



上海交通大学
Shanghai Jiao Tong University

基于第一性原理的粲重子 衰变常数计算

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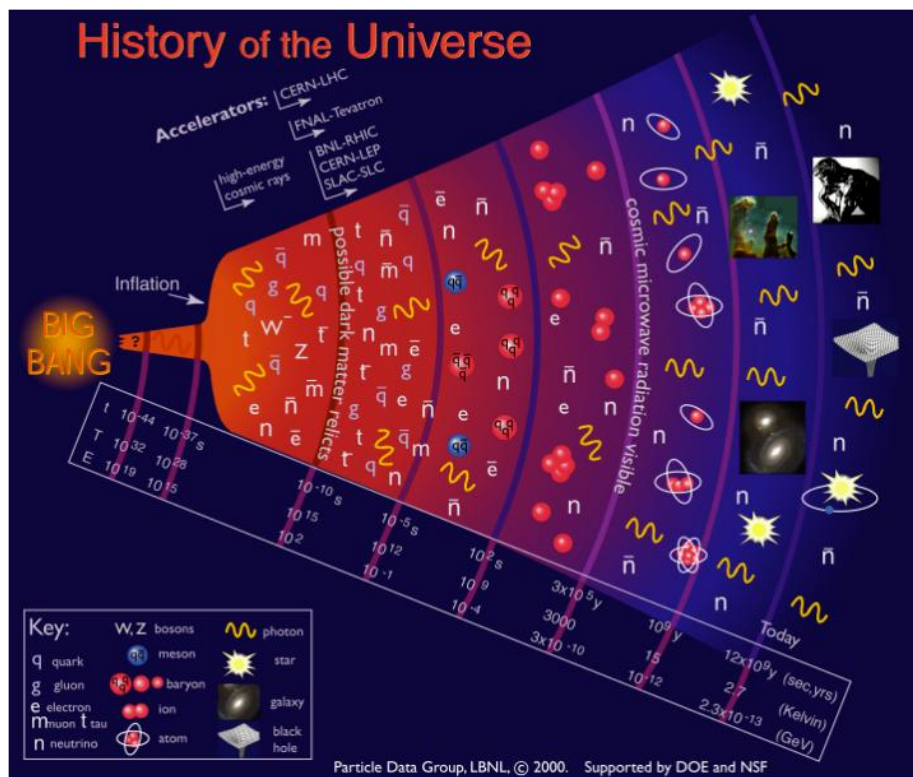
二、衰变矩阵元的计算

三、重整化常数的计算

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一、研究背景：宇宙中正反物质不对称之谜

Particle Data Group, LBNL © 2000



萨哈罗夫三条件:

1、C与CP破坏

2、重子数不守恒

3、热平衡的偏离

粲重子相关

A. D. Sakharov, Pisma Zh. Eksp. Teor. Fiz. 5, 32 (1967)

图1、宇宙的历史

1、C与CP破坏

半轻衰变过程可以抽取CKM矩阵元 V_{cb} :

$$\text{Br}(\Lambda_b^0 \rightarrow \Lambda_c^+ l \bar{\nu}) = 6.2_{-1.3}^{+1.4} \%$$

两体非轻衰变过程可以用来寻找新的CP破坏:

$$\text{Br}(\Lambda_b^0 \rightarrow \Lambda_c^+ \pi^-) = 6.7_{-2.2}^{+3.2} {}_{-2.8}^{+0.3+2.5} \%$$

$\Lambda_b^0 \rightarrow \Lambda_c^+$ 形状因子的因子化:

$$\langle \Lambda_c | iA | \Lambda_b \rangle \sim C \otimes T \otimes \underbrace{\langle \Lambda_c | O_{\Lambda_c} | 0 \rangle}_{f_{\Lambda_c} \phi_{\Lambda_c}} \otimes \langle 0 | O_{\Lambda_b} | \Lambda_b \rangle$$

粲重子衰变常数的定义:

$$\langle 0 | \epsilon_{ijk} (q_{1,i}^T C \Gamma_1 q_{2,j}) \Gamma_2 c_{3,k} | \mathcal{B}_c \rangle = m_{\mathcal{B}_c} f_{\mathcal{B}_c} u_{\mathcal{B}_c}$$

Y.Miao, Y.L. Shen, et al. Chin. Phys. C 46, no.11, 113107 (2022)

C. Q. Zhang, Z. Rui, et al. Phys. Rev. D 105, 073005 (2022)

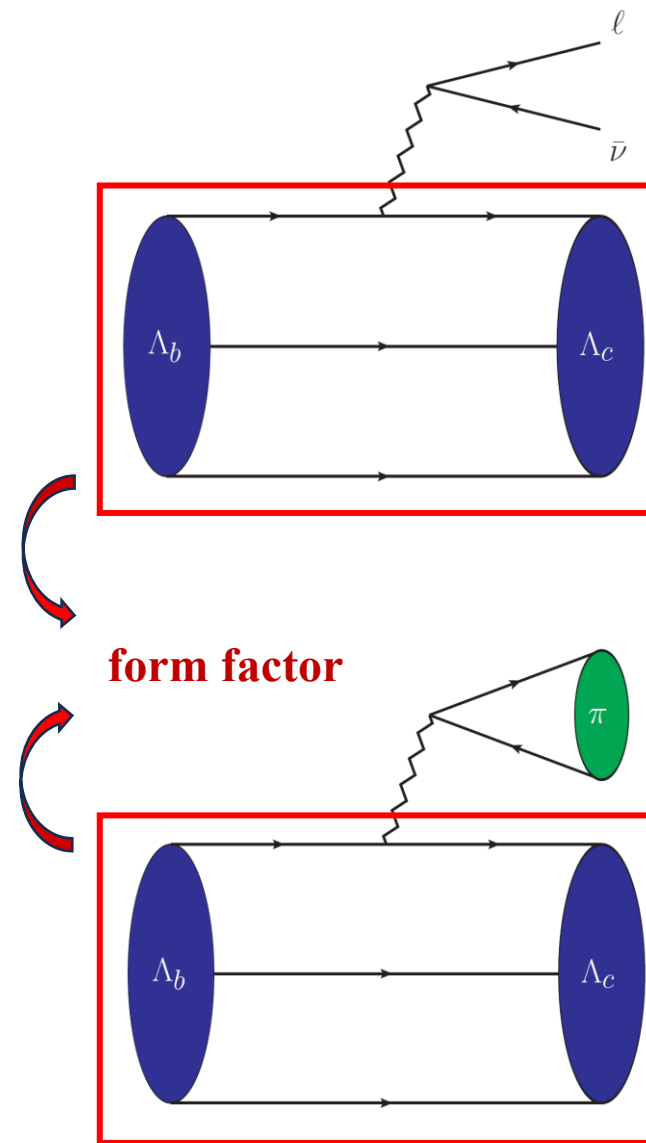


图2. 两体非轻和半轻衰变

2、重子数不守恒

P. del Amo Sanchez, et al. [BaBar],
Phys. Rev. D 83, 091101 (2011)

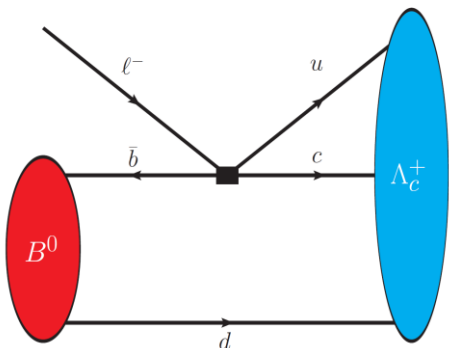


图3、大统一: $B^0 \rightarrow \Lambda_c^+ l^-$

C.O.Dib, et al. JHEP 02, 224 (2023)

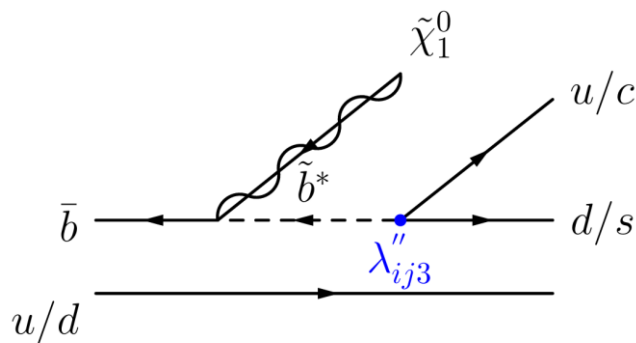


图4、超对称: $B^+ \rightarrow \Lambda_c^+ \tilde{\chi}_1^0$

J.P.Lees, et al. [BaBar],
Phys. Rev. D 111, L031101 (2025)

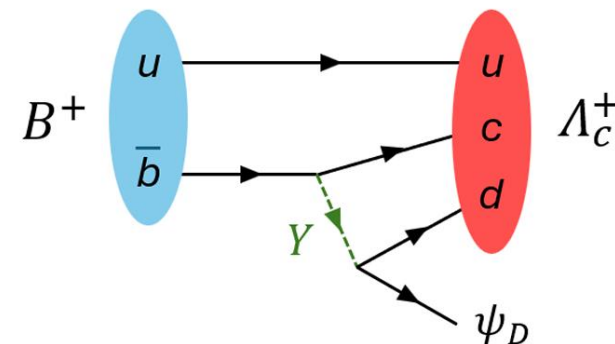


图5、重子生成模型: $B^+ \rightarrow \Lambda_c^+ \psi_D$

新物理形状因子 $B \rightarrow \Lambda_c^+$ 的因子化:

$$\langle \Lambda_c | iA | B \rangle \sim C_{new} \otimes T \otimes \underbrace{\langle \Lambda_c | O_{\Lambda_c} | 0 \rangle}_{f_{\Lambda_c} \phi_{\Lambda_c}} \otimes \langle 0 | O_B | B \rangle$$

实验 + 形状因子 $\rightarrow$ 新物理模型参数进行限制

非微扰输入: 粲重子衰变常数

3、粲重子衰变常数研究现状

目前只有QCD sum rule 给出结果:

Baryon	Decay constants (10^{-2}GeV^2)				
	Λ_c^+	Ξ_c	Ξ'_c	Σ_c	Ω_c^0
Z.G.Wang 【1】	-	-	2.13(62)	1.83(61)	3.45(85)
Z.G.Wang 【2】	0.96(35)	1.09(32)	-	-	-
A.Khodjamirian et al. 【3】	$1.19^{+0.19}_{-0.28}$	-	-	$3.08^{+0.49}_{-0.74}$	-
Y.J.Shi et al. 【4】	-	-	-	-	3.30(48)
Z.X.Zhao et al. 【5】	0.50(4)	0.93(4)	-	-	-

【1】 Z.G.Wang, Phys. Lett. B 685, 59 (2010),

【2】 Z.G.Wang, Eur. Phys. J. C 68, 479 (2010),

【3】 A. Khodjamirian, Y. M. Wang, et al. JHEP 09, 106,

【4】 Y.J. Shi, W. Wang, and Z. X. Zhao, Eur. Phys. J. C 80,568 (2020).

【5】 Z.X.Zhao, R. H. Li, Y. L.Shen, Y.J.Shi and Y.S.Yang, Eur. Phys. J. C 80, no.12, 1181 (2020)

精度在15%-20%

第一性原理的计算是有必要的

二、衰变矩阵元计算：抽取粲重子裸衰变常数

1、粲重子算符构造

di-quark的SU(3) 对称性:
 $3 \otimes 3 = \bar{3} \oplus 6$

$$\bar{3} : \begin{cases} O_{\Lambda_c^+} = \epsilon_{ijk}(u_i^T C \gamma_5 d_j) c_k, \\ O_{\Xi_c^+} = \epsilon_{ijk}(u_i^T C \gamma_5 s_j) c_k, \\ O_{\Xi_c^0} = \epsilon_{ijk}(d_i^T C \gamma_5 s_j) c_k, \end{cases} \quad 6 : \begin{cases} O_{\Xi_c'^+} = \epsilon_{ijk}(u_i^T C \gamma_\mu s_j) \gamma^\mu \gamma_5 c_k, \\ O_{\Xi_c'^0} = \epsilon_{ijk}(d_i^T C \gamma_\mu s_j) \gamma^\mu \gamma_5 c_k, \\ O_{\Sigma_c^+} = \epsilon_{ijk}(u_i^T C \gamma_\mu d_j) \gamma^\mu \gamma_5 c_k, \\ O_{\Sigma_c^0} = \frac{1}{\sqrt{2}} \epsilon_{ijk}(d_i^T C \gamma_\mu d_j) \gamma^\mu \gamma_5 c_k, \\ O_{\Sigma_c^{++}} = \frac{1}{\sqrt{2}} \epsilon_{ijk}(u_i^T C \gamma_\mu u_j) \gamma^\mu \gamma_5 c_k, \\ O_{\Omega_c^0} = \frac{1}{\sqrt{2}} \epsilon_{ijk}(s_i^T C \gamma_\mu s_j) \gamma^\mu \gamma_5 c_k. \end{cases}$$

赝标流

矢量流

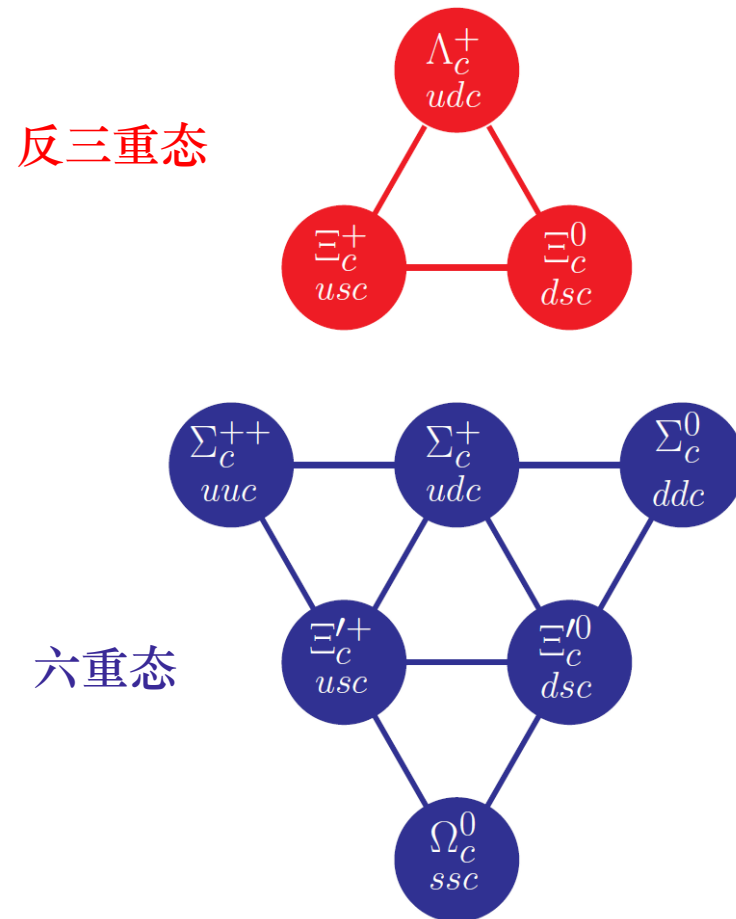


图6. 粲重子算符根据SU(3)对称性分类

A. Khodjamirian and Y.M.Wang, et al. JHEP 09, 106 (2011).

H. Liu, W. Wang, and Q.A. Zhang et al. Phys. Lett. B 841, 137941(2023)

2、两点关联函数计算

两点关联函数: $C_{2pt}(t) = \sum_x e^{-ip \cdot x} \langle 0 | O_{B_c}(x), \bar{O}_{B_c}(0) | 0 \rangle$

正宇称投影: $C_{2pt}^+(t) = \text{Tr}\{C_{2pt}(t)P_+\}$

插入强子完备集: $1 = \int \frac{d^3 \vec{p}}{(2\pi)^3} \frac{|n(p)\rangle \langle n(p)|}{2E_n}$

根据衰变常数的定义: $\langle 0 | O_{B_c} | B_c \rangle = m_{B_c} f_{B_c} u_{B_c}$

强子层次的关联函数进行参数化:

$$C_{2pt}^+(t) = 2m_{B_c}^2 f_{B_c}^{(0)2} e^{-m_{B_c} \cdot t} (1 + \Delta C \cdot e^{\Delta m \cdot t})$$

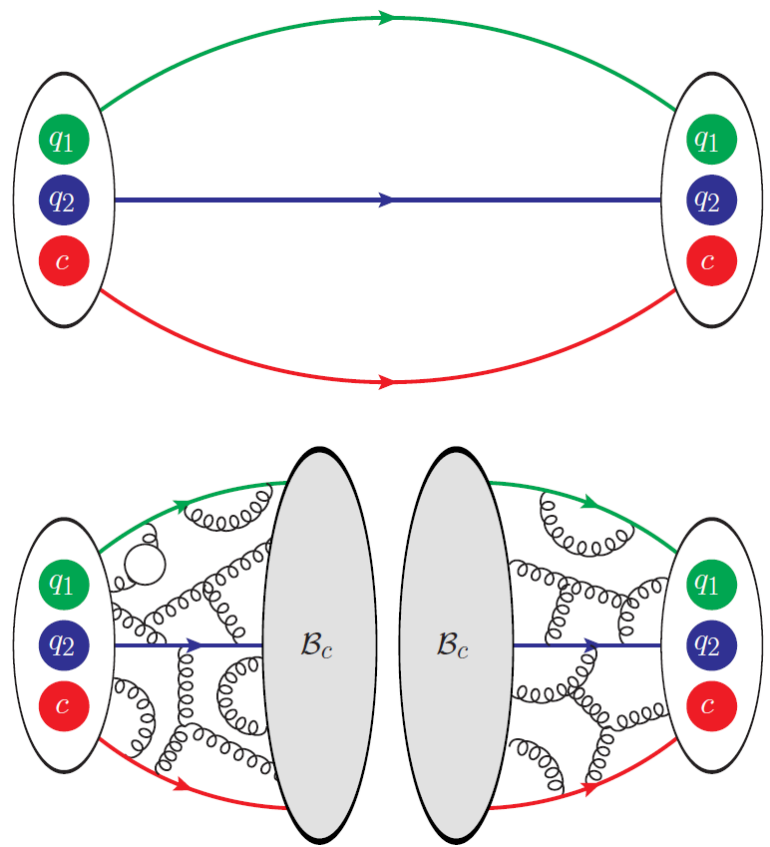


图7、夸克与强子层次的两点关联函数

3、格点set-up

CLQCD合作组产生的组态：

格距： [0.05, 0.105] fm → 连续极限外推

Pion： [135, 320] MeV → 物理质量外推



表1、CLQCD组态信息

Ensemble	$L^3 \times T$	a (fm)	Pion (MeV)	Nconf	Nsrc
C24P29	$24^3 \times 72$	0.105	290	864	2×10
C32P29	$32^3 \times 64$	0.105	290	984	2×10
C48P14	$48^3 \times 96$	0.105	135	187	2×20
C32P23	$32^3 \times 64$	0.105	230	451	2×10
F32P30	$32^3 \times 96$	0.077	300	777	2×10
H48P32	$48^3 \times 144$	0.052	320	550	2×6

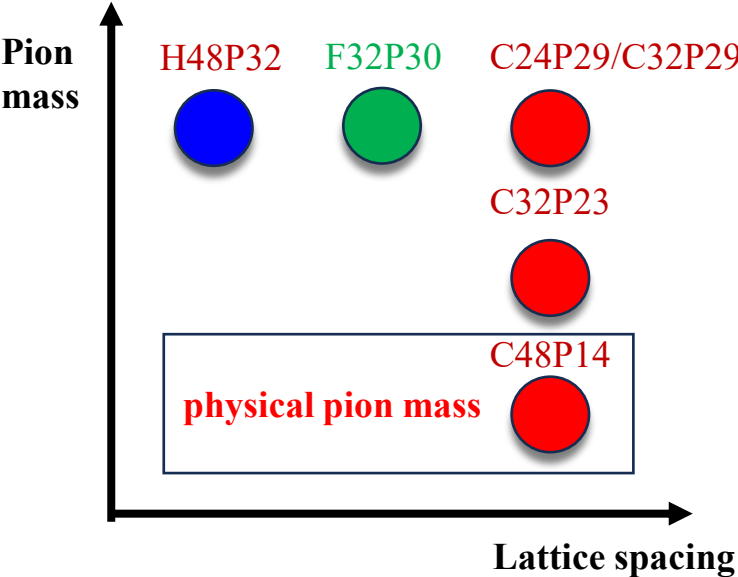


图8、CLQCD组态

4、有效质量

Ξ_c 和 Ξ'_c 的差别在洛伦兹结构:

$$O_{\Xi_c} = \epsilon_{ijk} (l_i^T C \gamma_5 s_j) c_k$$

$$O_{\Xi'_c} = \epsilon_{ijk} (l_i^T C \gamma_\mu s_j) \gamma_5 \gamma^\mu c_k$$

SU(3)对称性的影响会体现在质量平台

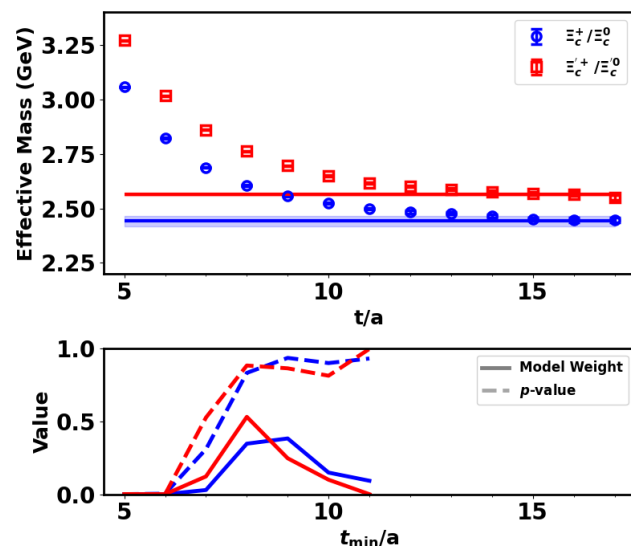


图9. Ξ_c 和 Ξ'_c 的有效质量图

两态拟合抽取有效质量和裸的衰变常数:

$$C_{2pt}^+(t) = 2m_{B_c}^2 f_{B_c}^{(0)2} e^{-m_{B_c} \cdot t} (1 + \Delta C \cdot e^{\Delta m \cdot t})$$

接下来手征极限外推和连续极限外推:

$$m_{B_c}(m_\pi, a) = m_{B_c, phy} + c_1(m_\pi^2 - m_{\pi, phy}^2) + c_2 a^2$$

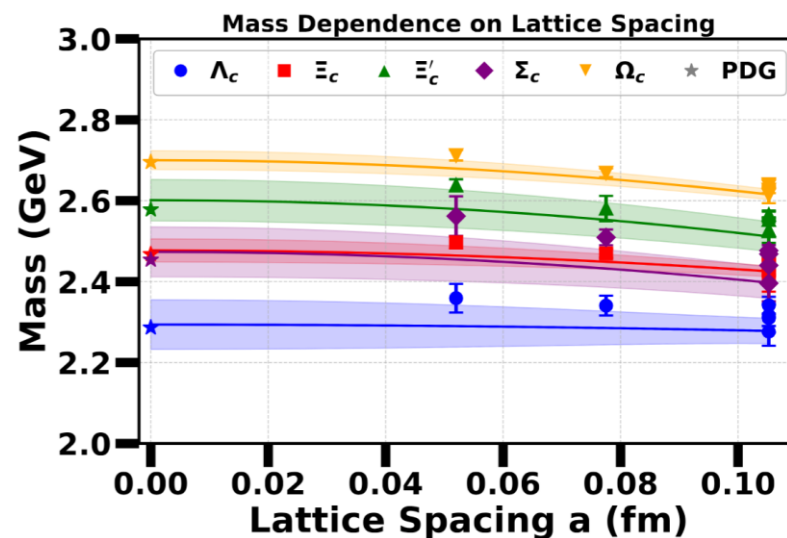


图10. 质量外推图

表2、粲重子裸的衰变常数汇总

	Decay constant (bare) (GeV^2)					
Baryon	C24P29	C32P23	C32P29	C48P14	F32P30	H48P32
Λ_c	0.0183(12)	0.0178(23)	0.0173(19)	0.0175(19)	0.0189(15)	0.0205(25)
Ξ_c	0.0170(19)	0.0167(21)	0.01782(77)	0.01709(90)	0.01957(81)	0.0220(11)
Ξ'_c	0.02656(73)	0.0232(36)	0.02547(54)	0.0240(36)	0.0279(38)	0.0348(17)
Σ_c	0.0246(11)	0.0223(33)	0.0241(11)	0.0202(32)	0.0272(18)	0.0329(55)
Ω_c	0.0274(15)	0.0263(11)	0.02679(60)	0.0276(18)	0.0309(19)	0.0365(21)

三、重整化常数：非微扰计算

格点中重子重整化的研究出现在质子衰变：

J. S. Yoo, et al, Phys. Rev. D 105, 074501 (2022).

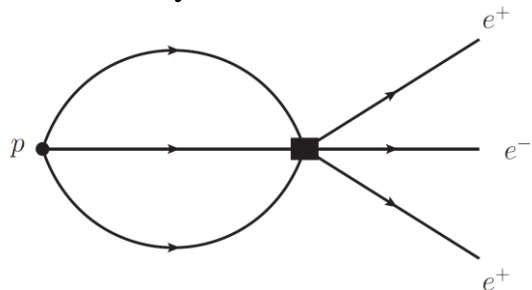


图11、 $p \rightarrow e^+e^-e^+$ 两点关联函数

推广到粲重子衰变常数的研究：

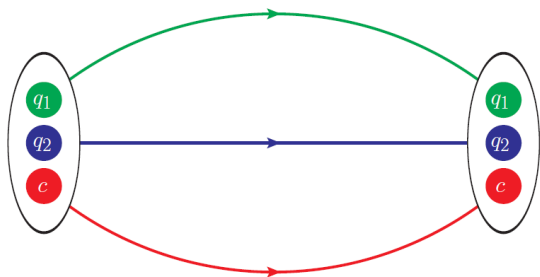


图12、粲重子两点关联函数

粲重子算符重整化常数为：

$$O = Z_{O_i} O^{(0)}$$

非微扰重整化条件：

$$Z_{O_i} Z_{q1}^{-1/2} Z_{q2}^{-1/2} Z_{q3}^{-1/2} \Gamma_{O_i} |_{\text{SMOM}_{\gamma_\mu}/\text{SYM3q}} = 1$$

夸克场重整化常数 顶点函数

SMOM_{γ_μ}：对称动量减除方案

SYM3q：三夸克对称动量减除方案

MOM3q vs SYM3q:

$$[\Lambda_{3q}^{\overline{\text{MS}}}]_{\text{MOM3q}} = 1 + (-4.06) \frac{\alpha_s}{4\pi} + O(\alpha_s^2)$$

$$[\Lambda_{3q}^{\overline{\text{MS}}}]_{\text{SYM3q}} = 1 + (0.989) \frac{\alpha_s}{4\pi} + O(\alpha_s^2)$$

J. S. Yoo, et al, Phys. Rev. D 105, 074501 (2022).

1、biliner算符顶点函数

Z_q 的计算 J. S. Yoo, et al. Phys. Rev. D 105, 074501 (2022):

$$Z_V \Gamma_V Z_q^{-1} |_{\text{SMOM}_{\gamma_\mu}} = 1$$

得到**矢量流**顶点函数：

$$\Gamma_V = \frac{1}{48} \text{Tr} \{ S_{in}^{-1}(p_2) G_V^\mu(p_1, p_2) S_{out}^{-1}(p_1) \gamma_\mu \}.$$

biliner算符： $O_V^\mu = \bar{\psi}_1 \gamma^\mu \psi_2$ 动量空间的格林函数：

$$G_V^\mu(p_1, p_2) = \langle \psi_1(p_1) O_V^\mu \bar{\psi}_2(p_1) \rangle = \frac{1}{N} S_{out}(p_1) \gamma^\mu S_{in}(p_2)$$

其中：

$$S_{out}(p_1) = \frac{1}{N} \sum_x e^{-ip_1 \cdot x} \langle \psi(x) \bar{\psi}(0) \rangle$$

$$S_{in}(p_2) = \frac{1}{N} \left(\gamma_5 \sum_x e^{-ip_2 \cdot x} \langle \psi(x) \bar{\psi}(0) \rangle \gamma_5 \right)^\dagger$$

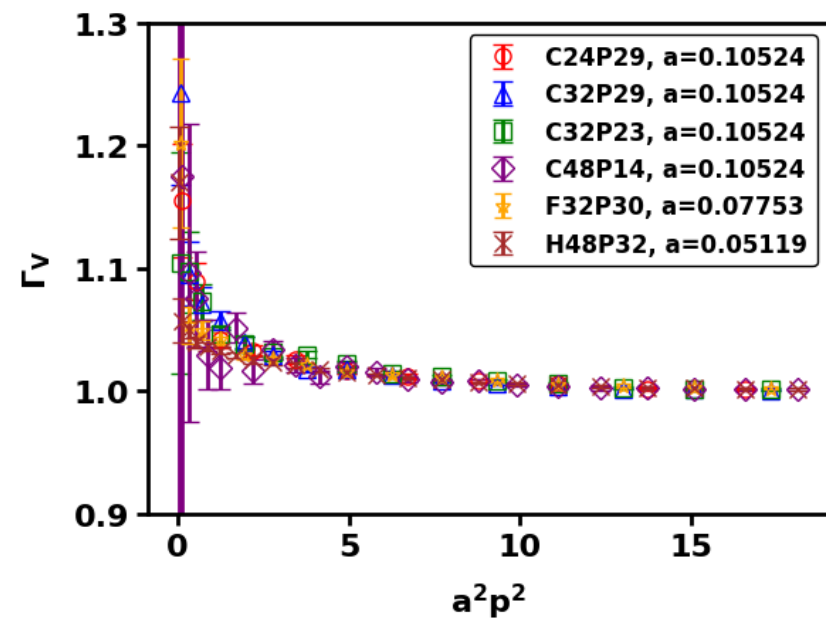


图13. 矢量流顶点函数 Γ_V

2、重子算符顶点函数

定义动量空间的**格林函数**:

$$G(x, y_1, y_2, y_3) = \langle O(x), \bar{q}_1(y_1) \bar{q}_2(y_2) \bar{q}_3(y_3) \rangle$$

动量空间的**截断格林函数**:

$$\Lambda(p_1, p_2, p_3) = G(p_1, p_2, p_3) S^{-1}(p_1) S^{-1}(p_2) S^{-1}(p_3)$$

粲重子**顶点函数**:

$$\Gamma(p_1, p_2, p_3) = (P_i)_{\beta\alpha\delta\gamma}^{ijk} \Lambda_{\alpha\beta\gamma\delta}^{ijk}(p_1, p_2, p_3)$$

投影算符:

$$(P_1)_{\beta\alpha\delta\gamma}^{ijk} = \frac{1}{96} \epsilon_{ijk} (\gamma_5 C)_{\beta\alpha} (1)_{\delta\gamma},$$

$$(P_2)_{\beta\alpha\delta\gamma}^{ijk} = \frac{1}{384} \epsilon_{ijk} (\gamma_\mu C)_{\beta\alpha} (\gamma_5 \gamma_\mu)_{\delta\gamma},$$

$$(P_3)_{\beta\alpha\delta\gamma}^{ijk} = \frac{1}{384\sqrt{2}} \epsilon_{ijk} (\gamma_\mu C)_{\beta\alpha} (\gamma_5 \gamma_\mu)_{\delta\gamma},$$

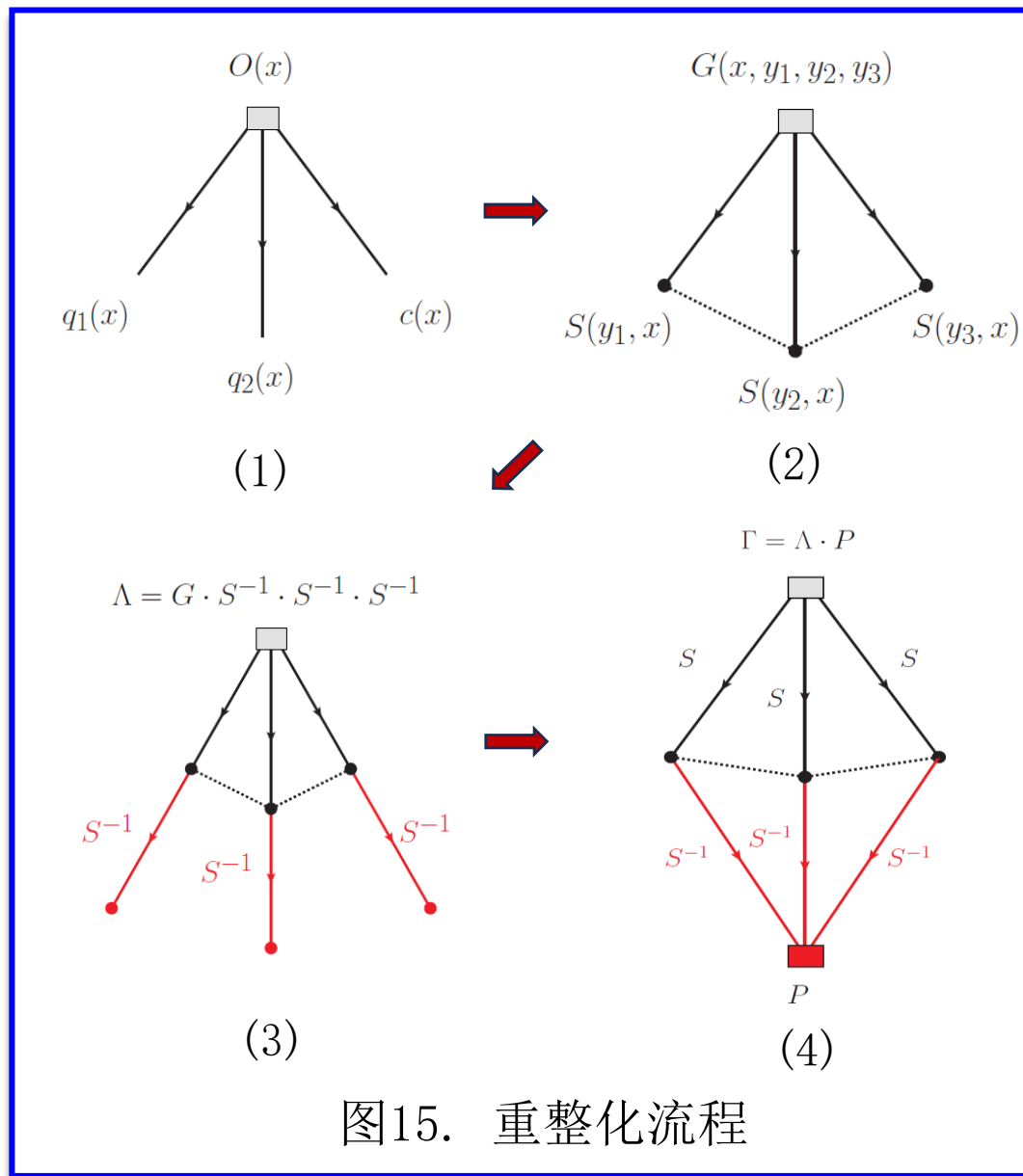


图15. 重整化流程

3、非微扰重整化方案

动量设置:

介子: SMOM_{γ_μ}

动量选择: $p_1 = \frac{2\pi}{L}(n, n, 0, 0)$, $p_2 = \frac{2\pi}{L}(n, 0, n, 0)$

重整化条件: $p_1^2 = p_2^2 = (p_1 - p_2)^2 = \mu^2$

重子: SYM3q

动量选择: $ap_1 = \frac{2\pi}{L}(n, n, 0, 0)$, $ap_2 = \frac{2\pi}{L}(-n, 0, n, 0)$,

$$ap_3 = \frac{2\pi}{L}(-n, 0, -n, 0)$$

重整化条件: $p_1^2 = p_2^2 = p_3^2 = \mu^2$, $p_1 + p_2 + p_3 = 0$

质量设置:

微扰: 转换因子计算忽略质量

格点: 算符采用轻夸克质量

再外推到零质量极限

$$O_1 = \epsilon_{ijk}(l_1^T C \gamma_5 l_2) l_3$$

$$O_2 = \epsilon_{ijk}(l_1^T C \gamma_\mu l_2) \gamma_\mu \gamma_5 l_3$$

$$O_3 = \epsilon_{ijk}(l_1^T C \gamma_\mu l_1) \gamma_\mu \gamma_5 l_3$$

粲重子算符九重态简并成三种:

$$O_1 = Z_{O1} O_1^{(0)}$$

$$O_2 = Z_{O2} O_2^{(0)}$$

$$O_3 = Z_{O3} O_3^{(0)}$$

4、非微扰重整化常数计算

矢量流算符 $O = \bar{\psi}_1 \gamma^\mu \psi_2$

重整化条件: $Z_V Z_q^{-1} \Gamma_V |_{\text{SMOM}_{\gamma_\mu}} = 1$,

顶点算符:

$$\Gamma_V = \frac{1}{48} \text{Tr} \{ \Lambda_\mu \gamma^\mu \}.$$

截腿格林函数 投影算符

$$Z_V = \frac{Z_q}{\Gamma_V},$$

重子流: $O = \epsilon_{ijk} (q_i^T C \Gamma_1 q_j) \Gamma_2 c_k$ 算符

重整化条件: $Z_{O_i} Z_q^{-3/2} \Gamma_{O_i} |_{\text{SYM3q}} = 1$

顶点算符:

$$\Gamma_{O_i}(p_1, p_2, p_3) = \Lambda_{xy,cz}^{\alpha\beta\gamma}(p_1, p_2, p_3) * P_{yx,zc}^{\alpha\beta\gamma}$$

截腿格林函数 投影算符

$$Z_{O_i} = \frac{Z_q^{3/2}}{\Gamma_{O_i}}$$

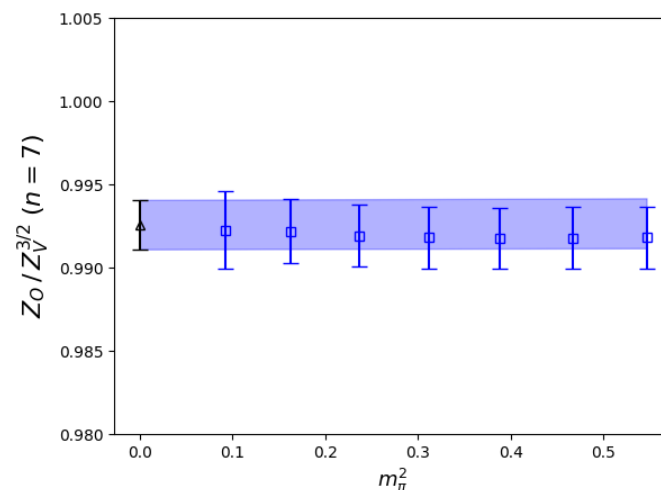
计算重整化常数的比值:

$$\frac{Z_{O_i}}{Z_V^{3/2}} |_{\text{SMOM}_{\gamma_\mu}/\text{SYM3q}} = \frac{\Gamma_V^{3/2}}{\Gamma_{O_i}} |_{\text{SMOM}_{\gamma_\mu}/\text{SYM3q}}$$

零质量极限外推公式:

$$\frac{Z_{O_i}}{Z_V^{3/2}}(m_q, a^2 p^2) = \frac{\Gamma_V^{3/2}}{\Gamma_{O_i}}(m_q, a^2 p^2) = \underline{A} + B a m_q$$

$m_q \sim m_\pi^2$



SMOM_{γ_μ}/SYM3q → $\overline{\text{MS}}$ 转换

转换到 $\overline{\text{MS}}$ scheme: $\frac{Z_{Oi}}{Z_V^{3/2}}(\mu, a^2 p^2) = \lim_{m_q \rightarrow 0} C^{\overline{\text{MS}} \leftarrow \text{SMOM}_{\gamma_\mu}/\text{SYM3q}} \frac{Z_{Oi}}{Z_V^{3/2}}$

粲重子算符转换因子: $C^{\overline{\text{MS}} \leftarrow \text{SMOM}_{\gamma_\mu}/\text{SYM3q}}(\mu) = [\Lambda^{\overline{\text{MS}}}]_{\text{SYM3q}} \left(C_q^{\overline{\text{MS}} \leftarrow \text{SMOM}_{\gamma_\mu}} \right)^{-3/2}$

夸克场的转换因子: $C_q^{\overline{\text{MS}} \leftarrow \text{SMOM}_{\gamma_\mu}} = 1 + \frac{4}{3} \left(\frac{\alpha_s}{4\pi} \right) + O(\alpha_s^2)$

赝标流算符的截腿格林函数: $[\Lambda^{\overline{\text{MS}}}]_{\text{SYM3q}} = 1 + 0.989426 \left(\frac{\alpha_s}{4\pi} \right) + O(\alpha_s^2)$

矢量流算符, 根据Fierz变换公式: $\epsilon_{ijk}(u_1^T C \gamma^\mu u_2) \gamma_5 \gamma_\mu d_3 = 2[\epsilon_{ijk}(u_1^T C u_2) d_3 - \epsilon_{ijk}(u_1^T C \gamma_5 u_2) \gamma_5 d_3]$

矢量流算符与赝标流算符一致。

动量外推

乘上转换因子并演化到 $\mu=2\text{GeV}$, a^2p^2 外推:

$$\frac{Z_{oi}(2\text{GeV}, a^2p^2)}{Z_V^{3/2}} = \boxed{\frac{Z_{oi}(2\text{GeV})}{Z_V^{3/2}}} + \frac{c_{-1}}{a^2p^2} + c_1(a^2p^2) + c_2(a^2p^2)^2$$

粲重子重整化常数: $Z_{oi}(2\text{GeV}) = \frac{Z_{oi}(2\text{GeV})}{Z_V^{3/2}} (Z_V^{q_1} Z_V^{q_2} Z_V^c)^{1/2}$

Z_V^l, Z_V^s, Z_V^c : H.Y. Du, Y. B. Yang et al. Phys. Rev. D 111, 054504 (2025).

表3. $\mu=2\text{GeV}$ 能标下的粲重子算符重整化常数

Ensemble	$Z_{\Lambda_c}^{\text{MS}}$	$Z_{\Xi_c}^{\text{MS}}$	$Z_{\Xi'_c}^{\text{MS}}$	$Z_{\Sigma_c}^{\text{MS}}$	$Z_{\Omega_c}^{\text{MS}}$
C24P29	0.9277(82)	0.9584(84)	0.9510(85)	0.9205(82)	0.9824(88)
C32P29	0.9252(91)	0.9557(95)	0.963(10)	0.9319(98)	0.994(10)
C32P23	0.9477(85)	0.9791(88)	0.9701(84)	0.9389(81)	1.0023(86)
C48P14	0.9481(86)	0.9796(89)	0.9783(80)	0.9469(77)	1.0108(83)
F32P30	0.8908(56)	0.9085(57)	0.9048(51)	0.8872(50)	0.9228(52)
H48P32	0.8400(27)	0.8492(28)	0.8513(26)	0.8420(26)	0.8607(27)

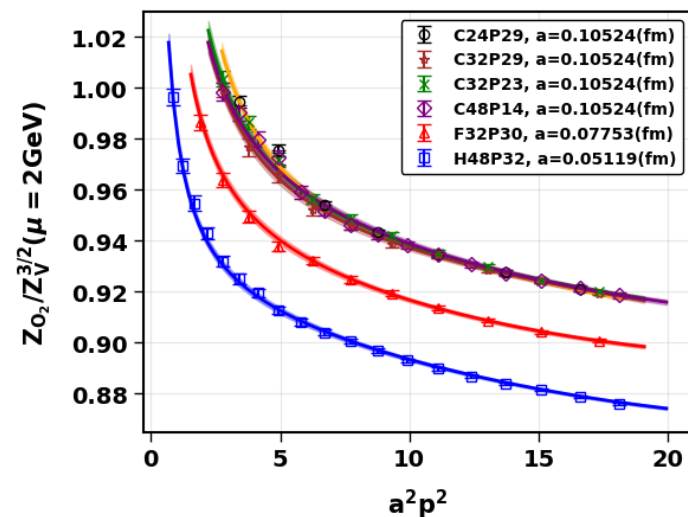
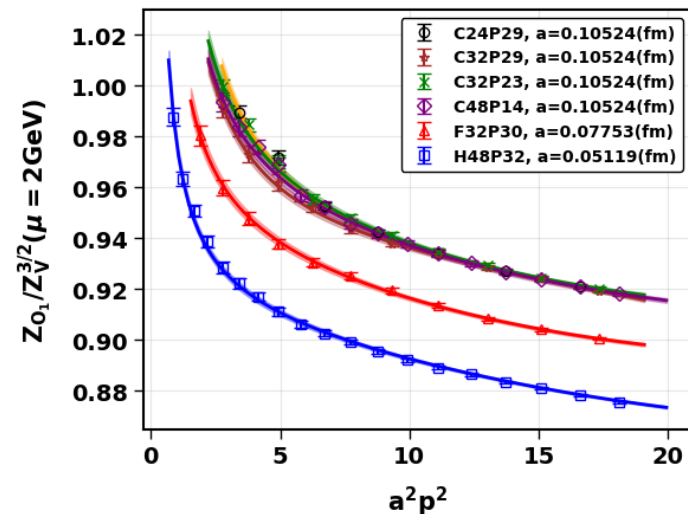


图16、重整化常数比值 a^2p^2 外推

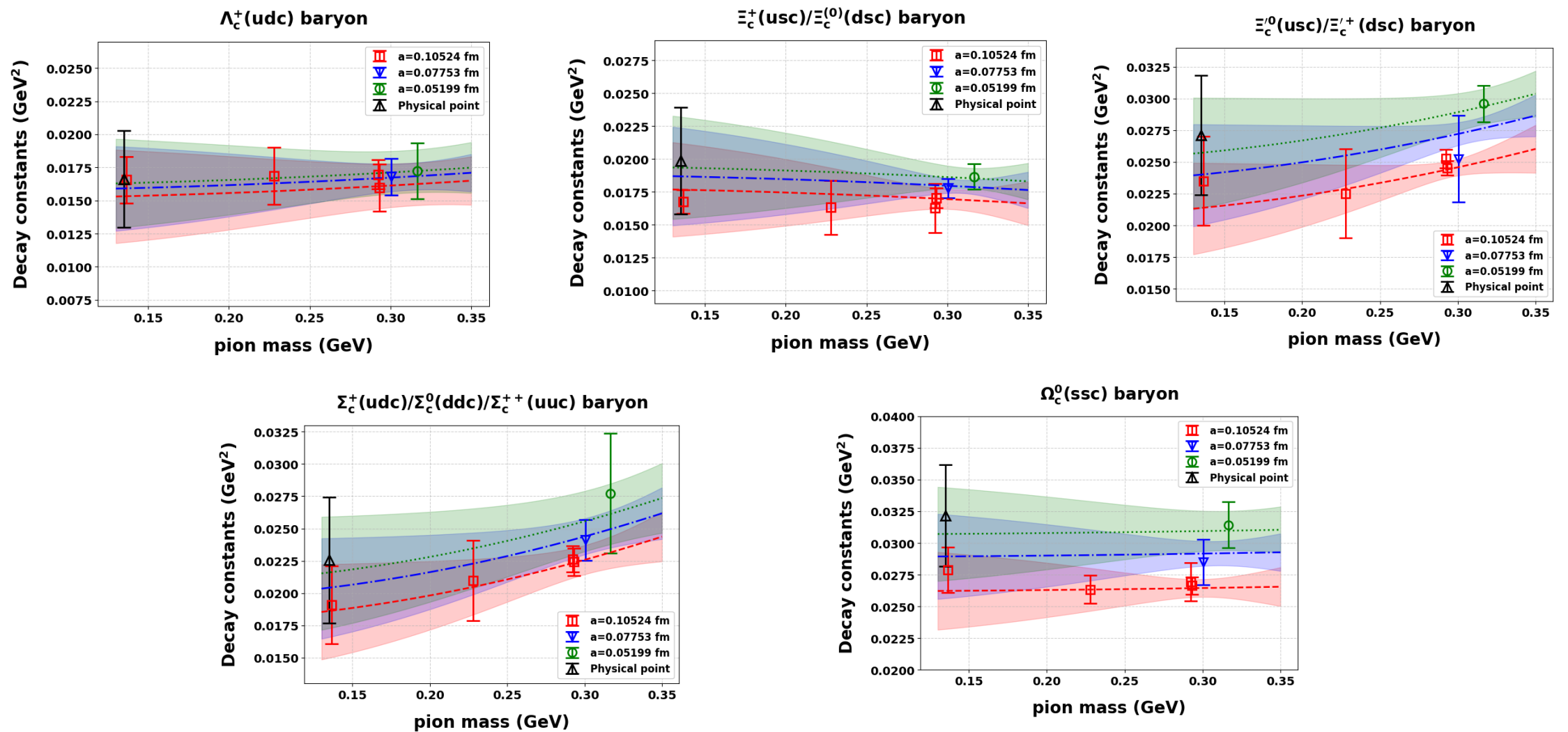


图18. 粲重子衰变常数外推

外推公式: $f_B(m_\pi, a) = f_{B,phy} + c_1(m_\pi^2 - m_{\pi,phy}^2) + c_2 a^2$

4、数值结果

$\mu=2\text{GeV}$ 能标下的粲重子衰变常数:

$$f_{\Lambda_c} = 1.71(35)_{\text{stat.}}(1)_{\text{syst.}} \times 10^{-2} \text{ GeV}^2,$$

$$f_{\Xi_c} = 1.89(20)_{\text{stat.}}(2)_{\text{syst.}} \times 10^{-2} \text{ GeV}^2,$$

$$f_{\Xi'_c} = 2.82(45)_{\text{stat.}}(2)_{\text{syst.}} \times 10^{-2} \text{ GeV}^2,$$

$$f_{\Sigma_c} = 2.27(48)_{\text{stat.}}(3)_{\text{syst.}} \times 10^{-2} \text{ GeV}^2,$$

$$f_{\Omega_c} = 3.01(33)_{\text{stat.}}(6)_{\text{syst.}} \times 10^{-2} \text{ GeV}^2.$$

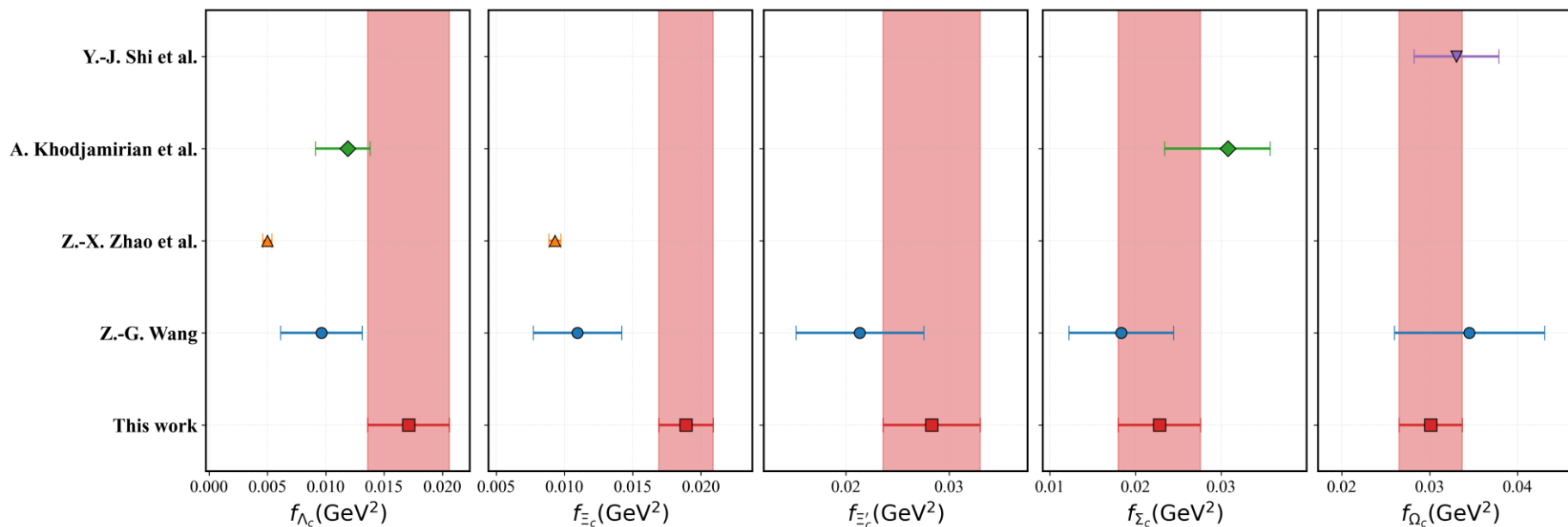


图19. 格点计算与QCD sum rule对比

格点计算的粲重子衰变常数的精度为11~21%

四、总结

- 首次基于第一性原理计算了粲重子衰变常数
- 对粲重子算符进行重整化计算
- 进行连续极限和物理质量外推，得到物理点的粲重子衰变常数

展望：

- 计算不同洛伦兹结构的粲重子算符
- 重味重子弱衰变的第一性原理计算

谢谢！