In medium radiation from a QCD antenna with realistic parton-medium interactions

Carlota Andres, Liliana Apolinário, Néstor Armesto,
André Cordeiro, Fabio Dominguez, Pablo Guerrero, Guilherme Milhano

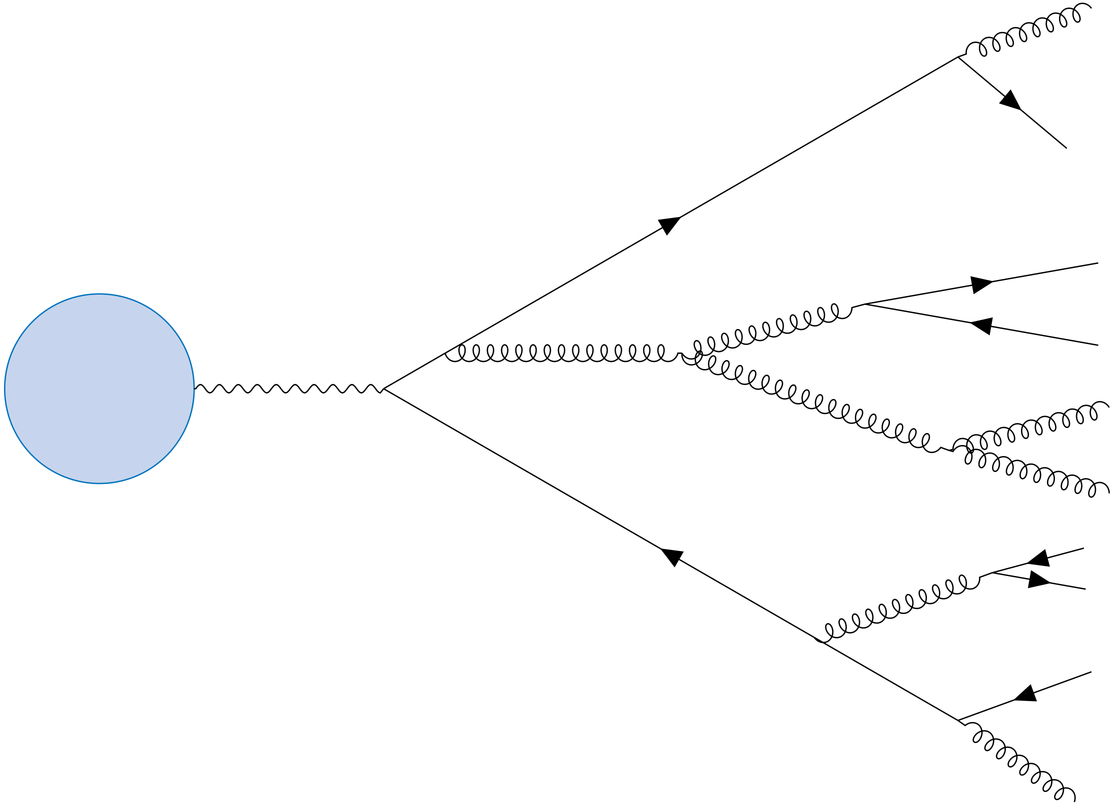
C3NT Workshop
Jet-soft dynamical medium interaction in high-energy heavy-ion collisions

TÉCNICO LISBOA

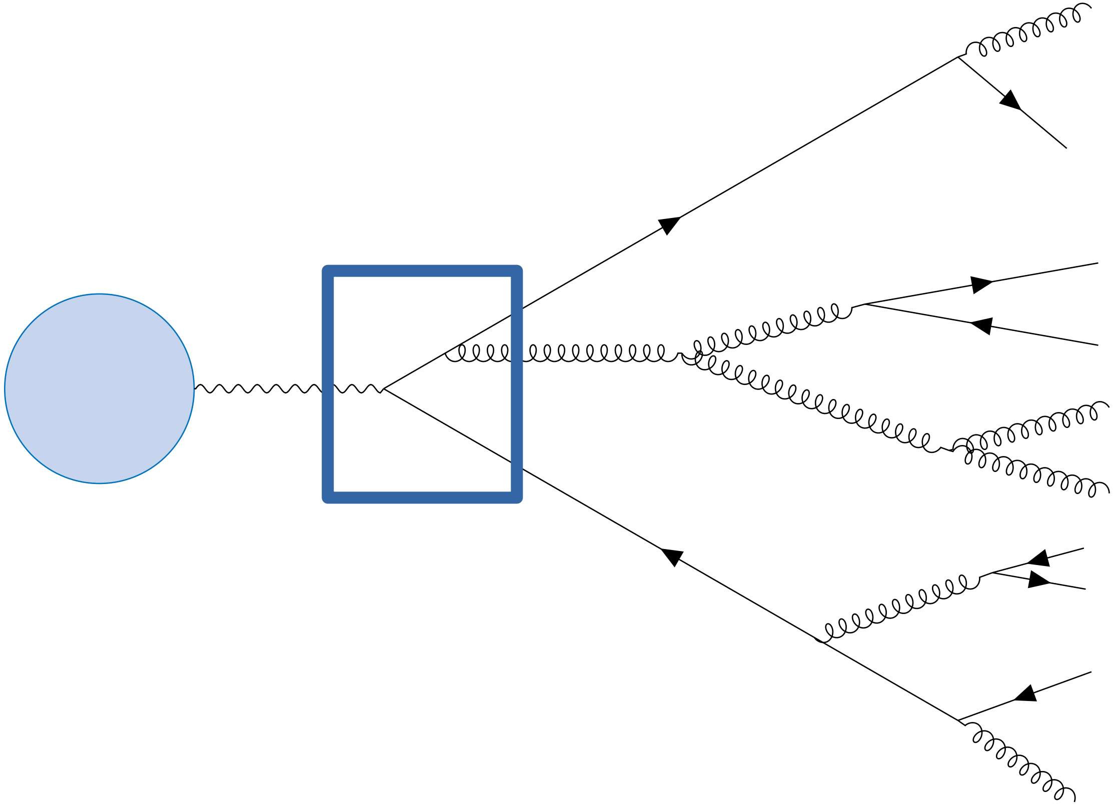
LIP

Friday, 27th March 2025
Wuhan, China

Why Study QCD Antennas

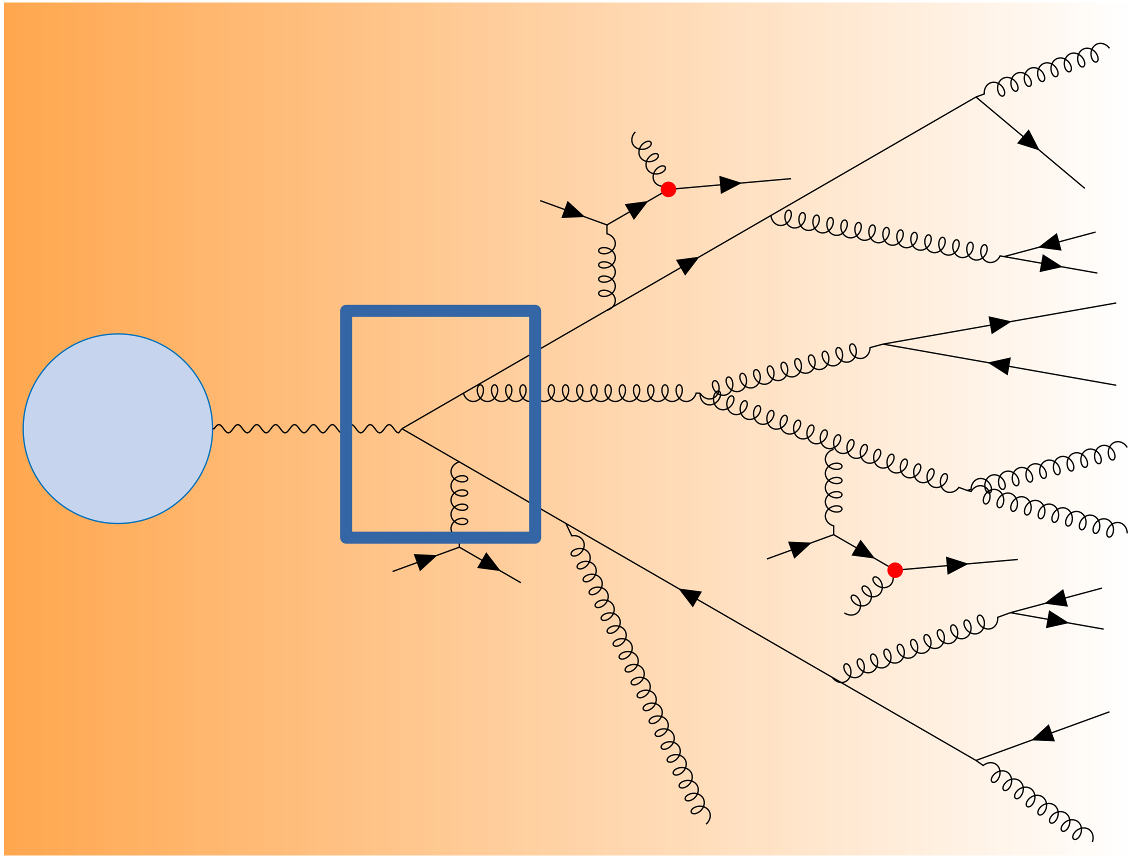


Why Study QCD Antennas



- 2 → 3 (coherent) processes are building blocks for Parton Cascades

Why Study QCD Antennas

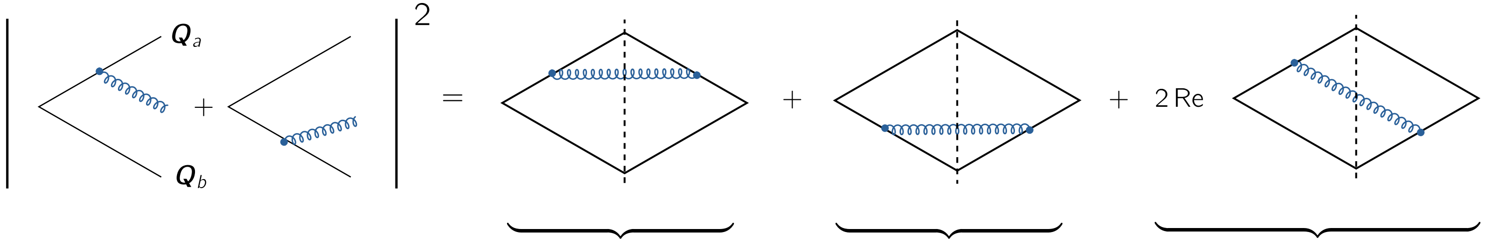


- 2 → 3 (coherent) processes are building blocks for Parton Cascades
- Coherence breakdown expected to drive medium modifications

Precision QGP studies require accurate fixed order calculations

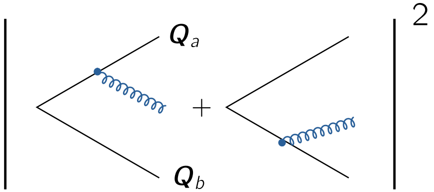
First: A look at vacuum antennas

QCD Coherence: Gauge Invariance



$$\boldsymbol{Q}_a^2\mathcal{R} \qquad \boldsymbol{Q}_b^2\bar{\mathcal{R}} \qquad 2\,\boldsymbol{Q}_a\cdot\boldsymbol{Q}_b\,\mathcal{J} = \left(\boldsymbol{Q}_{a+b}^2 - \boldsymbol{Q}_a^2 - \boldsymbol{Q}_b^2\right)\mathcal{J}$$

QCD Coherence: Gauge Invariance

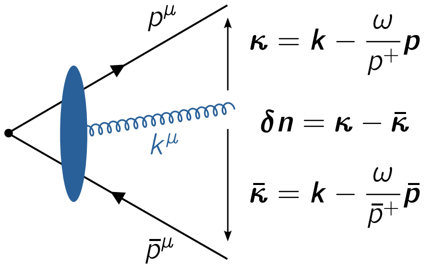


$$= \boldsymbol{Q}_a^2\left(\mathcal{R}-\mathcal{J}\right)+\boldsymbol{Q}_b^2\left(\bar{\mathcal{R}}-\mathcal{J}\right)+\boldsymbol{Q}_{a+b}^2\,\mathcal{J}$$

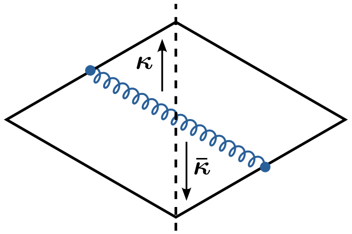
Radiation 'from' (a)

Radiation 'from' (b)

Radiation 'from' (a+b)



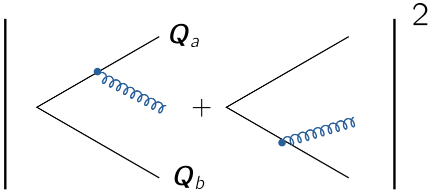
$$\mathcal{J}(\boldsymbol{\kappa},\bar{\boldsymbol{\kappa}}) =$$



(All other terms can be recovered from J)

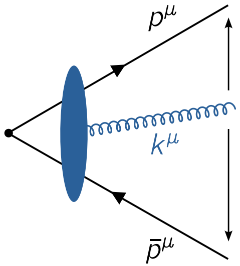
$$\mathcal{R}(\boldsymbol{\kappa}) = \mathcal{J}(\boldsymbol{\kappa},\boldsymbol{\kappa})$$
$$\bar{\mathcal{R}}(\bar{\boldsymbol{\kappa}}) = \mathcal{J}(\bar{\boldsymbol{\kappa}},\bar{\boldsymbol{\kappa}})$$

QCD Coherence: Gauge Invariance



$$= \quad \boldsymbol{Q}_a^2\,(\mathcal{R} - \mathcal{J}) + \boldsymbol{Q}_b^2\,(\bar{\mathcal{R}} - \mathcal{J}) + \boldsymbol{Q}_{a+b}^2\,\mathcal{J}$$

Radiation 'from' (a) | **Radiation 'from' (b)** | **Radiation 'from' (a+b)**

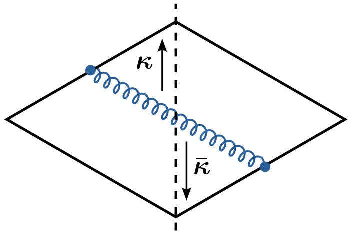


$$\boldsymbol{\kappa} = \boldsymbol{k} - \frac{\omega}{p^+}\boldsymbol{p}$$

$$\boldsymbol{\delta n} = \boldsymbol{\kappa} - \bar{\boldsymbol{\kappa}}$$

$$\bar{\boldsymbol{\kappa}} = \boldsymbol{k} - \frac{\omega}{\bar{p}^+}\bar{\boldsymbol{p}}$$

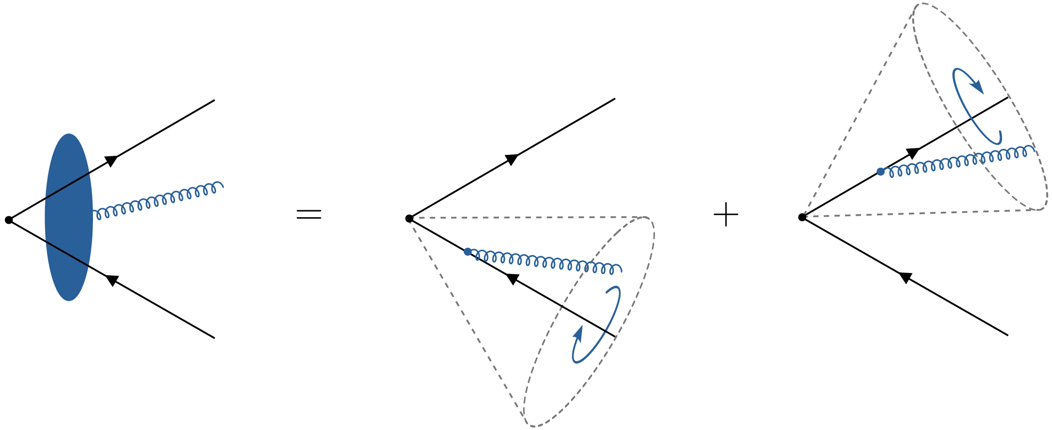
$\mathcal{J}(\boldsymbol{\kappa}, \bar{\boldsymbol{\kappa}}) =$



(All other terms can be recovered from J)

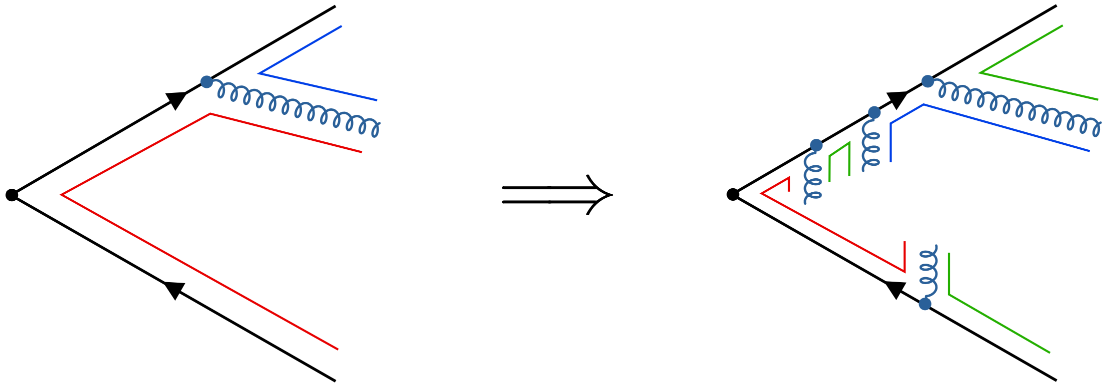
$$\mathcal{R}(\boldsymbol{\kappa}) = \mathcal{J}(\boldsymbol{\kappa}, \boldsymbol{\kappa})$$
$$\bar{\mathcal{R}}(\bar{\boldsymbol{\kappa}}) = \mathcal{J}(\bar{\boldsymbol{\kappa}}, \bar{\boldsymbol{\kappa}})$$

Interference effects* imply angular ordering:



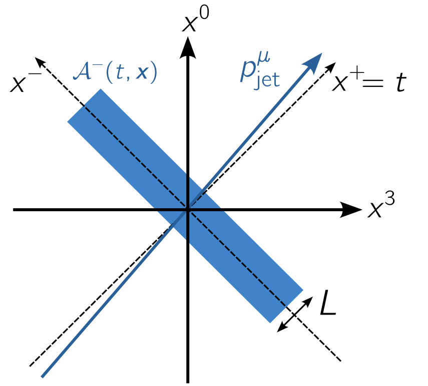
*** after azimuthal averaging:** $\displaystyle\int \frac{\mathrm{d}\varphi_{\kappa p}}{2\pi}\mathcal{J}(\boldsymbol{\kappa}, \bar{\boldsymbol{\kappa}}) = \mathcal{R}(\kappa)\Theta(\kappa/\omega - \delta n)$

What Changes inside the QGP?



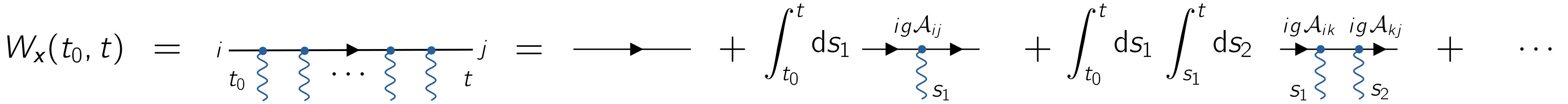
Colour exchanges with medium → Broken colour connections

Parton Propagation In-Medium



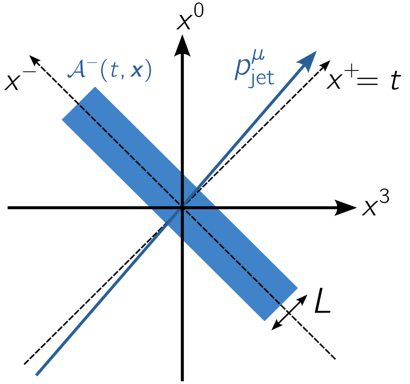
Consider a coloured field: $\mathcal{A}_{ij} = A^{-\,a}(t, \boldsymbol{x})\, T^a_{ij}$

Add up an infinite number of scatterings → **Wilson Lines**

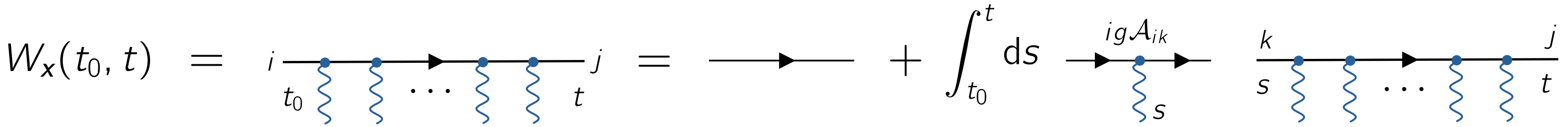


Parton Propagation In-Medium

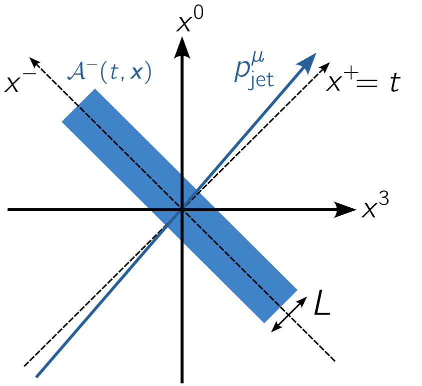
Consider a coloured field: $\mathcal{A}_{ij} = A^{-\,a}(t, \boldsymbol{x})\, T^a_{ij}$



Add up an infinite number of scatterings → **<u>Wilson Lines</u>**

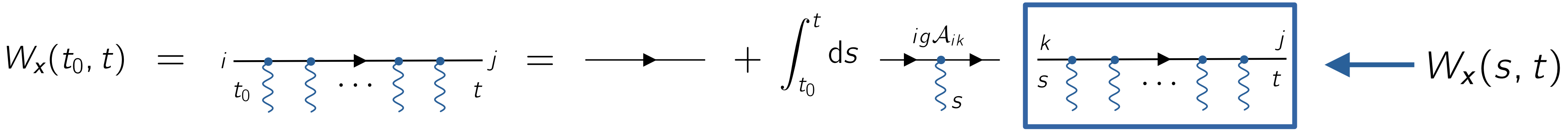


Parton Propagation In-Medium

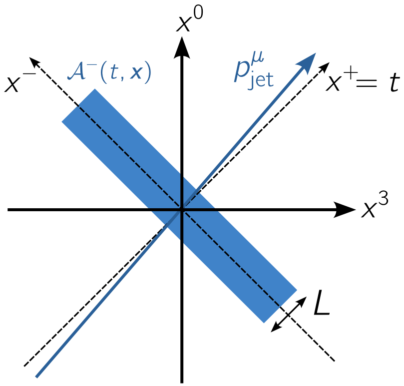


Consider a coloured field: $\mathcal{A}_{ij} = A^{-\,a}(t, \mathbf{x})\, T^a_{ij}$

Add up an infinite number of scatterings → **<u>Wilson Lines</u>**

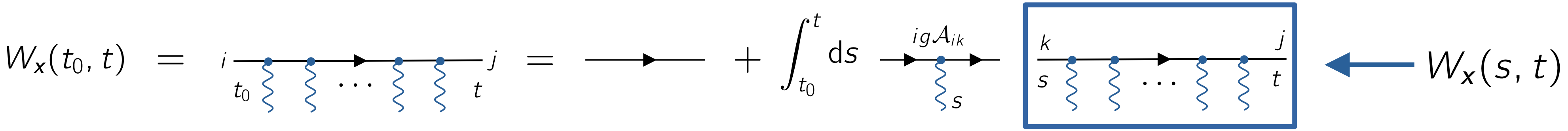


Parton Propagation In-Medium



Consider a coloured field: $\mathcal{A}_{ij} = A^{-\,a}(t,\boldsymbol{x})\, T^a_{ij}$

Add up an infinite number of scatterings → **Wilson Lines**



For a fixed transverse coordinate: $W_{\boldsymbol{x}}(t, t_0) = \mathcal{P} \exp\left\{ ig \int_{t_0}^{t} \mathrm{d}s\, \mathcal{A}(s, \boldsymbol{x}) \right\}$

(Rotation in colour space) (Path-ordering)

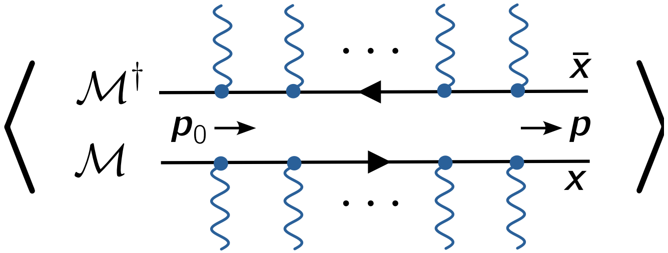
Allowing transverse diffusion: $\mathcal{G}(t, t_0) = \int \mathcal{D}\boldsymbol{r} \exp\left\{ \frac{p^+}{2} \int_{t_0}^{t} \dot{\boldsymbol{r}}^2(s) \mathrm{d}s \right\} W_{\boldsymbol{r}}(t, t_0)$

(Brownian motion) (Path Integral)

These are the building blocks for in-medium calculations

Momentum Broadening

$$\int_{\boldsymbol{x},\bar{\boldsymbol{x}}} e^{i(\boldsymbol{p}_0-\boldsymbol{p})\cdot(\boldsymbol{x}-\bar{\boldsymbol{x}})} \left\langle \quad \right\rangle \overset{\text{def}}{=} \mathcal{P}(t, t_0; \boldsymbol{p}_0 - \boldsymbol{p})$$



$\mathcal{P}(\boldsymbol{x} - \bar{\boldsymbol{x}}; t, t_0)$

Momentum Broadening

(due to colour rotation over different lines)

Medium Averages: According to MV-Model (Gaussian White Noise)

$$\langle A^a(t, \boldsymbol{x})\, A^{\bar{a}}(\bar{t}, \bar{\boldsymbol{x}})\rangle = V(\bar{\boldsymbol{x}} - \boldsymbol{x})\, n(t)\, \delta(t - \bar{t})\, \delta^{a\bar{a}}$$

Scattering Rate **Medium Density** → **Modelling choices**

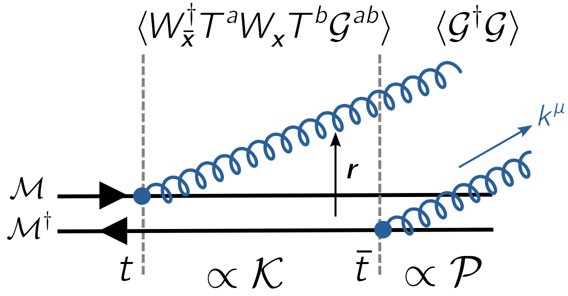
From the Wilson Line Evolution Equation:

$$\partial_t \mathcal{P}(\boldsymbol{r}; t, t_0) = -\frac{1}{2} n(t)\sigma(\boldsymbol{r})\mathcal{P}(\boldsymbol{r}; t, t_0) \implies \mathcal{P}(\boldsymbol{r}; t, t_0) = \exp\left\{-\int_{t_0}^{t} \mathrm{d}s\, \frac{n(s)}{2}\, \sigma(\boldsymbol{r})\right\}$$

Main phenomenological object: 'Dipole Cross-section'

$$\boxed{\sigma(\boldsymbol{r}) = 2g^2\,[V(\boldsymbol{0}) - V(\boldsymbol{r})]}$$

Medium-Induced Emission



Induced emissions encoded in the Kernel

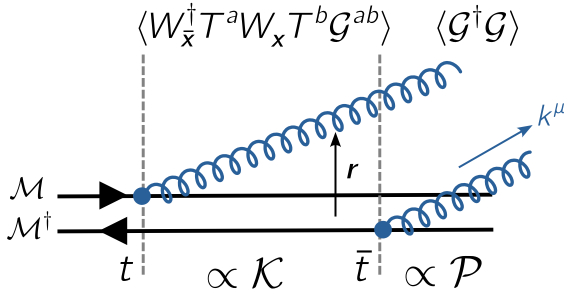
$$\mathcal{K}(\boldsymbol{r}_i, t_i; \boldsymbol{r}_f, t_f) = \int_{\boldsymbol{r}(t_i)=\boldsymbol{r}_i}^{\boldsymbol{r}(t_f)=\boldsymbol{r}_f} \mathcal{D}\boldsymbol{r} \exp\left\{ i \int_{t_i}^{t_f} \mathrm{d}s \left(\frac{\omega}{2}\dot{\boldsymbol{r}}^2(s) + i\frac{n(s)}{2}\sigma(\boldsymbol{r}) \right) \right\}$$

Gluon emission rate:

$$\omega \frac{\mathrm{d}I}{\mathrm{d}\omega \mathrm{d}^2\boldsymbol{k}} = \frac{\alpha_s C_F}{(2\pi)^2\omega^2} 2\mathrm{Re} \int_0^\infty \mathrm{d}\bar{t} \int_0^{\bar{t}} \mathrm{d}t \int_{\boldsymbol{pq}} \boldsymbol{p}\cdot\boldsymbol{q}\, \mathcal{K}(\boldsymbol{q}, \bar{t}; \boldsymbol{p}, t) \mathcal{P}(\infty, \boldsymbol{k}; \bar{t}, \boldsymbol{q})$$

Barata, Mehtar-Tani, Soto-Ontoso, Tywoniuk :: JHEP 09 (2021) 153

Medium-Induced Emission



Induced emissions encoded in the Kernel

$$\mathcal{K}(\boldsymbol{r}_i, t_i; \boldsymbol{r}_f, t_f) = \int_{\boldsymbol{r}(t_i)=\boldsymbol{r}_i}^{\boldsymbol{r}(t_f)=\boldsymbol{r}_f} \mathcal{D}\boldsymbol{r} \exp\left\{ i \int_{t_i}^{t_f} \mathrm{d}s \left(\frac{\omega}{2} \dot{\boldsymbol{r}}^2(s) + i\frac{n(s)}{2}\sigma(\boldsymbol{r}) \right) \right\}$$

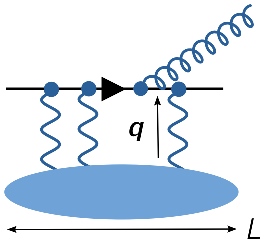
Gluon emission rate:

$$\omega \frac{\mathrm{d}I}{\mathrm{d}\omega \mathrm{d}^2\boldsymbol{k}} = \frac{\alpha_s C_F}{(2\pi)^2\omega^2} 2\mathrm{Re} \int_0^\infty \mathrm{d}\bar{t} \int_0^{\bar{t}} \mathrm{d}t \int_{\boldsymbol{pq}} \boldsymbol{p}\cdot\boldsymbol{q}\, \mathcal{K}(\boldsymbol{q}, \bar{t}; \boldsymbol{p}, t) \mathcal{P}(\infty, \boldsymbol{k}; \bar{t}, \boldsymbol{q})$$

Usual approach: Harmonic Oscillator

$$n(s)\sigma(\boldsymbol{r}) \to \frac{\hat{q}(s)}{2}\boldsymbol{r}^2$$

$$\hat{q} \simeq \frac{\langle \boldsymbol{k}^2_{\text{acquired}} \rangle}{L}$$

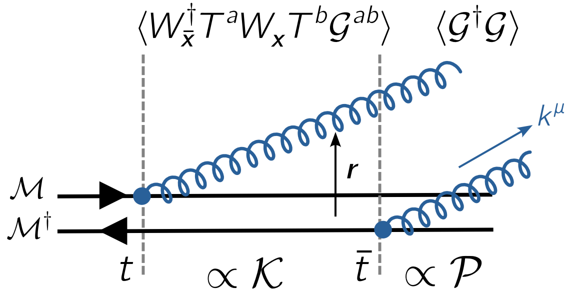


→ Adds up multiple interactions

→ Covers only small 'r' $(\boldsymbol{q}^2 \lesssim T^2_{\text{medium}})$

Barata, Mehtar-Tani, Soto-Ontoso, Tywoniuk :: JHEP 09 (2021) 153

Medium-Induced Emission



Induced emissions encoded in the Kernel

$$\mathcal{K}(\boldsymbol{r}_i, t_i; \boldsymbol{r}_f, t_f) = \int_{\boldsymbol{r}(t_i)=\boldsymbol{r}_i}^{\boldsymbol{r}(t_f)=\boldsymbol{r}_f} \mathcal{D}\boldsymbol{r} \exp\left\{ i \int_{t_i}^{t_f} \mathrm{d}s \left(\frac{\omega}{2}\dot{\boldsymbol{r}}^2(s) + i\frac{n(s)}{2}\sigma(\boldsymbol{r}) \right) \right\}$$

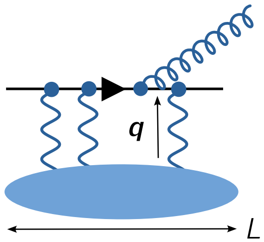
Gluon emission rate:

$$\omega \frac{\mathrm{d}I}{\mathrm{d}\omega \mathrm{d}^2\boldsymbol{k}} = \frac{\alpha_s C_F}{(2\pi)^2\omega^2} 2\mathrm{Re} \int_0^{\infty} \mathrm{d}\bar{t} \int_0^{\bar{t}} \mathrm{d}t \int_{\boldsymbol{pq}} \boldsymbol{p}\cdot\boldsymbol{q}\, \mathcal{K}(\boldsymbol{q}, \bar{t}; \boldsymbol{p}, t)\mathcal{P}(\infty, \boldsymbol{k}; \bar{t}, \boldsymbol{q})$$

Usual approach: Harmonic Oscillator

$$n(s)\sigma(\boldsymbol{r}) \to \frac{\hat{q}(s)}{2}\boldsymbol{r}^2$$

$$\hat{q} \simeq \frac{\langle \boldsymbol{k}^2_{\text{acquired}} \rangle}{L}$$

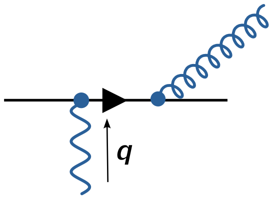


→ **Adds up multiple interactions**

→ **Covers only small 'r'** $(q^2 \lesssim T^2_{\text{medium}})$

Alternative: GLV (Opacity Expansion)

$$e^{-\int \mathrm{d}s n(s)\sigma(\boldsymbol{r})} \sim 1 - \int \mathrm{d}s n(s)\sigma(\boldsymbol{r})$$



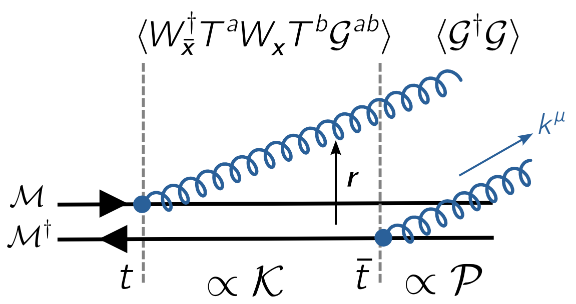
"Single Scattering"

→ **Allows for full momentum range**

→ **Misses many-scattering contribution**

Barata, Mehtar-Tani, Soto-Ontoso, Tywoniuk :: JHEP 09 (2021) 153

Medium-Induced Emission



Induced emissions encoded in the Kernel

$$\mathcal{K}(\boldsymbol{r}_i, t_i; \boldsymbol{r}_f, t_f) = \int_{\boldsymbol{r}(t_i)=\boldsymbol{r}_i}^{\boldsymbol{r}(t_f)=\boldsymbol{r}_f} \mathcal{D}\boldsymbol{r} \exp\left\{ i \int_{t_i}^{t_f} \mathrm{d}s \left(\frac{\omega}{2} \dot{\boldsymbol{r}}^2(s) + i \frac{n(s)}{2} \sigma(\boldsymbol{r}) \right) \right\}$$

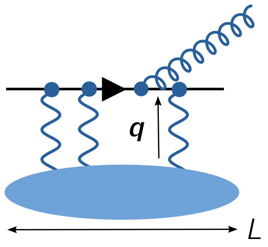
Gluon emission rate:

$$\omega \frac{\mathrm{d}I}{\mathrm{d}\omega \mathrm{d}^2\boldsymbol{k}} = \frac{\alpha_s C_F}{(2\pi)^2 \omega^2} 2\mathrm{Re} \int_0^\infty \mathrm{d}\bar{t} \int_0^{\bar{t}} \mathrm{d}t \int_{\boldsymbol{pq}} \boldsymbol{p} \cdot \boldsymbol{q}\, \mathcal{K}(\boldsymbol{q}, \bar{t}; \boldsymbol{p}, t) \mathcal{P}(\infty, \boldsymbol{k}; \bar{t}, \boldsymbol{q})$$

Usual approach: Harmonic Oscillator

$$n(s)\sigma(\boldsymbol{r}) \to \frac{\hat{q}(s)}{2} \boldsymbol{r}^2$$

$$\hat{q} \simeq \frac{\langle \boldsymbol{k}^2_{\text{acquired}} \rangle}{L}$$



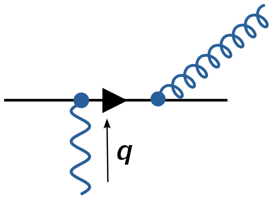
→ **Adds up multiple interactions**

→ **Covers only small 'r'** $(q^2 \lesssim T^2_{\text{medium}})$

Alternative: GLV (Opacity Expansion)

$$e^{-\int \mathrm{d}s n(s)\sigma(\boldsymbol{r})} \sim 1 - \int \mathrm{d}s n(s) \sigma(\boldsymbol{r})$$

"Single Scattering"



→ **Allows for full momentum range**

→ **Misses many-scattering contribution**

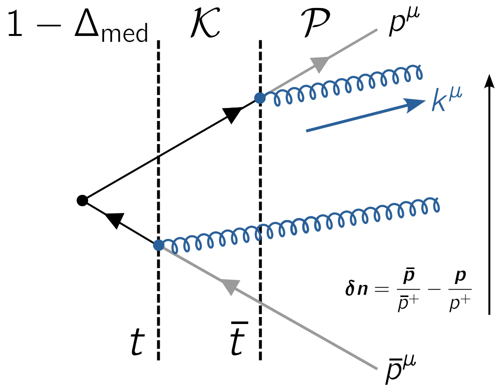
Analytical approaches possible, but limited in phase-space

*Mixed approach: Improved Opacity Expansion

Antenna Decoherence

Mehtar-Tani, Tywoniuk :: JHEP 01 (2013) 031
Casalderrey-Solana, Iancu :: JHEP 08 (2011) 015
Mehtar-Tani, Salgado, Tywoniuk :: JHEP 10 (2012) 197

* $\mathcal{M}$ and $\mathcal{M}^\dagger$ overlayed



<u>The interference term:</u>

$$\mathcal{J}(\boldsymbol{\kappa}, \bar{\boldsymbol{\kappa}}) = \mathrm{Re} \int_0^\infty \mathrm{d}t \int_t^\infty \mathrm{d}\bar{t} \int_{\boldsymbol{q}_1 \boldsymbol{q}_2} e^{it\left(\frac{\omega}{2}\boldsymbol{\delta n}^2 - \boldsymbol{q}_1 \cdot \boldsymbol{\delta n}\right)} \boldsymbol{q}_2 \cdot (\boldsymbol{q}_1 - \omega \boldsymbol{\delta n})$$

$$\mathcal{P}(\boldsymbol{\kappa} - \boldsymbol{q}_2; \infty, \bar{t}) \mathcal{K}(\boldsymbol{q}_2, \bar{t}; \boldsymbol{q}_1, t)[1 - \Delta_{\rm med}(t)] + (\mathrm{sym.})$$

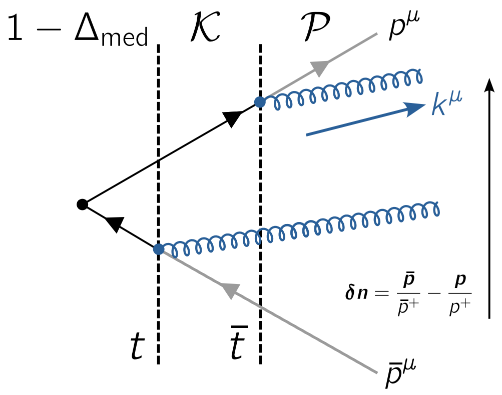
Gluon Broadening — **Gluon Decoherence** — **Antenna Decoherence**

<u>What physical picture underlies the decoherence factor?</u>

Antenna Decoherence

Mehtar-Tani, Tywoniuk :: JHEP 01 (2013) 031
Casalderrey-Solana, Iancu :: JHEP 08 (2011) 015
Mehtar-Tani, Salgado, Tywoniuk :: JHEP 10 (2012) 197

* $\mathcal{M}$ and $\mathcal{M}^\dagger$ overlayed



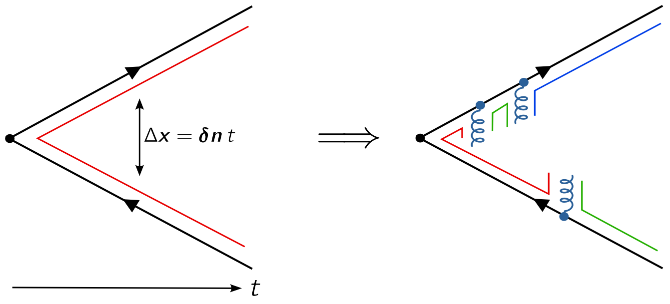
The interference term:

$$\mathcal{J}(\boldsymbol{\kappa}, \bar{\boldsymbol{\kappa}}) = \text{Re} \int_0^\infty \mathrm{d}t \int_t^\infty \mathrm{d}\bar{t} \int_{\boldsymbol{q}_1 \boldsymbol{q}_2} e^{it\left(\frac{\omega}{2}\boldsymbol{\delta n}^2 - \boldsymbol{q}_1 \cdot \boldsymbol{\delta n}\right)} \boldsymbol{q}_2 \cdot (\boldsymbol{q}_1 - \omega \boldsymbol{\delta n})$$

$$\mathcal{P}(\boldsymbol{\kappa} - \boldsymbol{q}_2; \infty, \bar{t})\mathcal{K}(\boldsymbol{q}_2, \bar{t}; \boldsymbol{q}_1, t)[1 - \Delta_{\text{med}}(t)] + (\text{sym.})$$

Gluon Broadening | **Gluon Decoherence** | **Antenna Decoherence**

Probability colour structure survives?



What physical picture underlies the decoherence factor?

Analogy: Radioactive decay!

'Single scattering probability': $\mathrm{d}P_{\text{1-scatter}} = n(t)\dfrac{\sigma(\boldsymbol{\delta n}t)}{2}\mathrm{d}t$

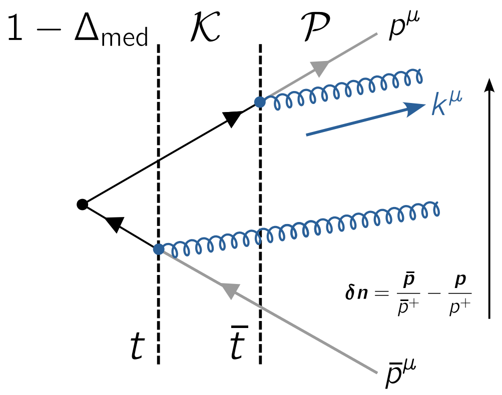
'Survival probability': $1 - \Delta_{\text{med}}(t, 0) = \exp\left\{-\int_0^t \mathrm{d}\xi\, n(\xi)\, \dfrac{\sigma(\boldsymbol{\delta n}\xi)}{2}\right\}$

The 'Decoherence Factor' quantifies colour correlations

Anti-Angular Ordering

Mehtar-Tani, Tywoniuk :: JHEP 01 (2013) 031
Casalderrey-Solana, Iancu :: JHEP 08 (2011) 015
Mehtar-Tani, Salgado, Tywoniuk :: JHEP 10 (2012) 197

* $\mathcal{M}$ and $\mathcal{M}^\dagger$ overlayed



The interference term:

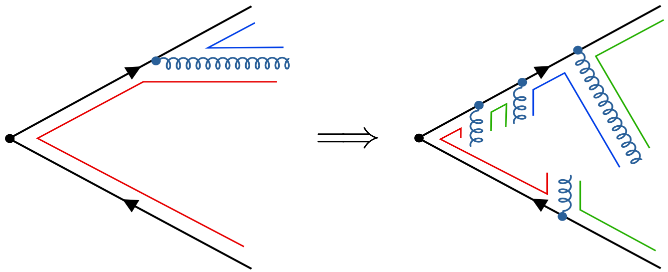
$$\mathcal{J}(\boldsymbol{\kappa},\bar{\boldsymbol{\kappa}}) = \mathrm{Re}\int_0^\infty \mathrm{d}t \int_t^\infty \mathrm{d}\bar{t} \int_{\boldsymbol{q}_1 \boldsymbol{q}_2} e^{it\left(\frac{\omega}{2}\boldsymbol{\delta n}^2 - \boldsymbol{q}_1\cdot\boldsymbol{\delta n}\right)} \boldsymbol{q}_2 \cdot (\boldsymbol{q}_1 - \omega\boldsymbol{\delta n})$$

$$\mathcal{P}(\boldsymbol{\kappa}-\boldsymbol{q}_2;\infty,\bar{t})\mathcal{K}(\boldsymbol{q}_2,\bar{t};\boldsymbol{q}_1,t)[1-\Delta_{\rm med}(t)] + (\mathrm{sym.})$$

Gluon Broadening | **Gluon Decoherence** | **Antenna Decoherence**

'Survival probability':

$$1-\Delta_{\rm med}(t,0) = \exp\left\{-\int_0^t \mathrm{d}\xi\, n(\xi)\, \frac{\sigma(\boldsymbol{\delta n}\,\xi)}{2}\right\}$$



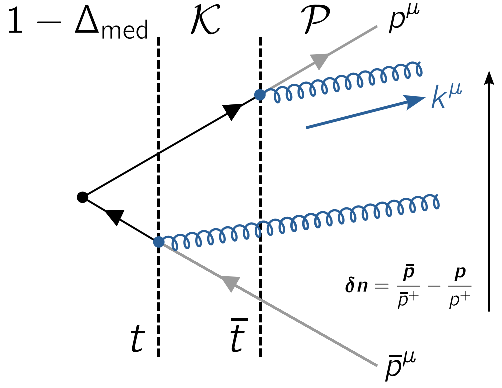
Destruction of colour structure

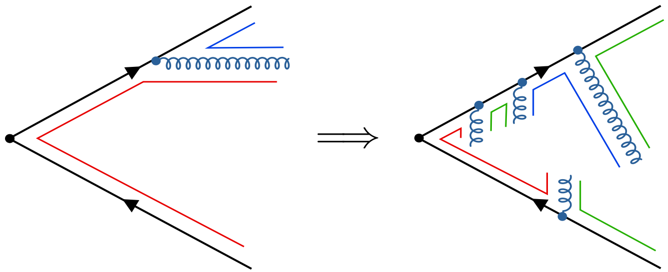
→ Disruption of Interference

Mehtar-Tani, Tywoniuk :: JHEP 01 (2013) 031
Casalderrey-Solana, Iancu :: JHEP 08 (2011) 015
Mehtar-Tani, Salgado, Tywoniuk :: JHEP 10 (2012) 197

Anti-Angular Ordering

* $\mathcal{M}$ and $\mathcal{M}^\dagger$ overlayed





The interference term:

$$\mathcal{J}(\boldsymbol{\kappa}, \bar{\boldsymbol{\kappa}}) = \mathrm{Re} \int_0^\infty \mathrm{d}t \int_t^\infty \mathrm{d}\bar{t} \int_{\boldsymbol{q}_1 \boldsymbol{q}_2} e^{it\left(\frac{\omega}{2}\boldsymbol{\delta n}^2 - \boldsymbol{q}_1 \cdot \boldsymbol{\delta n}\right)} \boldsymbol{q}_2 \cdot (\boldsymbol{q}_1 - \omega \boldsymbol{\delta n})$$

$$\mathcal{P}(\boldsymbol{\kappa} - \boldsymbol{q}_2; \infty, \bar{t}) \mathcal{K}(\boldsymbol{q}_2, \bar{t}; \boldsymbol{q}_1, t)[1 - \Delta_{\rm med}(t)] + (\mathrm{sym.})$$

Gluon Broadening | **Gluon Decoherence** | **Antenna Decoherence**

'Survival probability': $1 - \Delta_{\rm med}(t, 0) = \exp\left\{ - \int_0^t \mathrm{d}\xi\, n(\xi) \frac{\sigma(\boldsymbol{\delta n}\, \xi)}{2} \right\}$

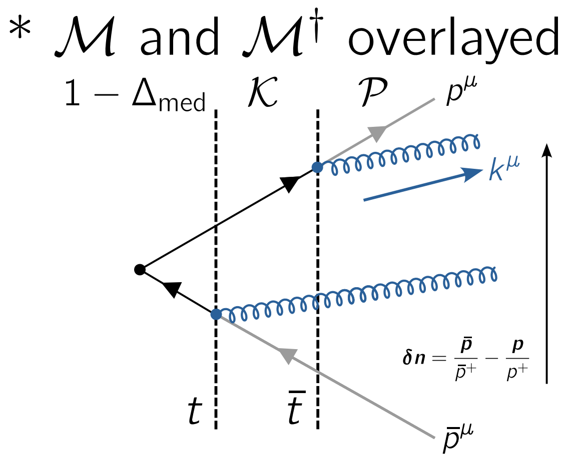
Destruction of colour structure

→ Disruption of Interference

Colour Decoherence breaks Angular Ordering

Mehtar-Tani, Tywoniuk :: JHEP 01 (2013) 031
Casalderrey-Solana, Iancu :: JHEP 08 (2011) 015
Mehtar-Tani, Salgado, Tywoniuk :: JHEP 10 (2012) 197

History of Antennas



The interference term:

$$\mathcal{J}(\boldsymbol{\kappa}, \bar{\boldsymbol{\kappa}}) = \text{Re} \int_0^\infty \mathrm{d}t \int_t^\infty \mathrm{d}\bar{t} \int_{\boldsymbol{q}_1 \boldsymbol{q}_2} e^{it\left(\frac{\omega}{2}\delta\boldsymbol{n}^2 - \boldsymbol{q}_1 \cdot \delta\boldsymbol{n}\right)} \boldsymbol{q}_2 \cdot (\boldsymbol{q}_1 - \omega\delta\boldsymbol{n})$$
$$\mathcal{P}(\boldsymbol{\kappa} - \boldsymbol{q}_2; \infty, \bar{t})\mathcal{K}(\boldsymbol{q}_2, \bar{t}; \boldsymbol{q}_1, t)[1 - \Delta_{\text{med}}(t)] + (\text{sym.})$$

Gluon Broadening — **Gluon Decoherence** — **Antenna Decoherence**

The story so far:

Standard Assumptions:

- Quark/Antiquark with no transverse diffusion
- Instantaneous antenna creation *
- Isotropic/Homogeneous medium **

People lifting these assumptions:

* Pablo Guerrero Rodríguez (Monday, 23 Mar, 14h30)
** Florian Lindenbauer (Tuesday, 24 Mar, 14h30)
** Andrey Sadofyev (Monday, 30 Mar, 09h00)

Some recent work:

Mapping collinear in-medium splittings
(Finite antenna formation time)
Dominguez, Milhano, Salgado, et al. :: EPJC 80 (2020) 1 11

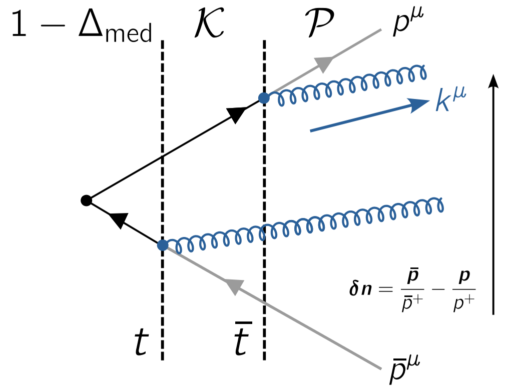
A generalized picture of colour decoherence in dense QCD media
(Decoherence as a function of antenna splitting)
Abreu, Mayo Lopéz, Milhano, Soto-Ontoso :: JHEP 03 (2025) 216

QCD antenna radiative spectrum in dense media within the Improved Opacity Expansion
Kuzmin, Silva :: JHEP12 (2025) 022

Our Goal

Mehtar-Tani, Tywoniuk :: JHEP 01 (2013) 031
Casalderrey-Solana, Iancu :: JHEP 08 (2011) 015
Mehtar-Tani, Salgado, Tywoniuk :: JHEP 10 (2012) 197

* $\mathcal{M}$ and $\mathcal{M}^\dagger$ overlayed

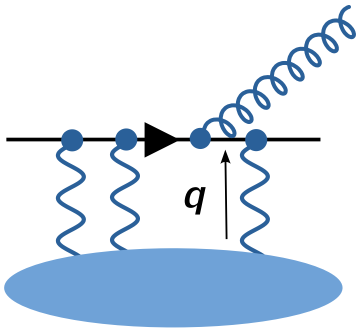


The interference term:

$$\mathcal{J}(\boldsymbol{\kappa}, \bar{\boldsymbol{\kappa}}) = \text{Re} \int_0^\infty \mathrm{d}t \int_t^\infty \mathrm{d}\bar{t} \int_{\boldsymbol{q}_1 \boldsymbol{q}_2} e^{it\left(\frac{\omega}{2}\delta \boldsymbol{n}^2 - \boldsymbol{q}_1 \cdot \delta \boldsymbol{n}\right)} \boldsymbol{q}_2 \cdot (\boldsymbol{q}_1 - \omega \delta \boldsymbol{n})$$
$$\mathcal{P}(\boldsymbol{\kappa} - \boldsymbol{q}_2; \infty, \bar{t}) \mathcal{K}(\boldsymbol{q}_2, \bar{t}; \boldsymbol{q}_1, t)[1 - \Delta_{\text{med}}(t)] + (\text{sym.})$$

Gluon Broadening | **Gluon Decoherence** | **Antenna Decoherence**

The story so far:



$$n(s)\sigma(\boldsymbol{r}) \to \frac{\hat{q}(s)}{2}\boldsymbol{r}^2$$

$$\sigma(\boldsymbol{r}) = \int_{\boldsymbol{\ell}} \left(1 - e^{i\boldsymbol{r}\cdot\boldsymbol{\ell}}\right) V(\boldsymbol{\ell})$$

Usual approach: Harmonic Oscillator Approximation

$$V^{\text{Yukawa}}(\boldsymbol{q}) = \frac{8\pi\mu^2}{(\boldsymbol{q}^2 + \mu^2)^2} \implies n\,\sigma(\boldsymbol{r}) \simeq \frac{\hat{q}}{2}\boldsymbol{r}^2$$

$$(\text{if } \boldsymbol{q}^2 \lesssim \mu^2)$$

This misses the high-momentum contribution ('hard perturbative tail')

In this talk: Realistic scattering rates

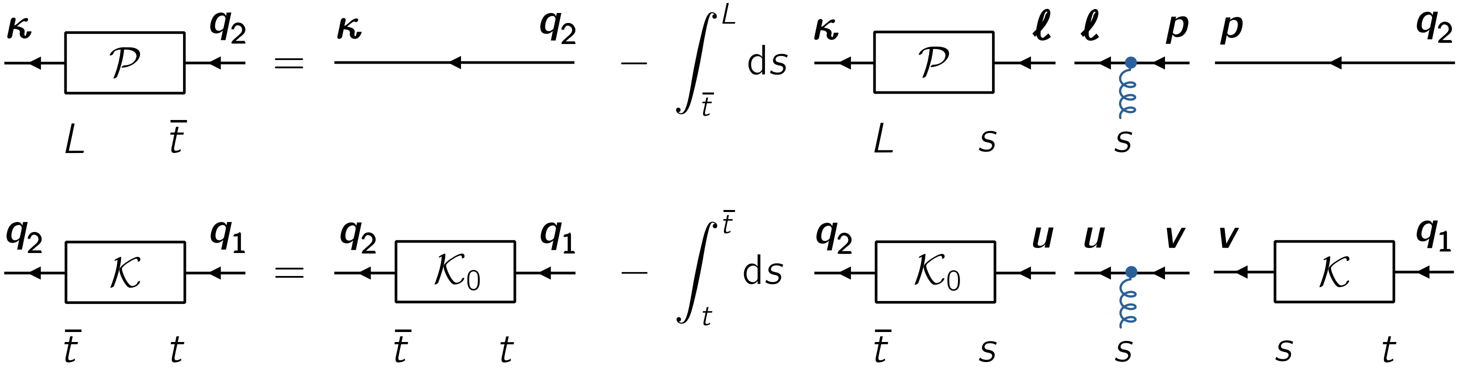
Caron-Huot, Gale :: Phys.Rev.C 82 (2010) 064902
Andres, Apolinário, Dominguez :: JHEP 07 (2020) 114
Schlichting, Soudi :: Phys.Rev.D 105 (2022) 7, 076002

Brief History of Rate Equations

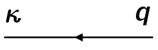
Previous idea: Consider both Emission Kernel and Broadening as Propagators

Vacuum-like contribution

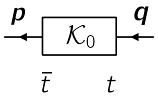
Medium-modified contribution



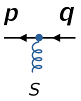
***where:**



$= (2\pi)^2\delta^{(2)}(\boldsymbol{p}-\boldsymbol{q})$



$= (2\pi)^2\delta^{(2)}(\boldsymbol{p}-\boldsymbol{q})\, e^{-i(\bar{t}-t)\frac{\boldsymbol{p}^2}{2\omega}}$



$= n(s)\,\frac{\sigma(\boldsymbol{p}-\boldsymbol{q})}{2}$

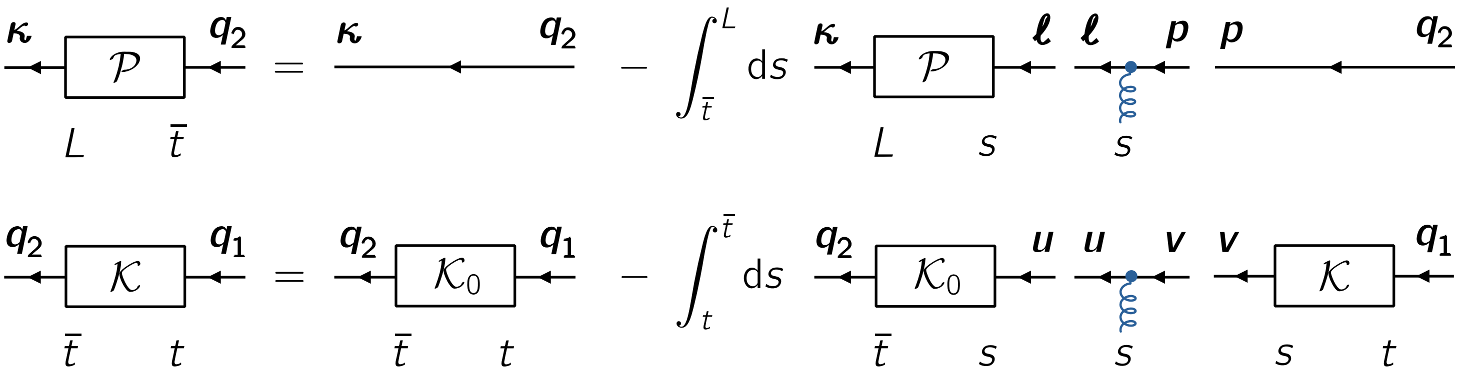
Brief History of Rate Equations

Caron-Huot, Gale :: Phys.Rev.C 82 (2010) 064902
Andres, Apolinário, Dominguez :: JHEP 07 (2020) 114
Schlichting, Soudi :: Phys.Rev.D 105 (2022) 7, 076002

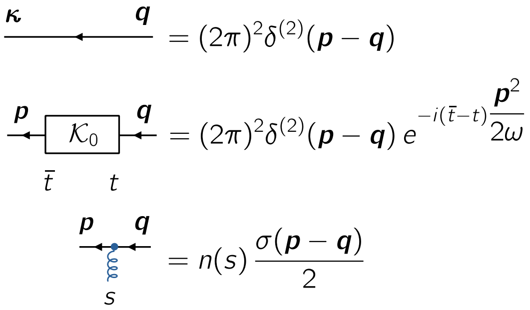
Previous idea: Consider both Emission Kernel and Broadening as Propagators

Vacuum-like contribution

Medium-modified contribution



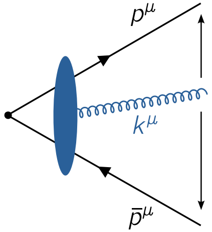
***where:**



Gluon spectrum framed as solution to differential equations

***Successfully implemented for single quark emission**

Interference from Propagators



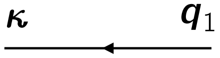
$$\boldsymbol{\kappa} = \boldsymbol{k} - \frac{\omega}{p^+}\boldsymbol{p}$$

$$\delta \boldsymbol{n} = \boldsymbol{\kappa} - \bar{\boldsymbol{\kappa}}$$

$$\bar{\boldsymbol{\kappa}} = \boldsymbol{k} - \frac{\omega}{\bar{p}^+}\bar{\boldsymbol{p}}$$

<u>Decoherence Factor must be handled</u>

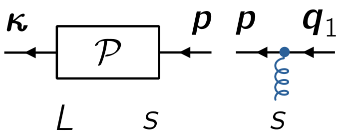
$$\mathcal{J}(\boldsymbol{\kappa}, \bar{\boldsymbol{\kappa}}) = \mathrm{Re} \int_0^\infty \mathrm{d}t \int_{\boldsymbol{q}_1} [1 - \Delta_{\mathrm{med}}(t)]\, e^{+it\frac{\bar{q}_1^2 - q_1^2}{2\omega}} \frac{2\omega}{i} \Bigg\{ \frac{\boldsymbol{\kappa}\cdot\bar{\boldsymbol{\kappa}}}{\boldsymbol{\kappa}^2}$$



(Close to) Vacuum-Like Contribution

Broadening: Uniquely for the J term

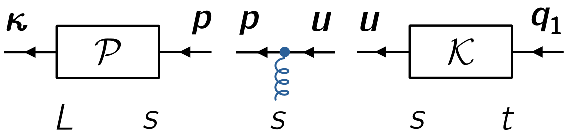
$$+ \int_t^L \mathrm{d}s$$



$$\frac{\boldsymbol{q}_1\cdot\bar{\boldsymbol{q}}_1}{\boldsymbol{q}_1^2}$$

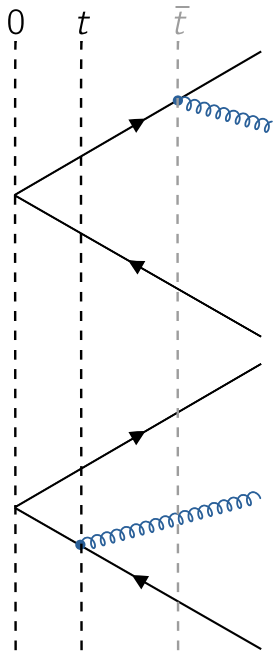
Kernel + Broadening: Remains for the R(κ) term

$$+ \int_t^L \mathrm{d}s$$

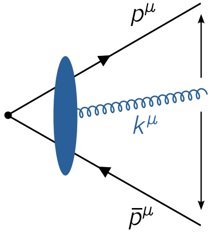


$$\left(\frac{\boldsymbol{u}}{\boldsymbol{u}^2} - \frac{\boldsymbol{p}}{\boldsymbol{p}^2}\right)\cdot\bar{\boldsymbol{q}}_1 \Bigg\}$$

$$+ \,(\mathrm{sym.})$$



Interference from Propagators



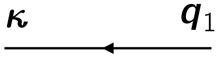
$\boldsymbol{\kappa} = \boldsymbol{k} - \frac{\omega}{p^+}\boldsymbol{p}$

$\delta \boldsymbol{n} = \boldsymbol{\kappa} - \bar{\boldsymbol{\kappa}}$

$\bar{\boldsymbol{\kappa}} = \boldsymbol{k} - \frac{\omega}{\bar{p}^+}\bar{\boldsymbol{p}}$

Decoherence Factor must be handled

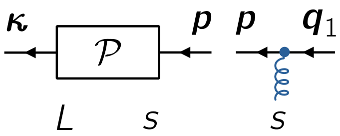
$$\mathcal{J}(\boldsymbol{\kappa},\bar{\boldsymbol{\kappa}}) = \mathrm{Re}\int_0^\infty \mathrm{d}t \int_{\boldsymbol{q}_1} [1-\Delta_{\mathrm{med}}(t)]\, e^{+it\frac{\bar{q}_1^2 - q_1^2}{2\omega}} \frac{2\omega}{i} \Bigg\{ \frac{\boldsymbol{\kappa}\cdot\bar{\boldsymbol{\kappa}}}{\boldsymbol{\kappa}^2}$$



(Close to) Vacuum-Like Contribution

Broadening: Uniquely for the J term

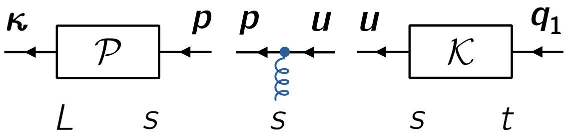
$$+ \int_t^L \mathrm{d}s$$



$$\frac{\boldsymbol{q}_1\cdot\bar{\boldsymbol{q}}_1}{\boldsymbol{q}_1^2}$$

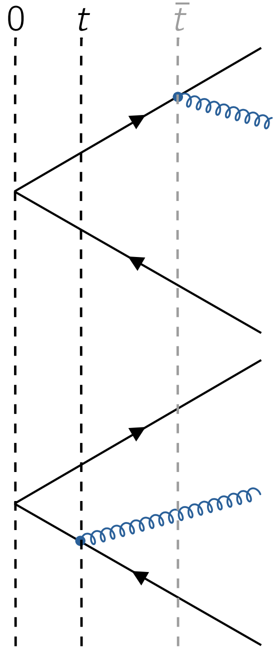
Kernel + Broadening: Remains for the R(κ) term

$$+ \int_t^L \mathrm{d}s$$



$$\left(\frac{\boldsymbol{u}}{\boldsymbol{u}^2} - \frac{\boldsymbol{p}}{\boldsymbol{p}^2}\right)\cdot\bar{\boldsymbol{q}}_1 \Bigg\}$$

$$+ (\mathrm{sym.})$$



Solve differential equations for Broadening and Kernel

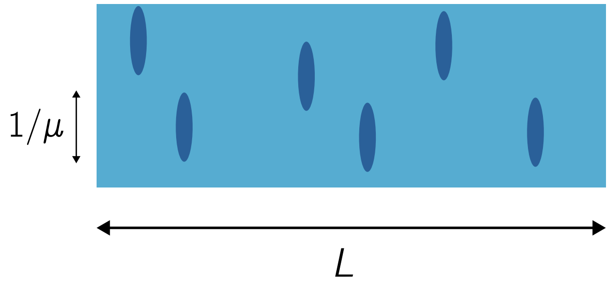
→ Build the gluon emission spectrum

Some (preliminary) results

Salgado, Wiedemann :: Phys.Rev.Lett. 89 (2002) 092303

Assumptions on the medium

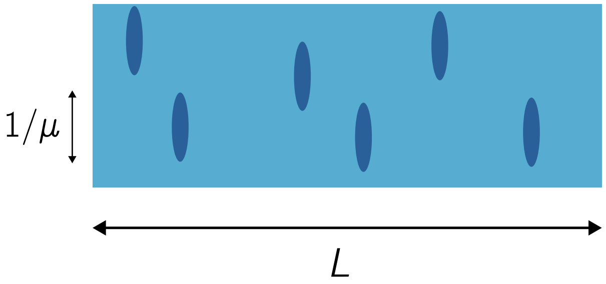
Simplified 'brick' model: $n(t) = n_0 \Theta(L - t)$



* Density of scattering centres
** Can be relaxed by *scaling laws*

Salgado, Wiedemann :: Phys.Rev.Lett. 89 (2002) 092303

Assumptions on the medium



Simplified 'brick' model: $n(t) = n_0 \Theta(L - t)$

* Density of scattering centres
** Can be relaxed by *scaling laws*

Characteristic Gluon Energy

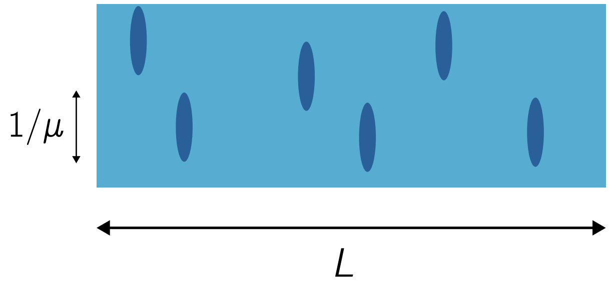
Medium Parameters: $(n_0, L, \mu) \rightarrow \left(n_0 L\, , \bar{\omega}_c = \mu^2 L/2\, , \bar{\theta}_c = 2/\mu L\right)$

Longitudinal density

Characteristic Angle

Salgado, Wiedemann :: Phys.Rev.Lett. 89 (2002) 092303

Assumptions on the medium



Simplified 'brick' model: $n(t) = n_0 \Theta(L - t)$

* Density of scattering centres
** Can be relaxed by *scaling laws*

Medium Parameters:

$$(n_0, L, \mu) \to \left(n_0 L\,, \bar{\omega}_c = \mu^2 L/2\,, \bar{\theta}_c = 2/\mu L\right)$$

Characteristic Gluon Energy

Longitudinal density

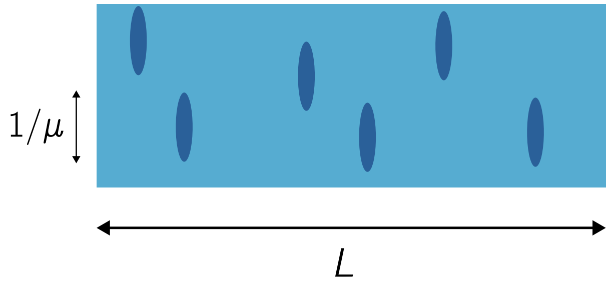
Characteristic Angle

Antenna Opening:

$$\delta n \to R_c = \delta n\, \mu L/2 = \delta n/\bar{\theta}_c$$

Salgado, Wiedemann :: Phys.Rev.Lett. 89 (2002) 092303

Assumptions on the medium



Simplified 'brick' model: $n(t) = n_0 \Theta(L - t)$

* Density of scattering centres
** Can be relaxed by *scaling laws*

Characteristic Gluon Energy

Medium Parameters: $(n_0, L, \mu) \to (n_0 L\,, \bar{\omega}_c = \mu^2 L/2\,, \bar{\theta}_c = 2/\mu L)$

Longitudinal density

Characteristic Angle

Antenna Opening: $\delta n \to R_c = \delta n\, \mu L/2 = \delta n/\bar{\theta}_c$

These set the typical scales for the gluon spectrum

The Decoherence Parameter

Antenna decoherence: Relevant before emission starts

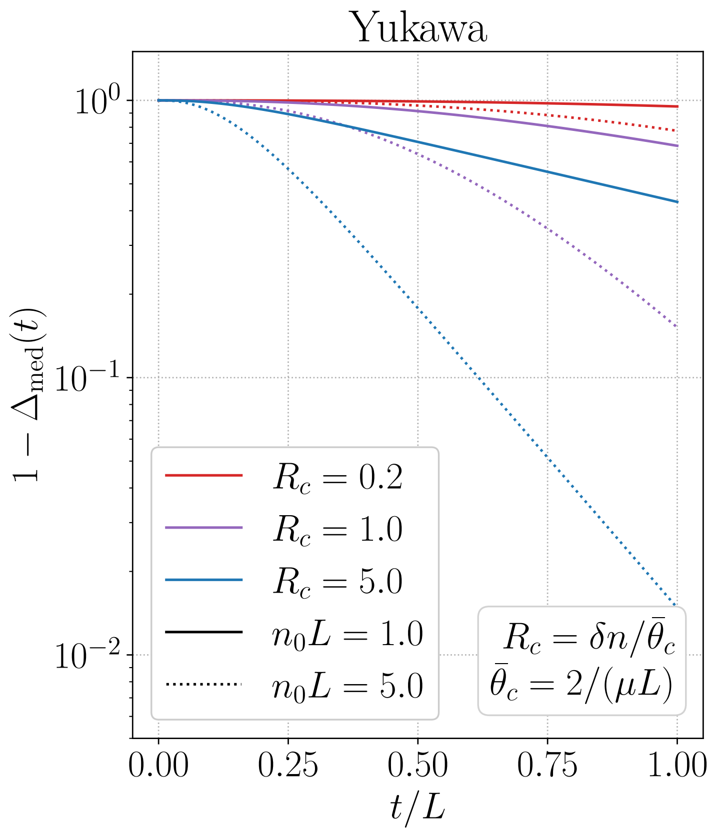
'Survival probability'

$$1 - \Delta_{\text{med}}(t, 0) = \exp\left\{ - \int_0^t \mathrm{d}\xi \, n(\xi) \, \frac{\sigma(\boldsymbol{\delta n} \, \xi)}{2} \right\}$$

Yukawa Rate:

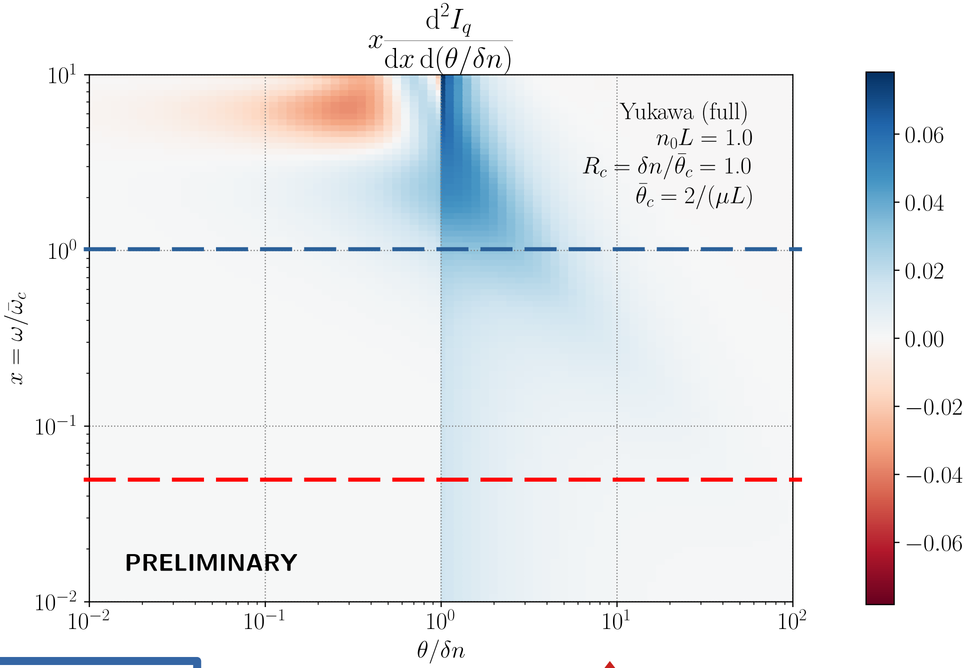
$$\sigma(\boldsymbol{r}) = \int_{\boldsymbol{\ell}} \left(1 - e^{i\boldsymbol{r}\cdot\boldsymbol{\ell}}\right) V(\boldsymbol{\ell})$$

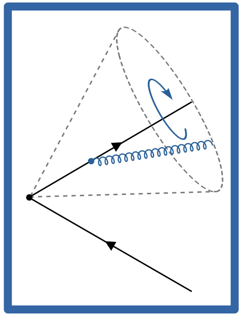
$$V^{\text{Yukawa}}(\boldsymbol{q}) = \frac{8\pi\mu^2}{(\boldsymbol{q}^2 + \mu^2)^2}$$

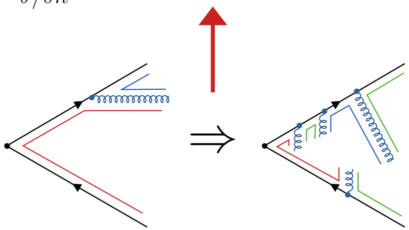


Qualitative behaviour:
Decoherence increases with antenna size and medium density

Covering the Phase Space

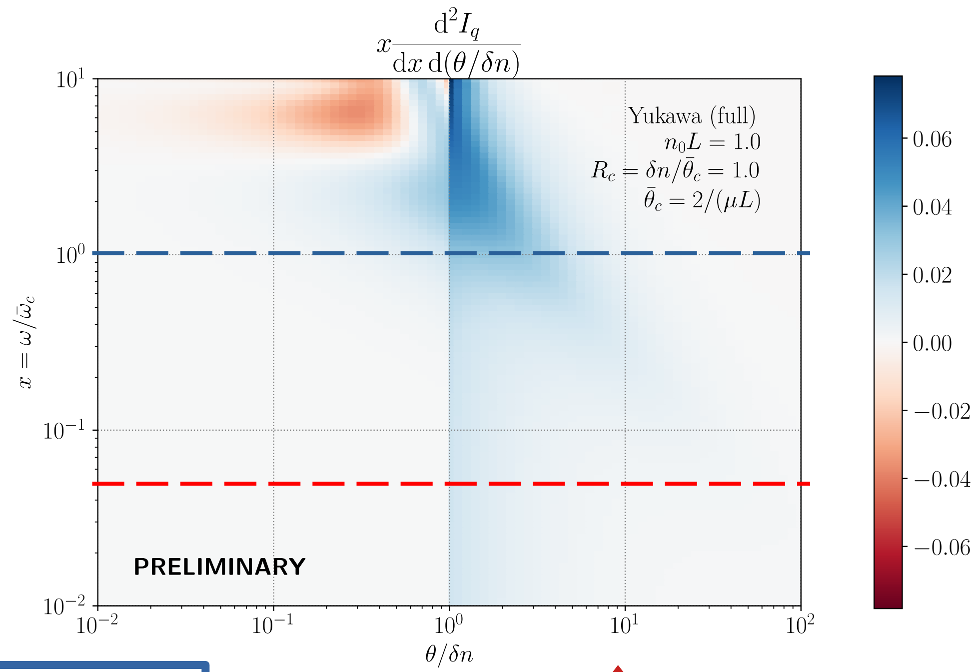


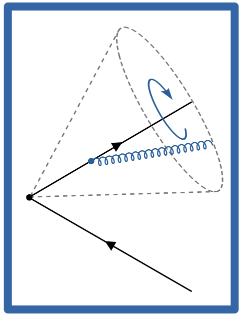


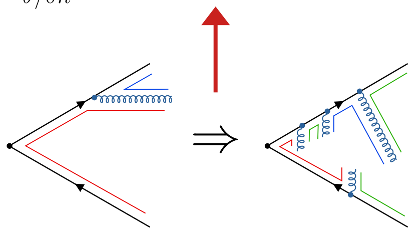


<u>Coherence break down</u>

Covering the Phase Space

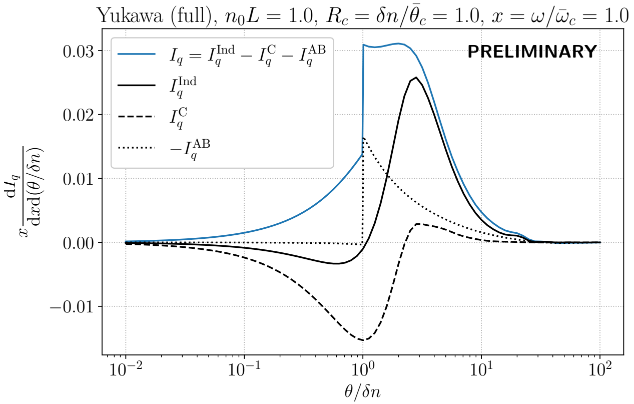


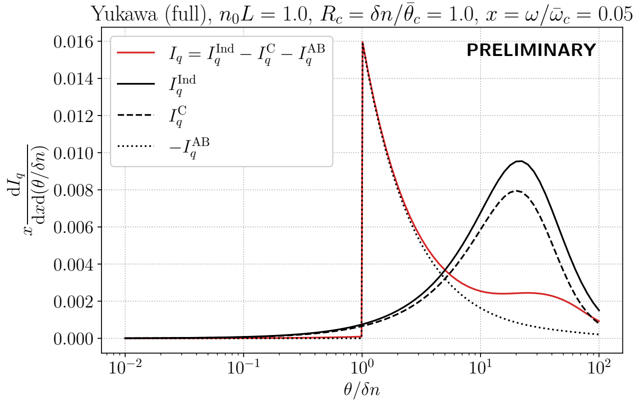




Coherence break down

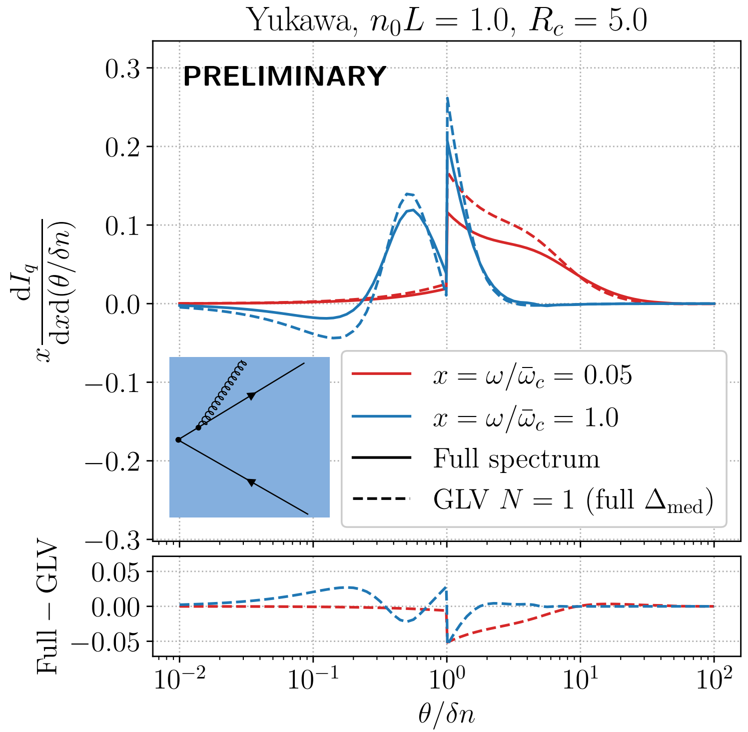
Slice the phase-space

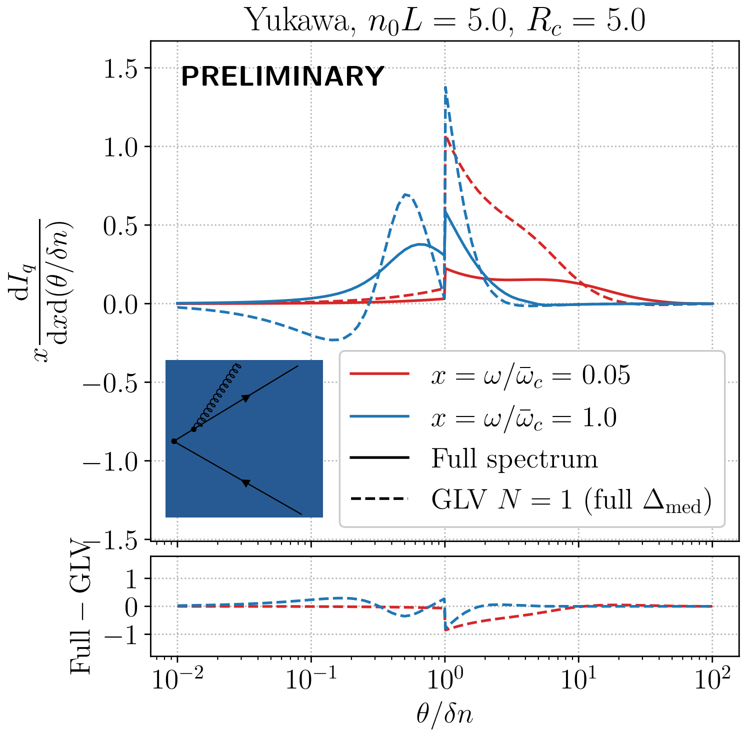




Note: BDMPS ($\kappa^2 \sim \mu^2$) and anti-angular ordered contributions

Yukawa Scattering: GLV vs Full Spectrum

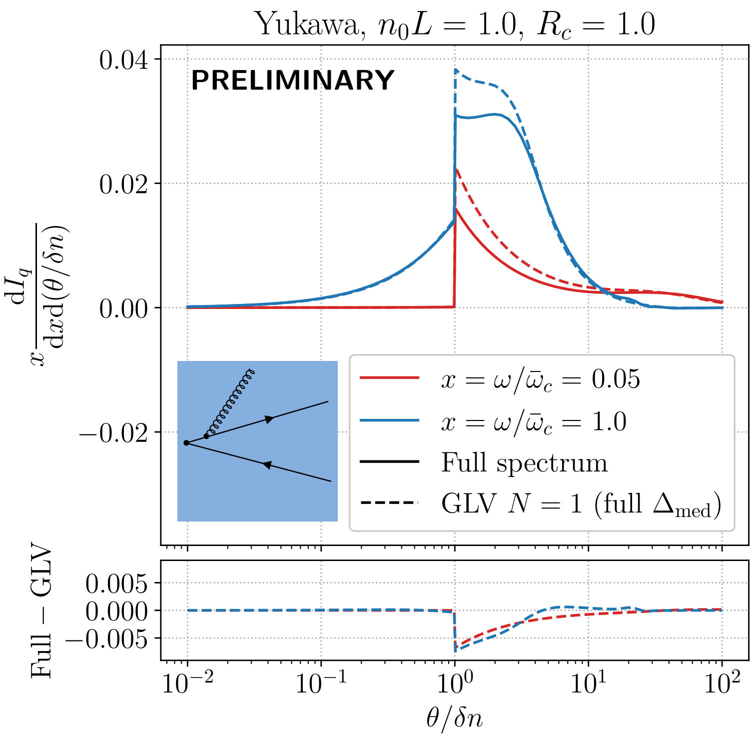


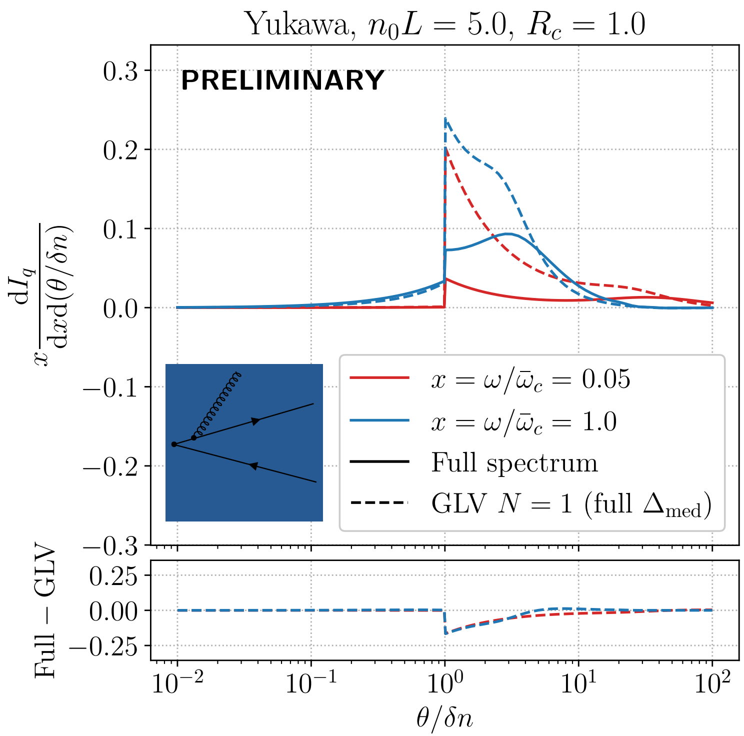


Radiation exclusively out-of-cone for soft gluons

Disgreement worsens for denser media

Yukawa Scattering: GLV vs Full Spectrum

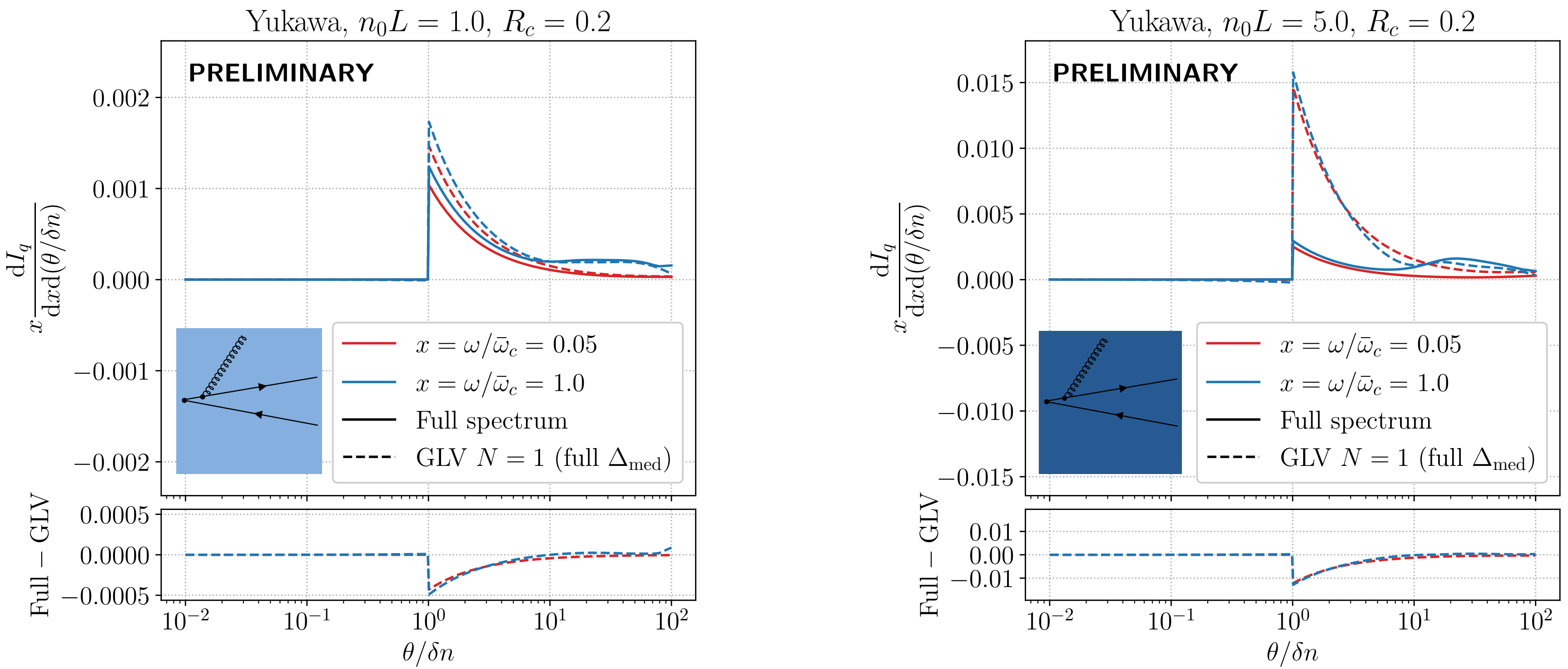




Radiation exclusively out-of-cone for soft gluons

Disgreement worsens for denser media

Yukawa Scattering: GLV vs Full Spectrum



Radiation exclusively out-of-cone for soft gluons

Disgreement worsens for denser media

Barata, Mehtar-Tani :: JHEP 10 (2020) 176

Varying the scattering rate

Our aim: Check dependence of the spectrum on modeling choices

Compare 'Yukawa' and 'Hard Thermal Loop' (*) scattering rates:

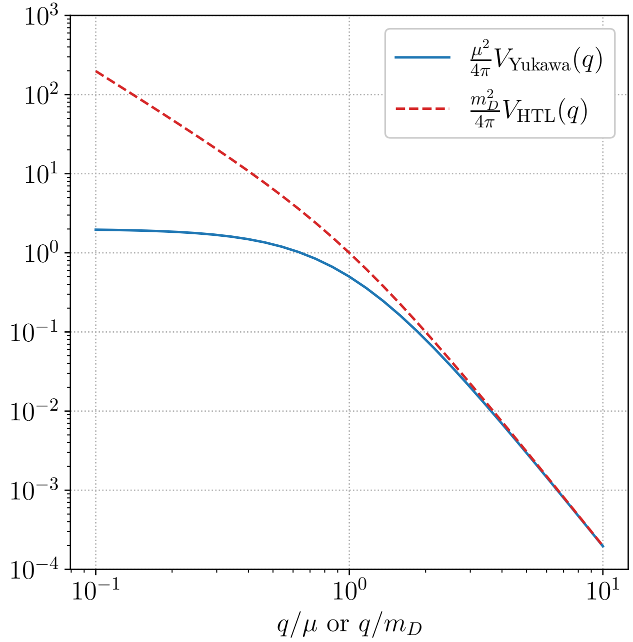
* Glossing over plenty of HTL subtleties

$$n(t)V_{\text{Yukawa}}(\boldsymbol{q}) \longleftrightarrow \alpha_s N_c T(t) V_{\text{HTL}}(\boldsymbol{q})$$

$$V_{\text{Yukawa}}(q) = \frac{8\pi\mu^2}{(\boldsymbol{q}^2+\mu^2)^2}$$

$$V_{\text{HTL}}(q) = \frac{8\pi m_D^2}{(\boldsymbol{q}^2+m_D^2)\boldsymbol{q}^2}$$

For a fair comparison: $\begin{cases} \alpha_s N_c T = n_0/e \\ m_D^2 = \mu^2 e \end{cases}$



Same 'Coulomb tail' at large 'q', but different infra-red behaviour

Decoherence for different rates

Antenna decoherence: Relevant before emission starts

'Survival probability'

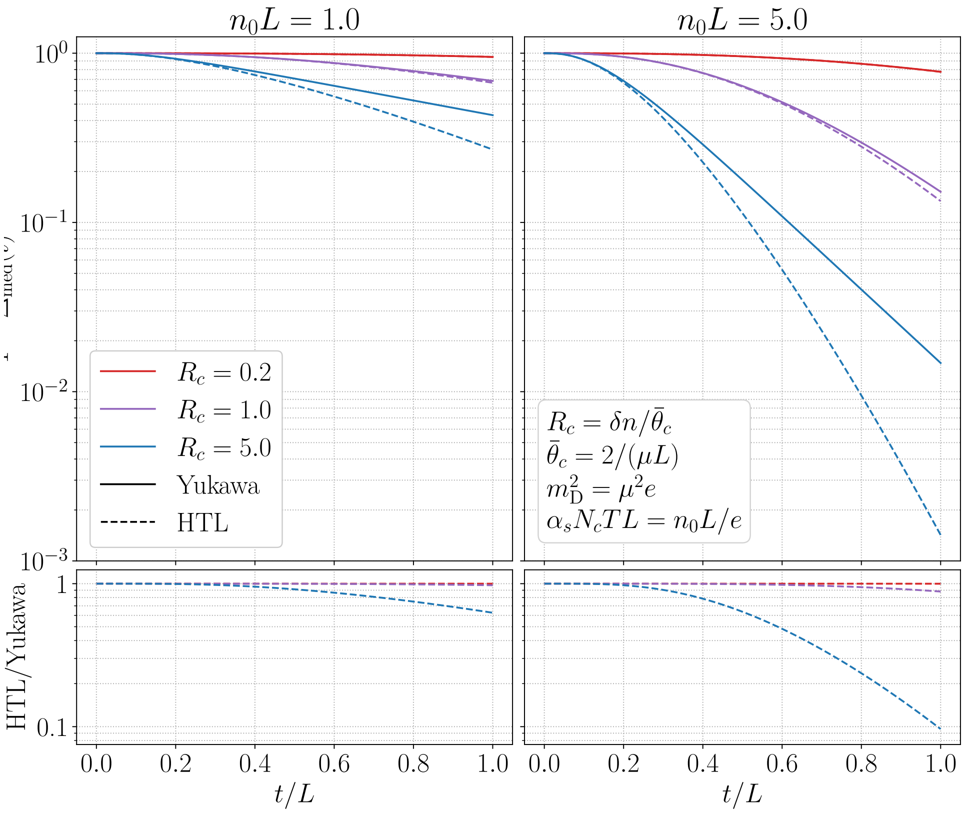
$$1 - \Delta_{\mathrm{med}}(t,0) = \exp\left\{ -\int_0^t \mathrm{d}\xi\, n(\xi)\, \frac{\sigma(\boldsymbol{\delta n}\,\xi)}{2} \right\}$$

$$\sigma(\boldsymbol{r}) = \int_{\boldsymbol{\ell}} \left(1 - e^{i\boldsymbol{r}\cdot\boldsymbol{\ell}}\right) V(\boldsymbol{\ell})$$

$$V_{\mathrm{Yukawa}}(q) = \frac{8\pi\mu^2}{(\boldsymbol{q}^2+\mu^2)^2}$$

$$V_{\mathrm{HTL}}(q) = \frac{8\pi m_D^2}{(\boldsymbol{q}^2+m_D^2)\boldsymbol{q}^2}$$

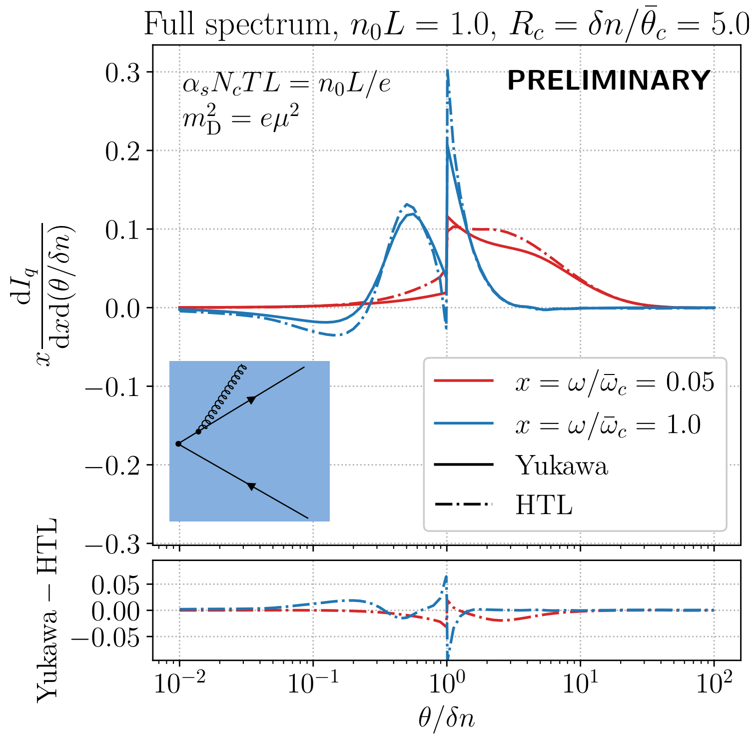
For a fair comparison: $\begin{cases} \alpha_s N_c T = n_0/e \\ m_D^2 = \mu^2 e \end{cases}$

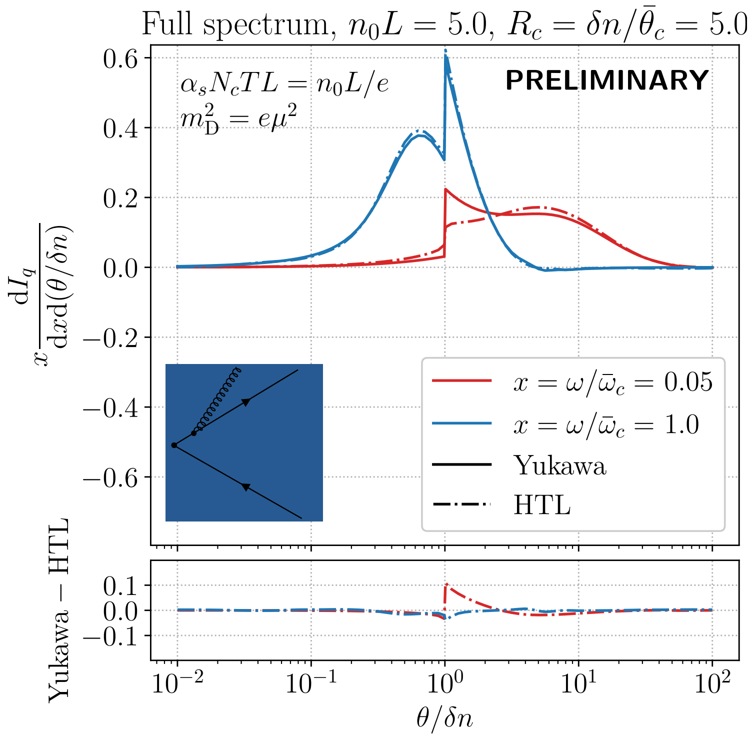


Same qualitative behaviour:

Decoherence increases with antenna size and medium density

Full Spectrum: Yukawa vs HTL

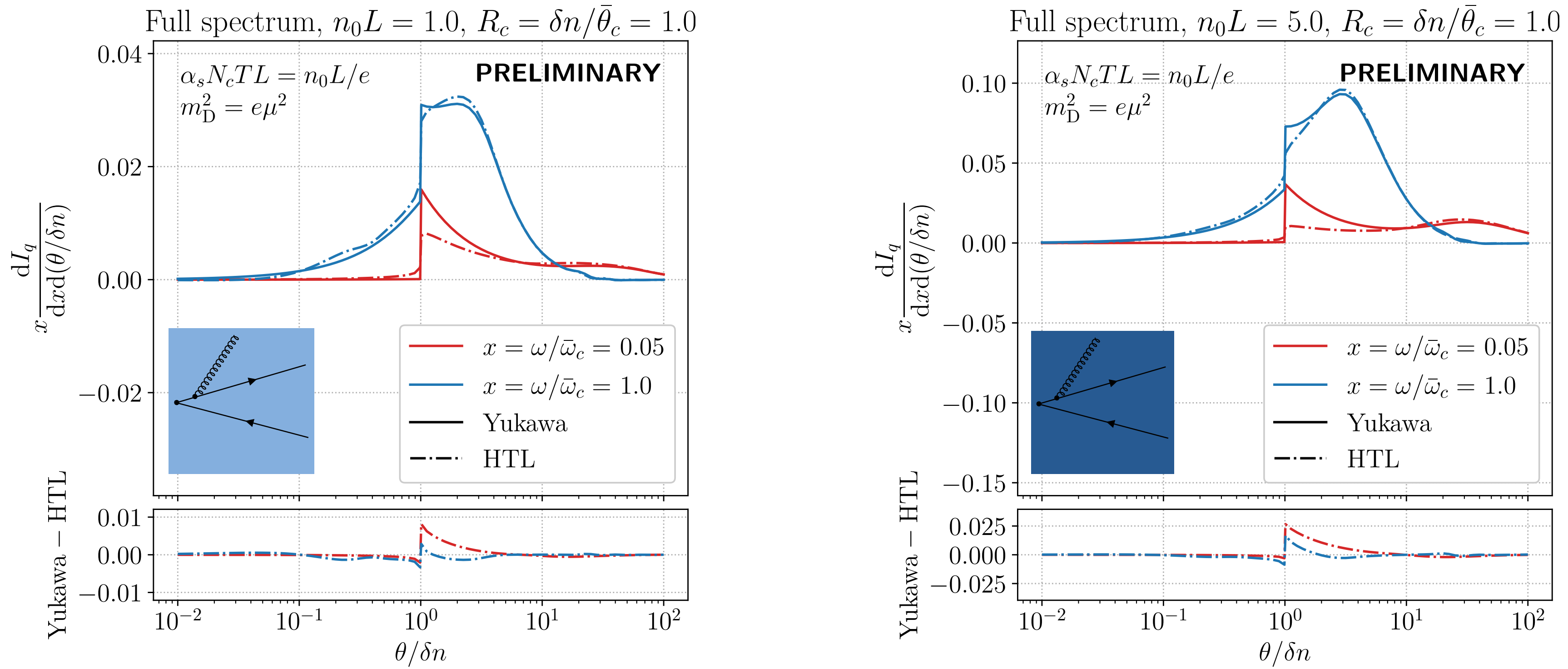




Smaller differences than between ‘Full’ and ‘GLV’ spectrum

Agreement improves for denser media

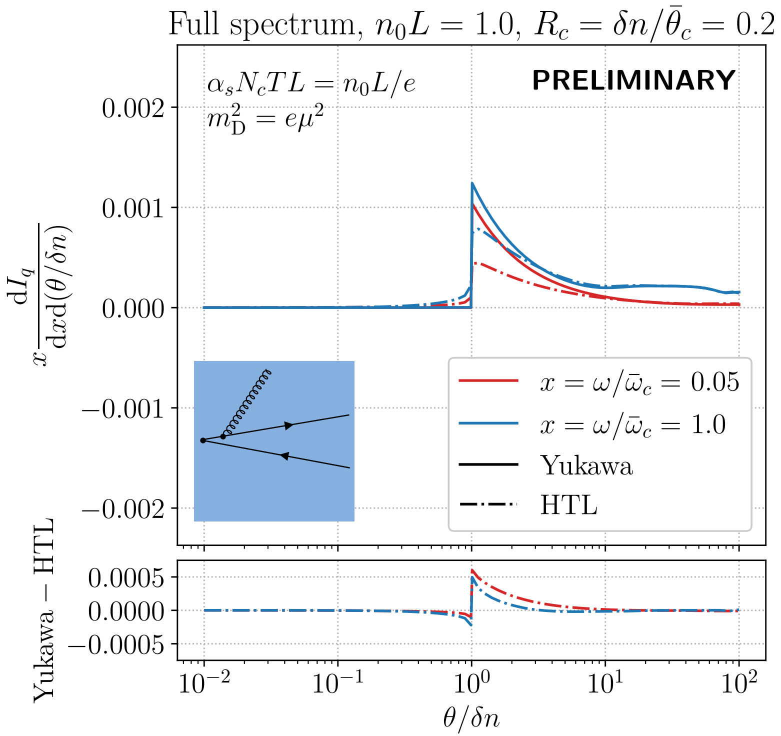
Full Spectrum: Yukawa vs HTL

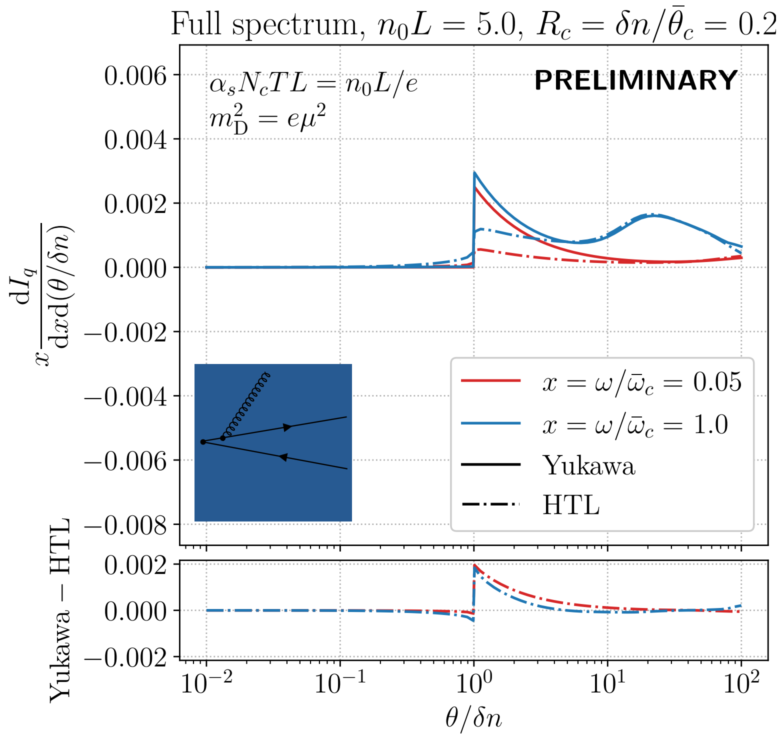


Smaller differences than between 'Full' and 'GLV' spectrum

Agreement improves for denser media

Full Spectrum: Yukawa vs HTL



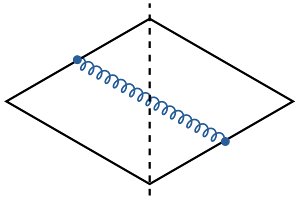


Smaller differences than between 'Full' and 'GLV' spectrum

Agreement improves for denser media

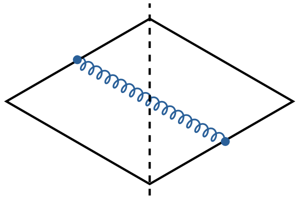
Summary

Summary



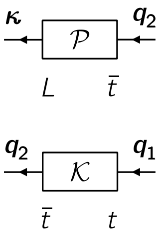
The radiation pattern of a quark-antiquark antenna is contained in the interference term

Summary

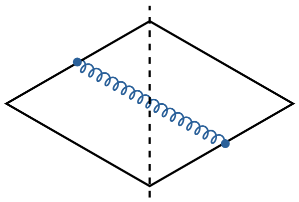


The radiation pattern of a quark-antiquark antenna is contained in the interference term

The harmonic oscillator approximation can be lifted with recourse to propagator (implicit) equations

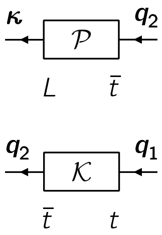


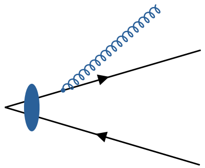
Summary

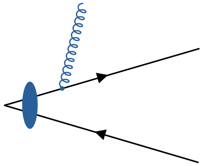


The radiation pattern of a quark-antiquark antenna is contained in the interference term

The harmonic oscillator approximation can be lifted with recourse to propagator (implicit) equations

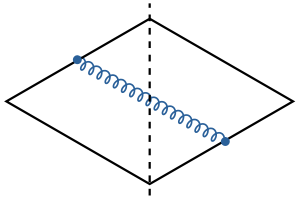






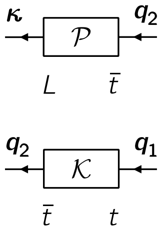
Quantifies the breakdown of colour coherence and the model dependence of antenna spectrum calculations

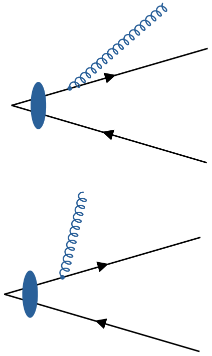
Summary



The radiation pattern of a quark-antiquark antenna is contained in the interference term

The harmonic oscillator approximation can be lifted with recourse to propagator (implicit) equations





Quantifies the breakdown of colour coherence and the model dependence of antenna spectrum calculations

A step towards precise computation of jet substructure

Acknowledgements

erc
European Research Council
Established by the European Commission

fct
Fundação
para a Ciência
e a Tecnologia

REPÚBLICA
PORTUGUESA

TÉCNICO
LISBOA

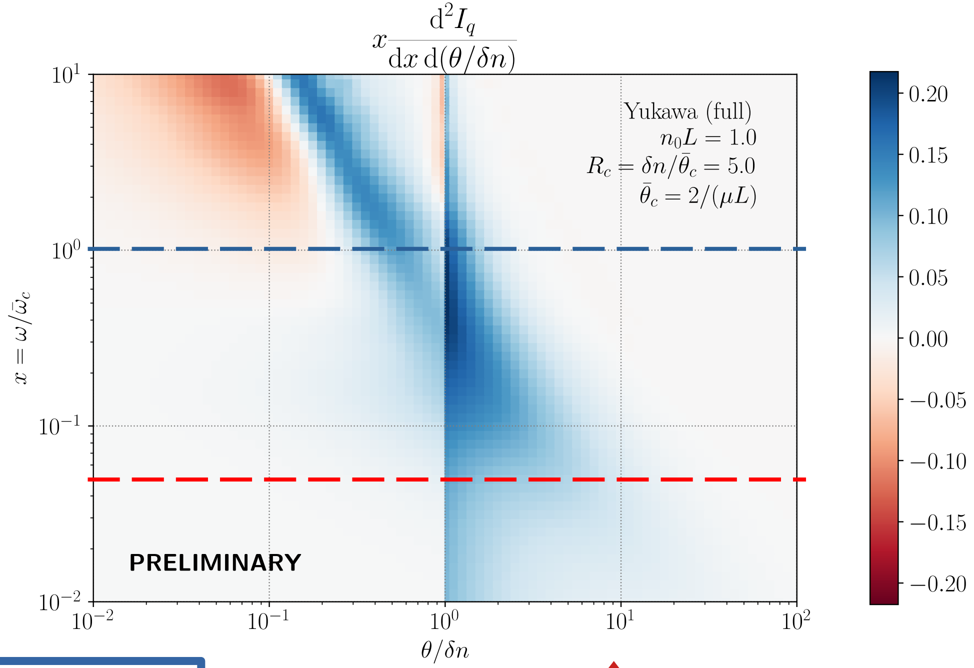
LIP

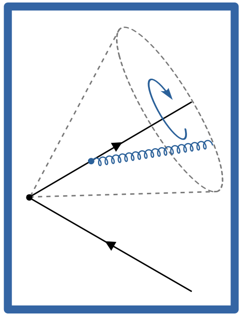
IGFAE
INSTITUTO GALEGO
DE FÍSICA
DE ALTAS ENERXÍAS

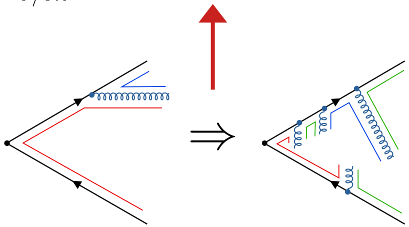
25
1999
2024

Backups

Covering the Phase Space

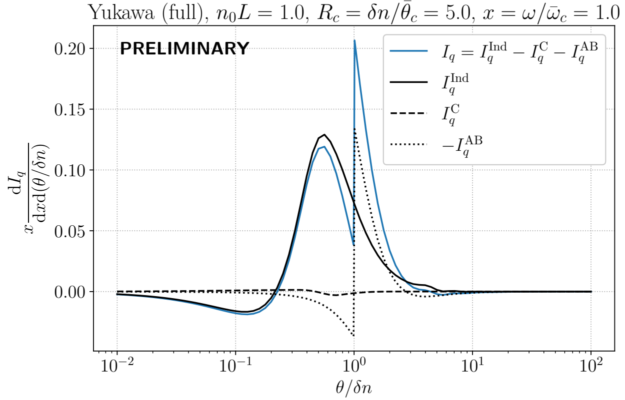


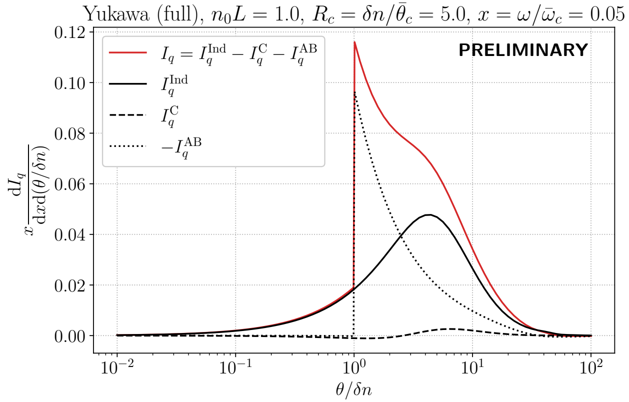




Coherence break down

Slice the phase-space

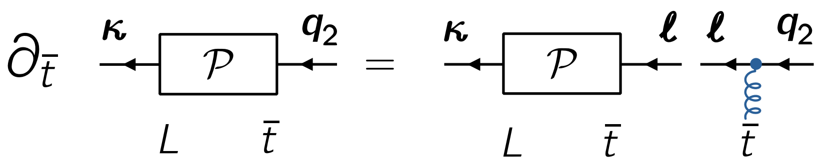


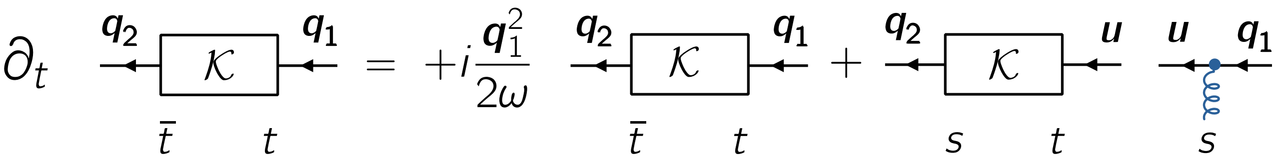


Note: BDMPS ($\kappa^2 \sim \mu^2$) and anti-angular ordered contributions

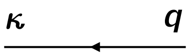
Propagator Equations: Details

Propagator equations in differential form

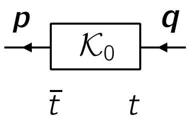




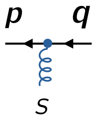
Some definitions:



$= (2\pi)^2\delta^{(2)}(\boldsymbol{p}-\boldsymbol{q})$

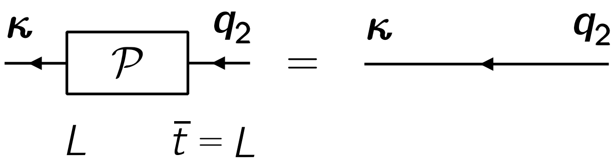


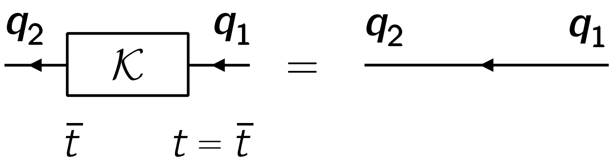
$= (2\pi)^2\delta^{(2)}(\boldsymbol{p}-\boldsymbol{q})\, e^{-i(\bar{t}-t)\frac{\boldsymbol{p}^2}{2\omega}}$



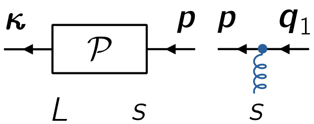
$= n(s)\,\frac{\sigma(\boldsymbol{p}-\boldsymbol{q})}{2}$

Some definitions:



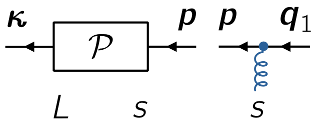


The Interference Term – With Propagators !

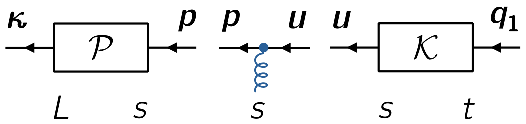
$$\mathcal{J}(\boldsymbol{\kappa},\bar{\boldsymbol{\kappa}}) - \mathcal{J}^{\text{vac}}(\boldsymbol{\kappa},\bar{\boldsymbol{\kappa}}) = +4\omega^2 \int_t^L \mathrm{d}s$$  $$\frac{\boldsymbol{q}_1\cdot\bar{\boldsymbol{q}}_1}{\boldsymbol{q}_1^2}\,\frac{1}{\bar{\boldsymbol{q}}_1^2-\boldsymbol{q}_1^2}$$

Broadening Only (I)

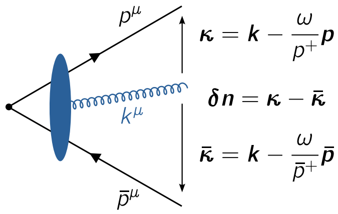
Broadening Only (II)

$$-4\omega^2\ \mathrm{Re}\int_0^\infty \mathrm{d}s\int_{\boldsymbol{q}_1}\left[1-\Delta_{\text{med}}(s)\right]e^{+is\frac{\bar{\boldsymbol{q}}_1^2-\boldsymbol{q}_1^2}{2\omega}}$$  $$\left\{\frac{\boldsymbol{q}_1\cdot\bar{\boldsymbol{q}}_1}{\boldsymbol{q}_1^2}\,\frac{1}{\bar{\boldsymbol{q}}_1^2-\boldsymbol{q}_1^2} - \frac{\boldsymbol{p}\cdot\bar{\boldsymbol{p}}}{\boldsymbol{p}^2}\,\frac{1}{\bar{\boldsymbol{p}}^2-\boldsymbol{p}^2}\right\}$$

Broadening + Kernel

$$+\,\mathrm{Re}\int_0^L \mathrm{d}s\int_0^s \mathrm{d}t\int_{\boldsymbol{q}_1}\left[1-\Delta_{\text{med}}(t)\right]e^{+it\frac{\bar{\boldsymbol{q}}_1^2-\boldsymbol{q}_1^2}{2\omega}}\,\frac{2\omega}{i}$$  $$\left(\frac{\boldsymbol{u}}{\boldsymbol{u}^2}-\frac{\boldsymbol{p}}{\boldsymbol{p}^2}\right)\cdot\bar{\boldsymbol{q}}_1$$

$$+\left(\boldsymbol{\kappa}\leftrightarrow\bar{\boldsymbol{\kappa}}\right)$$



<u>**** Azimuthal averaging is tricky!**</u>

Initial Value Problem (A+B)

$$-\omega \frac{I^{\mathrm{AB}}}{\mathrm{d}\omega \mathrm{d}^2(\boldsymbol{\theta}/\delta n)} = \frac{\alpha_s C_F}{(2\pi)^2} \int_0^L \mathrm{d}s\, n(s)\, \psi_{\mathrm{AB}}(\boldsymbol{\kappa} = \omega\boldsymbol{\theta}, \omega; \tau = L, s)\,,$$

Definitions

$$\psi_{\mathrm{AB}}(\boldsymbol{\kappa}, \omega; \tau, s) \stackrel{\mathrm{def}}{=} \int_{\boldsymbol{\ell}} \mathcal{P}(\boldsymbol{\kappa} - \boldsymbol{\ell}; \tau, s)\left[\Psi(\boldsymbol{\ell}, \omega; s = 0) - \mathcal{S}_{\mathrm{med}}(s)\Psi(\boldsymbol{\ell}, \omega; s)\right]\,,$$

$$\Psi(\boldsymbol{\ell}, \omega; s) \stackrel{\mathrm{def}}{=} \delta n^2 \int_{\boldsymbol{q}} \frac{V(\boldsymbol{\ell} - \boldsymbol{q})}{2}\left[\mathcal{J}^{\mathrm{vac}}(\boldsymbol{\ell}, \bar{\boldsymbol{\ell}}) - \mathcal{J}^{\mathrm{vac}}(\boldsymbol{q}, \bar{\boldsymbol{q}})\right] \mathrm{Re}\; e^{is\frac{\boldsymbol{q}^2 - \bar{\boldsymbol{q}}^2}{2\omega}}\,,$$

Initial Value Problems

$$\partial_\tau \psi_{\mathrm{AB}}(\boldsymbol{\kappa}; \tau, s) = -\frac{1}{2} n(\tau) \int_{\boldsymbol{y}} \sigma(\boldsymbol{\kappa} - \boldsymbol{y})\psi_{\mathrm{AB}}(\boldsymbol{y}; \tau, s)\,,$$

$$\psi_{\mathrm{AB}}(\boldsymbol{\kappa}; \tau = s, s) = \Psi(\boldsymbol{\kappa}; 0) - \Psi(\boldsymbol{\kappa}; s)\mathcal{S}_{\mathrm{med}}(s)$$

Note: Plotted spectra are the azimuthal average of this spectrum
→ Requires analytical work

Initial Value Problem (C)

$$\omega\frac{\mathrm{d}^3 I^{\mathrm{C}}}{\mathrm{d}\omega\mathrm{d}^2\boldsymbol{\theta}} = \frac{\alpha_s C_F}{(2\pi)^2}\mathrm{Re}\,2\omega i\int_0^L \mathrm{d}s\, n(s)\int_0^s \mathrm{d}t\mathcal{S}_{\mathrm{med}}(t)\int_{\boldsymbol{u}} e^{-i\left(s\frac{\boldsymbol{u}^2}{2\omega}-t\frac{\bar{\boldsymbol{u}}^2}{2\omega}\right)}\boldsymbol{\psi}_{\mathrm{I}}(\boldsymbol{\kappa};s;\boldsymbol{u},t)\cdot\bar{\boldsymbol{u}}.$$

$$\omega\frac{\mathrm{d}^3 I^{\mathrm{Ind}}}{\mathrm{d}\omega\mathrm{d}^2\boldsymbol{\theta}} = \omega\frac{\mathrm{d}^3 I^{\mathrm{C}}}{\mathrm{d}\omega\mathrm{d}^2\boldsymbol{\theta}}\bigg|_{\delta n=0}$$

Definitions:

$$\boldsymbol{\phi}(\boldsymbol{\kappa},\tau;\boldsymbol{q},s)\overset{\mathrm{def}}{=}\int_{\boldsymbol{\ell}}\mathcal{P}(\boldsymbol{\kappa}-\boldsymbol{\ell};\tau,s)\sigma(\boldsymbol{\ell}-\boldsymbol{q})\left(\frac{\boldsymbol{\ell}}{\boldsymbol{\ell}^2}-\frac{\boldsymbol{q}}{\boldsymbol{q}^2}\right),$$

$$\boldsymbol{\psi}(\boldsymbol{\kappa};s;\boldsymbol{u},t)\overset{\mathrm{def}}{=}\int_{\boldsymbol{q}}\boldsymbol{\phi}(\boldsymbol{\kappa},\tau=L;\boldsymbol{q},s)\mathcal{K}(\boldsymbol{q},s;\boldsymbol{u},t),$$

Initial Value Problems

$$\partial_\tau\boldsymbol{\phi}(\boldsymbol{\kappa},\tau;\boldsymbol{q},s) = -\frac{1}{2}n(\tau)\int_{\boldsymbol{y}}\sigma(\boldsymbol{\kappa}-\boldsymbol{y})\boldsymbol{\phi}(\boldsymbol{y},\tau;\boldsymbol{q},s),$$

$$\boldsymbol{\phi}(\boldsymbol{\kappa},\tau=s;\boldsymbol{q},s) = \sigma(\boldsymbol{\kappa}-\boldsymbol{q})\left(\frac{\boldsymbol{\kappa}}{\boldsymbol{\kappa}^2}-\frac{\boldsymbol{q}}{\boldsymbol{q}^2}\right),$$

$$\partial_t\boldsymbol{\psi}_{\mathrm{I}}(\boldsymbol{\kappa};s;\boldsymbol{u},t) = +\frac{1}{2}n(t)\int_{\boldsymbol{y}}\boldsymbol{\psi}_{\mathrm{I}}(\boldsymbol{\kappa};s;\boldsymbol{y},t)\sigma(\boldsymbol{y}-\boldsymbol{u})e^{i(s-t)\frac{\boldsymbol{u}^2-\boldsymbol{y}^2}{2\omega}},$$

$$\boldsymbol{\psi}_{\mathrm{I}}(\boldsymbol{\kappa};s;\boldsymbol{u},t=s) = \boldsymbol{\phi}(\boldsymbol{\kappa},\tau=L;\boldsymbol{u},s),$$

Note: Plotted spectra are the azimuthal average of this spectrum
→ Requires analytical work