MASSACHUSETTS INSTITUTE OF TECHNOLOGY
MIT CENTER FOR THEORETICAL PHYSICS – A LEINWEBER INSTITUTE

FWF Österreichischer Wissenschaftsfonds

How the nonequilibrium initial stages influence medium-induced radiation

PLB 852 (2024) [Boguslavski, Kurkela, Lappi, FL, Peuron]
arXiv:2509.03868 [Altenburger, Boguslavski, FL]
arXiv:2511.07519 [Barata, Boguslavski, FL, Sadofyev]

Florian Lindenbauer (MIT Center for Theoretical Physics – a Leinweber Institute)

March 24, 2026, C3NT Workshop, Wuhan:
Jet-soft dynamical medium interactions in high-energy heavy-ion collisions

Outline

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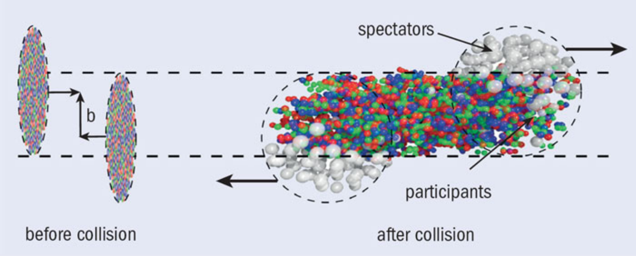
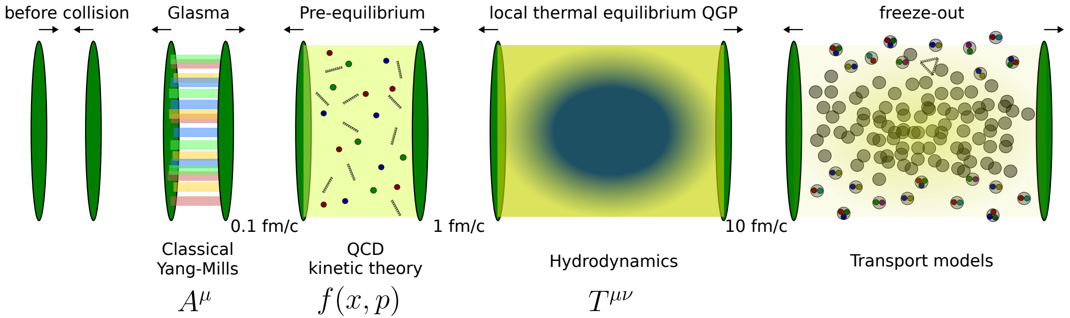
Heavy-ion collisions and the quark-gluon plasma

- Study high-temperature properties of the strong interaction (QCD)
- Collision of atomic nuclei at LHC or RHIC
- Creates high-temperature QCD matter = Quark-Gluon plasma (QGP)



[Alberica Toia 2013, CERN COURIER]

Time-evolution of the QGP in heavy-ion collisions



Interested in pre-equilibrium stages ("Initial stages")

$\rightarrow$ **QCD out of equilibrium** $\rightarrow$ Use **QCD kinetic theory** to simulate plasma evolution

[Rev.Mod.Phys. 93 (2021) [Berges, Heller, Mazeliauskas, Venugopalan]]

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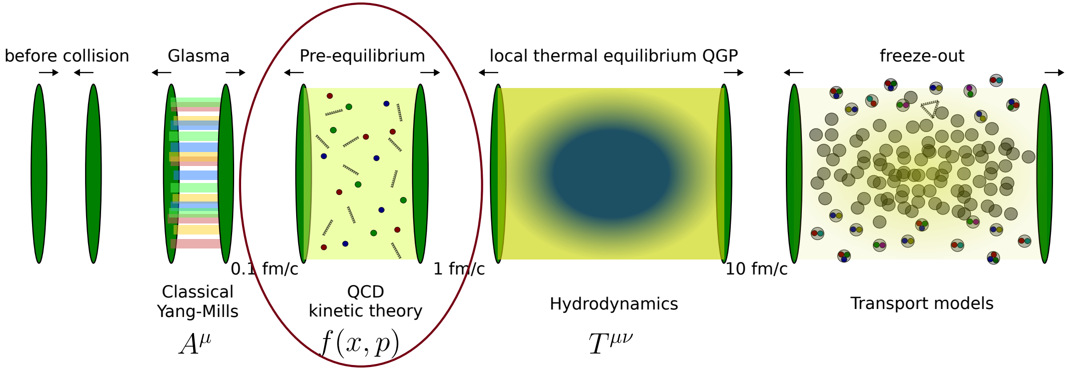

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Effective kinetic theory (EKT) description of the QGP

How many particles/gluons

- **Microscopic description**
- Gluons with **distribution function** $f(t, \mathbf{x}, \mathbf{p})$

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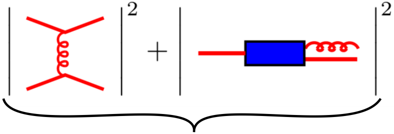
Effective kinetic theory (EKT) description of the QGP

How many particles/gluons

- **Microscopic description**
- Gluons with **distribution function** $f(t, \mathbf{x}, \mathbf{p})$
- Time evolution described by **Boltzmann equation** at leading-order[1]

$$(\partial_t + \boldsymbol{v} \cdot \boldsymbol{\nabla}) f =$$



Collision term

- Azimuthal symmetry around beam axis $\hat{z}$,
  Bjorken expansion, homogeneous in transverse plane

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Observables in EKT

- Fundamental quantity: Distribution function $f(\mathbf{p})$
- **Energy-Momentum tensor**:

$$T^{\mu\nu} = \nu_g \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3} \frac{p^\mu p^\nu}{p} f(\mathbf{p}) \qquad \leftarrow \text{"Moments of distribution function"}$$

- Longitudinal pressure $P_L = T_{zz}$
- Transverse pressure $P_T = T_{xx} = T_{yy}$

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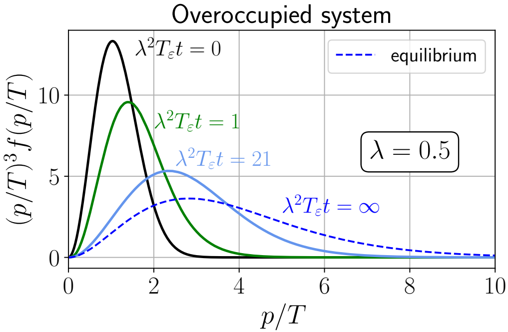
- Longitudinal pressure $P_L = T_{zz}$
- Transverse pressure $P_T = T_{xx} = T_{yy}$
- How many ‘hard’ particles

$$\frac{\langle pf \rangle}{\langle p \rangle} = \frac{\int \mathrm{d}^3\mathbf{p}\, p\, f(\mathbf{p})^2}{\int \mathrm{d}^3\mathbf{p}\, p\, f(\mathbf{p})} \qquad \leftarrow \text{“particle number weighted with energy”}$$

Thermalization in isotropic plasmas



- (Left): Initially over-occupied system
- (Right): Underocc. system: Soft thermal bath formed (bottom-up thermalization)

First studied in Phys.Rev.Lett. 133 (2024) [Kurkela, Lu], see also Phys.Rev.D 105 (2022) [Fu, Ghiglieri, Iqbal, Kurkela]

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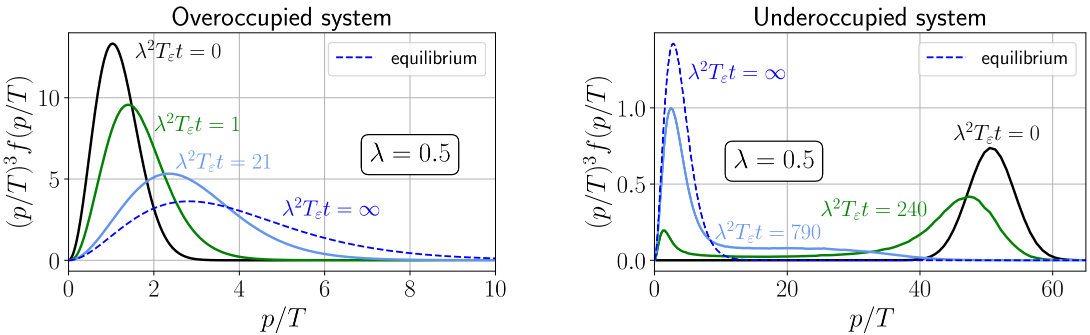

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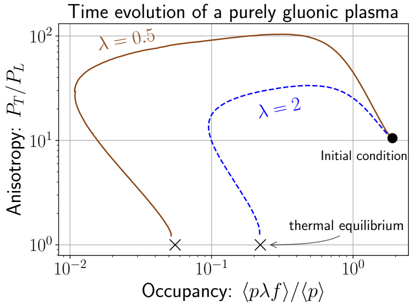
Bottom-up thermalization in heavy-ion collisions

Initial condition[2], with $\lambda = g^2 N_{\mathrm{C}}$:

$f(p_\perp, p_z) =$ "squeezed" $\frac{1}{\lambda} \times \exp\left(-p^2\right)/p$



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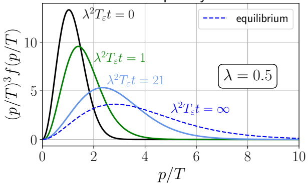

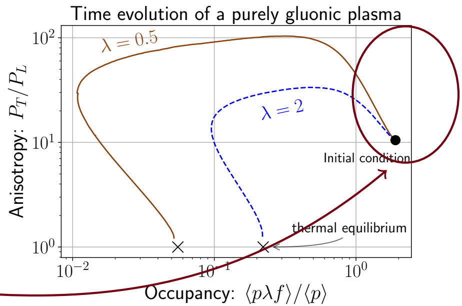

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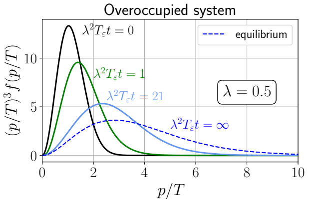

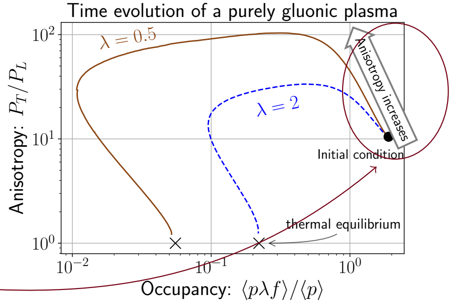

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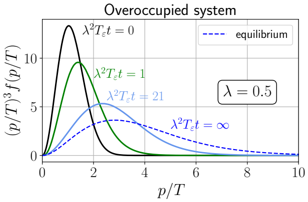

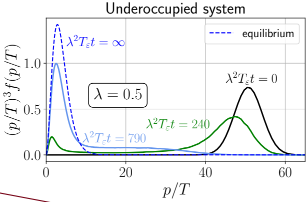

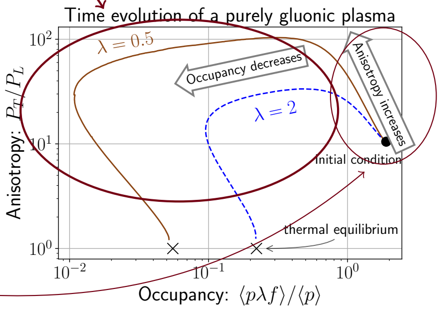

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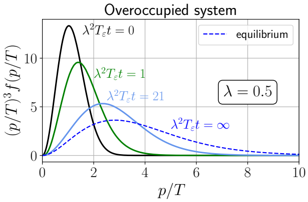

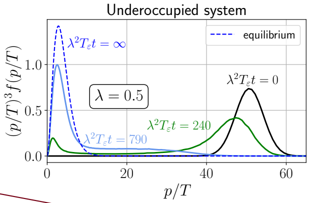

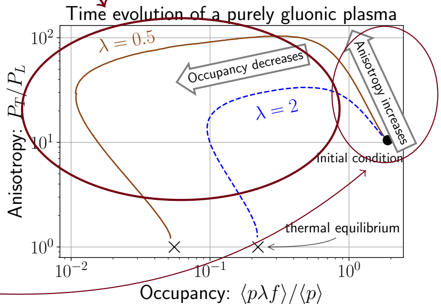

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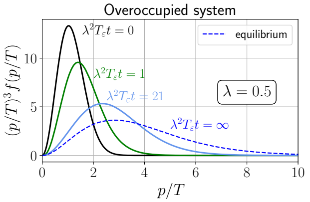

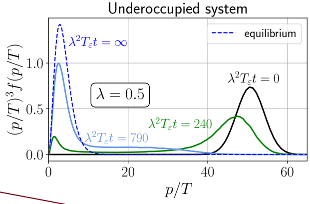

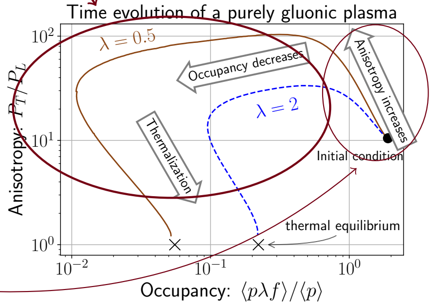

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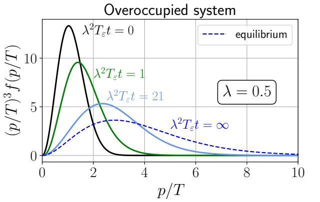

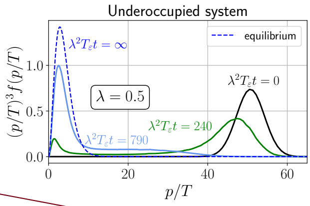

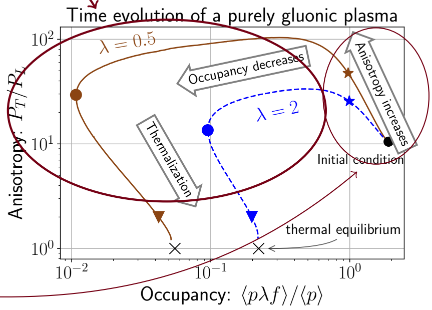

- thermalizes at[3] $\tau_\mathrm{BMSS} = \left(\frac{\lambda}{12\pi}\right)^{-13/5}/Q_s$,
- **Markers** represent **different stages**

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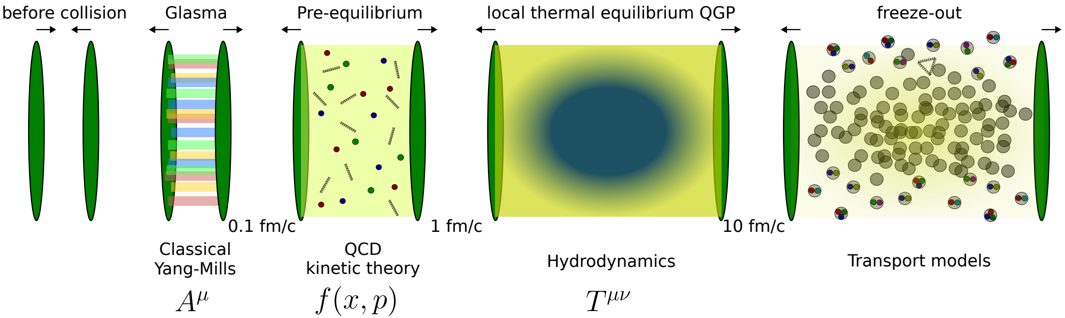

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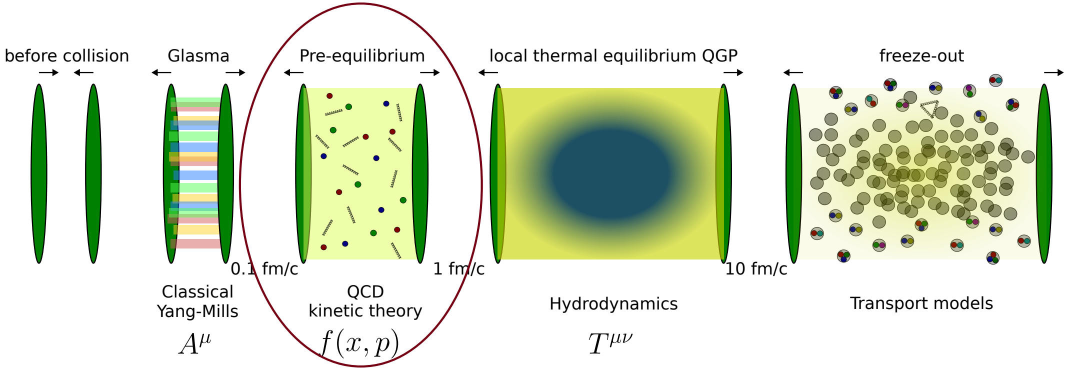

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Possible probes of initial stages

- Produced early (energetic or heavy):
  - Jets

    Phys.Lett.B 803 (2020) [Andres, Armesto, Niemi, Paatelainen, Salgado], JHEP 08 (2023) 027 [Hauksson, Iancu], Phys.Lett.B 850 (2024) [Boguslavski, Kurkela, Lappi, FL, Peuron], JHEP 12 (2024) [Barata, Salgado, Silva] arXiv:2509.03868 [Altenburger, Boguslavski, FL], arXiv:2509.19430 [Pablos, Takacs], arXiv:2511.07519 [Barata, Boguslavski, FL, Sadofyev], arXiv:2512.17009 [Barata, Milhano, Sadofyev, Silva], . . .
  - Heavy quarks, quarkonium

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- Weakly interacting with background:
  - Lepton production

    Nucl.Phys.A 1030 (2023) [Coquet, Du, Ollitrault, Schlichting, Winn], Phys.Rev.D 111 (2025) [Garcia-Montero, Plaschke, Schlichting]
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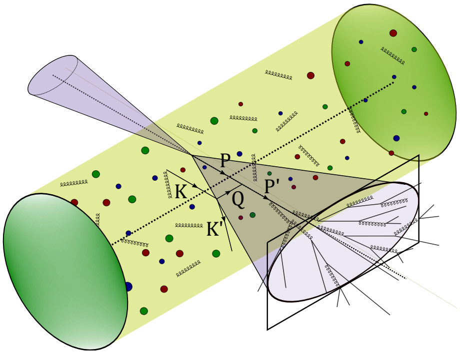
- **Highly energetic partons** created in initial collision
- **Interact** with nonequilibrium plasma
- Resulting particle shower measured



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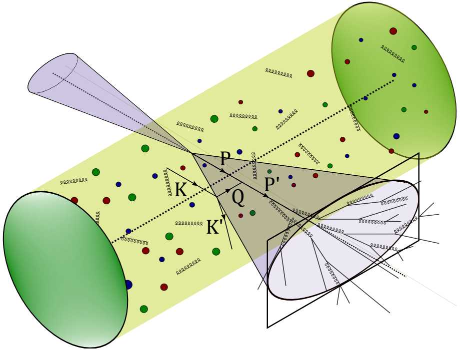
→ Imprints of **medium interactions**



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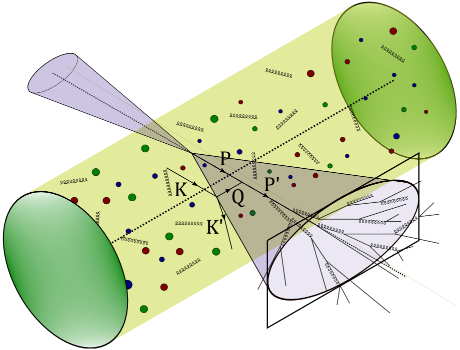
$\rightarrow$ Imprints of **medium interactions**

- Many event generators exist, rely on simplifying assumptions (isotropic, thermal, . . .)

$\rightarrow$ Describe **jet quenching** in **out-of-equilibrium plasma**!

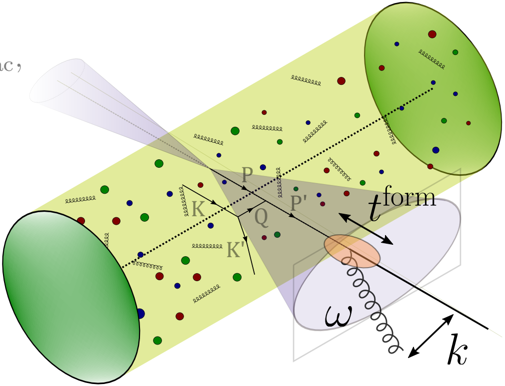


Obtaining the gluon emission spectrum

Phys.Rev.D 79 (2009) [Arnold]
JETP Lett. 65 (1997) [Zakharov]
Nucl.Phys.B 483 (1997) [Baier, Dokshitzer, Mueller, Peigne, Schiff]

- **Energy loss** dominated by **gluon radiation**

$$\omega \frac{\mathrm{d}I}{\mathrm{d}\omega} \sim \mathrm{Re} \int_{t1,t2} \partial_{\mathbf{x}} \cdot \partial_{\mathbf{y}} G(\mathbf{x}, t_2; \mathbf{y}, t_1)_{\mathbf{y}=0} - G_{\mathrm{vac}},$$



- $\mathrm{d}I/\mathrm{d}\omega$: **Spectrum**
  =Probability of emitting gluon
  with energy ω (relative to vacuum)
- ω: emitted gluon energy
- k: transverse momentum
- $t^{\mathrm{form}} \sim \sqrt{\omega/\hat{q}}$: formation time

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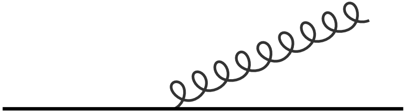

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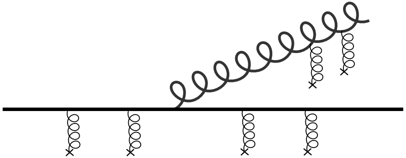

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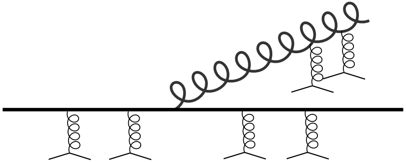

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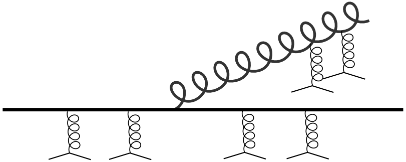

- Green's function solves

energy shift

$$(i\partial_t - \delta E + iC(\mathbf{x}, t))\, G(\mathbf{x}, t; \mathbf{y}, t_1) = \delta^2(\mathbf{x} - \mathbf{y})$$

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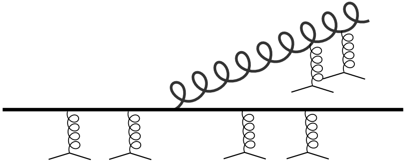

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$$\left(i\partial_t - \delta E + iC(\mathbf{x}, t)\right) G(\mathbf{x}, t; \mathbf{y}, t_1) = \delta^2(\mathbf{x} - \mathbf{y})$$

- Input: **dipole cross section** = potential in Schrödinger equation

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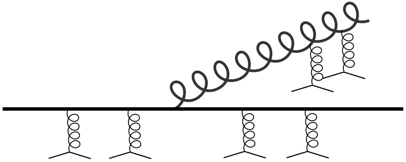
Obtaining the gluon emission spectrum

Phys.Rev.D 79 (2009) [Arnold]
JETP Lett. 65 (1997) [Zakharov]
Nucl.Phys.B 483 (1997) [Baier, Dokshitzer, Mueller, Peigne, Schiff]

- **Energy loss** dominated by **gluon radiation**

$$\omega \frac{\mathrm{d}I}{\mathrm{d}\omega} \sim \mathrm{Re} \int_{t1,t2} \partial_{\mathbf{x}} \cdot \partial_{\mathbf{y}} G(\mathbf{x}, t_2; \mathbf{y}, t_1)_{\mathbf{y}=0} - G_{\mathrm{vac}},$$



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$$(i\partial_t - \delta E + iC(\mathbf{x}, t))\, G(\mathbf{x}, t; \mathbf{y}, t_1) = \delta^2(\mathbf{x} - \mathbf{y})$$

'broadening probability'
Probability of receiving momentum kick $\mathbf{q}_\perp$

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- **Harmonic approximation** (for jets $|\mathbf{x}| \ll 1$ important!)

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- $\hat{q}$ quantifies **momentum broadening**, "jet quenching parameter"
- Schrödinger equation for harmonic oscillator $\rightarrow$ analytic solution!

Results for $\hat{q}$: Broadening

Phys.Rev.D 110 (2024) [Boguslavski, Kurkela, Lappi, FL, Peuron]
Phys.Lett.B 850 (2024) [Boguslavski, Kurkela, Lappi, FL, Peuron]]

- Obtain **Jet quenching parameter** from kinetic theory, input $f(\mathbf{k})$,

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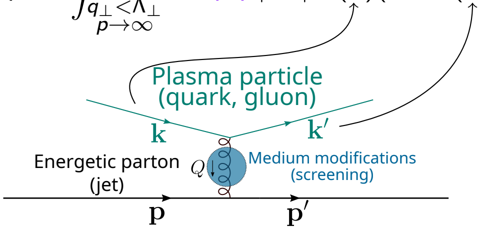

NB: Isotropic screening to prevent plasma instabilities! Phys.Lett.B 214 (1988) [Mrowczynski], Phys.Rev.D 68 (2003) [Romatschke, Strickland]

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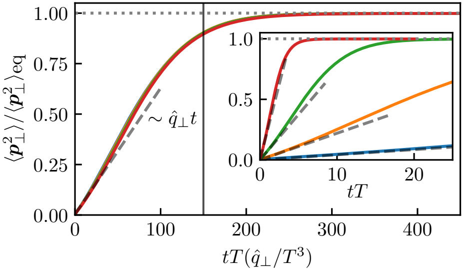
- Simulations of jet equilibration

→ $\hat{q}$ accurately captures transverse momentum broadening

→ **Remarkable collapse** of different couplings $\lambda \in (2, 20)$!

Jet evolution in thermal equilibrium[4]



→See talk by F. Zhou on Wednesday 10:30

[4] [arXiv:2510.25669 [Boguslavski, FL, Mazeliauskas, Takacs, Zhou]]

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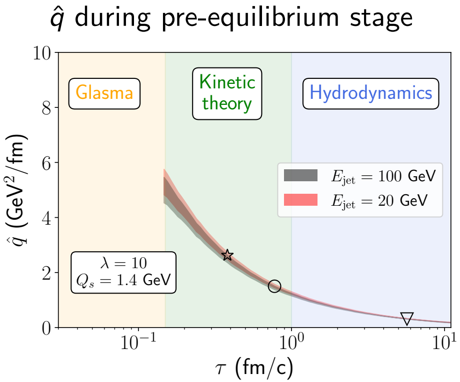
- Little jet energy dependence
- Supports **large values** from **Glasma**[4] and lower values in hydrodynamic stage



[4] [Phys.Lett.B 810 (2020) [Ipp, Müller, Schuh]]

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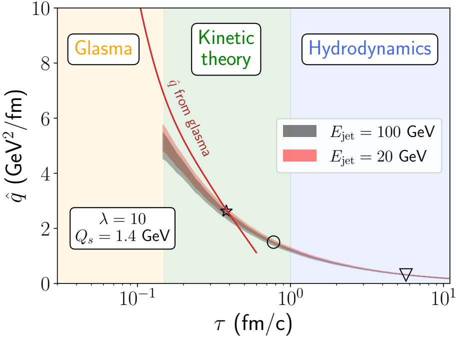

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Problems with $\hat{q}$ for radiation

- Recall: $\hat{q}$ appeared in small-distance expansion:

expansion parameter

physical quantity, enters equations as potential

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Need cutoff $\Lambda_\perp$ to restrict momentum transfer $q_\perp < \Lambda_\perp$ (needed in eikonal limit $p \to \infty$)

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Can we do better? $\to$ Consider full kernel $C(\mathbf{q}_\perp)$

Outline

Momentum broadening kernel vs. jet quenching parameter

Momentum broadening kernel:

$$C(\mathbf{q}_\perp) = \int \mathrm{d}\Gamma_{\mathrm{PS}}\, |\mathcal{M}|^2 f(\mathbf{k})(1 + f(\mathbf{k}-\mathbf{q}))$$

Compare to $\hat{q}$,

weighted with
momentum transfer

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different
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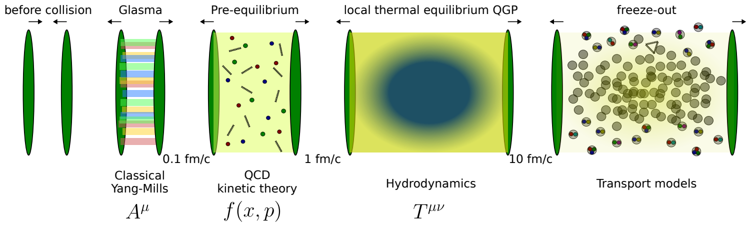
$$\hat{q}^{ij} = \int \frac{\mathrm{d}^2\mathbf{q}_\perp}{(2\pi)^2} q^i q^j C(\mathbf{q}_\perp),$$

- $C(q_\perp)$ is **more general** than $\hat{q}$

Recall: Kinetic theory and bottom-up thermalization



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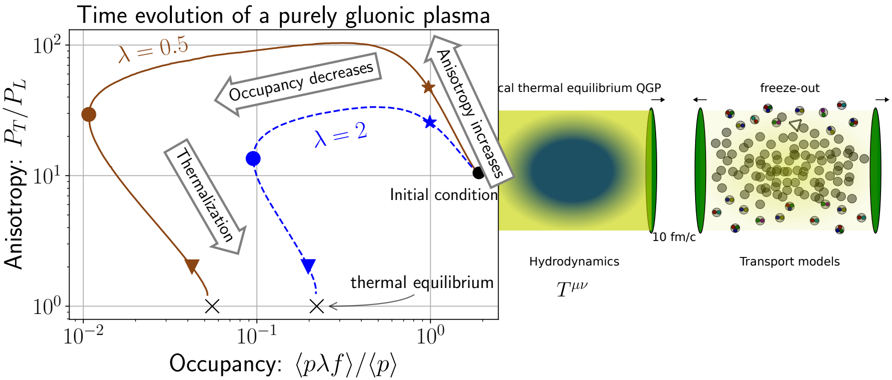
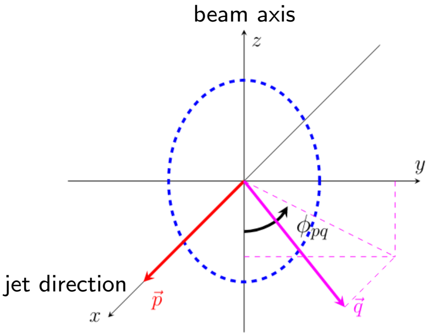
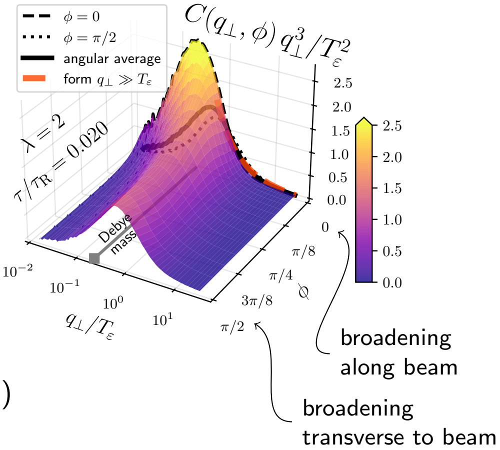


Angular dependence and contribution to $\hat{q}$

arXiv:2509.03868 [Altenburger, Boguslavski, FL]





- $C(q_\perp, \phi)$ is 2D function
- Contribution to $\hat{q} = \int \mathrm{d}^2\mathbf{q}_\perp\, q_\perp^2\, C(\mathbf{q}_\perp)/(2\pi)$

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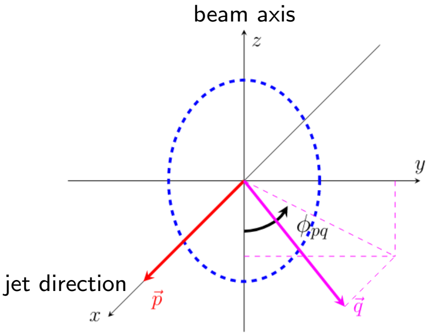

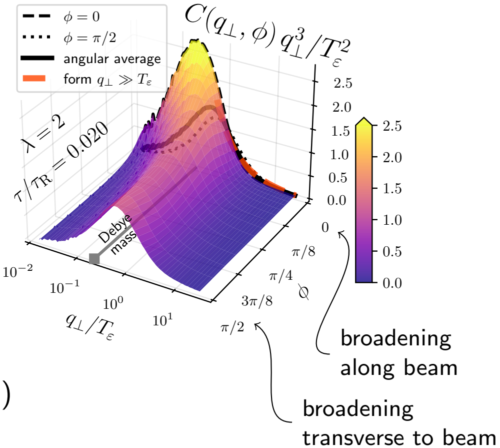

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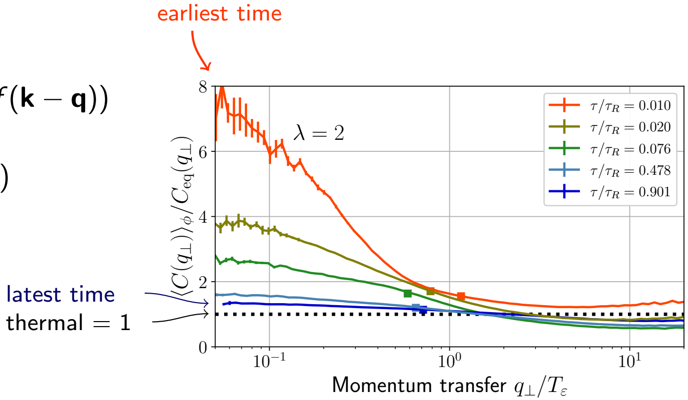
- Contribution to $\hat{q} = \int \mathrm{d}^2\mathbf{q}_\perp\, q_\perp^2\, C(\mathbf{q}_\perp)/(2\pi)$
- **Peaked at Debye mass** ■ for later times
- Along beam ($\phi_{pq} = 0$): Much larger and different form at early times

Angular average

arXiv:2509.03868 [Altenburger, Boguslavski, FL]

$$C(\mathbf{q}_\perp) = \int \mathrm{d}\Gamma_{\mathrm{PS}}\, |\mathcal{M}|^2\, f(\mathbf{k})(1 + f(\mathbf{k}-\mathbf{q}))$$

Normalize using (Landau-matched) thermal kernel



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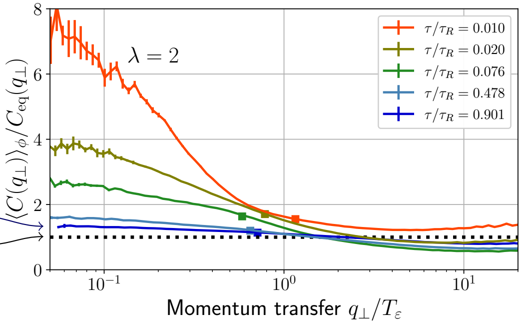
- Angular average
  $C(q_\perp) = \frac{1}{2\pi} \int_0^{2\pi} \mathrm{d}\phi\, C(q_\perp, \phi)$

earliest time

latest time

thermal $= 1$



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- Late times $\rightarrow$ thermal
- **Small-momentum transfer enhanced**

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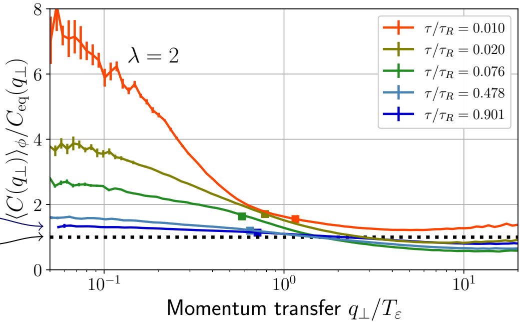
earliest time



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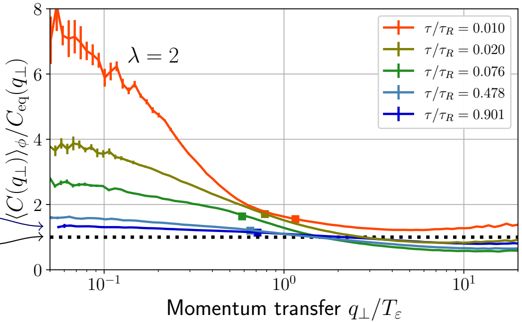
- **Small-momentum transfer enhanced**
- What to do with jet quenching?

earliest time

latest time

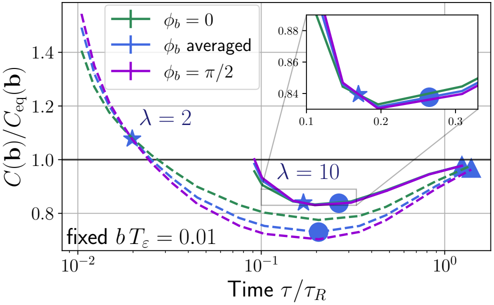
thermal $= 1$



Small-distance behavior of $C(\mathbf{x})$

arXiv:2509.03868 [Altenburger, Boguslavski, FL]

- Obtained via Fourier trafo
  $C(\mathbf{x}) = \int \frac{d^2\mathbf{q}_\perp}{(2\pi)^3}(1 - e^{i\mathbf{q}_\perp\cdot\mathbf{x}})C(\mathbf{q}_\perp)$
- Small-distance behavior important for jet quenching!
- For $\lambda = 10$, always smaller than in equilibrium



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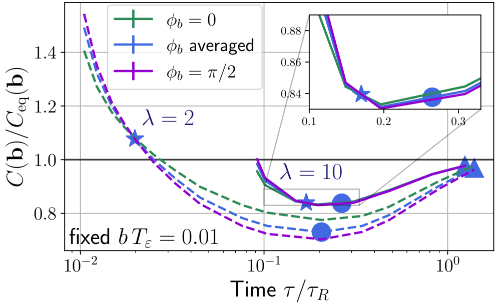
- Take small distance behavior and calculate jet emission spectrum

arXiv:2511.07519 [Barata, Boguslavski, FL, Sadofyev]



Outline

Calculating the spectrum from small-distance behavior

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$\rightarrow$ Differential Eq. for spectrum reduces to **harmonic oscillator**:

$$\left(i\partial_t - \delta E + iC(\mathbf{x},t)\right) G(\mathbf{x},t;\mathbf{y},t_1) = \delta^2(\mathbf{x}-\mathbf{y})$$

Green's function known!

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- Disadvantage: $\hat{q}$ **not well-defined** (cutoff-dependence)

Small-distance form of $C(x)$

- Proper expansion for small distance:

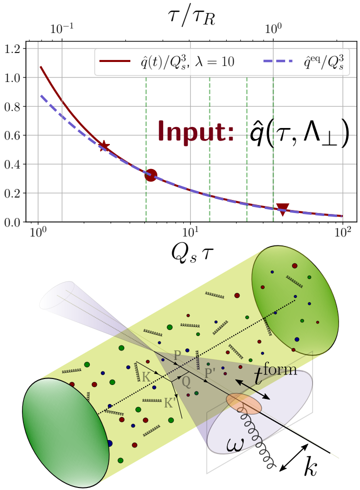
$$C(\mathbf{x},t) = \frac{1}{4}\hat{q}_0(t)\mathbf{x}^2 \log\frac{1}{\mathbf{x}^2\mu_*^2(t)}$$

- **Expand** perturbatively **around harmonic oscillator solution**
$\rightarrow$ "improved opacity expansion" JHEP 09 (2021) [Barata, Mehtar-Tani, Soto-Ontoso, Tywoniuk]

$$C(\mathbf{x},t) = \underbrace{\frac{\hat{q}_0(t)}{4}\mathbf{x}^2 \log\frac{Q^2}{\mu_*^2}}_{\text{harmonic approximation}} - \underbrace{\frac{\hat{q}_0(t)}{4}\mathbf{x}^2 \log(Q^2\mathbf{x}^2)}_{\text{perturbation}}$$

Obtaining the spectrum – Schematic

arXiv:2511.07519 [Barata, Boguslavski, FL, Sadofyev]



$\hat{q}_0(\tau)$, $\mu_*(\tau)$

Small-distance form of potential

"Improved opacity expansion"

Output: Spectrum $\frac{\mathrm{d}I}{\mathrm{d}\omega\,\mathrm{d}^2\mathbf{k}}$

Small-distance form of $C(x)$

- Proper expansion for small distance:

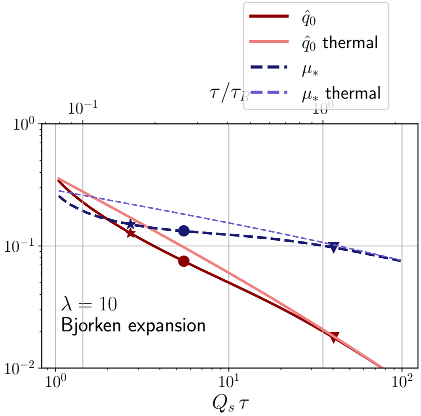
$$C(\mathbf{x}, t) = \frac{1}{4}\hat{q}_0(t)\mathbf{x}^2 \log \frac{1}{\mathbf{x}^2 \mu_*^2(t)} + \mathcal{O}(\hat{q}_0 \mathbf{x}^4 \mu_*^2)$$

- μ_* and $\hat{q}_0$ are medium-parameters uniquely determined from large-cutoff form of $\hat{q}(\Lambda_\perp)$,

$$\hat{q}(t) \approx 2\hat{q}_0(t) \log \frac{\Lambda_\perp}{Q_s} + b(t)$$

- Obtain μ_* via

$$\mu_*^2(t) = \frac{1}{4} \exp(-b(t)/\hat{q}_0(t) + 2\gamma_E - 2)$$



Comparing to thermal system

arXiv:2511.07519 [Barata, Boguslavski, FL, Sadofyev]

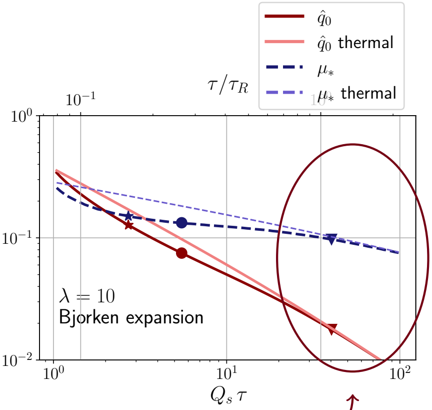
- Recall: $\hat{q}_0$, μ_* obtained from large cutoff behavior of $\hat{q}$
- Compare with an evolving thermal system

$$T_\varepsilon(t) = \left(\frac{30\epsilon(t)}{\pi^2}\right)^{1/4}$$

with energy density from kinetic theory,

$$\epsilon = \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3}|\mathbf{p}|f(\mathbf{p})$$

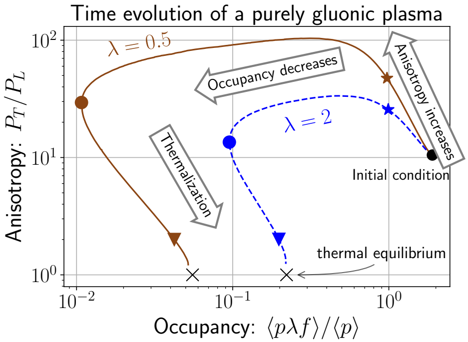
- Thermal forms of $\hat{q}_0$, μ_* known[5]



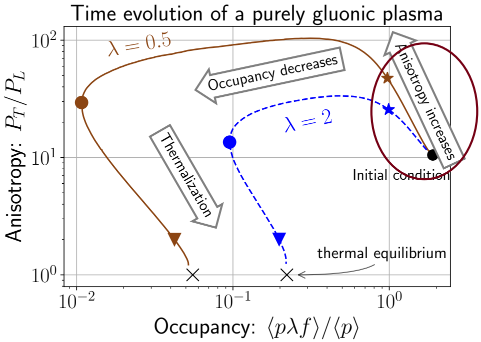
Late time approache to thermal

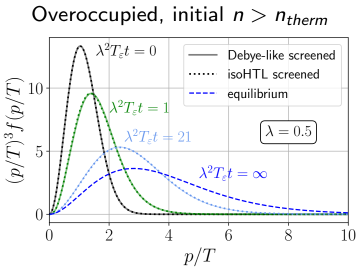
[5] [Phys.Rev.D 78 (2008) [Arnold, Xiao]]

Recall: Kinetic theory and bottom-up thermalization

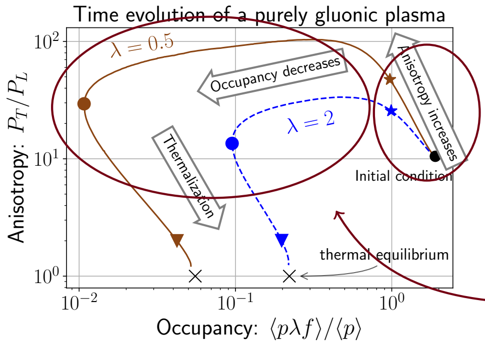


Recall: Kinetic theory and bottom-up thermalization

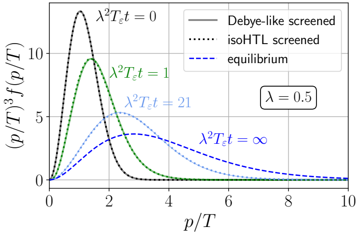




Recall: Kinetic theory and bottom-up thermalization



Overoccupied, initial $n > n_{therm}$



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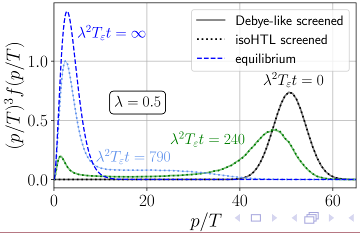
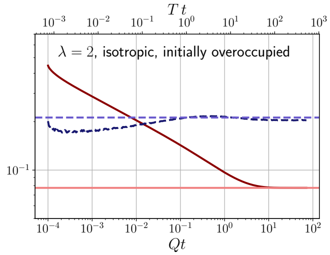
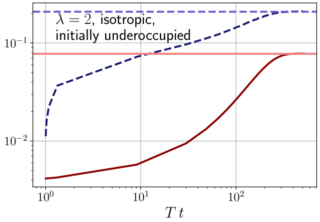
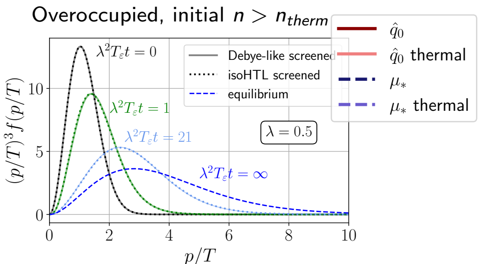


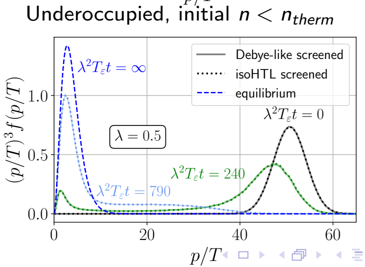
Isotropic toy models

arXiv:2511.07519 [Barata, Boguslavski, FL, Sadofyev]



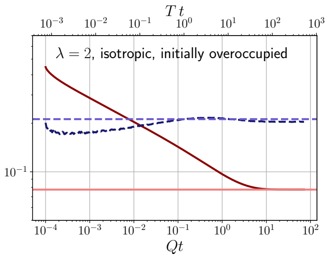


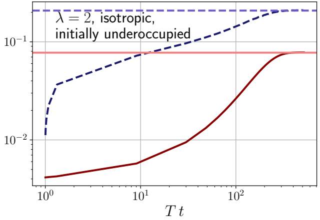


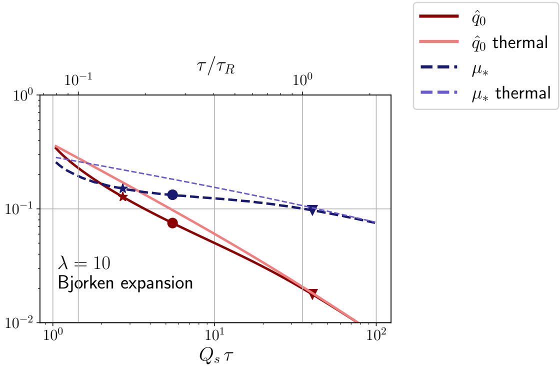


Comparison with isotropic toy models

arXiv:2511.07519 [Barata, Boguslavski, FL, Sadofyev]







- Expanding evolution is combination of over- and underoccupied evolution
- Better visible at smaller couplings!

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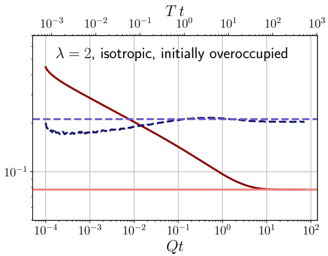

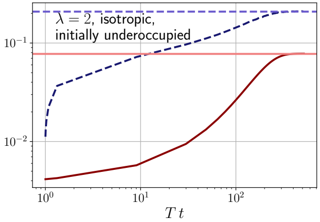

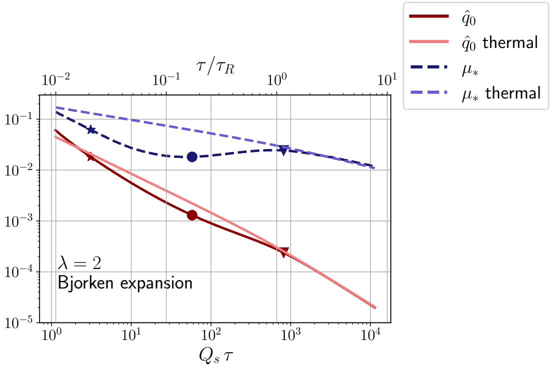

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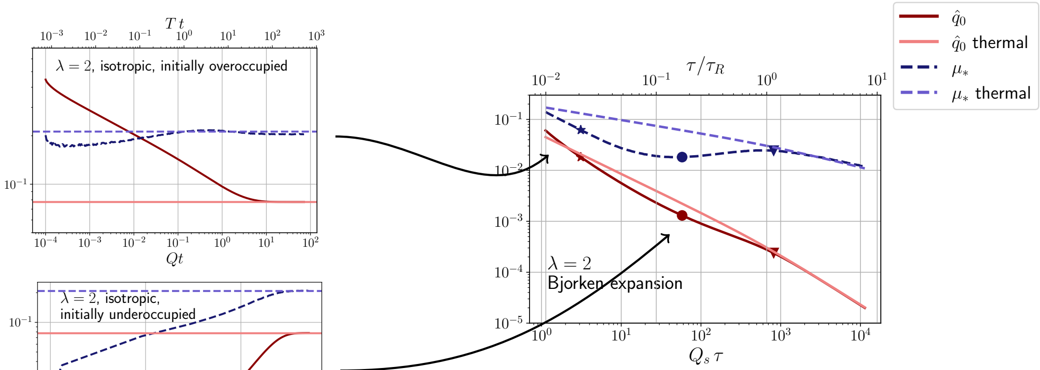

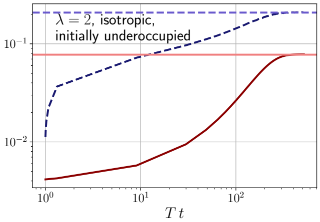

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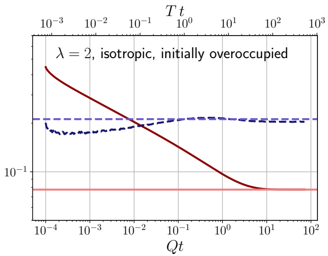

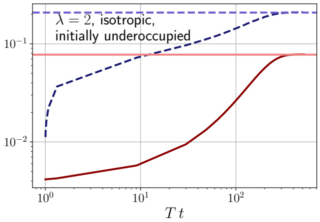

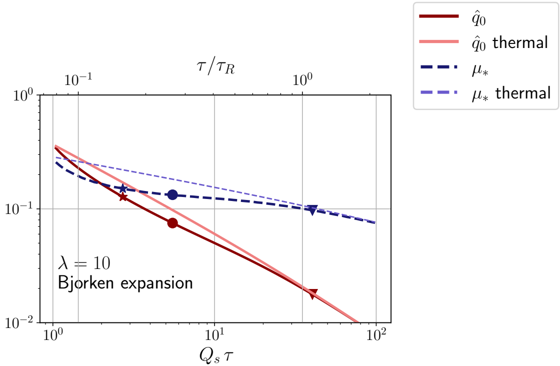

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Gluon spectrum

arXiv:2511.07519 [Barata, Boguslavski, FL, Sadofyev]

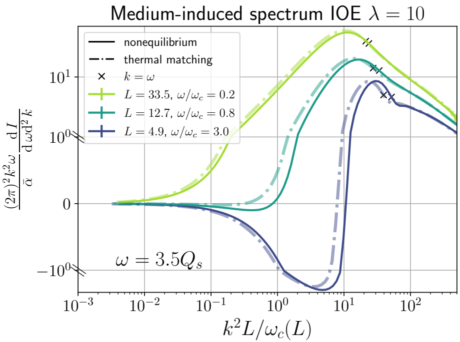
- Fixed gluon energy $\omega = 3.5Q_s$
- varying length (color in plot)

$$\hat{q}(\tau) = \begin{cases} \hat{q}^{\mathrm{sim}}(\tau), & t_{min} \leq \tau \leq t_{min} + L \\ 0, & \tau > t_{min} + L\,, \end{cases}$$

- Form of spectrum dominated by ratio ω/ω_c

$$\omega_c = \int_{t_{min}}^{t_{min}+L} \mathrm{d}t\,(t - t_{min})\hat{q}(t)$$

- All information, difficult to analyze



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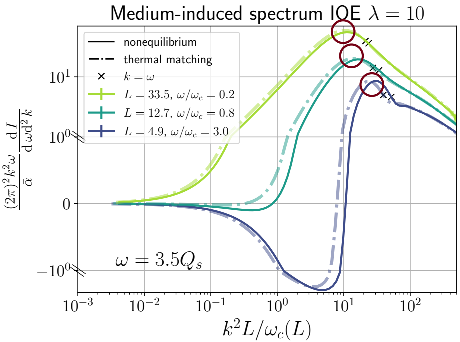
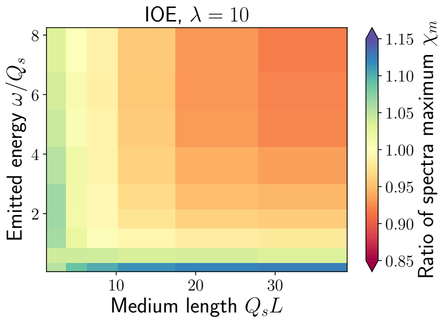


Comparison to equilibrium

- Comparison to equilibrium: Maximum of spectrum plot

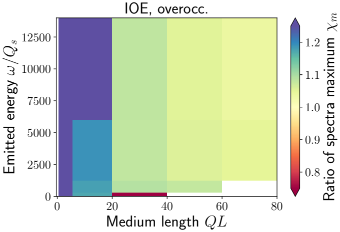
$$\chi_m = \frac{\text{Maximum spectrum nonequ.}}{\text{Maximum spectrum equ.}}$$

- Clear systematics visible
- Does not approach 1 even for $L \to \infty$!

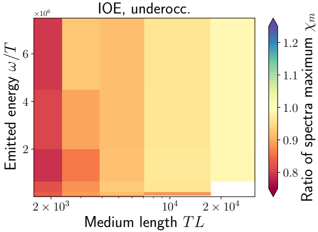


Comparison to equilibrium II

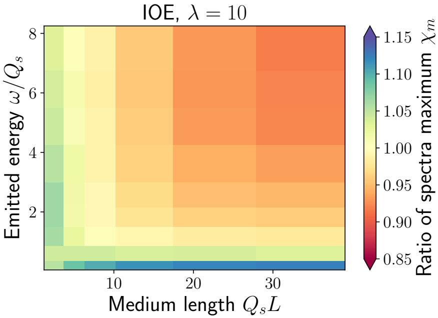
arXiv:2511.07519 [Barata, Boguslavski, FL, Sadofyev]



initially overoccupied
$\chi > 1$



then underoccupied
$\chi < 1$

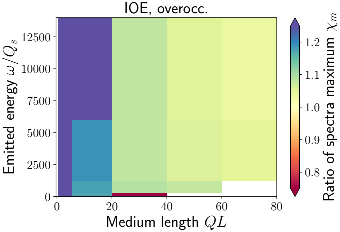


- For fixed L:
  $\chi > 1 \rightarrow \chi < 1$

$\rightarrow$ Can be explained with toy models

Comparison to equilibrium II

arXiv:2511.07519 [Barata, Boguslavski, FL, Sadofyev]



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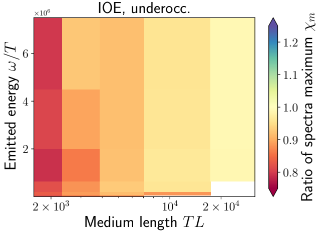

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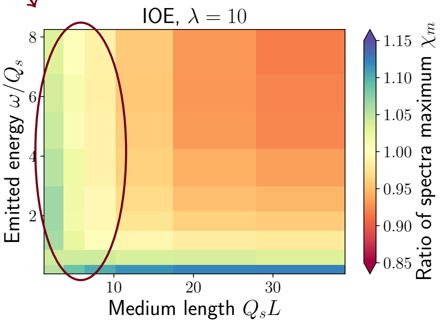

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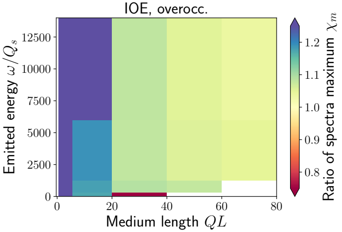

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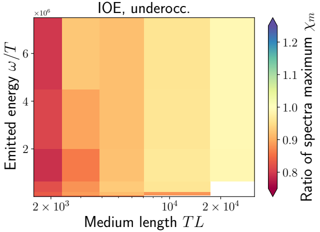

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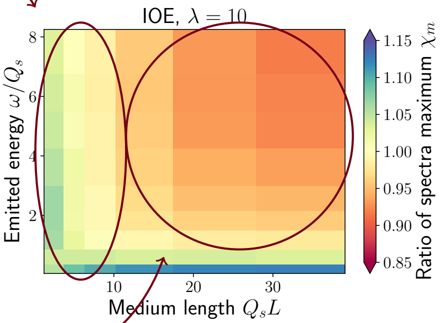

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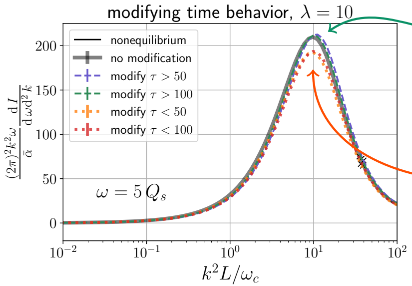
Effect of modifications

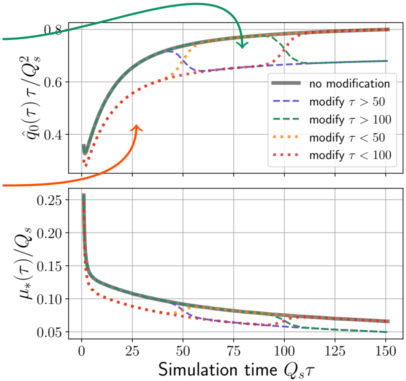
Gluon spectrum



Input $\hat{q}_0$ and μ_*

late-time modification

early-time modification



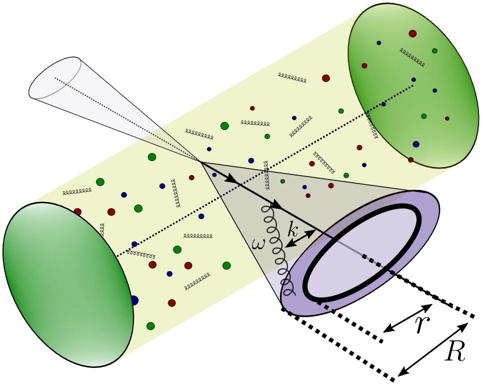
- Only modifications at early time influence spectrum!

Jet shape

arXiv:2511.07519 [Barata, Boguslavski, FL, Sadofyev]

- Energy deposited in *annulus* with radii r and R

$$\frac{\mathrm{d}\rho(r,R)}{\mathrm{d}\omega} = \int_{r\omega}^{R\omega} \mathrm{d}^2\mathbf{k}\,\omega\frac{\mathrm{d}I}{\mathrm{d}\omega\,\mathrm{d}^2\mathbf{k}}$$



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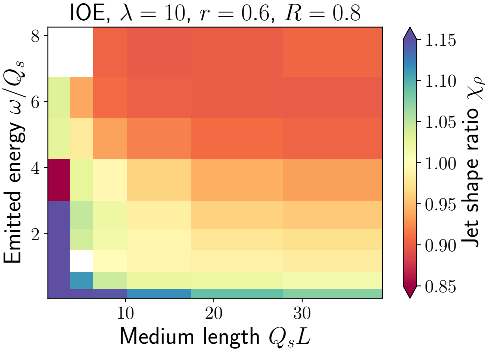
- Qualitatively same behavior as spectra maximum



Conclusions

- Use **QCD kinetic theory** to describe initial **nonequilibrium QCD plasma** in heavy-ion collisions
- Calculate **momentum broadening of jets** $\rightarrow \hat{q}$ and $C(\mathbf{q}_\perp)$
- Taken as input to calculate gluon emission spectrum
- Clear differences to thermal medium evolution, even at **large medium lengths**

Outlook

- Prediction/effects for specific more differential observables
- General structure, scaling of spectra?
- Improve QCD kinetic theory simulations of background e.g., machine learning Eur.Phys.J.C 85 (2025) [Barrera Cabodevila, Kurkela, FL]

Thank you very much for your attention!

Spectrum more explicit

arXiv:2511.07519 [Barata, Boguslavski, FL, Sadofyev]

Spectrum is obtained via

$$\frac{\mathrm{d}I^{(0)}}{\mathrm{d}\omega\,\mathrm{d}^2k} = \frac{2\bar{\alpha}}{\pi\omega}\mathrm{Re}\left\{k^{-2}\left(e^{-i\frac{k^2}{2\omega\mathrm{Cot}(L,t_0)}} - 1\right) + \int_{t_0}^{L}\mathrm{d}t\,\frac{\mathrm{Cot}(t,t_0)}{\hat{P}^2(t,t_0)}e^{-\frac{k^2}{\hat{P}^2(t,t_0)}}\right\},$$

$$\begin{aligned}\frac{\mathrm{d}I^{(1)}}{\mathrm{d}\omega\,\mathrm{d}^2k} = \frac{\bar{\alpha}\pi}{k^4}\mathrm{Re}\int_{t_0}^{L}\mathrm{d}t_2\Bigg\{\frac{i}{2\omega}\int_{t_0}^{t_2}\mathrm{d}t_1\,\frac{\hat{q}_0(t_1)}{\hat{R}_{21}^2}e^{-\frac{k^2}{\hat{P}_{21}^2}}\left[Q_s^2(L,t_2)I_a\left(\frac{k^2}{\hat{K}_{21}},\frac{k^2\hat{R}_{21}^2}{Q_r^2}\right) + 2C_{12}\hat{R}_{21}k^2 I_b\left(\frac{k^2}{\hat{K}_{21}},\frac{k^2\hat{R}_{21}^2}{Q_r^2}\right)\right] - \mathrm{Cot}_{20}\,Q_{s0}^2(L,t_2)\,I_a\left(\frac{k^2}{\hat{P}^2(t_2,t_0)},\frac{k^2}{Q_b^2}\right)\\ + 2\,\hat{q}_0(t_2)C^2(t_2,L)I_b\left(\frac{ik^2}{2\omega C^2(t_2,L)\left(\mathrm{Cot}_{20} - \frac{1}{C(t_2,L)S(L,t_2)} - \mathrm{Cot}(t_2,L)\right)};\frac{k^2}{Q_r^2C^2(t_2,L)}\right)e^{-\frac{ik^2}{2\omega\mathrm{Cot}(L,t_2)}}\Bigg\}\end{aligned}$$

And requires solving a differential equation

$$\left[\frac{\mathrm{d}^2}{\mathrm{d}t^2} + \Omega^2(t)\right]F(t,t_0) = 0\,, \qquad \Omega^2(t) = -i\frac{\hat{q}_r(t)}{\omega}, \qquad \hat{q}_r(t) = \hat{q}_0(t)\log\frac{Q_r^2}{\mu_*^2(t)}$$

medium input, obtained from $\hat{q}(\Lambda_\perp)$

scale, obtained self-consistently

with specific boundary conditions: $S(t_0,t_0) = \partial_t C(t,t_0)_{t=t_0} = 0$, $\partial_t S(t,t_0)_{t=t_0} = C(t_0,t_0) = 1$

Small-distance form of $C(x)$

arXiv:2511.07519 [Barata, Boguslavski, FL, Sadofyev]

- Introduce cutoff

$$C(\mathbf{x}) = \underbrace{\int_{|\mathbf{q}_\perp|<\Lambda} \frac{\mathrm{d}^2\mathbf{q}_\perp}{(2\pi)^2} C(\mathbf{q}_\perp) \underbrace{\left(1 - e^{i\mathbf{x}\cdot\mathbf{q}_\perp}\right)}_{\text{expand: } \sim\frac{1}{4}x^2 q_\perp^2 + \dots}} + \underbrace{\int_{|\mathbf{q}_\perp|>\Lambda_\perp} \frac{\mathrm{d}^2\mathbf{q}_\perp}{(2\pi)^3} C(\mathbf{q}_\perp)(1 - e^{i\mathbf{x}\cdot\mathbf{q}_\perp})}$$

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- $\hat{q}_0$ is constant related to large $q_\perp$ behavior of $C(q_\perp \to \infty) \to \frac{\hat{q}_0}{2\pi}\frac{1}{q_\perp^4}$

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- We know $\hat{q}(\Lambda) = \hat{q}_0 \log\frac{\Lambda^2}{Q^2} + b$ (for large enough Λ)

$$C(\mathbf{x}, t) = \frac{1}{2}x^2 b(t) + \frac{x^2\hat{q}_0(t)}{2}\left(1 - \gamma_E - \log\frac{x\,Q}{2}\right)$$

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- We know $\hat{q}(\Lambda) = \hat{q}_0 \log\frac{\Lambda^2}{Q^2} + b$ (for large enough Λ)

$$C(\mathbf{x},t) = \frac{1}{2}x^2 b(t) + \frac{x^2\hat{q}_0(t)}{2}\left(1-\gamma_E-\log\frac{x\,Q}{2}\right) = \frac{1}{4}\hat{q}_0(t)\mathbf{x}^2\log\frac{1}{\mathbf{x}^2\mu_*^2(t)}$$

- **Expand** perturbatively **around harmonic oscillator solution** $\rightarrow$ "improved opacity expansion" JHEP 09 (2021) [Barata, Mehtar-Tani, Soto-Ontoso, Tywoniuk]

Small-distance form of $C(x)$

arXiv:2511.07519 [Barata, Boguslavski, FL, Sadofyev]

- Introduce cutoff

$$C(\mathbf{x}) = \underbrace{\int_{|\mathbf{q}_\perp|<\Lambda} \frac{\mathrm{d}^2\mathbf{q}_\perp}{(2\pi)^2} C(\mathbf{q}_\perp) \underbrace{\left(1 - e^{i\mathbf{x}\cdot\mathbf{q}_\perp}\right)}_{\text{expand: } \sim\frac{1}{4}x^2 q_\perp^2 + \ldots}}_{\frac{1}{4}x^2\hat{q}(\Lambda)} + \underbrace{\int_{|\mathbf{q}_\perp|>\Lambda_\perp} \frac{\mathrm{d}^2\mathbf{q}_\perp}{(2\pi)^3} C(\mathbf{q}_\perp)\left(1 - e^{i\mathbf{x}\cdot\mathbf{q}_\perp}\right)}_{\text{expand: } \frac{x^2\hat{q}_0}{4}\left(1-\gamma_E-\log\frac{b\Lambda_\perp}{2}\right)+\mathcal{O}(x^4)}$$

- $\hat{q}_0$ is constant related to large $q_\perp$ behavior of $C(q_\perp \to \infty) \to \frac{\hat{q}_0}{2\pi}\frac{1}{q_\perp^4}$
- We know $\hat{q}(\Lambda) = \hat{q}_0 \log\frac{\Lambda^2}{Q^2} + b$ (for large enough Λ)

medium input, obtained from $\hat{q}(\Lambda_\perp)$

$$C(\mathbf{x}, t) = \frac{1}{2}x^2 b(t) + \frac{x^2\hat{q}_0(t)}{2}\left(1 - \gamma_E - \log\frac{x\,Q}{2}\right) = \frac{1}{4}\hat{q}_0(t)\mathbf{x}^2 \log\frac{1}{\mathbf{x}^2\mu_*^2(t)}$$

- **Expand** perturbatively **around harmonic oscillator solution**
  $\to$ "improved opacity expansion" JHEP 09 (2021) [Barata, Mehtar-Tani, Soto-Ontoso, Tywoniuk]

$\hat{q}_0$ and μ_* from EKT

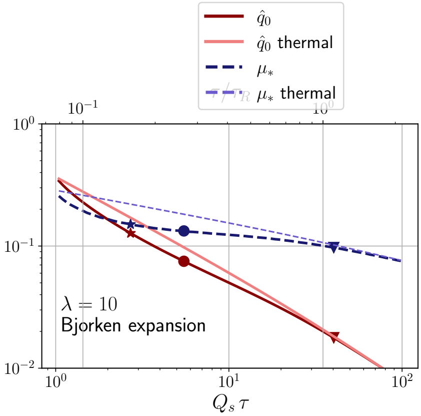
- Recall: $\hat{q}_0$, μ_* characterize small-x of $C(\mathbf{x})$,
- Get $\hat{q}_0$ from large-cutoff behavior, $\hat{q}(t) \approx 2\hat{q}_0(t)\log\frac{\Lambda_\perp}{Q_s} + b(t)$
- Get μ_* via

$$\mu_*^2(t) = \frac{1}{4}\exp(-b(t)/\hat{q}_0(t) + 2\gamma_E - 2)$$

- Thermal:

$$\hat{q}_0 = T^3 \frac{N_C C_R g^4 \zeta(3)}{2\pi^3}$$

$$\mu_*^2 = \frac{m_D^2}{4}\left(\frac{m_D}{2T}\right)^{2\zeta(2)/\zeta(3)-2} e^{2\gamma_E - 2 - \left(\frac{\zeta(2)}{\zeta(3)}-1\right)(1-2\gamma_E)+\frac{\sigma_+}{\pi\zeta(3)}}$$



Outline

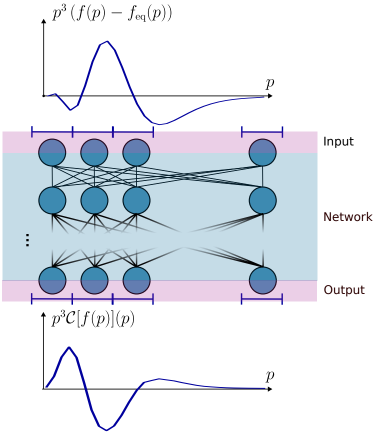
Basic idea

Eur.Phys.J.C 85 (2025) [Barrera Cabodevila, Kurkela, FL]

$$v^\mu \partial_\mu f(x^\mu, \mathbf{p}) = \mathcal{C}[f(x^\mu, \mathbf{p})]$$

- $\mathcal{C}$ is **local** in position space, but **nonlocal** in time
- Train at one point $\rightarrow$ parallelize over different points!
- On finite momentum grid:

$$\mathcal{C} : \mathbb{R}^{n_X} \rightarrow \mathbb{R}^{n_X}$$



Conformal transformation and rest frame

Eur.Phys.J.C 85 (2025) [Barrera Cabodevila, Kurkela, FL]

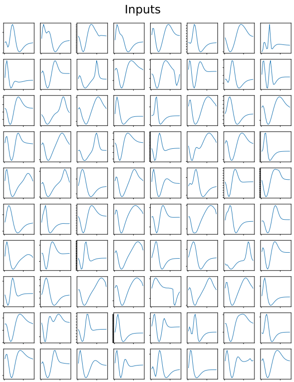
- In 3D, $f(\mathbf{p})$ direction-dependent
- Boost to restframe, where

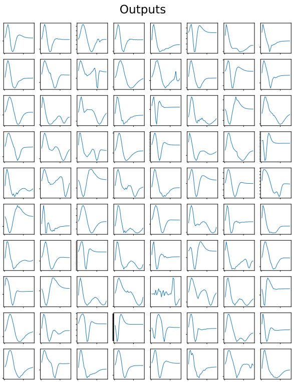
$$T^{\mu\nu} = \nu_g \int \frac{\mathrm{d}^3\mathbf{p}}{(2\pi)^3} \frac{p^\mu p^\nu}{p^0} f(\mathbf{p})$$

  is diagonal
- Rotate coordinate system such that $P_z < P_y < P_x$
- Rescale units such that $T = 1$

Input-Outpus

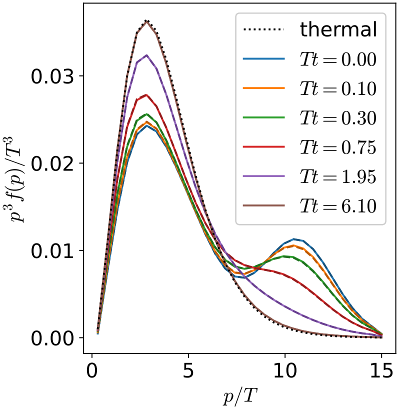
Eur.Phys.J.C 85 (2025) [Barrera Cabodevila, Kurkela, FL]

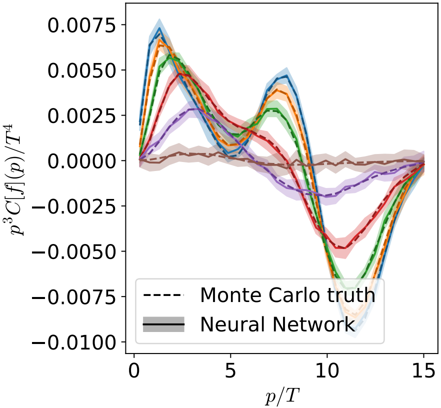




Isotropic systems

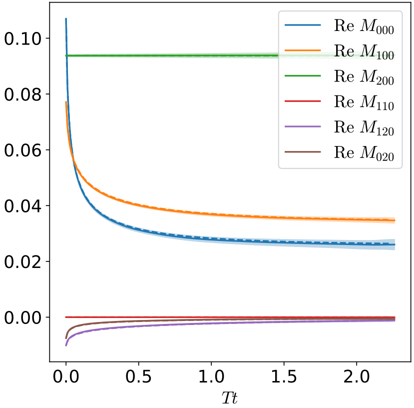
Eur.Phys.J.C 85 (2025) [Barrera Cabodevila, Kurkela, FL]

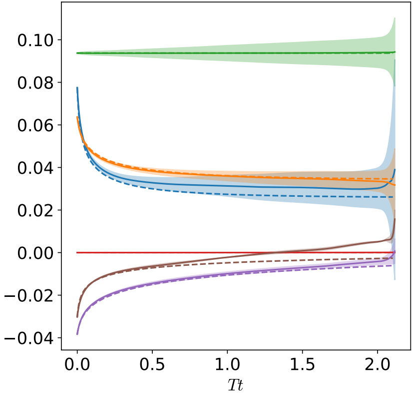




Anisotropic systems

Eur.Phys.J.C 85 (2025) [Barrera Cabodevila, Kurkela, FL]





Conclusions

- Time evaluation of collision kernel costly

$\rightarrow$ Train neural network

- Good results for isotropic and anisotropic systems
- Problems with staying at equilibrium