Polarization Probes of New Physics in Lepton-Flavor-Violating Hyperon Production from $e^- N \to \tau^- Y$ Scattering

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Contents

Motivation

Lepton flavor violation serves as one of the most important avenues for probing new physics. In this work, the focus is on processes such as $e^- \longrightarrow \tau^-$, especially $e^- d \longrightarrow \tau^- s$.

- **High-energy regime:**
  - LHC. *EPJC* **80**, 641 (2020)
  - In the future, EIC. *JHEP* **03**, 256 (2021)

- **Low-energy regime:**
  - $\tau^- \longrightarrow e^- K_S$ *PLB* **692**, 4 (2010)
  - $\tau^- \longrightarrow e^- K^{*0}$ *JHEP* **06**, 118 (2023)
  - $\tau^- \longrightarrow e^- \pi^- K^+$ *PLB* **719**, 346 (2013)

To address this gap, the quasi-elastic (QE) scattering process $e^- + N(n, p) \longrightarrow \tau^- + Y(\Sigma^+, \Sigma^0, \Lambda^0)$ is proposed to probe the baryonic sector of these LFV interactions.

The general low-energy effective Lagrangian involving $e^- d \to \tau^- s$ transitions can be written as

$$\mathcal{L}_{\text{eff}} = -\frac{4G_F}{\sqrt{2}} \sum_{\alpha} g_\alpha \mathcal{O}_\alpha + \text{h.c.}\,,$$

where G_F is the Fermi constant, $\mathcal{O}_\alpha$ is the semi-leptonic operator listed in Table, and g_α is the corresponding effective WC.

Coeff.	Operator	Coeff.	Operator
g_V^{LL}	$(\bar{\tau}\gamma_\mu P_L e)(\bar{s}\gamma^\mu P_L d)$	g_V^{RR}	$(\bar{\tau}\gamma_\mu P_R e)(\bar{s}\gamma^\mu P_R d)$
g_V^{LR}	$(\bar{\tau}\gamma_\mu P_L e)(\bar{s}\gamma^\mu P_R d)$	g_V^{RL}	$(\bar{\tau}\gamma_\mu P_R e)(\bar{s}\gamma^\mu P_L d)$
g_S^{LL}	$(\bar{\tau} P_L e)(\bar{s} P_L d)$	g_S^{RR}	$(\bar{\tau} P_R e)(\bar{s} P_R d)$
g_S^{LR}	$(\bar{\tau} P_L e)(\bar{s} P_R d)$	g_S^{RL}	$(\bar{\tau} P_R e)(\bar{s} P_L d)$
g_T^{LL}	$(\bar{\tau}\sigma_{\mu\nu} P_L e)(\bar{s}\sigma^{\mu\nu} P_L d)$	g_T^{RR}	$(\bar{\tau}\sigma_{\mu\nu} P_R e)(\bar{s}\sigma^{\mu\nu} P_R d)$

Theoretical Framework:Polarization vectors

The production density matrix ρ can be expanded in terms of the Pauli matrices σ^i as

$$\rho_{\lambda\lambda'} = \delta_{\lambda\lambda'} C + \sum_i \sigma^i_{\lambda\lambda'} \Sigma_i = C\left(\delta_{\lambda\lambda'} + \sum_i \sigma^i_{\lambda\lambda'} P_i\right) .$$

To investigate the dependence of the polarization on the beam energy, we introduce the averaged polarization component along direction i.

$$\langle P_i^{\tau,Y} \rangle = \frac{\int_{q^2_{\text{min}}}^{q^2_{\text{max}}} P_i^{\tau,Y}(q^2) \frac{d\sigma}{dq^2} dq^2}{\int_{q^2_{\text{min}}}^{q^2_{\text{max}}} \frac{d\sigma}{dq^2} dq^2} .$$

Theoretical Framework:Two Forms of the Form Factor

Two forms of the form factor:

- Scattering process: $N \longrightarrow Y$
- Decay process: $Y \longrightarrow N$

The hadronic matrix elements $H_a^{L,R}$ for the $N \longrightarrow Y$ transitions can be parameterized in terms of form factors. For the vector and axial-vector currents, their matrix elements are given by:

$$\langle Y(p')|\bar{s}\gamma_\mu d|N(p)\rangle = \bar{u}(p')\left[\gamma_\mu f_1(q^2) + i\sigma_{\mu\nu}\frac{q^\nu}{2m_Y}f_2(q^2) + \frac{q_\mu}{m_Y}f_3(q^2)\right]u(p)\,,$$

$$\langle Y(p')|\bar{s}\gamma_\mu\gamma_5 d|N(p)\rangle = \bar{u}(p')\left[\gamma_\mu g_1(q^2) + i\sigma_{\mu\nu}\frac{q^\nu}{m_Y}g_2(q^2) + \frac{q_\mu}{m_Y}g_3(q^2)\right]\gamma_5 u(p)\,.$$

$$\langle Y(p')|\bar{s}d|N(p)\rangle = \bar{u}(p')\left[\frac{m_Y - m_N}{m_s - m_d}f_1(q^2) + \frac{q^2}{M_Y(m_s - m_d)}f_3(q^2)\right]u(p),$$

$$\langle Y(p')|\bar{s}\gamma_5 d|N(p)\rangle = \bar{u}(p')\left[\frac{m_Y + m_N}{m_d + m_s}g_1(q^2) + \frac{q^2}{M_Y(m_d + m_s)}g_3(q^2)\right]\gamma_5 u(p)\,.$$

M. S. Athar, *The Physics of Neutrino Interactions*

Theoretical Framework: Two Forms of the Form Factor

$f_i(q^2)$ are related to the form factors $G^{p,n}_{E,M}$, and $G^{p,n}_{E,M}$ can be parameterized in the following three ways:

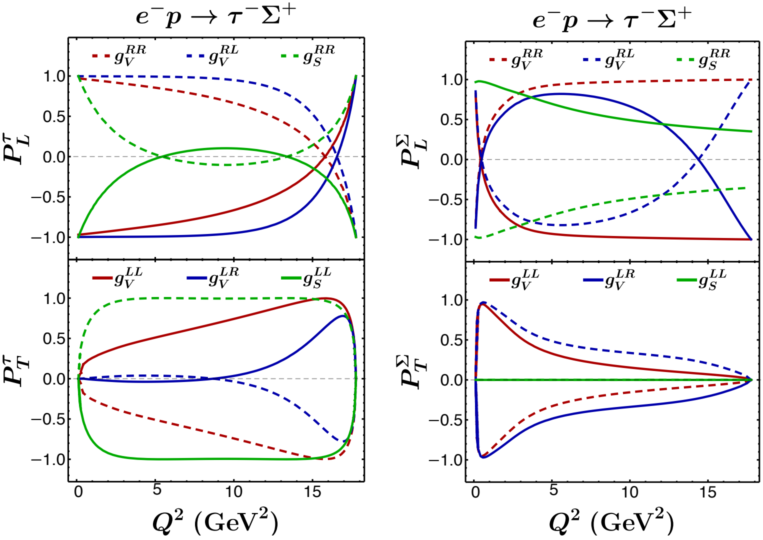
- Galster *NPB* **32**, 221 (1971)
- BBBA *NPB* **159**, 127 (2006)
- BHLT *PRD* **102**, 074012 (2020)

For the hadronic matrix elements of $Y \longrightarrow N$, the following two parameterizations are considered:

- QCDSR *JHEP* **06**, 122 (2024)
- χPT *JHEP* **04**, 104 (2019)

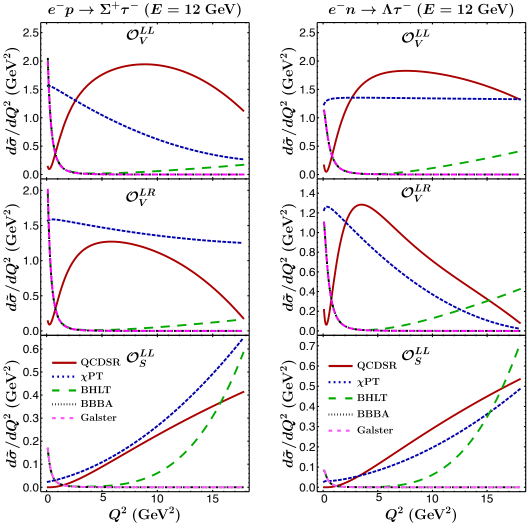
Form factors	f_1	f_2	f_3	g_1	g_2	g_3
BBBA	✓	✓	0	✓	0	✓
BHLT	✓	✓	0	✓	0	✓
Galster	✓	✓	0	✓	0	✓
QCDSR	✓	✓	0	✓	0	0
χPT	✓	0	0	✓	0	✓

Phenomenological Analysis: Generic features of polarization observables



The electron beam energy of $E = 12$ GeV, $e^- p \to \tau^- \Sigma^+$, BBBA parametrization.

Phenomenological Analysis: Differential cross section



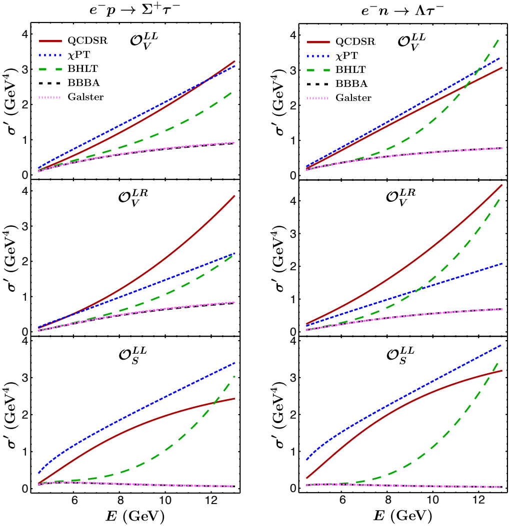
The scaling factor κ.

$p \to \Sigma^+$		
NP	QCDSR	χPT
g_V^{LL}	10	2
g_V^{LR}	10	2
g_S^{LL}	1000	10

$n \to \Lambda$		
NP	QCDSR	χPT
g_V^{LL}	20	5
g_V^{LR}	15	5
g_S^{LL}	2000	10

$\sigma = \kappa G_F^2 |g_\alpha|^2 \tilde{\sigma}/(16\pi m_N^2)$, κ is a dimensionless scaling factor.

Phenomenological Analysis: Total cross section



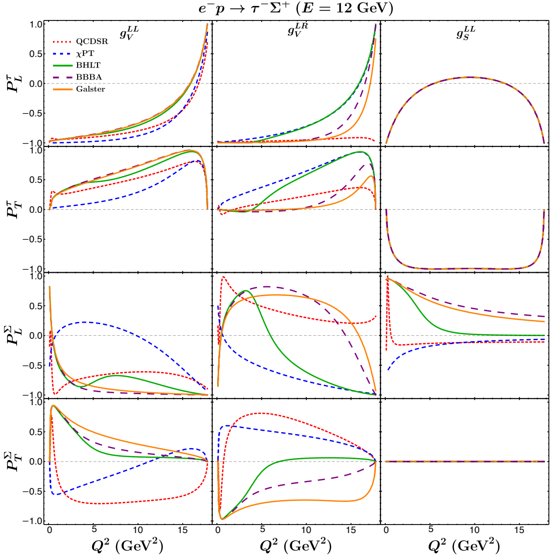
The scaling factor κ'.

	$p \to \Sigma^+$	
NP	QCDSR	χPT
g_V^{LL}	100	10
g_V^{LR}	50	25
g_S^{LL}	1500	15

	$n \to \Lambda$	
NP	QCDSR	χPT
g_V^{LL}	200	40
g_V^{LR}	50	25
g_S^{LL}	3000	10

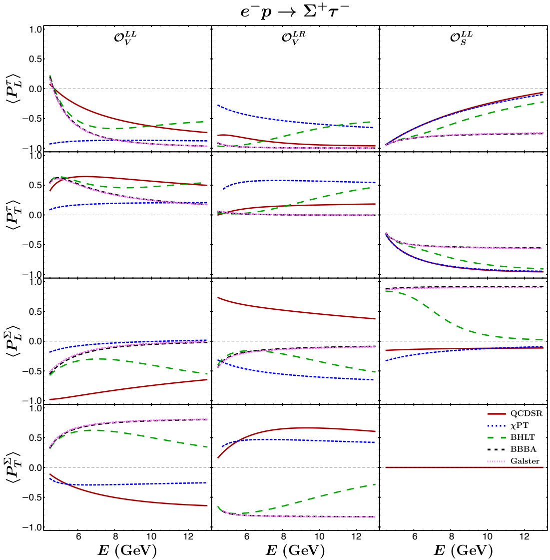
$\sigma = \kappa' G_F^2 |g_\alpha|^2 \tilde{\sigma}/(16\pi m_N^2)$, κ' is a dimensionless scaling factor.

Phenomenological Analysis: Polarization observables



Polarization observables $P_{L,T}$ of τ^- and Σ^+ in the process $e^- p \longrightarrow \tau^- \Sigma^+$.

Phenomenological Analysis: Average polarizations



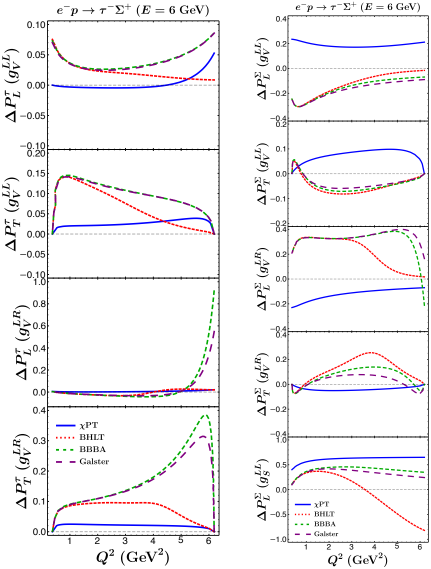
The average polarization observables $\langle P_{L,T} \rangle$ at an electron beam energy of $E = 12$ GeV.

Phenomenological Analysis: Average polarizations

Identification of vector NP operators, and distinction between distinct model families: {QCDSR, χPT} and {BBBA, BHLT, Galster}.

Model	NP	Sign of $\langle P_L^\tau \rangle$	Sign of $\langle P_T^\Sigma \rangle$
QCDSR, χPT	g_V^{LL}	$-$	$-$
	g_V^{LR}	$-$	$+$
	g_V^{RR}	$+$	$+$
	g_V^{RL}	$+$	$-$
BBBA, BHLT, Galster	g_V^{LL}	$-$	$+$
	g_V^{LR}	$-$	$-$
	g_V^{RR}	$+$	$-$
	g_V^{RL}	$+$	$+$

Phenomenological Analysis:Effects of g_3



Effects of g_3, $\Delta P^\tau_{L,T} \equiv P^\tau_{L,T} - P^\tau_{L,T}(g_3 = 0)$.

Experimental Setup

- JLab SoLID, $\mathcal{L} = 1.2 \times 10^{37}\,\mathrm{cm}^{-2}\mathrm{s}^{-1}$. *PRC* **109**, 065206 (2024)
- $dN/dt = \mathcal{L}\sigma$.

Constraints on Wilson Coefficients

- $\mathcal{B}\left(\tau^- \to e^- K_S\right) < 2.6 \times 10^{-8}$ *PLB* **692**, 4 (2010)
- $\mathcal{B}\left(\tau^- \to e^- K^{*0}\right) < 1.9 \times 10^{-8}$ *JHEP* **06**, 118 (2023)
- $\mathcal{B}\left(\tau^- \to e^- \pi^- K^+\right) < 3.1 \times 10^{-8}$ *PLB* **719**, 346 (2013)

The $\tau^- \to e^- K^{*0}$ decay provides the most restrictive constraint on the vector coefficients: *PLB* **797**, 134842 (2019)

$$|g_V^\alpha| < 2.77 \times 10^{-4} \quad (\alpha = \mathrm{LL},\ \mathrm{LR},\ \mathrm{RL},\ \mathrm{RR})\,.$$

On the other hand, the strongest constraint on the scalar coefficients comes from the $\tau^- \to e^- \pi^- K^+$ decay: *PLB* **797**, 134842 (2019)

$$|g_S^\alpha| < 4.09 \times 10^{-4} \quad (\alpha = \mathrm{LL},\ \mathrm{LR},\ \mathrm{RL},\ \mathrm{RR})\,.$$

Experimental Prospects:Event Rate Projections

$e^- p \longrightarrow \tau^- \Sigma^+$					
Model	QCDSR	χPT	BHLT	BBBA	Galster
$g_V^{LL,RR}$	6.73	0.668	0.0473	0.0206	0.0211
$g_V^{LR,RL}$	3.90	1.20	0.0425	0.0185	0.0190
g_S^{α}	183	2.46	0.117	0.00372	0.00389

$e^- n \longrightarrow \tau^- \Lambda$					
Model	QCDSR	χPT	BHLT	BBBA	Galster
$g_V^{LL,RR}$	13.3	2.91	0.0731	0.0181	0.0181
$g_V^{LR,RL}$	4.61	1.28	0.0758	0.0159	0.0159
g_S^{α}	481	1.88	0.136	0.00208	0.00209

Summary

- **Theoretical Framework and Model Dependence**
  - Cross section
  - Polarization observables
- **Resolving Theoretical Ambiguities Using Polarization Observables**
  - Distinguish different form factor models
  - Identify potential vector operators
- **Impact of the g_3 Form Factor**
  - Scalar operators
- **Experimental Outlook**
  - Projected annual event rates

Acknowledgements

Thank you!

Back up: τ- and Y-production density matrix

Within the framework of the effective Lagrangian $\mathcal{L}_{\text{eff}}$, the scattering amplitude $\mathcal{M}$ for the QE processes $e^- + N \to \tau^- + Y$, with nucleons $N = n, p$ and hyperons $Y = \Lambda, \Sigma^0, \Sigma^+$, takes the general form

$$\mathcal{M} = -\frac{4G_F}{\sqrt{2}} \sum_{a=S,V,T} \left(H_a^L L_a^L + H_a^R L_a^R \right) .$$

Starting from this amplitude, we construct the spin density matrix for the produced τ lepton.

$$\rho^{\tau}_{\lambda,\lambda'} = \sum_{r,s,t} \mathcal{M}(\lambda)\mathcal{M}^*(\lambda') = 8G_F^2 \sum_{r,s,t} \sum_{a,a'} \Big\{ H_a^L H_{a'}^{L*} L_a^L(\lambda) L_{a'}^{L*}(\lambda') + H_a^R H_{a'}^{R*} L_a^R(\lambda) L_{a'}^{R*}(\lambda') \Big\} .$$

To explicit evaluate the $L_a^{L,R}(\lambda) L_{a'}^{L,R*}(\lambda')$, we employ the Bouchiat-Michel formula.

$$u_{\lambda'}(k')\bar{u}_{\lambda}(k') = \frac{1}{2}\left[\delta_{\lambda\lambda'} + \sum_{i=1}^{3} \gamma_5 \not{s}_i \sigma^i_{\lambda\lambda'}\right](\not{k}' + m_\tau) .$$

Back up:Two Forms of the Form Factor

In the vector channel, the functions $D^V_{1,2}(q^2)$ and $F^V_{1,2}(q^2)$ are given by

$$F^V_{1,2}(q^2) = f^p_{1,2}(q^2) + \frac{1}{2} f^n_{1,2}(q^2)\,, \qquad D^V_{1,2}(q^2) = -\frac{3}{2} f^n_{1,2}(q^2)\,.$$

Here, $f^{p,n}_{1,2}(q^2)$ represent the Dirac ($i = 1$) and Pauli ($i = 2$) form factors for the proton and neutron. Each $f^{p,n}_{1,2}(q^2)$ is, in turn, composed of the standard electromagnetic form factors, $F^{p,n}_{1,2}(q^2)$, and a contribution from the strange-quark vector current, $F^s_{1,2}(q^2)$:

$$f^{n,p}_{1,2} = \left(\frac{1}{2} - 2\sin^2\theta_W\right) F^{n,p}_{1,2} - \frac{1}{2} F^{p,n}_{1,2} - \frac{1}{2} F^s_{1,2}\,.$$

where θ_W is the weak mixing angle.

The electromagnetic pieces are then related to the Sachs' electric and magnetic form factors via

$$F^{p,n}_1(q^2) = \frac{G^{p,n}_E(q^2) + \tau\, G^{p,n}_M(q^2)}{1+\tau}\,, \qquad F^{p,n}_2(q^2) = \frac{G^{p,n}_M(q^2) - G^{p,n}_E(q^2)}{1+\tau}\,.$$

where $\tau = -q^2/(4m_N^2)$ and m_N denotes the nucleon mass.

Back up:Two Forms of the Form Factor

Each form factor can be decomposed into two functions, $D(q^2)$ and $F(q^2)$, weighted by the appropriate Clebsch-Gordan coefficients,

$$f_i(q^2) = aF_i^V(q^2) + bD_i^V(q^2)\,,$$
$$g_i(q^2) = aF_i^A(q^2) + bD_i^A(q^2)\,, \quad i = 1, 2, 3,$$

where the coefficients a and b for each $N \to Y$ transition are listed in Table.

Transitions	a	b
$n \to \Lambda$	$-\frac{3}{\sqrt{6}}$	$-\frac{1}{\sqrt{6}}$
$n \to \Sigma^0$	$\frac{1}{\sqrt{2}}$	$-\frac{1}{\sqrt{2}}$
$p \to \Sigma^+$	-1	1

Back up:Two Forms of the Form Factor

The electromagnetic current which is a vector current is written using the charge operator $e = I_3 + \frac{Y}{2} = \lambda_3 + \frac{1}{2\sqrt{3}}\lambda_8$ as:

$$j_\mu^{\text{em}} = V_\mu^3 + \frac{1}{\sqrt{3}} V_\mu^8$$

In general, the expression for the matrix element of the transition between the two states of baryons (say B_i and B_k), through the SU(3) octet (V_j or A_j) of currents can be written as:

$$\langle B_i | V_j | B_k \rangle = i f_{ijk} F^V + d_{ijk} D^V$$

$$\langle B_i | A_j | B_k \rangle = i f_{ijk} F^A + d_{ijk} D^A$$

F^V and D^V are determined from the experimental data on the electromagnetic form factors, and F^A and D^A are determined from the experimental data on semileptonic decays of neutron and hyperons.

Back up:Two Forms of the Form Factor

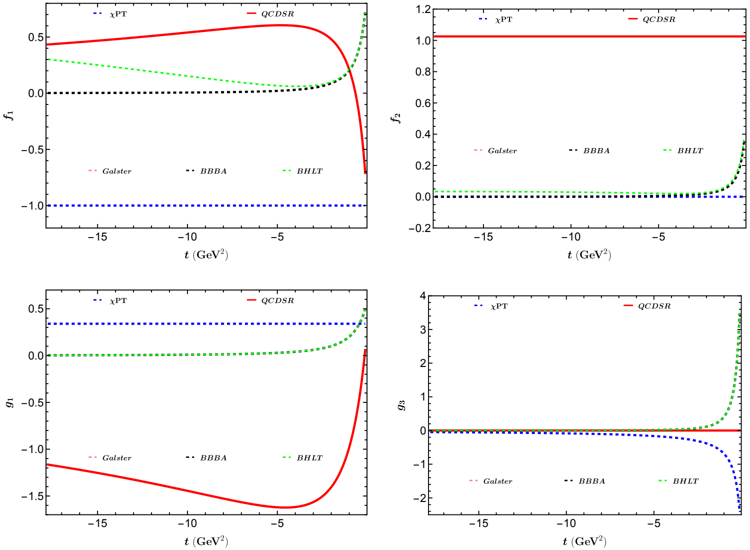
The physical baryon octet states are written in terms of their octet state B_i as:

$$p = \frac{1}{\sqrt{2}}(B_4 + iB_5), \qquad n = \frac{1}{\sqrt{2}}(B_6 + iB_7),$$
$$\Sigma^\pm = \frac{1}{\sqrt{2}}(B_1 \pm iB_2), \qquad \Xi^- = \frac{1}{\sqrt{2}}(B_4 - iB_5),$$
$$\Xi^0 = \frac{1}{\sqrt{2}}(B_6 - iB_7), \qquad \Sigma^0 = B_3, \quad \Lambda^0 = B_8.$$

The matrix element for the electromagnetic transition between two octet states B_i and B_k is defined as:

$$\langle B_i | V_3 + \frac{1}{\sqrt{3}} V_8 | B_k \rangle = i \left[f_{i3k} + \frac{1}{\sqrt{3}} f_{i8k} \right] F^V + \left[d_{i3k} + \frac{1}{\sqrt{3}} d_{i8k} \right] D^V$$

Back up:Two Forms of the Form Factor



Form factor:$e^- p \longrightarrow \tau^- \Sigma^+$.