QCD LCDA of Heavy Mesons from bHQET

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Beneke, Finauri, Vos and **YBW**: 2305.06401

Deng, Wang, **YBW** and Zeng: 2409.00632

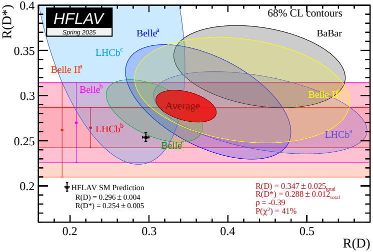
Outline

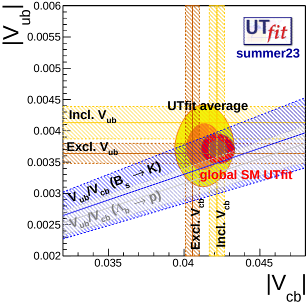
- LCDA of heavy mesons
- Introduction to bHQET
- Factorization of QCD LCDA
- Numeric applications

B Physics

New physics beyond the SM

- Indirect search: flavour physics
- BaBar, Belle
- LHC, Belle-II
- HL-LHC



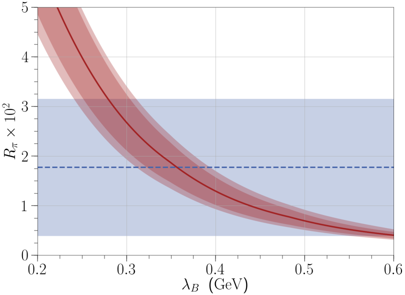


- CPV: $\Lambda_b^0 \to p K^- \pi^+ \pi^-$ [LHCb, 25'], [Wang and Yu, 24']

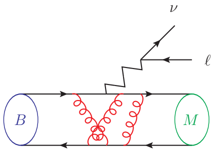
  B decays [Wang, Wang and **YBW**, 21']
- $R(D^{(*)})$: [Wang, **YBW**, Shen and Lü, 17']
- $\left|V_{ub/cb}\right|$: [Shen and **YBW**, 21']

HQET LCDA of heavy meson

HQET LCDA: Most important function in exclusive B decays



[Belle, 18'], [Beneke, Braun, Ji, and **YBW**, 18']



- Non-leptonic decays: $B \to \pi K \cdots$
- Semi-leptonic decays: $B \to D^{(*)}\ell\nu \cdots$
- Radiative decay: $B \to \gamma\ell\nu \cdots$

Direct study of HQET LCDA

HQET LCDA: non-perturbative physics

- Inverse moments: [Wang and Shen, 18'], [Beneke, Braun, Ji, and **YBW**, 18'], [Belle, 18'], [Galda, Neubert and Wang, 22'], [Han, et al., 24']
- RG evolution properties: [Braun, Ji and Manashov, 19'], [Galda and Neubert, 20'], [Cui, Shen, Wang and **YBW**, 23']
- EOM [Kawamura, Kodaira, Qiao and Tanaka, 02'], [Lü, Shen, Wang and **YBW**, 18']
- generalized LCDA [Beneke, Böer, Toelstede, and Vos, 22'], [Qin, Shen, Wang and Wang, 22'], [Huang, Ji, Shen, Wang and Wang, 23']

Direct determination:

- Calculate with LQCD [LPC collaboration, 22']
- Extract from the experiments: clean process $B \to \gamma \ell \nu$ [Wang and Shen, 18'], [Beneke, Braun, Ji, and **YBW**, 18'], [Belle, 18'], [Galda, Neubert and Wang, 22']

Indirect study of HQET LCDA

Indirect determination:

From QCD LCDA to HQET LCDA [Ishaq, Jia, Xiong and Yang, 19'], [Zhao, 19'], [Beneke, Finauri, Vos and **YBW**, 23'], [Deng, Wang, **YBW** and Zeng, 24']

$$\phi(u) = \mathcal{J}(u,\omega) \otimes \varphi_+(\omega)$$

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$$\phi(u) = \mathcal{J}(u,\omega) \otimes \varphi_+(\omega)$$

The heavy meson QCD LCDA [Braun and Filyanov, 89']

$$\langle H(p_H)|\bar{Q}(0)\not{n}_+\gamma^5[0,tn_+]q(tn_+)|0\rangle = -if_H n_+\cdot p_H \int_0^1 du\, e^{iutn_+\cdot p_H}\,\phi(u)$$

The leading-twist heavy-meson HQET LCDA [Grozin and Neubert, 96']

$$\langle H_v|\bar{h}_v(0)\not{n}_+\gamma^5[0,tn_+]q_s(tn_+)|0\rangle = -i\widetilde{f}_H(\mu)\, n_+\cdot v \int_0^\infty d\omega\, e^{i\omega tn_+\cdot v}\varphi_+(\omega)$$

Boosted HQET

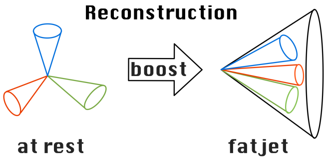
The heavy quark field in bHQET [Fleming, Hoang, Mantry and Stewart, 07'], [Dai, Kim and Leibovich, 21'], [Finauri: PhD thesis]

$$h_n(x) \equiv \sqrt{\frac{2}{n_+ v}}\, e^{i m_Q v\cdot x} \frac{\not{n}_- \not{n}_+}{4} Q(x)$$

For heavy particle with large energy

- Top-quark production [Fleming, Hoang, Mantry and Stewart, 07']
- B-meson leptonic decay with τ lepton [Huang, Shen, Zhao and Zhou, 23']

※① **Systematic power counting**: boosted frame



$$\begin{tikzcd} \text{QCD} \arrow[d] \arrow[r, phantom, "\xrightarrow{\text{boost}}"] & \text{SCET} \arrow[d] \\ \text{HQET} & \text{bHQET} \end{tikzcd}$$

The LP operators

$$\hat{\mathcal{O}}_k = \frac{1}{n_+v}\sqrt{\frac{n_+v}{2}}\,\bar{h}_n \slashed{n}_+ \left(n_+v\,\frac{i\slashed{D}_\perp}{in_+D}\right)^k \gamma^5 \xi_{sc}$$

Only operators $\hat{\mathcal{O}}_0$ and $\hat{\mathcal{O}}_1$ will appear at LP [Beneke, Finauri, Vos and **YBW**, 23']

Factorization of the QCD LCDA

※② **Cut** δ to QCD LCDA: avoid end-point singularity

The factorization formula

$$\phi(u) = \begin{cases} u \sim \delta : & \mathcal{J}_p(u,\omega) \otimes \varphi_+(\omega)\,, \qquad \mathcal{O}_{\rm QCD} = \mathcal{J}_p \otimes \mathcal{O}_0, \\ u \sim 1 : & \mathcal{J}_{\rm tail}(u)\,, \qquad \mathcal{O}_{\rm QCD} = \mathcal{J}_0 \mathcal{O}_0 + \mathcal{J}_1 \mathcal{O}_1\,, \end{cases}$$

The jet function $\mathcal{J}$: $\mathcal{O}(m_Q)$, HQET LCDA φ_+: $\mathcal{O}(\Lambda_{\rm QCD})$

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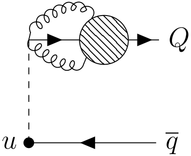
End-point singularity: refactorization-based subtraction (RBS)

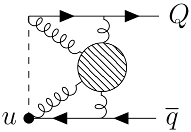
- NLP in $h \to \gamma\gamma$: [Liu and Neubert, 19']
- Off-diagonal parton scattering, NLP in gluon thrust: [Beneke et. al., 20', 22']
- $B_s \to \mu^+\nu_\mu$ [Cornella, König and Neubert, 22']

Short distance function



Peak region:

$$\mathcal{J}_p(u,\omega) = \theta(m_Q - \omega)\delta\Big(u - \frac{\omega}{m_Q}\Big)\Big[1 + \frac{\alpha_s C_F}{4\pi}\Big(\frac{L^2}{2} + \frac{L}{2} + \frac{\pi^2}{12} + 2\Big)\Big], \quad L = \ln\frac{\mu^2}{m_Q^2}$$



Tail region:

$$\mathcal{J}_{\text{tail}}(u) = \frac{\alpha_s C_F}{4\pi}\frac{2\bar{u}}{u}\Big((1+u)[L - 2\ln u] - u + 1\Big)$$

Evolution of the LCDAs: φ_+ and ϕ

Two-step evolutions: $\Lambda_{\rm QCD} \to m_Q \to Q$

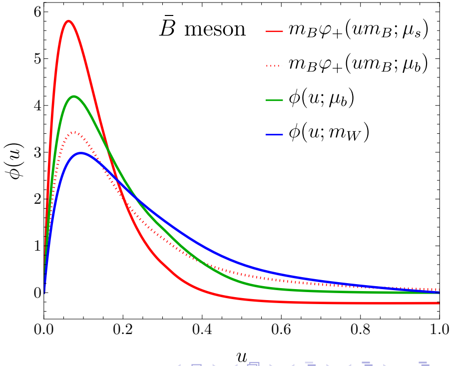
$\Lambda_{\rm QCD}$ $\qquad$ m_Q $\qquad$ Q

$\phi(m_Q) = \mathcal{J} \otimes \varphi_+(m_Q)$ $\qquad$ ERBL $\qquad$ $\phi(Q)$

$\varphi_+(\Lambda_{\rm QCD})$ $\qquad$ LN $\qquad$ $\varphi_+(m_Q)$

$\phi(m_Q) \to \phi(Q)$:
evolve of the Gegenbauer moments

$$\frac{a_n(\mu_h)}{a_n(\mu)} = \left(\frac{\alpha_s(\mu_h)}{\alpha_s(\mu)}\right)^{\frac{\gamma_n}{2\beta_0}}$$



Heavy-quark spin symmetry

Heavy-quark symmetry [Grozin and Neubert, 97']: relates LCDAs of P and V heavy mesons

Factorization formulas

$$\phi_i(u) = \frac{\tilde{f}_H}{f_i} m_H \mathcal{J}^i_{\mathrm{peak}}(m_H)\varphi_+(um_H), \qquad i = P,\ \|,\ \perp$$

EOM of the quarks, method of region [Deng, Wang, **YBW** and Zeng, 24']

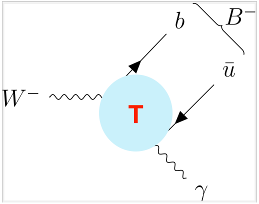
$$\mathcal{J}^i_{\mathrm{peak}}(u,\omega) = 1 + \frac{\alpha_s C_F}{4\pi}\Big(\frac{L^2}{2} + \frac{L}{2} + \frac{\pi^2}{12} + 2\Big) + \mathcal{O}(\alpha_s^2)$$

Estimate NLP corrections

$W \to B\gamma$: branch ratio

Rare decay $W^- \to B^-\gamma$ [Grossman, König and Neubert, 15']

$$A(W^- \to B^-\gamma) = \int_0^1 du\, T(u)\, \phi(u)$$



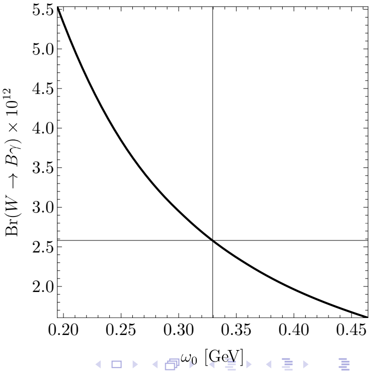
[Grossman, König and Neubert, 15']:
QCD LCDA

$$\mathrm{Br} = (1.99 \pm 0.17_{\mathrm{in}}\,{}^{+0.03}_{-0.06}\,{}_{\mu_h}\,{}^{+2.48}_{-0.80}\,{}_{\lambda_B}) \cdot 10^{-12}$$

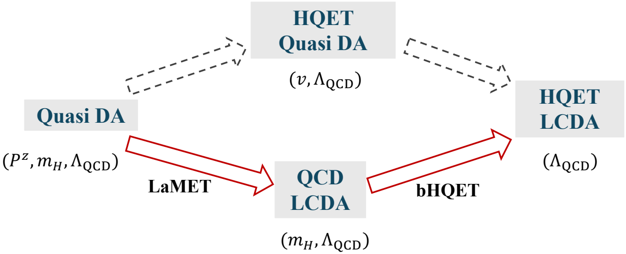
Our result: with $\ln \Lambda_{\mathrm{QCD}}/m_Q$ resummation

$$\mathrm{Br} = (2.58 \pm 0.21_{\mathrm{in}}\,{}^{+0.05}_{-0.08}\,{}_{\mu_h}\,{}^{+2.95}_{-0.98}\,{}_{\lambda_B}) \cdot 10^{-12}$$

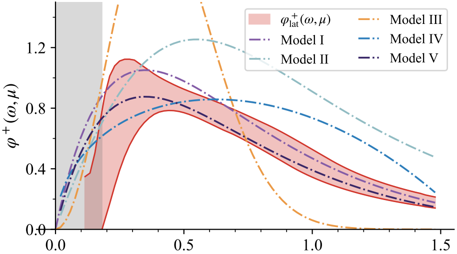
30% increase



Lattice QCD



LaMET [Ji, 13']: quasi DA $\to$ QCD LCDA $\to$ HQET LCDA [Han, et al. 24']



First inverse moment
$\lambda_B = 0.376(63)$ GeV

Future plans

- NLP operators in bHQET
- From NLL $\rightarrow$ NNLL
- Endpoint singularity in RBS scheme

Summary

※ Introduction to the QCD LCDA and HQET LCDAs

※ Match QCD LCDA to the HQET LCDA

- ① Systematic power counting in bHQET
- ② Cut δ: avoid endpoint singularity

※ $W^- \to B^- \gamma$ decay: 30% enhancement

Thank you!