

Minijet thermalization and jet transport coefficients in QCD kinetic theory

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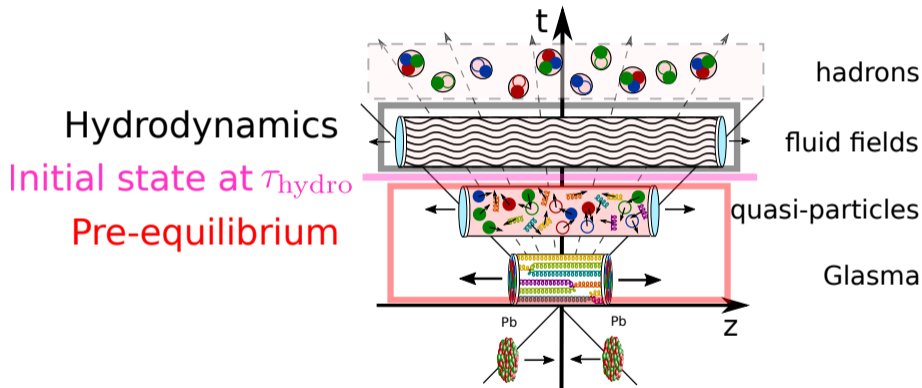


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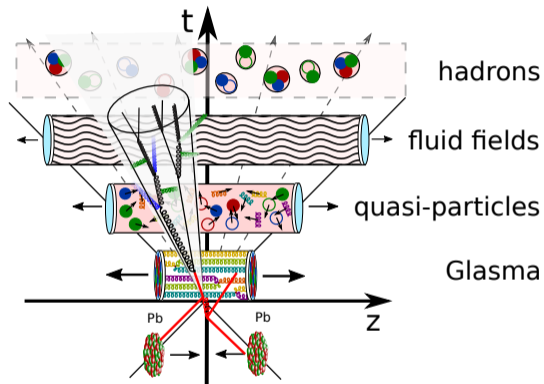


with K. Boguslavski, F. Lindembauer, A. Mazeliauskas and A. Takacs, arXiv:2510.25669
with J. Brewer and A. Mazeliauskas, JHEP, 2402.09298

Overview



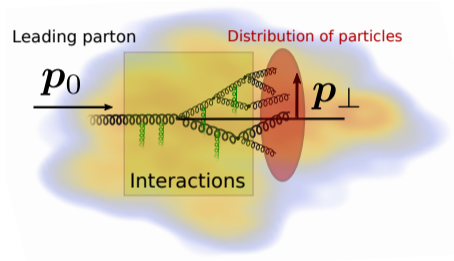
Overview



- ▶ study minijet thermalisation with kinetic theory

Overview

- ▶ Jet transport coefficient $\hat{q} \rightarrow$ medium property



- 1 bring together transport coefficients and minijet evolution
- 2 estimate thermalisation time of a jet

Outline

Linearised kinetic theory

Transport coefficients and minijet evolution

Equilibration time

Expanding QGP

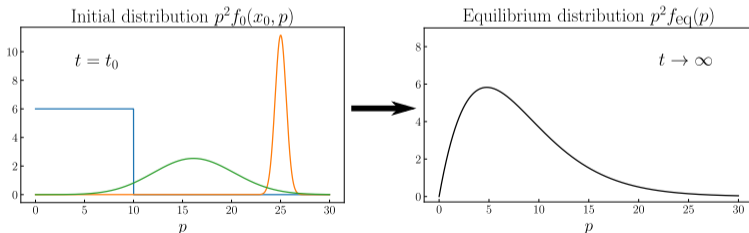
Linearised kinetic theory

Framework

- ▶ Effective Kinetic Theory for high temperature gauge theories [AMY \(2003\), JHEP 0209353](#)
- ▶ weakly coupled quasi-particle picture, $\lambda = 4\pi\alpha_s N_c$

→ phase space distribution $f(\tau, \mathbf{x}, \mathbf{p}) \rightarrow$ homogeneous system

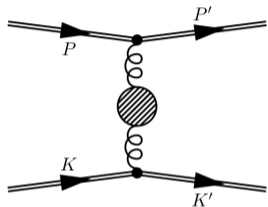
$$\partial_\tau f(\tau, \mathbf{p}) = -C[f]$$



Out-of-equilibrium initial state is transported to equilibrium

Collision kernel

$C_{2\leftrightarrow 2}$

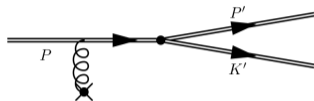


- ▶ small momentum transfer $q = |\mathbf{p}' - \mathbf{p}| \ll 1$

isotropic HTL screening

Boguslavski, Lindenbauer, PRD 2407.09605

$C_{1\leftrightarrow 2}$



- ▶ medium induced radiation of gluons
- ▶ $g \rightarrow q\bar{q}$ splittings
- ▶ LO: strictly collinear

Linearised EKT

- ▶ thermal background + linear perturbation

$$f(t, \mathbf{p}) = f_{\text{eq}}(\mathbf{p}) + \delta f_{\text{jet}}(t, \mathbf{p})$$

- ▶ linearized Boltzmann eq.

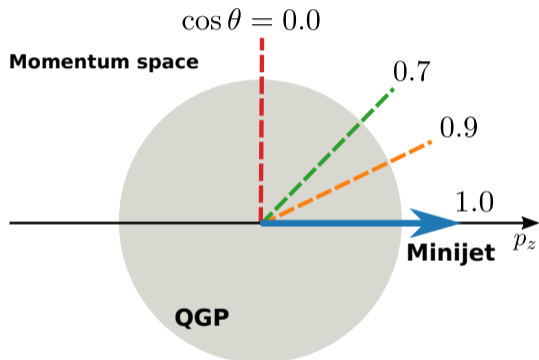
$$\partial_t \delta f = -\delta C[\delta f, f_{\text{eq}}]$$

Mehtar-Tani, Schlichting, Soudi, JHEP 2209.10569

Barrera, HP2024

Brewer, Mazeliauskas, FZ, JHEP 2402.09298

- ▶ Jet-like initial conditions



Initial conditions

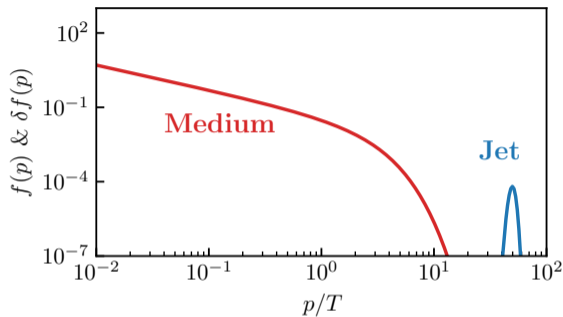
► Evolution

$$\partial_t \delta f = -\delta C[\delta f, f_{\text{eq}}]$$

► Initial jet distribution

$$\delta f_{\text{Jet}}(t_0, \mathbf{p}) \propto \delta(\mathbf{p} - \mathbf{p}_0)$$

$$\mathbf{p}_0 = (0, 0, E)$$



"single" parton in a thermal brick

transport coefficients + momentum broadening of $\delta f_{\text{jet}}(t_0, \mathbf{p})$

Boguslavski, Lindenbauer, Mazeliauskas, Takacs, FZ, 2510.25669

Transport coefficients and minijet evolution

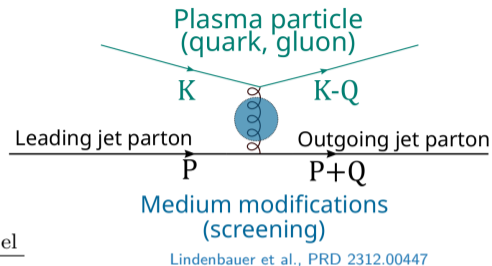
Transport coefficient \hat{q} - Literature

- ▶ Conventionally

$$\hat{q} = \int d^2 \mathbf{q}_\perp \mathbf{q}_\perp^2 \frac{d\Gamma_{\text{el}}}{d^2 \mathbf{q}_\perp}$$

- ▶ broadening of 1 parton with energy p_0

$$\hat{q}(p_0) = \frac{1}{2p_0} \int d\Omega |\mathcal{M}|^2 f_{\text{eq}}(k) (1 + f_{\text{eq}}(k')) \mathbf{p}'_\perp{}^2$$



Transport coefficient \hat{q} - This work

$$\hat{q} = \frac{d}{dt} \langle p_T^2 \rangle = \frac{d}{dt} \left(\int_{\mathbf{p}} p_T^2 \delta f_{\text{Jet}}(t, \mathbf{p}) \right)$$

- ▶ broadening of the "jet **distribution**"

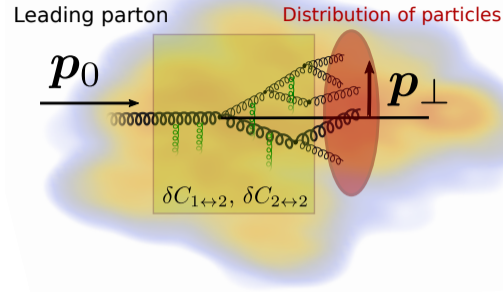
$$\hat{q} = - \int_{\mathbf{p}} p_T^2 \delta C[\delta f, f_{\text{eq}}]$$

- ▶ **single hit approximation** $\delta f = \delta f^{(0)} + \delta f^{(1)} + \dots$

$$\hat{q}^{(1)}(p_0) = \frac{1}{2p_0} \int d\Omega |\mathcal{M}|^2 f_{\text{eq}}(k) (1 + f_{\text{eq}}(k')) \left(\mathbf{p}'_{\perp}{}^2 + \underbrace{\mathbf{k}'_{\perp}{}^2 - \mathbf{k}_{\perp}{}^2}_{\text{additional terms}} \right)$$

cf. AMY, JHEP 0010177

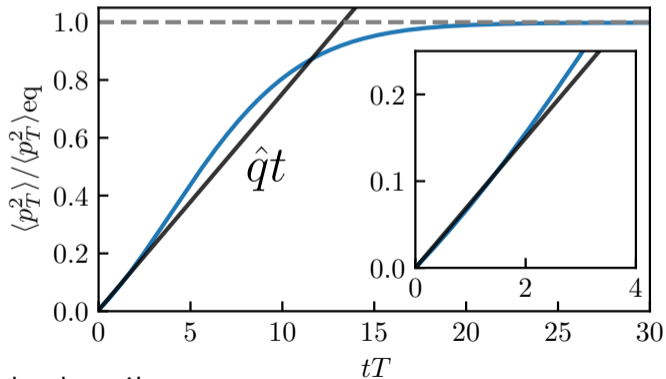
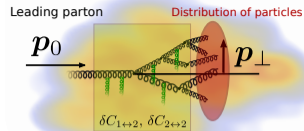
EKT takes into account effects from the recoiling medium



Momentum broadening from minijet evolution

- ▶ transverse momentum

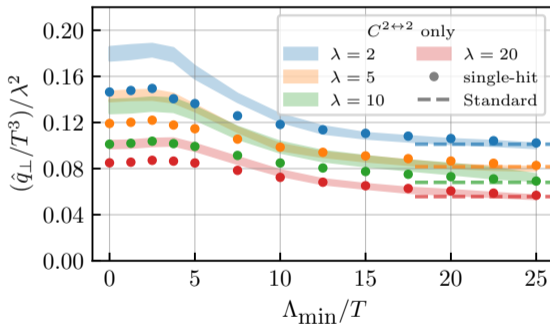
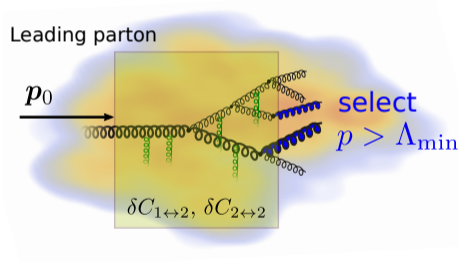
$$\langle p_T^2 \rangle = \int_{\mathbf{p}} p_T^2 \delta f_{\text{Jet}}(t, \mathbf{p}) \approx \hat{q}t$$



- ▶ **Extraction** of the slope \hat{q} !

Medium effects

- ▶ exclude soft momenta using lower momentum cutoff Λ_{\min}
- ▶ single-hit $\hat{q}^{(1)}$ vs extraction from minijet: $\hat{q} = \frac{d}{dt} \langle p_T^2 \rangle_{p > \Lambda_{\min}}$



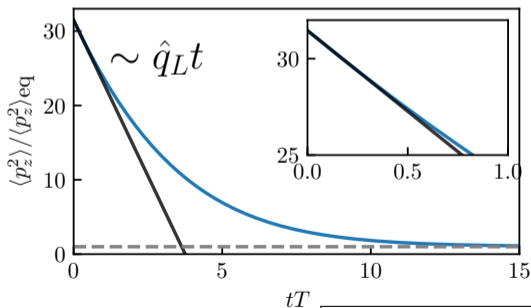
Λ_{\min} removes contributions from the **broadening of the medium!**

Momentum broadening from minijet evolution

- ▶ longitudinal broadening $\langle p_z^2 \rangle$

$$\langle p_z^2 \rangle = \int p_z^2 \delta f_{\text{Jet}}(t, \mathbf{p}) d^3 \mathbf{p}$$

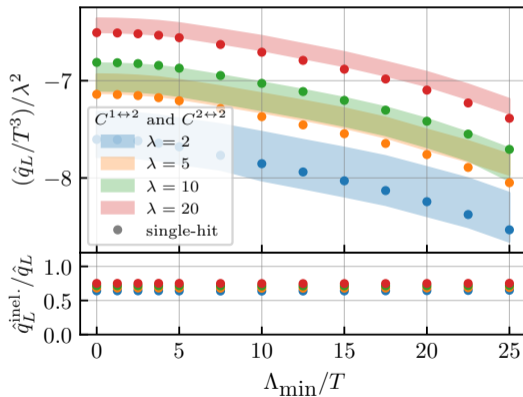
$$\approx \langle p_z^2 \rangle_0 + \hat{q}_L t$$



Important contributions from $C_{1 \leftrightarrow 2}$!

- ▶ extract \hat{q}_L

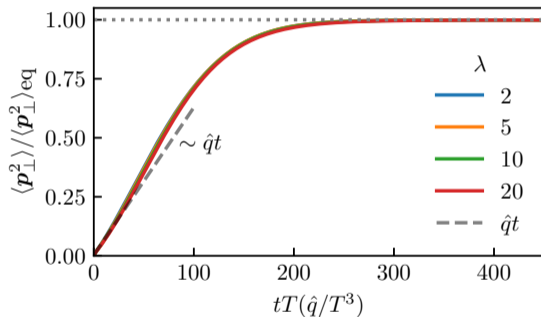
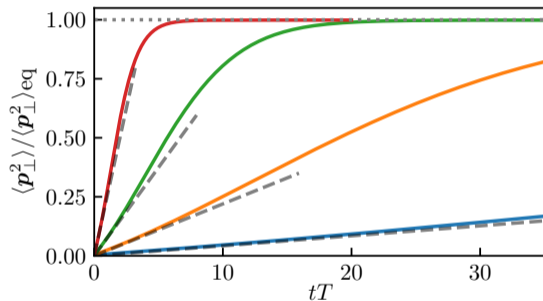
$$\hat{q}_L = \frac{d}{dt} \langle p_z^2 \rangle_{p > \Lambda_{\min}}$$



Equilibration time

Scaling of transverse broadening

- ▶ time evolution of $\langle p_{\perp}^2 \rangle$

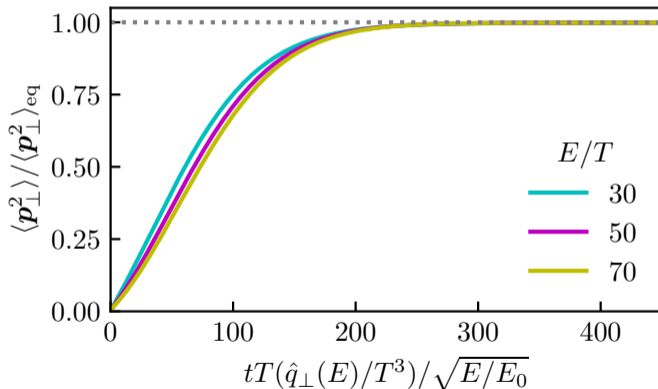


- ▶ Collapse for different couplings λ !
- ▶ Coupling dependence controlled by \hat{q} , "early time quantity"

Energy dependence

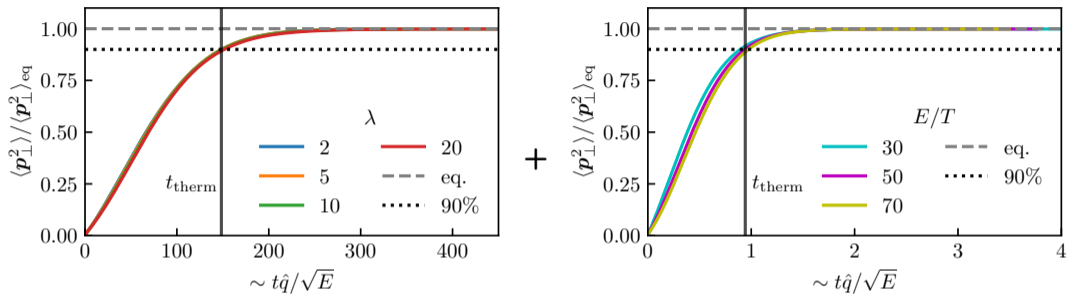
- **Equilibration** controlled by timescale of the first hard splitting $\sim \sqrt{E}$

isotropic perturbation: [Kurkela, Lu, PRL 1405.6318](#)



- Collapse for different jet energies E !

Thermalisation time of minijets



Scaling with \hat{q} and \sqrt{E}

$$t_{\text{therm}} = 16.43 \text{ fm} \left(\frac{T}{0.3 \text{ GeV}} \right)^{-2} \left(\frac{\hat{q}/T^3}{6} \right)^{-1} \left(\frac{E}{15 \text{ GeV}} \right)^{1/2}$$

Expanding QGP

Non-equilibrium medium

$$(\partial_\tau - \frac{p_z}{\tau} \partial_{p_z}) \delta f(\tau, \mathbf{p}) = -\delta C[\delta f, \bar{f}]$$

$$(\partial_\tau - \frac{p_z}{\tau} \partial_{p_z}) \bar{f}(\tau, \mathbf{p}) = -C[\bar{f}]$$

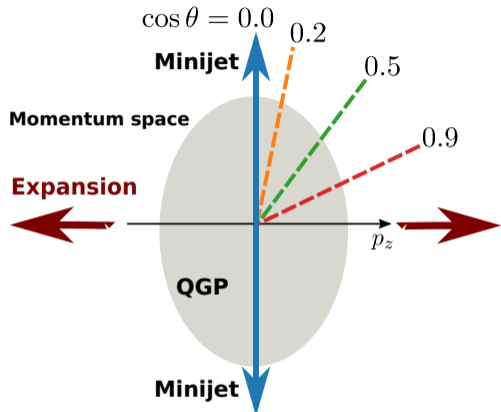
Kurkela and Zhu, PRL 1506.06647

Kurkela and Mazeliauskas,

PRD 1811.03068, PRL 1811.03040

- ▶ non-thermal initial condition

$$\bar{f}(\tau_0, \mathbf{p}) \propto \exp\left(-\frac{2}{3} \frac{p_\perp^2 + \xi^2 p_z^2}{Q^2}\right)$$



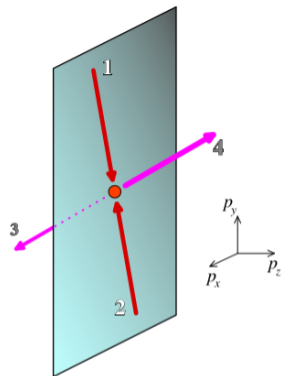
$\bar{f}(\tau, \mathbf{p})$: approach to hydrodynamics!

Out-of-plane scattering

- ▶ $f \gg 1 \rightarrow$ classical approximation
- ▶ elastic kernel $C_{2 \leftrightarrow 2}[f](\mathbf{p})$

$$\partial_t f_4 \sim f_1 f_2 (f_3 + f_4) - f_3 f_4 (f_1 + f_2) \quad \text{classical}$$
$$\quad \quad \quad \underline{+ f_1 f_2} - f_3 f_4 \quad \text{quantum}$$

- ▶ initial background suppressed in p_z -direction $\rightarrow f(\tau_0, \mathbf{p}_{3,4}) \approx 0$

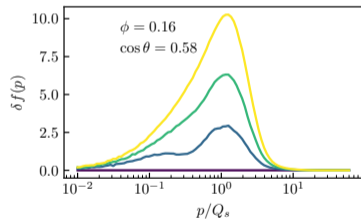
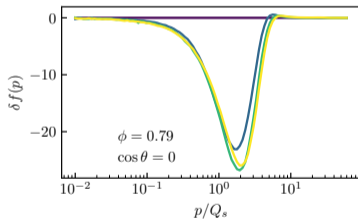
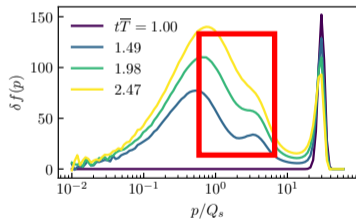
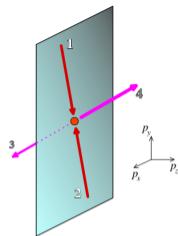


Wu et al.,
JHEP 1506.05580

Quantum terms responsible for isotropisation!

Out-of-plane scattering

- ▶ early times
- ▶ particles are scattered out of the transverse plane

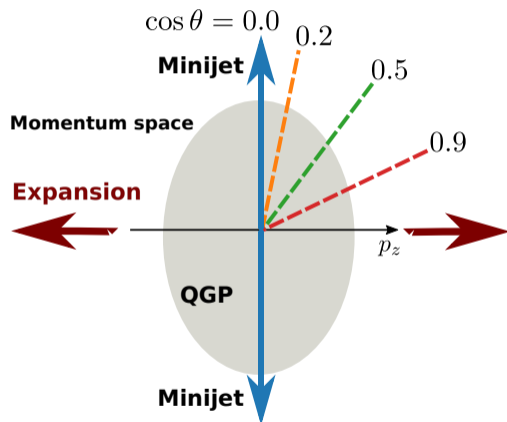
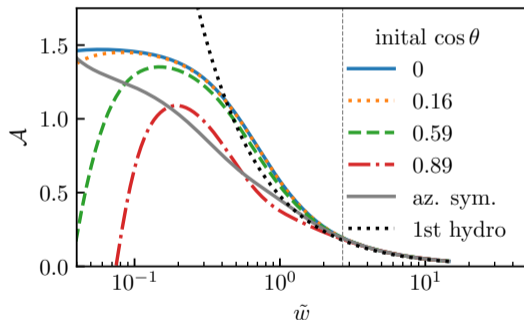


Depletion in the **transverse plane** \leftrightarrow **Isotropisation**

Hydrodynamisation

- ▶ jets with different initial $\cos\theta$

- ▶ anisotropy $\mathcal{A} = \frac{\delta P_T - \delta P_L}{\delta e/3}$



Indistinguishability around $\tilde{w}_{\text{hydro}} \approx 2.7 \rightarrow$ **Hydrodynamisation**

Summary

Minijets as linear perturbations

Transport coefficients:

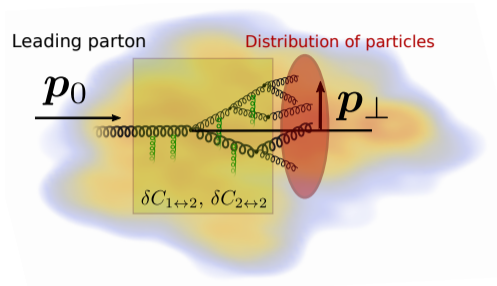
- ▶ cutoff Λ_{\min} exhibits effects of the medium

Equilibration:

- ▶ Universal evolution by scaling with \hat{q} and \sqrt{E}
- ▶ $t_{\text{therm}} \sim 16 \text{ fm}$ for a jet of 15 GeV (weakly coupled picture)
- ▶ jet perturbation becomes part of the hydrodynamical background

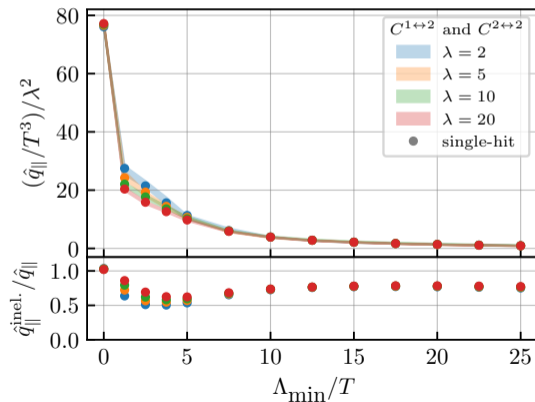
Outlook:

- ▶ Non-equilibrium $\hat{q}(t)$



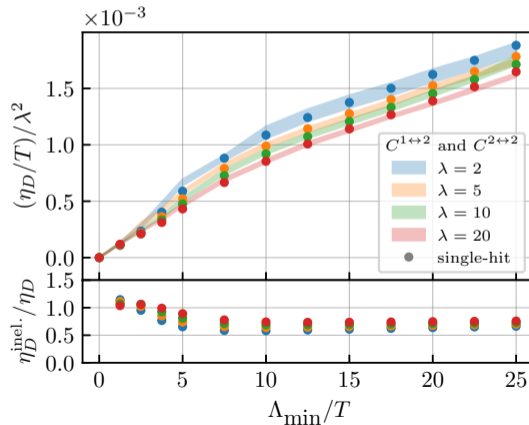
► relative momentum broadening

$$\hat{q}_{\parallel} = \frac{d}{dt} \langle (p_z - \langle p_z \rangle)^2 \rangle_{p > \Lambda_{\min}}$$



► drag coefficient

$$\eta_D = -\frac{1}{\langle p_z \rangle} \frac{d}{dt} \langle p_z \rangle_{p > \Lambda_{\min}}$$



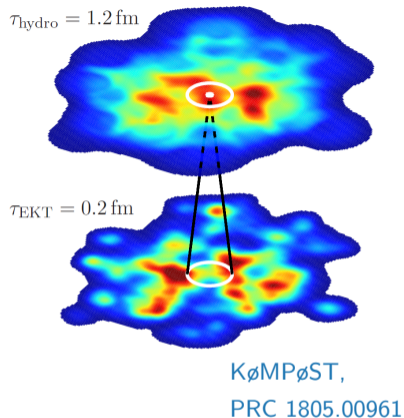
Expanding QGP

- ▶ longitudinal expansion
- ▶ approximate boost invariance
- ▶ homogeneity in the transverse plane

$$\left(\partial_\tau - \frac{p_z}{\tau} \partial_{p_z} \right) f(\tau, \mathbf{p}) = -C[f]$$

- ▶ leading order elastic and inelastic scattering processes

$$C[f](\mathbf{p}) = C_{2 \leftrightarrow 2}[f](\mathbf{p}) + C_{1 \leftrightarrow 2}[f](\mathbf{p})$$



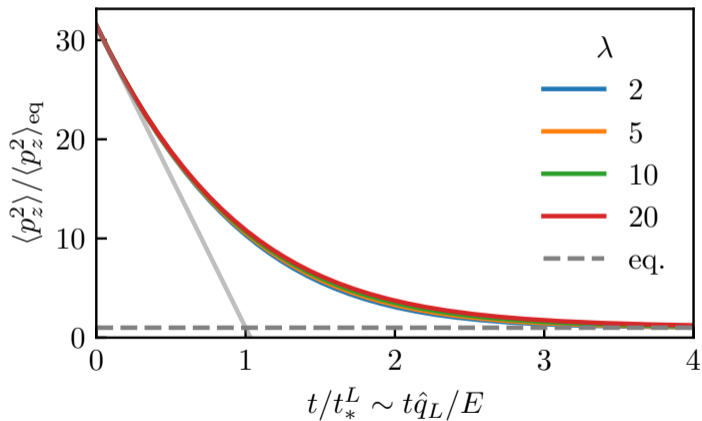
Minijet hydrodynamisation

- ▶ timescale of minijet quenching

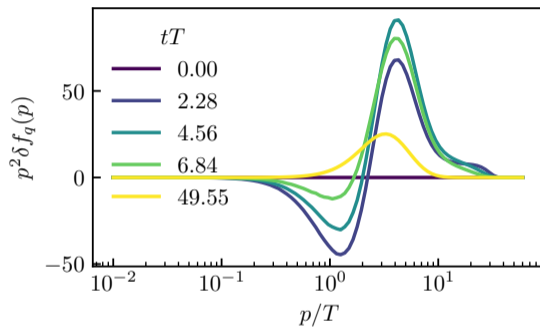
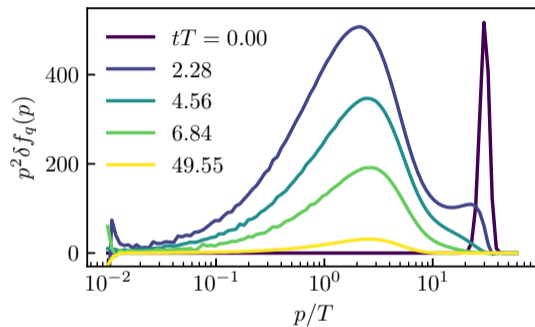
$$\tau_{\text{mjh}} = 5.1 \text{ fm} \left(\frac{4\pi\eta/s}{2} \right)^{3/2} \left(\frac{E}{31 \text{ GeV}} \right)^{1/2}$$

Longitudinal broadening

- ▶ longitudinal timescale $t_*^L = \frac{\langle p_z^2 \rangle_{\text{eq}} - \langle p_z^2 \rangle_0}{\hat{q}_L}$



Quark perturbation

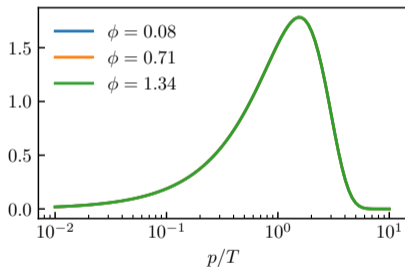


- ▶ early times: depletion of quarks around $p \sim T$
 - hard gluon scatters off of the soft background

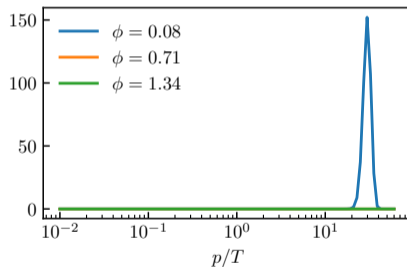
Comparison with background-like perturbation

- ▶ azimuthal symmetric (ϕ): $\delta f_{\text{sym}}^{\text{az}}(t_0, \mathbf{p}) = \epsilon \bar{f}(t_0, \mathbf{p})$

$$\bar{f} + \delta f_{\text{sym}}^{\text{az}} = (1 + \epsilon) \bar{f}$$



(a) $p^2 \delta f_{\text{sym}}^{\text{az}}(t_0, p, \phi)$

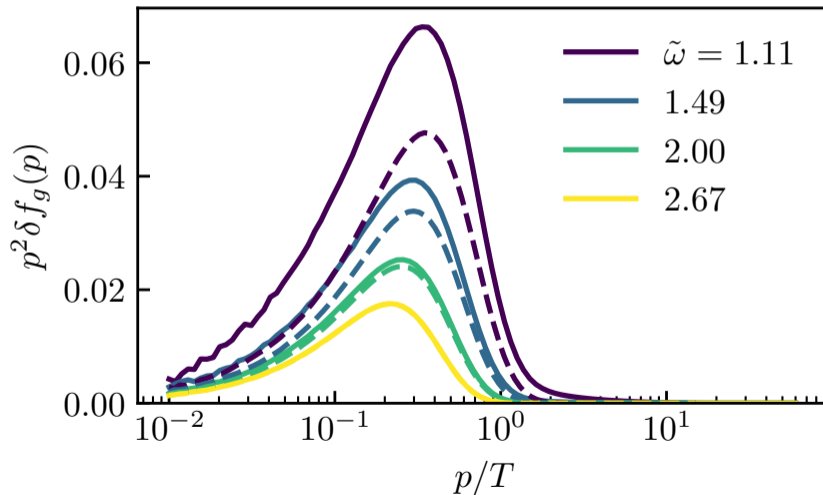


(b) $p^2 \delta f_{\text{Jet}}(t_0, p, \phi)$

- ▶ Hydrodynamization occurs, if both perturbations are indistinguishable

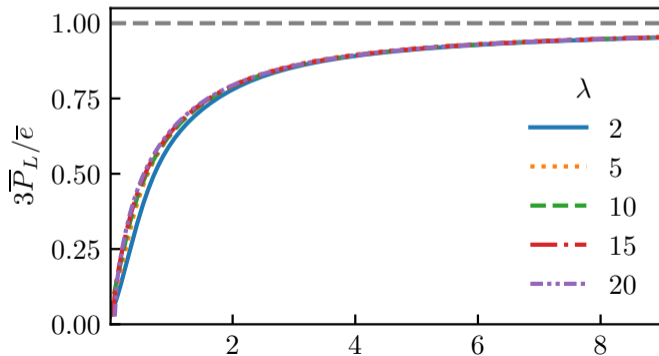
Hydrodynamisation

- ▶ anisotropy $\mathcal{A} = \frac{\delta P_T - \delta P_L}{\delta e/3}$



Pressure equilibration background

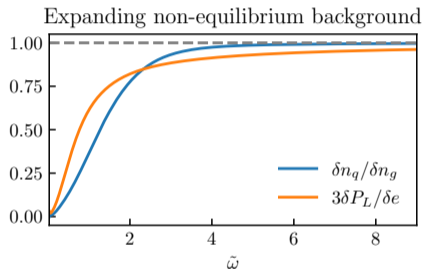
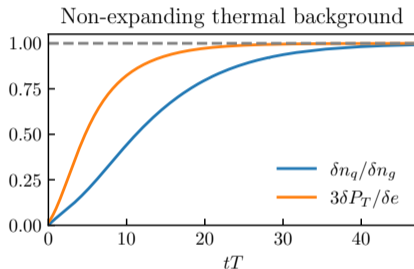
- ▶ energy momentum tensor $T^{\mu\nu} = \int_{\mathbf{p}} \frac{p^\mu p^\nu}{p} f(\tau, \mathbf{p})$
- ▶ $T_{\text{eq}}^{\mu\nu} = \text{diag}(e, P, P, P)$ with $P = e/3$
- ▶ Background longitudinal pressure $\bar{P}_L = \bar{T}^{zz}$



scaled time $\tilde{t} = \sigma T / (4\pi m/c)$

Chemical equilibration

- ▶ compare with kinetic equilibrium (isotropy of pressure)



- ▶ in chemical equilibrium more fermionic degrees of freedom
- ▶ chem. equilibration not affected by expansion

Backup

Equations of motion from 2PI effective action

$$\begin{aligned} & \left[iG_{0,ac}^{-1,\mu\gamma}(x; \mathcal{A}) + \Pi_{ac}^{(0),\mu\gamma}(x) \right] \rho_{\gamma\nu}^{cb}(x, y) \\ &= - \int_{y^0}^{x^0} dz \Pi_{ac}^{(\rho),\mu\gamma}(x, z) \rho_{\gamma\nu}^{cb}(z, y) \end{aligned} \quad (1)$$

$$\begin{aligned} & \left[iG_{0,ac}^{-1,\mu\gamma}(x; \mathcal{A}) + \Pi_{ac}^{(0),\mu\gamma}(x) \right] F_{\gamma\nu}^{cb}(x, y) \\ &= - \int_{t_0}^{x^0} dz \Pi_{ac}^{(\rho),\mu\gamma}(x, z) F_{\gamma\nu}^{cb}(z, y) + \int_{t_0}^{y^0} dz \Pi_{ac}^{(F),\mu\gamma}(x, z) \rho_{\gamma\nu}^{cb}(z, y) \end{aligned} \quad (2)$$

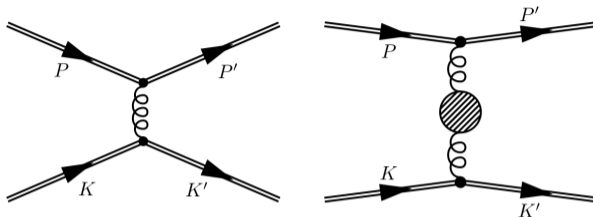
Backup

- ▶ local homogeneity \rightarrow relative coordinate $s^\mu = x^\mu - y^\mu$ and center coordinate $X^\mu = \frac{1}{2}(x^\mu + y^\mu)$
- ▶ gradient expansion in X^μ
- ▶ to lowest order, spectral function ρ is on shell
 \rightarrow quasi-particle picture
- ▶ non-equilibrium distribution function $f(X, p)$:

$$F(X, p) = -i \left[\frac{1}{2} \pm f(X, p) \right] \rho(X, p)$$

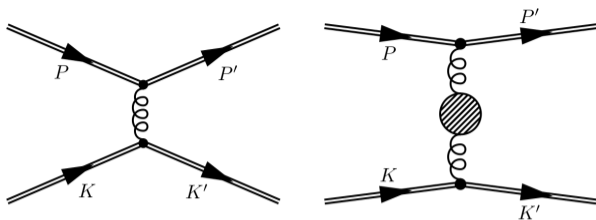
$$\Rightarrow p^\mu \partial_\mu f(X, p) = -C[f]$$

2 ↔ 2



Hard (left) and soft (right) medium regulated scattering

$$\begin{aligned}
 C_{2 \leftrightarrow 2}[f](\mathbf{p}) &= \frac{1}{4p\nu} \int_{\mathbf{k}, \mathbf{p}', \mathbf{k}'} (2\pi)^4 \delta^{(4)}(p^\mu + k^\mu - p'^\mu - k'^\mu) \\
 &\times |\mathcal{M}|^2 \left\{ \underbrace{f_{\mathbf{p}} f_{\mathbf{k}} (1 \pm f_{\mathbf{p}'}) (1 \pm f_{\mathbf{k}'})}_{\text{loss}} - \underbrace{f_{\mathbf{p}'} f_{\mathbf{k}'} (1 \pm f_{\mathbf{p}}) (1 \pm f_{\mathbf{k}})}_{\text{gain}} \right\}
 \end{aligned} \tag{3}$$



Hard (left) and soft (right) medium regulated scattering

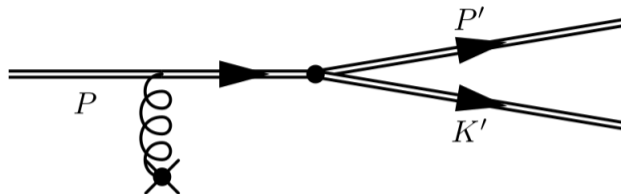
$$|\mathcal{M}|^2 = 2\lambda^2\nu \left(9 + \frac{(s-t)^2}{u^2} + \frac{(u-s)^2}{t^2} + \frac{(t-u)^2}{s^2} \right)$$

- small momentum transfer $q = |\mathbf{p}' - \mathbf{p}| \ll 1$ regulated by

$$\frac{1}{q^2} \rightarrow \frac{1}{q^2 + m_{\text{eff}}^2}$$

$$m_{\text{eff}}^2 = 2g^2 \int \frac{d^3\mathbf{p}}{(2\pi)^3 p} \left[N_c f_{\mathbf{p}}^g + N_f f_{\mathbf{p}}^q \right]$$

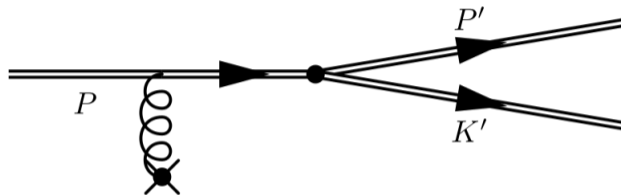
1 ↔ 2



effective 1 ↔ 2 process

$$\begin{aligned}
 C_{1\leftrightarrow 2}[f](\mathbf{p}) &= \frac{1}{2} \frac{1}{\nu} (2\pi)^3 \int_{\tilde{\mathbf{p}}, \mathbf{p}', \mathbf{k}'} (2\pi)^4 \delta^{(4)}(\tilde{\mathbf{p}}^\mu - \mathbf{p}'^\mu - \mathbf{k}'^\mu) \\
 &\times \left[\delta^{(3)}(\mathbf{p} - \tilde{\mathbf{p}}) - \delta^{(3)}(\mathbf{p} - \mathbf{p}') - \delta^{(3)}(\mathbf{p} - \mathbf{k}') \right] \\
 &\times \gamma \left\{ \underbrace{f_{\mathbf{p}} (1 \pm f_{\tilde{\mathbf{p}}}) (1 \pm f_{\mathbf{k}'})}_{\text{loss}} - \underbrace{f_{\mathbf{p}'} f_{\mathbf{k}'} (1 \pm f_{\tilde{\mathbf{p}}})}_{\text{gain}} \right\}
 \end{aligned} \tag{4}$$

$1 \leftrightarrow 2$



effective $1 \leftrightarrow 2$ process

- ▶ LO \rightarrow strictly collinear
- ▶ medium induced radiation of gluons
- ▶ $N + 1 \leftrightarrow N + 2$ effectively $1 \leftrightarrow 2$

1 ↔ 2

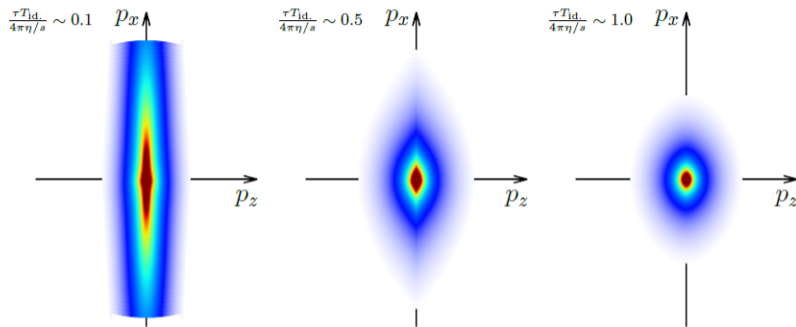


Parton going through the medium, Figure from [Jasmine]

- ▶ hard parton receiving multiple kicks
- ▶ formation time $\tau_f \sim E$
- ▶ BH: $l_f \ll l_{\text{mfp}}$, independent emissions
- ▶ LPM: $l_f \sim l_{\text{mfp}}$, destructive interference \rightarrow suppression

Bottom-up thermalization

Baier, Müller, Schiff, Son (2001)



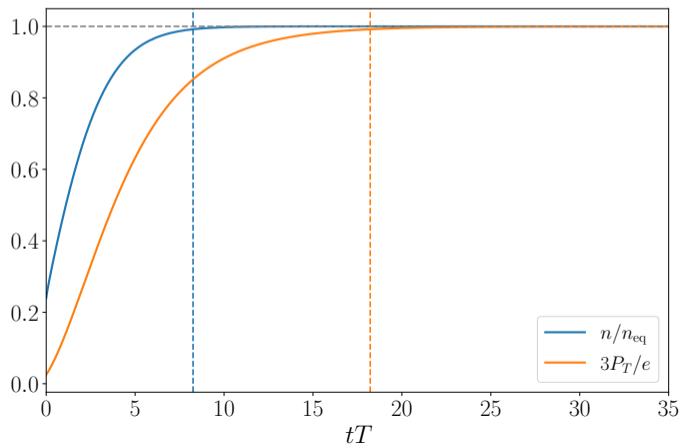
Isotropization of the distribution, Figure from [Kurkela_2019]

- ▶ 1: overoccupied system getting more anisotropic
- ▶ 2: population of soft gluons
- ▶ 3: inverse energy cascade

Radiation vs. elastic scattering

▶ particle number $n = \int_{\mathbf{p}} f(\tau, \mathbf{p}) \rightarrow C_{1 \leftrightarrow 2}[f]$

▶ transverse pressure $P_T = \frac{1}{2} \int_{\mathbf{p}} p_{\perp}^2 / p f(\tau, \mathbf{p}) \rightarrow C_{2 \leftrightarrow 2}[f]$



Equilibrium distribution

- ▶ equilibrated jet \rightarrow change in temperature δT and velocity δu^z

$$\delta f_{\text{eq}}(\mathbf{p}) = (\delta T \partial_T + \delta u^z \partial_{u^z}) n_{\text{BE}}(p_\mu u^\mu / T) \Big|_{u^z=0}$$

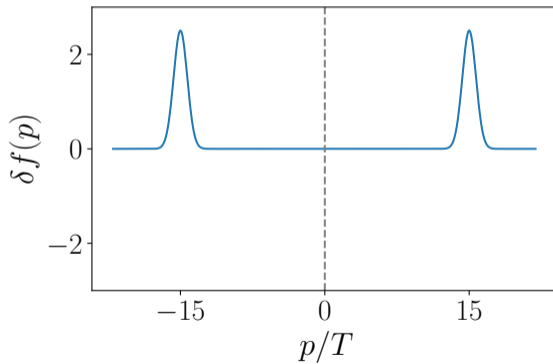
$$\delta f_{\text{eq}}(p, \theta) = \left(\delta u^z \cos \theta + \frac{\delta T}{T} \right) F(p/T)$$

- ▶ both contributions can be disentangled

Equilibrium distribution

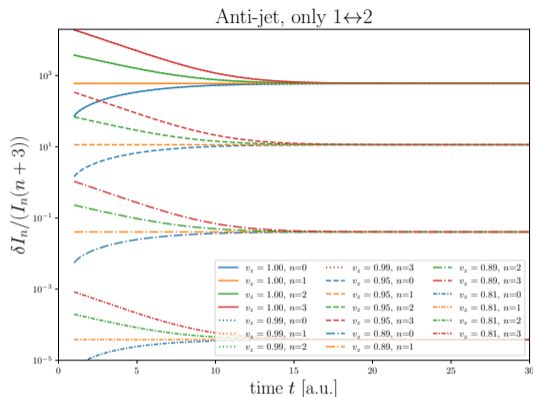
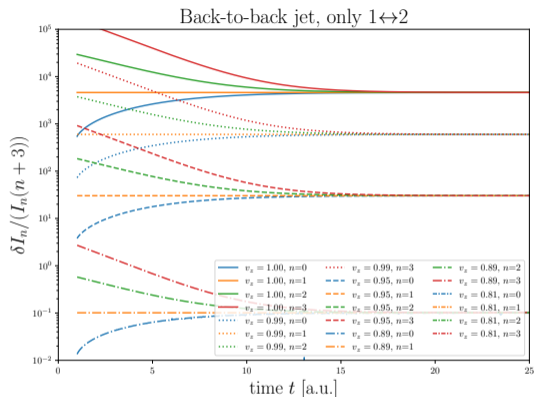
- ▶ back-to-back jet conserves net momentum:

$$\delta f_{\text{eq}}(p, \theta) = \left(\cancel{\delta u^z \cos \theta} + \frac{\delta T}{T} \right) F(p/T)$$



Initial condition: back-to-back jet

Only $1 \leftrightarrow 2$



- ▶ similar timescales of equilibration
⇒ $2 \leftrightarrow 2$ contribute more to equilibration of the anti-jet

Moments of δf

- ▶ angular effective temperature

$$I_n(\theta) \equiv 4\pi \int \frac{p^2 dp}{(2\pi)^3} p^n f(p, \theta) = \mathcal{N}_n \times T(\theta)^{n+3} \quad (5)$$

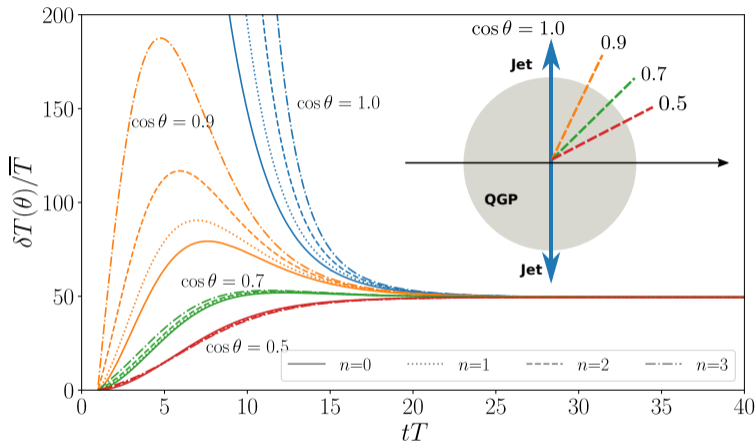
$$T(\theta) = \bar{T} + \delta T(\theta)$$

- ▶ temperature perturbation

$$\frac{\delta T(\theta)}{\bar{T}} = \frac{\delta I_n(\theta)}{(n+3)\bar{I}_n(\theta)}$$

- ▶ look at time evolution!

Moments of δf



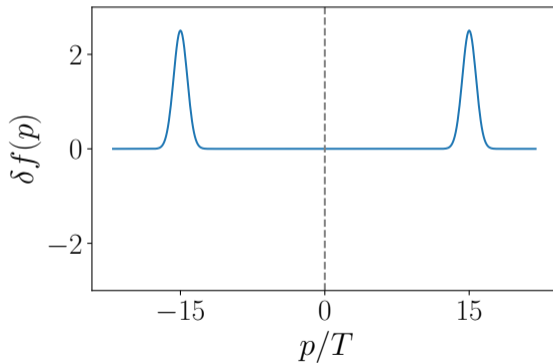
Moments of the back-to-back jet

- ▶ different moments agree before different angles do!

Equilibrium distribution

- ▶ back-to-back jet conserves net momentum:

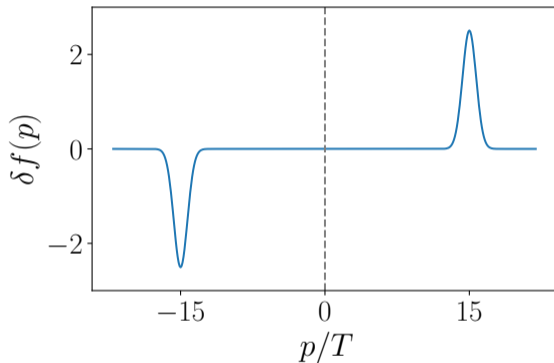
$$\delta f_{\text{eq}}(p, \theta) = \left(\cancel{\delta u^z \cos \theta} + \frac{\delta T}{T} \right) F(p/T)$$



Initial condition: back-to-back jet

Anti-jet

- ▶ introduce anti-jet → no energy deposited, **only** net momentum

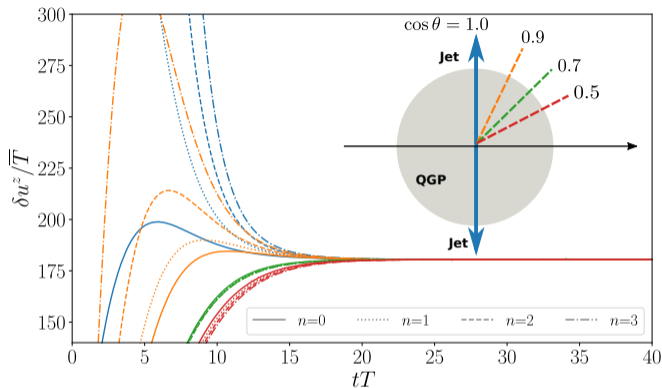


Particle and "hole"

$$\delta f_{\text{eq}}(p, \theta) = \left(\delta u^z \cos \theta + \frac{\delta T}{T} \right) F(p/T)$$

- ▶ allows us to study the build up of δu^z

Moments of δf_{anti}



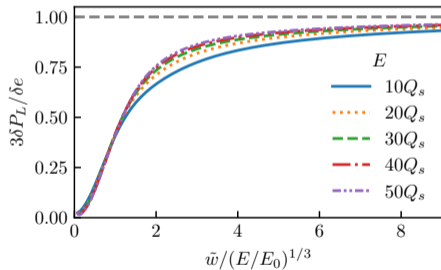
Collapse of different θ much earlier!

$$\delta f_{\text{eq}}(p, \theta) = (\delta u^z \cos \theta) F(p/T)$$

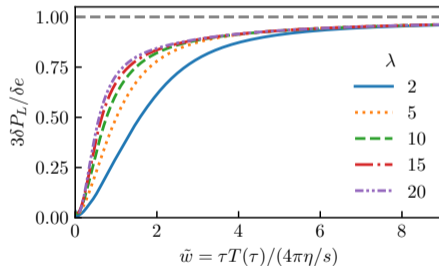
- ▶ θ -dependence \rightarrow faster reached by elastic processes
- ▶ single jet: velocity builds up faster than temperature

Pressure equilibration

- ▶ scaled time $\tilde{\omega} = \tau/\tau_R$ with $\tau_R = \frac{4\pi\eta/s}{T(\tau)}$
- ▶ effective temperature from $e(\tau) = \nu_{\text{eff}} \frac{\pi^2}{30} T(\tau)^4$



(c) Scaling with jet energy E



(d) coupling λ