QCD Antenna—Soft Correlations in Heavy Ion Collisions

Pablo Guerrero Rodríguez

Based on work in collaboration with Carlos A. Salgado and Fabio Domínguez

arXiv:2604.XXXXX

March 23rd 2026

C3NT Workshop: "Jet-soft dynamical medium interaction in high-energy heavy-ion collisions"
Wuhan (China)

Synopsis

➡ Our goal: to improve the theoretical description of **parton cascades within a QCD medium**

➡ Fundamental to achieve a **higher-precision description of jet observables**

➡ For this, we revisit the **in-medium soft gluon emission off a singlet quark-antiquark antenna**

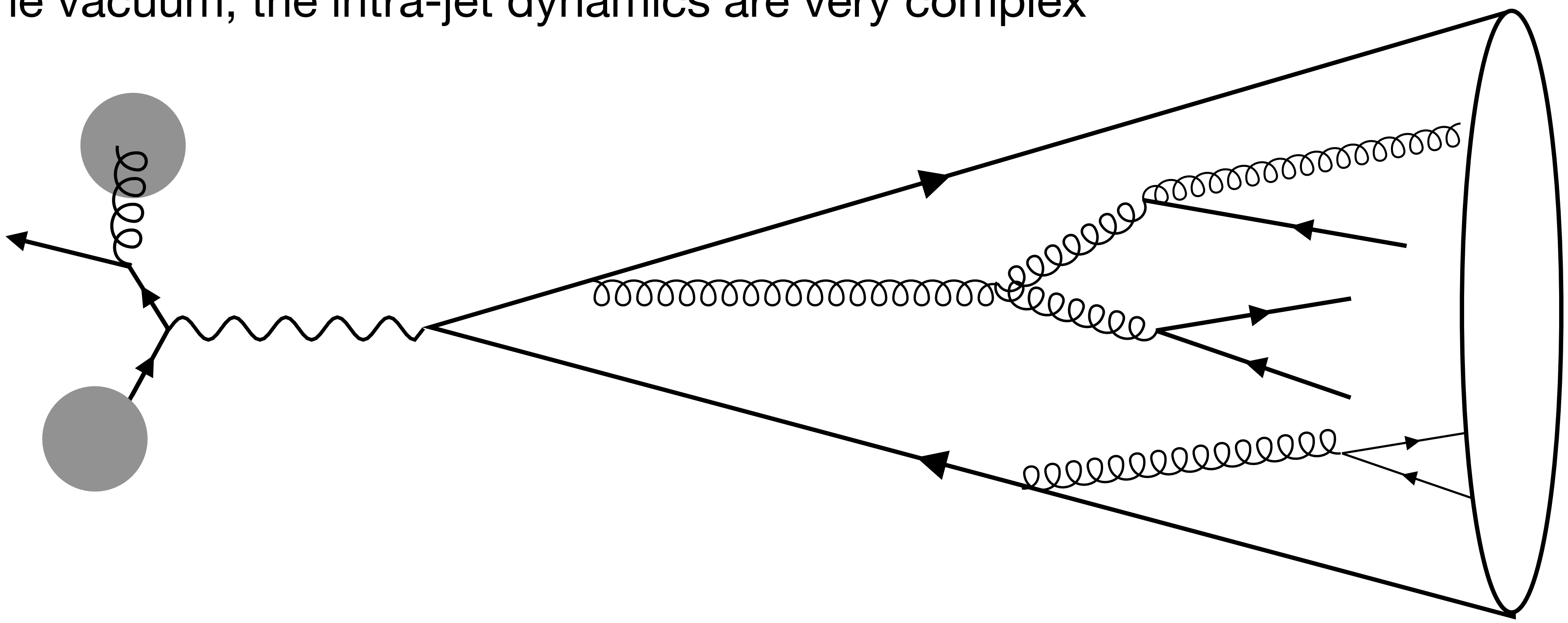
➡ The methodology: **relaxing/avoiding some of the standard approximations**

- Instantaneous antenna formation
- Very soft (i.e. *external*) gluon emission

The $q\bar{q}$ antenna as a QCD laboratory

- *How do you make a jet?*

Even in the vacuum, the intra-jet dynamics are very complex



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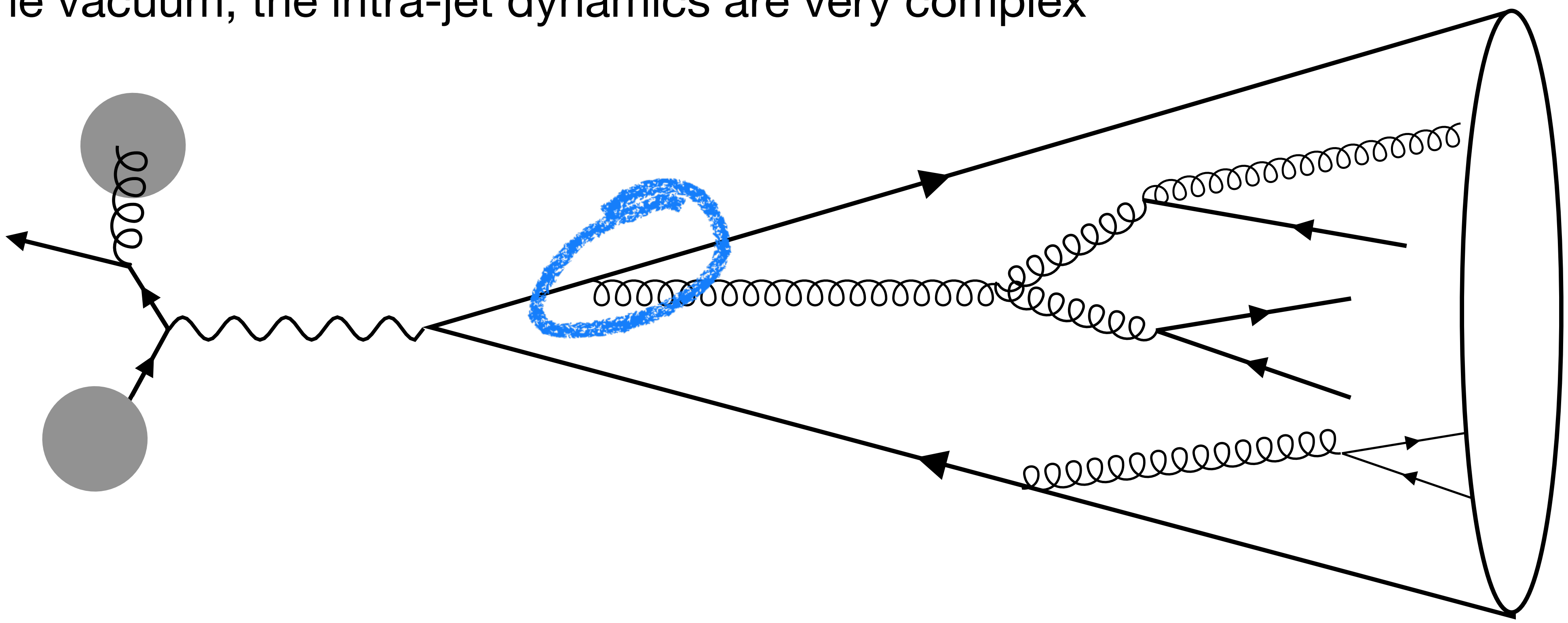
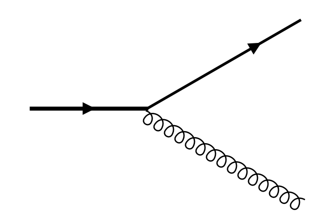


We focus on simple fundamental processes that can be used to model/simulate the whole cascade:

➡ The **1->2 process** (inclusive single gluon radiation off a fast quark or gluon) does not address coherence effects between multiple emitters



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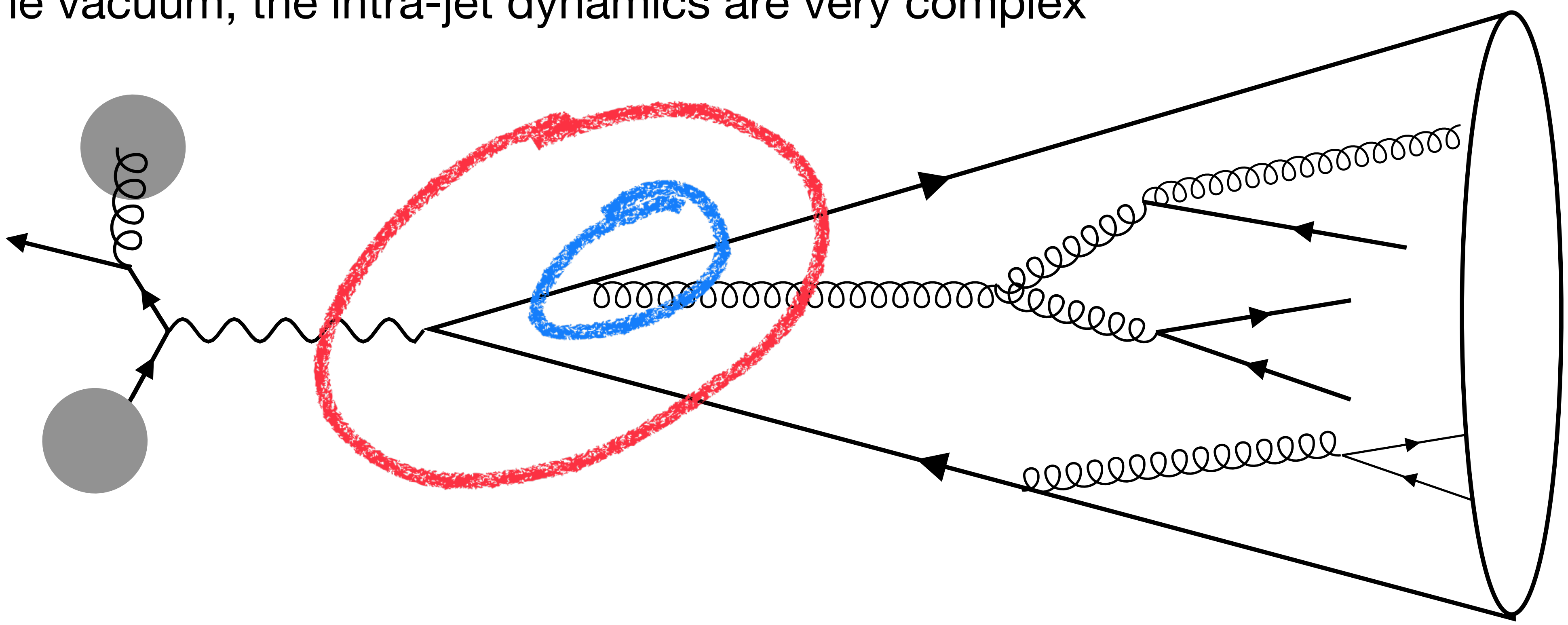

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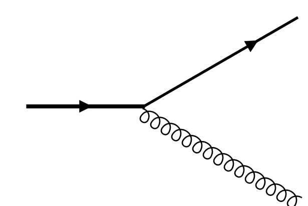
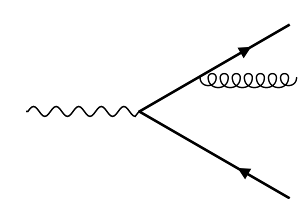


➡ The **2->3 process** (**singlet quark-antiquark antenna**) does give rise to interference phenomena and captures the main features of the cascade



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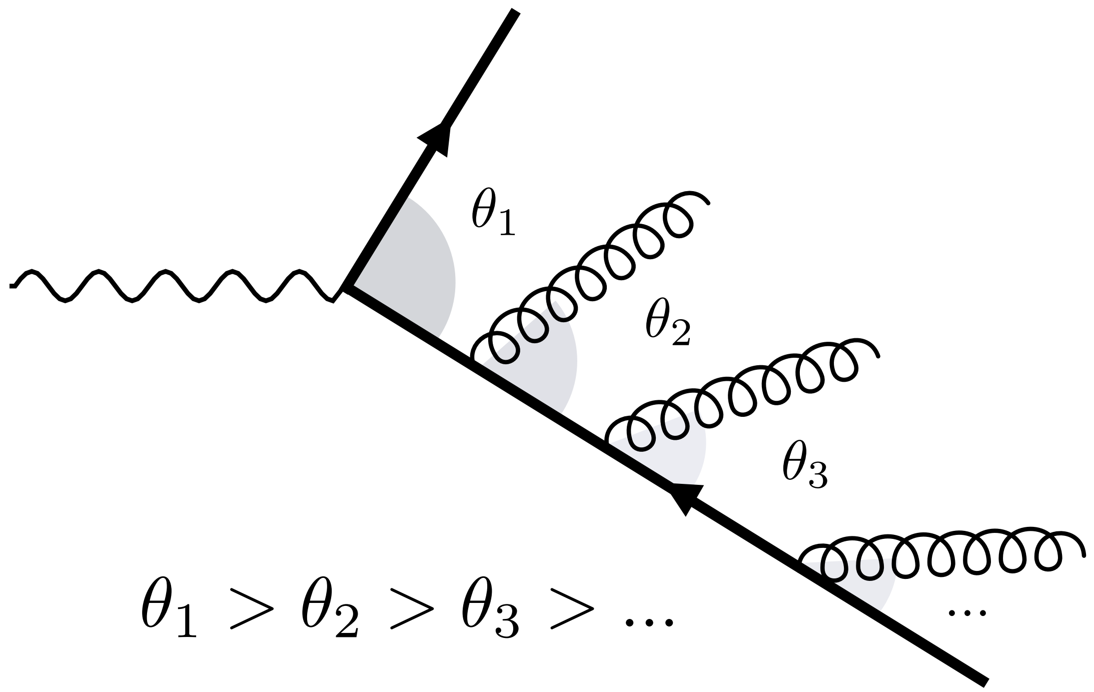
$$dN = \frac{dE_g}{E_g}\frac{\sin\theta_g d\theta_g}{1-\cos\theta_g}\frac{\alpha_s C_F}{2\pi}\Theta(\theta_{q\bar{q}} - \theta_g)$$

Angular ordering!

➡ Iterated to next emission(s): emission angles keep getting smaller



➡ An essential property for event generators!

Vacuum radiation, angular ordering

Y. L. Dokshitzer, V. A. Khoze, A. H. Mueller, S. I. Troian; Basics of perturbative QCD, 1991

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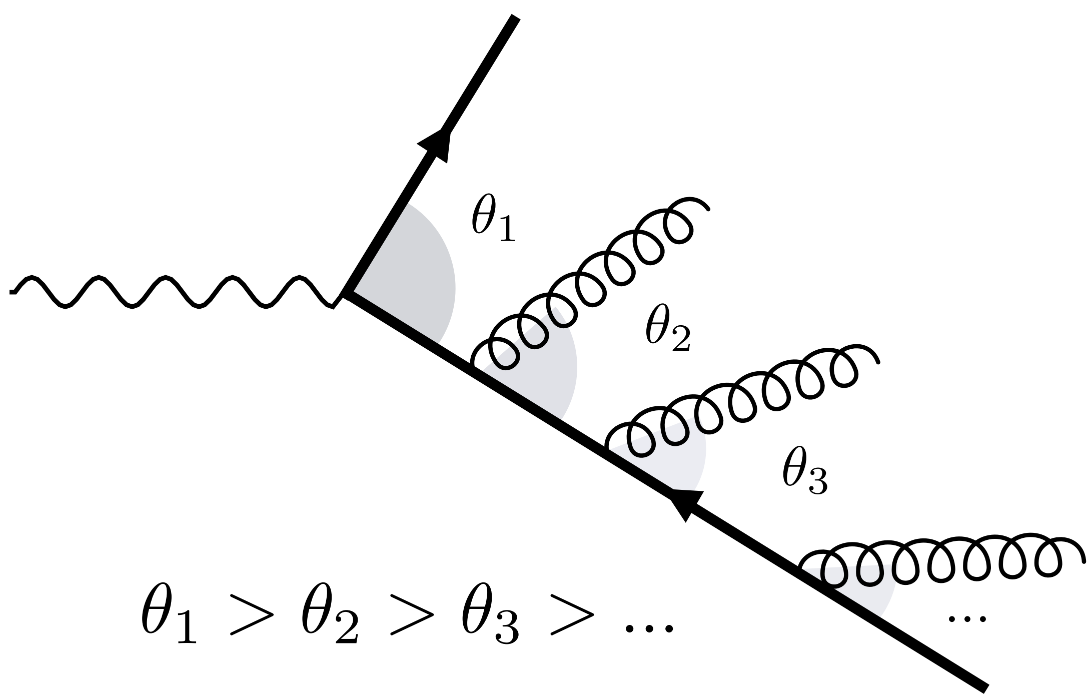

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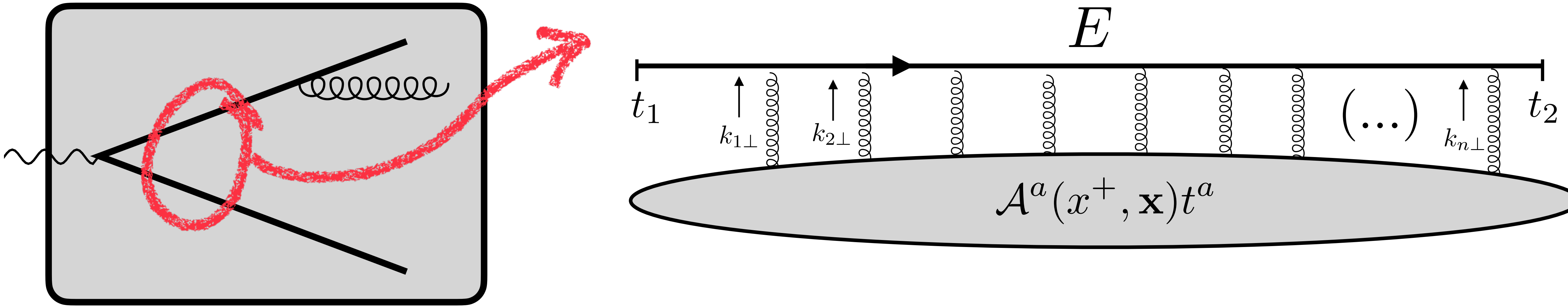
- *How is the picture of color coherence modified in the presence of a soft medium?*

In-medium propagation

R. Baier, Yu.L. Dokshitzer, A.H. Mueller, S. Peigné, D. Schiff; Nucl. Phys. B 483 (1997) 291-320

➡ For the antenna legs we apply the **eikonal approximation** ($E \gg k_{i\perp}$), where the only effect of each interaction with the medium is a **color rotation**:



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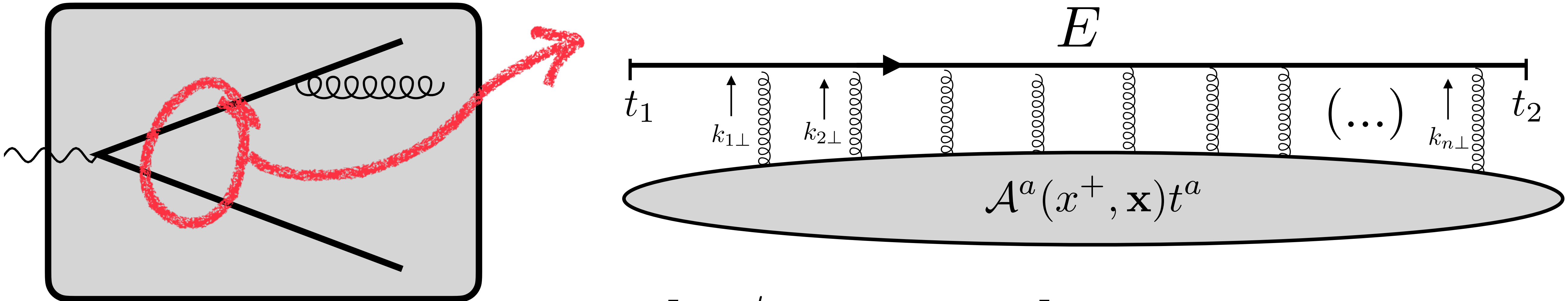

We have $|out\rangle = S|in\rangle$ with: $S_{ij} = \mathrm{P}\exp\left[ig\int_{t_1}^{t_2} dt\, \mathcal{A}^a(t, \mathbf{x}(t))t^a_{ij}\right] \equiv V_{ij}(\mathbf{x}) \in \mathrm{SU}(N_c)$ Wilson Line

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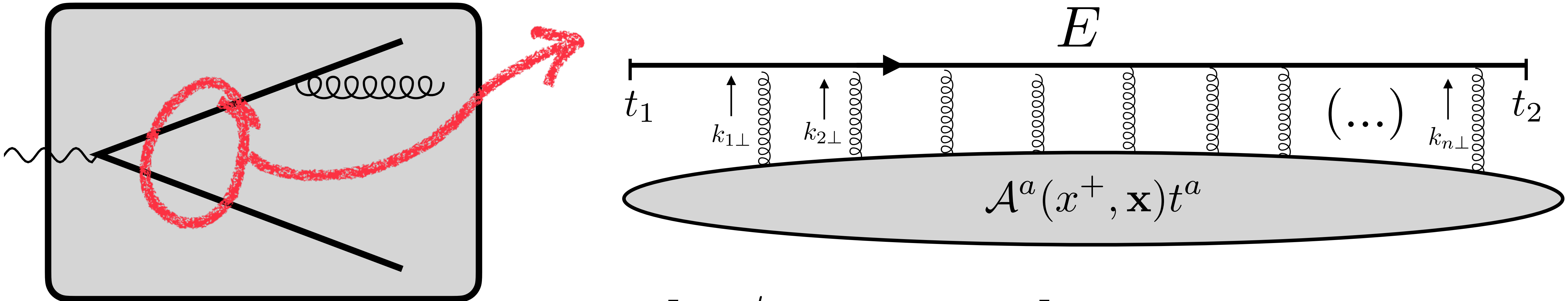

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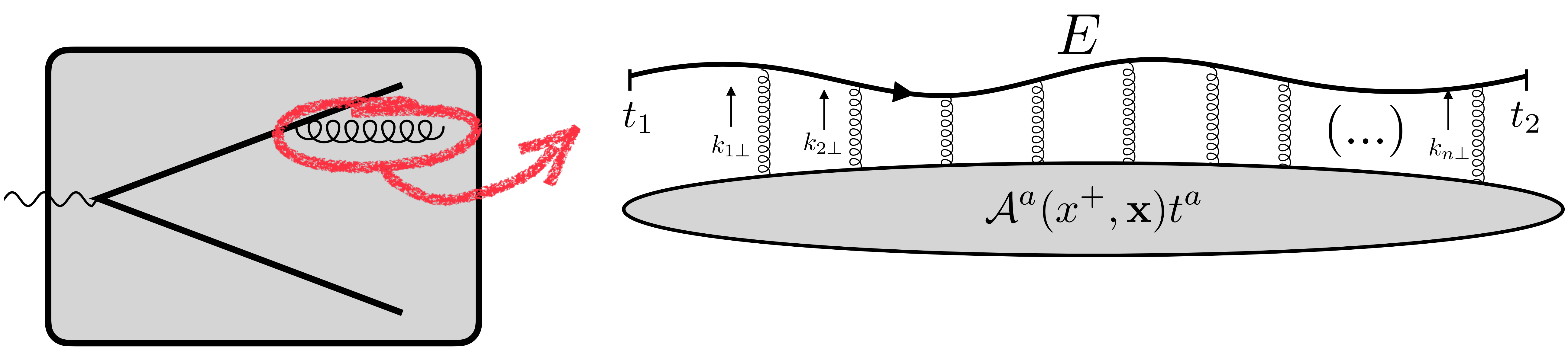

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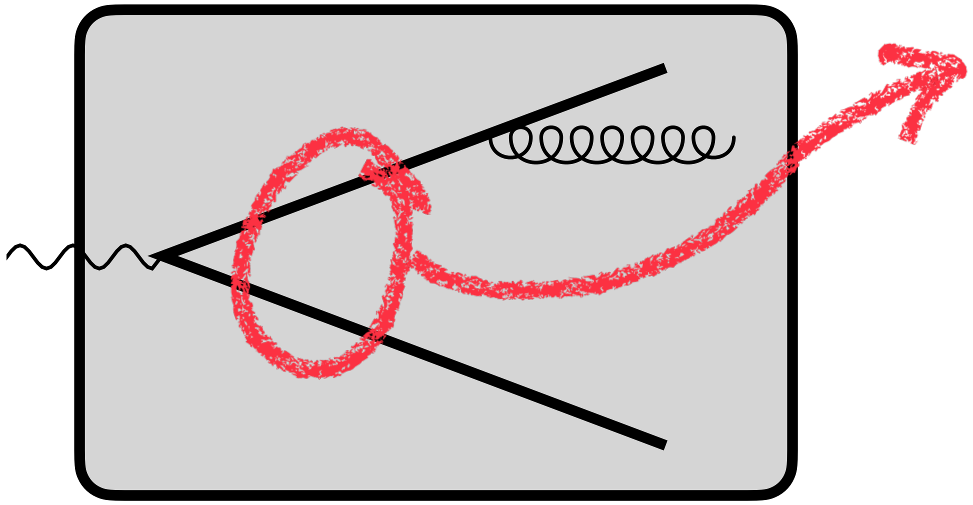

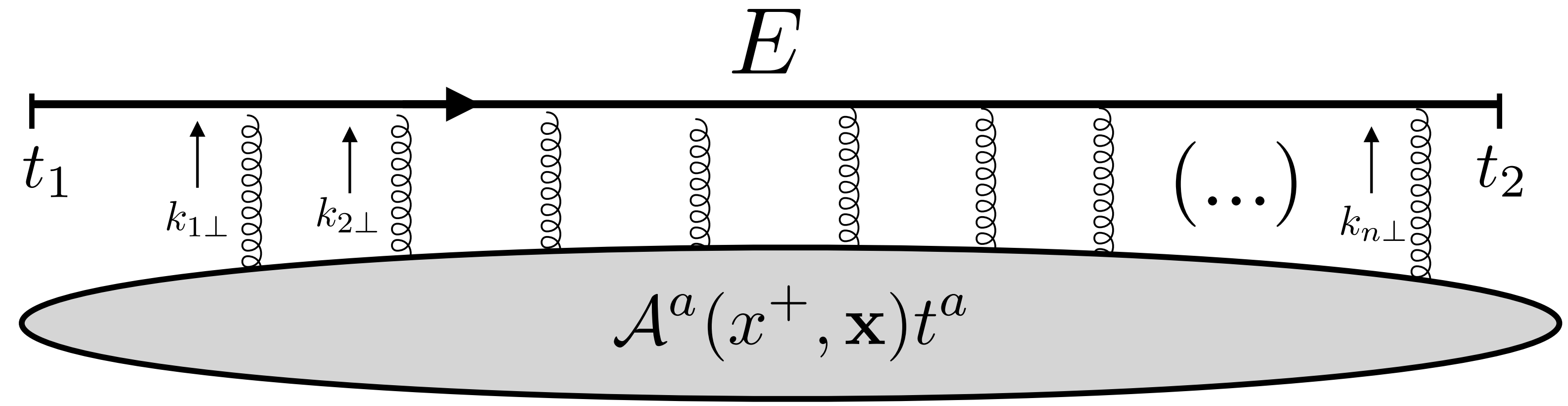

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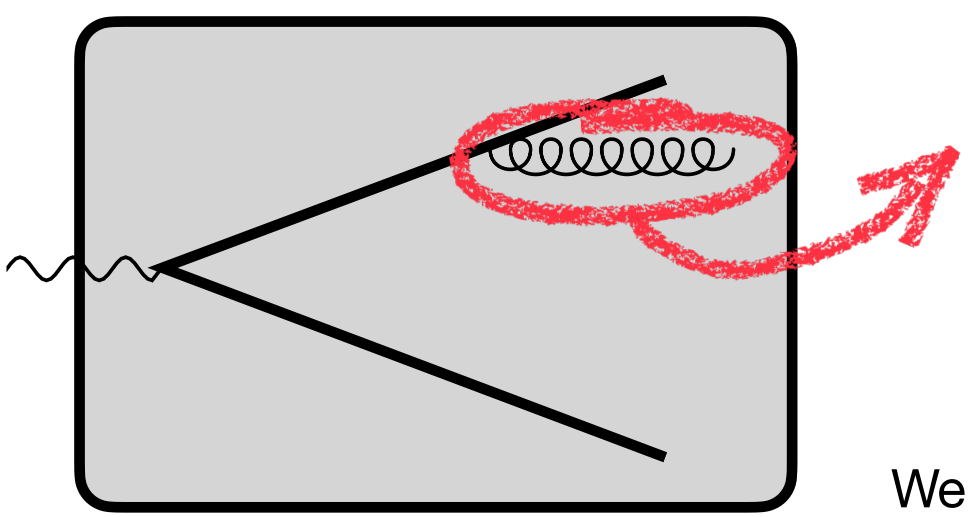
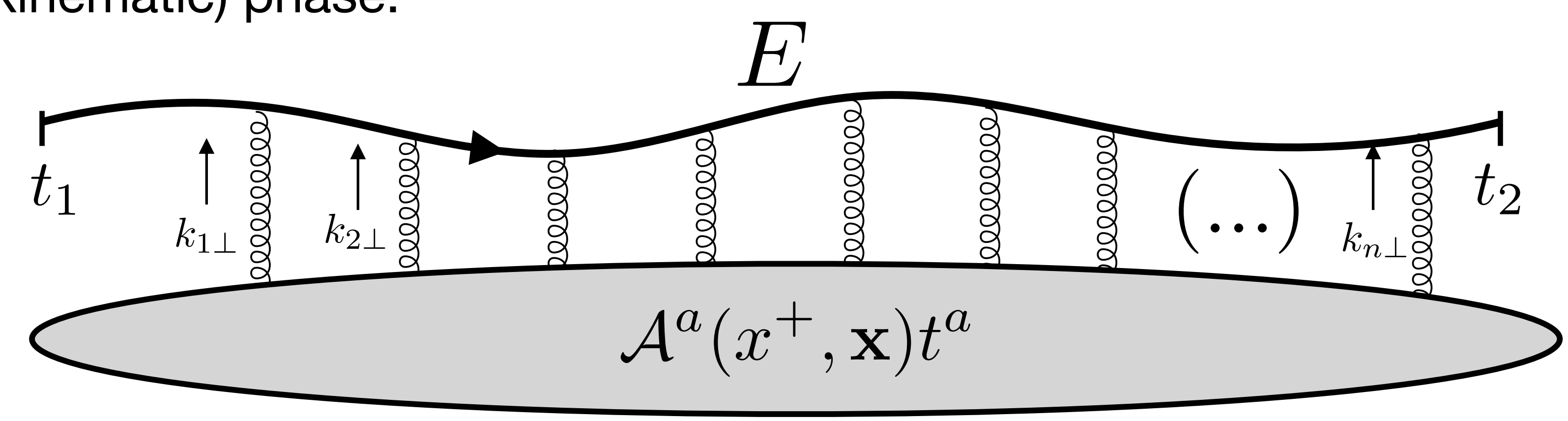
➡ For the emitted gluons we allow small, random changes in the transverse position (**Brownian motion**), which makes them pick an additional (kinematic) phase:



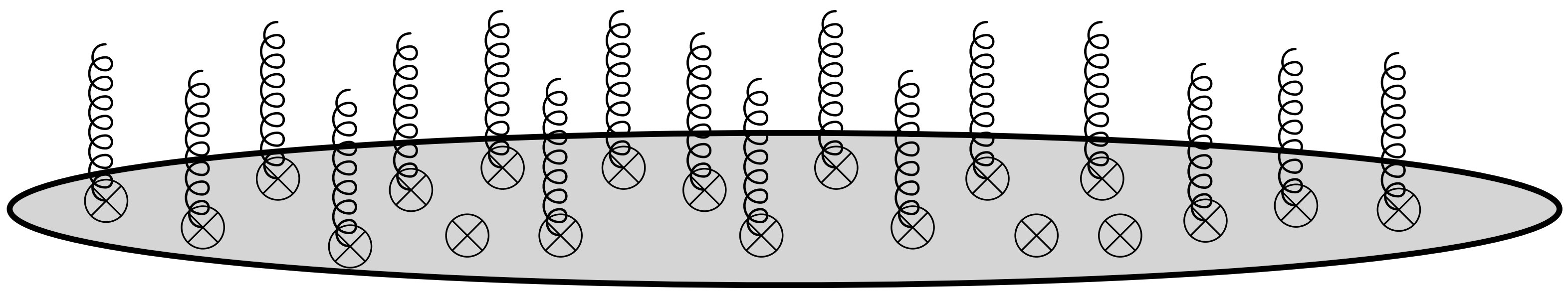


We have $\mathcal{G}_{ij} = \int \mathcal{D}\mathbf{x} \exp\left\{\frac{E}{2}\int_{t_1}^{t_2} dt\, \dot{\mathbf{x}}^2(t)\right\} V_{ij}(\mathbf{x})$ "Dressed" propagator

Medium averages: Gaussian white noise

➡ The in-medium propagators $V_{ij}, \mathcal{G}_{ij}$ are defined for a given configuration of the medium field $\mathcal{A}$

➡ $\mathcal{A}$ is generated by a **random ensemble of color sources:**

$\mathcal{A}[\rho(\otimes)] =$ 

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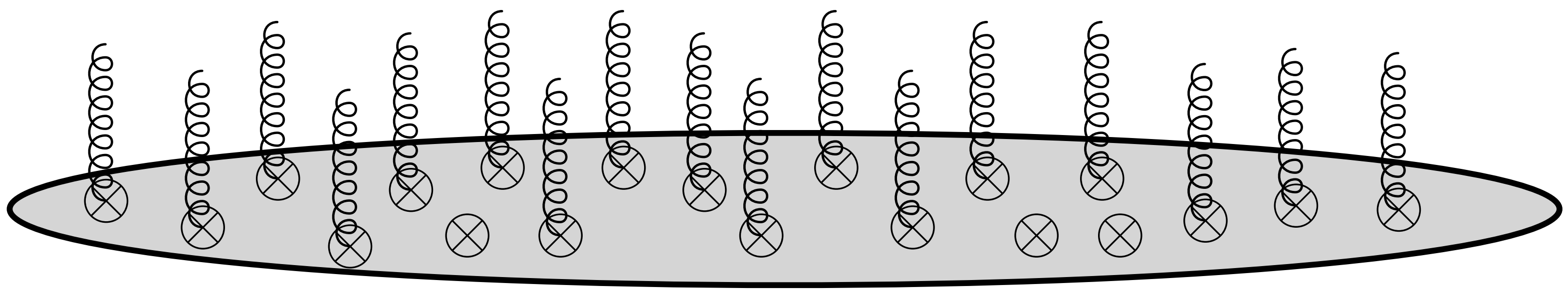

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- We assume that the field follows **Gaussian statistics**:

$$\langle \mathcal{A}^a(t,\mathbf{x})\mathcal{A}^b(\bar{t},\bar{\mathbf{x}})\rangle = \delta^{ab}\delta(t-\bar{t})\gamma(\mathbf{x}-\bar{\mathbf{x}})n(t)$$

$\sim$ Scattering rate

Density of scattering centers

Realistic parton-medium interactions: talk by André Cordeiro (Thursday 26)

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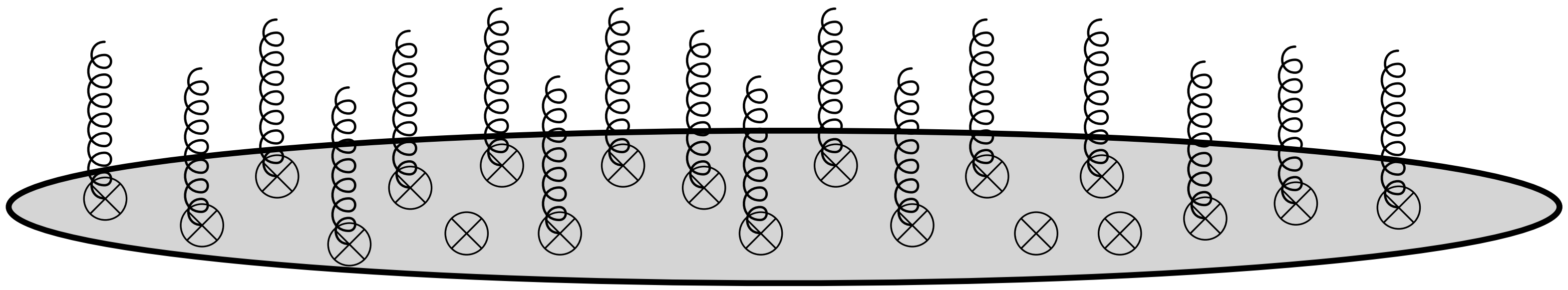

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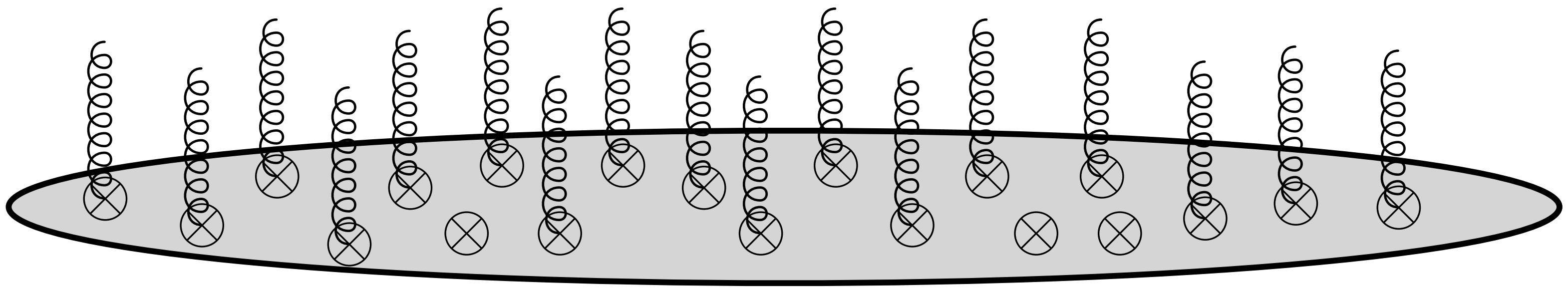

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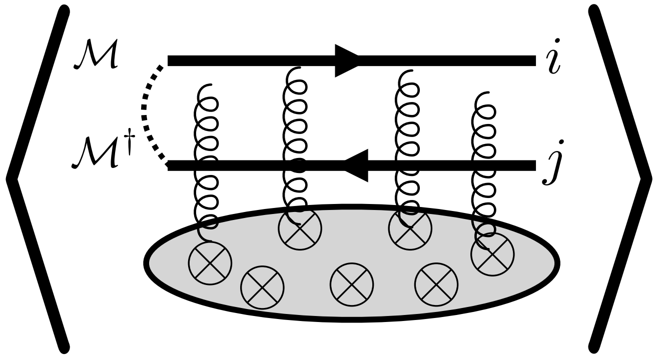

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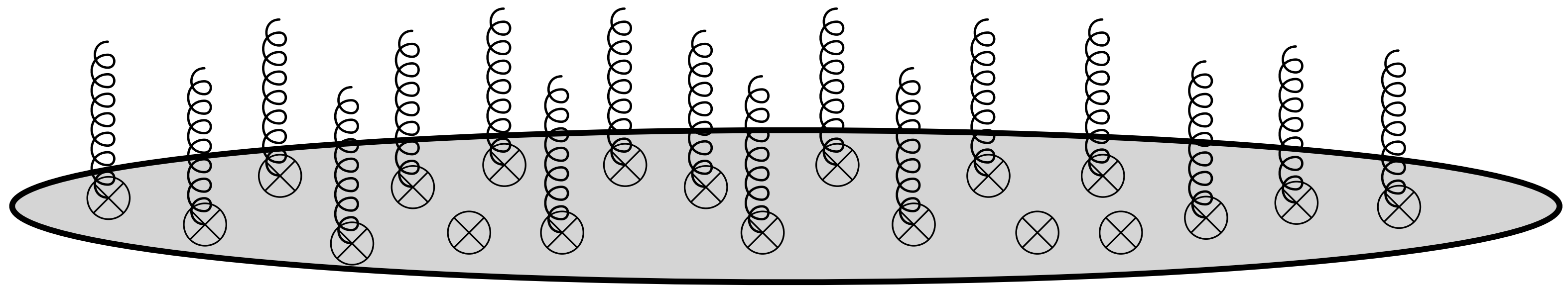

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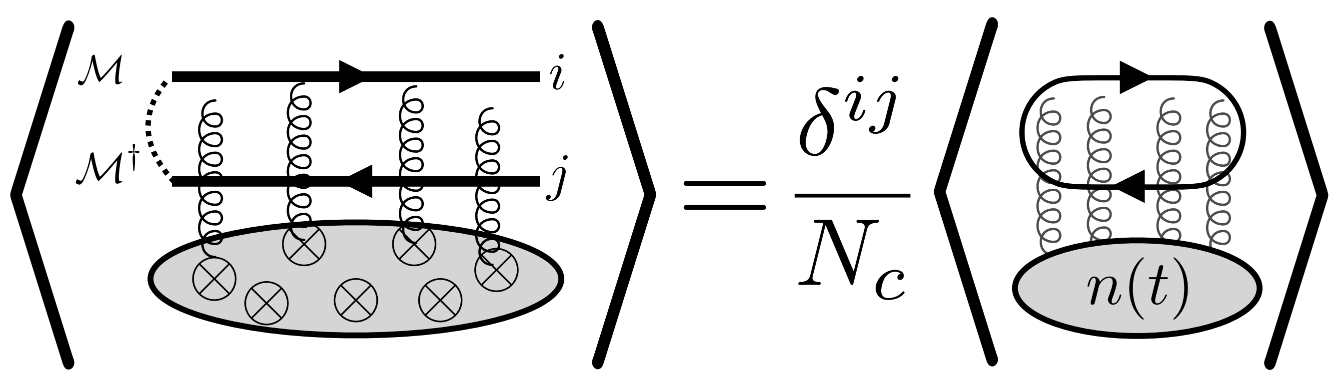
- Within this picture, **the system is in a singlet state at all times**, e.g. $\left\langle \left[V(t',t)\bar{V}^\dagger(t',t)\right]_{ij}\right\rangle = \frac{\delta^{ij}}{N_c}\left\langle \mathrm{Tr}\left\{V\bar{V}^\dagger\right\}\right\rangle$



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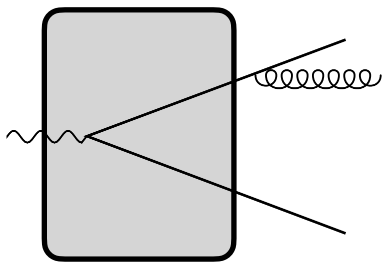

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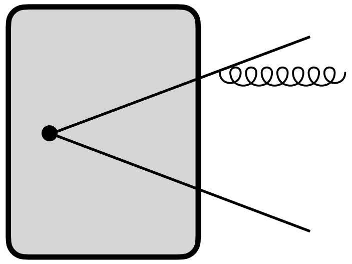

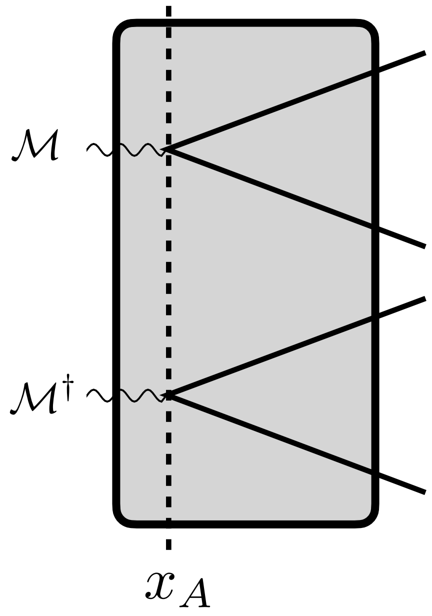

Y. Mehtar-Tani, C.A. Salgado and K. Tywoniuk; Phys. Rev. Lett. 106 (2011) 122002
Y. Mehtar-Tani, C.A. Salgado and K. Tywoniuk; Phys. Lett. B 707 (2012) 156-159

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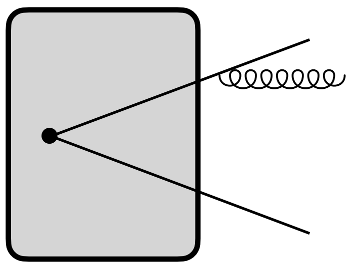
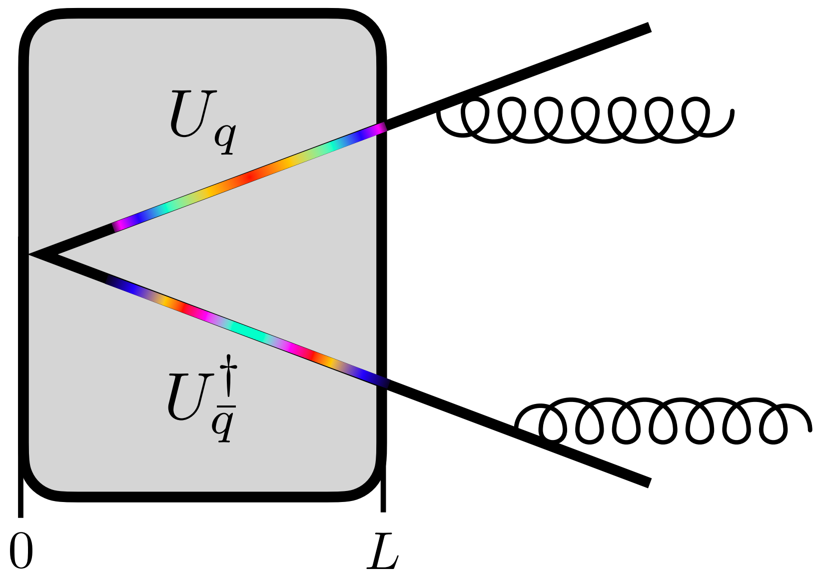


Under these approximations, the in-medium soft gluon spectrum is not so different to the vacuum:

$$dN = \frac{dE_g}{E_g}\frac{d\Omega}{2\pi}\frac{\alpha_s C_F}{2\pi}\left[\mathcal{R}_q + \mathcal{R}_{\bar{q}} - (1-\Delta_{\rm med})2\mathcal{J}\right]$$

Here, $\Delta_{\rm med} = 1 - \left\langle \mathrm{Tr}\{U_q(L,0)U^\dagger_{\bar{q}}(0,L)\}\right\rangle$, with $U^{ab} = \mathrm{Tr}\{t^a V^\dagger t^b V\}$ (WL in adjoint representation)



- **Dilute medium**: $\Delta_{\rm med} \to 0$

  $dN \sim \mathcal{R}_q + \mathcal{R}_{\bar{q}} - 2\mathcal{J}$ (Angular ordering)
- **Opaque medium:** $\Delta_{\rm med} \to 1$

  $dN \sim \mathcal{R}_q + \mathcal{R}_{\bar{q}}$ (Independent emitters)

➡ Intuitive picture: as the color of each parton rotates independently, **the initial correlation is broken**

($\mathcal{M}$ and $\mathcal{M}^\dagger$ overlayed in this cartoon)

Y. Mehtar-Tani, C.A. Salgado and K. Tywoniuk; Phys. Rev. Lett. 106 (2011) 122002

Y. Mehtar-Tani, C.A. Salgado and K. Tywoniuk; Phys. Lett. B 707 (2012) 156-159

Towards more realistic antennas

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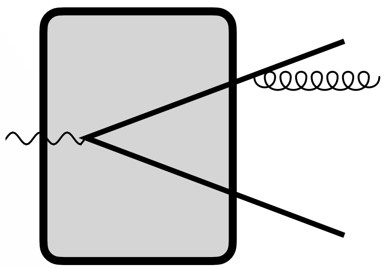

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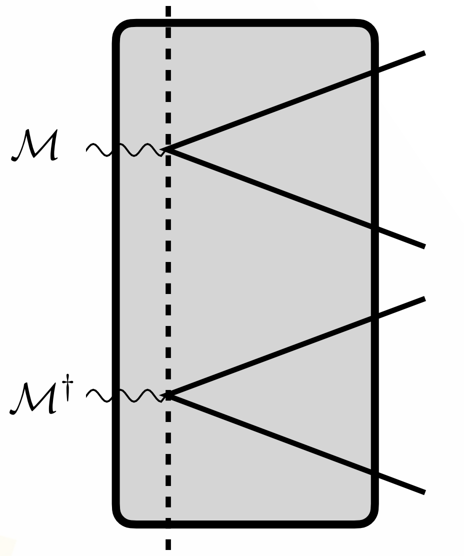

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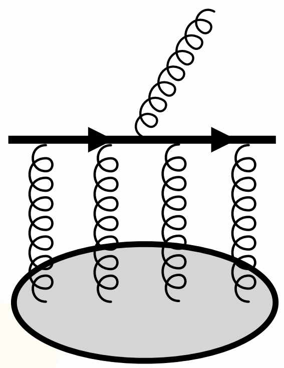

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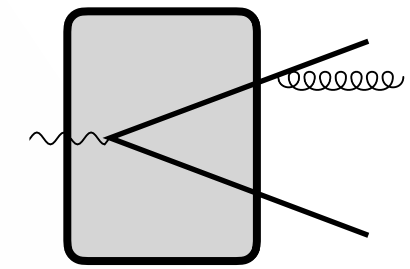

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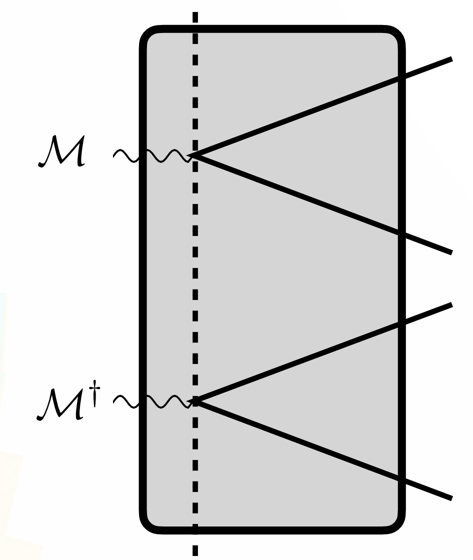

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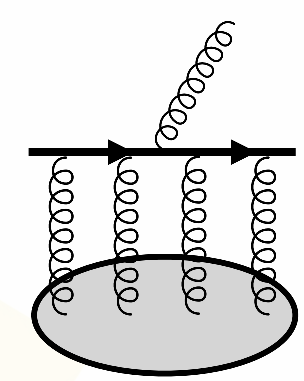

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DE ABSENCE THESE APPROXIMATIONS

Towards more realistic antennas

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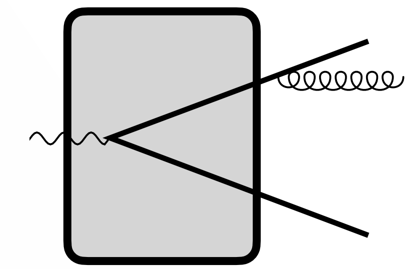

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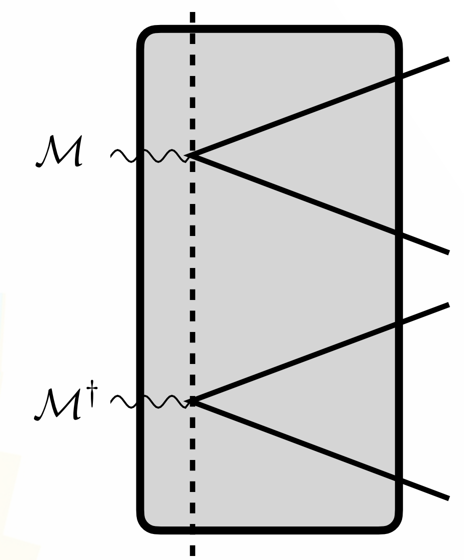
This assumption is abandoned in:

J. Casalderrey-Solana and E. Iancu; JHEP 08 (2011) 015

Y. Mehtar-Tani and K. Tywoniuk; JHEP 10 (2012) 197

Y. Mehtar-Tani and K. Tywoniuk; JHEP 01 (2013) 031



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This approximation is dropped in:

F. Domínguez, J.G. Milhano, C.A. Salgado, K. Tywoniuk, V. Vila; Eur.Phys.J.C 80 (2020) 1, 11

(for $\gamma \to q\bar{q}$ splitting)

S. Abreu, X. Mayo López, G. Milhano and A. Soto-Ontoso; JHEP 03 (2025) 216

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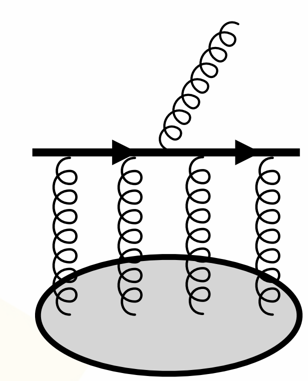

➡ **Eikonal approximation:** antenna legs are perfectly straight lines

This approximation is relaxed in:

J-P. Blaizot, F. Dominguez , E. Iancu, Y. Mehtar-Tani; JHEP 01 (2013) 143

L. Apolinário, N. Armesto, J.G. Milhano, C.A. Salgado; JHEP 02 (2015) 119

- *How is the picture of color coherence modified in the ~~presence~~ of ~~a medium~~?*

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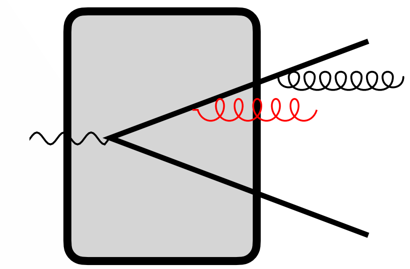

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  J. Casalderrey-Solana and E. Iancu; JHEP 08 (2011) 015

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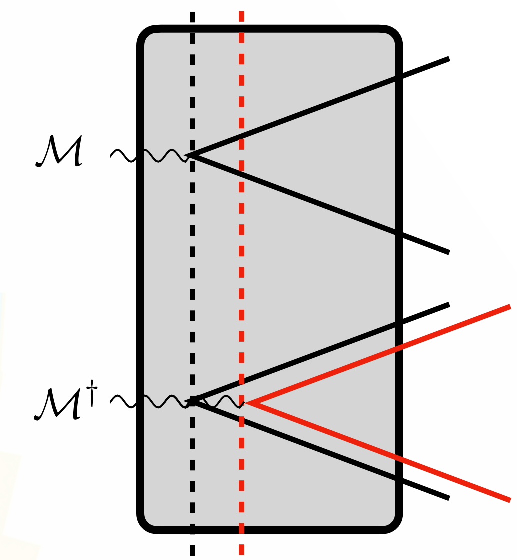
  Y. Mehtar-Tani and K. Tywoniuk; JHEP 01 (2013) 031

OUR WORK



- ➡ ~~**Instantaneous antenna formation**~~

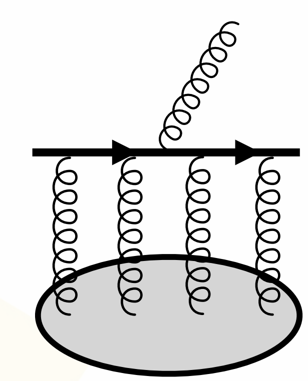
  This approximation is dropped in:

  F. Domínguez, J.G. Milhano, C.A. Salgado, K. Tywoniuk, V. Vila; Eur.Phys.J.C 80 (2020) 1, 11

  (for $\gamma \to q\bar{q}$ splitting)

  S. Abreu, X. Mayo López, G. Milhano and A. Soto-Ontoso; JHEP 03 (2025) 216

  (for $\gamma \to q\bar{q}g$ splitting)



- ➡ **Eikonal approximation:** antenna legs are perfectly straight lines

  This approximation is relaxed in:

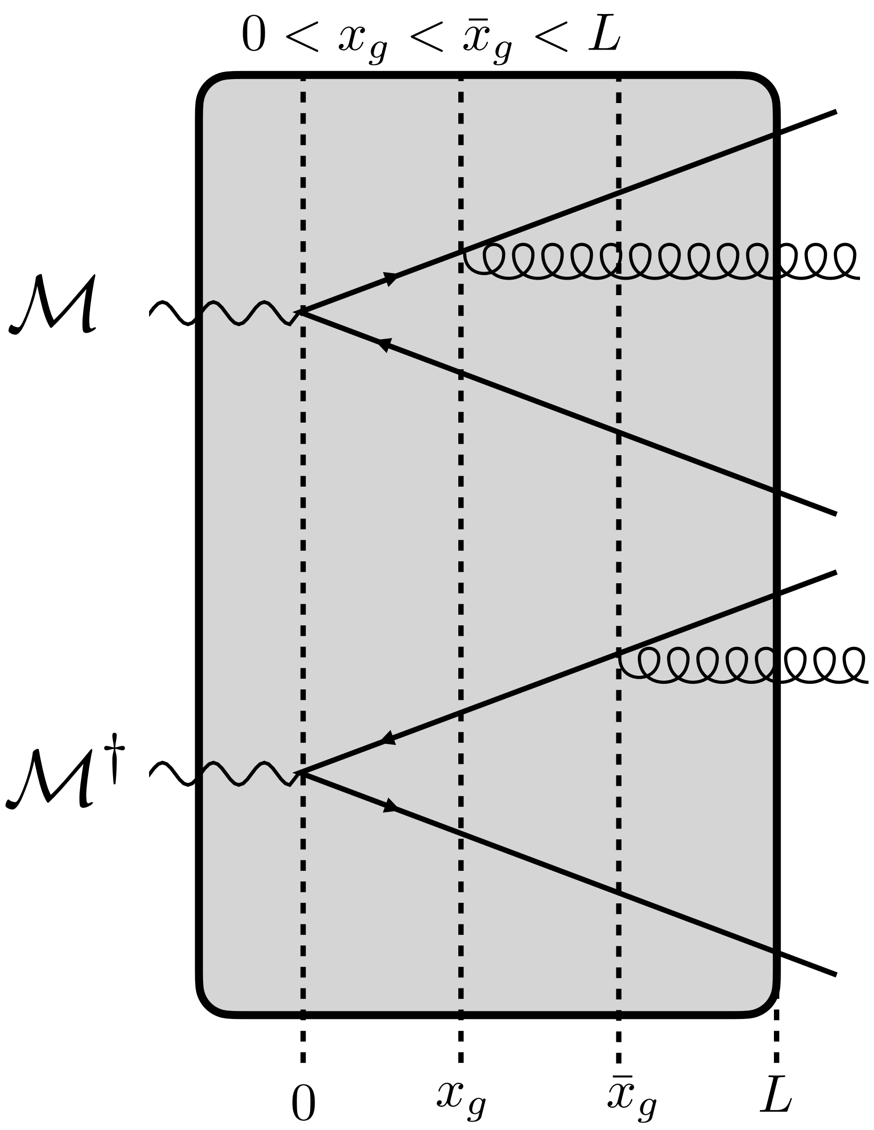
  J-P. Blaizot, F. Dominguez , E. Iancu, Y. Mehtar-Tani; JHEP 01 (2013) 143

  L. Apolinário, N. Armesto, J.G. Milhano, C.A. Salgado; JHEP 02 (2015) 119

- *How is the picture of color coherence modified in the ~~presence~~ of ~~a medium~~?*

DE ABSENCE THESE APPROXIMATIONS

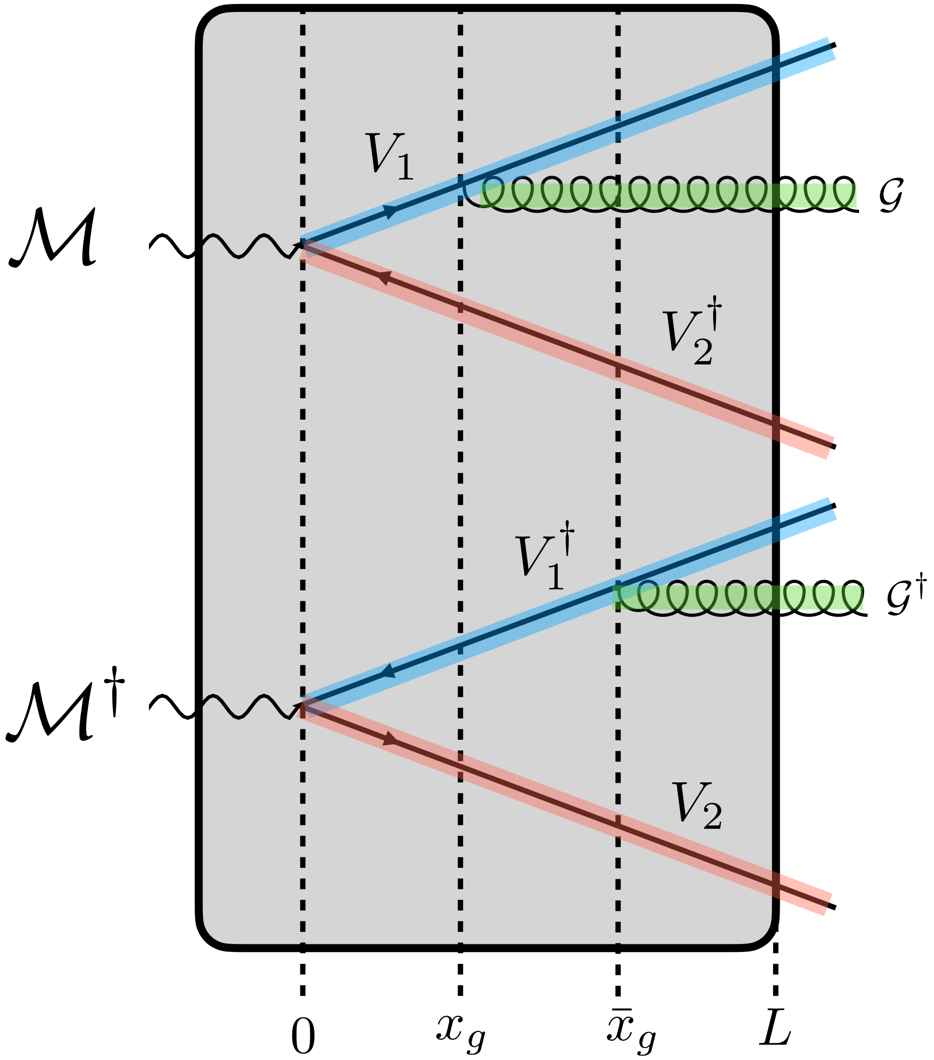
Y. Mehtar-Tani and K. Tywoniuk; JHEP 10 (2012) 197
Y. Mehtar-Tani and K. Tywoniuk; JHEP 01 (2013) 031



➡ As we introduce the gluon inside of the medium, new elements appear in the calculation

$$|\mathcal{M}_q|^2 \propto \int_{0<x_g<\bar{x}_g<L} \mathcal{G}^{ab\dagger}(\bar{x}_g, L)\mathcal{G}^{bc}(L, x_g)\mathrm{Tr}\left\{V_1^\dagger(0,\bar{x}_g)t^a V_1^\dagger(\bar{x}_g, L)V_1(L, x_g)t^c V_1(x_g, 0)V_2^\dagger(0, L)V_2(L, 0)\right\}$$

Gluons — Quarks — Antiquarks



Step I: In-medium gluon emission

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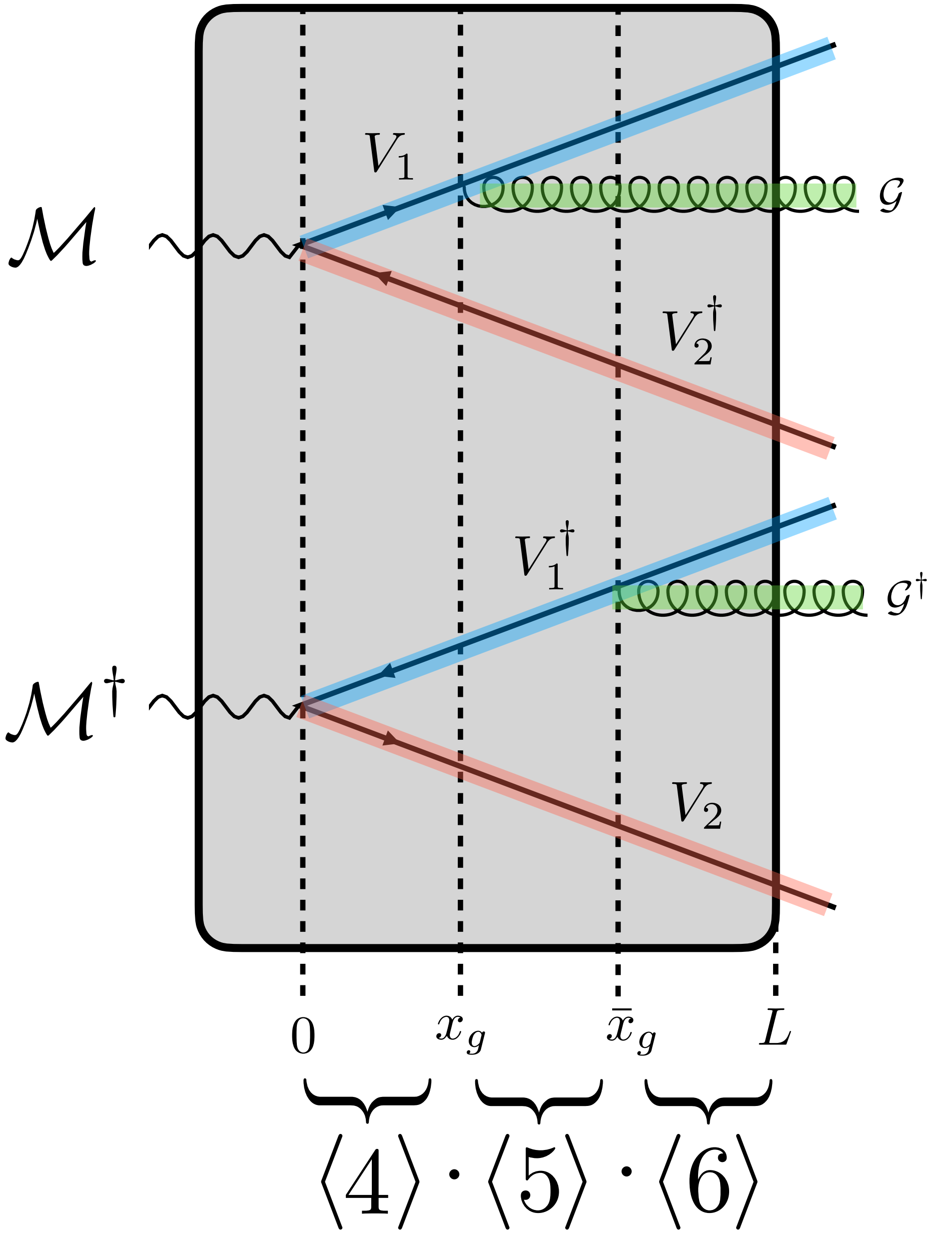
$0 < x_g < \bar{x}_g < L$



➡ We assume the medium average to be **local in time and color**:

$$\langle \mathcal{A}^a(t)\mathcal{A}^b(\bar{t})\rangle \sim \delta^{ab}\delta(t-\bar{t})$$

And thus, the correlator can be decomposed into **color-singlet regions**:

$$\langle|\mathcal{M}_q|^2\rangle \propto \int \langle \mathrm{Tr}\{\mathcal{G}(\bar{x}_g, x_g)U(\bar{x}_g, x_g)\}\rangle \langle \mathrm{Tr}\{\mathcal{G}(L,\bar{x}_g)\mathcal{G}(L,\bar{x}_g)\}\rangle$$

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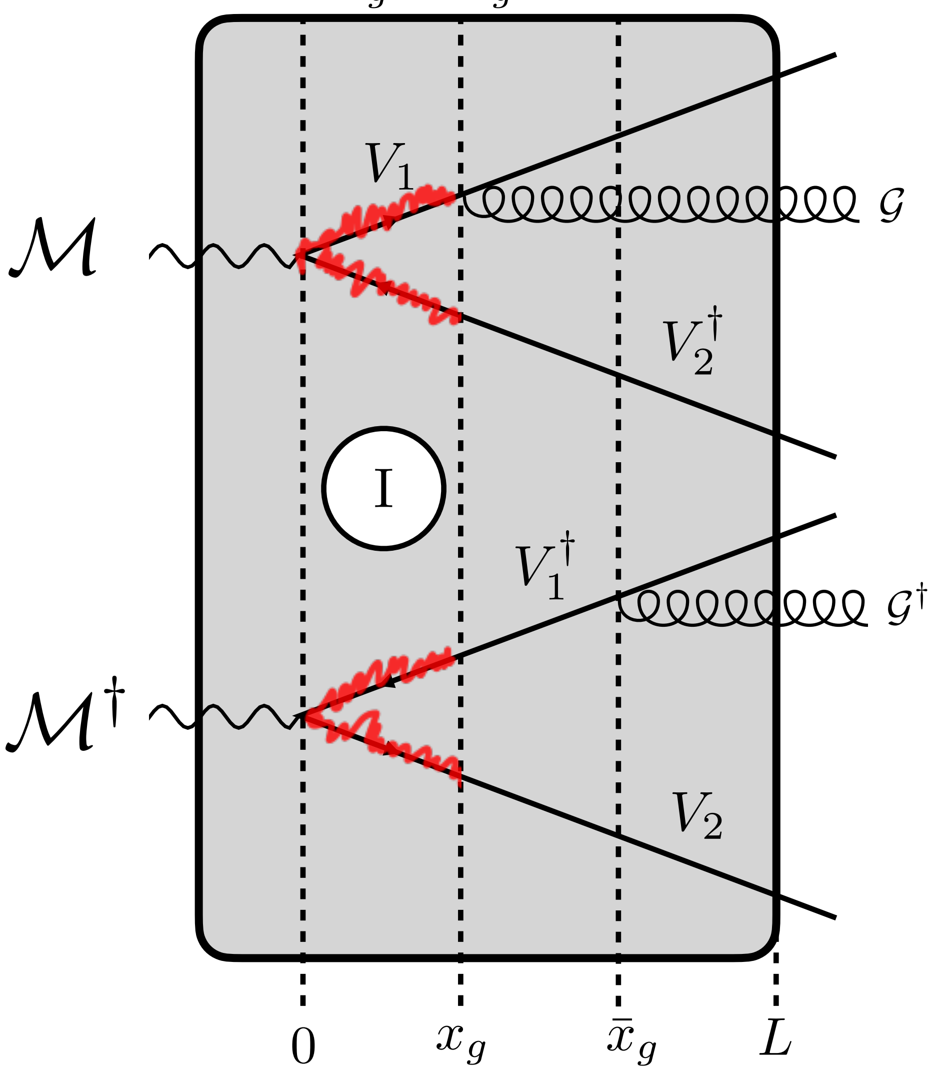

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- **REGION I**: Trivial (color rotations in $\mathcal{M}$ and $\mathcal{M}^\dagger$ **cancel**)

$$V_2^\dagger(0, x_g) V_2(x_g, 0) = \mathbb{I} \;, \quad V_1(x_g, 0) V_1^\dagger(0, x_g) = \mathbb{I}$$

Y. Mehtar-Tani and K. Tywoniuk; JHEP 10 (2012) 197
Y. Mehtar-Tani and K. Tywoniuk; JHEP 01 (2013) 031

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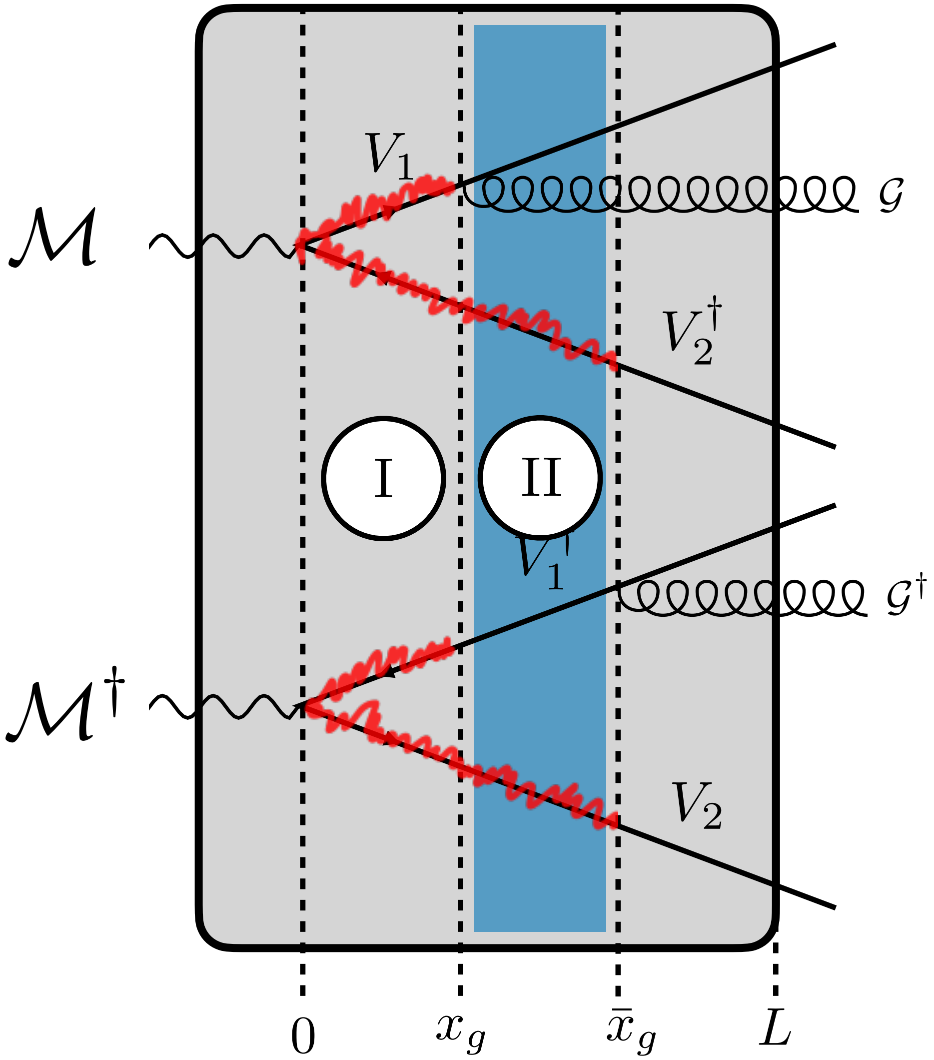

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- **REGION II**: Gluon decoherence, encoded in:

$$\mathcal{K}(\mathbf{x},\bar{x}_g;\mathbf{y},x_g) \equiv \langle \mathrm{Tr}\left\{\mathcal{G}(\bar{x}_g,x_g)U(\bar{x}_g,x_g)\right\}\rangle$$
$$= \int_{\mathbf{r}(x_g)=\mathbf{y}}^{\mathbf{r}(\bar{x}_g)=\mathbf{x}} \mathcal{D}\mathbf{r}\,\exp\left\{i\int_{x_g}^{\bar{x}_g} ds\left[\frac{p^+}{2}\dot{\mathbf{r}}^2 + in(s)\sigma(\mathbf{r})\right]\right\}$$

where $\sigma(\mathbf{r}) = 2g^2(\gamma(\mathbf{0}) - \gamma(\mathbf{r}))$ is the dipole cross section

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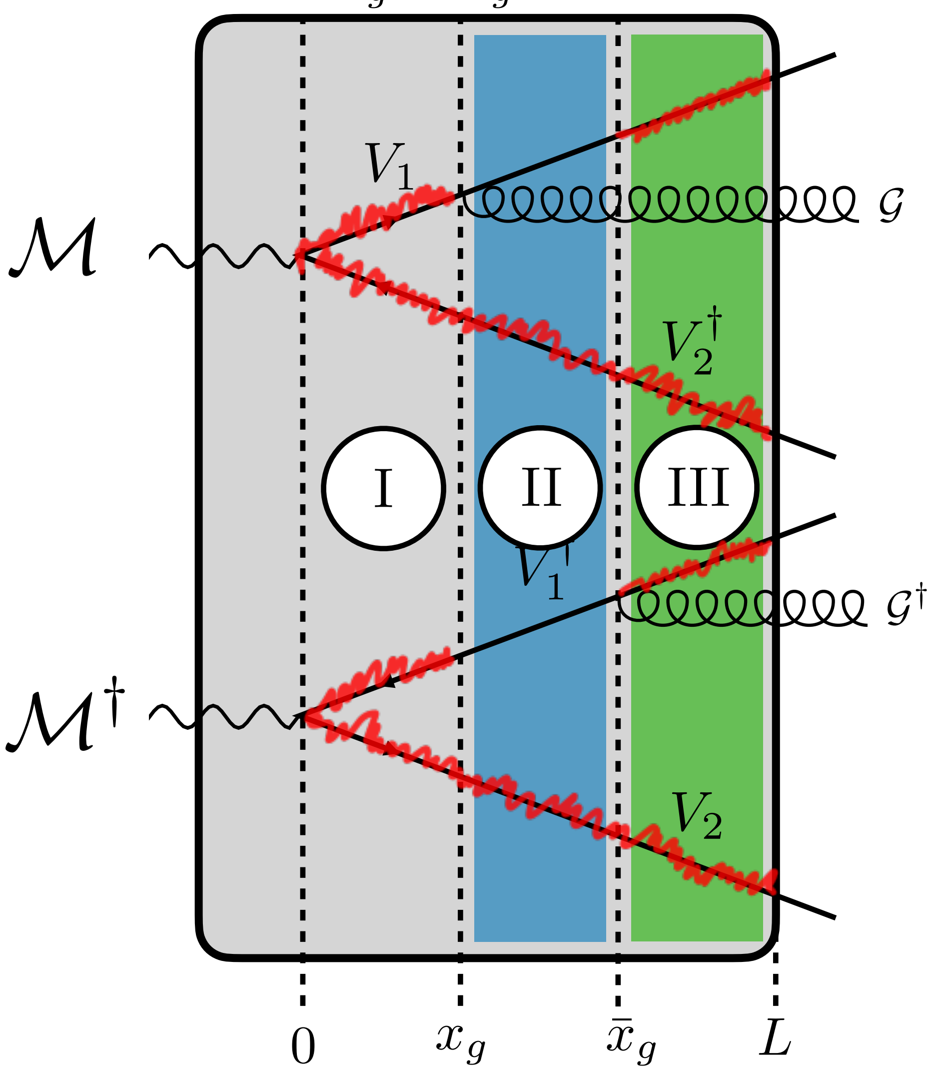

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- **REGION III**: Gluon momentum broadening

$$\langle \mathrm{Tr}\left\{\mathcal{G}(L,\bar{x}_g)\mathcal{G}(L,\bar{x}_g)\right\}\rangle = \int d\mathbf{r}\, e^{i(\mathbf{p}-\mathbf{q})\cdot\mathbf{r}} \exp\left\{-\frac{1}{2}\int_{\bar{x}_g}^{L} ds\, n(s)\sigma(\mathbf{r})\right\}$$

Y. Mehtar-Tani and K. Tywoniuk; JHEP 10 (2012) 197
Y. Mehtar-Tani and K. Tywoniuk; JHEP 01 (2013) 031

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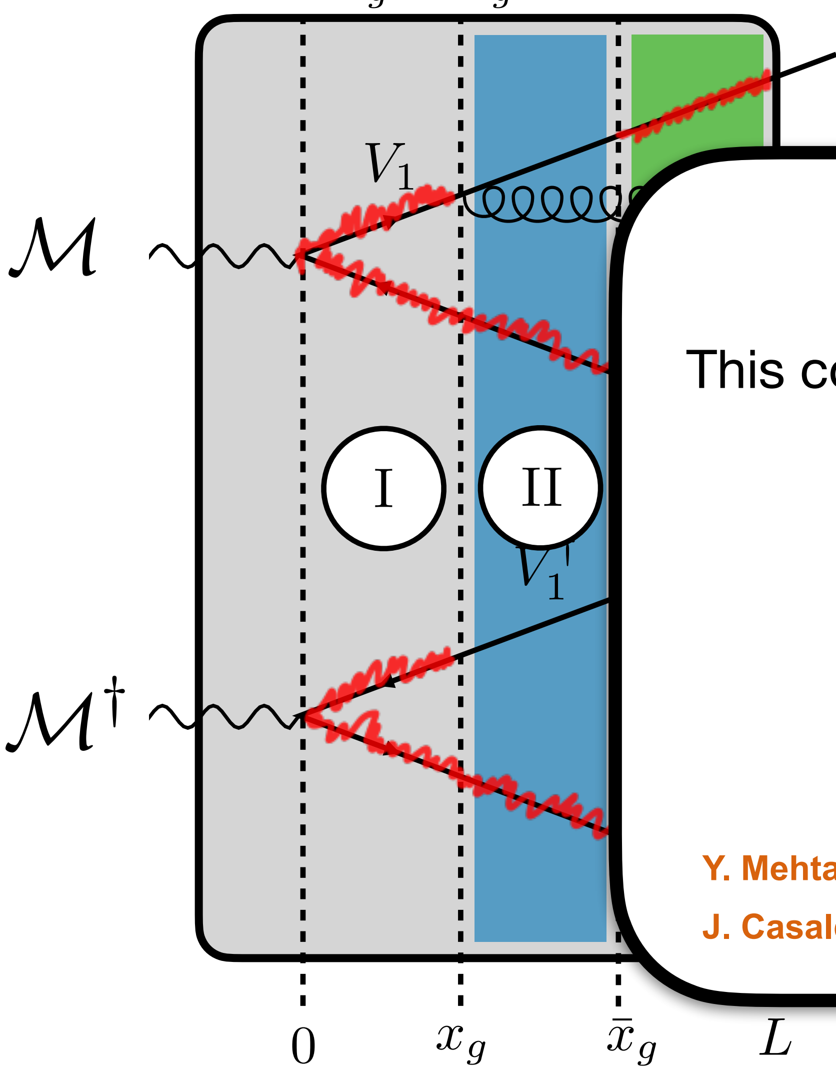

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...osed into **color-singlet**

$\langle\mathrm{Tr}\{\mathcal{G}(L,\bar{x}_g)\mathcal{G}(L,\bar{x}_g)\}\rangle$

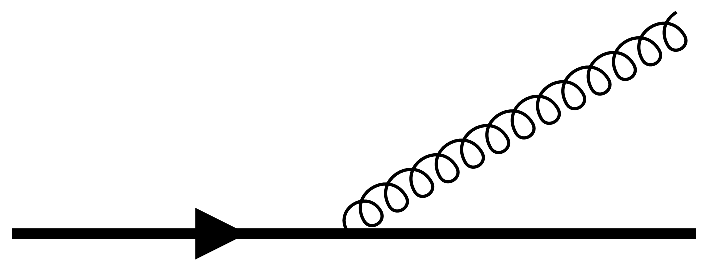
... and $\mathcal{M}^\dagger$ **cancel**)

$\ldots, 0)V_1^\dagger(0,x_g) = \mathbb{I}$

...ded in:

The antiquark plays no role!

This contribution thus reduces to BDMPS-Z spectrum:



Y. Mehtar-Tani and K. Tywoniuk; JHEP 01 (2013) 031
J. Casalderrey-Solana and E. Iancu; JHEP 08 (2011) 015

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Step II: Finite antenna formation time

➡ As we allow a finite formation time for the antenna, the Wilson lines in $\mathcal{M}$ and $\mathcal{M}^\dagger$ become mis-aligned:

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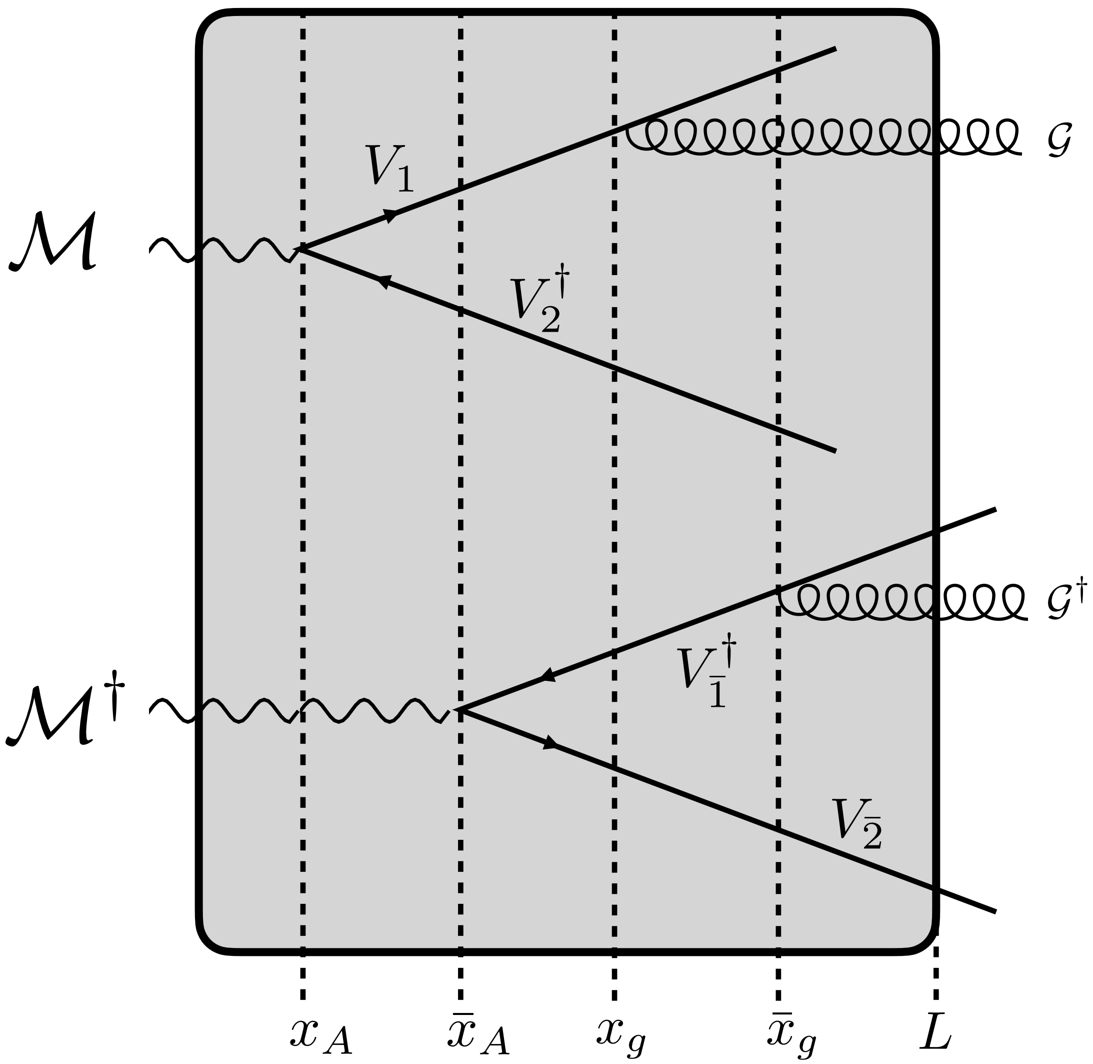
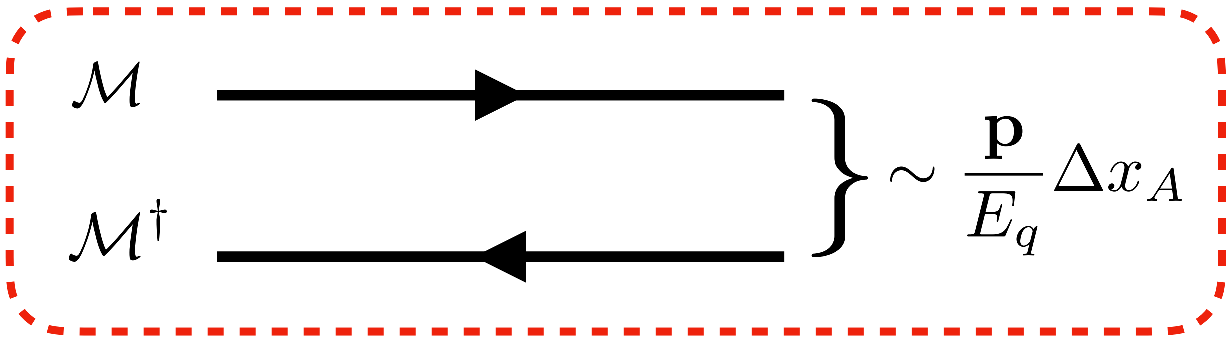


Wilson line cancellations are spoiled!



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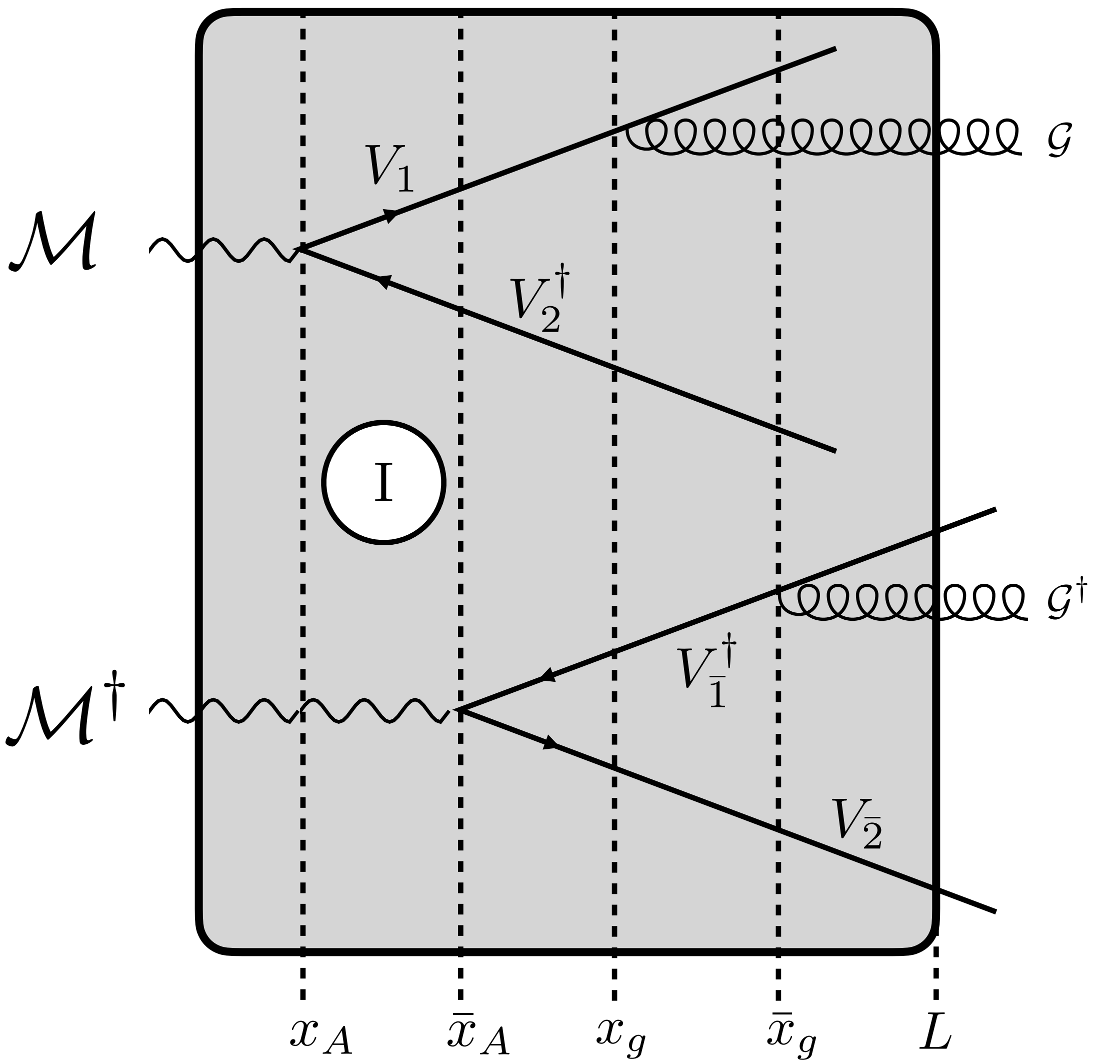

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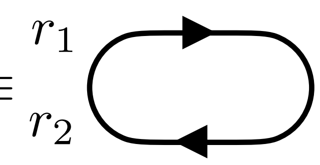
- **REGION I**: Antenna formation region: only 1 singlet

$$\left\langle \mathrm{Tr}\left\{V_1(\bar{x}_A,x_A)V_2^\dagger(x_A,\bar{x}_A)\right\}\right\rangle\equiv$$



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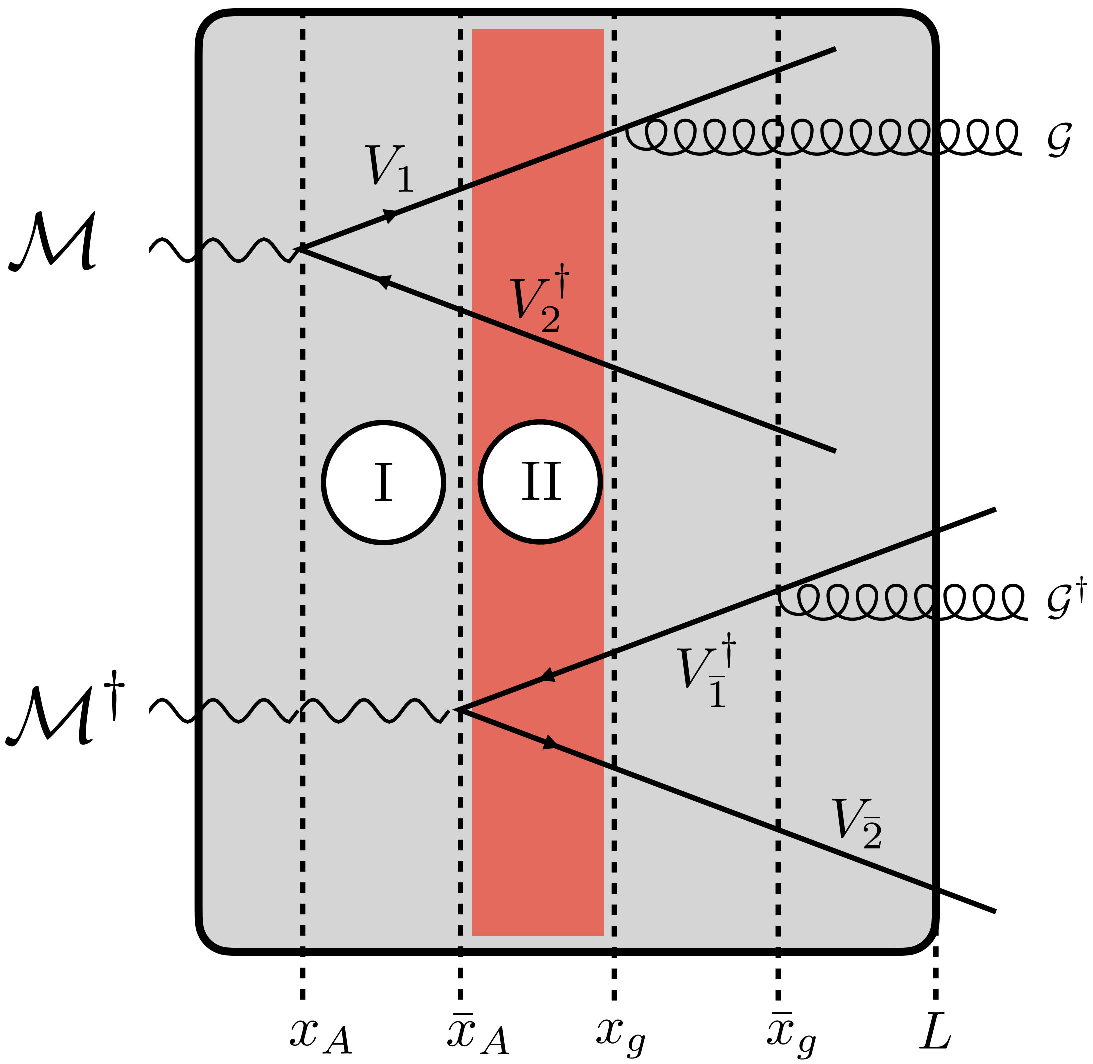

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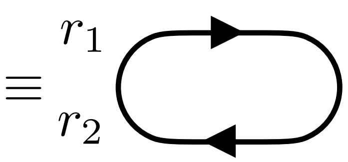

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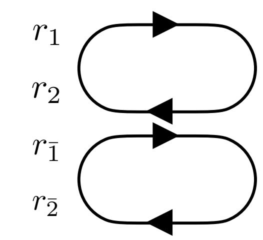

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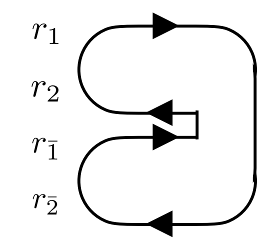

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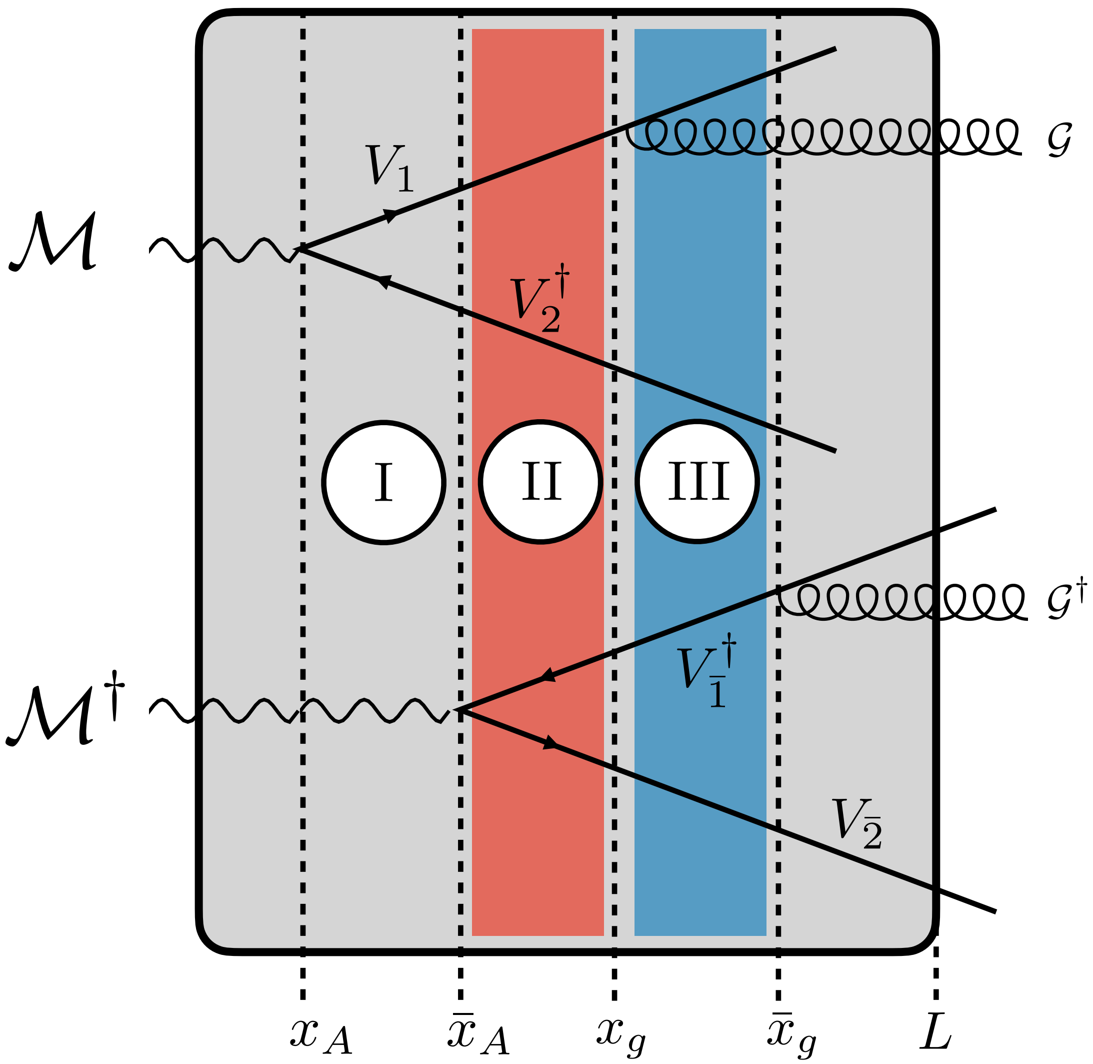

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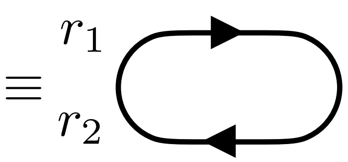

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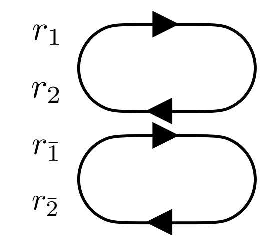

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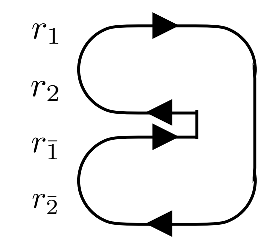
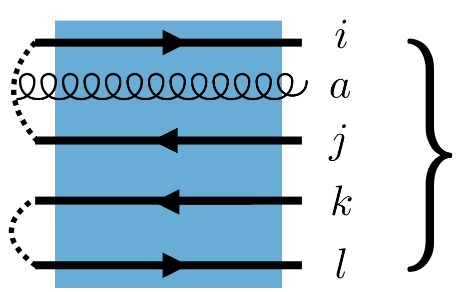


- **REGION III**: 4 (fundamental) + 1 (adjoint) open color indices



Projection into a dimension-6 singlet basis

Step II: Finite antenna formation time

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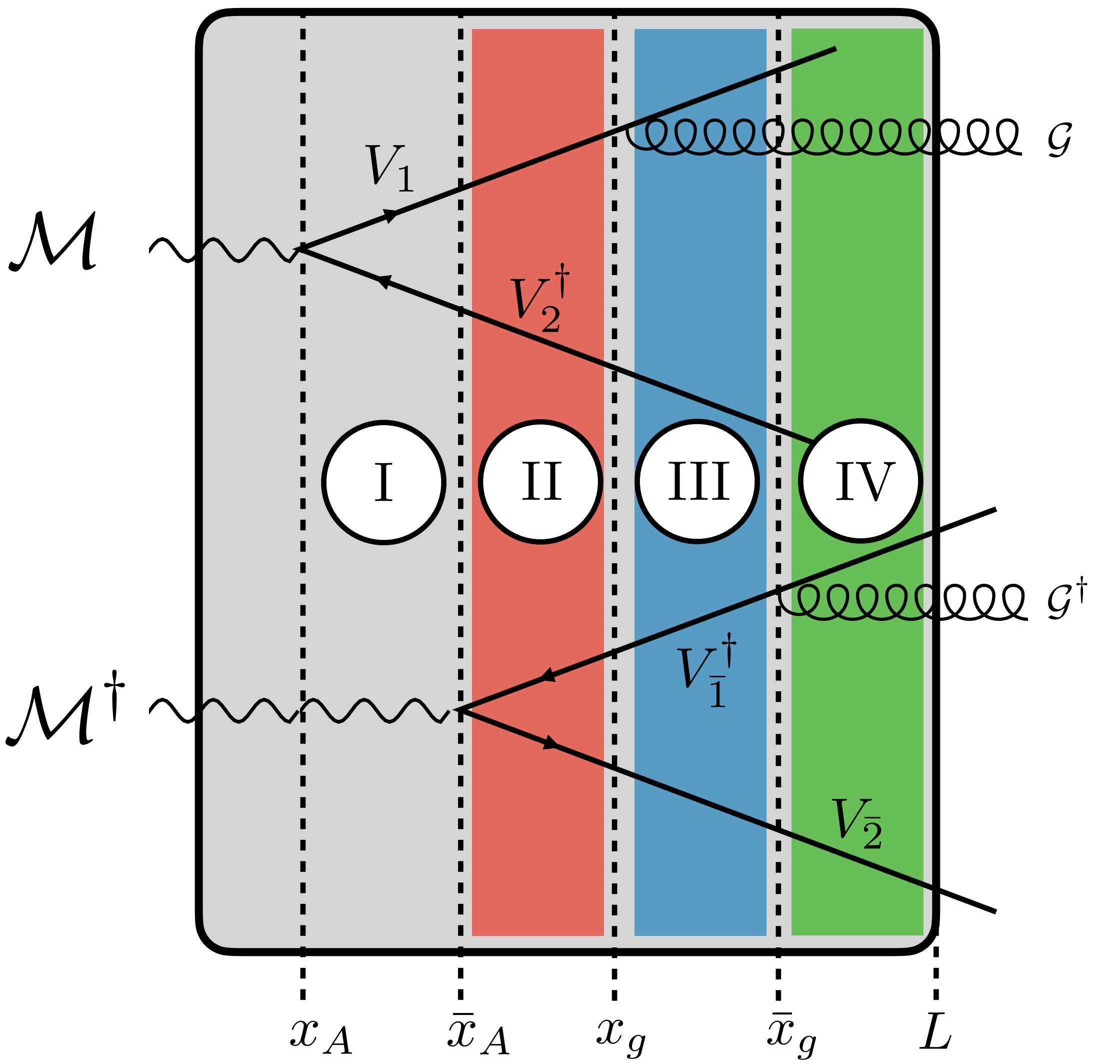

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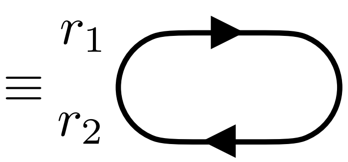

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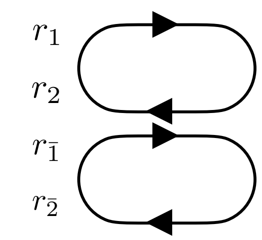

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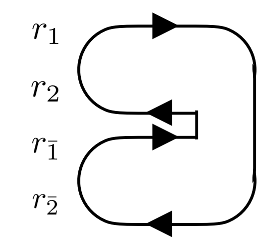
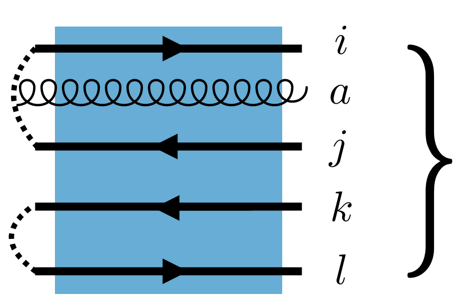
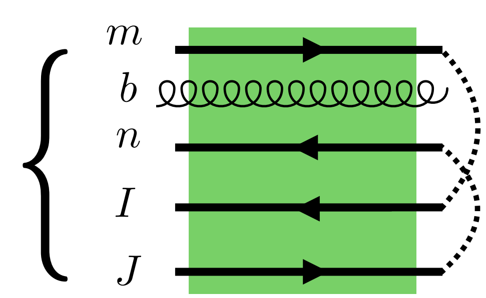
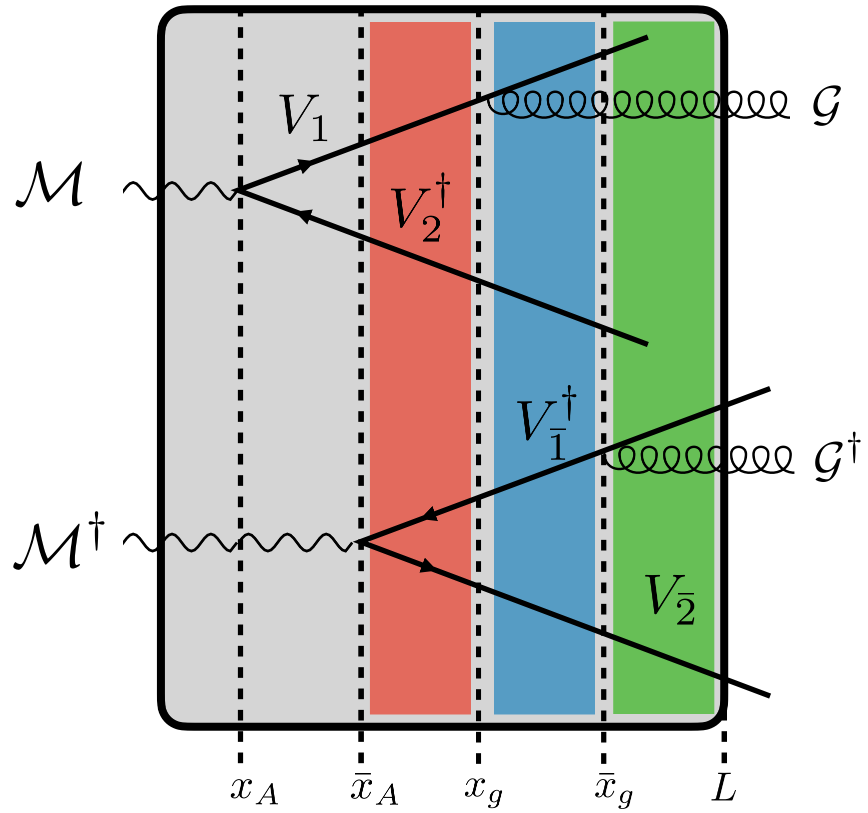


- **REGIONS III & IV**: 4 (fundamental) + 1 (adjoint) open color indices



Projection into a dimension-6 singlet basis





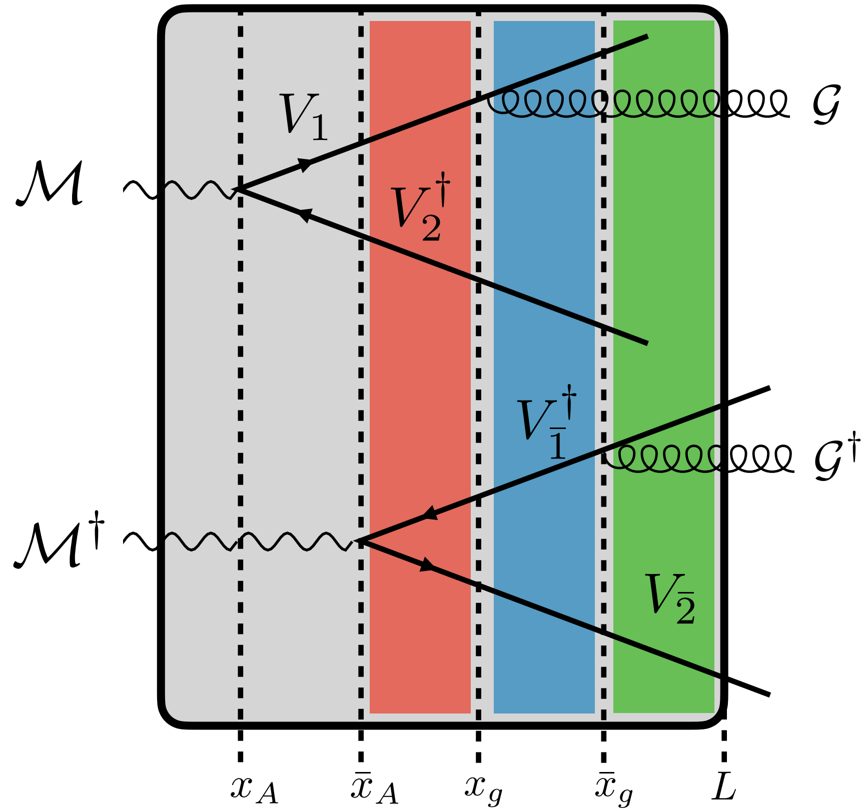
➡ After extensive algebra, one finally arrives at:

$$\left\langle |\mathcal{M}_q|^2 \right\rangle \propto \int \left\langle \mathrm{Tr}\left\{ V_1 V_2^\dagger \right\} \right\rangle_{(\bar{x}_A, x_A)} \langle \mathbf{A} \rangle_{(x_g, \bar{x}_A)} \cdot \mathbf{g}_2 \cdot \left\langle \mathcal{G}^{ab} \mathbf{B}^{ba} \right\rangle_{(\bar{x}_g, x_g)} \cdot \mathbf{g}_4 \cdot \left\langle \mathcal{G}^{cd} \mathbf{C}^{de} \mathcal{G}^{\dagger ec} \right\rangle_{(L, \bar{x}_g)}$$

with:

$$\mathbf{A}^{\mathrm{T}} = \begin{pmatrix} \mathrm{Tr}\{V_1 V_2^\dagger\}\mathrm{Tr}\{V_{\bar{2}} V_{\bar{1}}^\dagger\} \\ \mathrm{Tr}\{V_1 V_2^\dagger V_{\bar{2}} V_{\bar{1}}^\dagger\} \end{pmatrix}, \quad (\mathbf{B}^{ab})^{\mathrm{T}} = \begin{pmatrix} \mathrm{Tr}\{t^a V_2^\dagger V_{\bar{2}} V_{\bar{1}}^\dagger t^b V_1\} & \mathrm{Tr}\{V_{\bar{1}}^\dagger t^b V_1 t^a\}\mathrm{Tr}\{V_{\bar{2}} V_2^\dagger\} \\ \mathrm{Tr}\{V_1 t^a V_2^\dagger\}\mathrm{Tr}\{V_{\bar{1}}^\dagger t^b V_{\bar{2}}\} & \mathrm{Tr}\{t^a V_{\bar{1}}^\dagger t^b V_{\bar{2}} V_2^\dagger V_1\} \\ \mathrm{Tr}\{V_2^\dagger t^b V_1 t^a\}\mathrm{Tr}\{V_{\bar{2}} V_{\bar{1}}^\dagger\} & \mathrm{Tr}\{t^a V_{\bar{1}}^\dagger V_{\bar{2}} V_2^\dagger t^b V_1\} \\ \mathrm{Tr}\{t^b V_{\bar{2}} V_{\bar{1}}^\dagger V_1 t^a V_2^\dagger\} & \mathrm{Tr}\{V_1 t^a V_{\bar{1}}^\dagger\}\mathrm{Tr}\{V_2^\dagger t^b V_{\bar{2}}\} \end{pmatrix}, \quad \mathbf{C}^{de} = \begin{pmatrix} \mathrm{Tr}\{t^e V_{\bar{1}}^\dagger V_1 t^d\}\mathrm{Tr}\{V_2^\dagger V_{\bar{2}}\} \\ \mathrm{Tr}\{t^e V_{\bar{1}}^\dagger V_1 V_2^\dagger V_{\bar{2}} t^d\} \\ \mathrm{Tr}\{t^e V_{\bar{1}}^\dagger V_1 t^d V_2^\dagger V_{\bar{2}}\} \\ \mathrm{Tr}\{V_1 t^e V_{\bar{1}}^\dagger\}\mathrm{Tr}\{V_{\bar{2}} t^d V_2^\dagger\} \end{pmatrix}$$

and: $$\mathbf{g}_2 = \begin{pmatrix} N_c & -1 \\ -1 & N_c \end{pmatrix}, \quad \mathbf{g}_4 = \begin{pmatrix} (N_c^2-2) & -N_c & -N_c & 2 \\ -N_c & (N_c^2-2) & 2 & -N_c \\ -N_c & 2 & (N_c^2-2) & -N_c \\ 2 & -N_c & -N_c & (N_c^2-2) \end{pmatrix}$$



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- *What is the leading-Nc contribution?*

Computing the large-Nc limit

➡ In the large-Nc limit, $\langle \mathrm{Tr}_1\{V^\dagger V ... V^\dagger V\} ... \mathrm{Tr}_n\{V^\dagger V ... V^\dagger V\}\rangle \longrightarrow \langle \mathrm{Tr}_1\{V^\dagger V ... V^\dagger V\}\rangle ... \langle \mathrm{Tr}_n\{V^\dagger V ... V^\dagger V\}\rangle \propto N_c^n$

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- In the fundamental representation:

$$\langle |\mathcal{M}_q|^2 \rangle \propto \int \mathcal{D}\mathbf{r}_3\, \mathcal{D}\mathbf{r}_{\bar{3}} \left\langle \mathrm{Tr}\left\{ V_1 V_2^\dagger \right\} \right\rangle_{(\bar{x}_A, x_A)} \langle \mathbf{A} \rangle_{(x_g, \bar{x}_A)} \cdot \mathbf{g}_2 \cdot \langle \mathcal{G}^{ab} \mathbf{B}_f^{ba} \rangle_{(\bar{x}_g, x_g)} \cdot \mathbf{g}_6 \cdot \langle \mathcal{G}^{cd} \mathbf{C}_f^{de} \mathcal{G}^{\dagger ec} \rangle_{(L, \bar{x}_g)}$$

with:

$$\mathbf{B}_f^{\mathrm{T}} = \begin{pmatrix} \mathrm{Tr}\{V_1 V_3^\dagger\}\mathrm{Tr}\{V_3 V_2^\dagger V_{\bar{2}} V_{\bar{1}}^\dagger\} & \mathrm{Tr}\{V_1 V_3^\dagger\}\mathrm{Tr}\{V_3 V_{\bar{1}}^\dagger\}\mathrm{Tr}\{V_{\bar{2}} V_2^\dagger\} \\ \mathrm{Tr}\{V_3 V_2^\dagger V_1 V_3^\dagger V_{\bar{2}} V_{\bar{1}}^\dagger\} & \mathrm{Tr}\{V_3 V_{\bar{1}}^\dagger\}\mathrm{Tr}\{V_1 V_3^\dagger V_{\bar{2}} V_2^\dagger\} \\ \mathrm{Tr}\{V_2^\dagger V_{\bar{2}} V_{\bar{1}}^\dagger V_1\} & \mathrm{Tr}\{V_1 V_{\bar{1}}^\dagger\}\mathrm{Tr}\{V_{\bar{2}} V_2^\dagger\} \\ \mathrm{Tr}\{V_1 V_3^\dagger\}\mathrm{Tr}\{V_3 V_2^\dagger\}\mathrm{Tr}\{V_{\bar{2}} V_{\bar{1}}^\dagger\} & \mathrm{Tr}\{V_1 V_3^\dagger\}\mathrm{Tr}\{V_3 V_{\bar{1}}^\dagger V_{\bar{2}} V_2^\dagger\} \\ \mathrm{Tr}\{V_3 V_2^\dagger\}\mathrm{Tr}\{V_1 V_3^\dagger V_{\bar{2}} V_{\bar{1}}^\dagger\} & \mathrm{Tr}\{V_3 V_{\bar{1}}^\dagger V_1 V_3^\dagger V_{\bar{2}} V_2^\dagger\} \\ \mathrm{Tr}\{V_1 V_2^\dagger\}\mathrm{Tr}\{V_{\bar{2}} V_{\bar{1}}^\dagger\} & \mathrm{Tr}\{V_{\bar{1}}^\dagger V_{\bar{2}} V_2^\dagger V_1\} \end{pmatrix}, \quad \mathbf{C}_f = \begin{pmatrix} \mathrm{Tr}\{V_3^\dagger V_{\bar{3}} V_{\bar{1}}^\dagger V_1\}\mathrm{Tr}\{V_2^\dagger V_{\bar{2}}\}\mathrm{Tr}\{V_{\bar{3}}^\dagger V_3\} \\ \mathrm{Tr}\{V_3^\dagger V_{\bar{3}} V_{\bar{1}}^\dagger V_1 V_2^\dagger V_{\bar{2}}\}\mathrm{Tr}\{V_{\bar{3}}^\dagger V_3\} \\ \mathrm{Tr}\{V_2^\dagger V_{\bar{2}}\}\mathrm{Tr}\{V_{\bar{1}}^\dagger V_1\} \\ \mathrm{Tr}\{V_3^\dagger V_{\bar{3}} V_{\bar{1}}^\dagger V_1\}\mathrm{Tr}\{V_2^\dagger V_{\bar{2}} V_{\bar{3}}^\dagger V_3\} \\ \mathrm{Tr}\{V_3^\dagger V_{\bar{3}} V_{\bar{1}}^\dagger V_1 V_{\bar{3}}^\dagger V_3 V_2^\dagger V_{\bar{2}}\} \\ \mathrm{Tr}\{V_{\bar{1}}^\dagger V_1 V_2^\dagger V_{\bar{2}}\} \end{pmatrix}$$

Nc-scaling = power of explicit Nc factor + number of Wilson line traces

- In the large-Nc limit, $\langle \mathrm{Tr}_1\{V^\dagger V ... V^\dagger V\} ... \mathrm{Tr}_n\{V^\dagger V ... V^\dagger V\}\rangle \longrightarrow \langle \mathrm{Tr}_1\{V^\dagger V ... V^\dagger V\}\rangle ... \langle \mathrm{Tr}_n\{V^\dagger V ... V^\dagger V\}\rangle \propto N_c^n$
- In the fundamental representation:

$$\langle |\mathcal{M}_q|^2\rangle \propto \int \mathcal{D}\mathbf{r}_3\, \mathcal{D}\mathbf{r}_{\bar{3}} \left\langle \mathrm{Tr}\left\{V_1 V_{\bar{2}}^\dagger\right\}\right\rangle_{(\bar{x}_A, x_A)} \langle \mathbf{A}\rangle_{(x_g, \bar{x}_A)} \cdot \mathbf{g}_2 \cdot \langle \mathcal{G}^{ab}\mathbf{B}_f^{ba}\rangle_{(\bar{x}_g, x_g)} \cdot \mathbf{g}_6 \cdot \langle \mathcal{G}^{cd}\mathbf{C}_f^{de}\mathcal{G}^{\dagger ec}\rangle_{(L, \bar{x}_g)}$$

with:

$$\mathbf{B}_f^{\mathrm{T}} = \begin{pmatrix} \mathrm{Tr}\{V_1 V_3^\dagger\}\mathrm{Tr}\{V_3 V_2^\dagger V_{\bar{2}} V_{\bar{1}}^\dagger\} & \mathrm{Tr}\{V_1 V_3^\dagger\}\mathrm{Tr}\{V_3 V_{\bar{1}}^\dagger\}\mathrm{Tr}\{V_{\bar{2}} V_2^\dagger\} \\ \mathrm{Tr}\{V_3 V_2^\dagger V_1 V_3^\dagger V_{\bar{2}} V_{\bar{1}}^\dagger\} & \mathrm{Tr}\{V_3 V_{\bar{1}}^\dagger\}\mathrm{Tr}\{V_1 V_3^\dagger V_{\bar{2}} V_2^\dagger\} \\ \mathrm{Tr}\{V_2^\dagger V_{\bar{2}} V_{\bar{1}}^\dagger V_1\} & \mathrm{Tr}\{V_1 V_{\bar{1}}^\dagger\}\mathrm{Tr}\{V_{\bar{2}} V_2^\dagger\} \\ \mathrm{Tr}\{V_1 V_3^\dagger\}\mathrm{Tr}\{V_3 V_2^\dagger\}\mathrm{Tr}\{V_{\bar{2}} V_{\bar{1}}^\dagger\} & \mathrm{Tr}\{V_1 V_3^\dagger\}\mathrm{Tr}\{V_3 V_{\bar{1}}^\dagger V_{\bar{2}} V_2^\dagger\} \\ \mathrm{Tr}\{V_3 V_2^\dagger\}\mathrm{Tr}\{V_1 V_3^\dagger V_{\bar{2}} V_{\bar{1}}^\dagger\} & \mathrm{Tr}\{V_3 V_{\bar{1}}^\dagger V_1 V_3^\dagger V_{\bar{2}} V_2^\dagger\} \\ \mathrm{Tr}\{V_1 V_2^\dagger\}\mathrm{Tr}\{V_{\bar{2}} V_{\bar{1}}^\dagger\} & \mathrm{Tr}\{V_{\bar{1}}^\dagger V_{\bar{2}} V_2^\dagger V_1\} \end{pmatrix}, \quad \mathbf{C}_f = \begin{pmatrix} \mathrm{Tr}\{V_3^\dagger V_{\bar{3}} V_{\bar{1}}^\dagger V_1\}\mathrm{Tr}\{V_2^\dagger V_{\bar{2}}\}\mathrm{Tr}\{V_{\bar{3}}^\dagger V_3\} \\ \mathrm{Tr}\{V_3^\dagger V_{\bar{3}} V_{\bar{1}}^\dagger V_1 V_2^\dagger V_{\bar{2}}\}\mathrm{Tr}\{V_{\bar{3}}^\dagger V_3\} \\ \mathrm{Tr}\{V_2^\dagger V_{\bar{2}}\}\mathrm{Tr}\{V_{\bar{1}}^\dagger V_1\} \\ \mathrm{Tr}\{V_3^\dagger V_{\bar{3}} V_{\bar{1}}^\dagger V_1\}\mathrm{Tr}\{V_2^\dagger V_{\bar{2}} V_{\bar{3}}^\dagger V_3\} \\ \mathrm{Tr}\{V_3^\dagger V_{\bar{3}} V_{\bar{1}}^\dagger V_1 V_{\bar{3}}^\dagger V_3 V_2^\dagger V_{\bar{2}}\} \\ \mathrm{Tr}\{V_{\bar{1}}^\dagger V_1 V_2^\dagger V_{\bar{2}}\} \end{pmatrix}$$

Nc-scaling = power of explicit Nc factor + number of Wilson line traces

- After some transformations, the leading Nc contribution can be reduced to **4 terms**:

$$\begin{aligned} \langle ... \rangle \underset{N_c \uparrow}{\rightarrow} \int \mathcal{D}\mathbf{r}_3\, \mathcal{D}\mathbf{r}_{\bar{3}}\, \langle S_{12}\rangle_{(\bar{x}_A, x_A)} \Big[& \langle S_{1\bar{1}}\, S_{\bar{2}2}\rangle_{(x_g, \bar{x}_A)} \langle S_{13} S_{3\bar{1}}\, S_{\bar{2}2}\rangle_{(\bar{x}_g, x_g)} \langle S_{2\bar{2}} S_{3\bar{3}}\, Q_{\bar{3}\,\bar{1}\,1\,3}\rangle_{(L, \bar{x}_g)} \\ & + \frac{2}{N_c} \langle S_{12}\, S_{\bar{2}\bar{1}}\rangle_{(x_g, \bar{x}_A)} \langle S_{13}\, S_{32}\, S_{\bar{2}\bar{1}}\rangle_{(\bar{x}_g, x_g)} \left\langle \mathrm{Tr}\{V_{\bar{2}} t^a V_2^\dagger\}\mathrm{Tr}\{V_3 t^a V_{\bar{3}}^\dagger\}\, Q_{\bar{3}\,\bar{1}\,1\,3}\right\rangle_{(L, \bar{x}_g)} \\ & + \frac{2}{N_c} \langle S_{12}\, S_{\bar{2}\bar{1}}\rangle_{(x_g, \bar{x}_A)} \left\langle \mathrm{Tr}\{V_3 t^a V_{\bar{1}}^\dagger\}\mathrm{Tr}\{V_{\bar{2}} t^a V_2^\dagger\}\, S_{13}\right\rangle_{(\bar{x}_g, x_g)} \langle S_{2\bar{2}} S_{3\bar{3}}\, Q_{\bar{3}\,\bar{1}\,1\,3}\rangle_{(L, \bar{x}_g)} \\ & + \frac{2}{N_c} \left\langle \mathrm{Tr}\{V_1 t^a V_{\bar{1}}^\dagger\}\mathrm{Tr}\{V_{\bar{2}} t^a V_2^\dagger\}\right\rangle_{(x_g, \bar{x}_A)} \langle S_{13} S_{3\bar{1}}\, S_{\bar{2}2}\rangle_{(\bar{x}_g, x_g)} \langle S_{2\bar{2}} S_{3\bar{3}}\, Q_{\bar{3}\,\bar{1}\,1\,3}\rangle_{(L, \bar{x}_g)} \Big] \end{aligned}$$

Where we defined: $S_{ij} = \frac{1}{N_c}\mathrm{Tr}\left\{V_i V_j^\dagger\right\}$ and $Q_{ijkl} = \frac{1}{N_c}\mathrm{Tr}\left\{V_i V_j^\dagger V_k V_l^\dagger\right\}$

Large Nc limit (I)

- *How can we interpret this result?*
  - ➡ Let us consider the first of these terms:

$$\langle S_{12}\rangle_{(\bar{x}_A,x_A)}\,\langle S_{1\bar{1}}\,S_{\bar{2}2}\rangle_{(x_g,\bar{x}_A)}\,\langle S_{13}S_{3\bar{1}}\,S_{\bar{2}2}\rangle_{(\bar{x}_g,x_g)}\langle S_{2\bar{2}}S_{3\bar{3}}\,Q_{\bar{3}\,\bar{1}\,1\,3}\rangle_{(L,\bar{x}_g)}$$

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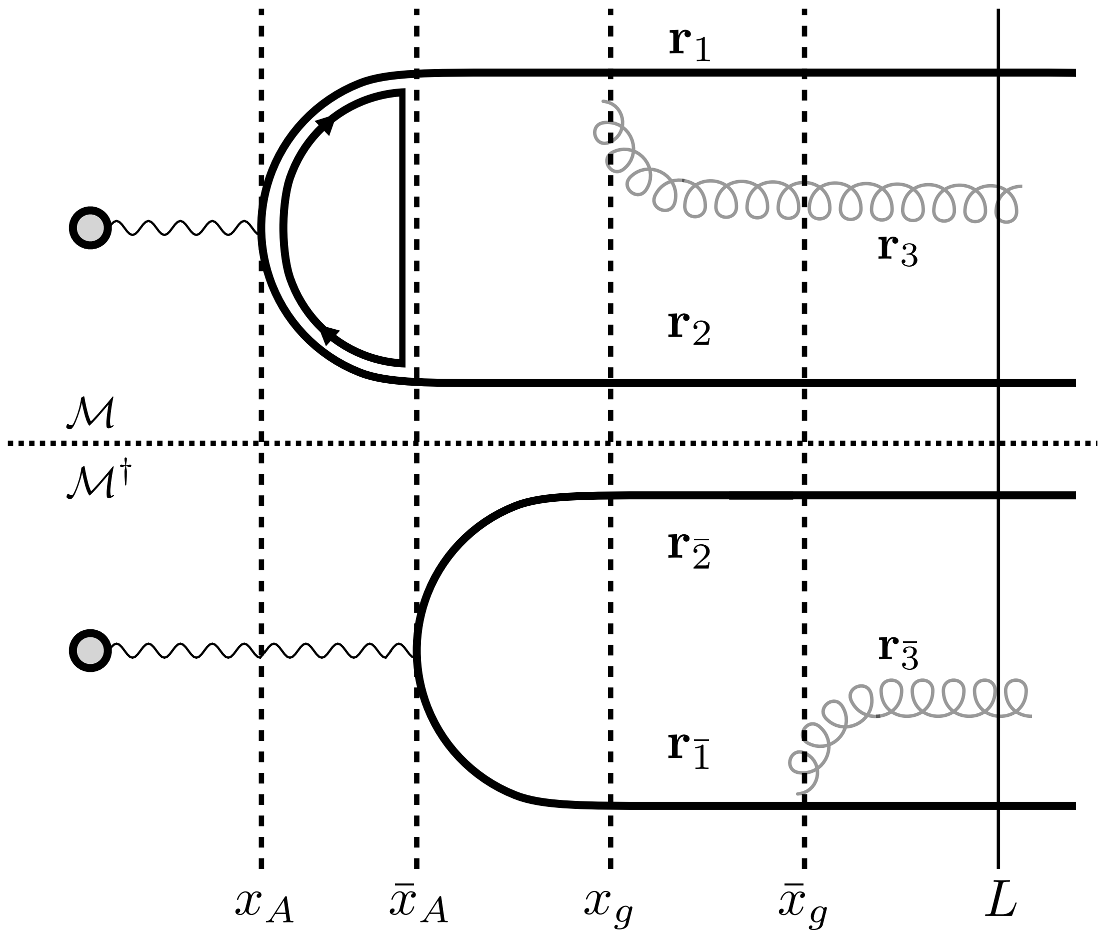
  - ➡ Let us consider the first of these terms:

$$\boxed{\langle S_{12}\rangle_{(\bar{x}_A,x_A)}}\langle S_{1\bar{1}}\, S_{\bar{2}2}\rangle_{(x_g,\bar{x}_A)}\,\langle S_{13}S_{3\bar{1}}\, S_{\bar{2}2}\rangle_{(\bar{x}_g,x_g)}\langle S_{2\bar{2}}S_{3\bar{3}}\, Q_{\bar{3}\,\bar{1}\,1\,3}\rangle_{(L,\bar{x}_g)}$$



(Note that the quark/antiquark positions are mirrored with respect to separation between $\mathcal{M}$ and $\mathcal{M}^\dagger$)

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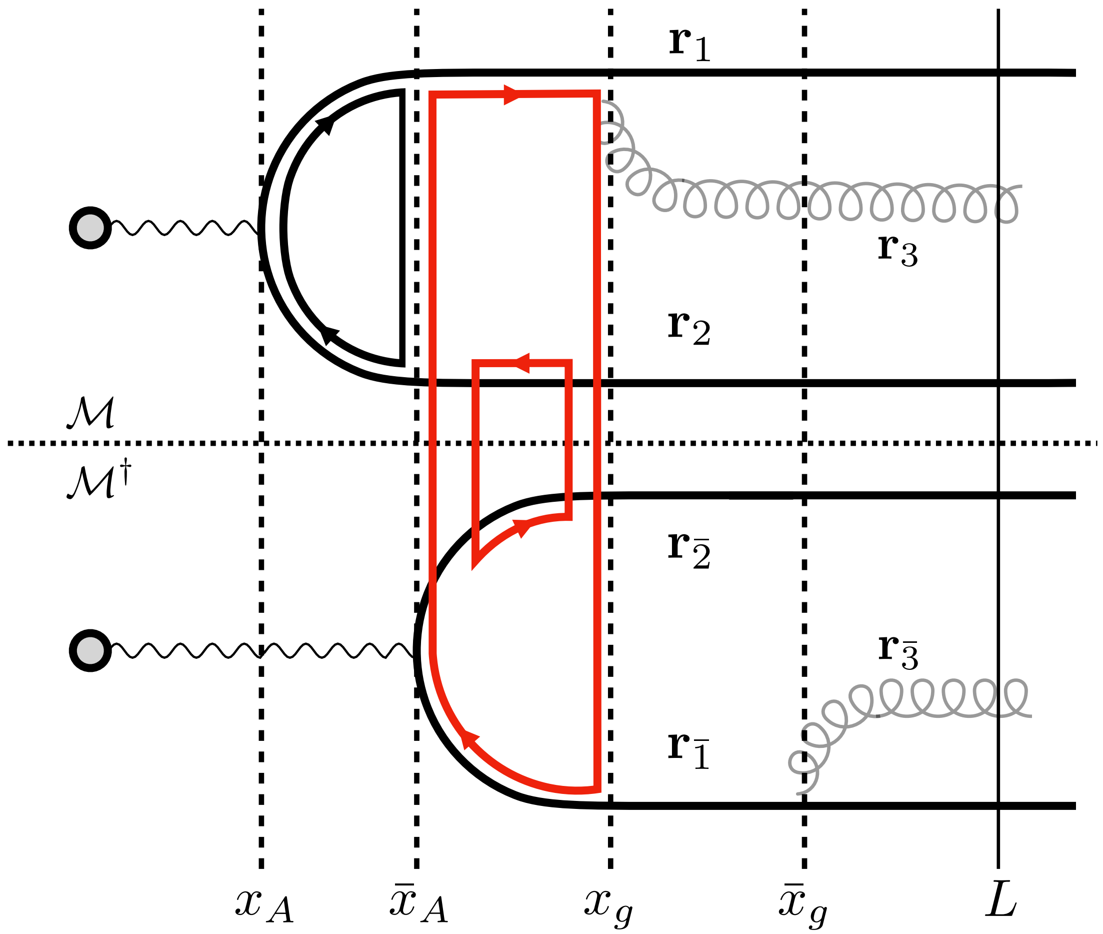
$$\langle S_{12}\rangle_{(\bar{x}_A,x_A)}\langle S_{1\bar{1}}\,S_{\bar{2}2}\rangle_{(x_g,\bar{x}_A)}\langle S_{13}S_{3\bar{1}}\,S_{\bar{2}2}\rangle_{(\bar{x}_g,x_g)}\langle S_{2\bar{2}}S_{3\bar{3}}\,Q_{\bar{3}\,\bar{1}\,1\,3}\rangle_{(L,\bar{x}_g)}$$



(Note that the quark/antiquark positions are mirrored with respect to separation between $\mathcal{M}$ and $\mathcal{M}^\dagger$)

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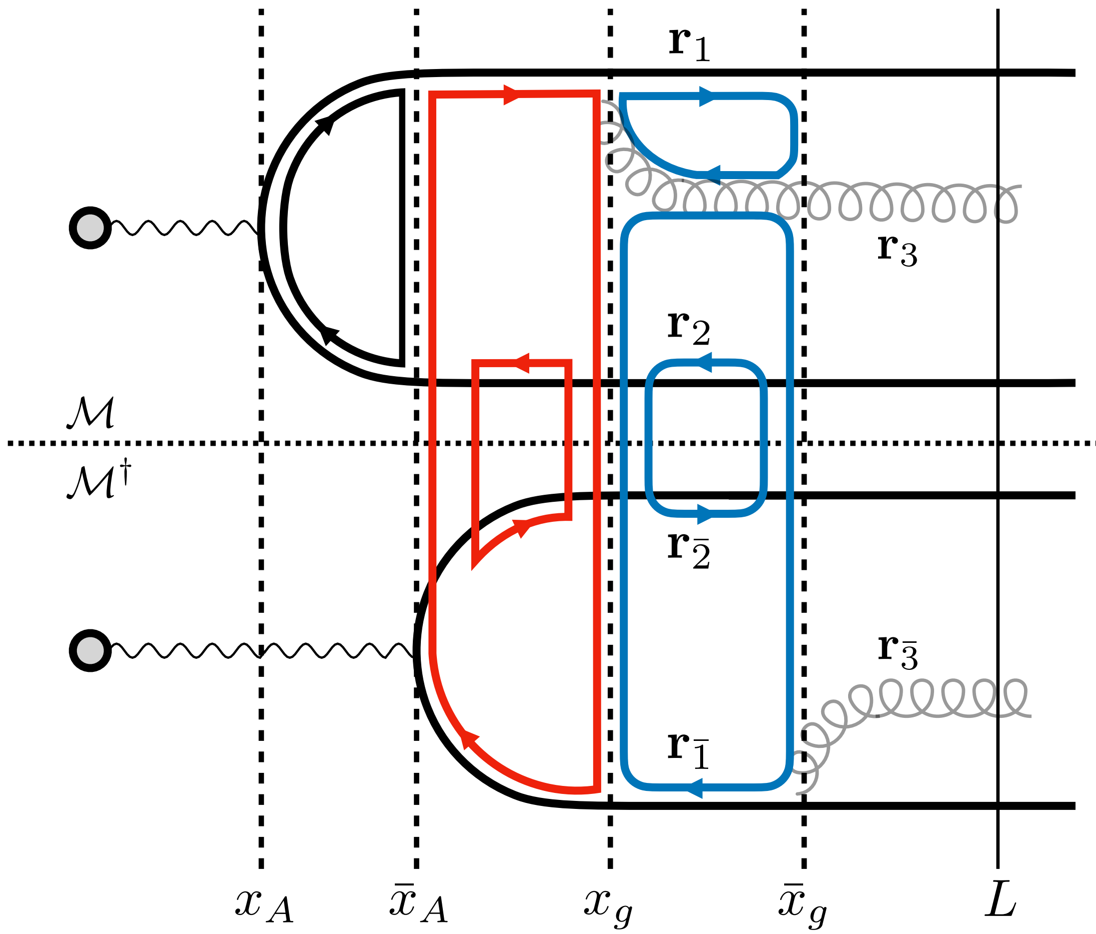

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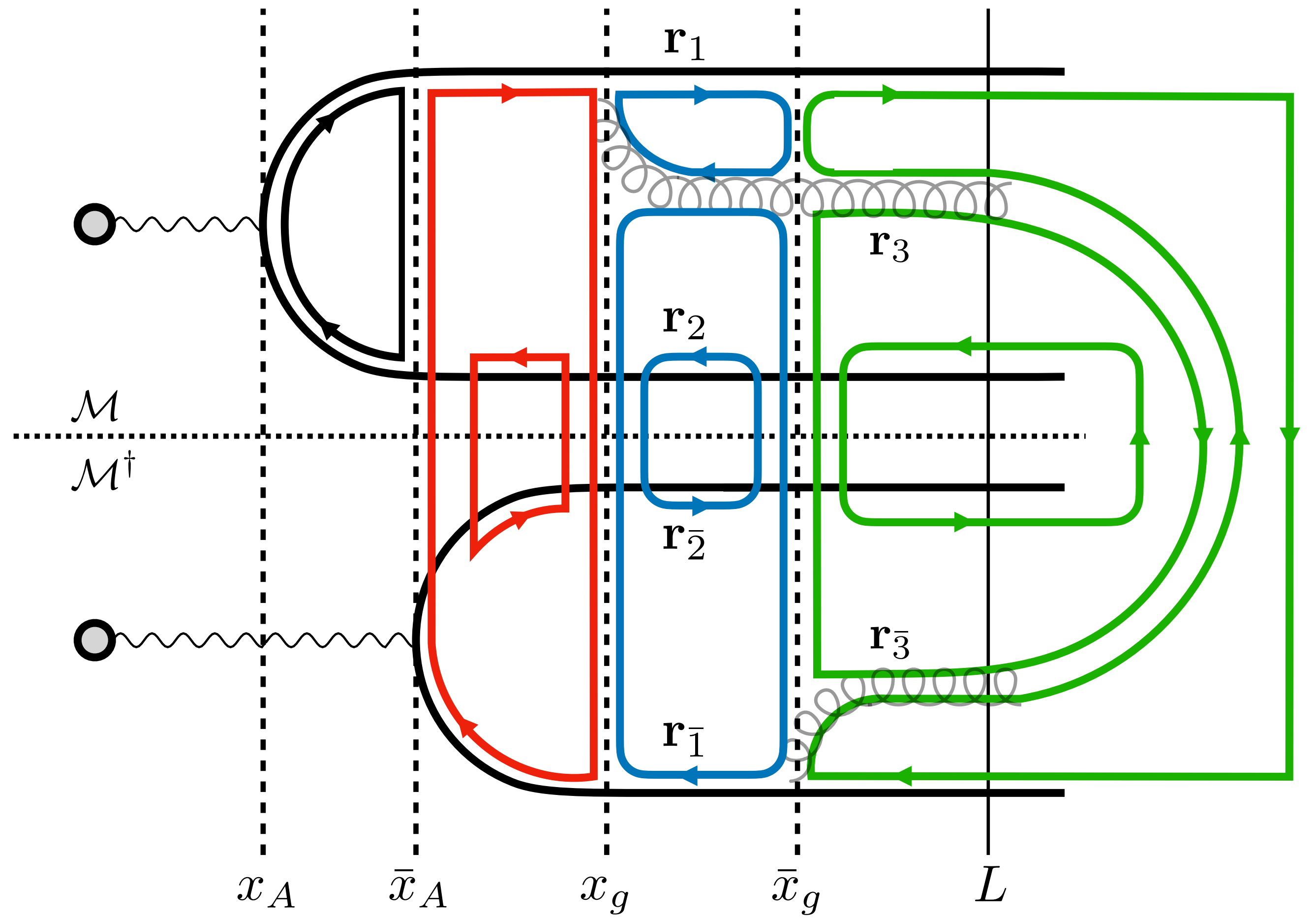

(Note that the quark/antiquark positions are mirrored with respect to separation between $\mathcal{M}$ and $\mathcal{M}^\dagger$)

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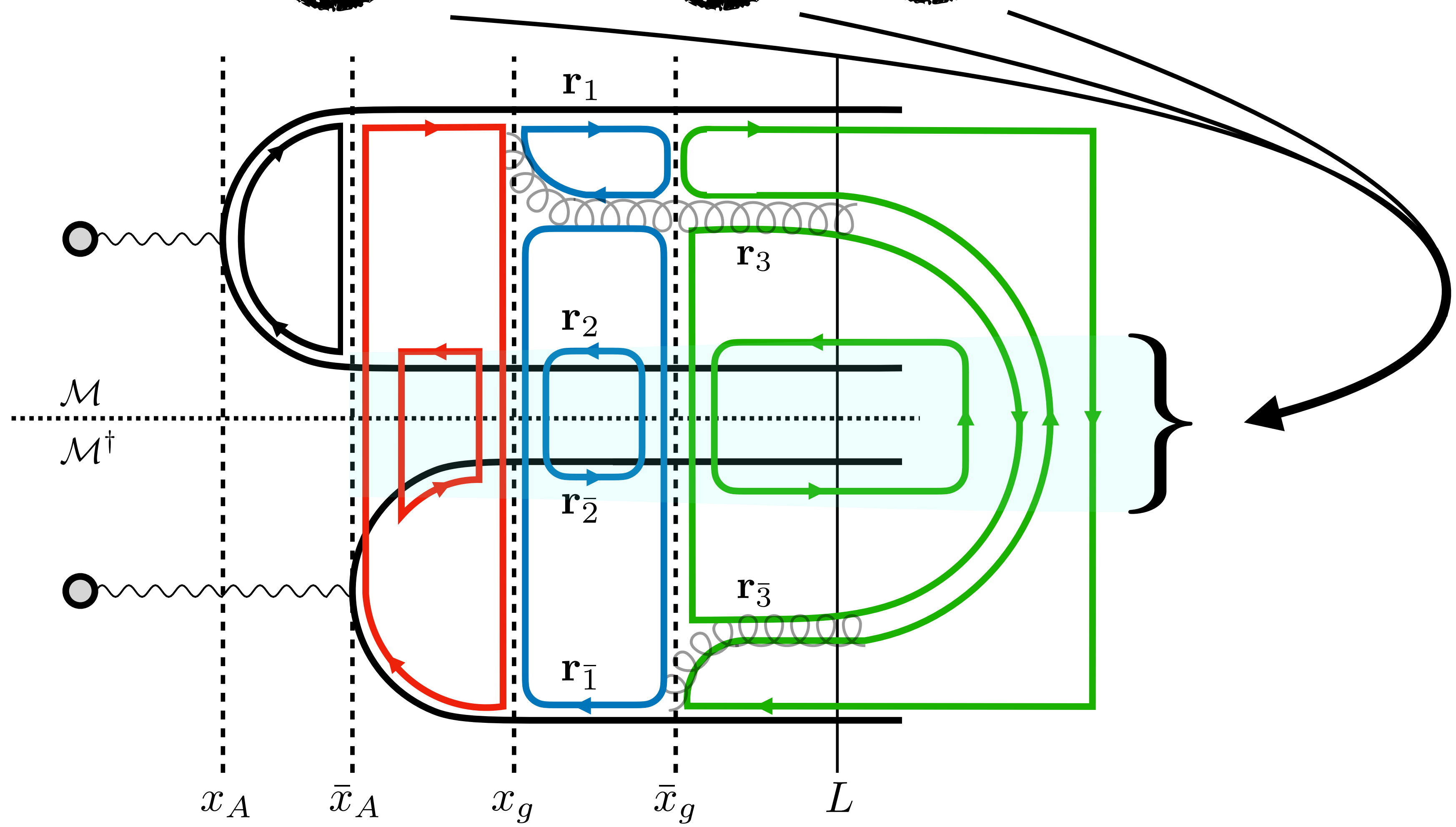
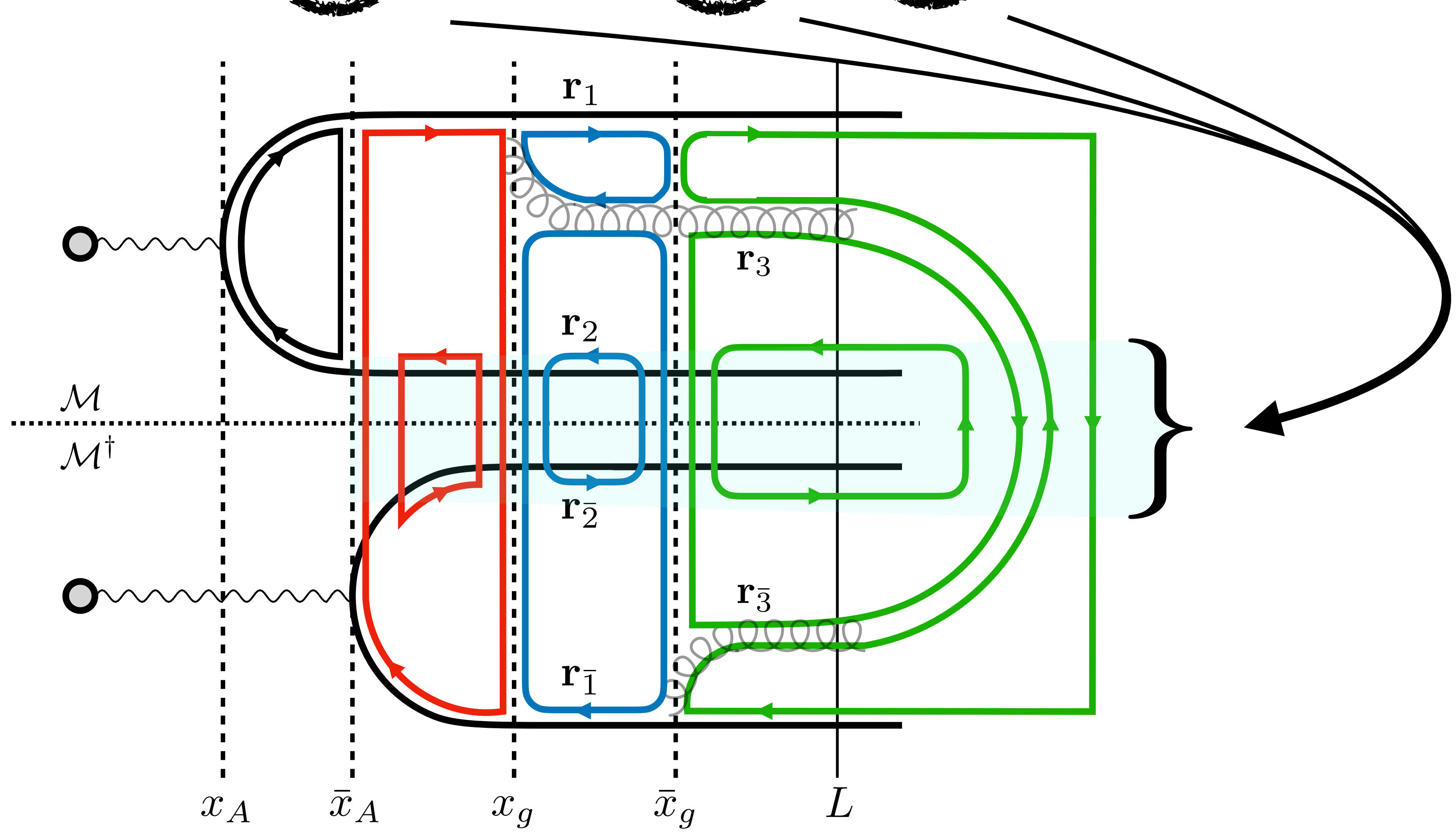


➡ In this contribution, **the spectator particle** (the antiquark in this case) **rotates color independently from the rest of the antenna**

(Note that the quark/antiquark positions are mirrored with respect to separation between $\mathcal{M}$ and $\mathcal{M}^\dagger$)

- *How can we interpret this result?*
  - ➡ Let us consider the first of these terms:

$$\langle S_{12}\rangle_{(\bar{x}_A,x_A)}\langle S_{1\bar{1}} S_{2\bar{2}}\rangle_{(x_g,\bar{x}_A)}\langle S_{13}S_{3\bar{1}} S_{2\bar{2}}\rangle_{(\bar{x}_g,x_g)}\langle S_{2\bar{2}} S_{3\bar{3}}\, Q_{\bar{3}\,\bar{1}\,1\,3}\rangle_{(L,\bar{x}_g)}$$



  - ➡ In this contribution, **the spectator particle** (the antiquark in this case) **rotates color independently from the rest of the antenna**
  - ➡ This term might be read as:

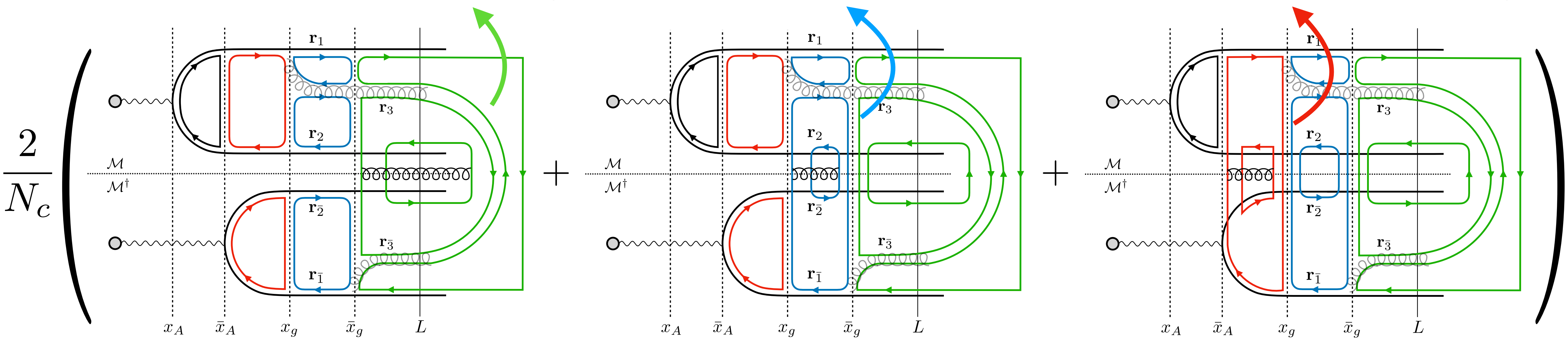
$$\underbrace{\langle S_{12}\rangle}_{\text{(Antenna decoherence)}}{}_{(\bar{x}_A,x_A)} \times (\text{BDMPS-Z})_{(L,\bar{x}_A)} \times \underbrace{\langle S_{2\bar{2}}\rangle}_{\text{(Spectator decoherence)}}{}_{(L,\bar{x}_A)}$$

(Note that the quark/antiquark positions are mirrored with respect to separation between $\mathcal{M}$ and $\mathcal{M}^\dagger$)

Large Nc limit (II)

- *How can we interpret this result?*
  - ➡ The remaining three terms contain **one effective gluon exchange** between the spectator and the rest of the antenna:

$$\left\langle \mathrm{Tr}\{V_{\bar{2}} t^a V_2^\dagger\}\mathrm{Tr}\{V_3 t^a V_{\bar{3}}^\dagger\}\, Q_{\bar{3}\,\bar{1}\,1\,3}\right\rangle_{(L,\bar{x}_g)} \qquad \left\langle \mathrm{Tr}\{V_3 t^a V_{\bar{1}}^\dagger\}\mathrm{Tr}\{V_{\bar{2}} t^a V_2^\dagger\}\, S_{13}\right\rangle_{(\bar{x}_g,x_g)} \qquad \left\langle \mathrm{Tr}\{V_1 t^a V_{\bar{1}}^\dagger\}\mathrm{Tr}\{V_{\bar{2}} t^a V_2^\dagger\}\right\rangle_{(x_g,\bar{x}_A)}$$

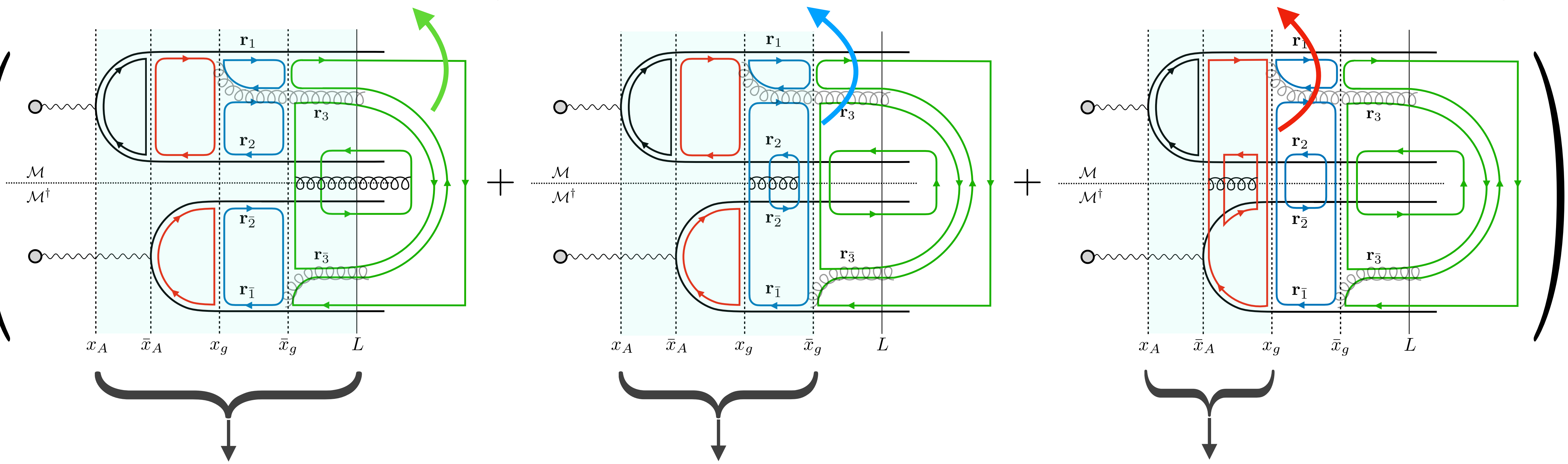


(Note that the quark/antiquark positions are mirrored with respect to separation between $\mathcal{M}$ and $\mathcal{M}^\dagger$)

- *How can we interpret this result?*

➡ The remaining three terms contain **one effective gluon exchange** between the spectator and the rest of the antenna:

$$\frac{2}{N_c}\Bigg(\left\langle \mathrm{Tr}\{V_{\bar 2} t^a V_2^\dagger\} \mathrm{Tr}\{V_3 t^a V_{\bar 3}^\dagger\}\, Q_{\bar 3 \bar 1 1 3} \right\rangle_{(L,\bar x_g)} + \left\langle \mathrm{Tr}\{V_3 t^a V_{\bar 1}^\dagger\} \mathrm{Tr}\{V_{\bar 2} t^a V_2^\dagger\}\, S_{13} \right\rangle_{(\bar x_g, x_g)} + \left\langle \mathrm{Tr}\{V_1 t^a V_{\bar 1}^\dagger\} \mathrm{Tr}\{V_{\bar 2} t^a V_2^\dagger\} \right\rangle_{(x_g,\bar x_A)} \Bigg)$$



Regions where the spectator particle exchanges color with other particles in the antenna

➡ These exchanges violate the conventional BDMPS-like picture obtained in the case of the instantaneously-formed antenna

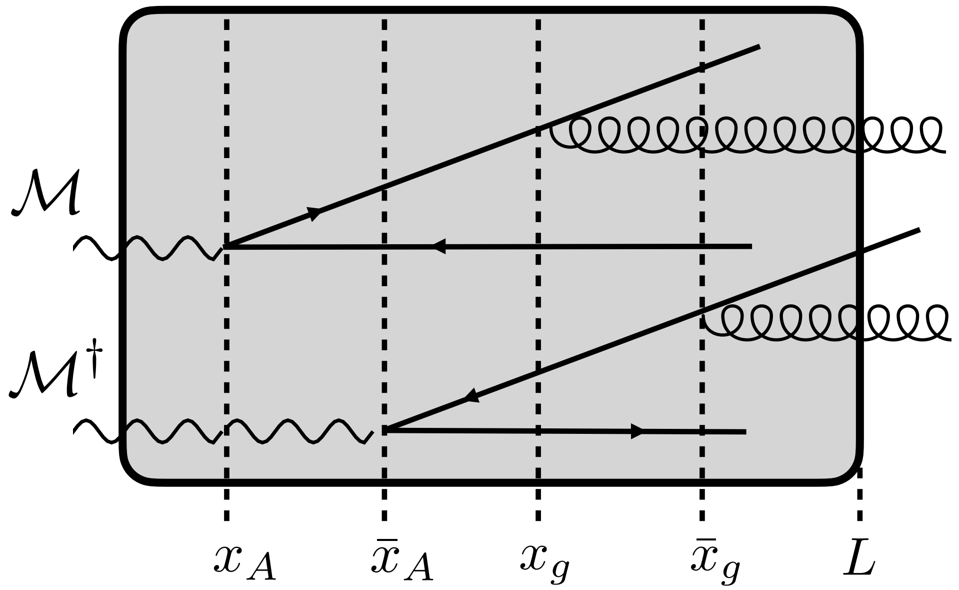
➡ **Even at large Nc**, this is a very substantial departure from the case where the antenna is formed instantaneously!

- *Why?*

(Note that the quark/antiquark positions are mirrored with respect to separation between $\mathcal{M}$ and $\mathcal{M}^\dagger$)

$z_{\bar{q}} \to 1$ limit: 'hard' spectator

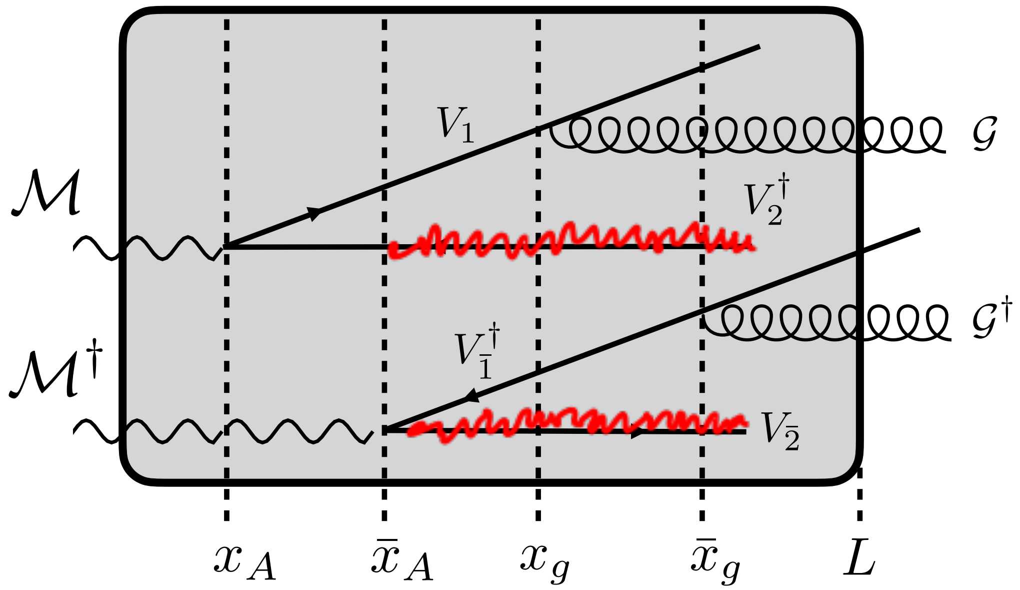
➡ We look at the limit where **the spectator leg of the antenna carries most of the energy of the photon**



➡ We look at the limit where **the spectator leg of the antenna carries most of the energy of the photon**

➡ The Wilson lines corresponding to the spectator leg are **approximately flat**: they cancel each other

$$V_2^\dagger\left[\mathbf{r}(s) = \frac{\bar{\mathbf{p}}}{E_{\bar{q}}}(s - x_A) \to \mathbf{0}\right] V_{\bar{2}}\left[\mathbf{r}(s) = \frac{\bar{\mathbf{p}}}{E_{\bar{q}}}(s - \bar{x}_A) \to \mathbf{0}\right] = \mathbb{I}$$



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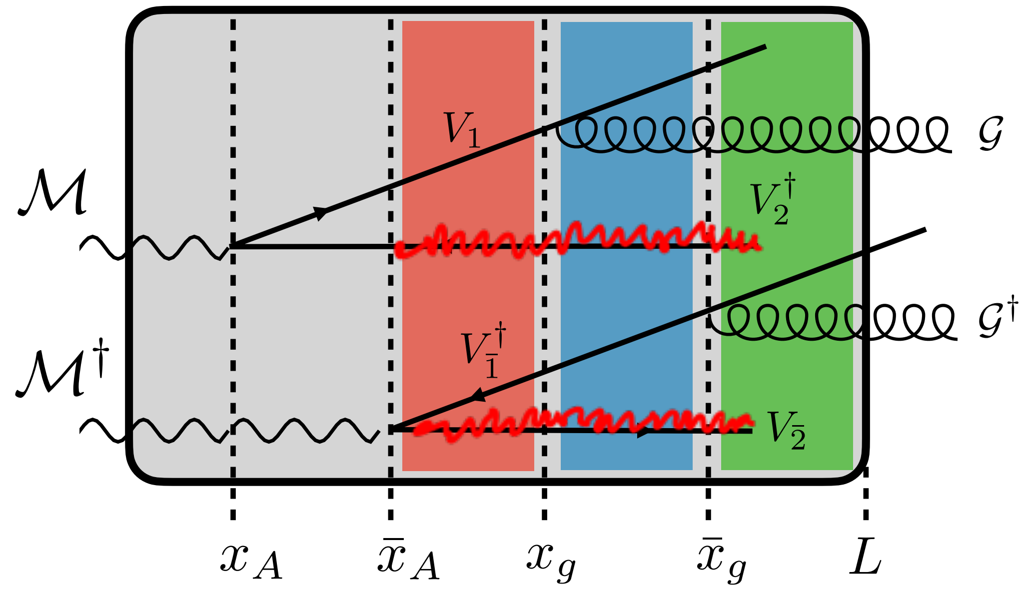


➡ In this limit, the only surviving term is a straightforward generalization of BDMPS-Z:

$$\langle|\mathcal{M}_q|^2\rangle \propto \int \left\langle \underline{\mathrm{Tr}\{V_1 V_2^\dagger\}} \right\rangle \left\langle \underline{\mathrm{Tr}\left\{V_1 V_{\bar{1}}^\dagger\right\}} \right\rangle \left\langle \underline{\mathcal{G}^{ab}\mathrm{Tr}\left\{V_{\bar{1}}^\dagger t^a V_1 t^b\right\}} \right\rangle \left\langle \underline{\mathcal{G}^{cd}\mathrm{Tr}\left\{t^d V_{\bar{1}}^\dagger V_1 t^e\right\}\mathcal{G}^{\dagger ec}} \right\rangle$$

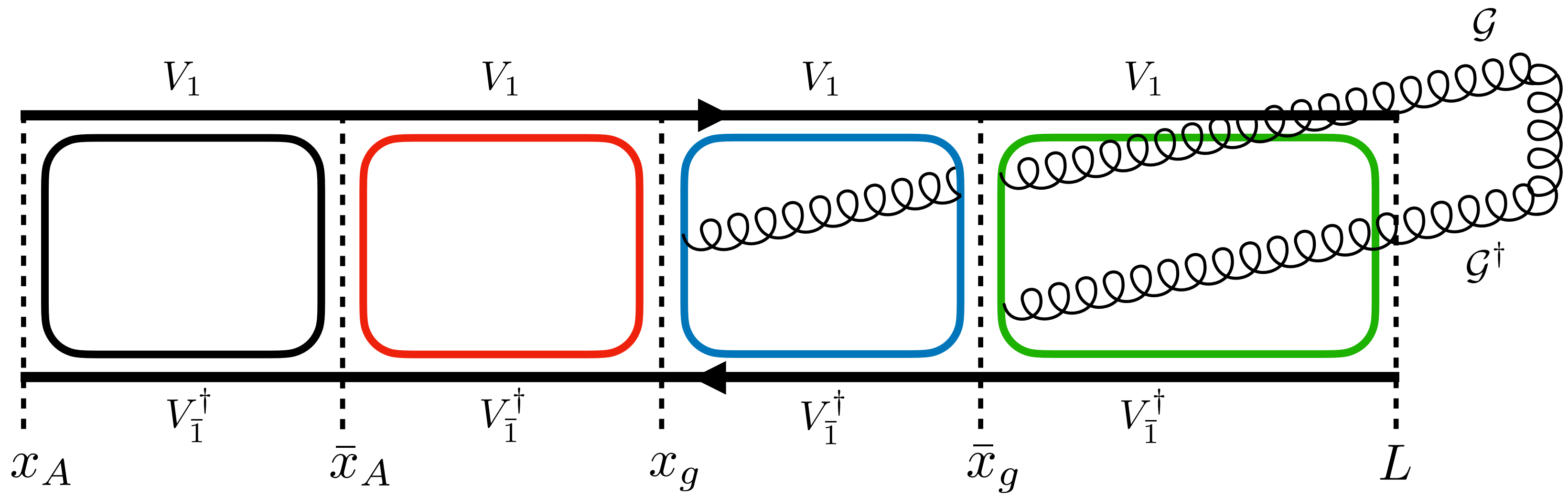
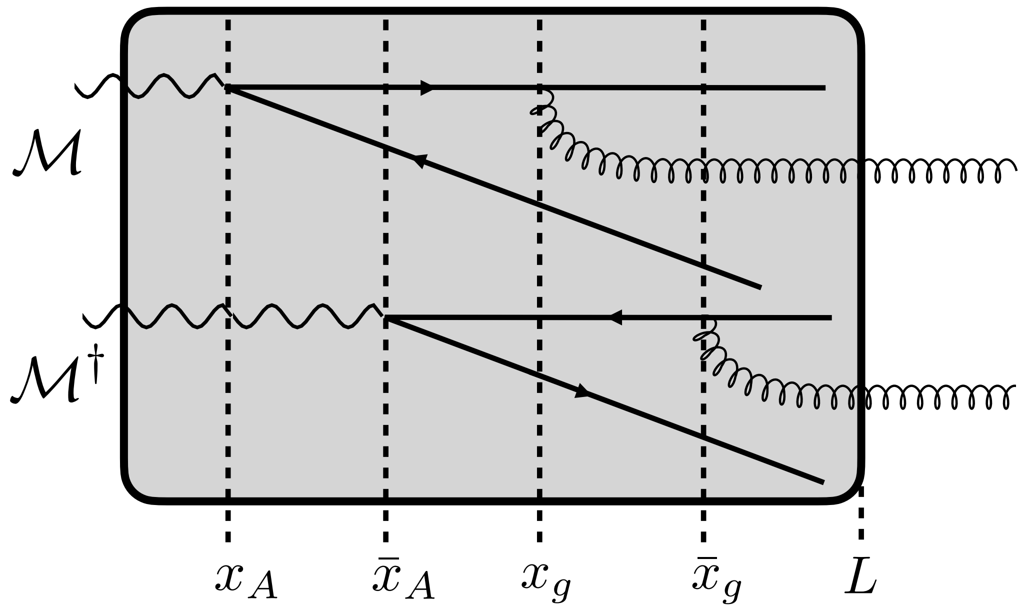


Diagram previously studied in B. Wu; JHEP 1110:029, 2011

➡ We now focus on the opposite situation, where the **emitter leg of the antenna carries most of the energy**:



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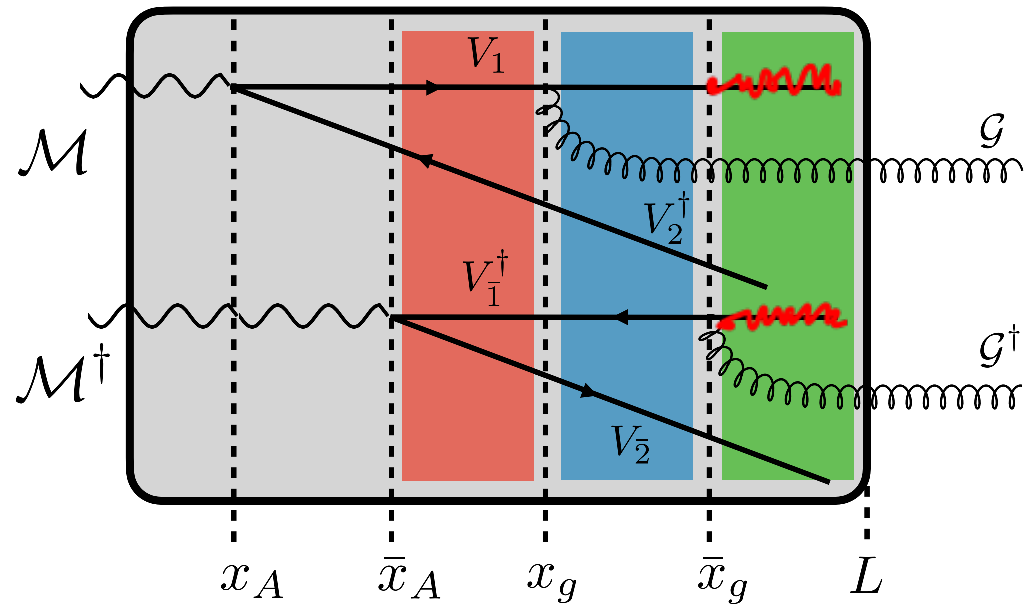

➡ Although this limit brings some cancellations, the final result remains highly complicated:

$$\langle|\mathcal{M}_q|^2\rangle \propto \int \left\langle \mathrm{Tr}\left\{V_1 V_{\bar{2}}^\dagger\right\}\right\rangle_{(\bar{x}_A,x_A)} \langle \mathbf{A}_0\rangle_{(x_g,\bar{x}_A)} \cdot \mathbf{g}_2 \cdot \left\langle \mathcal{G}^{ab}\mathbf{B}_0^{ba}\right\rangle_{(\bar{x}_g,x_g)} \cdot \mathbf{g}_4 \cdot \left\langle \mathcal{G}^{cd}\mathbf{C}_0^{de}\mathcal{G}^{\dagger ec}\right\rangle_{(L,\bar{x}_g)}$$

with:

$$\mathbf{A}_0^{\mathrm{T}} = \begin{pmatrix} \mathrm{Tr}\{V_1V_2^\dagger\}\mathrm{Tr}\{V_{\bar{2}}V_{\bar{1}}^\dagger\} \\ \mathrm{Tr}\{V_2^\dagger V_{\bar{2}}\}\end{pmatrix}, (\mathbf{B}_0^{ab})^{\mathrm{T}} = \begin{pmatrix} U_1^{bb'}\mathrm{Tr}\{t^a V_2^\dagger V_{\bar{2}} t^{b'}\} & U_1^{ba}\mathrm{Tr}\{V_{\bar{2}}V_2^\dagger\}/2 \\ \mathrm{Tr}\{V_1 t^a V_2^\dagger\}\mathrm{Tr}\{V_{\bar{1}}^\dagger t^b V_{\bar{2}}\} & \mathrm{Tr}\{t^b V_{\bar{2}} V_2^\dagger t^{a'}\}U_1^{a'a} \\ \mathrm{Tr}\{V_2^\dagger t^b V_1 t^a\}\mathrm{Tr}\{V_{\bar{2}}V_{\bar{1}}^\dagger\} & \mathrm{Tr}\{t^{a'}V_{\bar{2}}V_2^\dagger t^b\}U_1^{a'a} \\ \mathrm{Tr}\{V_2^\dagger t^b V_{\bar{2}} t^a\} & 0 \end{pmatrix}, \mathbf{C}_0^{de} = \begin{pmatrix} \delta^{ed}\mathrm{Tr}\{V_2^\dagger V_{\bar{2}}\}/2 \\ \mathrm{Tr}\{t^e V_2^\dagger V_{\bar{2}} t^d\} \\ \mathrm{Tr}\{t^d V_2^\dagger V_{\bar{2}} t^e\} \\ 0 \end{pmatrix}$$

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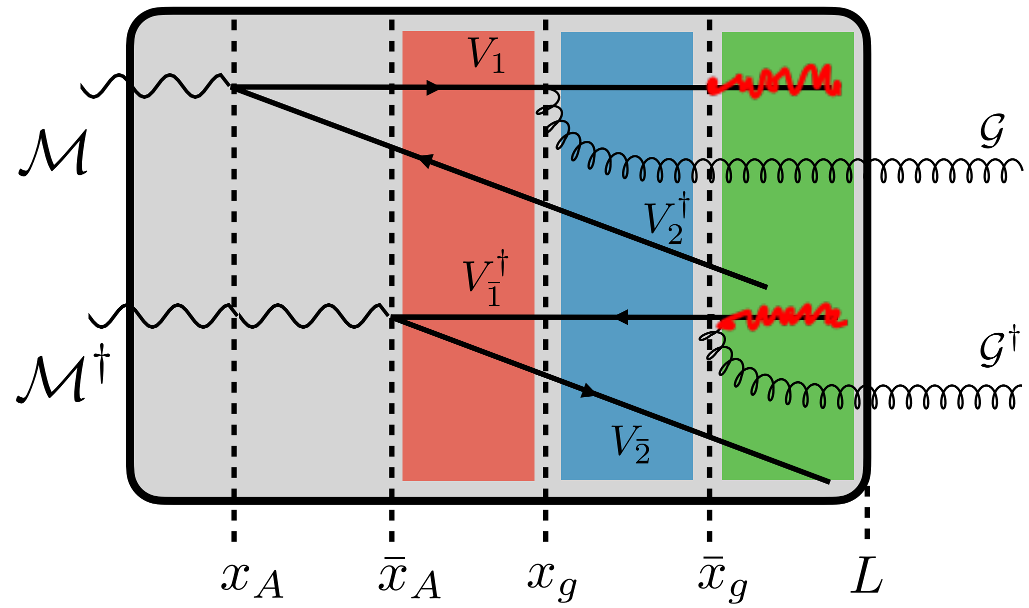


➡ Although this limit brings some cancellations, the final result remains highly complicated. In the large-N_c:

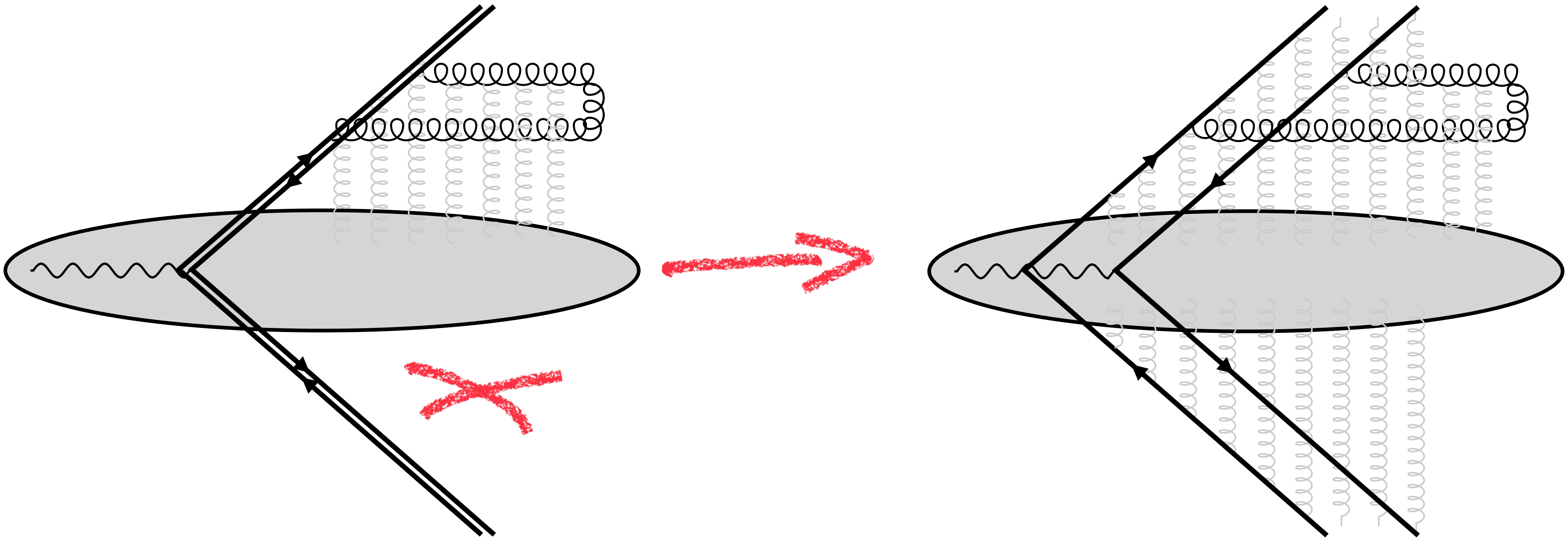
$$\langle \ldots \rangle \underset{\substack{N_c\uparrow \\ \mathbf{n}\to 0}}{\to} \int \mathcal{D}\mathbf{r}_3\, \mathcal{D}\mathbf{r}_{\bar{3}}\, \langle S_{12}\rangle_{(\bar{x}_A, x_A)} \Big[\langle S_{\bar{2}2}\rangle_{(x_g,\bar{x}_A)}\, \langle S_{13}S_{31}\, S_{\bar{2}2}\rangle_{(\bar{x}_g,x_g)} \langle S_{2\bar{2}}S_{3\bar{3}}\, S_{\bar{3}3}\rangle_{(L,\bar{x}_g)}$$
$$+ \frac{2}{N_c}\langle S_{12}\, S_{\bar{2}1}\rangle_{(x_g,\bar{x}_A)}\, \langle S_{13}\, S_{32}\, S_{\bar{2}1}\rangle_{(\bar{x}_g,x_g)} \Big\langle \mathrm{Tr}\{V_{\bar{2}} t^a V_2^\dagger\} \mathrm{Tr}\{V_3 t^a V_{\bar{3}}^\dagger\}\, S_{\bar{3}3}\Big\rangle_{(L,\bar{x}_g)}$$
$$+ \frac{2}{N_c}\langle S_{12}\, S_{\bar{2}1}\rangle_{(x_g,\bar{x}_A)} \Big\langle \mathrm{Tr}\{V_3 t^a V_1^\dagger\} \mathrm{Tr}\{V_{\bar{2}} t^a V_2^\dagger\}\, S_{13}\Big\rangle_{(\bar{x}_g,x_g)} \langle S_{2\bar{2}}S_{3\bar{3}}\, S_{\bar{3}3}\rangle_{(L,\bar{x}_g)} \Big].$$

Conclusions, future prospects

➡ The **naive picture of decoherence** inside a QCD medium **breaks down** upon departure from the **instantaneous antenna formation** limit

- Multiple **new contributions** emerge
- Some of these terms are relevant **even at leading-N_c order**

➡ This is primarily caused by previously absent **color exchanges with the spectator leg of the antenna**



($\mathcal{M}$ and $\mathcal{M}^\dagger$ overlayed in this cartoon)

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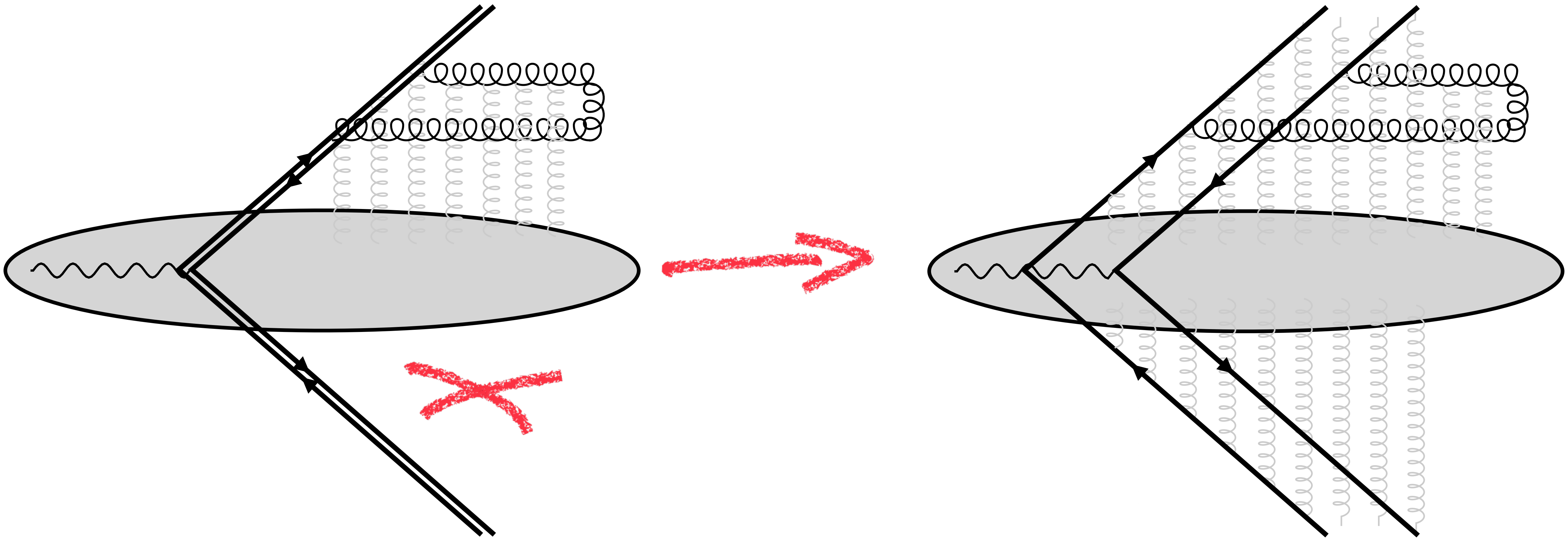


➡ **FURTHER RESEARCH**:

- Investigate similar effects on interference terms
- Quantify the impact of these correlations on the gluon spectrum
- Relax eikonal approximation

($\mathcal{M}$ and $\mathcal{M}^\dagger$ overlayed in this cartoon)

Thank you for your attention

➡ Our starting point is: $\left\langle \left[\mathcal{G}^\dagger(\bar{x}_g,L)\mathcal{G}(L,x_g)\right]^{b'b} \mathrm{Tr}\left\{V_{\bar{1}}^\dagger(\bar{x}_A,\bar{x}_g)t^{b'}V_{\bar{1}}^\dagger(\bar{x}_g,L)V_1(L,x_g)t^bV_1(x_g,x_A)V_2^\dagger(x_A,L)V_{\bar{2}}(L,\bar{x}_A)\right\}\right\rangle$

➡ We split this object into regions:

$$\Big\langle \mathcal{G}^{\dagger b'a}(\bar{x}_g,L)\mathcal{G}^{ac}(L,\bar{x}_g)\mathcal{G}^{cb}(\bar{x}_g,x_g)\mathrm{Tr}\Big\{V_{\bar{1}}^\dagger(\bar{x}_A,x_g)V_{\bar{1}}^\dagger(x_g,\bar{x}_g)t^{b'}V_{\bar{1}}^\dagger(\bar{x}_g,L)$$
$$V_1(L,\bar{x}_g)V_1(\bar{x}_g,x_g)t^bV_1(x_g,\bar{x}_A)V_1(\bar{x}_A,x_A)V_2^\dagger(x_A,\bar{x}_A)V_2^\dagger(\bar{x}_A,x_g)V_2^\dagger(x_g,\bar{x}_g)$$
$$V_2^\dagger(\bar{x}_g,L)V_{\bar{2}}(L,\bar{x}_g)V_{\bar{2}}(\bar{x}_g,x_g)V_{\bar{2}}(x_g,\bar{x}_A)\Big\}\Big\rangle.$$

➡ We separate the dressed propagators from their Wilson lines: $\mathcal{G}^{ab}(\bar{t},t) = \int_{\mathbf{r}(t)=\mathbf{x}}^{\mathbf{r}(\bar{t})=\mathbf{y}} \mathcal{D}\mathbf{r}(\xi)\,\exp\left\{\frac{ip^+}{2}\int ds\left(\frac{d\mathbf{r}}{ds}\right)^2\right\}U^{ab}(\bar{t},t;[\mathbf{r}(\xi)])$

➡ and then transform then to fundamental representation by applying: $U^{ab}t^b_{ij} = \left[V^\dagger t^a V\right]_{ij}$

➡ This results in: $U_{\bar{3}}^{\dagger b'a}(\bar{x}_g,L)t^{b'}_{ij} = U_{\bar{3}}^{ab'}t^{b'}_{ij} = \left[V_{\bar{3}}^\dagger(\bar{x}_g,L)t^aV_{\bar{3}}(L,\bar{x}_g)\right]_{ij}$

$$U_3^{ab}(L,x_g)t^b_{kl} = \left[V_3^\dagger t^a V_3\right]_{kl} = \left[V_3^\dagger(x_g,\bar{x}_g)V_3^\dagger(\bar{x}_g,L)t^aV_3(L,\bar{x}_g)V_3(\bar{x}_g,x_g)\right]_{kl}$$

➡ By substituting and applying the Fierz identity, $t^a_{ij}t^a_{kl} = \frac{1}{2}\left(\delta_{il}\delta_{jk} - \frac{1}{N_c}\delta_{ij}\delta_{kl}\right)$, we obtain:

$$\frac{1}{2N_c}\Big\langle\left(\mathrm{Tr}\left\{V_1V_2^\dagger\right\}\right)_{(\bar{x}_A,x_A)}\left(\left[V_1V_2^\dagger\right]_{ij}\left[V_{\bar{2}}V_{\bar{1}}^\dagger\right]_{kl}\right)_{(x_g,\bar{x}_A)}\left(\left[V_1V_3^\dagger\right]_{mn}(V_2^\dagger)_{jI}(V_{\bar{2}})_{Jk}(V_{\bar{1}}^\dagger)_{lI_1}(V_3)_{I_2i}\right)_{(\bar{x}_g,x_g)}$$
$$\left(\left[V_{\bar{3}}^\dagger V_{\bar{3}}V_{\bar{1}}^\dagger V_1\right]_{nm}\left[V_2^\dagger V_{\bar{2}}\right]_{IJ}\left[V_{\bar{3}}^\dagger V_3\right]_{I_1I_2}\right)_{(L,\bar{x}_g)}\Big\rangle$$
$$-\frac{1}{2N_c^2}\Big\langle\left(\mathrm{Tr}\left\{V_1V_2^\dagger\right\}\right)_{(\bar{x}_A,x_A)}\left(\left[V_1V_2^\dagger\right]_{ij}\left[V_{\bar{2}}V_{\bar{1}}^\dagger\right]_{kl}\right)_{(x_g,\bar{x}_A)}\left((V_2^\dagger)_{jI}(V_{\bar{2}})_{Jk}(V_{\bar{1}}^\dagger)_{ln}(V_1)_{mi}\right)_{(\bar{x}_g,x_g)}$$
$$\left(\left[V_{\bar{1}}^\dagger V_1\right]_{nm}\left[V_2^\dagger V_{\bar{2}}\right]_{IJ}\right)_{(L,\bar{x}_g)}\Big\rangle.$$

BACK-UP: Color Algebra (color singlet bases)

➡ The singlet 'basis' for a correlator with 2 open (fundamental) color indices is: $s_1^{ij} = \frac{1}{N_c}\delta^{ij}$

➡ In the case of a correlator with 4 open color indices:

K. Fukushima, Y. Hidaka; JHEP 06 (2007) 040

K. Fukushima, Y. Hidaka; JHEP 11 (2017) 114

$$s_1^{ijkl} = \frac{1}{N_c}\delta^{ij}\delta^{kl}$$

$$s_2^{ijkl} = \frac{1}{\sqrt{N_c^2-1}}\left(\delta^{il}\delta^{jk} - \frac{1}{N_c}\delta^{ij}\delta^{kl}\right)$$

➡ And finally, for one with 6 open color indices:

$$s_1^{ijklmn} = \frac{1}{\sqrt{N_c^3}}\delta^{ij}\delta^{kl}\delta^{mn}$$

$$s_2^{ijklmn} = \frac{1}{\sqrt{N_c(N_c^2-1)}}\left(\delta^{il}\delta^{jk} - \frac{1}{N_c}\delta^{ij}\delta^{kl}\right)\delta^{mn}$$

$$s_3^{ijklmn} = \frac{1}{\sqrt{N_c(N_c^2-1)}}\left(\delta^{in}\delta^{jm} - \frac{1}{N_c}\delta^{ij}\delta^{mn}\right)\delta^{kl}$$

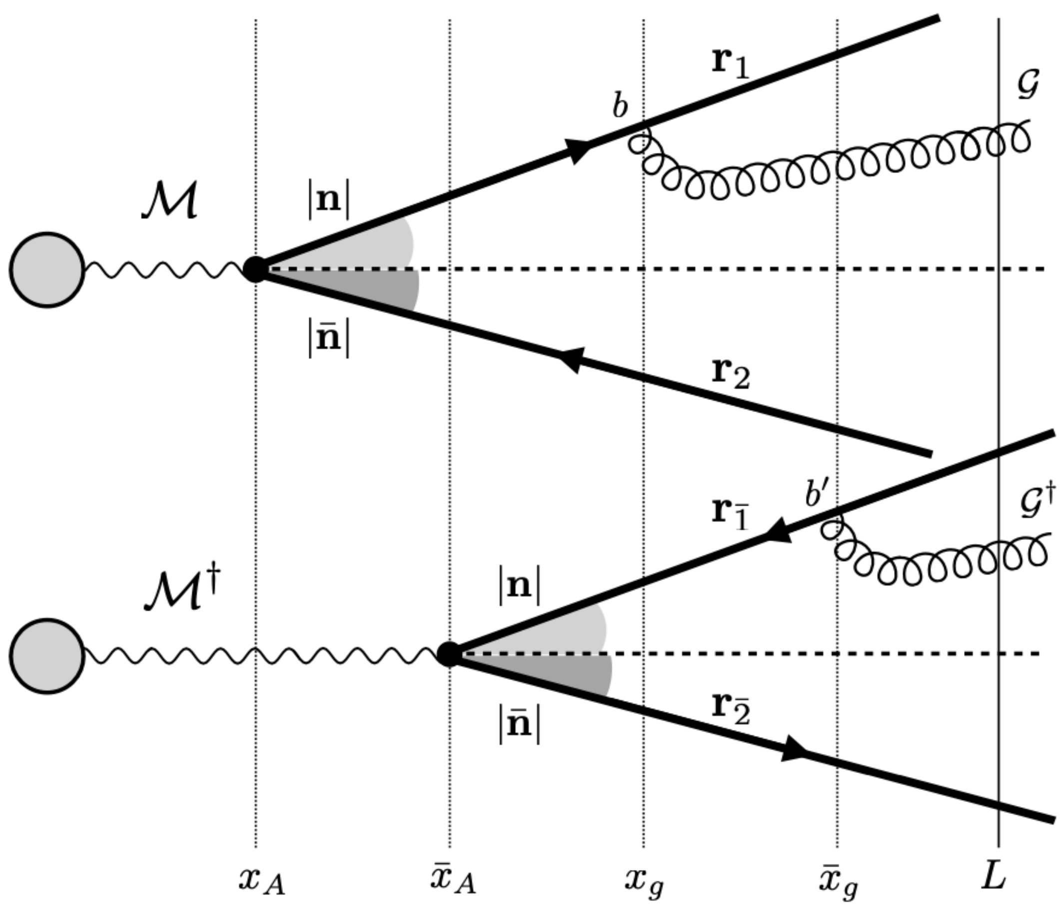
$$s_4^{ijklmn} = \frac{1}{\sqrt{N_c(N_c^2-1)}}\left(\delta^{kn}\delta^{lm} - \frac{1}{N_c}\delta^{mn}\delta^{kl}\right)\delta^{ij}$$

$$s_5^{ijklmn} = \frac{1}{\sqrt{2N_c(N_c^2-1)}}\left(\delta^{il}\delta^{kn}\delta^{mj} - \delta^{in}\delta^{ml}\delta^{jk}\right)$$

$$s_6^{ijklmn} = \sqrt{\frac{N_c}{2(N_c^2-4)(N_c^2-1)}}\Bigg(\delta^{il}\delta^{kn}\delta^{mj} + \delta^{in}\delta^{ml}\delta^{jk} - \frac{2}{N_c}\left(\delta^{il}\delta^{jk}\delta^{mn} + \delta^{in}\delta^{jm}\delta^{kl} + \delta^{kn}\delta^{lm}\delta^{ij}\right) + \frac{4}{N_c^2}\delta^{ij}\delta^{kl}\delta^{mn}\Bigg)$$

T. Lappi, H. Mäntysaari, A. Ramnath; Phys. Rev. D 102 (2020) 074027

BACK-UP: Squared Amplitude (full expression)



➡ The full expression for the specific contribution considered in this work reads:

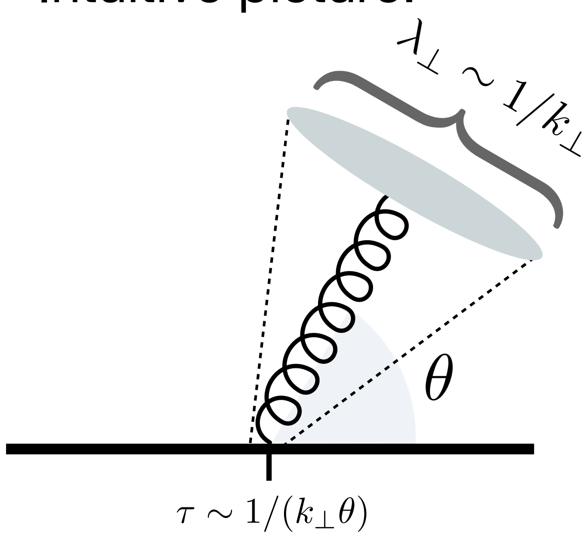
$$\langle|\mathcal{M}_q|^2\rangle = \mathcal{M}^2_{q\bar{q}}\left(\frac{2\,g}{E_g}\right)^2\int_0^L\frac{dx_A}{t_A}\int_{x_A}^L\frac{d\bar{x}_A}{t_A}\,e^{i\frac{(x_A-\bar{x}_A)}{t_A}}\int_{\bar{x}_A}^L dx_g\int_{x_g}^L d\bar{x}_g\,e^{i\frac{E_g}{2}\mathbf{n}^2(x_g-\bar{x}_g)}\int_{\mathbf{z},\bar{\mathbf{z}}}e^{-i\mathbf{k}\cdot(\mathbf{z}-\bar{\mathbf{z}})}$$
$$(i\partial_{\boldsymbol{\alpha}}-E_g\mathbf{n})\cdot(-i\partial_{\boldsymbol{\beta}}-E_g\mathbf{n})\frac{1}{N_c}\Big\langle\left[\mathcal{G}^\dagger(\bar{x}_g,\boldsymbol{\beta};L,\bar{\mathbf{z}})\mathcal{G}(L,\mathbf{z};x_g,\boldsymbol{\alpha})\right]^{b'b}\Big|_{\substack{\boldsymbol{\alpha}=\mathbf{n}x_g\\ \boldsymbol{\beta}=\mathbf{n}\bar{x}_g}}$$
$$\mathrm{Tr}\left\{V^\dagger_{\bar{1}}(\bar{x}_A,\bar{x}_g)t^{b'}V^\dagger_{\bar{1}}(\bar{x}_g,L)V_1(L,x_g)t^bV_1(x_g,x_A)V^\dagger_{2}(x_A,L)V_{\bar{2}}(L,\bar{x}_A)\right\}\Big\rangle$$

where: $\mathcal{M}^2_{q\bar{q}} = \dfrac{e^2t_A}{E_\gamma}P_{\gamma\to q\bar{q}}(z_q)$, with $P_{\gamma\to q\bar{q}}$ the Altarelli-Parisi splitting function.

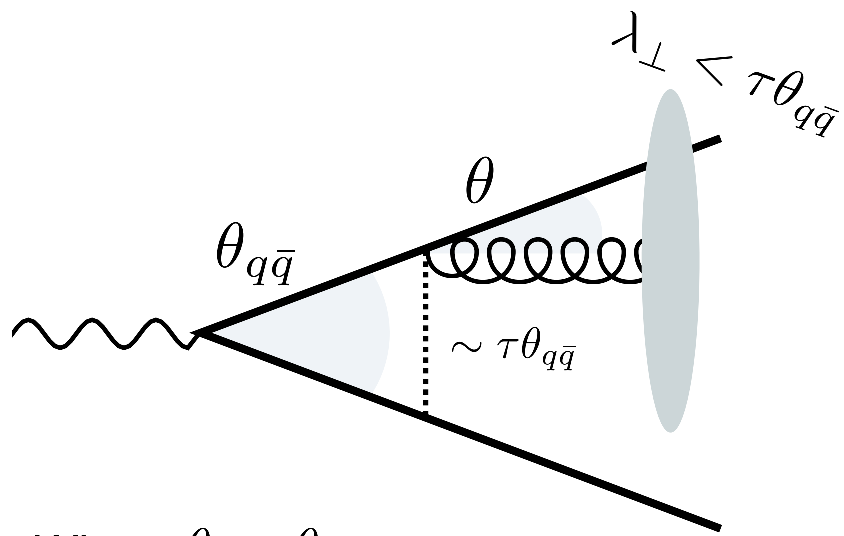
- *What are coherence effects?*

Y. L. Dokshitzer, V. A. Khoze, A. H. Mueller, S. I. Troian; Basics of perturbative QCD, 1991

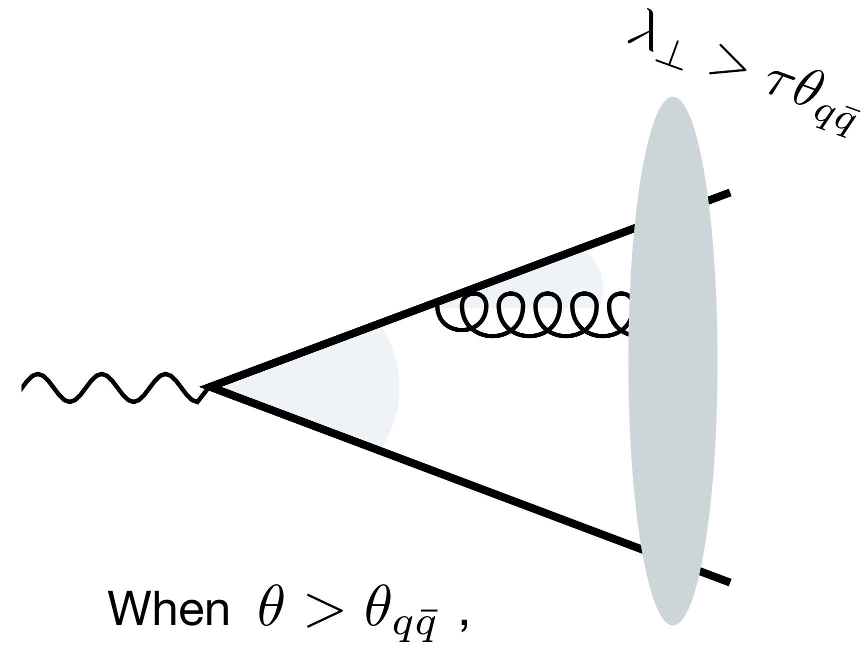
➡ Intuitive picture:



The emitted gluon has a transverse resolution, $\lambda_\perp$

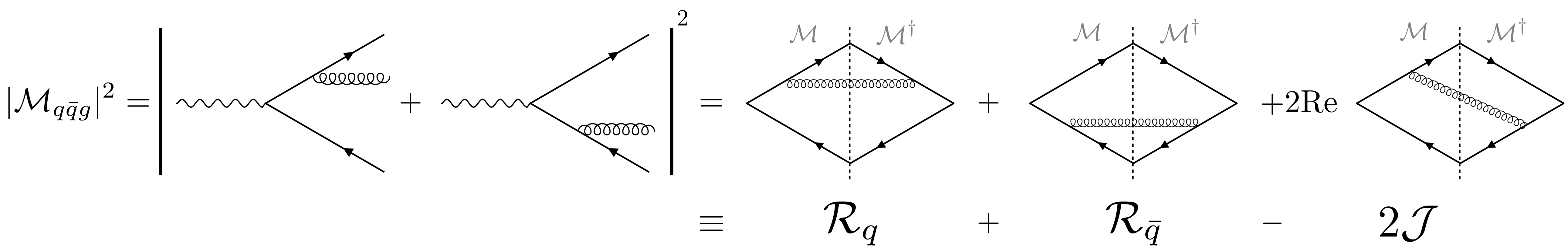


When $\theta < \theta_{q\bar{q}}$, the radiation is sensitive to the color charge of the parent



When $\theta > \theta_{q\bar{q}}$, it is sensitive to the charge of the antenna (0 in singlet case)

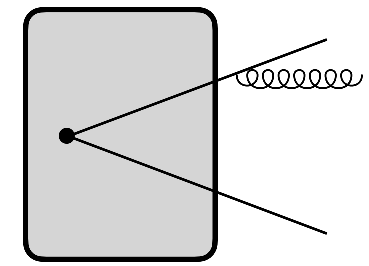
➡ Squared amplitude for $\gamma \to q\bar{q}g$ splitting:

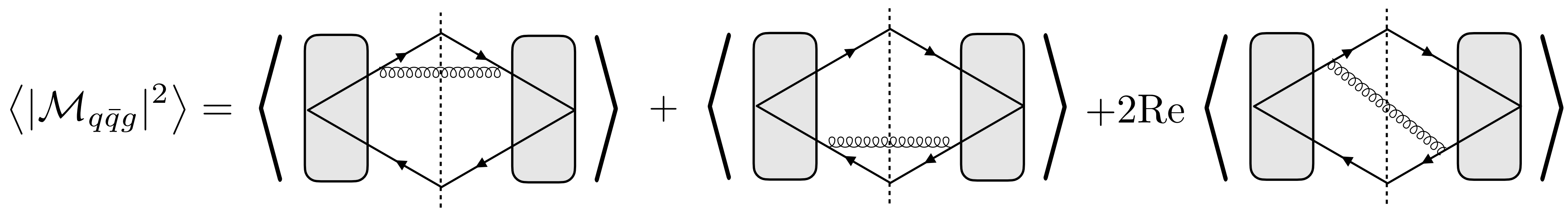


Soft gluon spectrum: $dN = \dfrac{dE_g}{E_g} \dfrac{\sin\theta_g d\theta_g}{1-\cos\theta_g} \dfrac{\alpha_s C_F}{2\pi} \Theta(\theta_{q\bar{q}} - \theta_g)$

Interference effects give rise to **angular ordering**

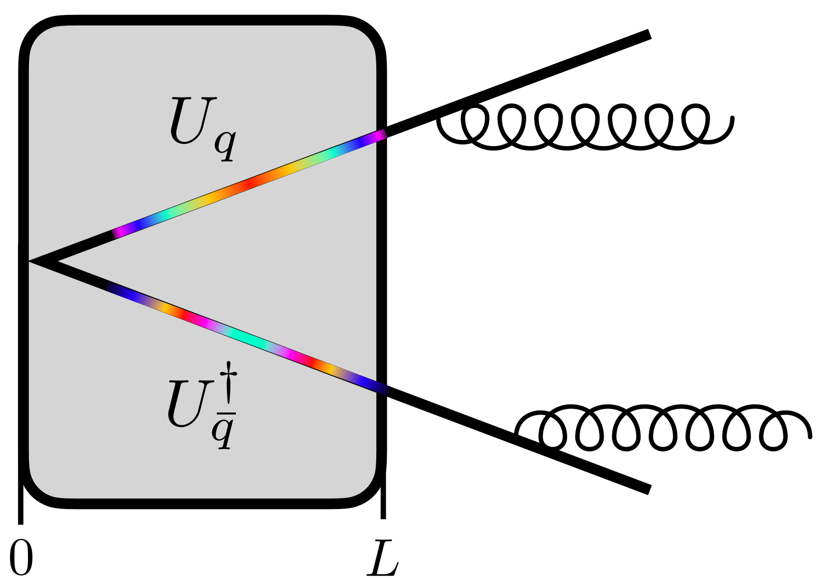
- *How is the picture of color coherence modified in the presence of a medium?*
  - ➡ **Initial approach**: we consider a *very* soft gluon (emitted outside of the medium)
  - ➡ **Also**: we assume a *very* virtual photon (antenna is formed instantaneously)
  - ➡ Squared amplitude for in-medium $\gamma \to q\bar{q}g$ splitting:





Soft gluon spectrum: $$dN = \frac{dE_g}{E_g}\frac{d\Omega}{2\pi}\frac{\alpha_s C_F}{2\pi}\left[\mathcal{R}_q + \mathcal{R}_{\bar{q}} - (1-\Delta_{\text{med}})2\mathcal{J}\right]$$

Where $\Delta_{\text{med}} = 1 - \left\langle \mathrm{Tr}\{U_q(L,0)U^\dagger_{\bar{q}}(0,L)\}\right\rangle$, with $U^{ab} = \mathrm{Tr}\{t^a V^\dagger t^b V\}$ (WL in adjoint representation)



- **Dilute medium**: $\Delta_{\text{med}} \to 0$

  $dN \sim \mathcal{R}_q + \mathcal{R}_{\bar{q}} - 2\mathcal{J}$ (Angular ordering)

- **Opaque medium:** $\Delta_{\text{med}} \to 1$

  $dN \sim \mathcal{R}_q + \mathcal{R}_{\bar{q}}$ (Independent emitters)

➡ Intuitive picture: as the color of each parton rotates independently, **the initial correlation is broken**

($\mathcal{M}$ and $\mathcal{M}^\dagger$ overlayed in this cartoon)

Y. Mehtar-Tani, C.A. Salgado and K. Tywoniuk; Phys. Rev. Lett. 106 (2011) 122002

Y. Mehtar-Tani, C.A. Salgado and K. Tywoniuk; Phys. Lett. B 707 (2012) 156-159