



How the nonequilibrium initial stages influence medium-induced radiation

PLB 852 (2024) [Boguslavski, Kurkela, Lappi, FL, Peuron]

arXiv:2509.03868 [Altenburger, Boguslavski, FL]

arXiv:2511.07519 [Barata, Boguslavski, FL, Sadofyev]

Florian Lindembauer (MIT Center for Theoretical Physics – a Leinweber Institute)

March 24, 2026, C3NT Workshop, Wuhan:

Jet-soft dynamical medium interactions in high-energy heavy-ion collisions

Outline

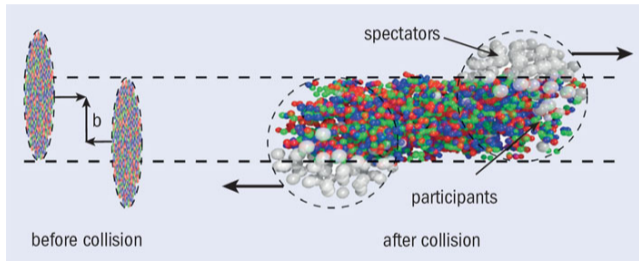
- 1 Introduction
- 2 QCD kinetic theory
- 3 Jets in heavy-ion collisions and harmonic approximation: \hat{q}
- 4 Beyond the jet quenching parameter: $C(q_{\perp})$
- 5 Jet spectrum from small-distance form of $C(x)$

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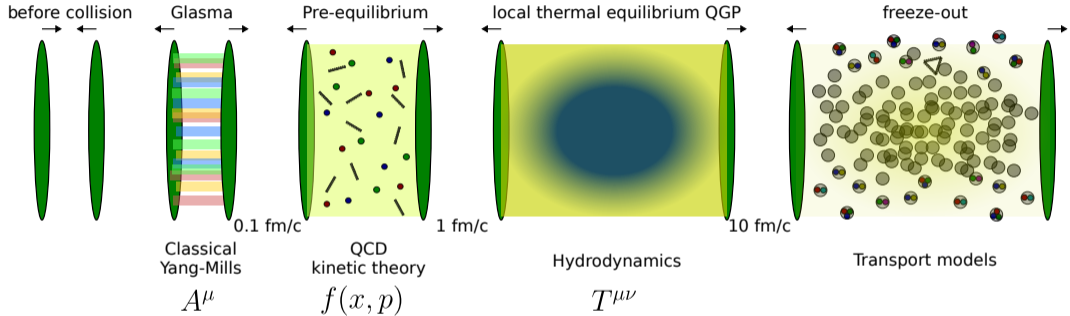
Heavy-ion collisions and the quark-gluon plasma

- Study high-temperature properties of the strong interaction (QCD)
- Collision of atomic nuclei at LHC or RHIC
- Creates high-temperature QCD matter = Quark-Gluon plasma (QGP)



[Alberica Toia 2013, CERN COURIER]

Time-evolution of the QGP in heavy-ion collisions

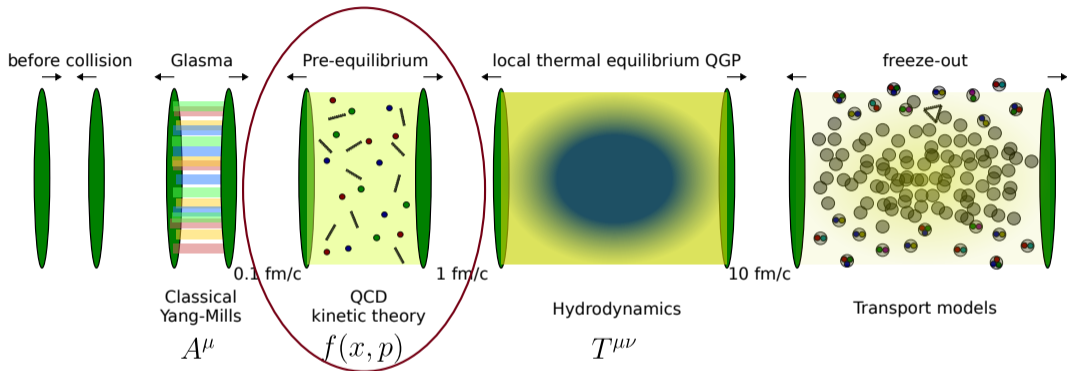


Interested in pre-equilibrium stages ("Initial stages")

→ **QCD out of equilibrium** → Use **QCD kinetic theory** to simulate plasma evolution

[Rev.Mod.Phys. 93 (2021) [Berges, Heller, Mazeliauskas, Venugopalan]]

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Effective kinetic theory (EKT) description of the QGP

- **Microscopic description**
- Gluons with **distribution function** $f(t, \mathbf{x}, \mathbf{p})$

How many particles/gluons



¹[JHEP 01 (2003) [Arnold, Moore, Yaffe], Int.J.Mod.Phys.E 16 (2007) [Arnold]]

Observables in EKT

- Fundamental quantity: Distribution function $f(\mathbf{p})$
- **Energy-Momentum tensor:**

$$T^{\mu\nu} = \nu_g \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{p^\mu p^\nu}{p} f(\mathbf{p}) \quad \leftarrow \text{“Moments of distribution function”}$$

- Longitudinal pressure $P_L = T_{zz}$
- Transverse pressure $P_T = T_{xx} = T_{yy}$

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- Transverse pressure $P_T = T_{xx} = T_{yy}$

- How many ‘hard’ particles

$$\frac{\langle pf \rangle}{\langle p \rangle} = \frac{\int d^3\mathbf{p} p f(\mathbf{p})^2}{\int d^3\mathbf{p} p f(\mathbf{p})} \quad \leftarrow \text{“particle number weighted with energy”}$$

Thermalization in isotropic plasmas

- (Left): Initially over-occupied system
- (Right): Underocc. system: Soft thermal bath formed (bottom-up thermalization)

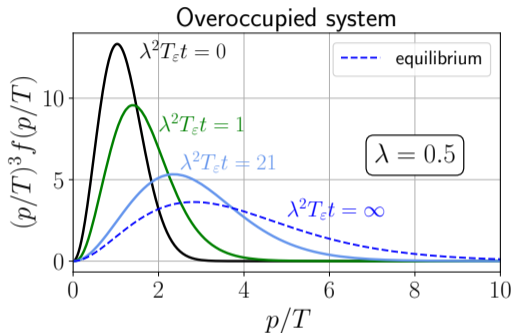
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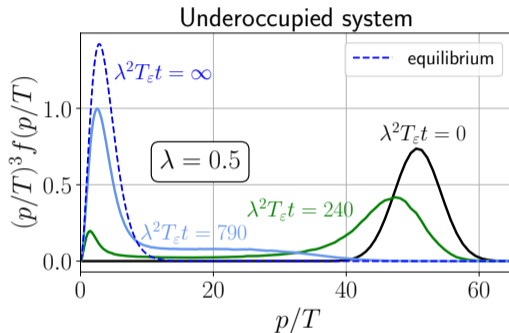
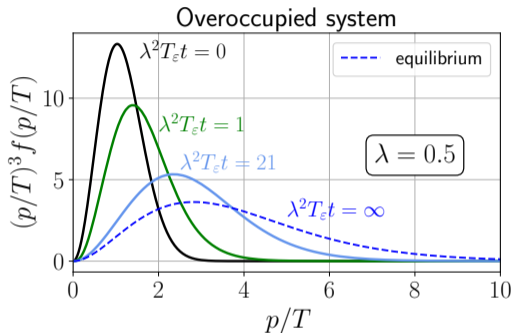
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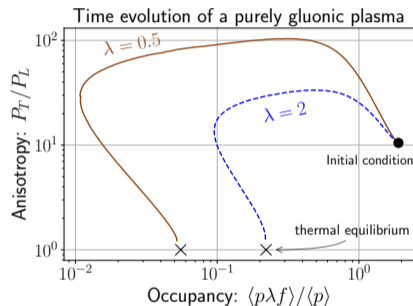
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Bottom-up thermalization in heavy-ion collisions

Initial condition², with $\lambda = g^2 N_C$:

$$f(p_\perp, p_z) = \text{“squeezed” } \frac{1}{\lambda} \times \exp(-p^2) / p$$



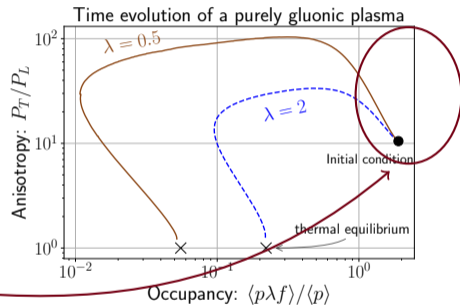
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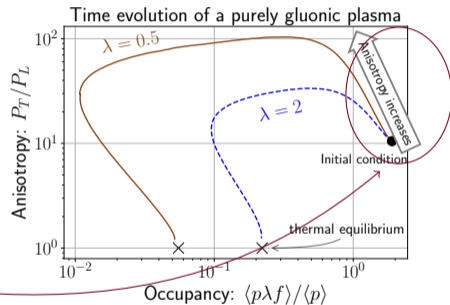
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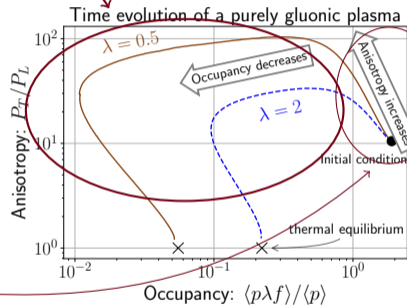
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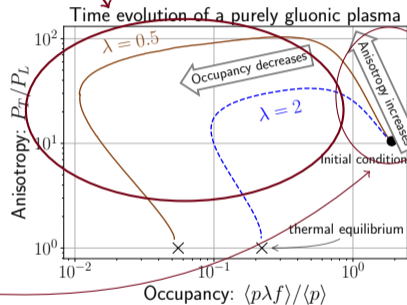
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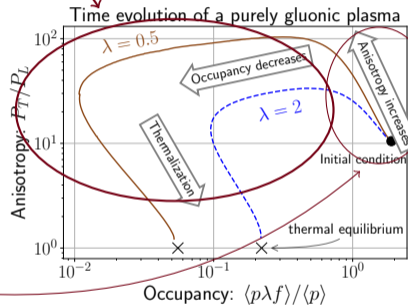
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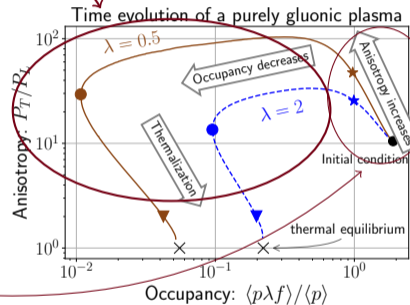
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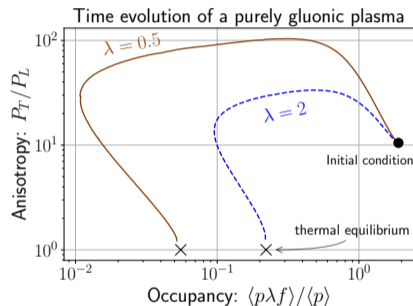
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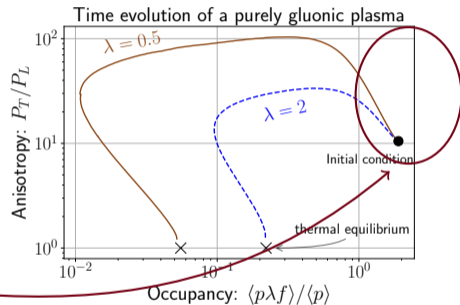
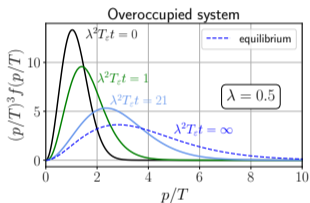
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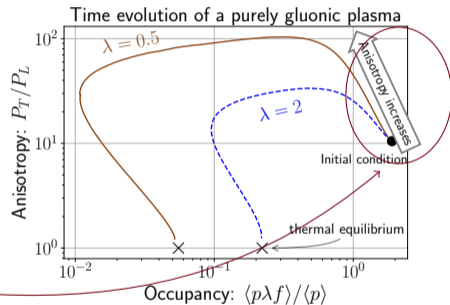
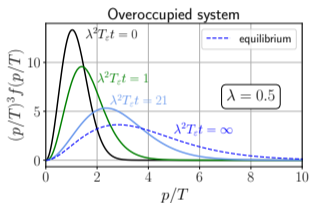
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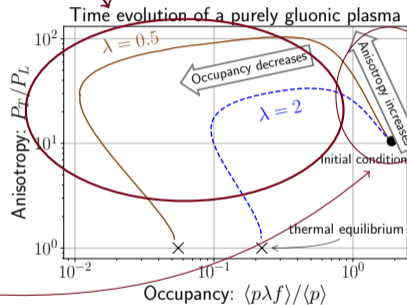
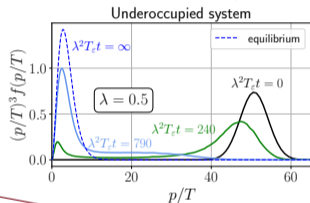
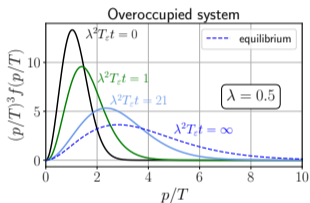
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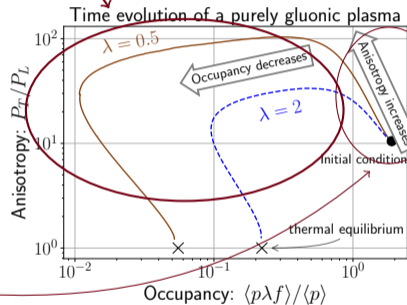
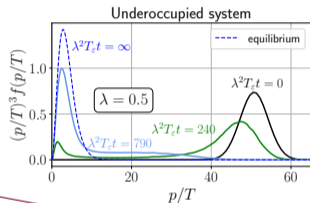
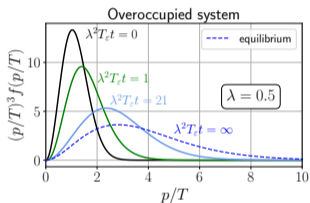
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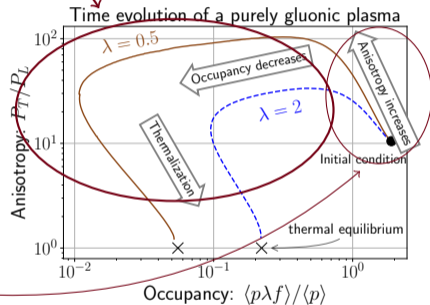
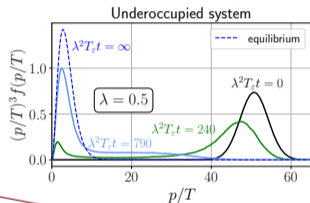
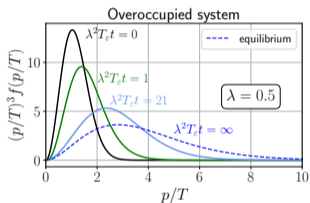
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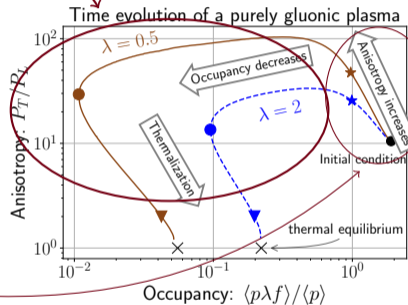
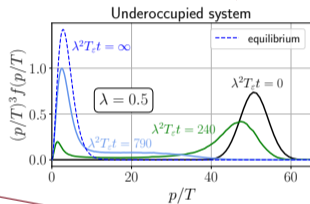
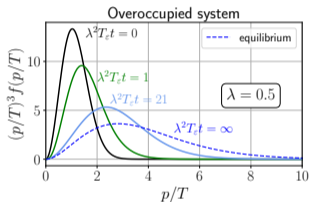
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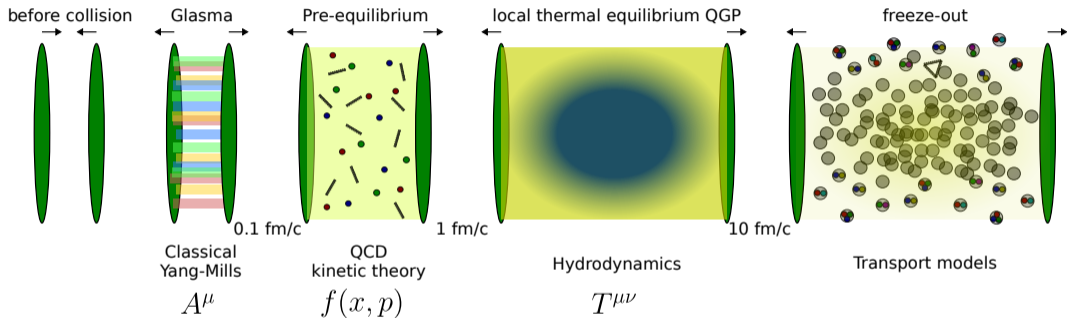
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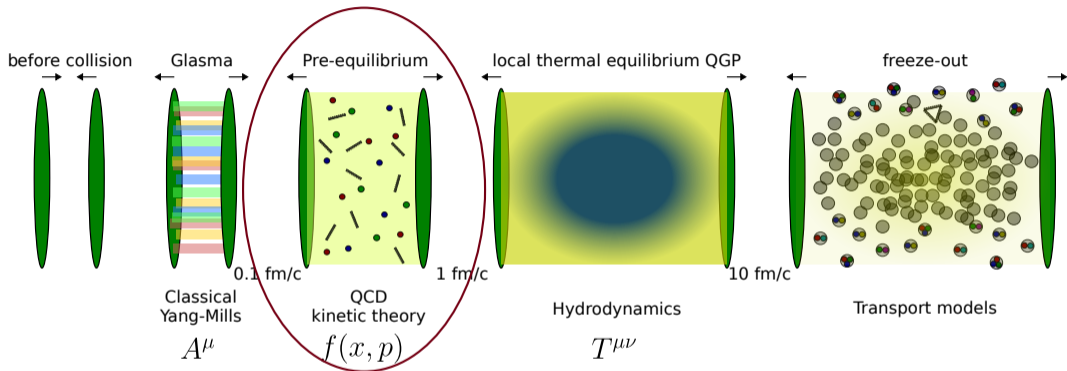


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Possible probes of initial stages

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- Heavy quarks, quarkonium

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- Weakly interacting with background:

- Lepton production

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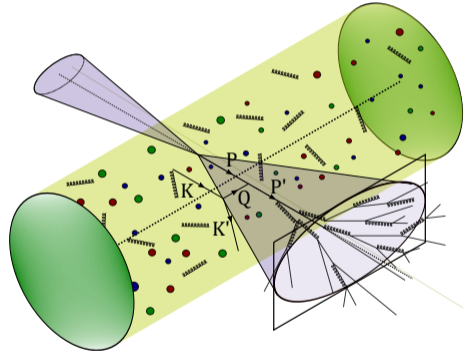
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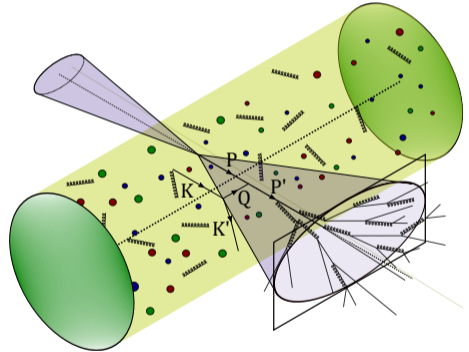
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- **Interact** with nonequilibrium plasma
- Resulting particle shower measured



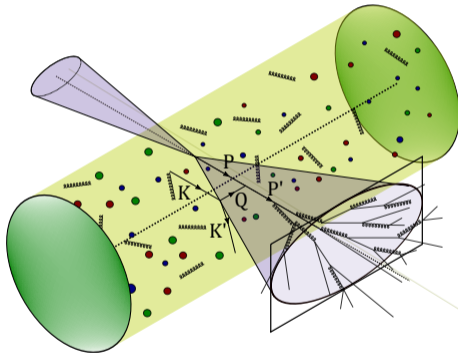
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- **Interact** with nonequilibrium plasma
- Resulting particle shower measured
- Imprints of **medium interactions**
- Many event generators exist, rely on simplifying assumptions (isotropic, thermal, ...)
- Describe **jet quenching** in **out-of-equilibrium plasma!**



Obtaining the gluon emission spectrum

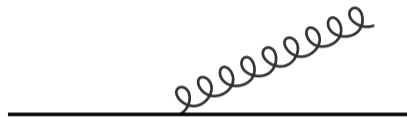
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- **Energy loss** dominated by **gluon radiation**

$$\omega \frac{dI}{d\omega} \sim \text{Re} \int_{t_1, t_2} \partial_{\mathbf{x}} \cdot \partial_{\mathbf{y}} G(\mathbf{x}, t_2; \mathbf{y}, t_1)_{\mathbf{y}=0} - G_{\text{vac}},$$



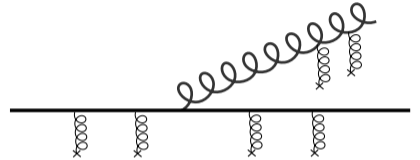
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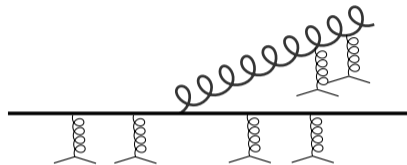
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$$\omega \frac{dI}{d\omega} \sim \text{Re} \int_{t_1, t_2} \partial_{\mathbf{x}} \cdot \partial_{\mathbf{y}} G(\mathbf{x}, t_2; \mathbf{y}, t_1)_{\mathbf{y}=0} - G_{\text{vac}},$$



Obtaining the gluon emission spectrum

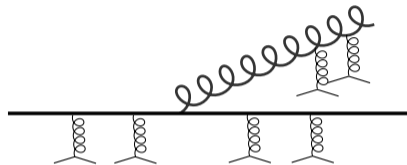
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$$(i\partial_t - \delta E + iC(\mathbf{x}, t)) G(\mathbf{x}, t; \mathbf{y}, t_1) = \delta^2(\mathbf{x} - \mathbf{y})$$

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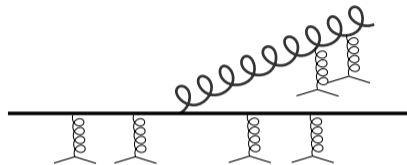
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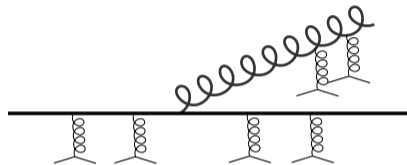
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energy shift

'broadening probability'
Probability of receiving
momentum kick \mathbf{q}_{\perp}

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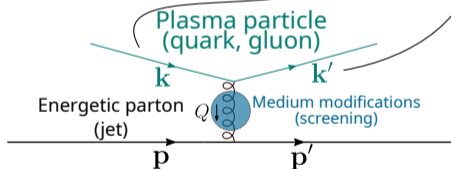
- \hat{q} quantifies **momentum broadening**, “jet quenching parameter”
- Schrödinger equation for harmonic oscillator \rightarrow analytic solution!

Results for \hat{q} : Broadening

Phys.Rev.D 110 (2024) [Boguslavski, Kurkela, Lappi, FL, Peuron]
Phys.Lett.B 850 (2024) [Boguslavski, Kurkela, Lappi, FL, Peuron]]

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$$\hat{q}^{ij} = \int_{\substack{q_{\perp} < \Lambda_{\perp} \\ p \rightarrow \infty}} d\Gamma q^i q^j |\mathcal{M}|^2 f(k)(1 + f(k'))$$



NB: Isotropic screening to prevent plasma instabilities! Phys.Lett.B 214 (1988)

[Mrowczynski], Phys.Rev.D 68 (2003) [Romatschke, Strickland]

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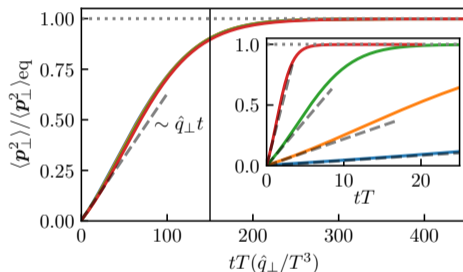
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- Simulations of jet equilibration
- \hat{q} accurately captures transverse momentum broadening
- **Remarkable collapse** of different couplings $\lambda \in (2, 20)$!

Jet evolution in thermal equilibrium⁶



→ See talk by F. Zhou
on Wednesday 10:30

⁶[arXiv:2510.25669 [Boguslavski, FL, Mazeliauskas, Takacs, Zhou]]

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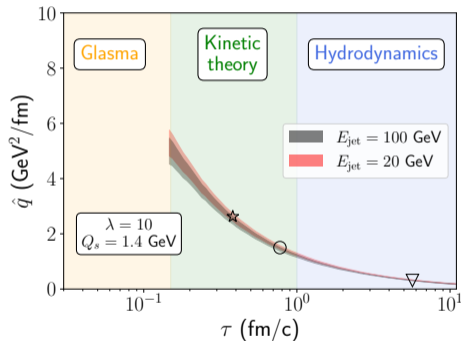
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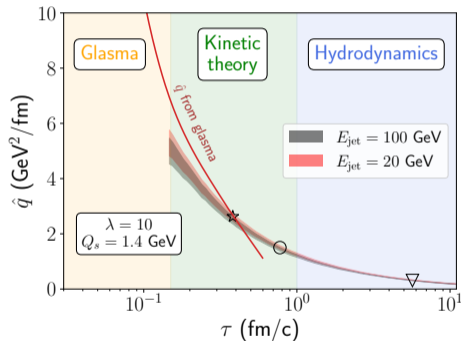
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- Recall: \hat{q} appeared in small-distance expansion:

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Can we do better? \rightarrow **Consider full kernel** $C(\mathbf{q}_\perp)$

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Momentum broadening kernel vs. jet quenching parameter

Momentum broadening kernel:

$$C(\mathbf{q}_\perp) = \int d\Gamma_{\text{PS}} |\mathcal{M}|^2 f(\mathbf{k})(1 + f(\mathbf{k} - \mathbf{q}))$$

Compare to \hat{q} , weighted with momentum transfer

$$\hat{q}^{jj} = \int_{\substack{q_\perp \leq \Lambda_\perp \\ p \rightarrow \infty}} d\Gamma q^i q^j |\mathcal{M}|^2 f(\mathbf{k})(1 + f(\mathbf{k} - \mathbf{q}))$$

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- $C(\mathbf{q}_\perp)$ is **more general** than \hat{q}

Compare to \hat{q} , weighted with momentum transfer

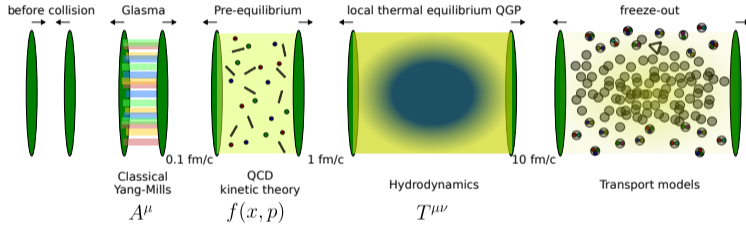
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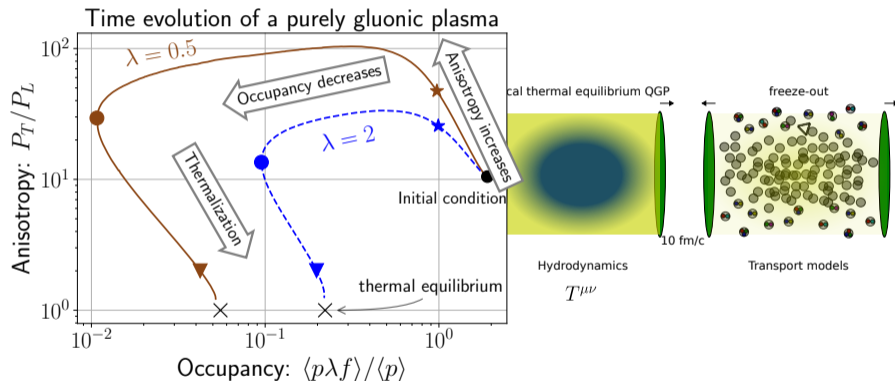
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Recall: Kinetic theory and bottom-up thermalization

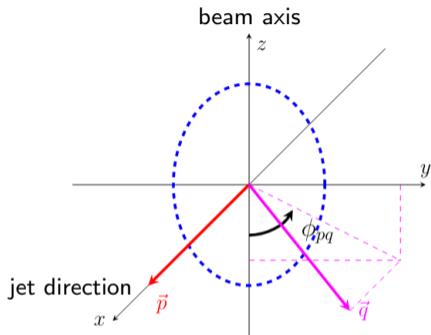


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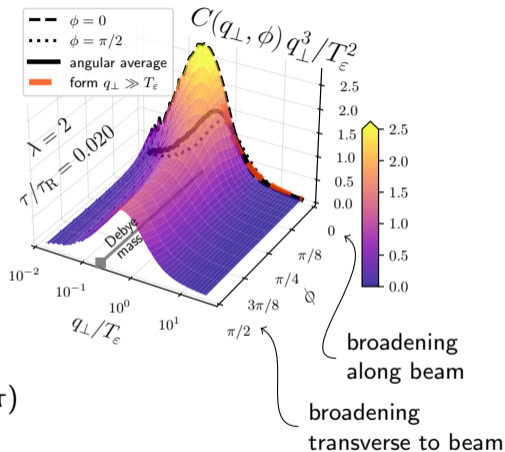


Angular dependence and contribution to \hat{q}

arXiv:2509.03868 [Altenburger, Boguslavski, FL]

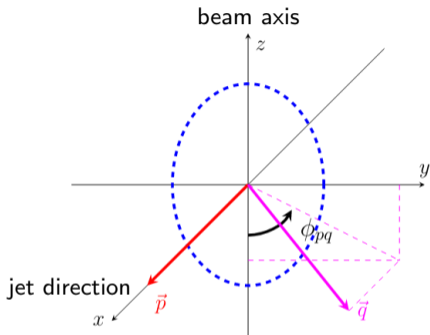


- $C(q_{\perp}, \phi)$ is 2D function
- Contribution to $\hat{q} = \int d^2\mathbf{q}_{\perp} q_{\perp}^2 C(\mathbf{q}_{\perp}) / (2\pi)$

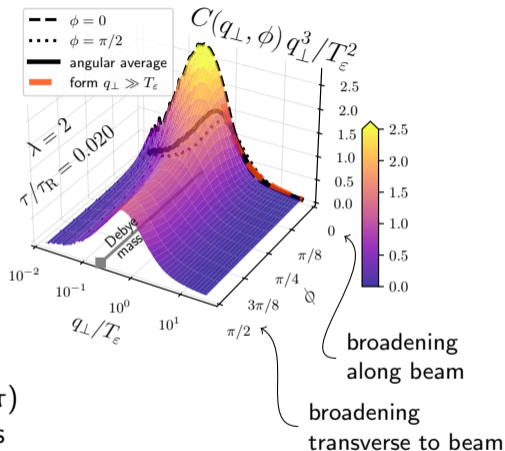


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- **Peaked at Debye mass** ■ for later times
- Along beam ($\phi_{pq} = 0$): Much larger and different form at early times



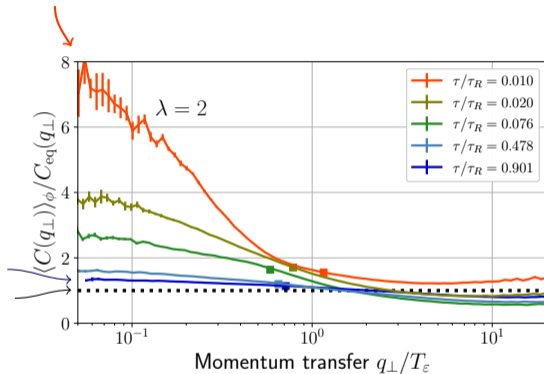
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Normalize using (Landau-matched)
thermal kernel

latest time
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earliest time



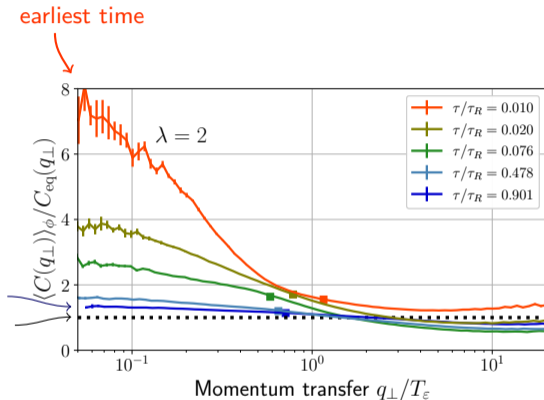
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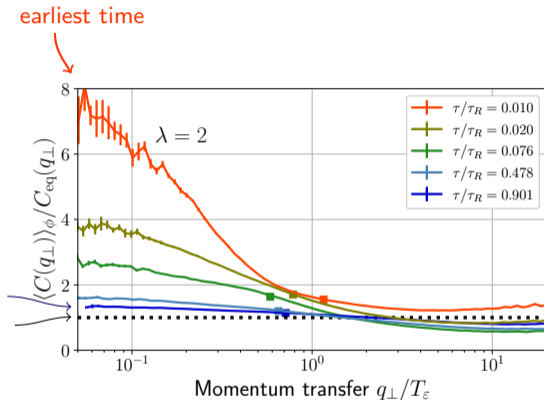
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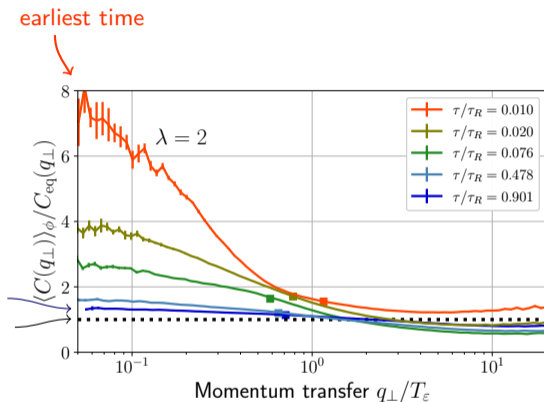
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- What to do with jet quenching?



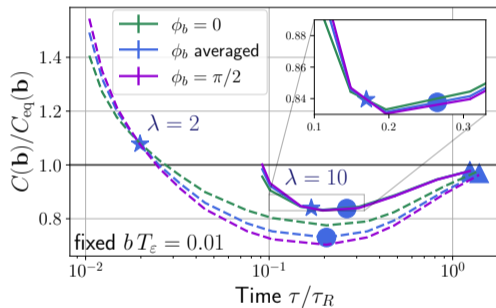
Small-distance behavior of $C(\mathbf{x})$

arXiv:2509.03868 [Altenburger, Boguslavski, FL]

- Obtained via Fourier trafo

$$C(\mathbf{x}) = \int \frac{d^2\mathbf{q}_\perp}{(2\pi)^3} (1 - e^{i\mathbf{q}_\perp \cdot \mathbf{x}}) C(\mathbf{q}_\perp)$$

- Small-distance behavior important for jet quenching!
- For $\lambda = 10$, always smaller than in equilibrium

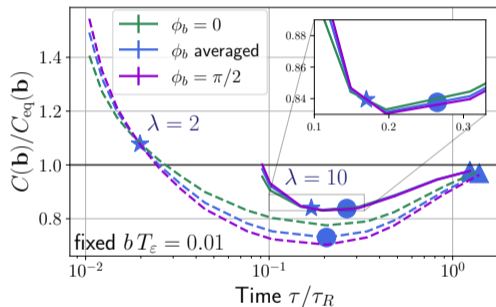


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- For $\lambda = 10$, always smaller than in equilibrium
- Take small distance behavior and calculate jet emission spectrum

arXiv:2511.07519 [Barata, Boguslavski, FL, Sadofyev]



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- Disadvantage: \hat{q} **not well-defined** (cutoff-dependence)

Small-distance form of $C(x)$

- Proper expansion for small distance:

$$C(\mathbf{x}, t) = \frac{1}{4} \hat{q}_0(t) \mathbf{x}^2 \log \frac{1}{\mathbf{x}^2 \mu_*^2(t)}$$

- **Expand** perturbatively **around harmonic oscillator solution**

→ “improved opacity expansion” JHEP 09 (2021) [Barata, Mehtar-Tani, Soto-Ontoso, Tywoniuk]

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- Proper expansion for small distance:

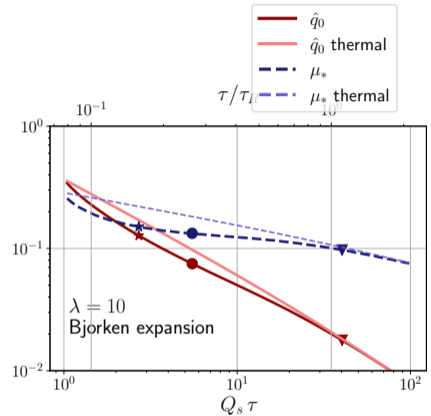
$$C(x, t) = \frac{1}{4} \hat{q}_0(t) x^2 \log \frac{1}{x^2 \mu_*^2(t)} + \mathcal{O}(\hat{q}_0 x^4 \mu_*^2)$$

- μ_* and \hat{q}_0 are medium-parameters uniquely determined from large-cutoff form of $\hat{q}(\Lambda_\perp)$,

$$\hat{q}(t) \approx 2\hat{q}_0(t) \log \frac{\Lambda_\perp}{Q_s} + b(t)$$

- Obtain μ_* via

$$\mu_*^2(t) = \frac{1}{4} \exp(-b(t)/\hat{q}_0(t) + 2\gamma_E - 2)$$



Comparing to thermal system

- Recall: \hat{q}_0, μ_* obtained from large cutoff behavior of \hat{q}
- Compare with an evolving thermal system

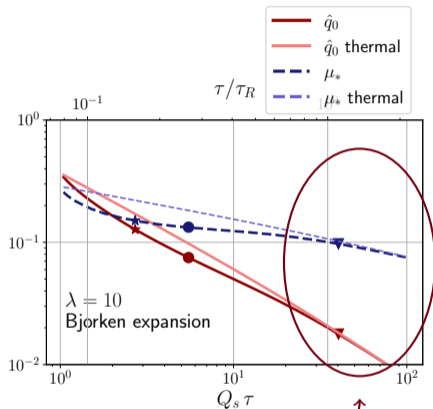
$$T_\epsilon(t) = \left(\frac{30\epsilon(t)}{\pi^2} \right)^{1/4}$$

with energy density from kinetic theory,

$$\epsilon = \int \frac{d^3\mathbf{p}}{(2\pi)^3} |\mathbf{p}| f(\mathbf{p})$$

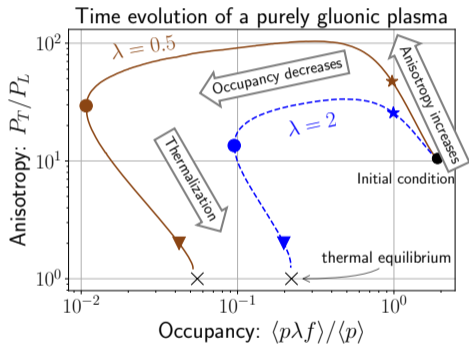
- Thermal forms of \hat{q}_0, μ_* known⁷

⁷[Phys.Rev.D 78 (2008) [Arnold, Xiao]]

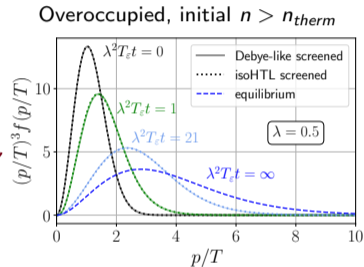
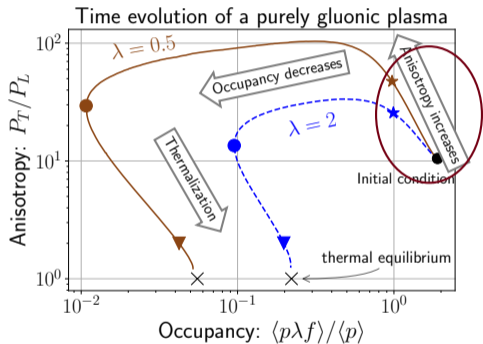


Late time
approach to thermal

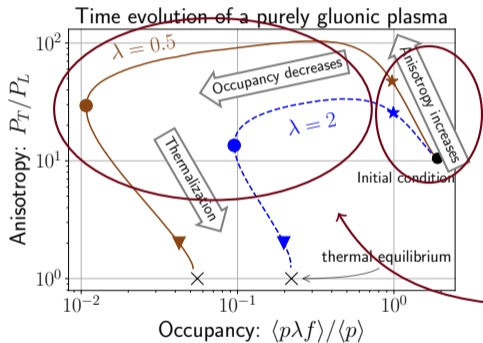
Recall: Kinetic theory and bottom-up thermalization



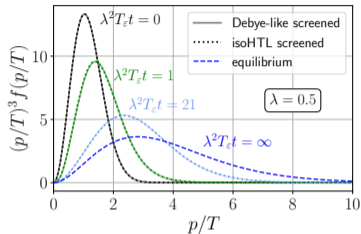
Recall: Kinetic theory and bottom-up thermalization



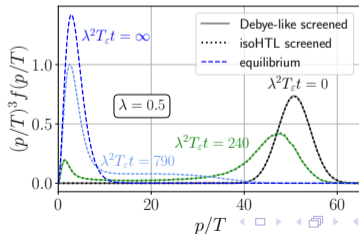
Recall: Kinetic theory and bottom-up thermalization



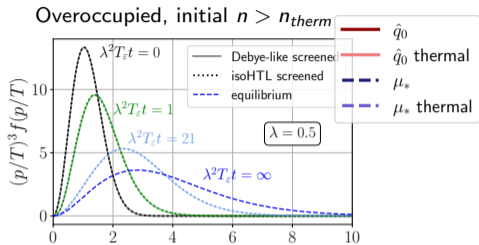
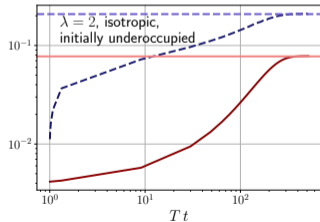
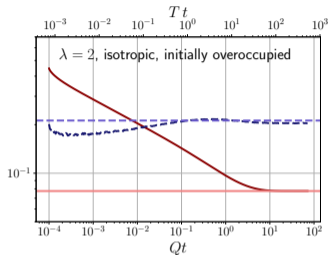
Overoccupied, initial $n > n_{therm}$



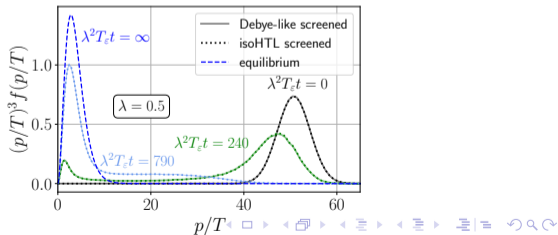
Underoccupied, initial $n < n_{therm}$



Isotropic toy models

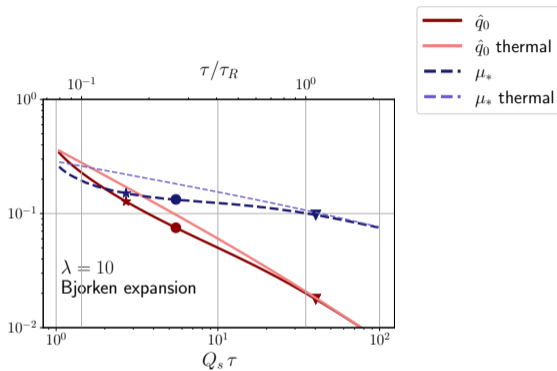
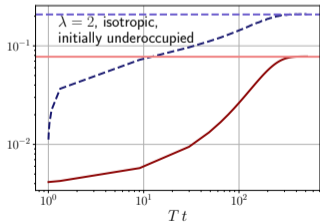
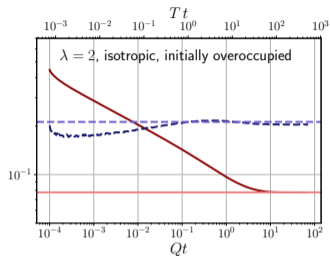


Underoccupied, initial $n < n_{therm}$



Comparison with isotropic toy models

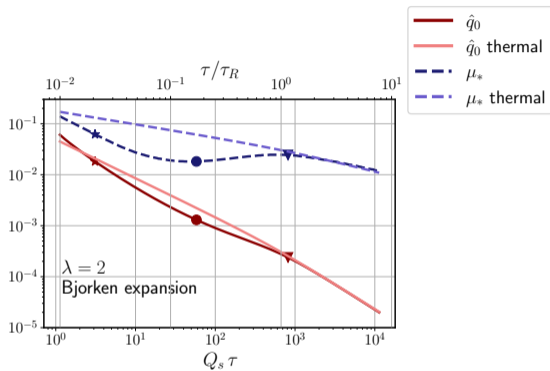
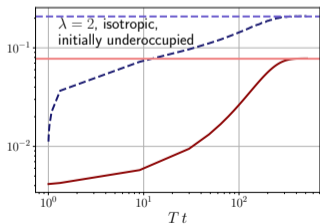
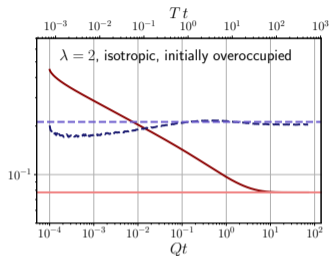
arXiv:2511.07519 [Barata, Boguslavski, FL, Sadofyev]



- Expanding evolution is combination of over- and underoccupied evolution
- Better visible at smaller couplings!

Comparison with isotropic toy models

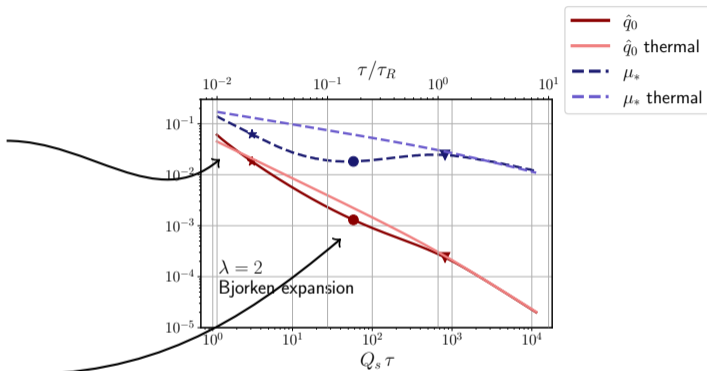
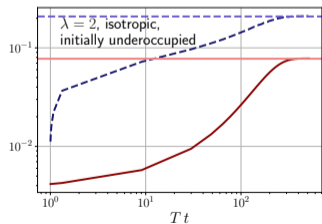
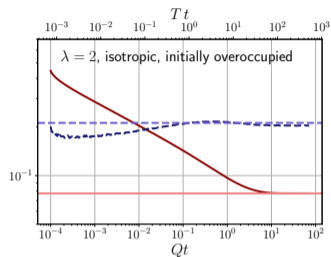
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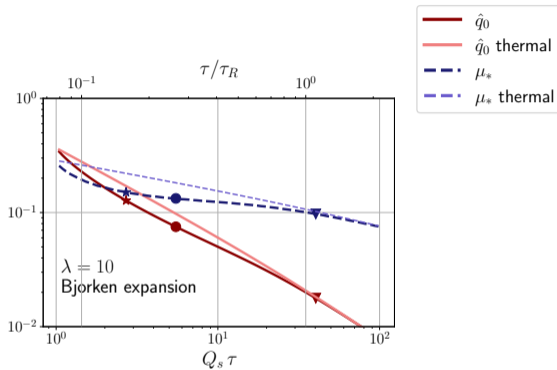
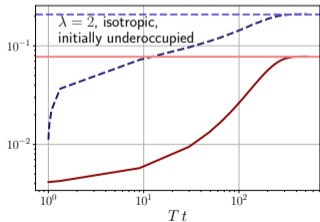
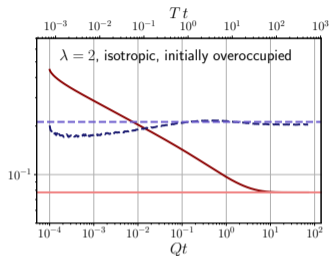
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Gluon spectrum

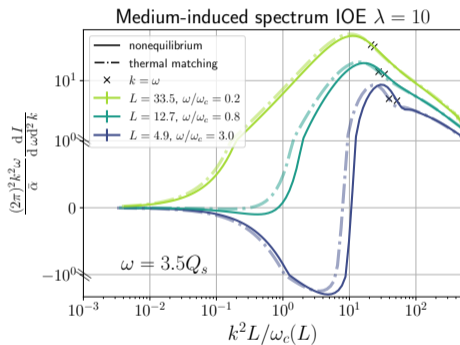
- Fixed gluon energy $\omega = 3.5Q_s$
- varying length (color in plot)

$$\hat{q}(\tau) = \begin{cases} \hat{q}^{\text{sim}}(\tau), & t_{\min} \leq \tau \leq t_{\min} + L \\ 0, & \tau > t_{\min} + L, \end{cases}$$

- Form of spectrum dominated by ratio ω/ω_c

$$\omega_c = \int_{t_{\min}}^{t_{\min}+L} dt (t - t_{\min}) \hat{q}(t)$$

- All information, difficult to analyze



Gluon spectrum

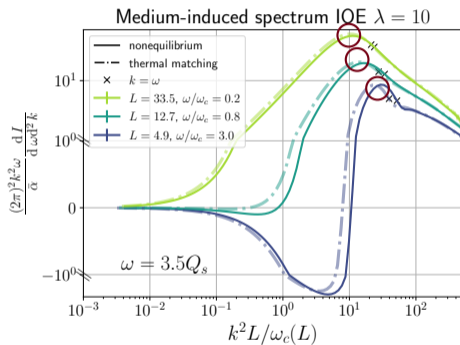
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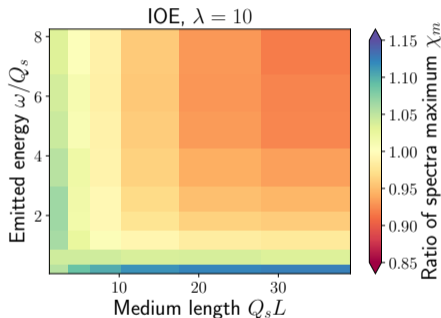


Comparison to equilibrium

- Comparison to equilibrium: Maximum of spectrum plot

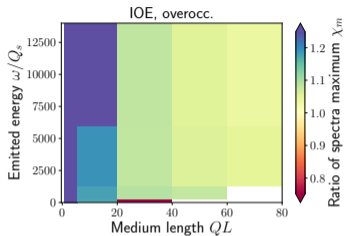
$$\chi_m = \frac{\text{Maximum spectrum nonequ.}}{\text{Maximum spectrum equ.}}$$

- Clear systematics visible
- Does not approach 1 even for $L \rightarrow \infty$!

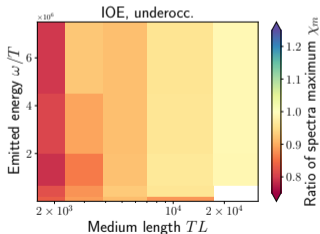
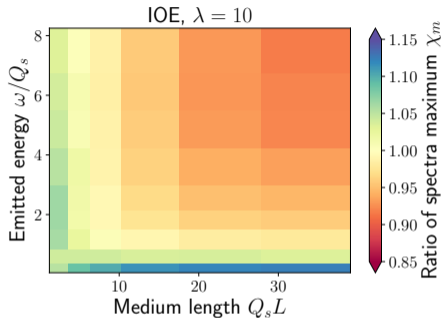


Comparison to equilibrium II

arXiv:2511.07519 [Barata, Boguslavski, FL, Sadofyev]



initially overoccupied
 $\chi > 1$

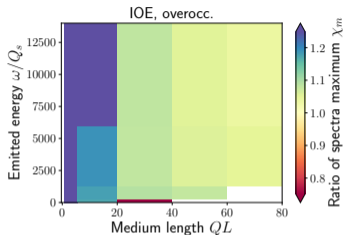


then underoccupied
 $\chi < 1$

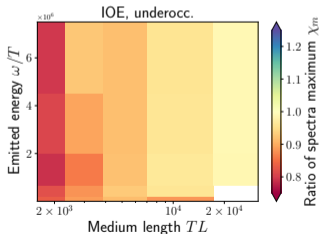
- For fixed L:
 $\chi > 1 \rightarrow \chi < 1$
- Can be explained with toy models

Comparison to equilibrium II

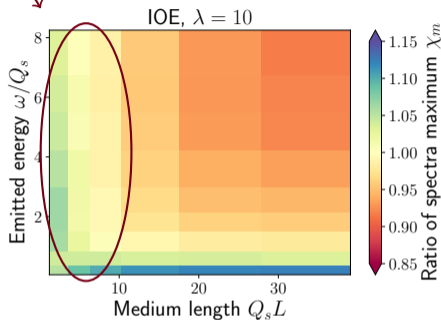
arXiv:2511.07519 [Barata, Boguslavski, FL, Sadofyev]



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 $\chi > 1$



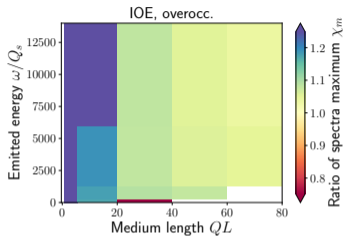
then underoccupied
 $\chi < 1$



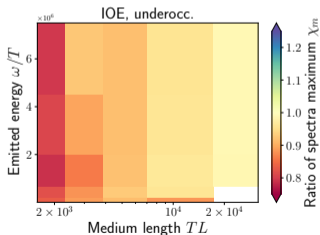
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Comparison to equilibrium II

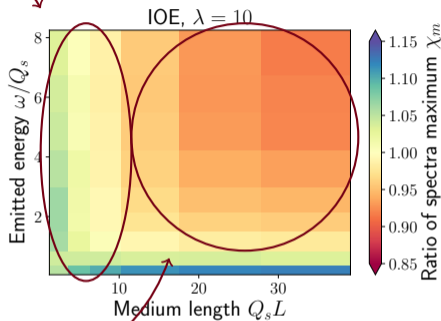
arXiv:2511.07519 [Barata, Boguslavski, FL, Sadofyev]



initially overoccupied
 $\chi > 1$



then underoccupied
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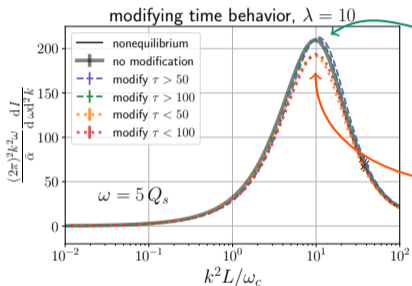
■ For fixed L:

$$\chi > 1 \rightarrow \chi < 1$$

→ Can be explained with toy models

Effect of modifications

Glueon spectrum

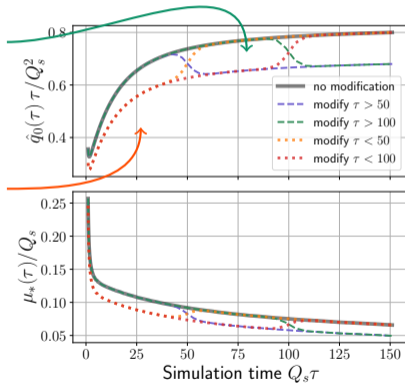


late-time
modification

early-time
modification

- Only modifications at early time influence spectrum!

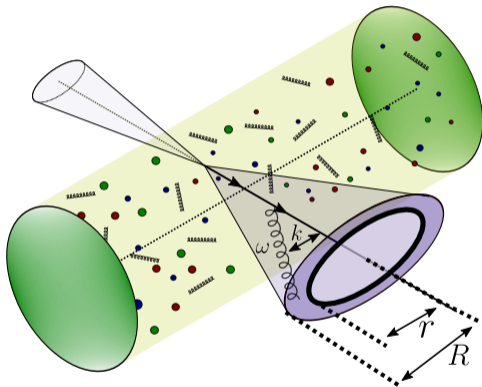
Input \hat{q}_0 and μ_*



Jet shape

- Energy deposited in *annulus* with radii r and R

$$\frac{d\rho(r, R)}{d\omega} = \int_{r\omega}^{R\omega} d^2\mathbf{k} \omega \frac{dI}{d\omega d^2\mathbf{k}}$$

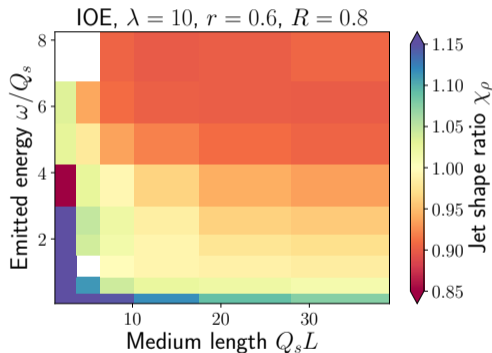


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$$\frac{d\rho(r, R)}{d\omega} = \int_{r\omega}^{R\omega} d^2\mathbf{k} \omega \frac{dI}{d\omega d^2\mathbf{k}}$$

- Qualitatively same behavior as spectra maximum



Conclusions

- Use **QCD kinetic theory** to describe initial **nonequilibrium QCD plasma** in heavy-ion collisions
- Calculate **momentum broadening of jets** $\rightarrow \hat{q}$ and $C(\mathbf{q}_\perp)$
- Taken as input to calculate gluon emission spectrum
- Clear differences to thermal medium evolution, even at **large medium lengths**

Outlook

- Prediction/effects for specific more differential observables
- General structure, scaling of spectra?
- Improve QCD kinetic theory simulations of background
e.g., machine learning Eur.Phys.J.C 85 (2025) [Barrera Cabodevila, Kurkela, FL]

Thank you very much for your attention!

Spectrum more explicit

Spectrum is obtained via

$$\frac{dI^{(0)}}{d\omega d^2k} = \frac{2\bar{\alpha}}{\pi\omega} \operatorname{Re} \left\{ k^{-2} \left(e^{-i\frac{k^2}{2\omega\cot(L,t_0)}} - 1 \right) + \int_{t_0}^L dt \frac{\cot(t,t_0)}{\hat{P}^2(t,t_0)} e^{-\frac{k^2}{\hat{P}^2(t,t_0)}} \right\},$$

$$\begin{aligned} \frac{dI^{(1)}}{d\omega d^2k} = & \frac{\bar{\alpha}\pi}{k^4} \operatorname{Re} \int_{t_0}^L dt_2 \left\{ \frac{i}{2\omega} \int_{t_0}^{t_2} dt_1 \frac{\hat{q}_0(t_1)}{\hat{R}_{21}^2} e^{-\frac{k^2}{\hat{R}_{21}^2}} \left[Q_s^2(L, t_2) I_a \left(\frac{k^2}{\hat{K}_{21}}, \frac{k^2 \hat{R}_{21}^2}{Q_r^2} \right) + 2C_{12} \hat{R}_{21} k^2 I_b \left(\frac{k^2}{\hat{K}_{21}}, \frac{k^2 \hat{R}_{21}^2}{Q_r^2} \right) \right] - \cot_{20} Q_{s0}^2(L, t_2) I_a \left(\frac{k^2}{\hat{P}^2(t_2, t_0)}, \frac{k^2}{Q_b^2} \right) \right. \\ & \left. + 2\hat{q}_0(t_2) C^2(t_2, L) I_b \left(\frac{ik^2}{2\omega C^2(t_2, L) (\cot_{20} - \frac{1}{C(t_2, L)S(L, t_2)} - \cot(t_2, L))}; \frac{k^2}{Q_r^2 C^2(t_2, L)} \right) e^{-\frac{k^2}{2\omega\cot(L, t_2)}} \right\} \end{aligned}$$

medium input,
obtained from $\hat{q}(\Lambda_{\perp})$

scale,
obtained self-consistently

And requires solving a differential equation

$$\left[\frac{d^2}{dt^2} + \Omega^2(t) \right] F(t, t_0) = 0, \quad \Omega^2(t) = -i \frac{\hat{q}_r(t)}{\omega}, \quad \hat{q}_r(t) = \hat{q}_0(t) \log \frac{Q_r^2}{\mu_*^2(t)}$$

with specific boundary conditions: $S(t_0, t_0) = \partial_t C(t, t_0)_{t=t_0} = 0$,
 $\partial_t S(t, t_0)_{t=t_0} = C(t_0, t_0) = 1$

Small-distance form of $C(x)$

- Introduce cutoff

$$C(\mathbf{x}) = \underbrace{\int_{|\mathbf{q}_\perp| < \Lambda} \frac{d^2 \mathbf{q}_\perp}{(2\pi)^2} C(\mathbf{q}_\perp)}_{\text{expand: } \sim \frac{1}{4} x^2 q_\perp^2 + \dots} \underbrace{(1 - e^{i\mathbf{x} \cdot \mathbf{q}_\perp})}_{\text{expand: } \sim \frac{1}{4} x^2 q_\perp^2 + \dots} + \underbrace{\int_{|\mathbf{q}_\perp| > \Lambda} \frac{d^2 \mathbf{q}_\perp}{(2\pi)^3} C(\mathbf{q}_\perp) (1 - e^{i\mathbf{x} \cdot \mathbf{q}_\perp})}_{\text{expand: } \sim \frac{1}{4} x^2 q_\perp^2 + \dots}$$

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arXiv:2511.07519 [Barata, Boguslavski, FL, Sadofyev]

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- \hat{q}_0 is constant related to large q_\perp behavior of $C(q_\perp \rightarrow \infty) \rightarrow \frac{\hat{q}_0}{2\pi} \frac{1}{q_\perp^4}$

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$$C(\mathbf{x}, t) = \frac{1}{2} x^2 b(t) + \frac{x^2 \hat{q}_0(t)}{2} \left(1 - \gamma_E - \log \frac{x Q}{2}\right)$$

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- Expand perturbatively **around harmonic oscillator solution**

→ “improved opacity expansion”

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medium input, obtained from $\hat{q}(\Lambda_\perp)$

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\hat{q}_0 and μ_* from EKT

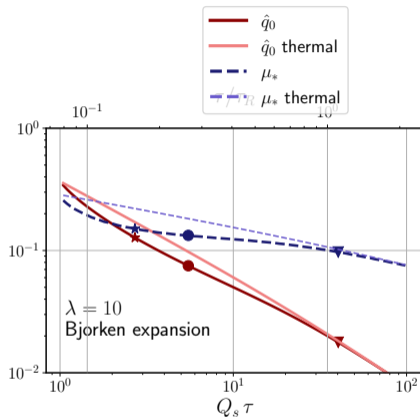
- Recall: \hat{q}_0 , μ_* characterize small- x of $C(x)$,
- Get \hat{q}_0 from large-cutoff behavior,
 $\hat{q}(t) \approx 2\hat{q}_0(t) \log \frac{\Lambda_{\perp}}{Q_s} + b(t)$
- Get μ_* via

$$\mu_*^2(t) = \frac{1}{4} \exp(-b(t)/\hat{q}_0(t) + 2\gamma_E - 2)$$

- Thermal:

$$\hat{q}_0 = T^3 \frac{N_C C_{RG}^4 \zeta(3)}{2\pi^3}$$

$$\mu_*^2 = \frac{m_D^2}{4} \left(\frac{m_D}{2T} \right)^{2\zeta(2)/\zeta(3)-2} e^{2\gamma_E - 2 - \left(\frac{\zeta(2)}{\zeta(3)} - 1 \right) (1 - 2\gamma_E) + \frac{\sigma_+}{\pi\zeta(3)}}$$



Outline

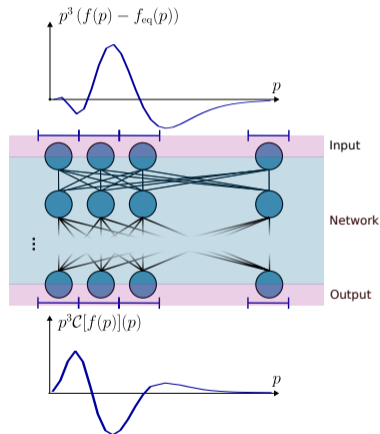
6 Machine learning and EKT

Basic idea

$$v^\mu \partial_\mu f(x^\mu, \mathbf{p}) = \mathcal{C}[f(x^\mu, \mathbf{p})]$$

- \mathcal{C} is **local** in position space, but **nonlocal** in time
- Train at one point \rightarrow parallelize over different points!
- On finite momentum grid:

$$\mathcal{C} : \mathbb{R}^{n_x} \rightarrow \mathbb{R}^{n_x}$$



Conformal transformation and rest frame

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- In 3D, $f(\mathbf{p})$ direction-dependent
- Boost to restframe, where

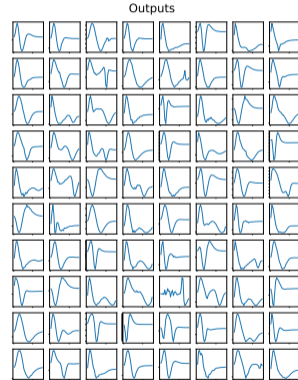
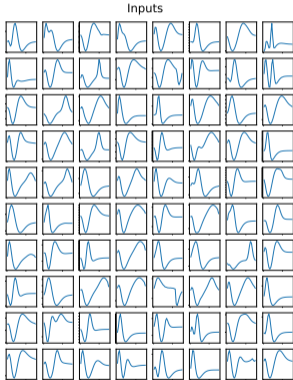
$$T^{\mu\nu} = v_g \int \frac{d^3\mathbf{p}}{(2\pi)^3} \frac{p^\mu p^\nu}{p^0} f(\mathbf{p})$$

is diagonal

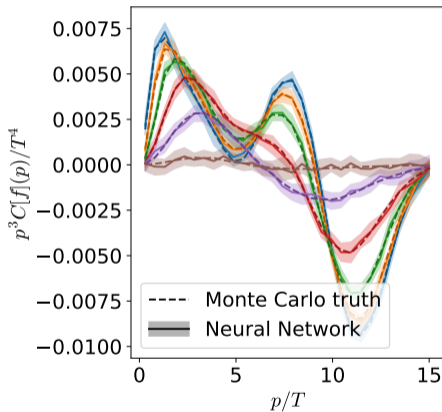
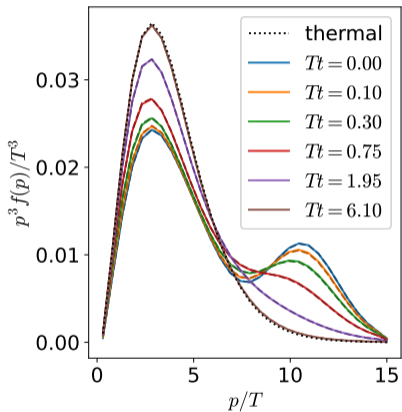
- Rotate coordinate system such that $P_z < P_y < P_x$
- Rescale units such that $T = 1$

Input-Output

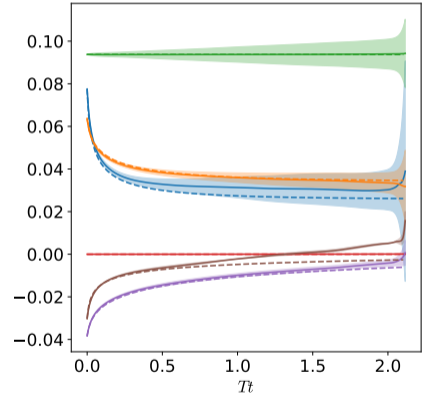
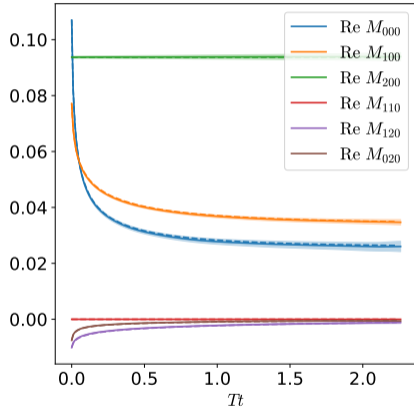
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Isotropic systems



Anisotropic systems



Conclusions

- Time evaluation of collision kernel costly
- Train neural network
- Good results for isotropic and anisotropic systems
- Problems with staying at equilibrium