Minijet thermalization and jet transport coefficients in QCD kinetic theory

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with K. Boguslavski, F. Lindenbauer, A. Mazeliauskas and A. Takacs, arXiv:2510.25669

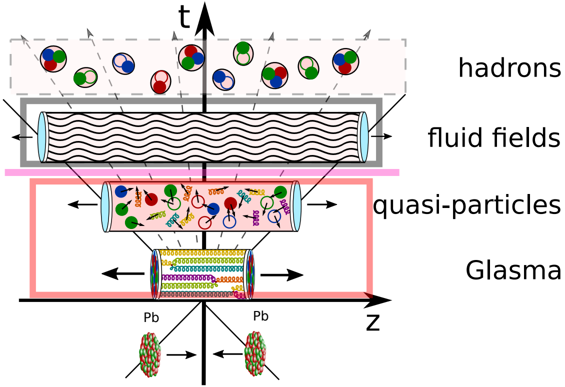
with J. Brewer and A. Mazeliauskas, JHEP, 2402.09298

Overview

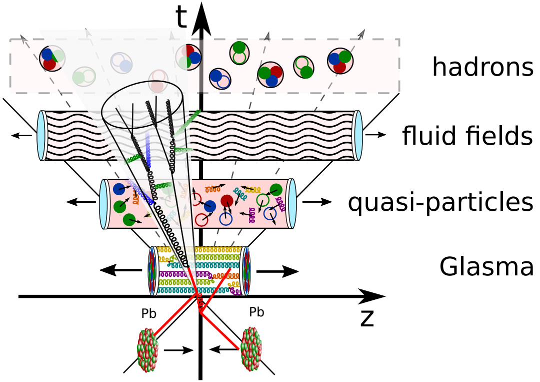
Hydrodynamics

Initial state at τ_{hydro}

Pre-equilibrium



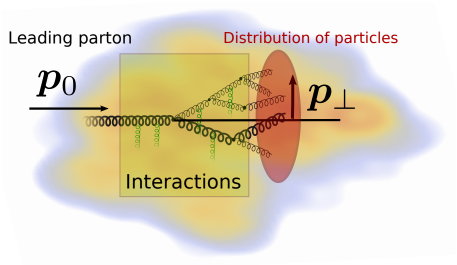
Overview



- study minijet thermalisation with kinetic theory

Overview

- Jet transport coefficient $\hat{q} \to$ medium property



1 bring together transport coefficients and minijet evolution

2 estimate thermalisation time of a jet

Outline

Linearised kinetic theory

Transport coefficients and minijet evolution

Equilibration time

Expanding QGP

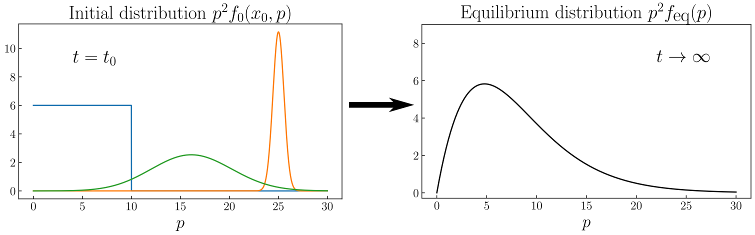
Linearised kinetic theory

Framework

- Effective Kinetic Theory for high temperature gauge theories AMY (2003), JHEP 0209353
- weakly coupled quasi-particle picture, $\lambda = 4\pi\alpha_s N_c$

  $\rightarrow$phase space distribution $f(\tau, \boldsymbol{x}, \boldsymbol{p}) \rightarrow$ homogeneous system

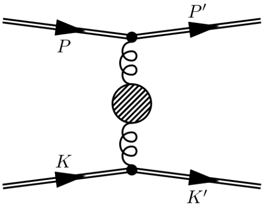
$$\partial_\tau f(\tau, \boldsymbol{p}) = -C[f]$$



Out-of-equilibrium initial state is transported to equilibrium

Collision kernel

$C_{2\leftrightarrow 2}$

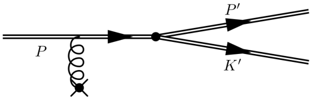


- small momentum transfer $q = |\boldsymbol{p}' - \boldsymbol{p}| \ll 1$

isotropic HTL screening

Boguslavski, Lindenbauer, PRD 2407.09605

$C_{1\leftrightarrow 2}$



- medium induced radiation of gluons
- $g \to q\bar{q}$ splittings
- LO: strictly collinear

Linearised EKT

- thermal background + linear perturbation

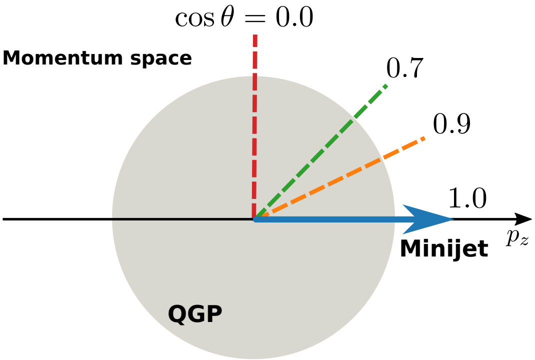
$$f(t,\boldsymbol{p}) = f_{\rm eq}(p) + \delta f_{\rm jet}(t,\boldsymbol{p})$$

- linearized Boltzmann eq.

$$\partial_t \delta f = -\delta C[\delta f, f_{\rm eq}]$$

Mehtar-Tani, Schlichting, Soudi, JHEP 2209.10569
Barrera, HP2024
Brewer, Mazeliauskas, FZ, JHEP 2402.09298

- Jet-like initial conditions



Initial conditions

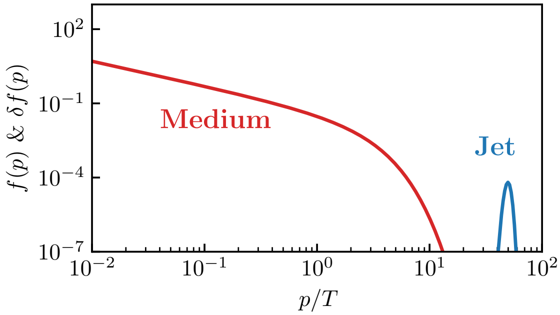
- Evolution

$$\partial_t \delta f = -\delta C[\delta f, f_{\rm eq}]$$

- Initial jet distribution

$$\delta f_{\rm Jet}(t_0, \boldsymbol{p}) \propto \delta(\boldsymbol{p} - \boldsymbol{p}_0)$$

$$\boldsymbol{p}_0 = (0, 0, E)$$



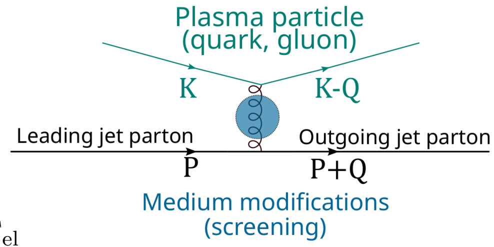
"single" parton in a thermal brick

transport coefficients + momentum broadening of $\delta f_{\rm jet}(t_0, \boldsymbol{p})$

Boguslavski, Lindenbauer, Mazeliauskas, Takacs, FZ, 2510.25669

Transport coefficients and minijet evolution

Transport coefficient $\hat{q}$ - Literature



Lindenbauer et al., PRD 2312.00447

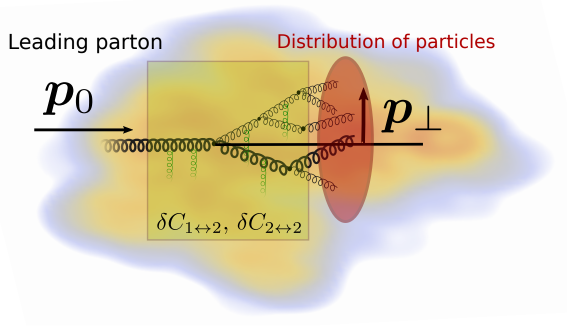
- Conventionally

$$\hat{q} = \int d^2\boldsymbol{q}_\perp \boldsymbol{q}_\perp^2 \frac{d\Gamma_{\rm el}}{d^2\boldsymbol{q}_\perp}$$

- broadening of 1 parton with energy p_0

$$\hat{q}(p_0) = \frac{1}{2p_0}\int d\Omega |\mathcal{M}|^2 f_{\rm eq}(k)(1+f_{\rm eq}(k'))\, \boldsymbol{p}'_\perp{}^2$$

Transport coefficient $\hat{q}$ - This work



$$\hat{q} = \frac{d}{dt}\langle p_T^2\rangle = \frac{d}{dt}\left(\int_{\boldsymbol{p}} p_T^2 \delta f_{\mathrm{Jet}}(t,\boldsymbol{p})\right)$$

- broadening of the "jet distribution"

$$\hat{q} = -\int_{\boldsymbol{p}} p_T^2 \delta C[\delta f, f_{\mathrm{eq}}]$$

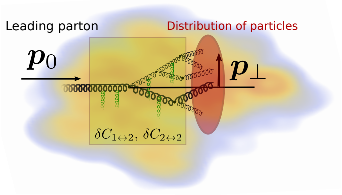
- **single hit approximation** $\delta f = \delta f^{(0)} + \delta f^{(1)} + \ldots$

$$\hat{q}^{(1)}(p_0) = \frac{1}{2p_0}\int d\Omega |\mathcal{M}|^2 f_{\mathrm{eq}}(k)(1+f_{\mathrm{eq}}(k'))\left(\boldsymbol{p}'^{\,2}_\perp + \underbrace{\boldsymbol{k}'^{\,2}_\perp - \boldsymbol{k}^2_\perp}_{\text{additional terms}}\right)$$

cf. AMY, JHEP 0010177

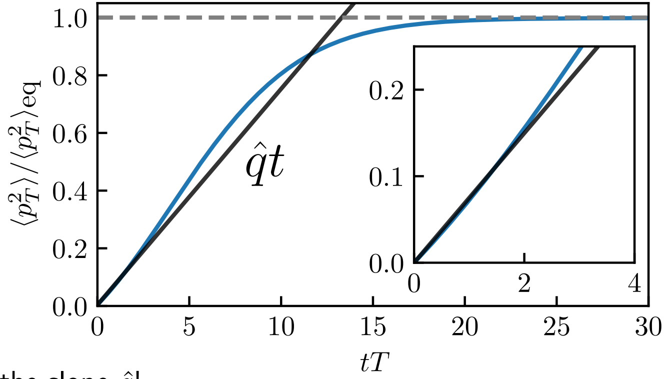
EKT takes into account effects from the recoiling medium

Momentum broadening from minijet evolution



- transverse momentum

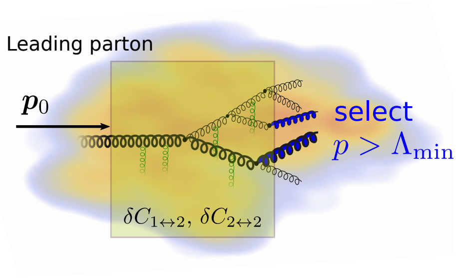
$$\langle p_T^2 \rangle = \int_{\boldsymbol{p}} p_T^2 \delta f_{\rm Jet}(t, \boldsymbol{p}) \approx \hat{q} t$$

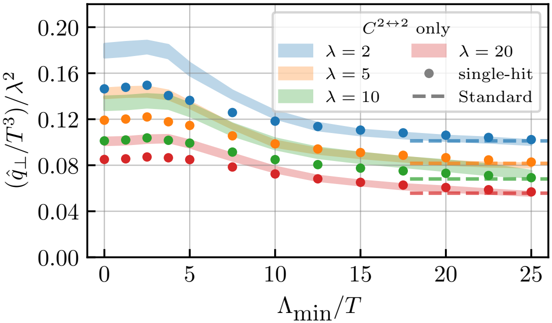


- **Extraction** of the slope $\hat{q}$!

Medium effects

- exclude soft momenta using lower momentum cutoff $\Lambda_{\min}$
- single-hit $\hat{q}^{(1)}$ vs extraction from minijet: $\hat{q} = \frac{d}{dt}\langle p_T^2\rangle_{p>\Lambda_{\min}}$



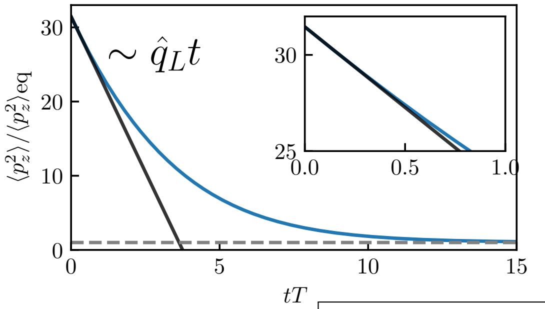


$\Lambda_{\min}$ removes contributions from the **broadening of the medium**!

Momentum broadening from minijet evolution

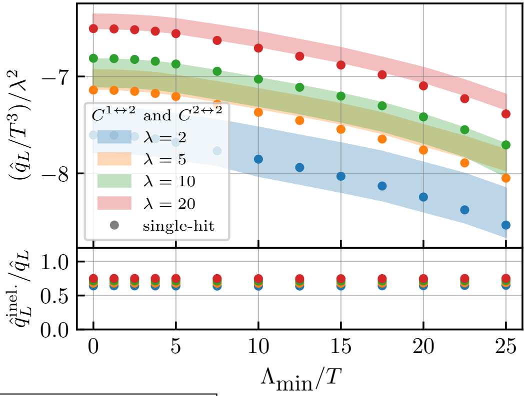
- longitudinal broadening $\langle p_z^2 \rangle$

$$\langle p_z^2 \rangle = \int p_z^2 \delta f_{\mathrm{Jet}}(t, \boldsymbol{p}) d^3\boldsymbol{p}$$
$$\approx \langle p_z^2 \rangle_0 + \hat{q}_L t$$



- extract $\hat{q}_L$

$$\hat{q}_L = \frac{d}{dt} \langle p_z^2 \rangle_{p > \Lambda_{\mathrm{min}}}$$

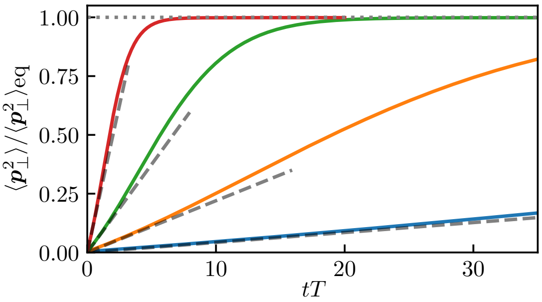


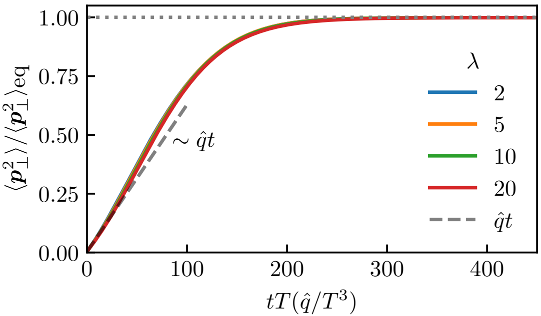
Important contributions from $C_{1\leftrightarrow 2}$!

Equilibration time

Scaling of transverse broadening

- time evolution of $\langle \boldsymbol{p}_\perp^2 \rangle$



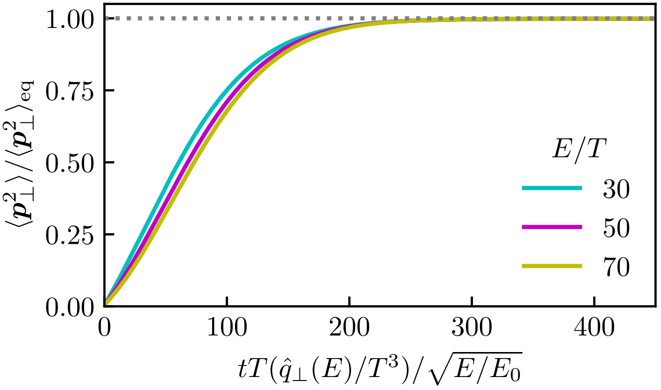


- Collapse for different couplings λ!
- Coupling dependence controlled by $\hat{q}$, "early time quantity"

Energy dependence

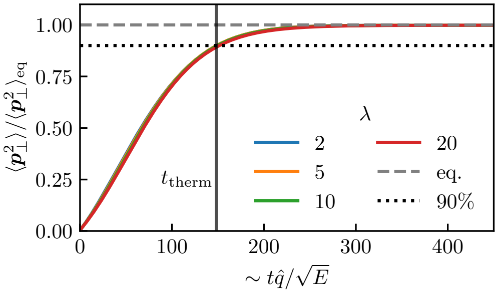
- **Equilibration** controlled by timescale of the first hard splitting $\sim \sqrt{E}$

  isotropic perturbation: Kurkela, Lu, PRL 1405.6318



- Collapse for different jet energies E!

Thermalisation time of minijets



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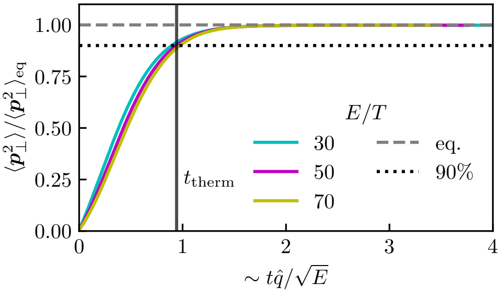


Scaling with $\hat{q}$ and $\sqrt{E}$

$$t_{\rm therm} = 16.43\,{\rm fm} \left(\frac{T}{0.3\,{\rm GeV}}\right)^{-2} \left(\frac{\hat{q}/T^3}{6}\right)^{-1} \left(\frac{E}{15\,{\rm GeV}}\right)^{1/2}$$

Expanding QGP

Non-equilibrium medium

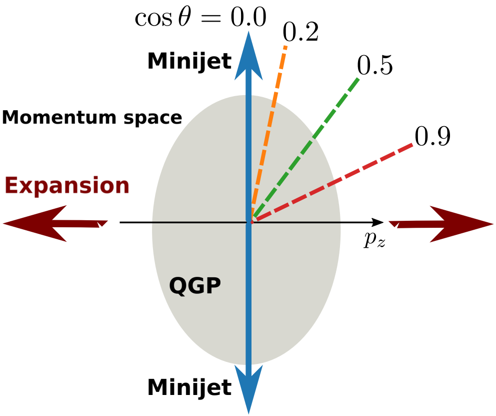
$$\left(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z}\right)\delta f(\tau,\mathbf{p}) = -\delta C[\delta f, \bar{f}]$$

$$\left(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z}\right)\bar{f}(\tau,\mathbf{p}) = -C[\bar{f}]$$

Kurkela and Zhu, PRL 1506.06647
Kurkela and Mazeliauskas,
PRD 1811.03068, PRL 1811.03040

- non-thermal initial condition

$$\bar{f}(\tau_0,\mathbf{p}) \propto \exp\left(-\frac{2}{3}\frac{p_\perp^2+\xi^2 p_z^2}{Q^2}\right)$$



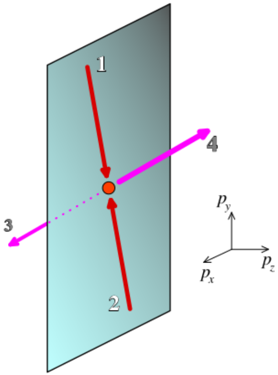
$\bar{f}(\tau,\mathbf{p})$: approach to hydrodynamics!

Out-of-plane scattering

- $f \gg 1 \rightarrow$ classical approximation
- elastic kernel $C_{2\leftrightarrow 2}[f](\mathbf{p})$

$$\partial_t f_4 \sim f_1 f_2(f_3 + f_4) - f_3 f_4(f_1 + f_2) \quad \text{classical}$$
$$\underline{+f_1 f_2} - f_3 f_4 \qquad \text{quantum}$$

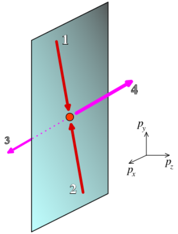
- initial background suppressed in p_z-direction $\rightarrow f(\tau_0, \boldsymbol{p}_{3,4}) \approx 0$



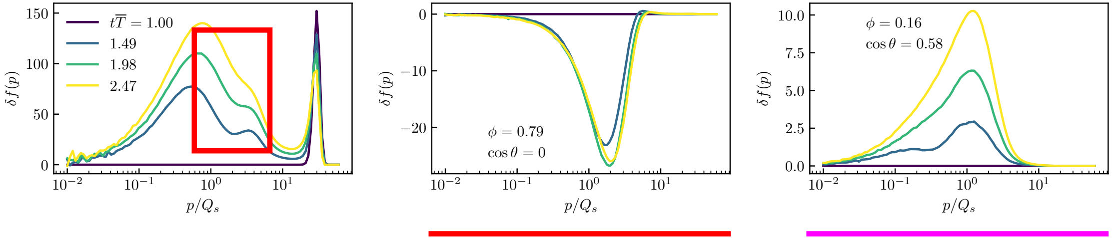
Wu et al., JHEP 1506.05580

Quantum terms responsible for isotropisation!

Out-of-plane scattering



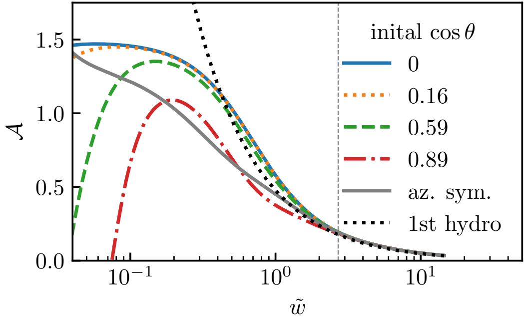
- early times
- particles are scattered out of the transverse plane

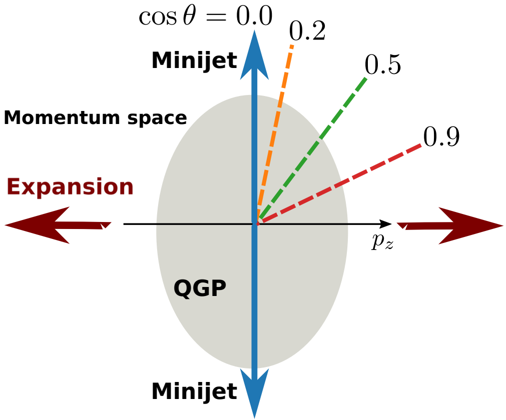


Depletion in the transverse plane $\leftrightarrow$ **Isotropisation**

Hydrodynamisation

- jets with different initial $\cos\theta$
- anisotropy $\mathcal{A} = \frac{\delta P_T - \delta P_L}{\delta e/3}$





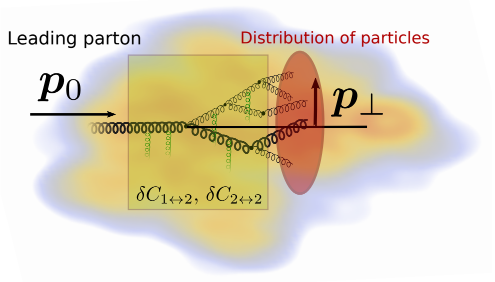
Indistinguishability around $\tilde{w}_{\text{hydro}} \approx 2.7$ **→ Hydrodynamisation**

Summary

Minijets as linear perturbations

Transport coefficients:

- cutoff Λ_{min} exhibits effects of the medium



Equilibration:

- Universal evolution by scaling with $\hat{q}$ and $\sqrt{E}$
- $t_{\text{therm}} \sim 16\,\text{fm}$ for a jet of $15\,\text{GeV}$ (weakly coupled picture)
- jet perturbation becomes part of the hydrodynamical background

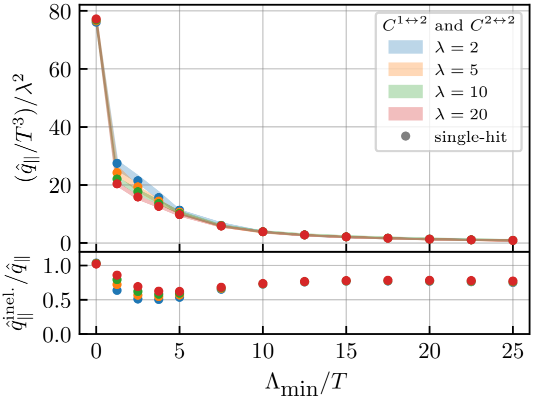
Outlook:

- Non-equilibrium $\hat{q}(t)$

Further transport coefficients Ghiglieri, Teaney, Int.J.Mod.Phys.E, 1502.03730

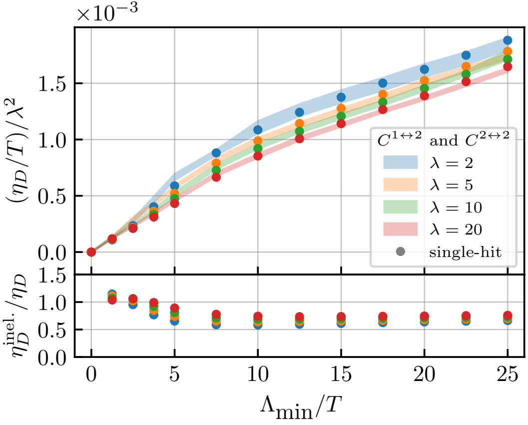
- relative momentum broadening

$$\hat{q}_{\parallel} = \frac{d}{dt}\langle (p_z - \langle p_z \rangle)^2 \rangle_{p>\Lambda_{\text{min}}}$$



- drag coefficient

$$\eta_D = -\frac{1}{\langle p_z \rangle}\frac{d}{dt}\langle p_z \rangle_{p>\Lambda_{\text{min}}}$$



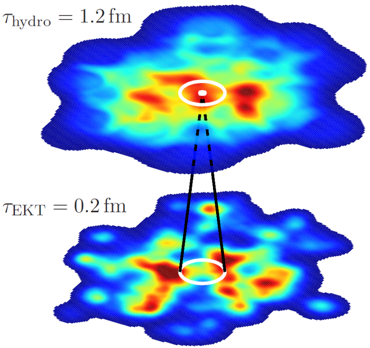
Expanding QGP

- longitudinal expansion
- approximate boost invariance
- homogeneity in the transverse plane

$$\left(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z}\right) f(\tau, \boldsymbol{p}) = -C[f]$$

- leading order elastic and inelastic scattering processes

$$C[f](\boldsymbol{p}) = C_{2\leftrightarrow 2}[f](\boldsymbol{p}) + C_{1\leftrightarrow 2}[f](\boldsymbol{p})$$



KøMPøST, PRC 1805.00961

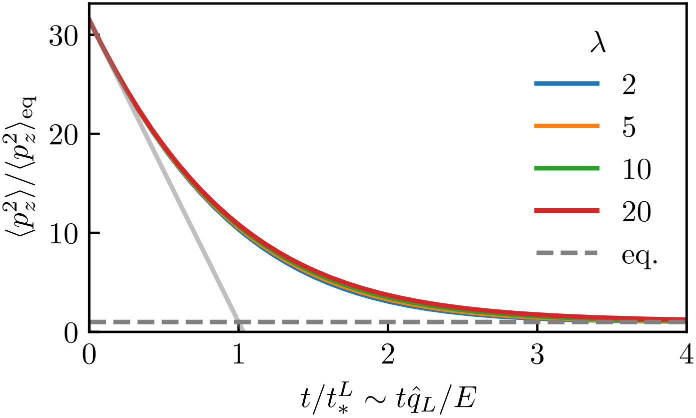
Minijet hydrodynamisation

- timescale of minijet quenching

$$\tau_{\mathrm{mjh}} = 5.1\,\mathrm{fm}\left(\frac{4\pi\eta/s}{2}\right)^{3/2}\left(\frac{E}{31\,\mathrm{GeV}}\right)^{1/2}$$

Longitudinal broadening

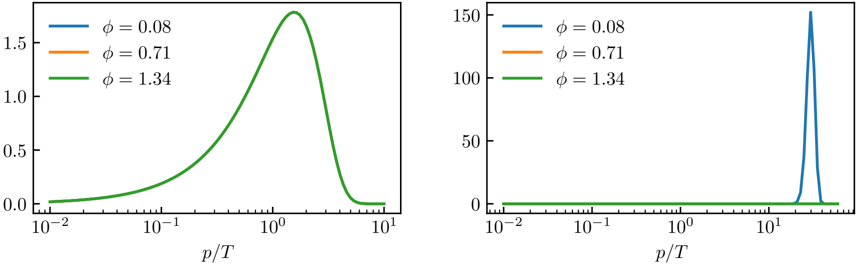
- longitudinal timescale $t_*^L = \frac{\langle p_z^2 \rangle_{\rm eq} - \langle p_z^2 \rangle_0}{\hat{q}_L}$



Comparison with background-like perturbation

- azimuthal symmetric (ϕ): $\delta f^{\mathrm{az}}_{\mathrm{sym}}(t_0, \boldsymbol{p}) = \epsilon \bar{f}(t_0, \boldsymbol{p})$

$$\bar{f} + \delta f^{\mathrm{az}}_{\mathrm{sym}} = (1+\epsilon)\bar{f}$$

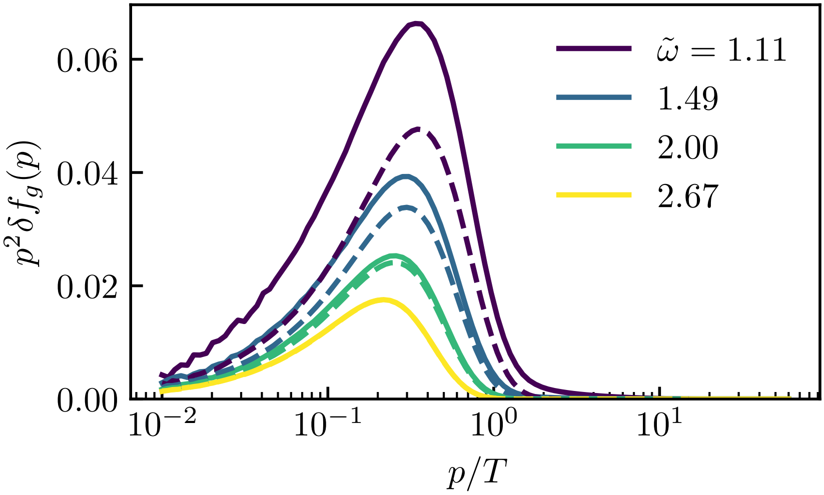


(a) $p^2 \delta f^{\mathrm{az}}_{\mathrm{sym}}(t_0, p, \phi)$ (b) $p^2 \delta f_{\mathrm{Jet}}(t_0, p, \phi)$

- Hydrodynamization occurs, if both perturbations are indistinguishable

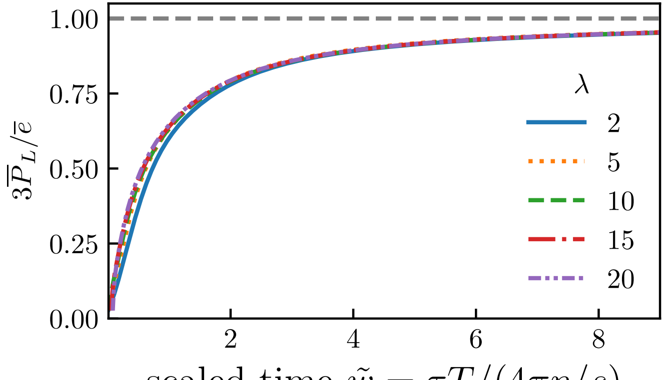
Hydrodynamisation

- anisotropy $\mathcal{A} = \frac{\delta P_T - \delta P_L}{\delta e/3}$



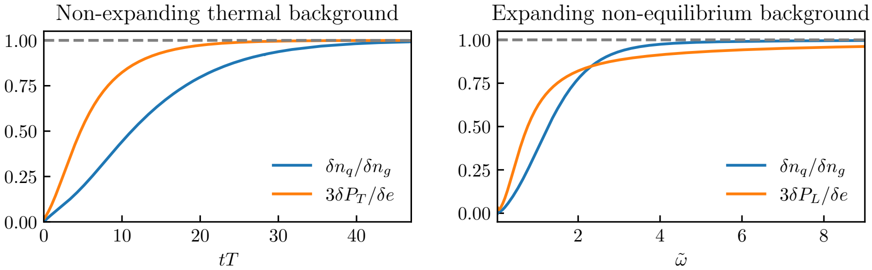
Pressure equilibration background

- energy momentum tensor $T^{\mu\nu} = \int_{\boldsymbol{p}} \frac{p^\mu p^\nu}{p} f(\tau, \boldsymbol{p})$
- $T^{\mu\nu}_{\text{eq}} = \text{diag}(e, P, P, P)$ with $P = e/3$
- Background longitudinal pressure $\overline{P}_L = \overline{T}^{zz}$



Chemical equilibration

- compare with kinetic equilibrium (isotropy of pressure)



- in chemical equilibrium more fermionic degrees of freedom
- chem. equilibration not affected by expansion

Backup

Equations of motion from 2PI effective action

$$\left[iG^{-1,\mu\gamma}_{0,ac}(x;\mathcal{A})+\Pi^{(0),\mu\gamma}_{ac}(x)\right]\rho^{cb}_{\gamma\nu}(x,y)$$
$$=-\int_{y^0}^{x^0}dz\Pi^{(\rho),\mu\gamma}_{ac}(x,z)\rho^{cb}_{\gamma\nu}(z,y) \tag{1}$$
$$\left[iG^{-1,\mu\gamma}_{0,ac}(x;\mathcal{A})+\Pi^{(0),\mu\gamma}_{ac}(x)\right]F^{cb}_{\gamma\nu}(x,y)$$
$$=-\int_{t_0}^{x^0}dz\Pi^{(\rho),\mu\gamma}_{ac}(x,z)F^{cb}_{\gamma\nu}(z,y)+\int_{t_0}^{y^0}dz\Pi^{(F),\mu\gamma}_{ac}(x,z)\rho^{cb}_{\gamma\nu}(z,y) \tag{2}$$

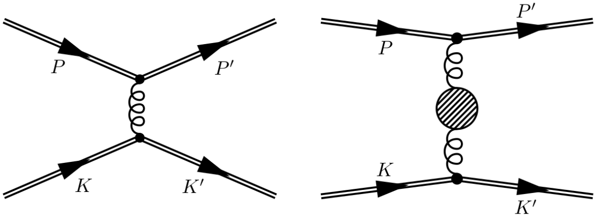
Backup

- local homogeneity $\rightarrow$ relative coordinate $s^\mu = x^\mu - y^\mu$ and center coordinate $X^\mu = \frac{1}{2}(x^\mu + y^\mu)$
- gradient expansion in X^μ
- to lowest order, spectral function ρ is on shell
  $\rightarrow$ quasi-particle picture
- non-equilibrium distribution function $f(X,p)$:

$$F(X,p) = -i\left[\frac{1}{2} \pm f(X,p)\right]\rho(X,p)$$

$$\Rightarrow p^\mu \partial_\mu f(X,p) = -C[f]$$

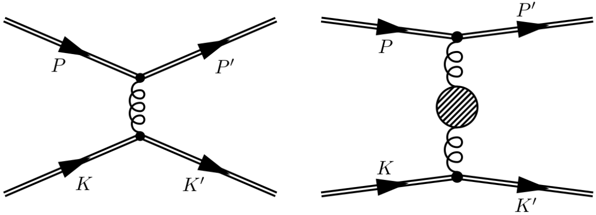
$2 \leftrightarrow 2$



Hard (left) and soft (right) medium regulated scattering

$$C_{2\leftrightarrow 2}[f](\boldsymbol{p}) = \frac{1}{4p\nu}\int_{\boldsymbol{k},\boldsymbol{p}',\boldsymbol{k}'}(2\pi)^4\delta^{(4)}\left(p^\mu + k^\mu - p'^\mu - k'^\mu\right)$$
$$\times\,|\mathcal{M}|^2\left\{\underbrace{f_{\boldsymbol{p}}f_{\boldsymbol{k}}\left(1\pm f_{\boldsymbol{p}'}\right)\left(1\pm f_{\boldsymbol{k}'}\right)}_{\text{loss}} - \underbrace{f_{\boldsymbol{p}'}f_{\boldsymbol{k}'}\left(1\pm f_{\boldsymbol{p}}\right)\left(1\pm f_{\boldsymbol{k}}\right)}_{\text{gain}}\right\} \quad (3)$$

$2 \leftrightarrow 2$



Hard (left) and soft (right) medium regulated scattering

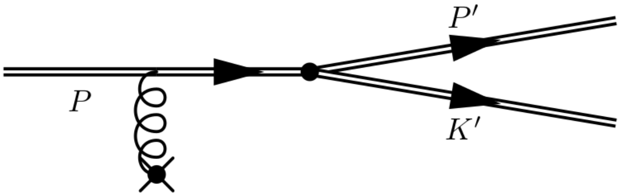
$$|\mathcal{M}|^2 = 2\lambda^2\nu\left(9 + \frac{(s-t)^2}{u^2} + \frac{(u-s)^2}{t^2} + \frac{(t-u)^2}{s^2}\right)$$

- small momentum transfer $q = |\boldsymbol{p}' - \boldsymbol{p}| \ll 1$ regulated by

$$\frac{1}{q^2} \to \frac{1}{q^2 + m_{\text{eff}}^2}$$

$$m_{\text{eff}}^2 = 2g^2 \int \frac{d^3\boldsymbol{p}}{(2\pi)^3 p}\left[N_c f_{\boldsymbol{p}}^g + N_f f_{\boldsymbol{p}}^q\right]$$

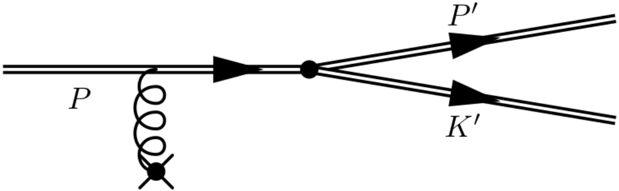
$1 \leftrightarrow 2$



effective $1 \leftrightarrow 2$ process

$$C_{1\leftrightarrow 2}[f](\boldsymbol{p}) = \frac{1}{2}\frac{1}{\nu}(2\pi)^3 \int_{\tilde{\boldsymbol{p}},\boldsymbol{p}',\boldsymbol{k}'} (2\pi)^4 \delta^{(4)}\left(\tilde{p}^\mu - p'^\mu - k'^\mu\right)$$
$$\times \left[\delta^{(3)}(\boldsymbol{p}-\tilde{\boldsymbol{p}}) - \delta^{(3)}\left(\boldsymbol{p}-\boldsymbol{p}'\right) - \delta^{(3)}\left(\boldsymbol{p}-\boldsymbol{k}'\right)\right]$$
$$\times \gamma\{\underbrace{f_{\boldsymbol{p}}\left(1 \pm f_{\tilde{\boldsymbol{p}}'}\right)\left(1 \pm f_{\boldsymbol{k}'}\right)}_{\text{loss}} - \underbrace{f_{\boldsymbol{p}'} f_{\boldsymbol{k}'}\left(1 \pm f_{\tilde{\boldsymbol{p}}}\right)}_{\text{gain}}\} \quad (4)$$

$1 \leftrightarrow 2$



effective $1 \leftrightarrow 2$ process

- LO $\rightarrow$ strictly collinear
- medium induced radiation of gluons
- $N+1 \leftrightarrow N+2$ effectively $1 \leftrightarrow 2$

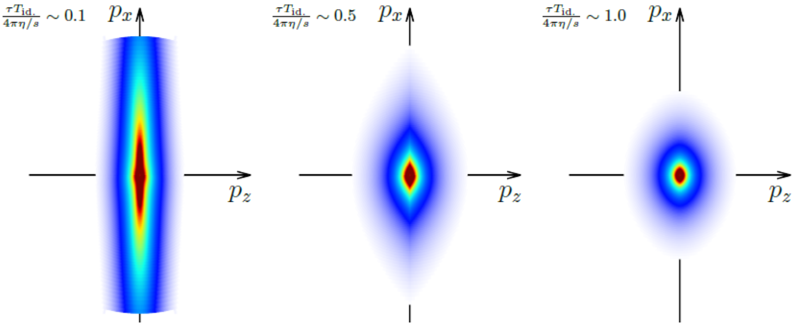
$1 \leftrightarrow 2$

Parton going through the medium, Figure from [**Jasmine**]

- hard parton receiving multiple kicks
- formation time $\tau_f \sim E$
- BH: $l_f \ll l_{\rm mfp}$, independent emissions
- LPM: $l_f \sim l_{\rm mfp}$, destructive interference $\rightarrow$ suppression

Bottom-up thermalization

Baier, Müller, Schiff, Son (2001)

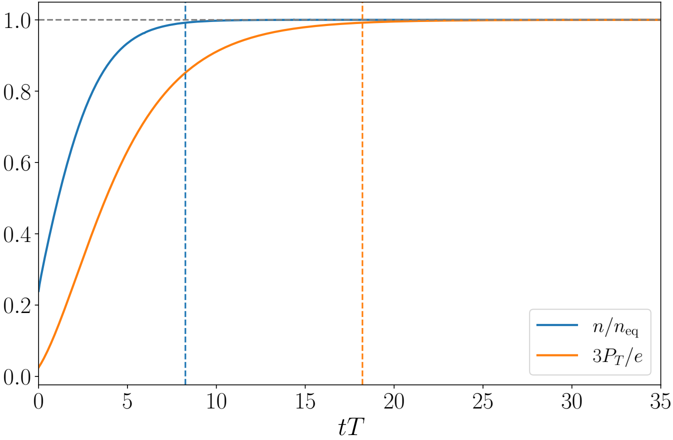


Isotropization of the distribution, Figure from [**Kurkela_2019**]

- 1: overoccupied system getting more anisotropic
- 2: population of soft gluons
- 3: inverse energy cascade

Radiation vs. elastic scattering

- particle number $n = \int_{\boldsymbol{p}} f(\tau, \boldsymbol{p}) \to C_{1\leftrightarrow 2}[f]$
- transverse pressure $P_T = \frac{1}{2}\int_{\boldsymbol{p}} p_\perp^2/p f(\tau, \boldsymbol{p}) \to C_{2\leftrightarrow 2}[f]$



Equilibrium distribution

- equilibrated jet $\to$ change in temperature δT and velocity δu^z

$$\delta f_{\rm eq}(\boldsymbol{p}) = (\delta T \partial_T + \delta u^z \partial_{u^z})\, n_{\rm BE}\,(p_\mu u^\mu / T)\Big|_{u^z=0}$$

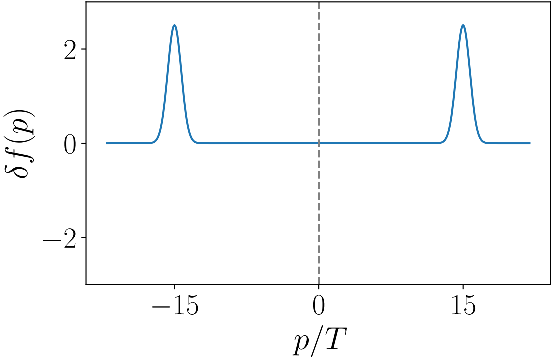
$$\delta f_{\rm eq}(p,\theta) = \left(\delta u^z \cos\theta + \frac{\delta T}{T}\right) F(p/T)$$

- both contributions can be disentangled

Equilibrium distribution

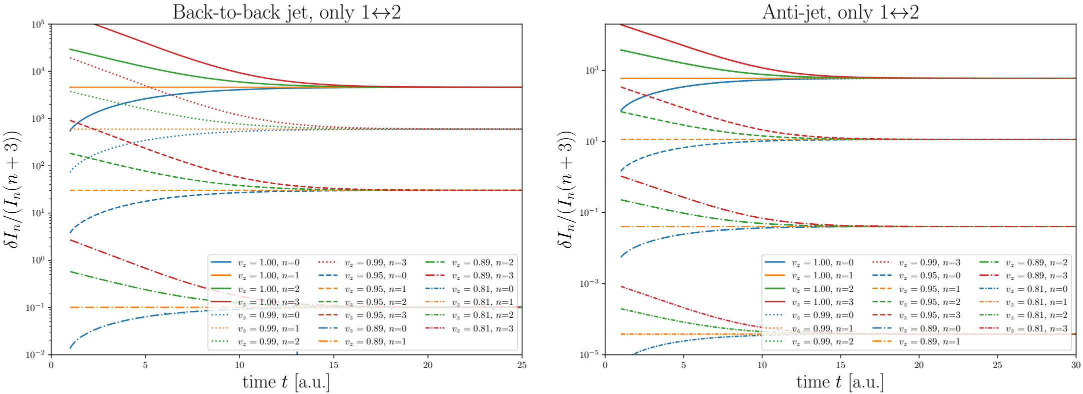
- back-to-back jet conserves net momentum:

$$\delta f_{\rm eq}(p,\theta) = \left(\cancel{\delta u^z \cos\theta} + \frac{\delta T}{T}\right) F(p/T)$$



Initial condition: back-to-back jet

Only $1 \leftrightarrow 2$



- similar timescales of equilibration
  $\Rightarrow$ 2↔2 contribute more to equilibration of the anti-jet

Moments of δf

- angular effective temperature

$$I_n(\theta) \equiv 4\pi \int \frac{p^2 dp}{(2\pi)^3} p^n f(p,\theta) = \mathcal{N}_n \times T(\theta)^{n+3} \tag{5}$$

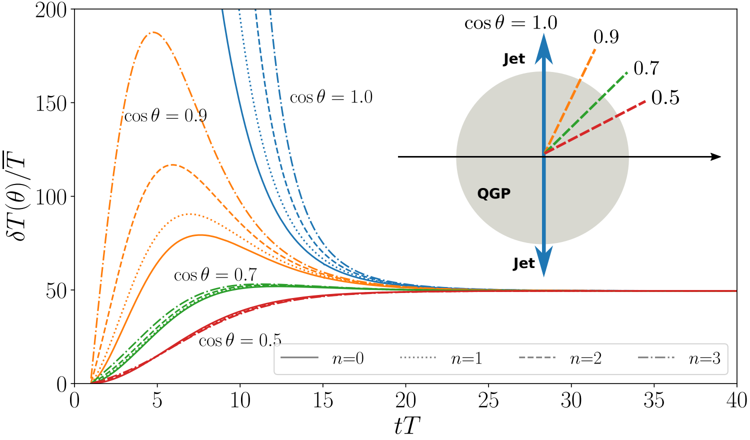
$$T(\theta) = \overline{T} + \delta T(\theta)$$

- temperature perturbation

$$\frac{\delta T(\theta)}{\overline{T}} = \frac{\delta I_n(\theta)}{(n+3)\bar{I}_n(\theta)}$$

- look at time evolution!

Moments of δf



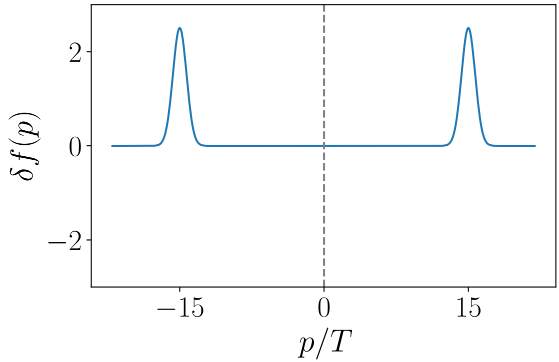
Moments of the back-to-back jet

- different moments agree before different angles do!

Equilibrium distribution

- back-to-back jet conserves net momentum:

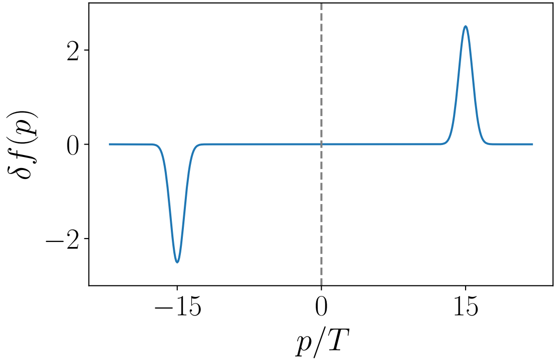
$$\delta f_{\text{eq}}(p,\theta) = \left(\cancel{\delta u^z \cos\theta} + \frac{\delta T}{T} \right) F(p/T)$$



Initial condition: back-to-back jet

Anti-jet

- introduce anti-jet $\rightarrow$ no energy deposited, **only** net momentum

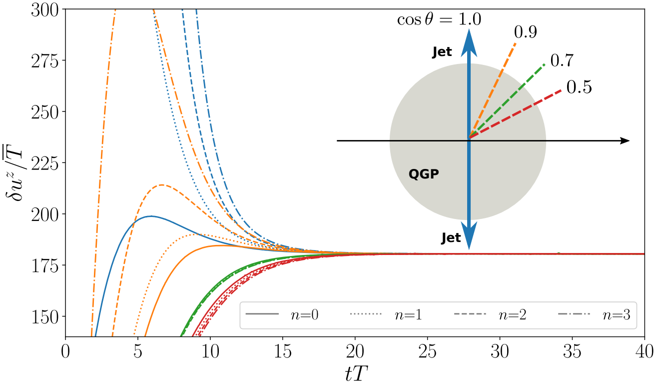


Particle and "hole"

$$\delta f_{\text{eq}}(p,\theta) = \left(\delta u^z \cos\theta + \cancel{\frac{\delta T}{T}}\right) F(p/T)$$

- allows us to study the build up of δu^z

Moments of δf_{anti}



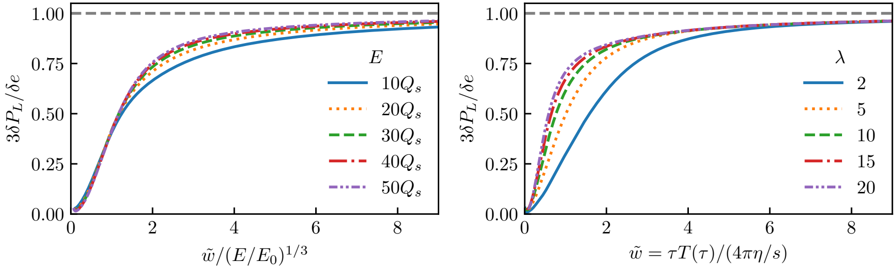
Collapse of different θ much earlier!

$$\delta f_{\mathrm{eq}}(p,\theta) = (\delta u^z \cos\theta)\, F(p/T)$$

- θ-dependence $\rightarrow$ faster reached by elastic processes
- single jet: velocity builds up faster than temperature

Pressure equilibration

- scaled time $\tilde{\omega} = \tau/\tau_R$ with $\tau_R = \frac{4\pi\eta/s}{T(\tau)}$
- effective temperature from $e(\tau) = \nu_{\text{eff}} \frac{\pi^2}{30} T(\tau)^4$



(c) Scaling with jet energy E

(d) coupling λ