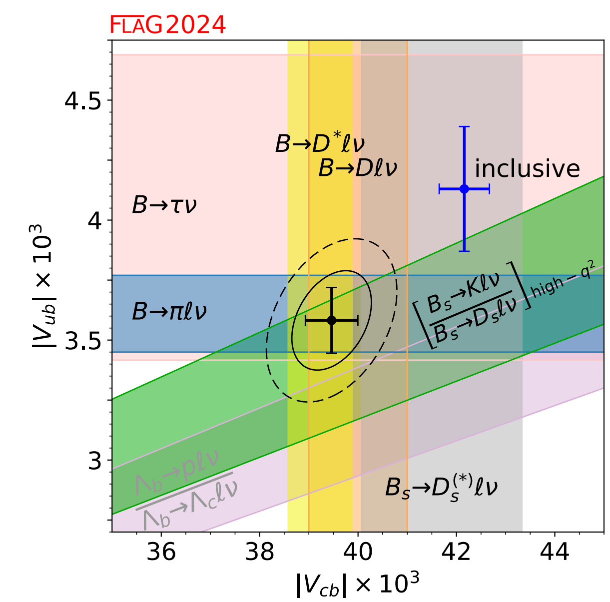
- $B_{d,s} \to \ell^+\ell^-$ is highly suppressed in the SM and can be computed with a good precision!

- $B_u \to \ell\,\nu$ for determination of $|V_{ub}|$ largely unaffected by hadronic uncertainties




$$\langle 0|\bar{u}\,\gamma^\mu\gamma_5\,b\,|B_u(p)\rangle = i\,f_B\,p^\mu$$

$$f_B = (190.0 \pm 1.3)\mathrm{MeV}$$ *[FLAG 2024]*



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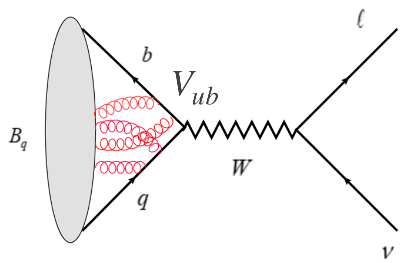


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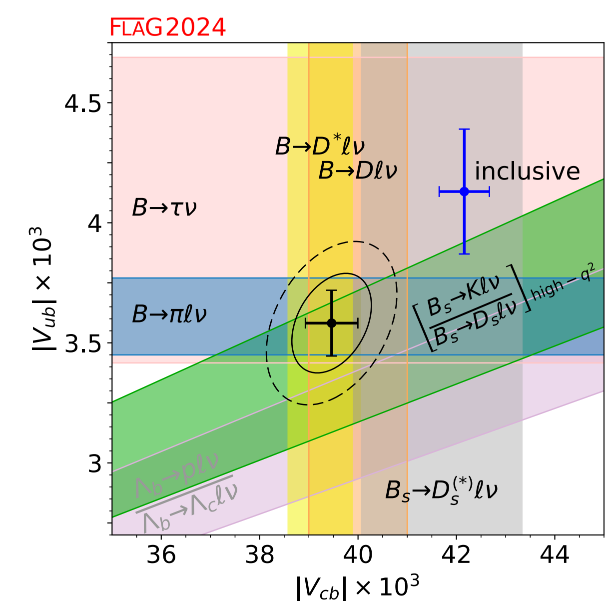
Especially for $B_u \to \tau\,\nu$ $(\mathcal{B}_\tau \sim 10^{-5},\quad \mathcal{B}_\mu \sim 10^{-7},\quad \mathcal{B}_\mu \sim 10^{-12})$

► Recently, Belle II measured its branching fracti*[arXiv: 2502.04885 ]*

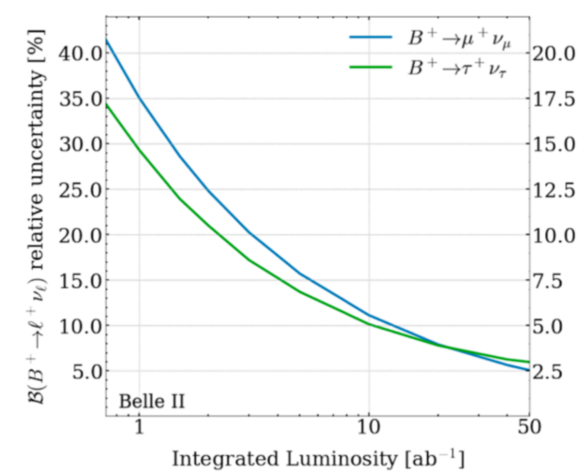
$$\mathcal{B}^{(\text{exp})}(B_u \to \tau\nu) = \left(1.24 \pm 0.41(stat.) \pm 0.19(syst.)\right) \times 10^{-4}$$

and updated the value $|V_{ub}| = \left(4.41^{+0.74}_{-0.89}\right) \times 10^{-3}$




► Belle II will measure the $\tau$ channels with $5-7\,\%$ uncertainty

*[Belle II Physics Book]*

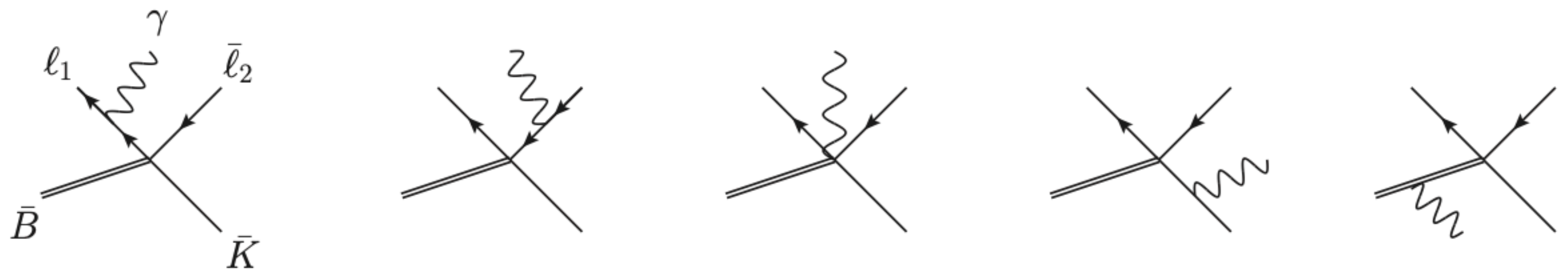



=> QED correction in $B_u \to \tau\,\nu$ become important

# ultra-soft photon approximation

- In most cases, QED analyses focused solely on **ultra-soft photon approximation**

e.g. $\bar{B} \to \bar{K}\ell^+\ell^-$ *[G. Isidori, etc. JHEP 12,104 (2020) ]*




- ► pointlike B meson when photons are extremely low-energy ("ultrasoft")
- ► ultra-soft photon approximation assume pointlike photon couplings to charged hadrons up to energy scales of $\mathcal{O}(m_B)$

- The result is just the pure-QCD amplitude timing with Bloch-Nordsieck factor

$$\sim \exp(-\frac{\alpha_{\rm em}}{\pi}\ln\frac{m_B}{\Delta E})$$

# Beyond ultra-soft photon approximation

- When scale $\Lambda_{\rm QCD} < \mu < m_b$ , the photon emitting from lepton can recoil against the light spectator quark which is then delocalized along the light cone

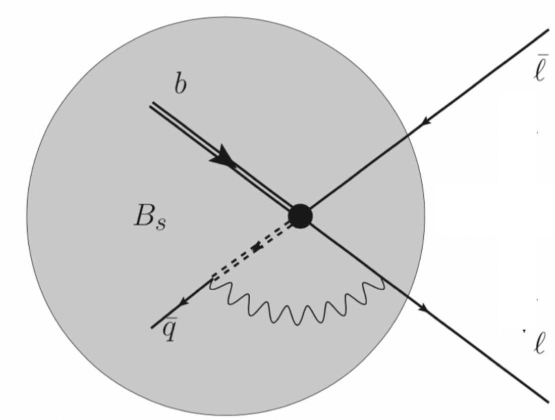
  - The hadronic currents become non-local, the corresponding hadronic matrix element becomes

    $$\langle 0 \,|\, \bar{q}_s(vn_-)\, Y(vn_-,0)\, \frac{\not{n}_+}{2}\, P_L\, h_v(0) \,|\, B \rangle$$

    is light cone distribution $\sim \phi_B(\omega)$, and it is the structure-dependent QED effects

$r_{\rm photon} \sim 1/\sqrt{m_b \Lambda_{\rm QCD}}$

$r_{\rm meson} \sim 1/\Lambda_{\rm QCD}$




*[Fig from Robert Szafron's talk]*

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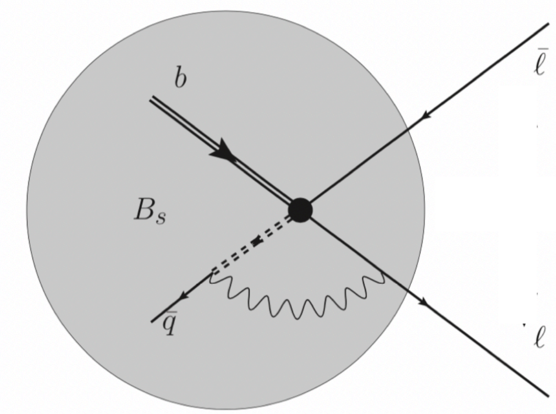



*[Fig from Robert Szafron's talk]*

  - Soft photon interacting with final charged lepton can decouple from lepton to introduce a **Wilson line** $S_{n_-}^{(\ell)\dagger}$

$$\bar{q}_s(vn_-)\, Y(vn_-,0)\, \frac{\not{n}_+}{2}\, P_L\, h_v(0) \rightarrow \bar{q}_s(vn_-)\, Y(vn_-,0)\, \frac{\not{n}_+}{2}\, P_L\, h_v(0)\, S_{n_-}^{(\ell)\dagger}$$

=> B-LCDAs need to be redefined *[M. Beneke, etc. 2022 ]*

- Thus charged leptonic B decays become as complicated as non-leptonic decays when photon in $\Lambda_{\mathrm{QCD}} < \mu < m_b$

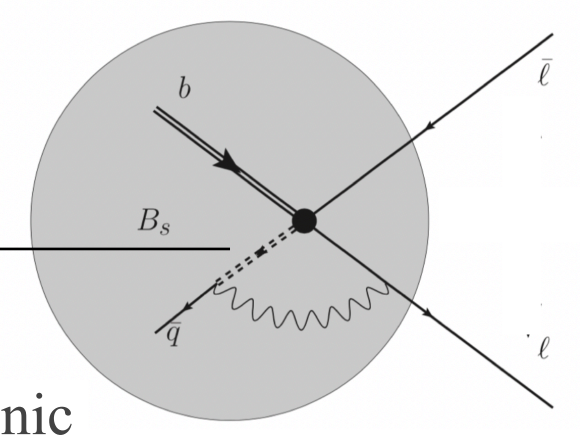
$$\boxed{f_B \rightarrow \text{B-LCDA} \rightarrow \text{QED-redefined B-LCDA}}$$

# Virtual QED corrections can reach $\mathcal{O}(1\%)$ accuracy

- When scale $\Lambda_{\text{QCD}} < \mu < m_b$ , the photon emitting from lepton can recoil against the light spectator quark which is then delocalized along the light cone

  ► Power enhanced effects arising from the propagator of the resolved spectator quark

$$\frac{1}{n\_k_q} \sim \frac{1}{\Lambda_{\text{QCD}}}$$




  ► More QED Logarithms appear besides logs involving final leptonic scale and soft photon cut $\Delta E$

$$\ln\frac{m_\ell}{\Delta E}, \quad \ln\frac{m_B}{m_\ell}, \quad \ln\frac{m_b}{\Lambda_{\text{QCD}}}$$

=> the expansion parameter is $\frac{\alpha_{\text{em}}}{\pi}\log^2$ , rather than just $\frac{\alpha_{\text{em}}}{\pi}$

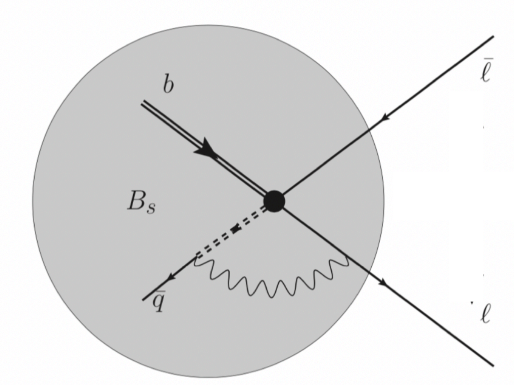
$$\boxed{\frac{1}{\Lambda_{\text{QCD}}} + \frac{\alpha_{\text{em}}}{\pi}\log^2}$$

# Power enhancement in $B_{d,s} \to \ell^+\ell^-$

- Power enhanced effects exist in $B_{d,s} \to \ell^+\ell^-$ and **overcome** its **helicity suppression** $m_\ell/m_b$

- Thus $B_{d,s} \to \ell^+\ell^-$ is a leading power process, and we only need to calculate this power enhancement term

  - Non-local time ordered products have to be evaluated




$$\left\langle 0 \right| \int d^4x\, e^{iqx}\, T\,\{j_{\rm QED}(x), \mathscr{L}_{\Delta B=1}(0)\}\left| B \right\rangle$$

  - The dynamical enhancement by a power of $m_b/\Lambda_{\rm QCD}$ and by large logarithms $\ln m_b\, \Lambda_{\rm QCD}/m_\ell^2$

=> 1% of Br $(B_s \to \mu^+\mu^-)$, four times the size of previous estimates of NLO QED effects $\alpha_{\rm em}/\pi \sim 0.3\ \%$ *[M. Beneke, etc. 17 & 19 ]*

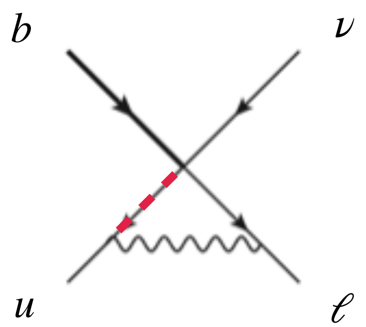
=> 0.4% of Br $(B_s \to \tau^+\tau^-)$ due to hard-collinear scale tau mass in $\ln m_b\, \Lambda_{\rm QCD}/m_\tau^2$ *[Y.K.Huang, Y.L.Shen, X.C.Zhao, SHZ 23]*

# $B_u \to \ell\,\nu$ is a subleading power process

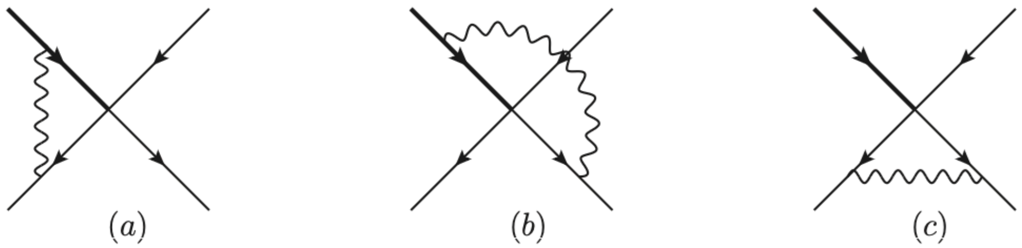
- Power enhancement happens for $B_{d,s} \to \ell^+\ell^-$ with $\gamma^\mu(1+\gamma_5)$, but not for $B_u \to \ell\,\nu$ with left-handed currents $\gamma^\mu(1-\gamma_5)$ by cancellation

$$[\bar{q}_s \gamma_\perp^\mu \gamma_\perp^\nu \frac{\not{n}_-}{2}(1-\gamma_5)\,h_v][\bar{\ell}_c\,\gamma_{\perp\mu}\gamma_{\perp\nu}(1-\gamma_5)\,\nu_{\bar{c}}] = 0$$

=> No power enhanced effect in $B_u \to \ell\,\nu$ !




- We need to consider all power suppressed contributions to $B_u \to \ell\,\nu$




NLP Local contribution    NL calculated to NLP

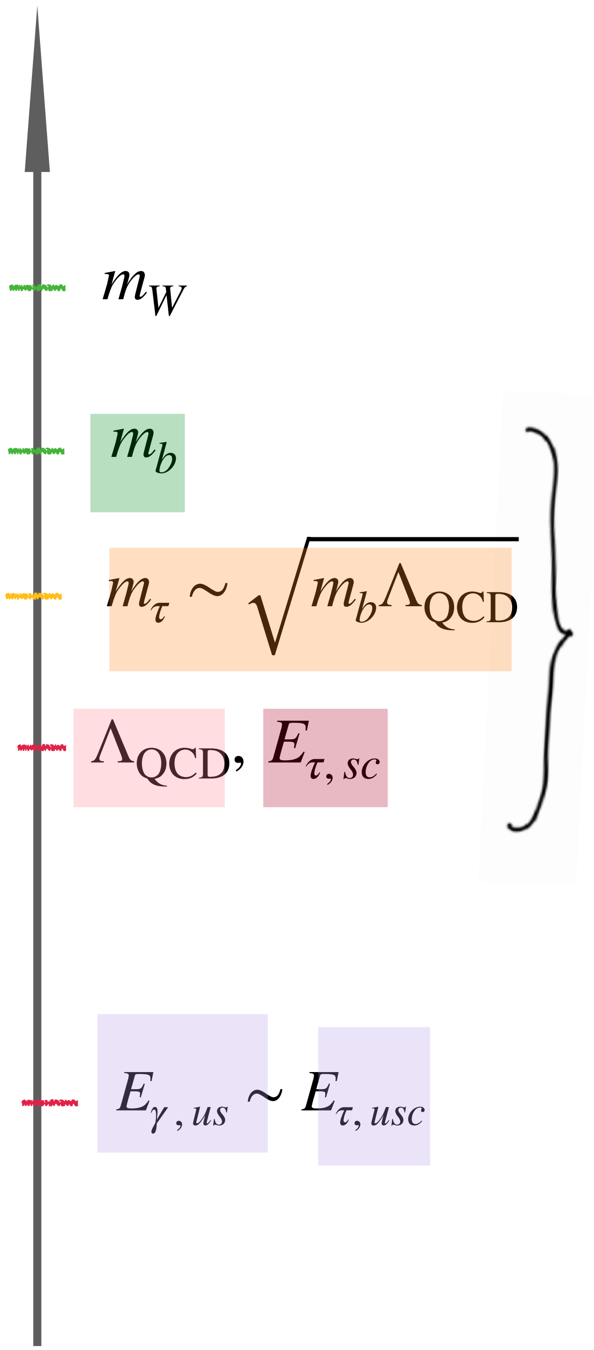
- mixed (QED+QCD) and NLP are the main challenges for EFTs in building the factorization theorem

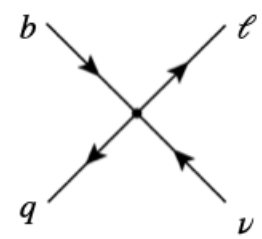
  - ▶ endpoint divergences in NLP ? *[Feldmann, Gubernari, Huber, Neubert, Seitz 2022; Hurth, Neubert, Szafron 2023, Neubert etc.2023 ]*

  - ▶ QED IR regulator in EFTs ?

# A muti-scale process

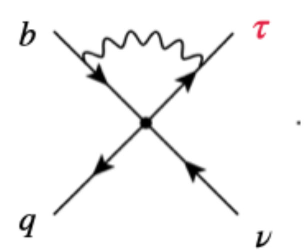
focus on $B_u \to \tau\,\nu$ new scales appear in the present of QED effects



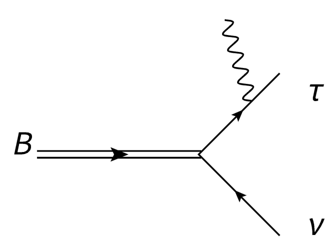

- For $\mu > m_b$ , Fermi theory

$$\mathscr{L}_{\rm eff} = -\frac{G_F}{\sqrt{2}} V_{ub}\,[\bar{q}\,\gamma_\mu\,(1-\gamma_5)\,b][\bar{\ell}\,\gamma^\mu\,(1-\gamma_5)\,\nu]$$




- Intermediate scale $\Lambda_{\rm QCD} < \mu < m_b$ , virtual photons can resolve the substructure of $B$ meson which be dependent on final lepton ($e$ , $\mu$ , or $\tau$)

different effective field-theory constructions for different final lepton scale




- $\mu \ll \Lambda_{\rm QCD}$ ultra-soft photon approximation

In this talk, we focus on the virtual QED corrections to $B_u \to \tau\,\nu$

# modes and EFTs

- Relevant momentum modes $p \sim (n_+p, n_-p, p_\perp)$ for **virtual** QED corrections

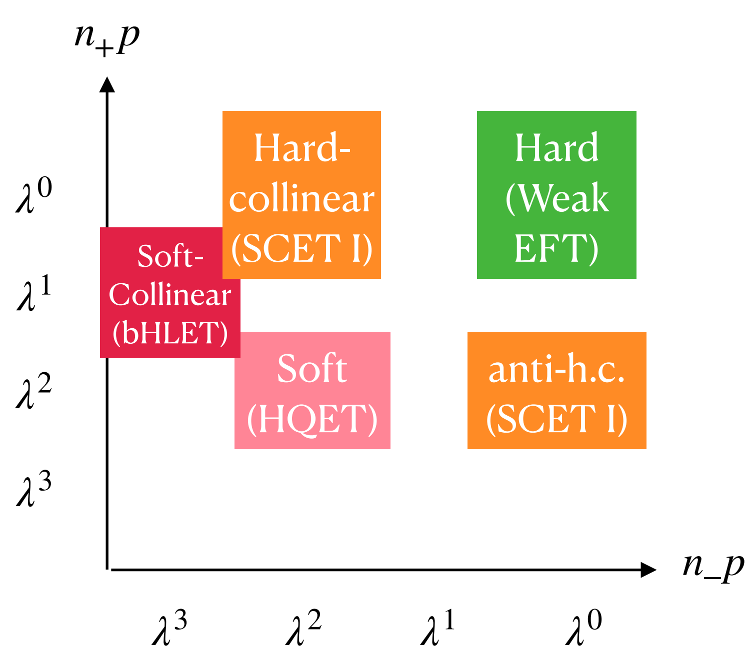
  - Hard $(\lambda^0, \lambda^0, \lambda^0)$
  - Hard-collinear $(\lambda^0, \lambda^2, \lambda)$
  - Soft $(\lambda^2, \lambda^2, \lambda^2)$
  - Soft-collinear $\lambda^2(1/b, b, 1) \sim (\lambda, \lambda^3, \lambda^2)$

Expansion parameters:

$$\lambda^2 = \frac{\Lambda_{\mathrm{QCD}}}{m_b}$$

$$b = \frac{m_\tau}{m_b} \sim \lambda$$

- Factorization requires a number of different EFT constructions




**Fermi theory**

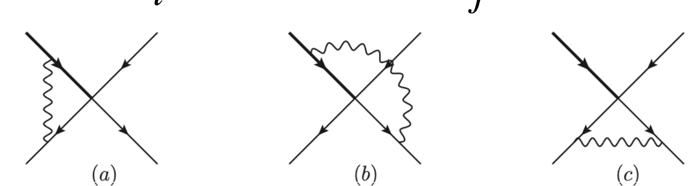
$\downarrow$

$\mathrm{HQET} \times \mathrm{SCET}_{\mathrm{I}}$

$\downarrow$

$\mathrm{HQET} \times \mathrm{bHLET}$

- The factorized amplitude can be expressed as $\mathcal{A}^{\text{virtual}} = \sum_i H_i J_i S_i + \sum_j \int_u H_j \int_\omega J_j S_j$

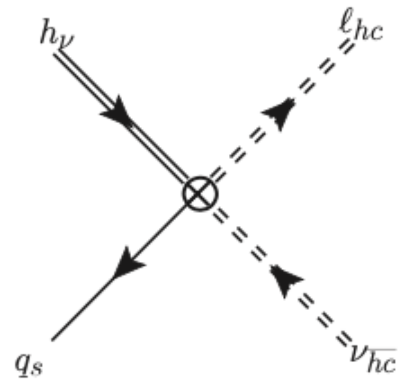


# Fermi theory $\to$ HQET $\times$ SCET$_\mathrm{I}$ $(\mu < m_b)$

**Integrating out hard modes to reach to HQET $\times$ SCET$_\mathrm{I}$**

- Field redefinition of heavy quark field *[Heavy Quark Physics]*

$$b(x) \to e^{-im_b v\cdot x}(1 + \mathcal{O}(\lambda^2))\, h_v(x)$$



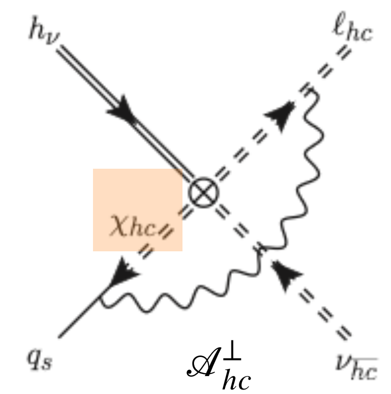

- Hard-collinear lepton fields are described in SCET$_\mathrm{I}$

*[Bauer, Fleming, Pirjol and Stewart, 01'], [Beneke,Chapovsky, Diehl and Feldmann, 02'], [Becher, Broggio and Ferroglia, 14']*

$$\ell(x) = \frac{\not{n}_- \not{n}_+}{4}\ell_C(x)\,, \qquad \nu(x) = \frac{\not{n}_+ \not{n}_-}{4}\nu_{\bar{C}}(x)$$

- Spectator light quark can also be hard-coll. firstly

$$q(x) = \frac{\not{n}_- \not{n}_+}{4}\chi_C(x)$$

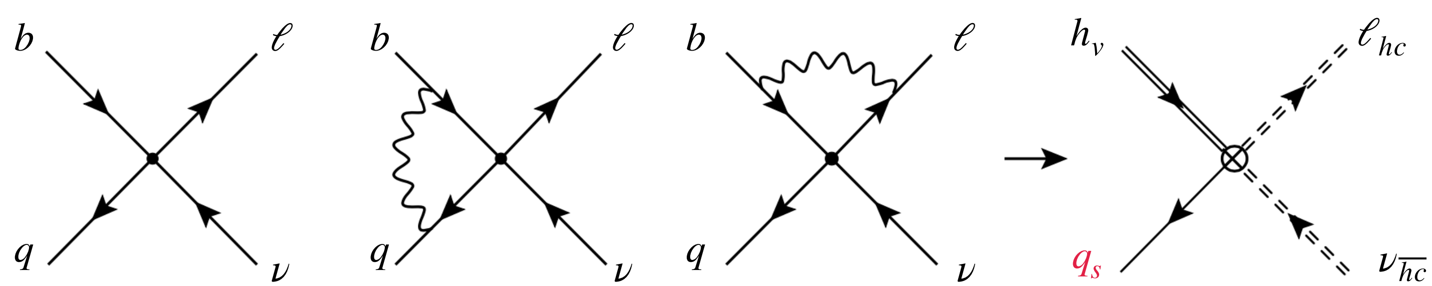



with power counting parameter :

$$h_v\,, q_s \sim \lambda^3 \qquad \ell_{hc}\,, \nu_{\overline{hc}}\,, \chi^{(q)}_{hc} \sim \lambda \qquad \mathscr{A}^\perp_{hc} \sim \lambda$$

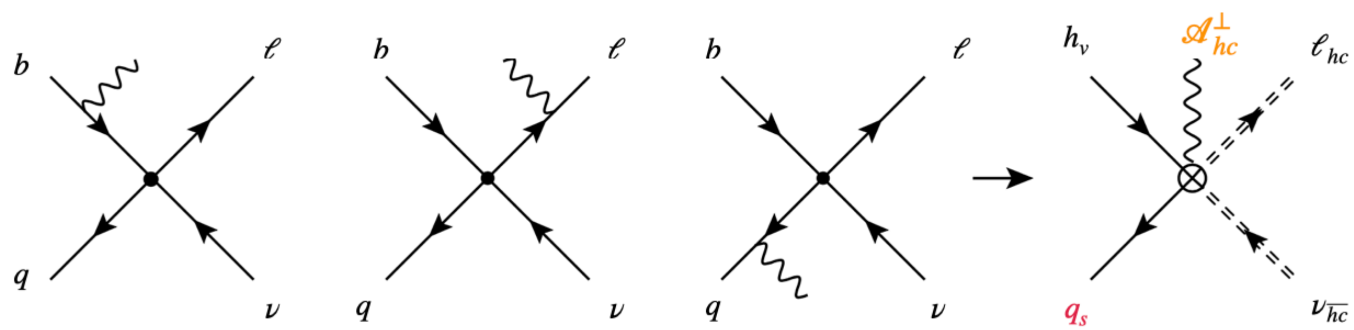
# Fermi theory $\rightarrow$ HQET $\times$ SCET$_\mathrm{I}$ $(\mu < m_b)$

- Construction of HQET $\times$ SCET$_\mathrm{I}$ operators: Local and Nonlocal operators
  - local operator with soft spectator $\mathcal{O}_A$



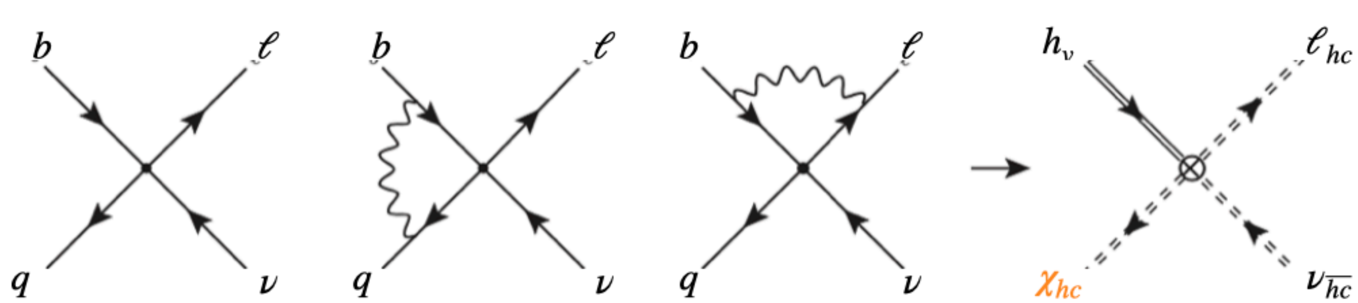

$$\mathcal{O}_A = [\bar{q}_s \ldots h_v]\,[\bar{\ell}_{hc} \ldots \nu_{\overline{hc}}]$$

  - local operator with soft spectator and hard-collinear photon $\mathcal{O}_B$




$$\mathcal{O}_B = [\bar{q}_s \ldots h_v]\,[\bar{\ell}_{hc} \ldots \nu_{\overline{hc}}] \quad \mathcal{A}^{\perp}_{hc}$$

  - nonlocal operator with hard-collinear spectator $\mathcal{O}_\chi$




$$\mathcal{O}_\chi = [\chi_{hc} \ldots h_v]\,[\bar{\ell}_{hc} \ldots \nu_{\overline{hc}}]$$

- hard functions $H_{i,j}$ are obtained from the above matching $\mathcal{A}^{\mathrm{virtual}} = \sum_i H_i J_i S_i + \sum_j \int_u H_j \int_\omega J_j S_j$

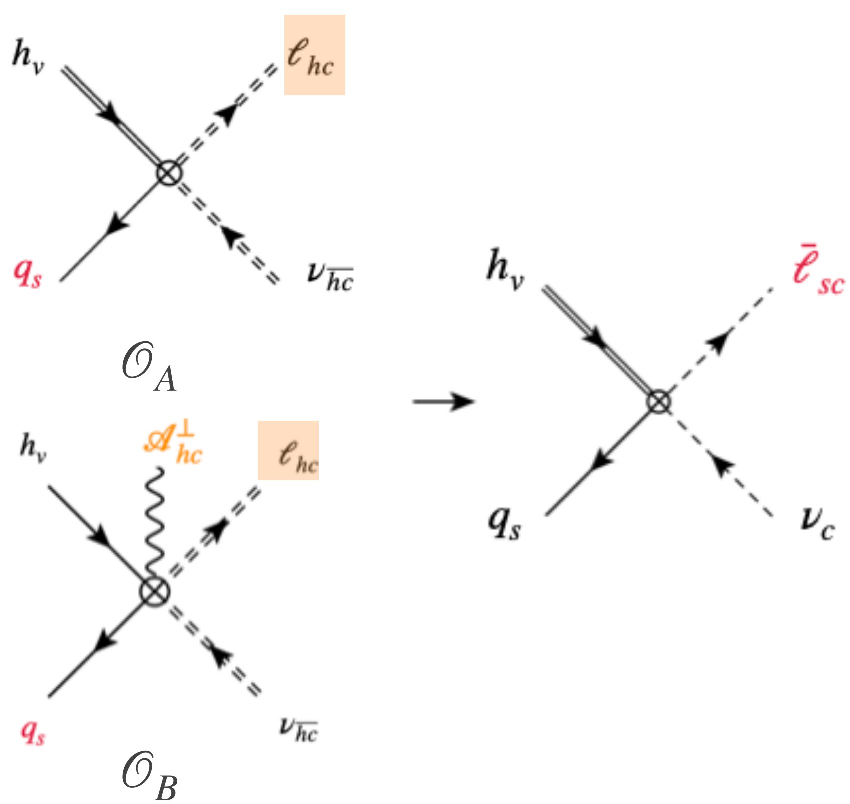
# SCET$_\text{I}$ $\to$ bHLET $(\mu < \sqrt{m_b \Lambda_{\text{QCD}}})$

**lower the virtuality to remove the hard-collinear mode to reach to bHLET**

- heavy tau filed become to a soft-collinear (sc) field in boosted HLET after integrating $m_\tau$ *[Fleming, Hoang, Mantry and Stewart, 07'], [Dai, Kim and Leibovich, 21']*

$$\ell_{hc} \to e^{-i m_\ell v_\ell \cdot x}\left(1 + b\,\frac{\not{n}_+}{2}\right)\underset{\downarrow}{\ell_{sc}}$$

soft scale to $\tau$, boosted in the $B$ frame with $b = \dfrac{m_\tau}{m_b}$




$$\mathcal{J}^{A,B}_m = [\bar{q}_s \frac{\not{n}_+}{2} P_L h_v]\, S^{(\ell)\dagger}_{n_-} [\bar{\ell}_{sc} P_L \nu_{\bar{c}}]$$

*Local operators*

- hard-collinear functions $J_i$ are obtained from the above matching

$$\mathscr{A}^{\text{virtual}} = \sum_i H_i J_i S_i + \sum_j \int_u H_j \int_\omega J_j S_j$$

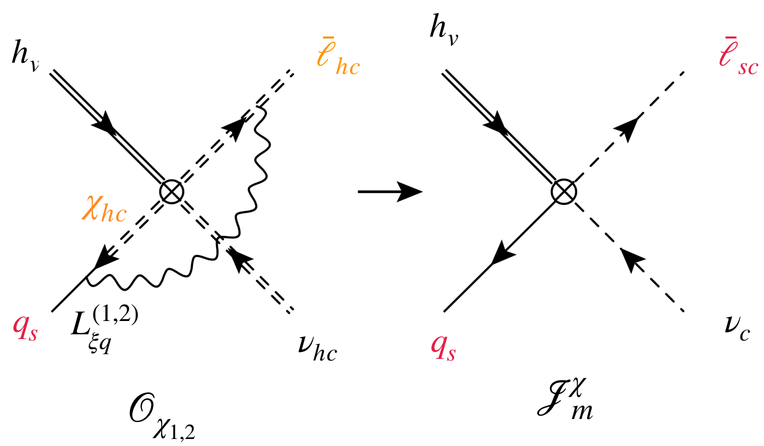
# $\mathrm{SCET_I} \to \mathrm{HQET} \times \mathrm{bHLET} \ (\mu < \sqrt{m_b \Lambda_{\mathrm{QCD}}})$

- $\chi_{hc}^{(q)} \to q_s$ by inserting the following NLP and NNLP interactions

*[Beneke, Garny, Szafron, Wang, 18']*

$$\mathcal{L}_{\xi q}^{(1)}(x) = \bar{q}_s(x_-)[W_{\xi C} W_C]^\dagger(x)\, i \not{D}_{C\perp}\, \xi_C(x) + \mathrm{h.c.}$$

$$\mathcal{L}_{\xi q}^{(2)}(x) = \bar{q}_s(x_-)\left[W_{\xi,hc} W_{hc}\right]^\dagger(x)\, \left(i n_- D_{hc} + i \not{D}_{hc\perp}(i n_+ D_{hc})^{-1} i \not{D}_{hc\perp}\right) \frac{\not{n}_+}{2} \xi_{hc}(x) + \ldots$$




$$\mathcal{J}_m^\chi(v) = [\bar{q}_s(v n_-)\, Y(v n_-, 0) \frac{\not{n}_+}{2} P_L\, h_v(0)]\, S_{n_-}^{(\ell)\dagger}(0)\, [\bar{\ell}_{sc}(0) P_L\, \nu_{\bar{c}}(0)]$$

*non-local operators*

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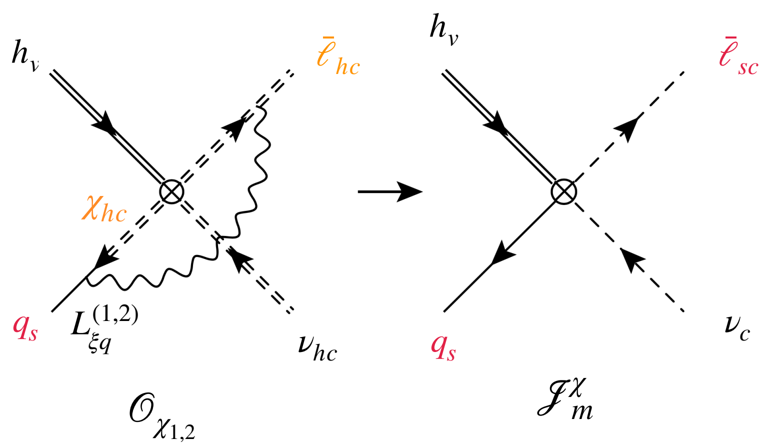


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*non-local operators*

- hard-collinear functions $J_j$ are obtained from the above matchings

$$\mathscr{A}^{\mathrm{virtual}} = \sum_i H_i J_i S_i + \sum_j \int_u H_j \int_\omega J_j S_j$$

endpoint div. $(1/u \to \infty$, when u$\to 0)$ ?

$B_{d,s} \to \mu^+ \mu^-$

$B_u \to \mu\,\nu$

*Neubert etc.2023*

$$J_{\chi,1}^{(1)} = \frac{\alpha_{\mathrm{em}}}{\pi} Q_\ell Q_u \frac{m_\ell}{n_+ p_\ell} u \left[\ln \frac{\mu^2}{\bar{u}^2 m_b\, n_- p_\ell} - \frac{1+r}{r}\ln(1+r)\right]\theta(u)\theta(\bar{u}) \quad r = \frac{u}{\bar{u}}\frac{\omega\, m_B}{m_\ell^2}$$

=> No endpoint div. in $B_u \to \tau\,\nu$ when convoluting to hard function due to massive tauon !

subtractions scheme independence

# Numerical prediction

- The non-radiative QED corrections to branching fraction of $B_u \to \tau\nu$ for central values of the parameters

$$\mathrm{Br}^{(0)}(B_u \to \tau\nu) = \left(0.89_{\color{red}{(\mathrm{LO})}} - 0.01_{\color{red}{(\mathrm{NLO})}}\right) \times 10^{-4}$$

**NLP+NLO+LL** QED virtual correction changes the branching fraction by $\sim 1\,\%$

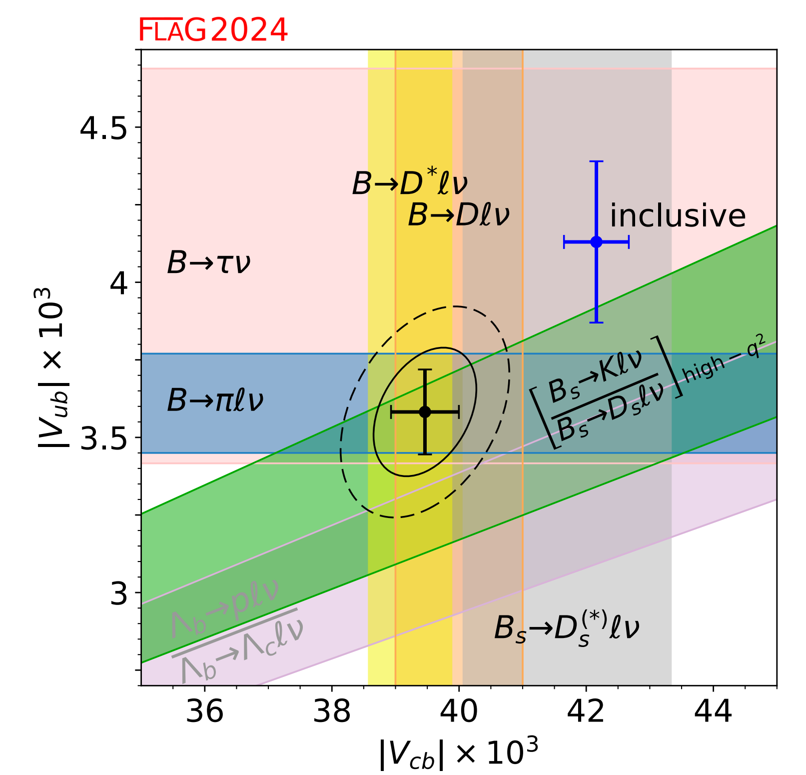
be comparable with QCD uncertainties $\delta f_B^2 \sim 1.4\,\%$

- Determining the CKM matrix element using the latest data from Belle II

$$|V_{ub}| = \left[\left(4.41 + {\color{red}{0.01_{[\delta_{\mathrm{QED}}]}}}\right) \pm 0.03(\mathrm{th.})^{+0.73}_{-0.91}(\mathrm{exp.})\right] \times 10^{-3}$$

the experimental uncertainty would reduce to $\sim 0.08 \times 10^{-3}$

based on $\sim$ 50 $\mathrm{ab}^{-1}$ of electron-positron collision data



# Summary

- QED corrections are important in high precision process

- QED corrections in $B_{d,s} \to \tau^+\tau^-$ , $B_u \to \tau\nu$

- Charged leptonic B decays become as complicated as non-leptonic decays

- Subleading power factorization formula for QED corrections to $B_u \to \tau\nu$ derived in SCET, HQET and bHLET

  - no endpoint divergences in this factorization at NLP (due to tau mass)
    => subtractions scheme independence

  - NLP+NLO+LL QED virtual correction changes the branching fraction by $\sim 1\,\%$ , are critical for $|V_{ub}|$ determination.



## Thank you

# Backup slides

# Generalized decay constant and LCDA

- After two-step matching, we reach to the factorized amplitude

$$A^{\text{virtual}}_{B\to\tau\nu} \sim H_{A,B}\, J_{A,B}\, \langle \tau^- \nu \,|\, \mathcal{J}^{A,B}_m \,|\, \bar{B}_u \rangle + \int_0^1 du\, H_\chi(u) \int_0^\infty d\omega\, J_\chi(u;\omega)\, \langle \tau^- \nu \,|\, \mathcal{J}^{\chi}_m \,|\, \bar{B}_u \rangle$$

$m_b$

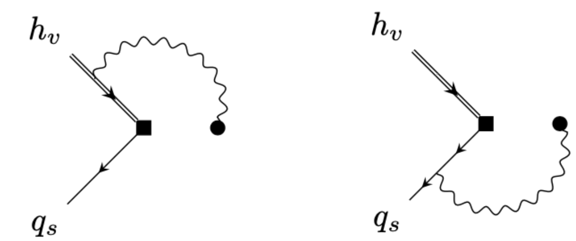
$m_\tau \sim \sqrt{m_b \Lambda_{\text{QCD}}}$

$\Lambda_{\text{QCD}}$, $E_{\tau,sc}$

HQET × bHLET operators

$$\mathcal{J}^{A,B}_m = [\bar{q}_s \frac{\not{n}_+}{2} P_L h_v]\, S^{(\ell)\dagger}_{n_-}\, [\bar{\ell}_{sc} P_L \nu_{\bar{c}}]$$

$$\mathcal{J}^{\chi}_m(v) = [\bar{q}_s(v\, n_-)\, Y(v\, n_-, 0) \frac{\not{n}_+}{2} P_L h_v(0)]\, S^{(\ell)\dagger}_{n_-}(0)\, [\bar{\ell}_{sc}(0) P_L \nu_{\bar{c}}(0)]$$




Soft photon decoupling from lepton

- QED-generalized decay constant and LCDA *[M. Beneke, etc. 2022 ]*

$$\mathcal{F}_B \equiv \frac{\langle 0 \,|\, \bar{q}_s \frac{\not{n}_+}{2} P_L h_v S^{(\ell)\dagger}_{n_-} \,|\, B \rangle}{\langle 0 \,|\, [\, S^{(B)}_{v_B}(0)\, S^{(\ell)\dagger}_{n_-}(0)\, ] \,|\, 0 \rangle} \begin{array}{l} \longrightarrow \text{IR div.} \\ \longrightarrow \text{Redefination} \end{array}$$

new nonperturbative hadronic parameters

$$\mathcal{F}_B \Phi_B(v) \equiv \frac{\langle 0 \,|\, \bar{q}_s(v\, n_-)\, Y(v\, n_-, 0) \frac{\not{n}_+}{2} P_L h_v(0)\, S^{(\ell)\dagger}_{n_-}(0) \,|\, B \rangle}{\langle 0 \,|\, [\, S^{(B)}_{v_B}(0)\, S^{(\ell)\dagger}_{n_-}(0)\, ] \,|\, 0 \rangle}$$

- It is interesting to determine in Lattice or to estimate in QCD SR

# Decay amplitude including virtual QED corrections at NLP+NLO

$$i\,\mathscr{A}^{\text{virtual}} = -\frac{i}{4}\, m_{B_u}\, \bar{u}_{sc}\,(p_\ell)\; P_L\, v_{\bar{c}}\,(p_\nu)\, \left[\sum_{i=A,1}^{B,(1,2)} H_i(\mu)\, J_i(\mu)\, \mathscr{F}_{B_u}(\mu)\; + \sum_{j=\chi,1}^{\chi,2} \int_0^1 du\, H_j(u,\mu) \int_0^\infty d\omega\, J_j(u;\omega,\mu)\, \mathscr{F}_{B_u}(\mu)\, \Phi_B(\omega,\mu)\right]$$

$$\begin{aligned}\mathcal{A}^{\text{virtual}} &= \frac{G_F}{\sqrt{2}} V_{ub}\, m_B\, f_B\, \bar{u}_{sc}\, P_L\, v_{\bar{c}} \times \frac{\alpha}{4\pi} Q_\ell\, b \left\{ 2\,Q_b \left[ (L - 2\ln s - s - 1)\, L - \frac{1}{2} L^2 + \right.\right. \\ & 2\ln^2 s + \frac{s^2-2}{s-1}\ln s + 2\,\mathrm{Li}_2(1-s) - s - 1 + \left.\frac{\pi^2}{12}\right] - 6\,(2\,Q_b - Q_\ell) - \\ & Q_\ell \left(\frac{1}{2} L'^2 + \frac{\pi^2}{12}\right) + 4 \int_0^1 \mathrm{d}u\, (\bar{u}\, Q_b + Q_u) \ln \frac{\mu^2}{\bar{u}^2\, m_\tau^2} - \\ & 4\,Q_u \int_0^1 \mathrm{d}u \int_0^\infty \mathrm{d}\omega\, \phi_B^+(\omega)\, (1+\bar{u}) \left.\left[ \ln \frac{\mu^2}{\bar{u}^2\, m_\tau^2} - \frac{1+r}{r} \ln(1+r) + \frac{1}{(1+\bar{u})\,\bar{u}\, r} \ln(1+r) \right] \right\}\end{aligned}$$

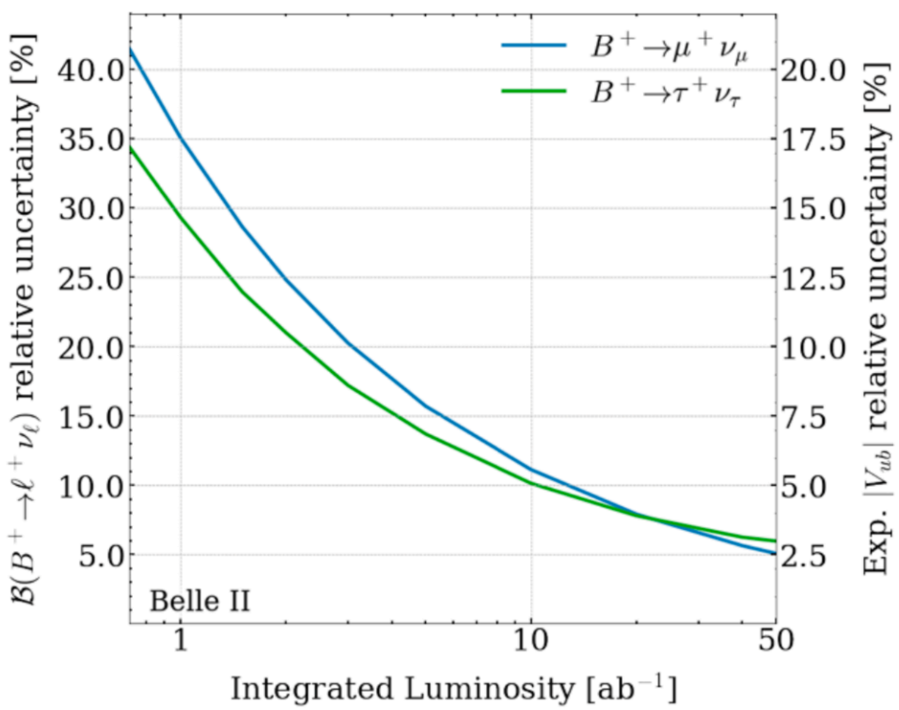
- Large (double) logarithms $L, L'$
- $\phi_B^+(\omega)$ demonstrates explicitly the structure-dependent QED effects

- $\phi_B^+(\omega)$ :

$$\phi_+^B(\omega,\mu) = N\,\frac{\omega}{\omega_0^2}\,e^{-\omega/\omega_0} + \theta(\omega-\omega_t)\,\frac{C_F\alpha_s}{\pi\omega} \times \left[\left(\frac{1}{2}-\ln\frac{\omega}{\mu}\right) + \frac{4\bar{\Lambda}_{\rm DA}}{3\omega}\left(2-\ln\frac{\omega}{\mu}\right)\right], \quad (29)$$

which exhibits a negative radiation tail
for $\omega \gg \mu$

Seung J. Lee and Matthias Neubert hep-ph/ 0509350




[Belle II Physics Book]

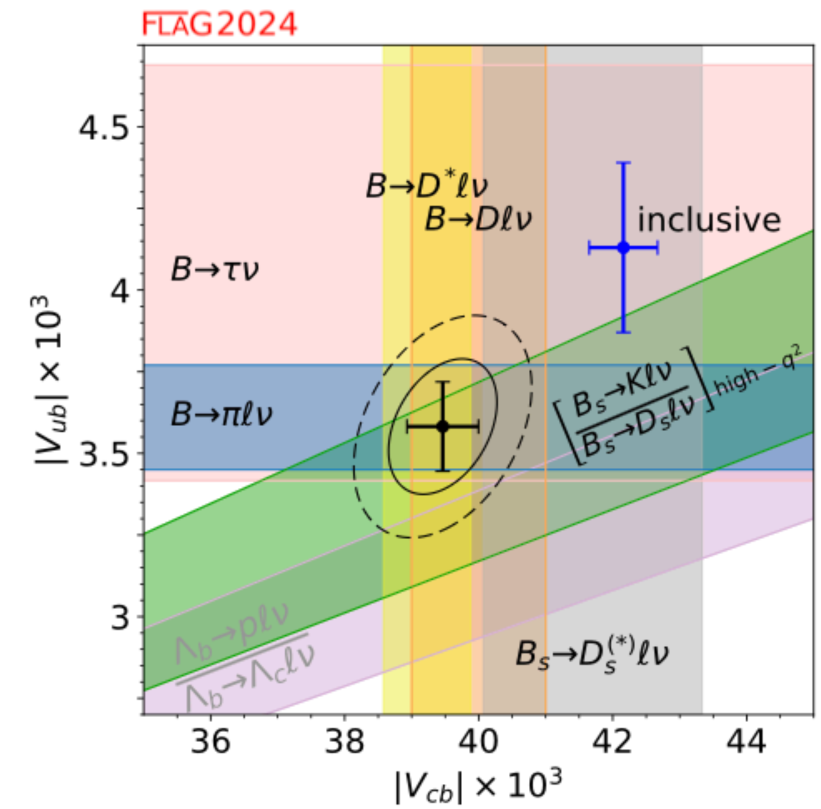
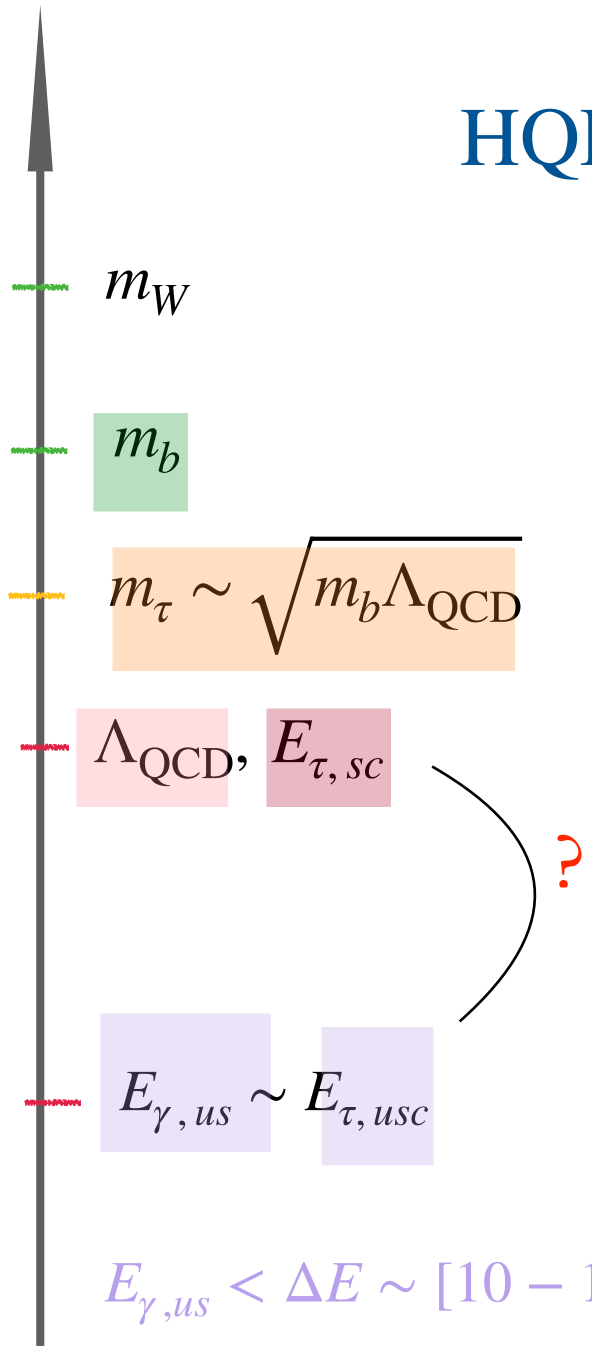



Figure 1: Projection of uncertainties on the branching fractions $\mathcal{B}(B^+ \to \mu^+ + \nu_\mu)$ and $\mathcal{B}(B^+ \to \tau^+ + \nu_\tau)$. The corresponding uncertainty on the experimental value of $|V_{ub}|$ is shown on the right-hand vertical axis.

# HQET × bHLET → **Low-energy theory** ($\mu < \Lambda_{\rm QCD}$)




$E_{\gamma,us} < \Delta E \sim [10-100]$ MeV

- $\mu < \Lambda_{\rm QCD}$, the hadronic $B$ meson can be described as a heavy scalar effective theory (HSET)

$$\Phi_B(x) \to \mathrm{e}^{-i m_B v_B \cdot x}\, h_{v_B}(x) \qquad m_B v_B \sim \mu_s$$

- $\mu < \mu_{sc}$ , soft-coll. (sc) field in bHLET turned into ultra-soft-coll. one (usc) in bHLET-2

$$\ell_{sc} \to \mathrm{e}^{-i m_\ell v'_\ell \cdot x}\, \ell_{usc} \qquad m_\ell v'_\ell \sim \mu_{sc}$$

- HQET × bHLET → HSET × bHLET$_{\rm II}$ $\quad \mu \sim \Lambda_{\rm QCD}$

power parameters: $\lambda_E^2 = \dfrac{E_{\gamma,us}}{m_b} \sim \dfrac{\Lambda_{\rm QCD}^2}{m_b^2} = \lambda^4$

- Ultra-soft $\quad p \sim (\lambda_E^2, \lambda_E^2, \lambda_E^2)$
- Ultra-soft-collinear $\quad p \sim \lambda_E^2 (1/b, b, 1)$

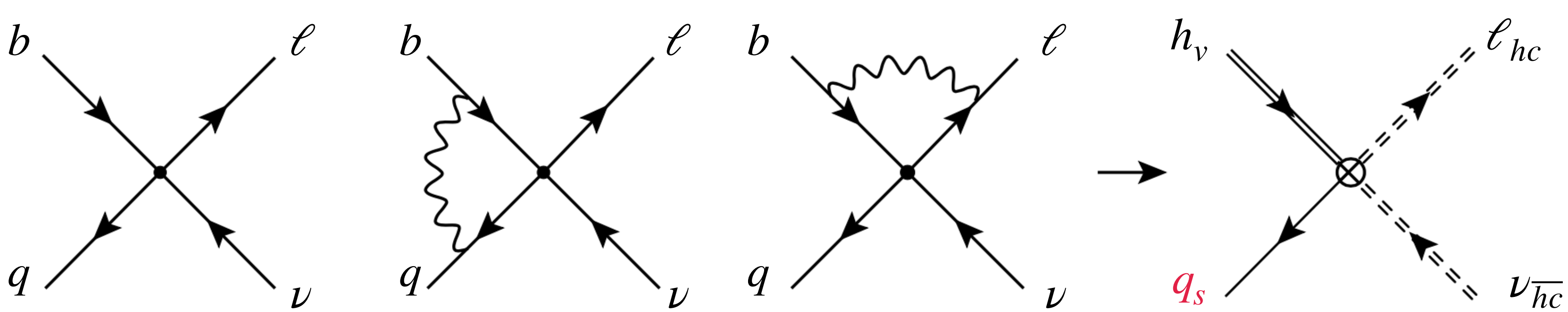
→ultrosoft scale to $\tau$ boosted in the $B$ frame

# Construction of HQET × SCET$_\mathrm{I}$ operator

1. local operator with soft spectator $\mathcal{O}_A$

only two irreducible Dirac structures

$$[\bar{\ell}_{hc}\,\Gamma_\ell\, P_L \nu_{\overline{hc}}] \ \text{ with } \ \Gamma_\ell = 1, \gamma_\mu^\perp$$



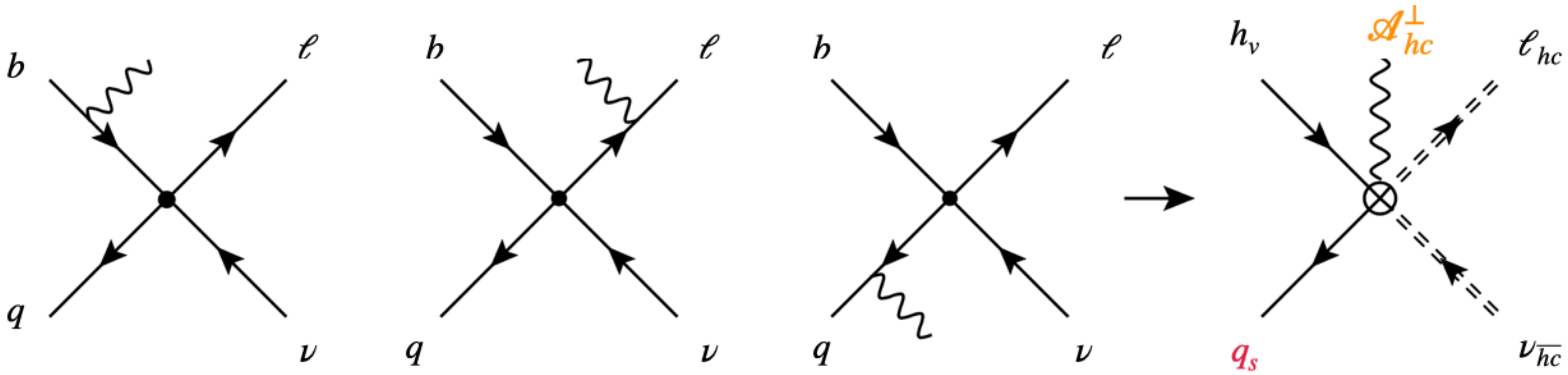

$$\mathcal{O}_A^{(\lambda)} = [\bar{q}_s \ldots h_v]\,[\bar{\chi}_{hc}^{(\ell)} \ldots \chi_{\overline{hc}}^{(\nu)}]$$

$$\mathcal{O}_{A,1}^{(9)} = m_\ell\,[\bar{q}_s \frac{\not{n}_+}{2} P_L\, h_v]\,[\bar{\ell}_{hc} \frac{1}{i\, n_+ \overleftarrow{\partial}_{hc}} P_L\, \nu_{\overline{hc}}]$$

$$\mathcal{O}_{A,2}^{(8)} = [\bar{q}_s \gamma_{\mu\perp} P_L\, h_v]\,[\bar{\ell}_{hc}\, \gamma_\perp^\mu P_L\, \nu_{\overline{hc}}]$$

# Construction of HQET $\times$ SCET$_{\mathrm{I}}$ operator

2. local operator with soft spectator and hard-collinear photon $\mathcal{O}_B$



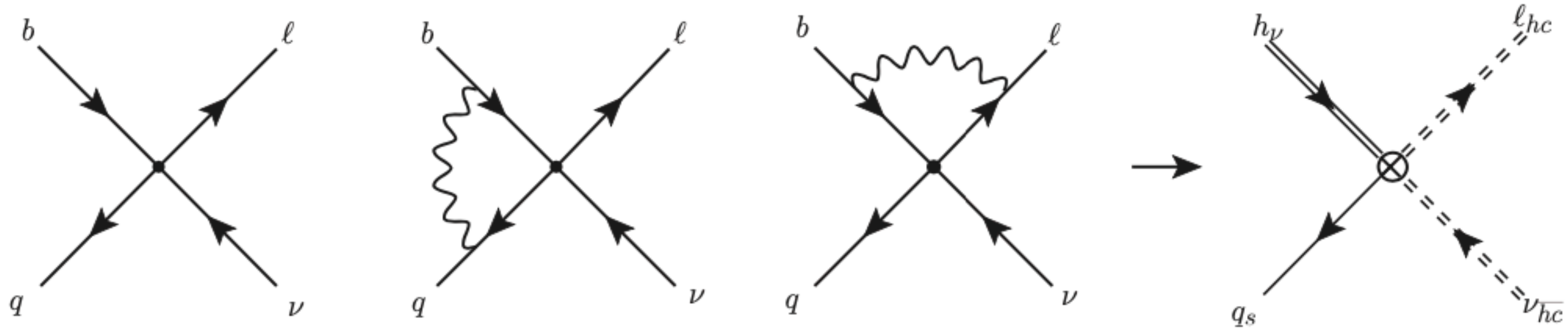

$$\mathcal{O}_B^{(\lambda)} = [\bar{q}_s \ldots h_v]\,[\bar{\ell}_{hc} \ldots \nu_{\overline{hc}}]\;\; \mathcal{A}^{\perp}_{hc}$$

$$\mathcal{O}^{(9)}_{B,1} = \frac{1}{i\,n_+\partial_{hc}}\,[\bar{q}_s \frac{\not{n}_-}{2}\,\gamma_{\mu\perp}\;\; \not{\mathcal{A}}^{(b)}_{hc\perp}\,P_L\,h_v]\,[\bar{\ell}_{hc}\,\gamma^{\mu}_{\perp}\,P_L\,\nu_{\overline{hc}}]$$

$$\mathcal{O}^{(9)}_{B,2} = [\bar{q}_s\;\; \not{\mathcal{A}}^{(q)}_{hc\perp} \frac{1}{i\,n_+\overleftarrow{\partial}_{hc}}\,\frac{\not{n}_+}{2}\,\gamma_{\mu\perp}\;P_L\,h_v]\,[\bar{\ell}_{hc}\;\gamma^{\mu}_{\perp}\,P_L\,\nu_{\overline{hc}}]$$

$$\mathcal{O}^{(9)}_{B,3} = \frac{1}{i\,n_+\partial_{hc}}\,[\bar{q}_s\,\frac{\not{n}_+}{2}\,P_L\,h_v]\,[\bar{\ell}_{hc}\;\; \not{\mathcal{A}}^{(\ell)}_{hc\perp}\,P_L\,\nu_{\overline{hc}}]$$

# Fermi theory $\rightarrow$ HQET $\times$ SCET$_{\mathrm{I}}$



$$Q_1 = [\bar{u}\,\gamma^\mu P_L\, b]\,[\bar{\ell}\,\gamma_\mu P_L\, \nu]$$

$$E_1 = [\bar{u}\,\gamma^\mu \gamma^\nu \gamma^\rho P_L\, b]\,[\bar{\ell}\,\gamma_\mu \gamma_\nu \gamma_\rho P_L\, \nu] - 16\, Q_1$$

evanescent operator

$$\mathcal{O}^{(9)}_{A,1} = m_\ell\, [\bar{q}_s \frac{\not{n}_+}{2} P_L\, h_v]\,[\bar{\ell}_{hc} \frac{1}{i\, n_+ \overleftarrow{\partial}_{hc}} P_L\, \nu_{\overline{hc}}]$$

$$\mathcal{O}_E = [\bar{q}_s \frac{\not{n}_+}{2} \gamma^\mu_\perp \gamma^\nu_\perp P_L\, h_v]\,[\bar{\ell}_{hc}\, \gamma_{\perp\nu} \gamma_{\perp\mu} P_L\, \nu_{\overline{hc}}]$$

$$\langle Q_1 \rangle = \langle \mathcal{O}_{A,1} \rangle$$

$$\langle E_1 \rangle = 6\,(D-4)\,\langle \mathcal{O}_{A,1} \rangle - 3\,\langle \mathcal{O}_E \rangle \sim \mathcal{O}(\epsilon)\, A^{(0)}_{E1,A1}$$

$$H^{(1)}_{A,1} = A^{(1)}_{1,(A,1)} + Z^{(1)}_{ext}\, A^{(0)}_{1,(A,1)} + Z^{(1)}_{(A,1)j}\, A^{(0)}_{j,(A,1)} - H^{(0)}_{A,1}\,(\mu_b)\, Z^{(1)}_{(A,1)(A,1)}$$

$$Z_{A1,E1} = \frac{1}{2\,\epsilon}\, Q_\ell\, (Q_\ell + 2\, Q_u)$$

$$Y(x,y) = \exp\left[i\, e\, Q_q \int_y^x dz_\mu\, A_s^\mu(z)\right] \mathcal{P} \exp\left[i\, g_s \int_y^x dz_\mu\, G_s^\mu(z)\right],$$

$$Y_\pm(x) = \exp\left[-i\, e\, Q_\ell \int_0^\infty ds\, n_\mp A_s\,(x + s n_\mp)\right].$$

$$S_r^{'(i)}(x) \;=\; \exp\left[-i\, e\, Q_i \int_0^\infty ds\, r \cdot A_{us}\,(x + s \cdot r)\right]$$

$$C_r^{'(i)}(x) \;=\; \exp\left[-i\, e\, Q_i \int_0^\infty ds\, r \cdot A_{usc}\,(x + s \cdot r)\right]$$

# Why QED corrections in flavor physics ?

- Compared to QCD corrections, QED corrections can lead to certain short-distance contribution which can mimic new physics.

  - e.g. QED corrections to $B_u \to \tau\nu$ have effects on $|V_{ub}|$ extraction

- B decay modes receive large uncertainties from the B-LCDA, QED and QCD at low energies introduce new LCDA.

- Theoretic calculation on soft photon radiation to consistent with phenomenological analyses of experimental data.

  - Soft real photons contribution can be simulated with MC tools such as PHOTOS in *[P. Golonka, Z. Was, hep-ph/0506026 ]*

# Summary

- QED corrections are important in high precision process

- Charged leptonic B decays become as complicated as non-leptonic decays

- Subleading power factorization formula for QED corrections to $B_u \to \tau\nu$ derived in SCET, HQET and bHLET

  - no endpoint divergences in this factorization at NLP (due to tau mass) => subtractions scheme independence

  - Structure depended QED corrections arising from hard, hard-collinear photons exchange $\to$ important source of large logarithmic corrections

  - Soft photon still resolve the B meson structure ( decoupling from leptonic field) to produce generalized $B$ decay constant and LCDA $\to$ new hadronic parameters

  - NLP+NLO+LL QED virtual correction changes the branching fraction by $\sim 1\ \%$ , are critical for $|V_{ub}|$ determination.



**Thank you**