HF jet quenching in pre-equilibrated medium

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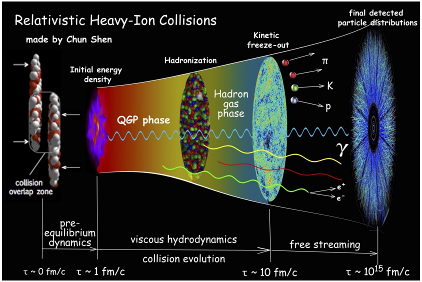
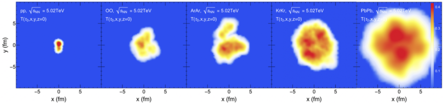
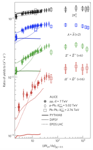
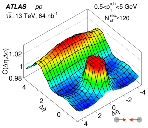
C3NT Workshop on Jet-Soft medium interactions, Mar 27th, 2026,
based on arXiv:2509.24316 and arXiv:2604.XXXXX

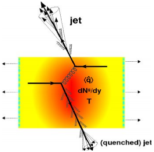
Motivation











QGP in A+A collisions

↓

p+p as a reference

↓

puzzling collectivity in p+p

↓

viscous hydro applicable in small systems?

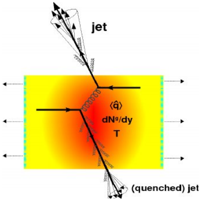
↓

medium tomography

Heavy flavor jets as early probes

Jet tomography → probe QGP properties:

- jet quenching (R_{AA})
- jet-hadron correlations
- jet-induced wake



Can it isolate pre-equilibrium dynamics?

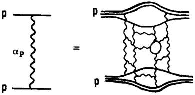
heavy-flavor jets	**early stages**	**test in pp?**
produced in initial hard-scattering ↓ don't thermalize with medium ↓ clean experimental tag	QGP models start at $\tau_0 \sim 0.6$ fm ↓ what happens *before* with the jet?	minimal QGP contribution ↓ pre-equilibrium effects *relatively* larger

Playground

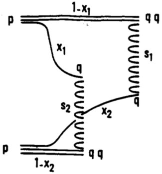
Pre-hydro stage: anisotropic, inhomogeneous, evolving

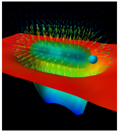
Option: color flux tubes (strings) between colliding partons

- pomeron exchanges
- longitudinal dynamics
- ebe fluctuations



[Capella, Phys.Rep. **236** (1994) 225]





[Varilly, Thesis (2006) MIT]

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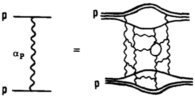

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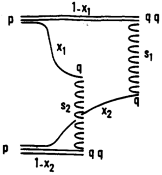
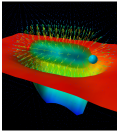
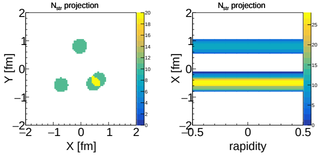


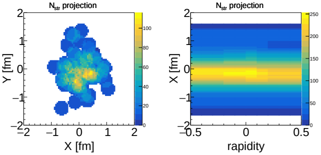


[Varilly, Thesis (2006) MIT]

Color strings → medium: energy density ε from overlapping strings



binned X-Y-rapidity space



[D. Prokhorova, E. Andronov, Physics 6 (2024) 264]

Medium properties: gluon gas with tunable anisotropy

Assumption: gluon-rich medium. In a cell covered by $k_{\rm cell}$ strings:

- energy density $\varepsilon_{\rm cell} = \dfrac{\sigma\sqrt{k_{\rm cell}}}{\pi r_{\rm str}^2}$, $\sigma = 1$ GeV/fm, $r_{\rm str} = 0.25$ fm
- gluon distribution $f(k)$ with momentum scale Λ and momentum anisotropy ξ [Romatschke, Strickland, Phys.Rev.D 68 (2003) 036004]

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Anisotropic $(\xi > 0)$

$$f_{\mathrm{aniso}}(k) = \frac{1}{e^{\frac{\sqrt{k_T^2+(1+\xi)k_z^2}}{\Lambda}} - 1}$$

$$\Lambda = \left(\frac{\varepsilon}{16\pi^2/60}\right)^{1/4} \cdot \mathcal{F}(\xi)$$

$$\mathcal{F}(\xi) = \left(\frac{1}{1+\xi} + \frac{\arctan\sqrt{\xi}}{\sqrt{\xi}}\right)^{-1/4}$$

$$\rho_{\mathrm{aniso}} = \frac{16\zeta(3)}{\pi^2}\frac{\Lambda^3}{\sqrt{1+\xi}}$$

ξ controls

anisotropy
↓
gluon density
↓
interaction rate

Isotropic $(\xi = 0)$

$$f_{\mathrm{iso}}(k) = \frac{1}{e^{k/T} - 1}$$

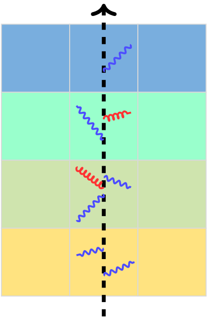
$$k = \sqrt{k_T^2 + k_z^2}$$

$$T = \left(\frac{\varepsilon}{16\pi^2/30}\right)^{1/4}$$

$$\rho_{\mathrm{iso}} = \frac{16\zeta(3)}{\pi^2}T^3$$

Heavy-flavor quark transport in granular gluon medium

HF trajectory



straight trajectory, no diffusion

Medium cell:

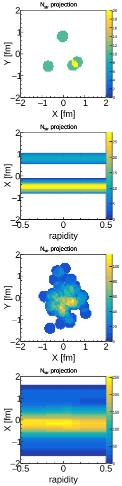
- dX, dY, dy - parameters
- k_{cell} strings/cell $\sim$ 1–100
- $\varepsilon_{\text{cell}} \sim$ 5–50 GeV/fm^3

Propagation:

- $t_{\max} = 1.5$ fm/c
- time step $\Delta t = 0.1$ fm/c

Interactions:

- elastic $q + g \to q + g$
- radiative $q \to q + g$



HF quark transport: collisional momentum loss

Momentum loss rate of HF quark in elastic scattering: $q + g \to q + g$

$$\frac{\mathrm{d}p}{\mathrm{d}t} = 16 \int \frac{\mathrm{d}^3 k}{(2\pi)^3} f(k)\, \Phi \int_{\hat{t}_{\min}}^{\hat{t}_{\max}} \mathrm{d}\hat{t} \frac{\mathrm{d}\sigma}{\mathrm{d}\hat{t}} \Delta p(\hat{t}, k, p),$$

$\Delta p(\hat{t}, k, p)$ - one-collision momentum loss, Φ - flux of HF quarks

elastic x-section [Thoma, Gyulassy, Nucl.Phys.B 351 (1991) 491]

$$\frac{\mathrm{d}\sigma}{\mathrm{d}\hat{t}} = \frac{2\pi\alpha_s^2}{(\hat{t} + \mu^2)\hat{t}}$$

CUJET3 **thermal running coupling** [Xu, Liao, Gyulassy, Chin.Phys.Lett. 32 (2015) 092501]

$$\alpha_s(Q^2) = \frac{\alpha_c}{1 + \frac{11}{4\pi}\alpha_c \ln(Q^2/T_c^2)}$$

critical parameters [Shi, Liao, Gyulassy, Chin.Phys. C 43 (2019) 4, 044101]

$$\alpha_c = 0.9, \ T_c = 0.16 \text{ GeV}$$

gluon **Debye screening** mass [Gossiaux, Aichelin, Phys.Rev. C 78 (2008) 014904]

$$\mu(T) = T\sqrt{4\pi \cdot \alpha_s(\mu^2)}$$

Collisional momentum loss rate (beyond high-energy limit)

Anisotropic medium: $k_T - k_z$ anisotropy parameter $\xi > 0$, Λ - scale

$$\frac{\mathrm{d}p}{\mathrm{d}t} = \frac{2}{3}\pi \cdot \alpha_s\left[\mu^2\right] \cdot \alpha_s\left[6E\Lambda\right] \cdot \ln\left[\frac{6E\Lambda - E^2(1-\varepsilon^2) + \frac{E^3(1-\varepsilon^2)^2}{12\varepsilon\Lambda} \cdot \ln\left[\frac{E(1-\varepsilon^2)+6\Lambda(1+\varepsilon)}{E(1-\varepsilon^2)+6\Lambda(1-\varepsilon)}\right]}{\mu^2}\right]$$

$$\times (1-\varepsilon) \cdot \left(\frac{\Lambda^2}{\xi+\varepsilon^2} \cdot \left\{\frac{\arctan\sqrt{\xi}}{\sqrt{\xi}} \cdot \left(\xi + \frac{1+\xi}{1-\varepsilon} - \frac{2\xi}{\gamma}\right) + 1 + \ln\left[\frac{1-\varepsilon}{1+\varepsilon}\right] \cdot \left(\frac{\varepsilon}{\gamma} + \frac{1}{2}\right)\right\}\right.$$

$$\left. + \frac{12\zeta(3)}{\pi^2} \cdot \frac{\Lambda^3}{E} \cdot \frac{1}{(\xi+1)^{\frac{3}{2}}} \cdot \left\{\frac{\operatorname{arctanh}\sqrt{\gamma}}{\gamma^{\frac{3}{2}}} - \frac{1}{\gamma}\right\}\right), \text{ where } \boxed{\varepsilon = \frac{p}{E}, \quad \gamma = \frac{\xi+\varepsilon^2}{\xi+1}}$$

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Isotropic medium: $\xi = 0$, T - temperature

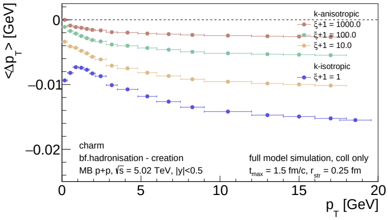
$$\frac{dp}{dt} = \frac{2}{3}\pi \cdot \alpha_s\left[\mu^2\right] \cdot \alpha_s\left[6ET\right] \cdot \ln\left[\frac{6ET - E^2(1-\varepsilon^2) + \frac{E^3(1-\varepsilon^2)^2}{12\varepsilon T} \cdot \ln\left[\frac{E(1-\varepsilon^2)+6T(1+\varepsilon)}{E(1-\varepsilon^2)+6T(1-\varepsilon)}\right]}{\mu^2}\right]$$

$$\times \left(\frac{T^2}{\varepsilon^3} \cdot \left\{(2-\varepsilon)\varepsilon - (2-\varepsilon-\varepsilon^2)\operatorname{arctanh}\left[\varepsilon\right]\right\} + \frac{12\zeta(3)}{\pi^2} \cdot \frac{T^3}{E} \cdot \frac{1-\varepsilon}{\varepsilon^3} \cdot \left(\operatorname{arctanh}\left[\varepsilon\right] - \varepsilon\right)\right)$$

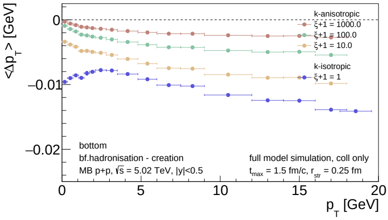
Collisional momentum loss: charm vs bottom quarks

elastic $q + g \to q + g$

$m_c = 1.5$ GeV



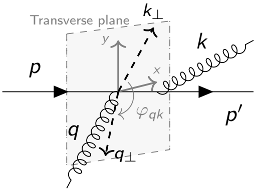
$m_b = 4.2$ GeV



- medium anisotropy $\xi > 0$ reduces **collisional loss** $\langle \Delta p_T \rangle$
- thermal limit ($\xi = 0$) at $p_T \sim 20$ GeV: $\langle \Delta p_T \rangle \sim 0.015$ GeV for both

HF quark transport: radiative momentum loss

Inelastic scattering: $q + \mathrm{medium} \rightarrow q + \mathbf{g} \leftarrow$ radiated gluon spectrum?



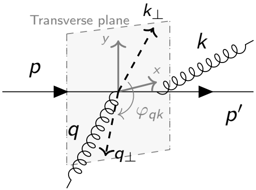
Djordjevic-Gyulassy-Levai-Vitev (**DGLV**) opacity expansion [Gyulassy, Levai, Vitev, Nucl. Phys. B, 571 (2000) 197; Djordjevic, Gyulassy, Nucl. Phys. A 733 (2004) 265]

- $n = 1$: single scattering $\rightarrow$ single gluon emission
- finite size medium

$$z \frac{d\Gamma^{\mathrm{n=1}}_{q\rightarrow qg}}{dz}(z, p, T, t) \propto \rho(T) \int d^2\mathbf{q}_\perp \, d^2\mathbf{k}_\perp \, \mathcal{K}(\mathbf{k}_\perp, \mathbf{q}_\perp, z, p, T, t)$$

HF quark transport: radiative momentum loss

Inelastic scattering: $q + \text{medium} \to q + \mathbf{g}$ ← radiated gluon spectrum?



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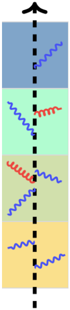


$$z \frac{d\Gamma^{\mathrm{n=1}}_{q\to qg}}{dz}(z, p, T, t) \propto \rho(T) \int d^2\mathbf{q}_\perp\, d^2\mathbf{k}_\perp\, \mathcal{K}(\mathbf{k}_\perp, \mathbf{q}_\perp, z, p, T, t)$$

- emission probability $P_{\mathrm{rad}} = \int dz \int dt\, \frac{d\Gamma}{dz}$, $z = \frac{k}{p}$
- if occurs in a cell, sample z, reduce quark momentum by zp
- reset time since last splitting

HF quark transport: radiative momentum loss

Single gluon emission spectrum at n = 1 DGLV opacity series [Shi, Yazdi, Gale, Jeon, Phys. Rev. C 107 (2023) 3, 034908]

$$z\frac{dN^{\mathrm{n}=1}_{q\to qg}}{dz}(p,T,\tau,z)=\frac{6}{\pi^2}\int_0^{\tau}dt\,\rho(T)\int d^2\mathbf{k}_\perp\,d^2\mathbf{q}_\perp\left\{\frac{\alpha_s^2\,[\mathbf{q}_\perp^2]}{\mathbf{q}_\perp^2(\mathbf{q}_\perp^2+\mu^2)}\right.\longleftarrow \text{elastic}$$

$$\text{gluon emission}\longrightarrow\times\,\alpha_s\left[\frac{\mathbf{k}_\perp^2}{z_+(1-z_+)}\right]\cdot\frac{z}{z_+}\cdot\left|\frac{dz_+}{dz}\right|$$

$$\text{interference}\longrightarrow\times\frac{-2}{(\mathbf{k}_\perp-\mathbf{q}_\perp)^2+\chi^2}\cdot\left(\frac{\mathbf{k}_\perp\cdot(\mathbf{k}_\perp-\mathbf{q}_\perp)}{\mathbf{k}_\perp^2+\chi^2}-\frac{(\mathbf{k}_\perp-\mathbf{q}_\perp)^2}{(\mathbf{k}_\perp-\mathbf{q}_\perp)^2+\chi^2}\right)$$

$$\text{LPM suppression}\longrightarrow\times\left.\left(1-\cos\left[\frac{(\mathbf{k}_\perp-\mathbf{q}_\perp)^2+\chi^2}{2z_+p}\cdot t\right]\right)\right\}$$

dead cone and screening:

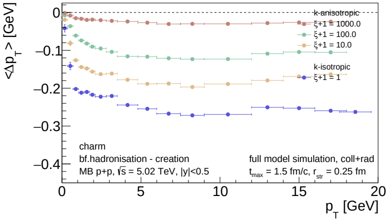
$$\chi^2(T)=m^2z_+^2+\frac{\mu^2(T)}{2}(1-z_+)$$

light cone $z+$ vs momentum fraction z:

$$z_+=\frac{1}{2}\left(z+\sqrt{z^2-(\mathbf{k}_\perp/p)^2}\right)$$

Collisional + radiative momentum loss: charm vs bottom quarks

$m_c = 1.5$ GeV



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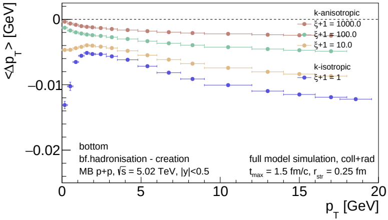
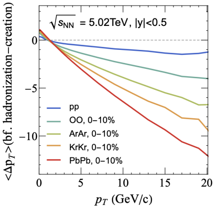
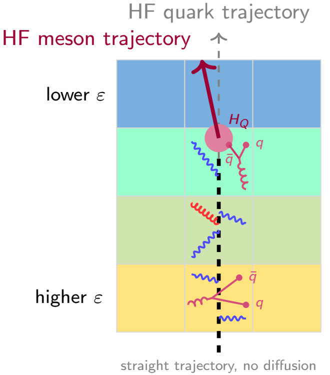




- medium anisotropy $\xi > 0$ reduces **total loss** $\langle \Delta p_T \rangle$
- thermal limit $\langle \Delta p_T \rangle_c \sim 0.3$ GeV at $p_T \sim 20$ GeV

← comparable to EPOS4HQ loss for charm [Zhao et al., Phys.Rev.C 111 (2025) 014907] in QGP spots

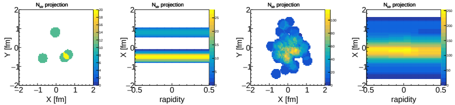
Naive hadronisation model for HF quark



Coalescence mechanism:

- only **cells along HF quark trajectory**
- each cell: gluons with known $f(k)$, density
- $g \to q\bar{q}$ fluctuation possible
- HF quark and q or $\bar{q}$ - momentum match?
- HF meson forms and escapes
- No further energy loss

[WORK IN PROGRESS]



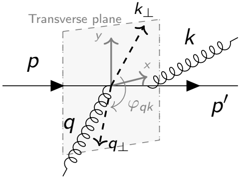
Summary I

[D. Prokhorova, S. Shi, E. Andronov]

Transport of HF quarks in pre-equilibrium color-string medium:

- Radiation $\gg$ collisional loss (20$\times$ for charm)
- **Dead cone effect**: $m_c < m_b \rightarrow \langle \Delta p_T \rangle_c^{\mathrm{rad}} > \langle \Delta p_T \rangle_b^{\mathrm{rad}}$
- **Medium anisotropy matters:** $\xi > 0$ drastically reduces loss
- Total **pre-equilibrium loss is comparable to EPOS4HQ loss** in QGP droplets [Zhao et al., Phys. Rev. C 111 (2025) 014907]

Angular structure of in-medium HF energy loss



So far: how much energy HF quarks lose.

Next: can we study the angular structure of radiation?

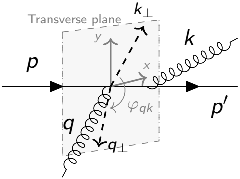
The DGLV emission kernel depends on 2D vectors $\mathbf{k}_\perp$ and $\mathbf{q}_\perp$

$$z\frac{d\Gamma^{\mathrm{n=1}}_{q\to qg}}{dz} \propto \rho(T)\int d^2\mathbf{q}_\perp d^2\mathbf{k}_\perp\,\mathcal{K}(\mathbf{k}_\perp,\mathbf{q}_\perp,t), \quad z=\frac{k}{p}$$

With $q_\perp \equiv |\mathbf{q}_\perp|$, $k_\perp \equiv |\mathbf{k}_\perp|$, and $\varphi_{kq} \equiv \widehat{\mathbf{k}_\perp,\mathbf{q}_\perp}$ we transform

$$z\frac{d\Gamma^{\mathrm{n=1}}_{q\to qg}}{dz} \propto \rho(T)\int dq_\perp dk_\perp\,\tilde{\mathcal{K}}(q_\perp,k_\perp)\int d\varphi_{kq}\,F\Big(q_\perp,k_\perp,\varphi_{kq};t\Big)$$

Angular structure of in-medium HF energy loss



So far: how much energy HF quarks lose.

Next: can we study the angular structure of radiation?

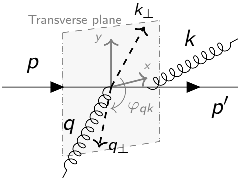
Then **angular dependence** is factorized

$$F(q_\perp, k_\perp, \varphi_{kq}; t) = \sum_{l=0}^{\infty}\sum_{n=0}^{\infty} R_l(q_\perp, k_\perp; t)\, A_n(q_\perp, k_\perp)\, T_l(\cos\varphi_{kq})\, T_n(\cos\varphi_{kq})$$

and the **analytical integration** over φ_{kq} can be performed using properties of Chebyshev polynomials

$$\int_0^\pi d\varphi_{kq}\, F\Big(q_\perp, k_\perp, \varphi_{kq}; t\Big) = \frac{\pi}{2}\sum_{l=0}^{\infty}(1+\delta_{l0})\, R_l\Big(q_\perp, k_\perp; t\Big) A_l(q_\perp, k_\perp)$$

Partial-wave expansion of the gluon emission rate



So far: how much energy HF quarks lose.

Next: can we study the angular structure of radiation?

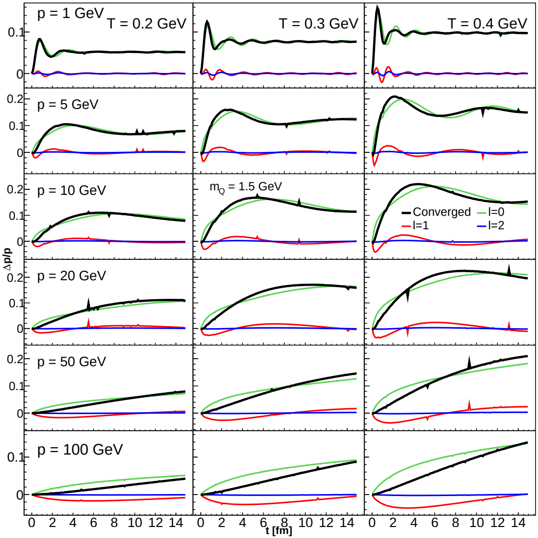
This leads to the **partial-wave expansion** of the gluon spectrum

$$z\frac{d\Gamma^{\mathrm{n=1}}_{q\to qg}}{dz}(p,T,t,z) \propto \rho(T)\sum_{l=0}^{\infty}\int dq_\perp dk_\perp\,\tilde{\mathcal{K}}(q_\perp,k_\perp)\,F_l\Big(q_\perp,k_\perp;t\Big)$$

and the **partial-wave expansion** of the **relative momentum loss**

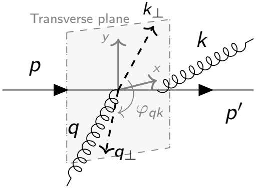
$$\frac{\Delta p}{p}(p,T,\tau) \propto \rho(T)\sum_{l=0}^{\infty}\int^{\tau} dt\int_0^1 dz\,\left(z\frac{d\Gamma^{\mathrm{n=1}}_{q\to qg}}{dz}(p,T,t,z)\right)_l$$

Partial-wave contributions of the relative momentum loss



Parton-level 3-point correlation

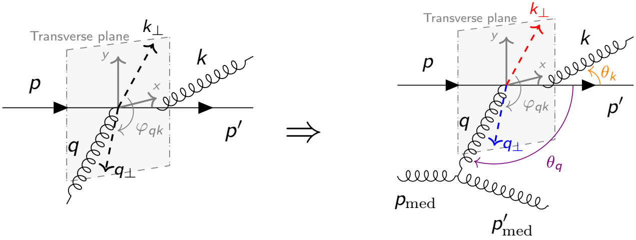
Now: angular structure of gluon emission kernel **is preserved**



Parton-level 3-point correlation

Now: angular structure of gluon emission kernel **is preserved**

Next: use it to calculate **3-point energy correlator**!



Parton-level 3-point correlation

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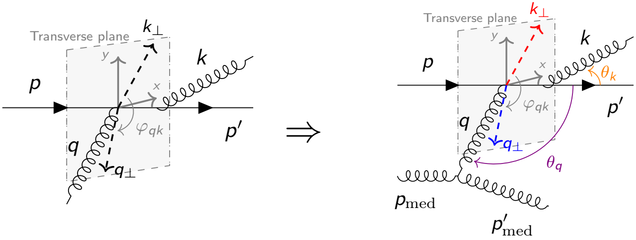


- restore 3D picture of DGLV single-gluon emission kernel
- integrate DGLV kernel with energy weights and tagged HF hadron

$$\mathrm{E3C} \propto \int dt\, d^3k\, d^3q\, \frac{d^3 p_{\rm med}}{(2\pi)^3} \left\{ \frac{1}{e^{\frac{p_{\rm med}}{T}} - 1} \times \frac{k\; p'\; p'_{\rm med}}{p^3} \times \frac{d\Gamma \cdot \delta(q_z - q_z^*)}{dk_\perp\; dz\; d\varphi_k\; dq_\perp\; dq_z\; d\varphi_q} \right\}$$

Partial-wave expansion of E3C

Then the **full 3-point energy correlator** can be represented as

$$\mathrm{E3C} \propto \sum_{l,n=0}^{\infty} \int dt \int dz \int dq_{\perp} \int dk_{\perp} \int \frac{d^3 p_{\mathrm{med}}}{(2\pi)^3} \left\{ \frac{1}{e^{\frac{p_{\mathrm{med}}}{T}} - 1} \times \frac{k\, p'\, p^{*'}_{\mathrm{med}}}{p^3} \right.$$

$$\left. \times\, \tilde{\mathcal{K}}\left(q_{\perp}, k_{\perp}\right) R_l\left(q_{\perp}, k_{\perp}; t\right) A_n\left(q_{\perp}, k_{\perp}\right) \int d\varphi_{kq}\, T_l\left(\cos\varphi_{kq}\right)\, T_n\left(\cos\varphi_{kq}\right) \right\}$$

and it gives us direct access to **partial-wave terms of E3C**.

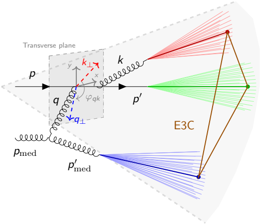
[WORK IN PROGRESS]

Partial-wave expansion of E3C

Then the **full 3-point energy correlator** can be represented as

$$\mathrm{E3C} \propto \sum_{l,n=0}^{\infty} \int dt \int dz \int dq_\perp \int dk_\perp \int \frac{d^3 p_\mathrm{med}}{(2\pi)^3} \left\{ \frac{1}{e^{\frac{p_\mathrm{med}}{T}} - 1} \times \frac{k\ p'\ p_\mathrm{med}^{*'}}{p^3} \right.$$
$$\left. \times \tilde{\mathcal{K}}(q_\perp, k_\perp)\, R_l(q_\perp, k_\perp; t)\, A_n(q_\perp, k_\perp) \int d\varphi_{kq}\, T_l(\cos\varphi_{kq})\ T_n(\cos\varphi_{kq}) \right\}$$

and it gives us direct access to **partial-wave terms of E3C**.



Next:

- take into account medium effects (smearing)
- estimate hadronisation influence
- which partial waves survive at certain parameters?

[WORK IN PROGRESS]

Summary II

Angular structure of the in-medium gluon emission:

- partial-wave expansion of the DGLV ($n = 1$) single gluon emission spectrum [D. Prokhorova, M. Kaygorodov]
- inherited partial-wave expansion of the 3-point energy correlator [D. Prokhorova, M. Kaygorodov, S. Shi, W. Ke]

THANK YOU!

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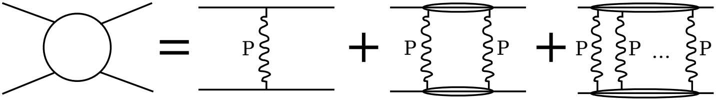
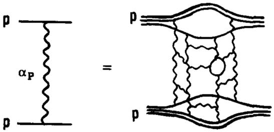
BACKUP

Advent of the colour string model of particle production

- pre-QCD Regge-Gribov: high-energy elastic scattering amplitude as multiple **Pomeron exchanges** [Gribov, JETP **53** (1967) 654]



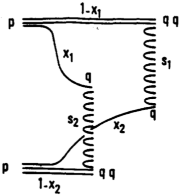
- dominant contribution of QCD topological expansion in **large** N_c **and** N_f **limits** – cylindrical diagram ↔ Pomeron exchange [Veneziano, Phys. Lett. B **52** (1974) 220; Nucl. Phys. B **117** (1976) 519]



[Capella, Phys. Rep. **236** (1994) 225]

Advent of the colour string model of particle production

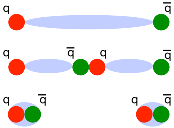
- **space-time localisation** of the cylindrical pomeron exchange diagram with unitarity cut: two-rapidity-chains fragmenting into soft particles [Capella et al. Phys. Lett. B **81** (1979) 68; Kaidalov, Phys. Lett. B **116** (1982) 459; Artru, Phys. Rep. **97** (1983) 147; Werner, Phys. Rep. **232** (1993) 87]



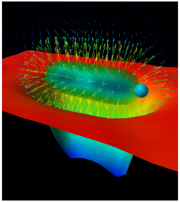
[Capella, Phys. Rep. **236** (1994) 225]

- **Cornell potential** between confined colour charges [Eichten et al. Phys. Rev. Lett. **34** (1975) 369], $q\bar{q}$ pair production

$$V(r) = -\frac{4}{3}\cdot\frac{\alpha_s}{r} + \sigma \cdot r,$$



  - α_s - QCD running coupling
  - σ - string tension



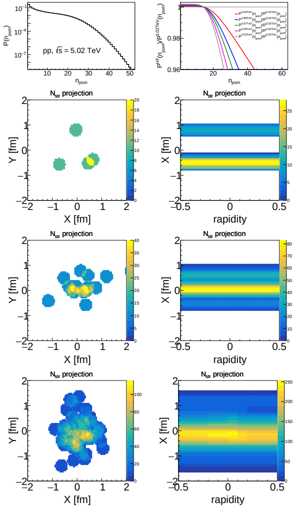
[Varilly, Thesis (2006) MIT]

Model workflow

1. HF quarks extracted from PYTHIA
2. pure longitudinal color strings formed between colliding partons, $n_{\mathrm{str}} = 2n_{\mathrm{pom}}$
3. strings overlap $\Rightarrow$ rearrangement of color fields
4. binned medium structure in X-Y-rapidity space
5. cell energy density from k_{cell} strings

$$\varepsilon_{\mathrm{cell}} = \frac{\sigma\sqrt{k_{\mathrm{cell}}}}{\pi r_{\mathrm{str}}^2},\ \sigma = 1\ \frac{\mathrm{GeV}}{\mathrm{fm}},\ r_{\mathrm{str}} = 0.25\ \mathrm{fm}$$

6. massive string endpoint oscillations $\frac{dp_z}{dt} = -\sigma$ $\Rightarrow$ fluctuating string medium
7. HF travels through it in straight trajectory defined by initial p_T (no diffusion)
8. deterministic energy-loss model
9. hadronisation (for future)



Heavy-flavor quark's propagation through medium

Macroscopic picture

Deterministic energy loss over quark trajectory through evolving medium

Microscopic process

- Collisional (elastic scattering)
- Radiative (gluon emission)

Approach

- Thoma–Gyulassy x-section
- DGLV inelastic parton splitting

Computational algorithm

1. **Step-wise propagation** through medium cells
2. **Collisional energy loss** computed in each cell to update p_T
3. **Radiative probability** evaluated cumulatively:
   - check gluon emission probability since last radiation
   - if emitted: instant energy loss, reset cumulative tracking

Model description

1. Hard vs soft regimes new!

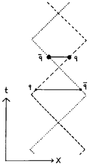
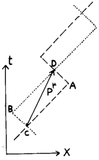
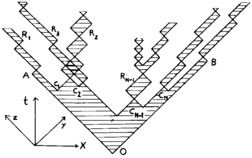
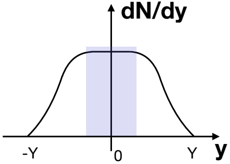
- new! extract HF quarks from proton-proton PYTHIA event: HardQCD: all, gg2ccbar, gg2bbbar, qqbar2ccbar, qqbar2bbbar = on
- new! remaining energy used for soft pomeron creation
- cut pomerons [A. Kaidalov et al. Phys. Lett. B **117** (1982) 247] $\rightarrow$ strings in an event: $n_{\rm str} = 2n_{\rm pom}$ [A. Capella et al. Phys. Rep. **236** (1994) 225] :

$$P\left(n_{\rm pom}\right) = C\left(z\right)\frac{1}{zn_{\rm pom}}\left(1 - \exp\left(-z\right)\sum_{l=0}^{n_{\rm pom}-1}\frac{z^l}{l!}\right), \quad z = \frac{2w\gamma s^{\Delta}}{R^2 + \alpha' \ln s}$$

$w = 1.5$, $\Delta = \alpha(0) - 1 = 0.2$, $\gamma = 1.035$ GeV^{-2}, $R^2 = 3.3$ GeV^{-2}, $\alpha' = 0.05$ GeV^{-2} from [V. Vechernin, S. Belokurova, J. Phys. Conf. Ser. **1690** (2020) 012088]

Color string evolution and fragmentation

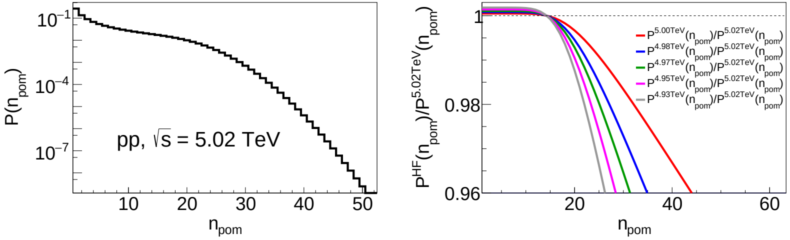
- in $1+1$ space-time: massless relativistic string is a **yo-yo mode** solution of $dp/dt = \pm\sigma_T$ equation of motion [X. Artru, Phys. Rep. **97** (1983) 147]
- probabilistic string fragmentation depends on hatched area spanned during quarks motion [B. Andersson, G. Gustafson, G. Ingelman, T. Sjostrand, Phys. Rep. **97** (1983) 31]
- colourless hadrons **uniformly** distributed over rapidity, $y = \frac{1}{2}\ln\left(\frac{p_0+p_z}{p_0-p_z}\right)$

- common approximation: **infinite** in rapidity strings
- convenient for finite experimental acceptances at **mid-rapidity** [S. Belokurova, V. Vechernin, Symmetry **12** (2020) 110]

Model description

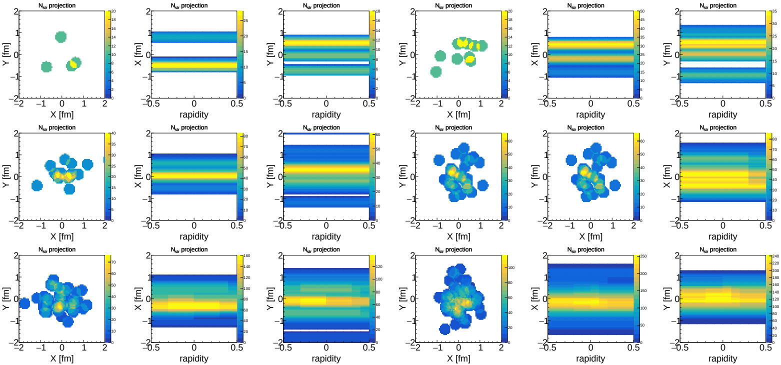
Examples of modified event-by-event $n_{\rm pom}$ distributions:



2. Proton composition:
   - inside each proton $N_{\rm part} = N_{\rm str} = 2 \cdot n_{\rm pom}$, which includes valence quark, valence diquark, sea pairs
   - partons' $p_Z = x \cdot p_{\rm beam}$ sampled from PDFs [A. Buckley et al. Eur. Phys. J. C **75** (2015) 132]

Model description

3. Strings formation
   - pure longitudinal strings between partons of two protons
   - in X-Y distributed according to Gaussian (0.0, 0.5) fm

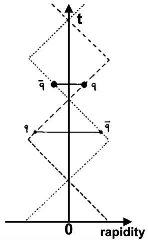


Model description

4. Strings evolution [C. Shen, B. Schenke, Phys. Rev. C **97** (2018) 024907]

For a **parton at string end** with m and $p_z = xp_{\mathrm{beam}}$:

- initial rapidity: $y_{\mathrm{init}} = \sinh^{-1}\left(\frac{p_z}{m}\right)$
- equations of motion: $\frac{dp_z}{dt} = -\sigma, \quad \frac{dE}{dz} = -\sigma$
- proper time before flip ($\tilde{y}$ in string rest frame): $\Delta\tau_{\max} = \frac{m}{\sigma}\sqrt{2\left(\cosh(\tilde{y}_{\mathrm{init}} - 1)\right)}$ – **periodicity!**



- relation between proper time $\Delta\tau$ and lab time Δt intervals:

$$\Delta t = \Delta\tau\left(\pm\frac{\sigma\Delta\tau}{2m}\sinh(y_{\mathrm{init}}) + \cosh(y_{\mathrm{init}})\sqrt{\frac{\sigma^2\Delta\tau^2}{4m^2} + 1}\right)$$

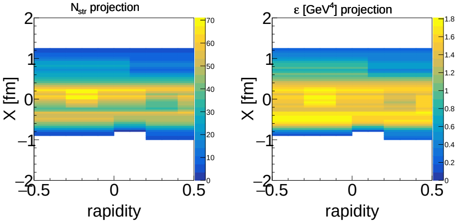
- change of rapidity: $y_{\mathrm{loss}} = \cosh^{-1}\left(\frac{\sigma^2\Delta\tau^2}{2m^2} + 1\right)$
- final rapidity after proper time $\Delta\tau$: $y_{\mathrm{fin}} = y_{\mathrm{init}} \pm y_{\mathrm{loss}}$

Model description

5. String medium characteristics

- finite transverse size → strings overlap → interplay of randomly oriented color fields [M. Braun et al. Int. J. Mod. Phys. A **14** (1999) 2689]
- fine granulation of **X-Y-rapidity space**, $\Delta t = 0.1$ fm/c **time steps**
- new! number of strings in a cell, k_{cell}, gives energy density, $\varepsilon_{\mathrm{cell}}$:

$$\varepsilon_{\mathrm{cell}} = \frac{\sigma\sqrt{k_{\mathrm{cell}}}}{\Delta S_{\mathrm{str}}}, \quad \sigma = 1\,\frac{\mathrm{GeV}}{\mathrm{fm}}, \quad \Delta S_{\mathrm{str}} = \pi r_{\mathrm{str}}^2, \quad r_{\mathrm{str}} = 0.25\ \mathrm{fm}$$

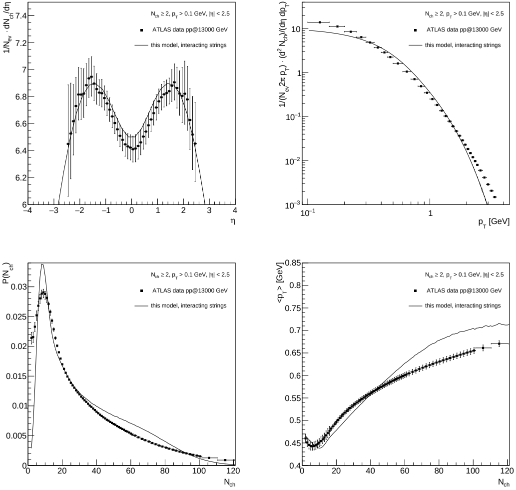


$$k_{\mathrm{cell}} \in [1, 100]$$

$$\varepsilon_{\mathrm{cell}} \in [5, 50]\ \frac{\mathrm{GeV}}{\mathrm{fm}^3}$$

$$\varepsilon_{\mathrm{cell}} \in [0.04, 0.4]\ \mathrm{GeV}^4$$

Model parameters using ATLAS $p + p$ data at $\sqrt{s} = 13$ TeV

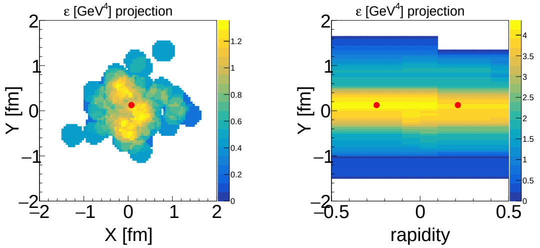


[D. Prokhorova, E. Andronov, Physics 6 (2024) 264]

Model description

6. HF production new!

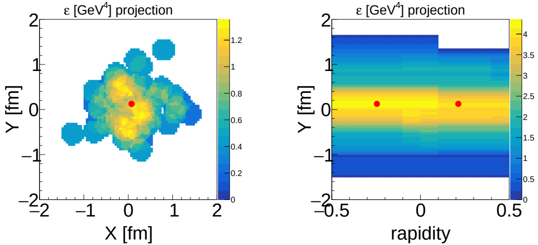
- PYTHIA provides mass and $\vec{p}$ of HF quarks but $X = Y = 0$
- at $t = 0$ fm/c HF quark pair inserted in string medium



Model description

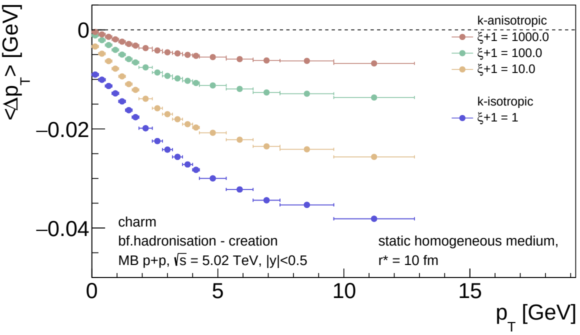
7. HF propagation new!

- every $\Delta t = 0.1$ fm HF quarks propagate in **changing string medium** during $t = 1.5$ fm
- deterministic energy-loss model (keeps initial p_T direction):
  - straight trajectories (no stochastic noise, no diffusion)
  - no large-angle scatterings
  - no change of rapidity slice (focus on transverse loss)



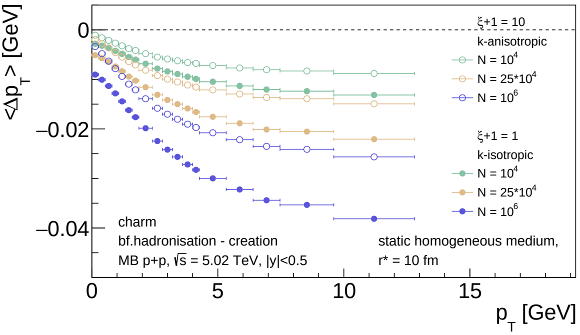
Brick test I

- static infinite homogeneous medium
- high string density equivalent to $k_{\mathrm{cell}} = 100$ (extremely HM events)
- constant quark propagation time $t_{\mathrm{max}} = 1.5$ fm/c
- medium gluon $k_z - k_T$ anisotropy $\xi + 1 = 1000, 100, 10, 1$



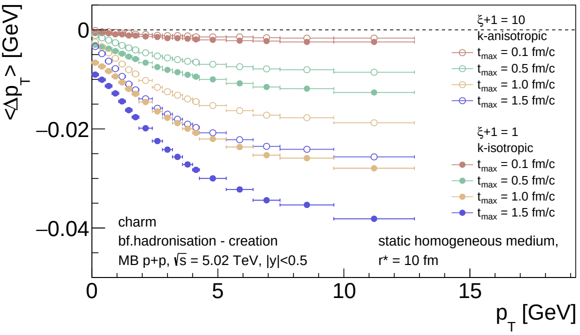
Brick test II

- static infinite homogeneous medium
- *varying string density*
- constant quark propagation time $t_{\max} = 1.5$ fm/c
- medium gluon $k_z - k_T$ anisotropy $\xi + 1 = 10$ and 1

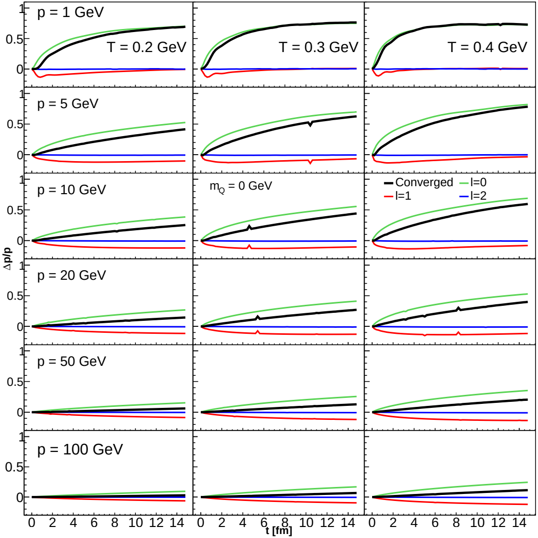


Brick test III

- static infinite homogeneous medium
- high string density equivalent to $k_{\text{cell}} = 100$ (extremely HM events)
- *varying quark propagation time*
- medium gluon $k_z - k_T$ anisotropy $\xi + 1 = 10$ and 1

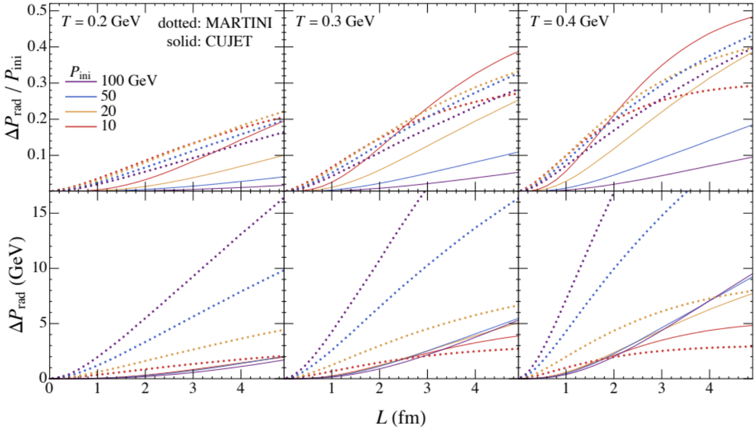


Relative momentum quenching for light quarks



Relative momentum quenching for light quarks (MARTINI, CUJET)

[Shi, Yazdi, Gale, Jeon, Phys. Rev. C 107 (2023) 3, 034908]



Relative momentum quenching for bottom quark