Quark production and momentum isotropization in the QGP thermalization

Sergio Barrera Cabodevila

in collaboration with

Xiaojian Du, Carlos A. Salgado and Bin Wu

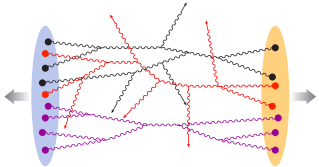
based on

Phys. Lett. B 871 (2025). SBC et al. & [arXiv:2604.XXXXX]

Outline

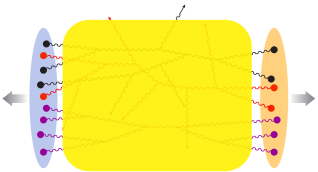
Outline

CGC



Kinetic theory

Hydro (QGP)



Ann. Rev. Nucl. Part. Sci. 60 (2010). Gelis et al.

- The QCD Boltzmann equation at leading order

$$\left(\partial_t + \underbrace{\frac{\mathbf{p}}{p}\cdot\nabla_{\mathbf{x}}}_{\text{Expansion}}\right) f^a(\mathbf{x};\mathbf{p};t) = C^a_{2\leftrightarrow 2}[f] + C^a_{1\leftrightarrow 2}[f] \quad , \quad a=\{g,q,\bar{q}\}$$

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- Longitudinal-boost invariant expansion + infinite transverse size

$$\left(\partial_\tau - \frac{p_z}{\tau}\partial_{p_z}\right) f^a = C^a_{2\leftrightarrow 2}[f] + C^a_{1\leftrightarrow 2}[f] \quad , \quad a=\{g,q,\bar{q}\}$$

- Macroscopic quantities evolving during the thermalization process

$$\hat{q} = \hat{q}[f] \qquad m_D^2 = m_D^2[f] \qquad T_* = \frac{\hat{q}}{\alpha_s N_c \mathcal{L} m_D^2}$$

Kinetic theory as a tool to study thermalization II

- In the weak-coupling limit → Bottom-up thermalization

Phys. Lett. B 502 (2001). Baier et al.

- Effective Kinetic Theory (EKT) has been widely used in this context

JHEP 01 (2003). Arnold, Moore, and Yaffe

Phys. Rev. Lett. 115.18 (2015). Kurkela and Zhu

Phys. Rev. Lett. 127.12 (2021). Du and Schlichting

- Boltzmann Equation in Diffusion Approximation (BEDA) keeps most of the relevant physics.

JHEP 06 (2024). SBC, Salgado, and Wu

Phys. Lett. B 871 (2025). SBC et al.

	BEDA	**EKT**
$C_{2\leftrightarrow 2}$	Only small angle scatterings ⇒ Fokker-Planck + source term	Full $2 \leftrightarrow 2$ QCD dynamics @LO
$C_{1\leftrightarrow 2}$	Deep LPM regime	Both LPM and Bethe-Heitler

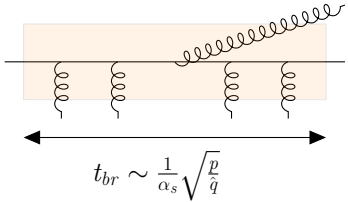
$$\hat{q}_a = 8\pi\alpha_s^2 C_a \log\frac{\langle p_t^2\rangle}{m_D^2}\int\frac{d^3p}{(2\pi)^3}\left[N_c f\,(1+f) + \frac{N_f}{2}F(1-F) + \frac{N_f}{2}\bar{F}(1-\bar{F})\right]$$

- Shows up when integrating the IR divergencies

Nucl. Phys. B 572 (2000). Mueller

- m_D^2 is given by BEDA, but $\langle p_t^2\rangle$ needs to be estimated by the broadening:

$$\langle p_t^2\rangle \sim \hat{q}\tau_{char} \sim \hat{q}t_{br}$$



$$\log\frac{\langle p_t^2\rangle}{m_D^2} = \log\left(\frac{\sqrt{\hat{q}p}}{\alpha_s m_D^2}\right) + \mathcal{O}(\log\log) \xrightarrow[\text{equilibrium}]{\text{thermal}} \log\frac{1}{\alpha_s}\ \checkmark$$

- Evolve from CGC-like initial conditions

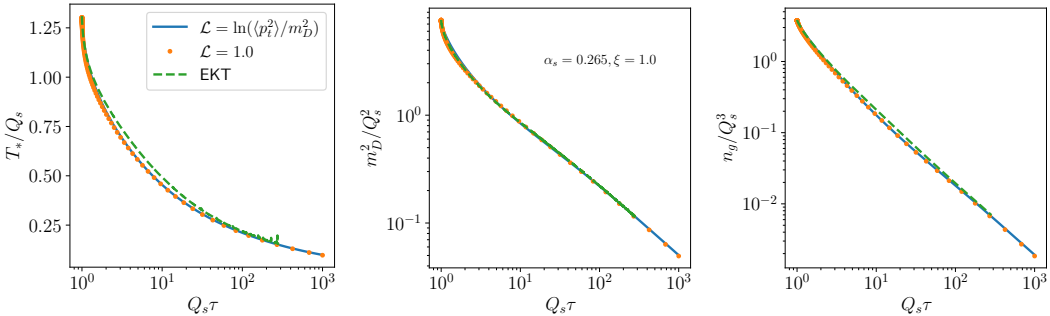
Phys. Rev. Lett. 115.18 (2015). Kurkela and Zhu

Phys. Rev. D 99.5 (2019). Kurkela and Mazeliauskas

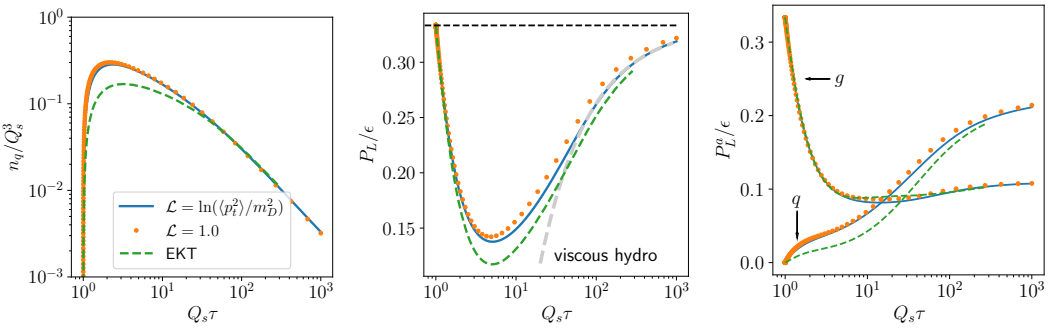
$$f_0 = \frac{A}{4\pi N_c \alpha_s} \frac{e^{-\frac{2}{3}\left((p_z\xi)^2 + p_\perp^2\right)/Q_0^2}}{\sqrt{\left((p_z\xi)^2 + p_\perp^2\right)/Q_0^2}}, \qquad F_0 = 0,$$

- $\xi \rightarrow$ Quantifies anisotropy (taken as 0 for this comparison)
- $Q_0, A \rightarrow$ Phenomenological parameters fitting CGC
- Initial time $\tau_0 = Q_s^{-1}$

Comparison with EKT II. $\alpha_s = 0.265$

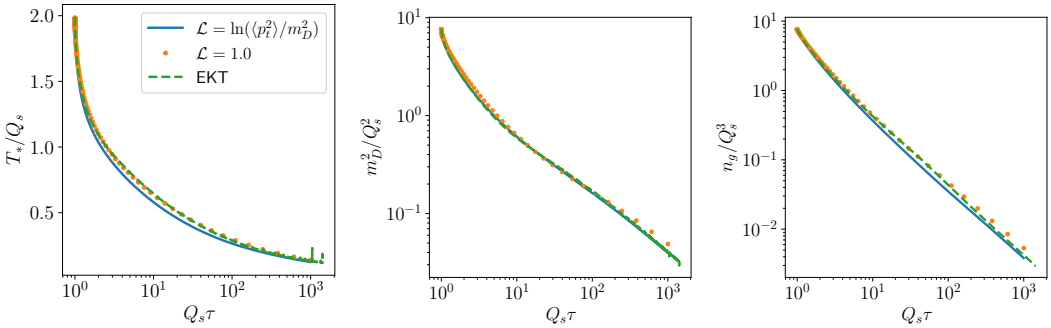


- Nice quantitative agreement during whole evolution
- Gluon number density dominated by expansion



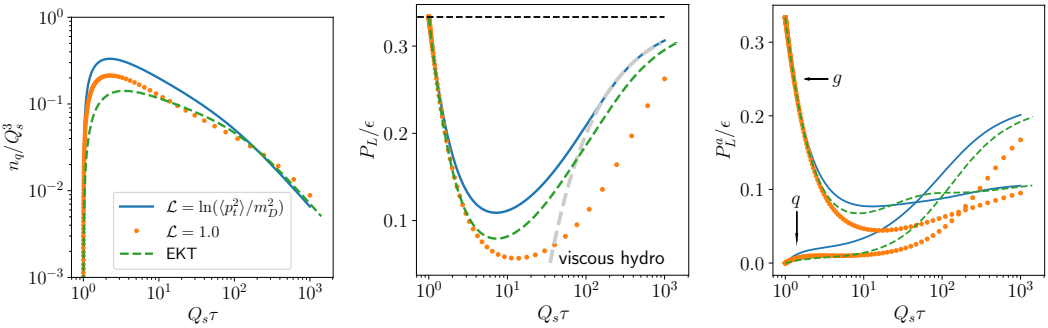
- Quantitative diferences in n_q due to differences in $g \to q\bar{q}$ splitting rates
- Higher n_q systematically leads to higher P_L/ϵ

Comparison with EKT II. $\alpha_s = 0.132$



- Still nice quantitative agreement

Comparison with EKT II. $\alpha_s = 0.132$



- The time dependent logarithm improves the accord with EKT

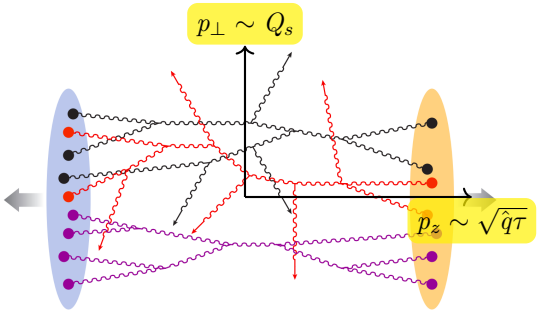
Outline

- Assume Bjorken expansion

  Phys. Rev. D 27 (1983). Bjorken

- Partons propagating along z-direction quickly fly away from the system
  $\rightarrow$ Typical momentum given by broadening $p \sim \sqrt{\hat{q}\tau} \Rightarrow$ **Soft partons**

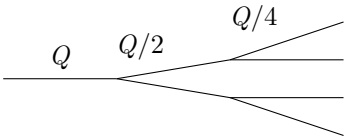
- Partons moving along the transverse plane remain in the system with $p \sim Q_s \Rightarrow$ **Hard partons**

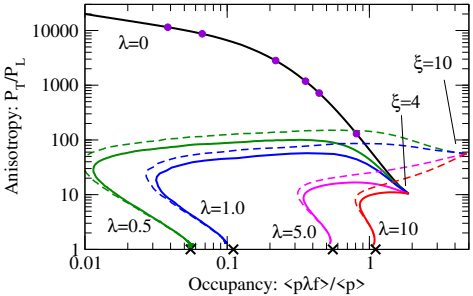


- There are three different stages for thermalization when considering boost-invariant expansion

Phys. Lett. B 502 (2001). Baier et al.

1. System dominated by expansion
2. Collision and expansion contribute similarly
3. Democratic cascade speeds up the thermalization





Phys. Rev. Lett. 115.18 (2015). Kurkela and Zhu

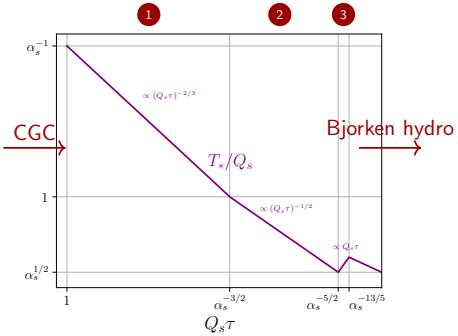
Set up the bottom-up thermalization

- Assume a parametrically highly populated and isotropic

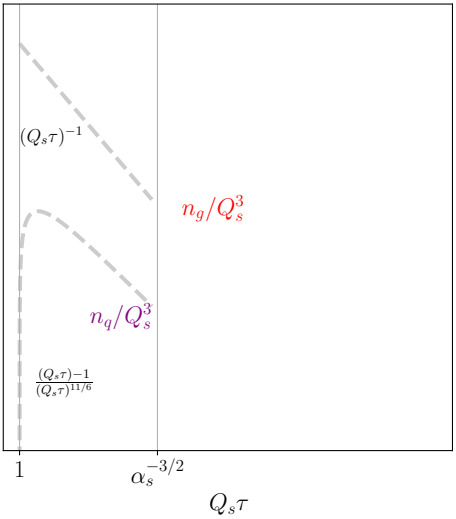
$$f_0 \sim \frac{1}{\alpha_s} \quad , \quad F = 0$$

- Parametric behaviour is well known for the pure gluon scenario

Phys. Lett. B 502 (2001). Baier et al.



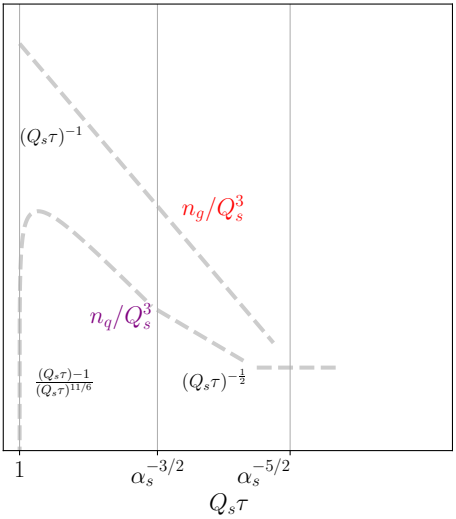
Number densities



First stage $1 \ll Q_s\tau \ll \alpha_s^{-3/2}$

- Gluon number density dominated by hard gluons decreases due to expansion
- Quark number density is dominated by $g \to q\bar{q}$ splitting
- Rapidly reaches its peak and then decreases with the expansion

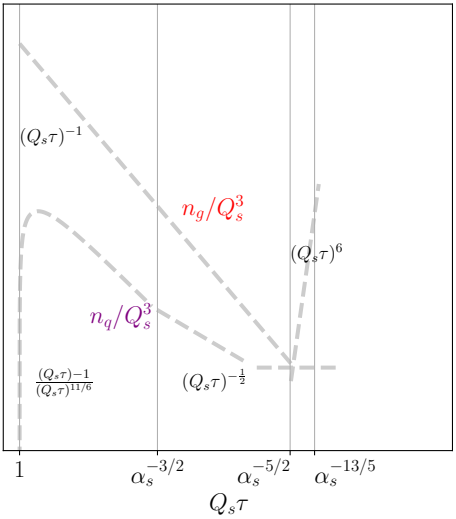
Number densities



Second stage $\alpha_s^{-3/2} \ll Q_s\tau \ll \alpha_s^{-5/2}$

- Most gluons are still hard $\rightarrow$ expansion dominated
- Change in $\hat{q}$ changes the scaling in quark production
- At $Q_s\tau \sim \alpha_s^{-2}$ quark production starts to be dominated by soft gluons via $g \rightarrow q\bar{q}$ and $gg \rightarrow q\bar{q}$ and keeps n_q constant

Number densities



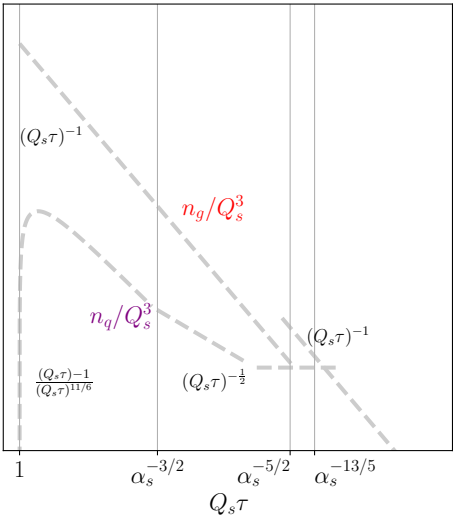
Third stage $\alpha_s^{-5/2} \ll Q_s\tau \ll \alpha_s^{-13/5}$

- Most partons are thermal

$$n_g \sim n_q \sim T^3$$

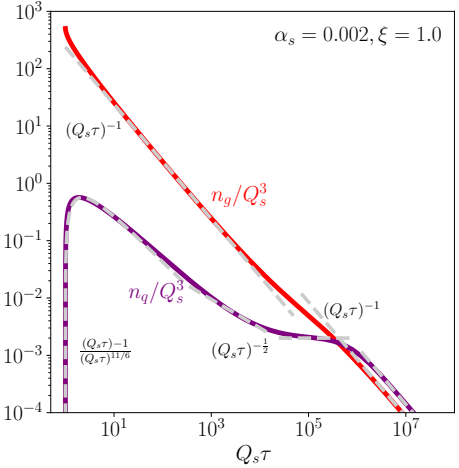
- Remaining hard partons reheat the QGP

Number densities



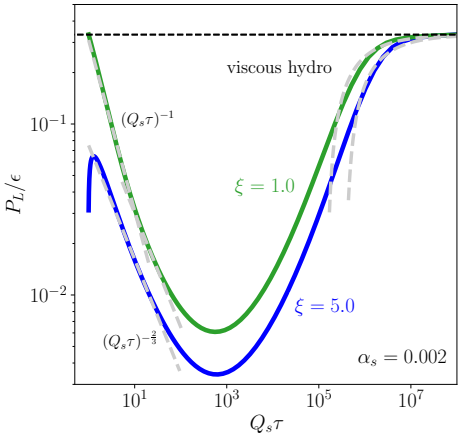
Bjorken hydro $\alpha_s^{-13/5} \ll Q_s\tau$

Number densities

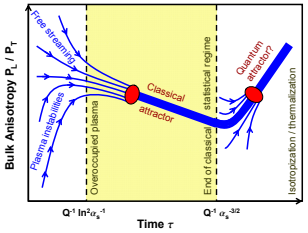


Comparison with numerics

- Nice agreement with numerical simulations
- Reheating stage missing since α_s is not small enough



- System quickly converges to non-thermal fixed point scaling at early times



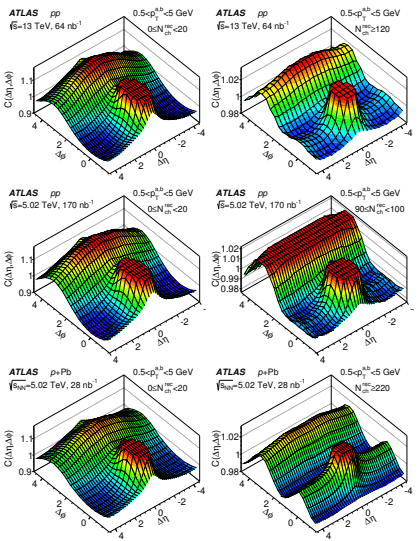
Phys. Rev. D 89.7 (2014). Berges et al.

- At later times, system approaches to viscous hydro

$$\frac{P_L}{\epsilon} = \frac{1}{3} - \frac{16}{9}\frac{\eta}{s}\frac{1}{T_{hydro}\tau}$$

Outline

Azimuthal anisotropies in small systems

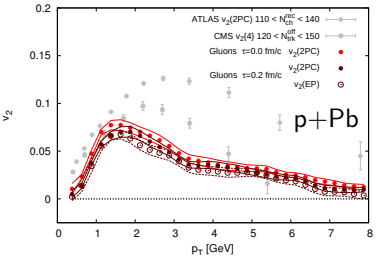


- Similar ridge to that in heavy-ion collision has been observed in small systems

  Phys. Rev. C 96.2 (2017). Aaboud et al.

- Hydrodynamics might not be the cause
- Initial state mechanisms have been proposed to explain it.

  Eur. Phys. J. A 56.8 (2020). Altinoluk and Armesto



Phys. Lett. B 747 (2015). Schenke, Schlichting, and Venugopalan

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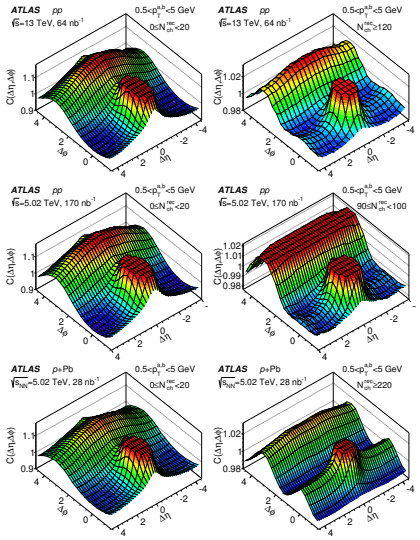

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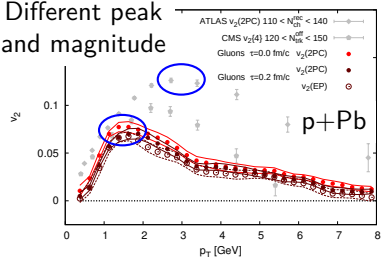
Eur. Phys. J. A 56.8 (2020). Altinoluk and Armesto

Different peak and magnitude



Phys. Lett. B 747 (2015). Schenke, Schlichting, and Venugopalan

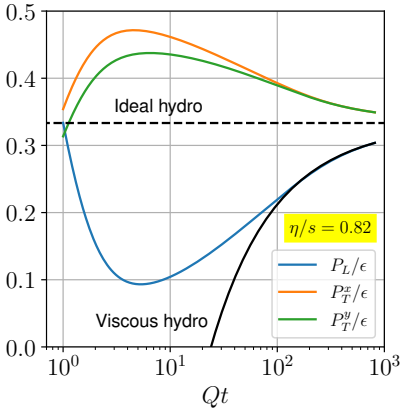
$$\frac{d^6N}{d^3\mathbf{x}d^3\mathbf{p}} \sim f(\mathbf{x};\mathbf{p};t_f) \Rightarrow \frac{d^3N}{d^3\mathbf{p}} \sim \int d^3\mathbf{x} f(\mathbf{x};\mathbf{p};t_f) = f(\mathbf{p};t_f)$$

- We study how initial anisotropies relax in Kinetic Theory
- Initial condition

$$f_0(\mathbf{p}) = f_0(\mathbf{p}_T)\left(1 + \sum_n v_n^0(\mathbf{p}_T)\cos(n\phi)\right)$$

- Harmonic coefficients computed in the Event Plane

$$v_n(\mathbf{p}_T;t) = \frac{\int d\phi \cos(n(\phi-\psi_n)) f(\mathbf{p};t)}{\int d\phi f(\mathbf{p};t)}$$



- Start initializing with constant anisotropy and explore fully integrated v_n

$$v_n^0(\mathbf{p}_T) = v_n^0 \qquad v_n(t) \equiv \frac{\int d^3\mathbf{p}\cos(n(\phi-\psi_n))f(\mathbf{p};t)}{\int d^3\mathbf{p} f(\mathbf{p};t)}$$

- Exploring the variation due to the $C_{2\leftrightarrow 2}$ kernel

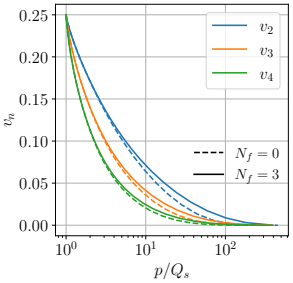
$$\partial_\tau v_n(\tau) = \frac{\hat{q}}{4n_g}\int \frac{dp\ d(\cos\theta)}{(2\pi)^2}p^2\Bigg[-\frac{n^2 v_n^2 \tilde{f}/2}{p^2(1-\cos^2\theta)} + \frac{1}{T_*}\left(\frac{2\tilde{f}}{p}+\partial_p\tilde{f}\right)v_n^2 + \frac{1}{T_*}\left(\frac{2\tilde{f}^2}{p}+\partial_p\tilde{f}^2\right)\left(v_n^2+\sum_{k,l}v_k v_l\int\frac{d\phi}{2\pi}\cos(n\phi)\cos(k\phi)\cos(l\phi)\right)\Bigg]$$

Higher n relaxes faster

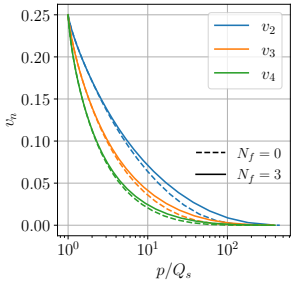
Produces higher n anisotropies

- For initial v_n, only v_{kn} with $k = 1, 2, 3\ldots$ are produced

Evolution of fully integrated coefficients II. Numerical results



- Higher n relaxes faster
- Including quarks in the system delays the isotropization



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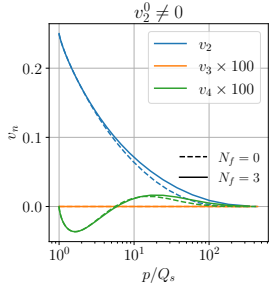
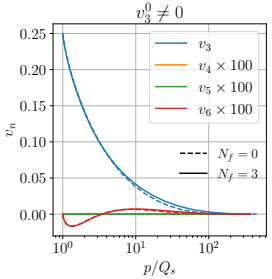


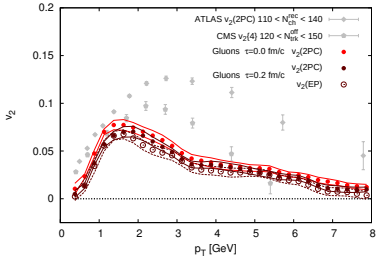


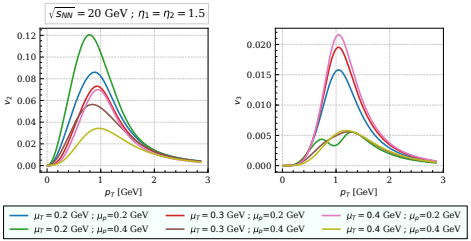
- Based on different results from initial state interactions we model

Eur. Phys. J. A 56.8 (2020). Altinoluk and Armesto

$$f_0(\mathbf{p}) = f_0(\mathbf{p}_T)\left(1 + \sum_n v_n^0(\mathbf{p}_T)\cos(n\phi)\right), \qquad v_n^0(p_T) = \tilde{v}_n \frac{p}{Q} e^{-|p|/Q}$$

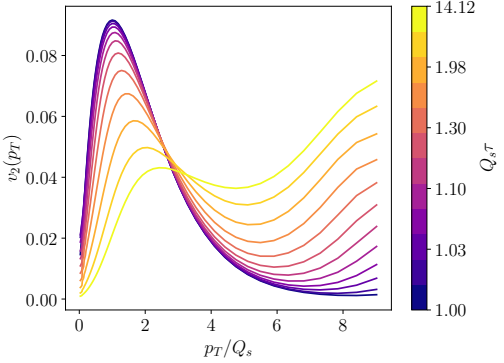
- In the following results we take $v_2 = 0.25$ and $v_{n\neq 2} = 0$





Phys. Lett. B 747 (2015). Schenke, Schlichting, and Venugopalan

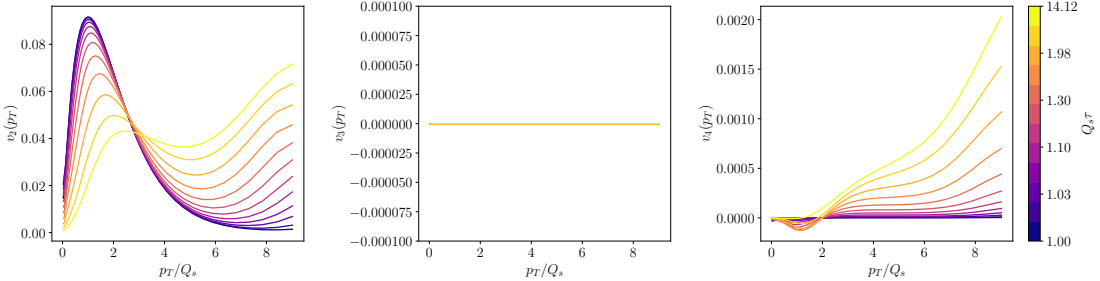
Eur. Phys. J. C 79.9 (2019). Agostini, Altinoluk, and Armesto

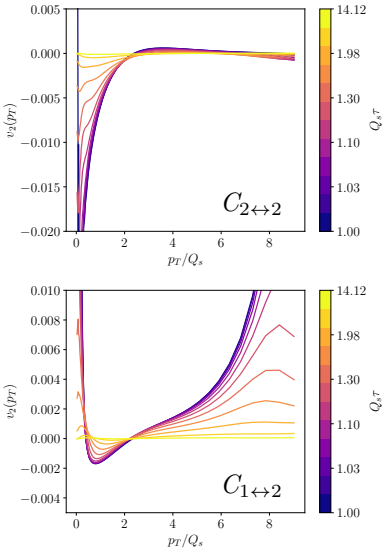


$$v_n(p_T;t) = \frac{\int dp_z d\phi \cos(n(\phi-\psi_n)) f(\mathbf{p};t)}{\int d\phi f(\mathbf{p};t)}$$

- The magnitude of the anisotropy decreases with time
- The position of the maximum is shifted to higher momenta
- Anisotropy at large p_T is produced easily because of the low occupancy in this region

- Initial $v_2 \Rightarrow$ only even harmonics are generated





- Infrarred:
  - $C_{1\leftrightarrow 2} \rightarrow$ Increases v_2 due to **collinear radiation** of partons from the more anisotropic regions of the phase space
  - $C_{2\leftrightarrow 2} \rightarrow$ Decreases the anisotropy more efficiently than $C_{1\leftrightarrow 2}$
- Ultraviolet:
  - $C_{1\leftrightarrow 2} \rightarrow$ Increases v_2 due to **collinear merging** of partons in the more anisotropy regions of the phase space
  - $C_{2\leftrightarrow 2} \rightarrow$ Modifies v_2 less efficiently

- Same initial condition as before

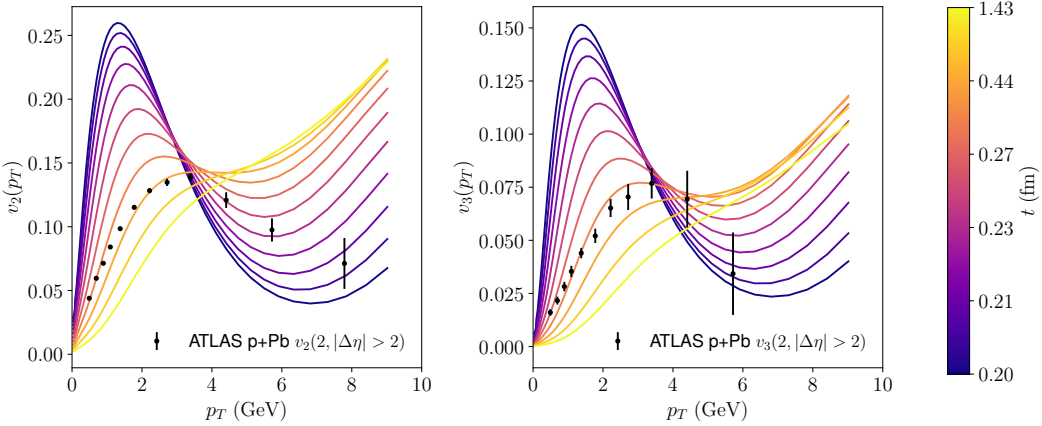
$$f_0(\mathbf{p}) = f_0(\mathbf{p}_T)\left(1 + \sum_n v_n^0(\mathbf{p}_T)\cos(n\phi)\right), \qquad v_n^0(p_T) = \tilde{v}_n \frac{p}{Q} e^{-|p|/Q}$$

- Parametrization to reproduce energy density from $p + Pb$ collisions at LHC

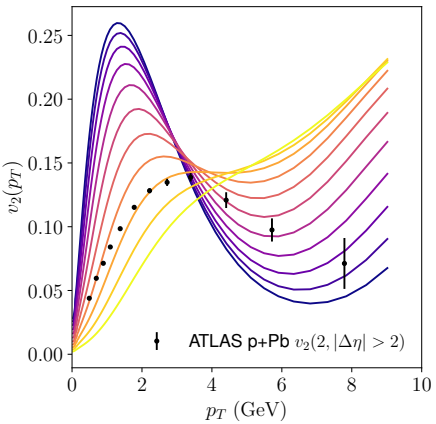
JHEP 11 (2021). Kurkela, Mazeliauskas, and Törnkvist

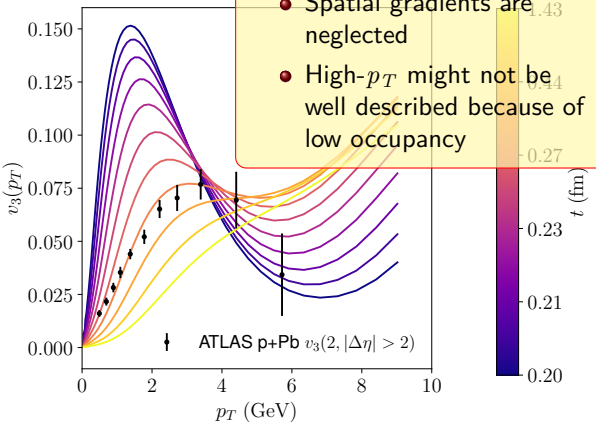
- Initial time for the kinetic stage set to $t_0 = 0.2$ fm
- Initialize anisotropy with $\tilde{v}_2 = 0.75$, $\tilde{v}_3 = 0.45$ and $\tilde{v}_{n\neq 2,3} = 0$
- Position of the maximum set to $Q \sim Q_s \approx 1.22$ GeV

- Experimental data can be mimicked at a time $t \sim 0.5$ fm



- Experimental data can be mimicked at a time $t \sim 0.5$ fm





Disclaimer

- Spatial gradients are neglected
- High-p_T might not be well described because of low occupancy

Conclusions

- We have shown that BEDA is qualitatively consistent with EKT
- Quark production has been parametrically understood in the bottom-up picture
- Both non-thermal fixed point and hydrodynamical attractor show up in the simulations
- Study of azimuthal isotropization in the BEDA framework indicate that final state interactions may be relevant if initial momentum anisotropies are present
- Final state interactions introduce a mechanism which shifts the maximum of the azimuthal anisotropy to higher p_T, as experimental data suggests
- Include spatial gradients will allow to perform a more complete study of the collective behavior of the bulk

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Thank you for your attention!

Back-up

- In diffusion approximation, the $2 \leftrightarrow 2$ collision kernel can be expressed as a Fokker-Planck equation plus an additional source term.

Phys. Lett. B 475 (2000). Mueller

Nucl. Phys. A 930 (2014). Blaizot, Wu, and Yan

$$C^a_{2\leftrightarrow 2} = \frac{1}{4}\hat{q}_a(t)\nabla_p \cdot \left[\nabla_p f^a + \frac{\mathbf{v}}{T^*(t)} f^a(1+\epsilon_a f^a)\right] + \mathcal{S}_a$$

$$\mathcal{S}_q = \frac{2\pi\alpha_s^2 C_F^2 \ln \frac{\langle p_t^2\rangle}{m_D^2}}{p}\left[\mathcal{I}_c f(1-F) - \bar{\mathcal{I}}_c F(1+f)\right],$$

$$\mathcal{S}_{\bar{q}} = \mathcal{S}_q|_{F\leftrightarrow\bar{F}}, \qquad \mathcal{S}_g = -\frac{N_f}{2C_F}(\mathcal{S}_q + \mathcal{S}_{\bar{q}}),$$

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Phys. Lett. B 475 (2000). Mueller

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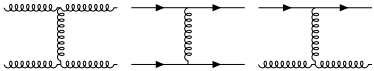
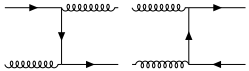
Fokker-Planck term



Source term



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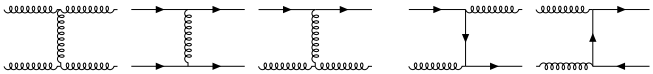
Fokker-Planck term

Source term



- The gluon distribution function is known to diverge at small p, $f \propto 1/p$, for over-occupied systems, which is interpreted as the onset of Bose-Einstein Condensation (BEC).

Nucl. Phys. A 920 (2013). Blaizot, Liao, and McLerran

- The presence of BEC can be study numerically by choosing the appropriate boundary conditions with[1] $\dot{n}_c \propto (\lim_{p\to 0} pf - T_*)$.

Nucl. Phys. A 930 (2014). Blaizot, Wu, and Yan

[1] $n_c \equiv$ number density of the BEC.

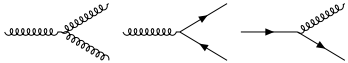
- The $1 \leftrightarrow 2$ kernel can be computed in the deep LPM regime

Nucl. Phys. B 483 (1997). Baier et al.

Phys. Rev. D 78 (2008). Arnold and Dogan

$$C^a_{1\leftrightarrow 2} = \int_0^1 \frac{dx}{x^3} \sum_{b,c} \left[\frac{\nu_c}{\nu_a} C^c_{ab}(p/x; p, p(1-x)/x) - \frac{1}{2} C^a_{bc}(p; xp, (1-x)p) \right]$$

- The $C^a_{bc}(p; xp, (1-x)p)$ describes the collinear splitting $a \leftrightarrow bc$.
- The three possible processes involved are the three QCD interaction vertices.



- Will the BEC still appear in initially over-populated system after including inelastic collisions?

- At small p, the $g \leftrightarrow gg$ and $g \leftrightarrow q\bar{q}$ are the dominant processes in the production of gluons and (anti)quarks, respectively.
- The distributions of gluons and quarks quickly fill a thermal distribution up to small soft momentum p_s

$$f^g(p) \approx \frac{T_*}{p} \qquad \text{for } p \lesssim p_g$$

$$f^q(p) \approx \frac{1}{e^{-\frac{\mu_*}{T_*}} + 1} \qquad \text{for } p \lesssim p_q$$

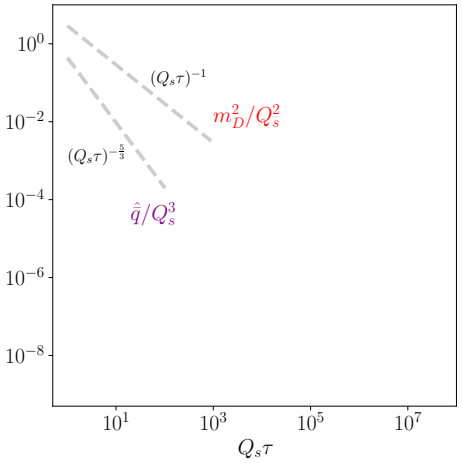
At early times, p_s is given by ($\mathcal{I}_c = \mathcal{I}_c[f]$)

$$p_g \equiv (\hat{q}_A m_D^4 t^2/2)^{\frac{1}{5}} \qquad p_q \equiv [\alpha_s C_F \pi (\mathcal{I}_c + \bar{\mathcal{I}}_c) t]^{\frac{2}{5}} \hat{q}_F^{\frac{1}{5}}$$

- This behavior implies that $\dot{n}_c = 0$, so no BEC is observed as in the pure gluon case.

Nucl. Phys. A 961 (2017). Blaizot, Liao, and Mehtar-Tani

JHEP 06 (2024). SBC, Salgado, and Wu



First stage $1 \ll Q_s\tau \ll \alpha_s^{-3/2}$

- Most partons in the system are hard and dominated by the expansion

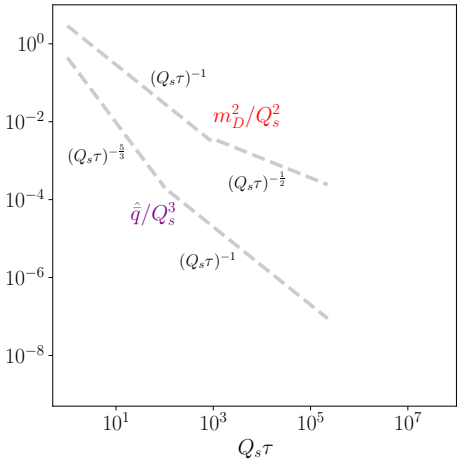
$$n_g \sim \frac{1}{\alpha_s}\frac{Q_s^3}{Q_s\tau} \quad , \quad f_g \gg 1$$

- Screening and $\hat{q}$ dominated by hard gluons

$$m_D^2 \sim \alpha_s \frac{n_g}{Q_s} \sim \frac{Q_s^2}{Q_s\tau}$$

$$\hat{q} \sim \alpha_s^2 n_g f_g$$

Time evolution of macroscopic quantities: $m_D^2,\ \hat{q}$

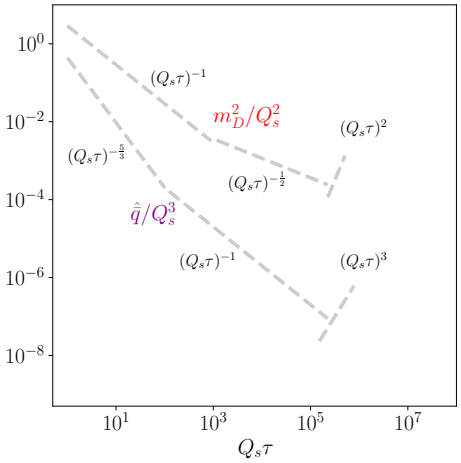


Second stage $\alpha_s^{-3/2} \ll Q_s\tau \ll \alpha_s^{-5/2}$

- Most of gluons are still hard but $f_g \ll 1$
- Screening is dominated by soft gluons
- $\hat{q}$ changes scaling due to Bose enhancement

$$\hat{q} \sim \alpha_s^2 n_g (1+f)$$

Time evolution of macroscopic quantities: $m_D^2,\ \hat{q}$



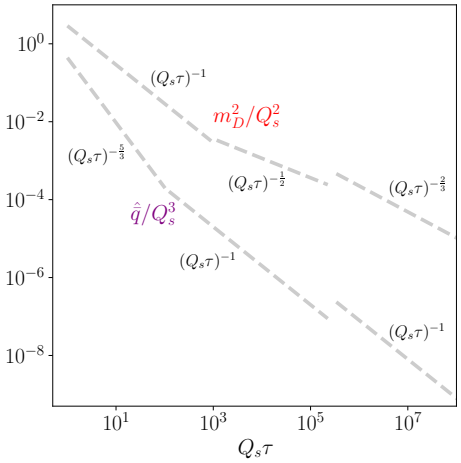
Third stage $\alpha_s^{-5/2} \ll Q_s\tau \ll \alpha_s^{-13/5}$

- Most partons in the system are soft and in thermal equilibrium

$$n_g \sim T^3 \quad m_D^2 \sim \alpha_s T^2 \quad \hat{q} \sim \alpha_s^2 T^3$$

- Remaining soft partons cascade and reheat the soft bath

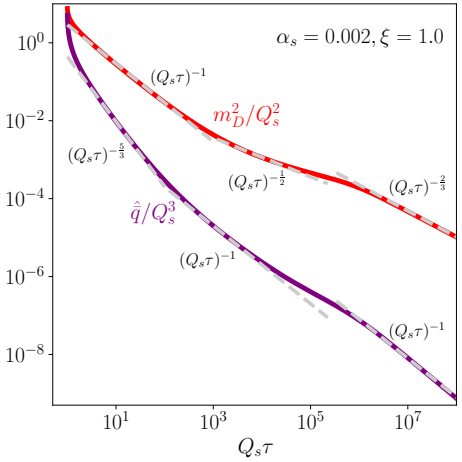
Time evolution of macroscopic quantities: $m_D^2,\ \hat{q}$



Bjorken hydro $\alpha_s^{-13/5} \ll Q_s\tau$

- Full system has thermalized and evolves as in Bjorken hydro

Time evolution of macroscopic quantities: $m_D^2,\ \hat{q}$



Comparison with numerics