Various Topics Tangentially Related to…

Implementation of Jet Drift in Monte Carlos

C3NT Jet-Soft Correlations 2026
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CCNU
30th March, 2026

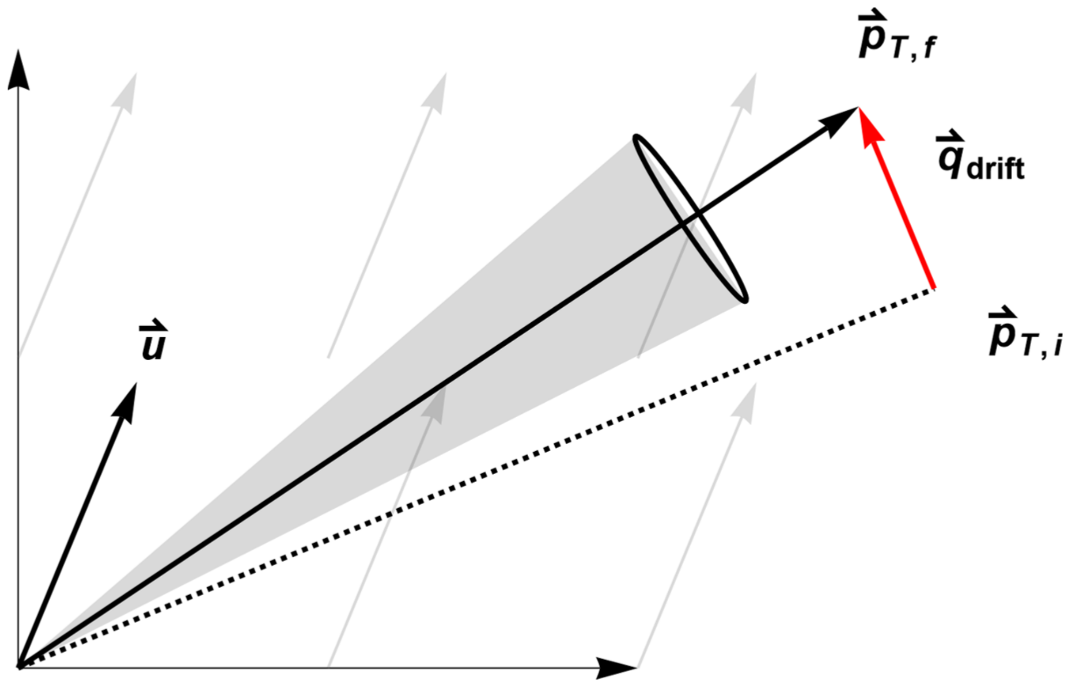
Outline

I. Introducing Flow Effects on Elastic Scatterings (part of "Jet Drift")

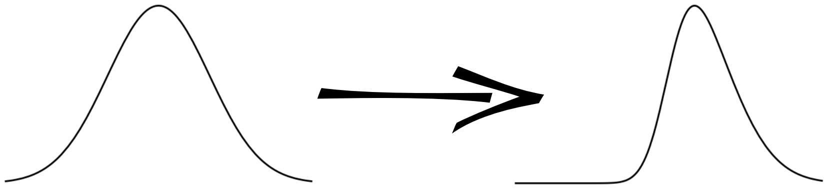
II. Velocity Field Ingredients, Gradient Expansion, & Implementation Issues

III. Toy Phenomenology

Flow-Induced Anisotropic Broadening ("Drift")



Derivation
A. V. Sadofyev,
M.D. Sievert, & I. Vitev,
Phys.Rev.D 104 (2021)
(arXiv:2104.09513)

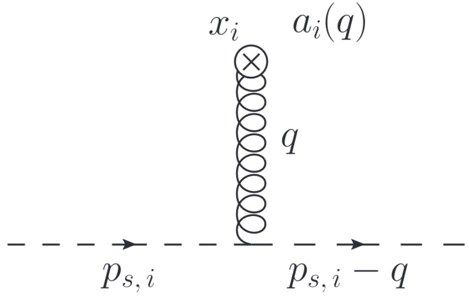


- Moving scattering centers skew broadening distribution
  - Hard partons deflect in direction of medium flow
- Traditionally accounted for with global boosts
- Recent advancement: velocity of scattering centers enters at the level of the potential

Drift Treatment

$$v_i(q) \equiv v_i\left(\boldsymbol{q}^2 - (\boldsymbol{u}_i \cdot \boldsymbol{q})^2\right) \equiv \frac{-g^2}{\boldsymbol{q}^2 + \mu_i^2 - (\boldsymbol{u}_i \cdot \boldsymbol{q})^2 - i\epsilon}$$

Directional scattering centers

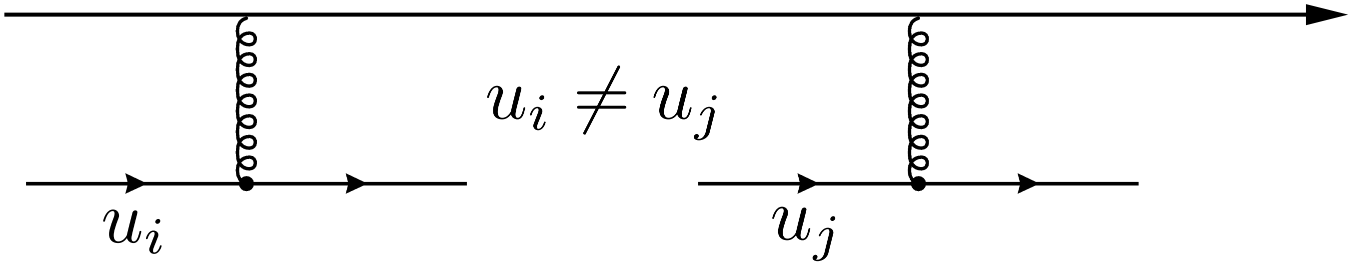


$$g\, a_i^{\mu a}(q) = t_i^a\, u_i^\mu\, v_i(q)\, (2\pi)\, \delta\left(q^0 - \boldsymbol{u}_i \cdot \boldsymbol{q}\right)$$

$$A_{ext}^{\mu a}(q) = \sum_i e^{iq\cdot x_i} a_i^{\mu a}(q)$$

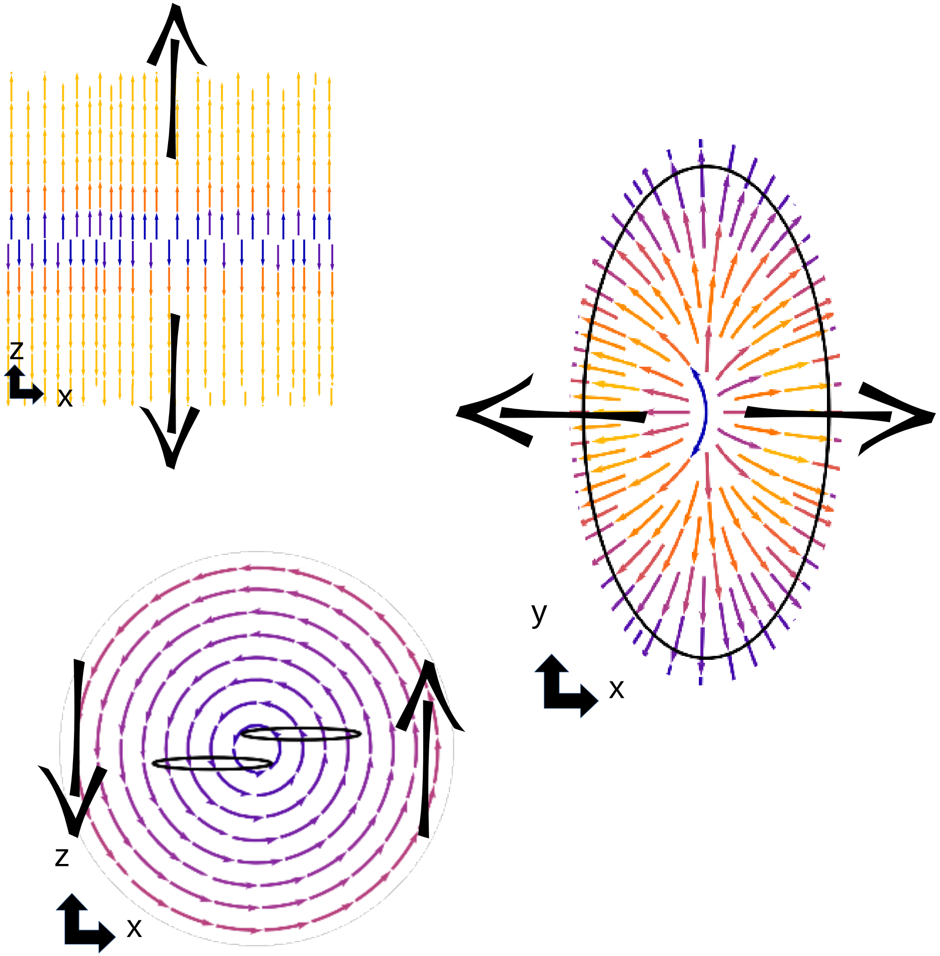
A. V. Sadofyev,
I. Vitev, & M. D. Sievert
Phys.Rev.D 104 (2021)
(arXiv:2104.09513)

Individual scattering centers have spatially varying velocity profile!



Importance for QGP Tomography

- Drift couples to preferred directions in the medium
  - Event plane (azimuthal)
  - Vorticity (rapidity)
- Accesses new information about the flow velocity of the plasma
  - Isotropic, eikonal-level interactions are insensitive to anisotropy

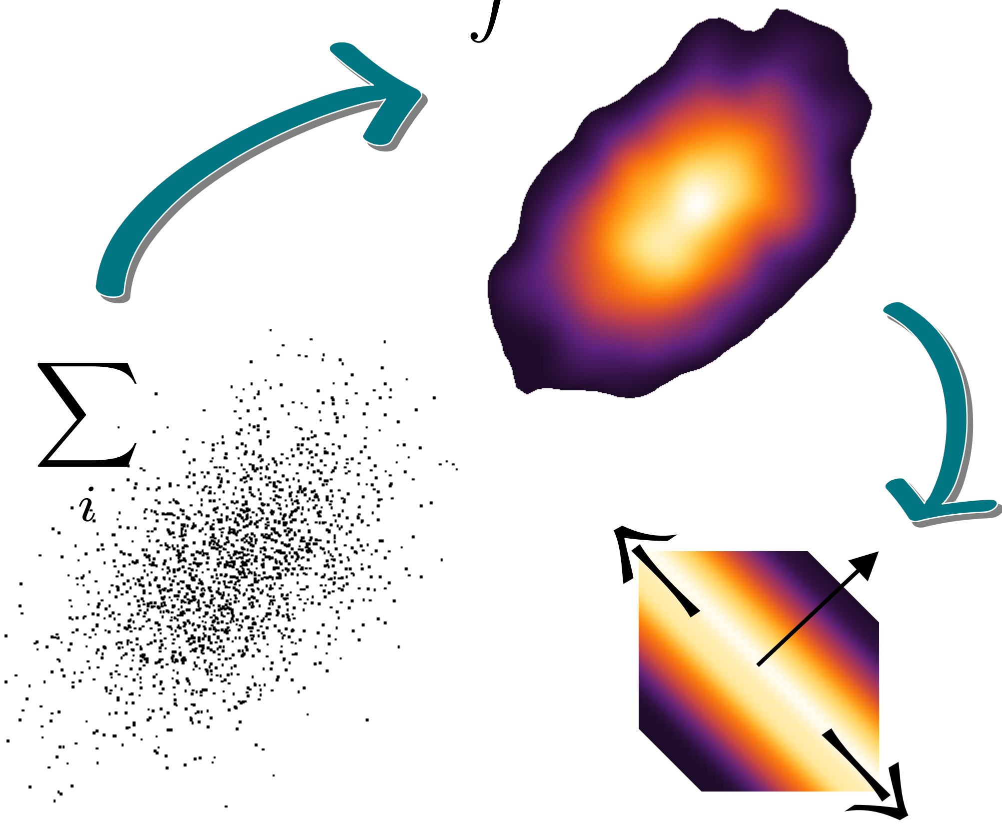


Velocity Field Ingredients

Treatment of Scattering Centers in Opacity Expansion

- Summation over scattering centers to spatial integral over density
- ***All*** scattering centers contribute to amplitude!
- Traditionally, transverse variation is neglected to simplify integration
- Longitudinal integration alone is analytically (simple models) or computationally (e.g. hydro) tractable
- **Imagine we perform the spatial integral exactly…**

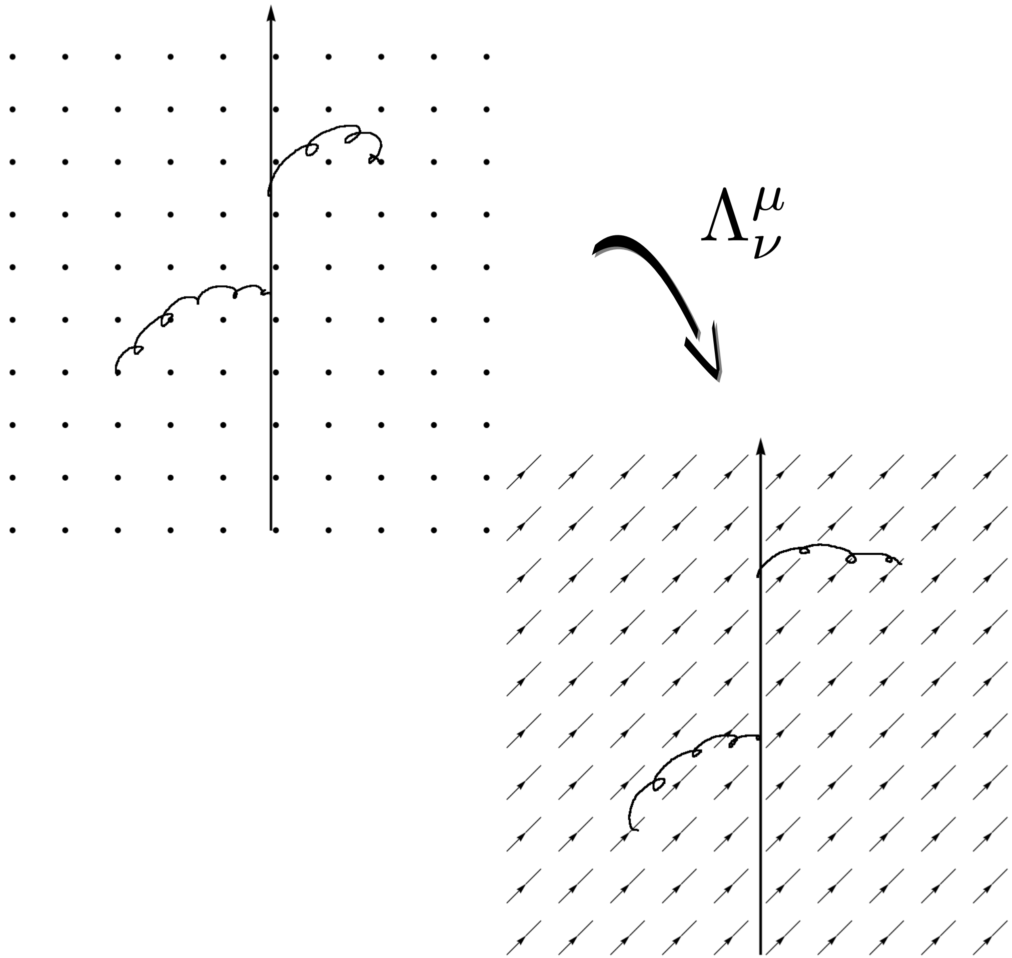
$$\int d^3x\, \rho(x)$$



$$\sum_i$$

$$e^{-i\alpha \vec{x}_\perp - i\beta\tau}$$

Global Boost Treatment

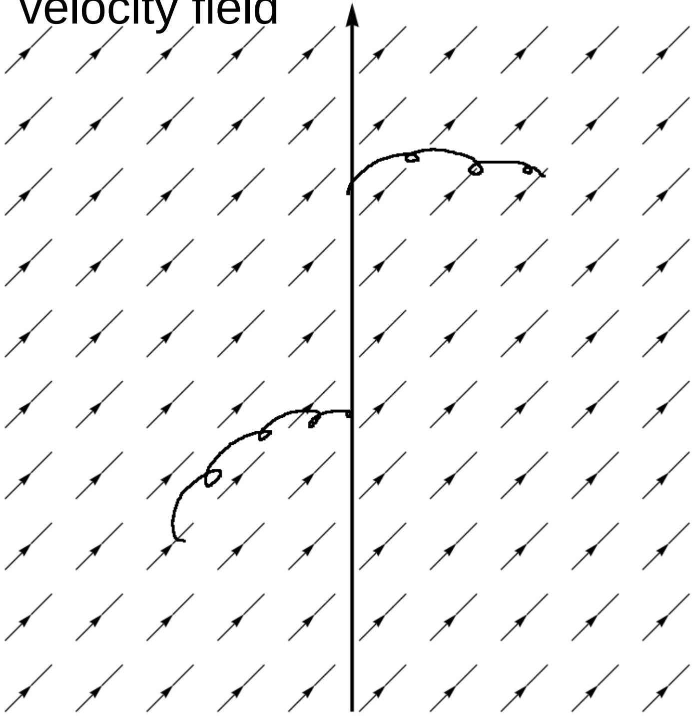


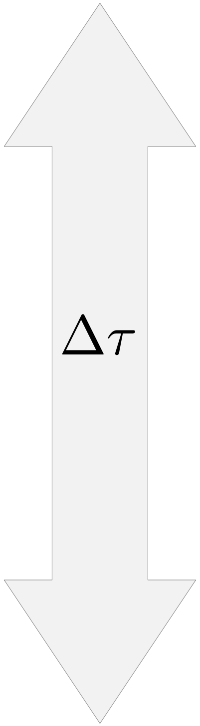
- Standard result neglects transverse variation of medium
- Calculate broadening in fluid rest frame, boost to moving frame
- Spatially *constant* velocity field

Velocity Fields

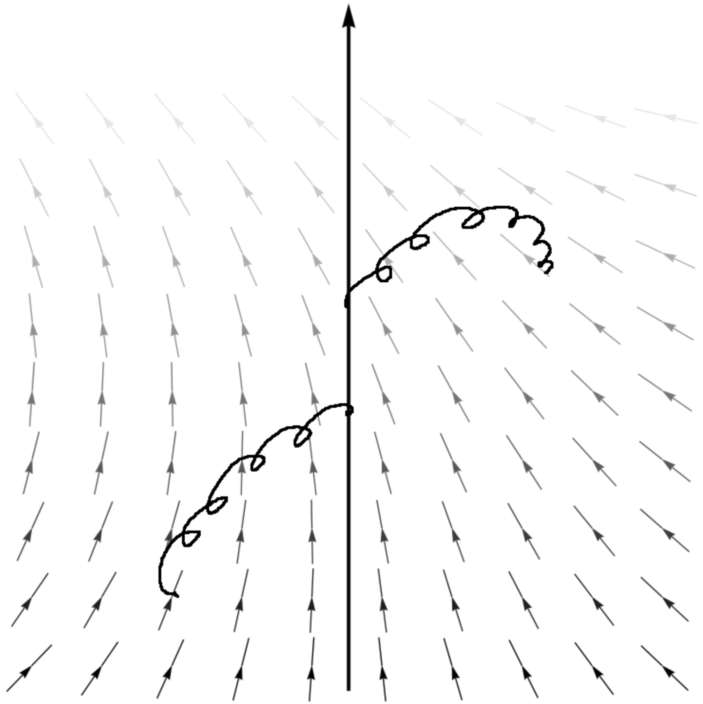
$\Delta\tau$

<u>Boost:</u>
Longitudinally uniform, transversely uniform velocity field





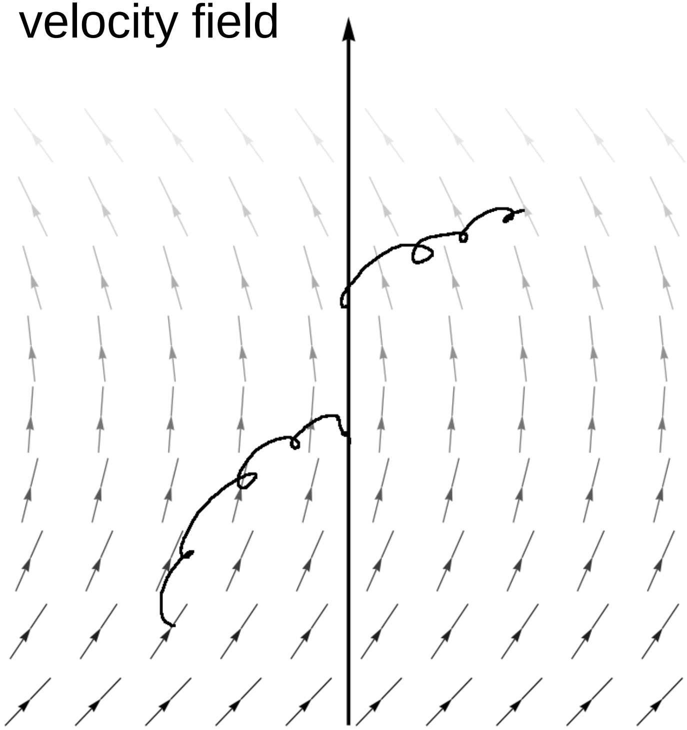
<u>Drift:</u>
Dynamic velocity field

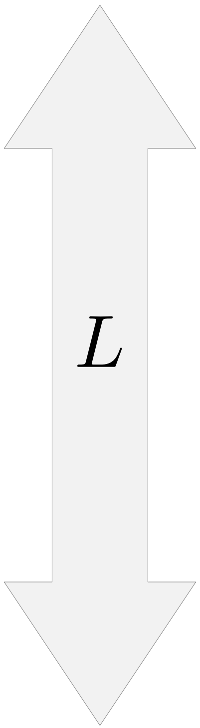


Velocity Fields

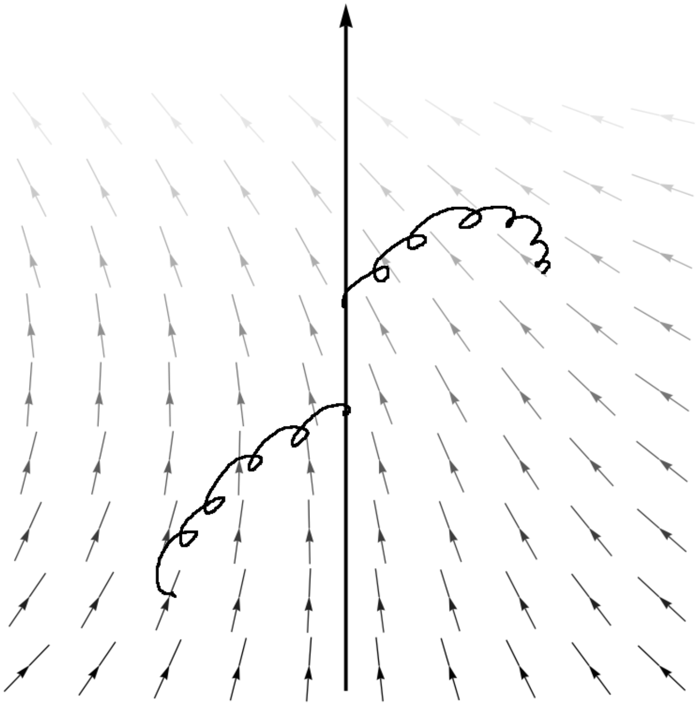
$$L = \int d\tau$$

<u>Boost:</u>
Longitudinally dynamic,
transversely uniform
velocity field





<u>Drift:</u>
Dynamic velocity field

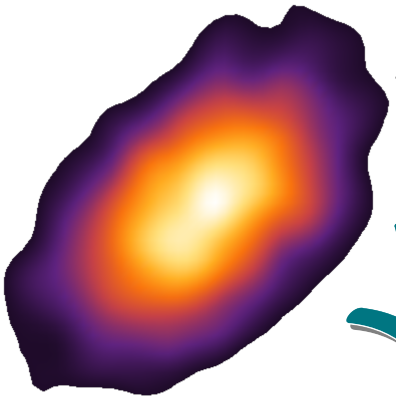


Gradient Expansion to Implement

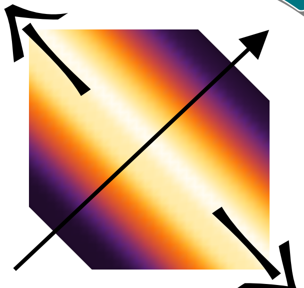
$$\int d^3x\,\rho(x)$$

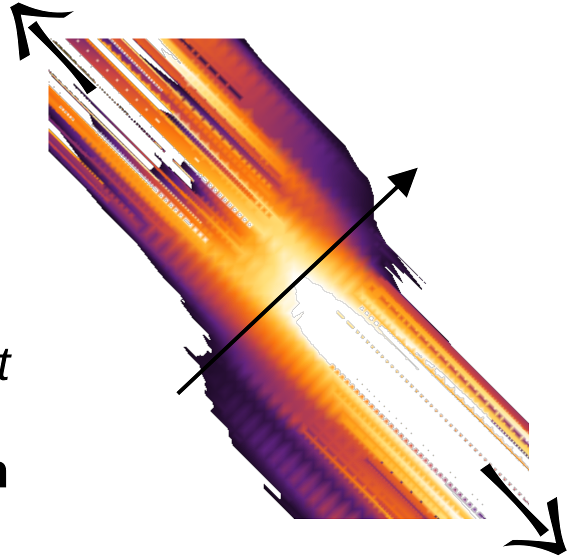
$$\int d\tau \int d^2x_\perp \left(\rho_0(\tau) + \vec{\nabla}_\perp \rho(\tau)\cdot(\vec{x}_\perp - \vec{x}_{\perp,0}(\tau))\right)$$

$$+\,\mathcal{O}(\vec{\nabla}^2_\perp \rho)$$



$$\sum_i$$

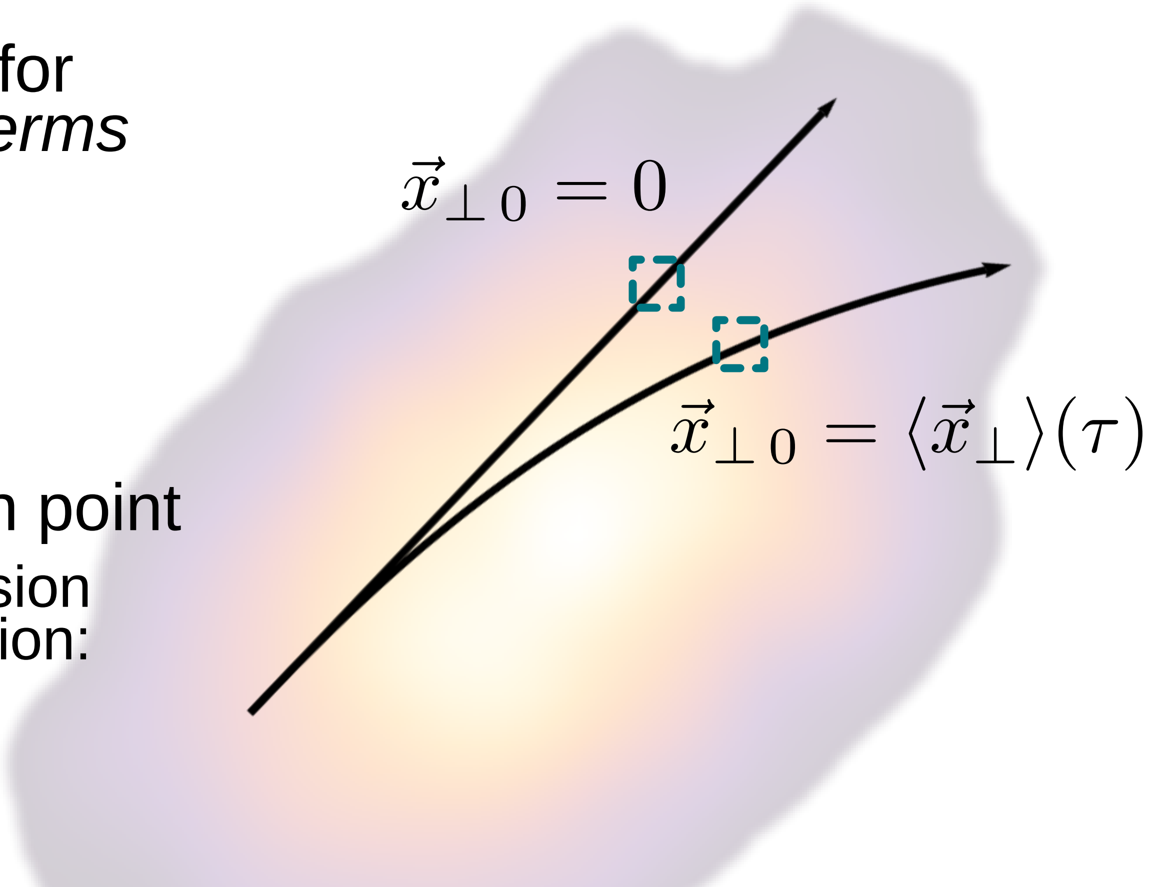




These Don't look good?
That's darn right!

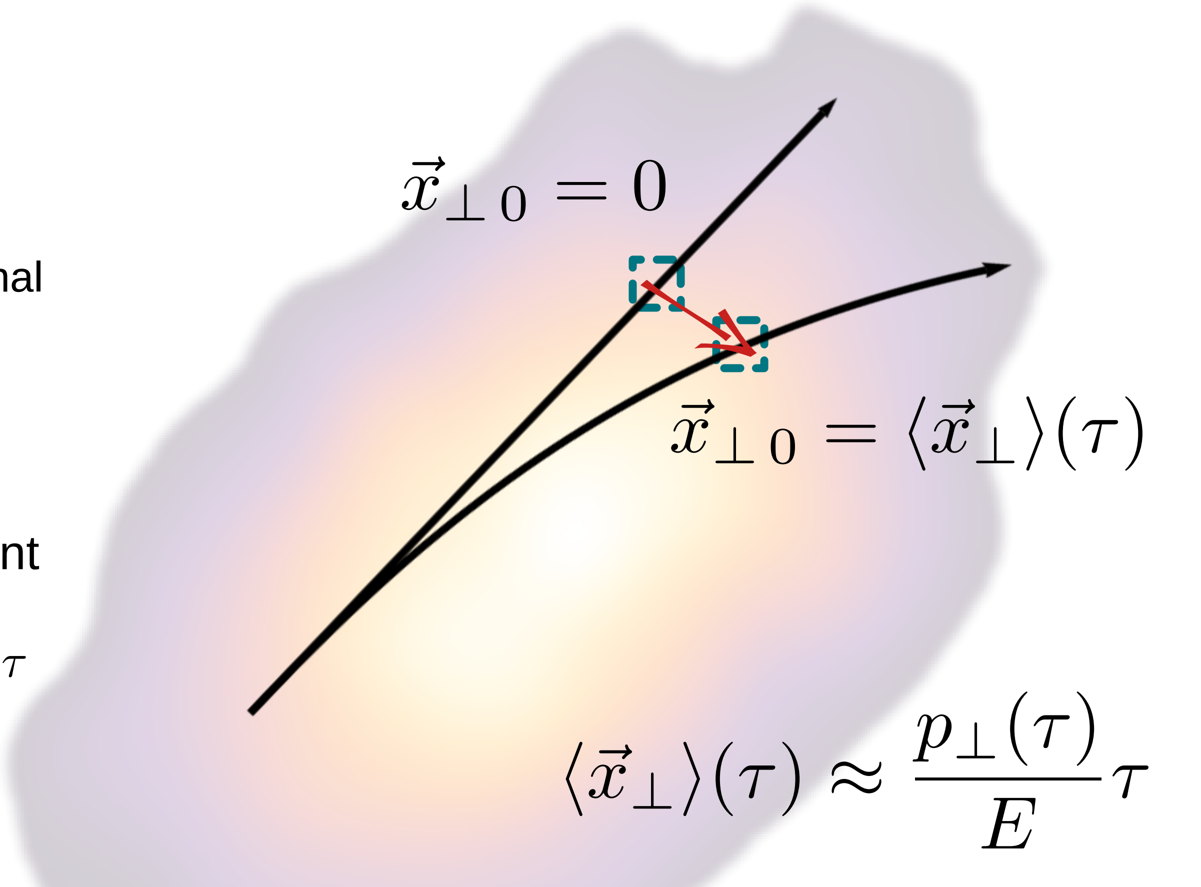
Gradients of… What and Where?

- Independent terms enter for gradients of… *separate terms* at linear order
  - Density
  - Debye Mass
  - Flow
- Free to choose expansion point
  - A more consistent expansion point for MC implementation:
    - Old – zero, straight traj.
    - New – Avg. transverse pos.



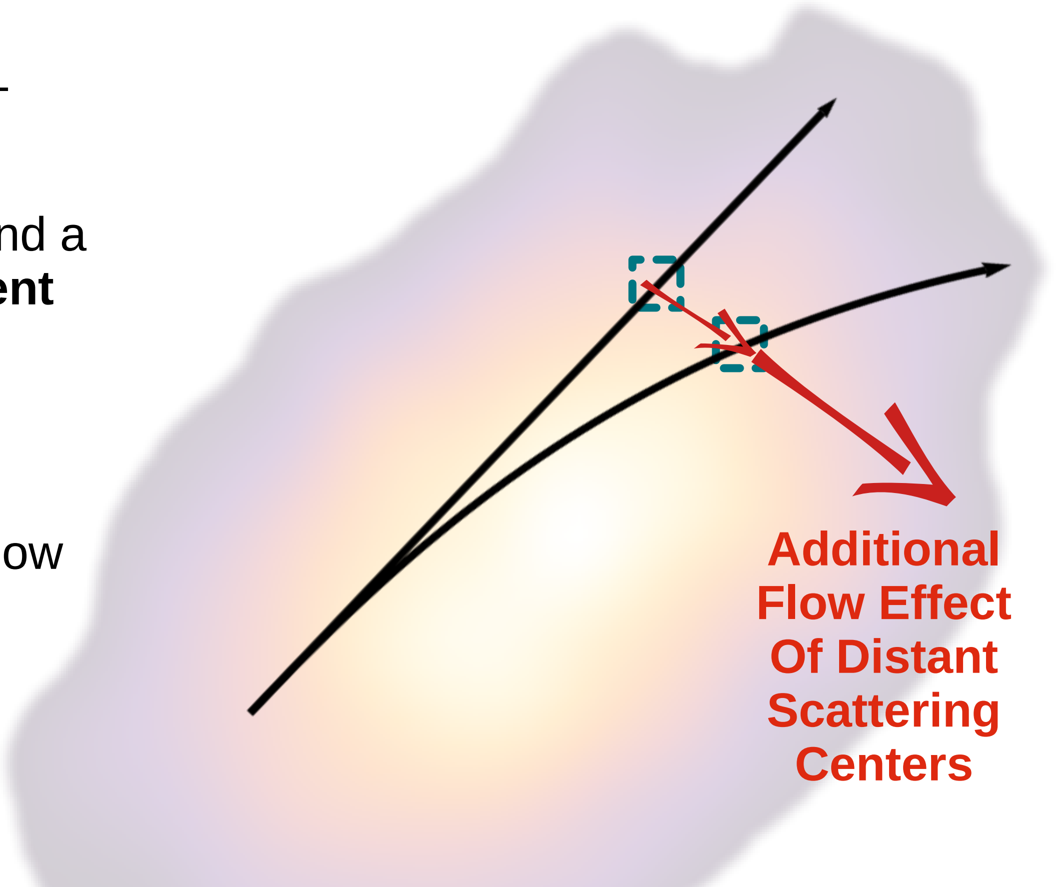
"Pure" Gradients: Pure Coordinate Transformation

- Linear order pure gradients (zero flow) are a coordinate transformation
  - Exact cancellation at first sub-eikonal order
- Centers the "waveform" about the mean $x_\perp$ position
  - In MCs, this is exactly equivalent to the dynamic updating of transverse position between $\Delta\tau$ steps

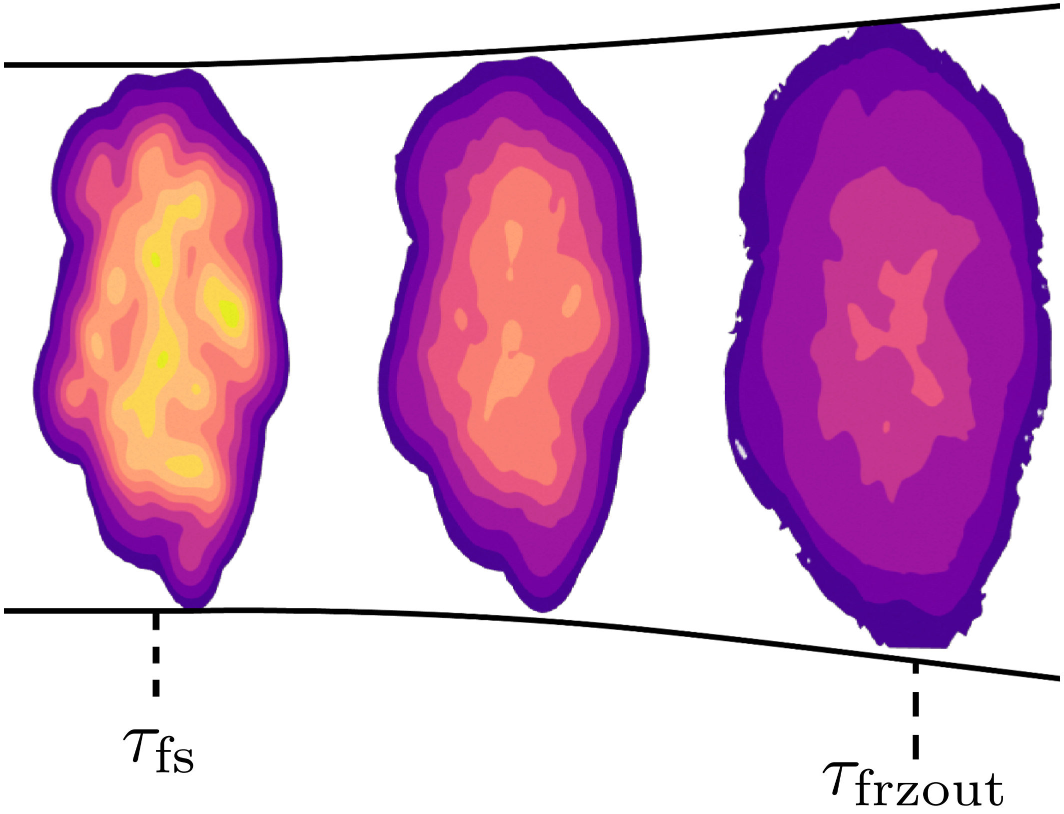


Flow x Gradients – Partial Coordinates, Partial Intrinsic

- Contains coordinate transformation + intrinsic correction
- Each can be zeroed expanding around a particular **energy and flow dependent** point
  - Not a necessary feature: special to gradient effects
- **Cannot** zero (flow x grad) + (flow x flow grad)
  - Unique interaction with flow of distant scattering centers



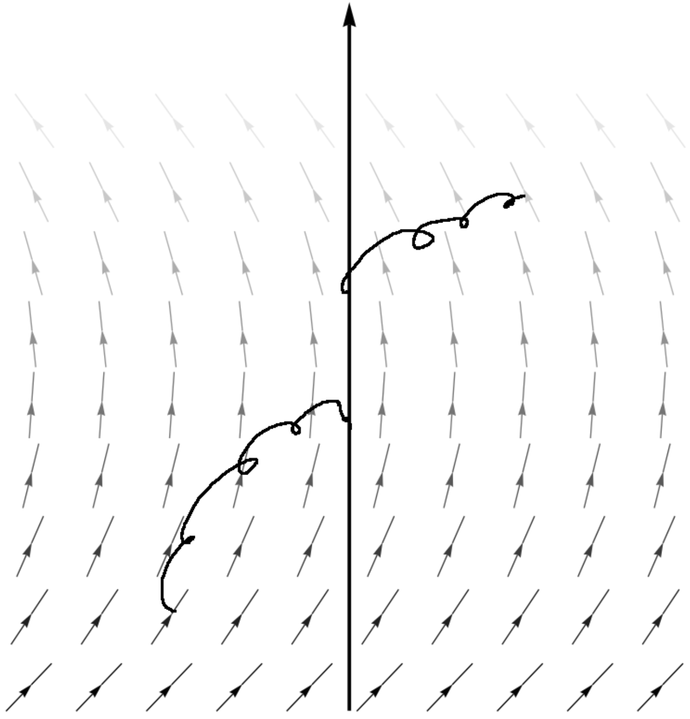
Are Gradients Small?



- Short answer:
  Not always
- Gradients are small…
  - at late times in hydro evolution
  - away from hot spots & edges
- Implementation in MC needs either:
  - careful event selection
  - higher order gradients
    (quickly becomes algebraically miserable)
- For the moment, let's put a pin in this problem...

Gradients & Boosts Punchlines

- Boost driven MCs:
  - Pure gradient effects are generally included to linear order via dynamic transverse position
- Jet Drift:
  - Locally-varying flow of scattering centers captured in higher order gradient terms
- Boosts and translations do not commute – Cannot form a global boost to replicate scattering-center level treatment
- **Boost includes no analogue of “flow x flow gradient” or SB / DB interference effects**



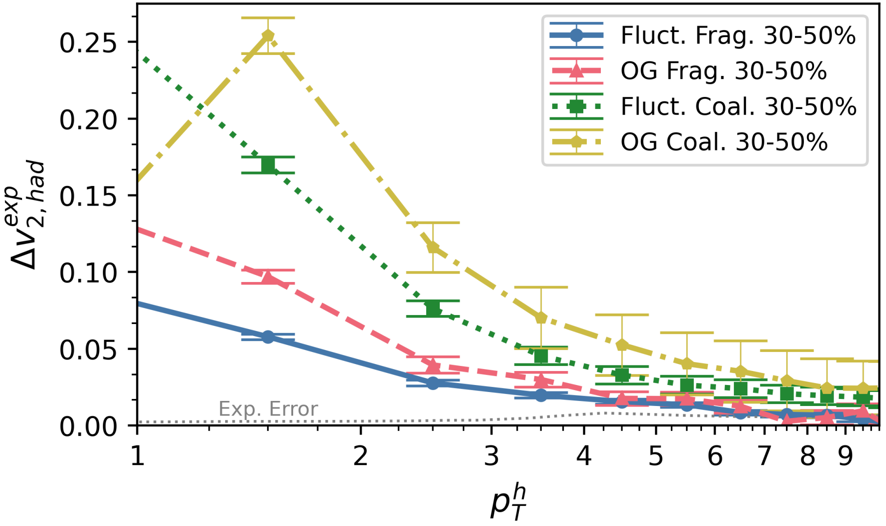
$$\vec{\nabla}_\perp u_\perp(z) = 0,\ \vec{\nabla}_\perp u_\tau(z) = 0$$

Toy Jet Substructure Phenomenology

My Goal:
Build excitement with experimentalists
about signed vector-coupled observables

Past Phenomenology

- Modulates distribution of hard particles, mainly at low & intermediate p_T
  - e.g. v_2 enhancement
- Magnitude of effect strongly dependent on hadronization and medium model details

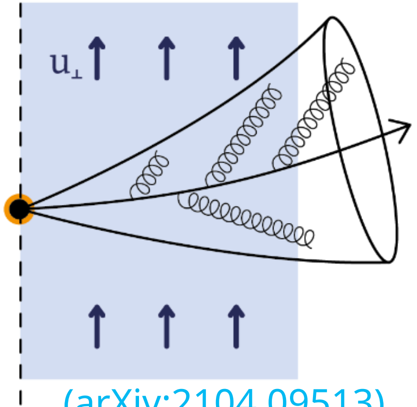


Analytic Pheno.
L. Antiporda, J. Bahder,
H. Rahman, & M. D. Sievert
Phys.Rev.D 105 (2022)
(arXiv:2110.03590)

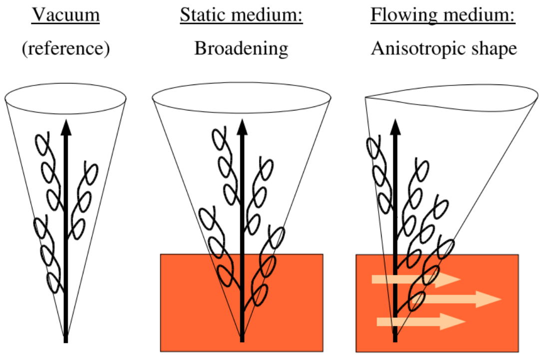
APE MC & Realistic Pheno.
J. Bahder, H. Rahman,
M. D. Sievert, & I. Vitev
PRR (2026)
(arXiv:2412.05474)

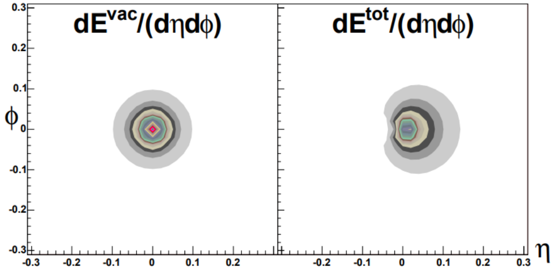
Many Thoughts on Flow Affecting Substructure

- People have been thinking about anisotropic radiation distribution for some time
  - Flow anisotropy as Lorentz boost (Armesto, Salgado, & Wiedemann)
  - Perturbative scalar calculation of radiation (Sievert, Sadofyev, Vitev)
  - Jet substructure harmonics (Barata, Milhano, & Sadofyev)
  - Perturbative real gluon calculation of radiation (Kuzmin & López)
- Elastic scatterings contribute to similar effects!

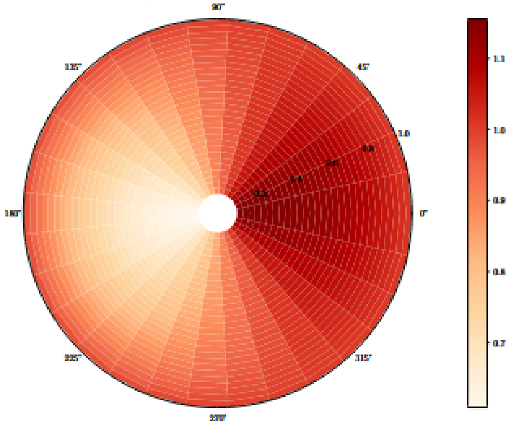


(arXiv:2104.09513)

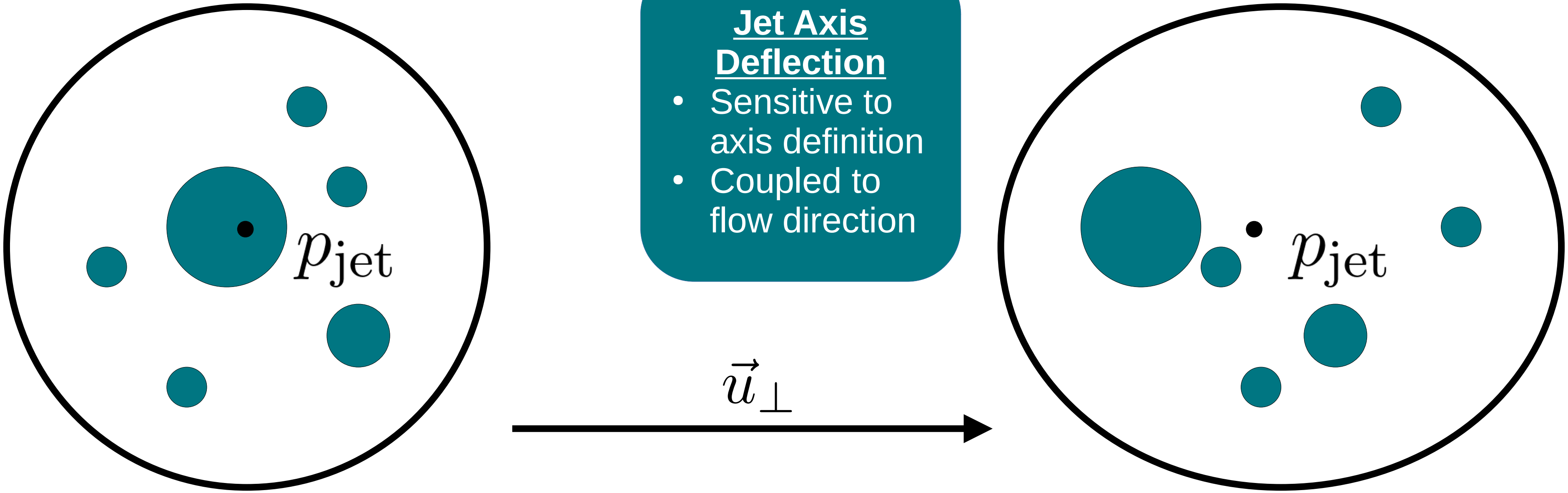




(arXiv:0405301)


(arXiv:2308.01294)

Jet Substructure Sensitivity to Drift



Jet Axis Deflection

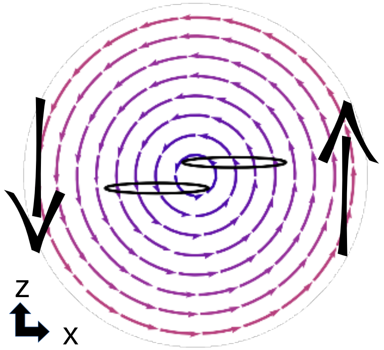
- Sensitive to axis definition
- Coupled to flow direction

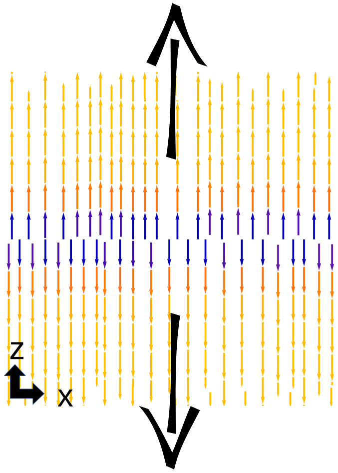
Jet Shape Distortion

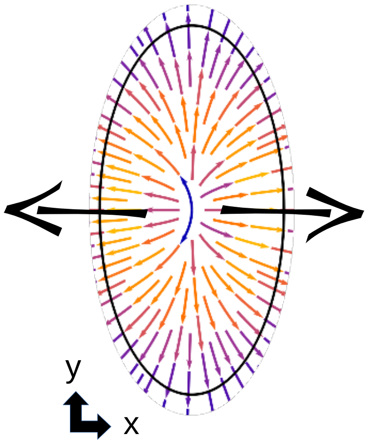
- $+v_1$ coupled to flow direction
- $+v_2$ coupled to flow direction axis

What Flow Do We Couple To?

- Longitudinal Flow
  - Expansion flow
  - Vorticity flow (neglected here)
- Transverse Plane Flow
  - Elliptic flow towards soft event plane







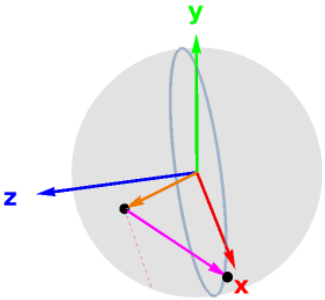
Coupling Vectors

"Towards the flow attractor" often ambiguous...

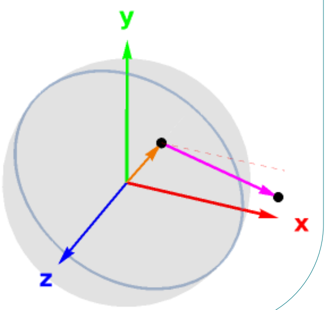
Transverse Plane Flow:

$$\vec{\alpha}_x = \text{sign}(p_{jet,\,x})\,(\hat{x} - (\hat{x}\cdot\hat{p}_{jet})\hat{p}_{jet})$$

$$\alpha_{x2} = \text{sign}(p_x^{jet} p_y^{jet})(\hat{p}_{jet} \times \hat{z})$$

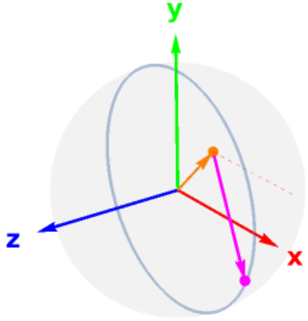


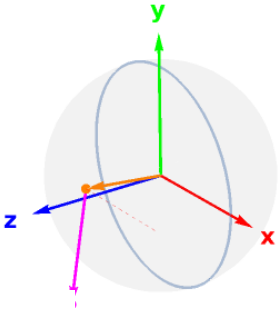
Must be near mid-rapidity and far from ψ_2 or $\psi_2 + \pi/2$

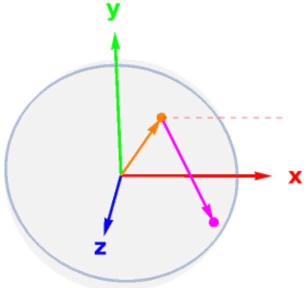


"Total" flow vector:

$$\alpha_{total} = \text{sign}(p_y)\,((\hat{y}\cdot\hat{p}_{jet})\hat{p}_{jet} - \hat{y})$$



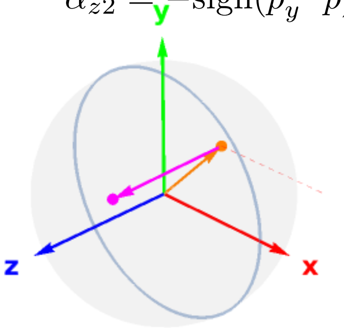




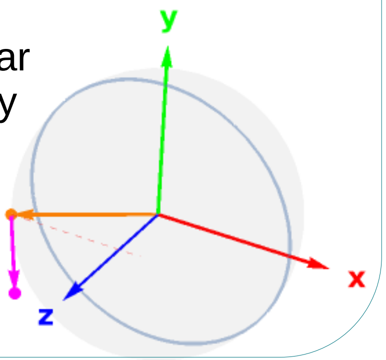
Longitudinal Plane Flow:

$$\vec{\alpha}_z = \text{sign}(p_{jet,\,z})\,(\hat{z} - (\hat{z}\cdot\hat{p}_{jet})\hat{p}_{jet})$$

$$\alpha_{z2} = -\text{sign}(p_y^{jet} p_z^{jet})(\hat{p}_{jet} \times \hat{x})$$



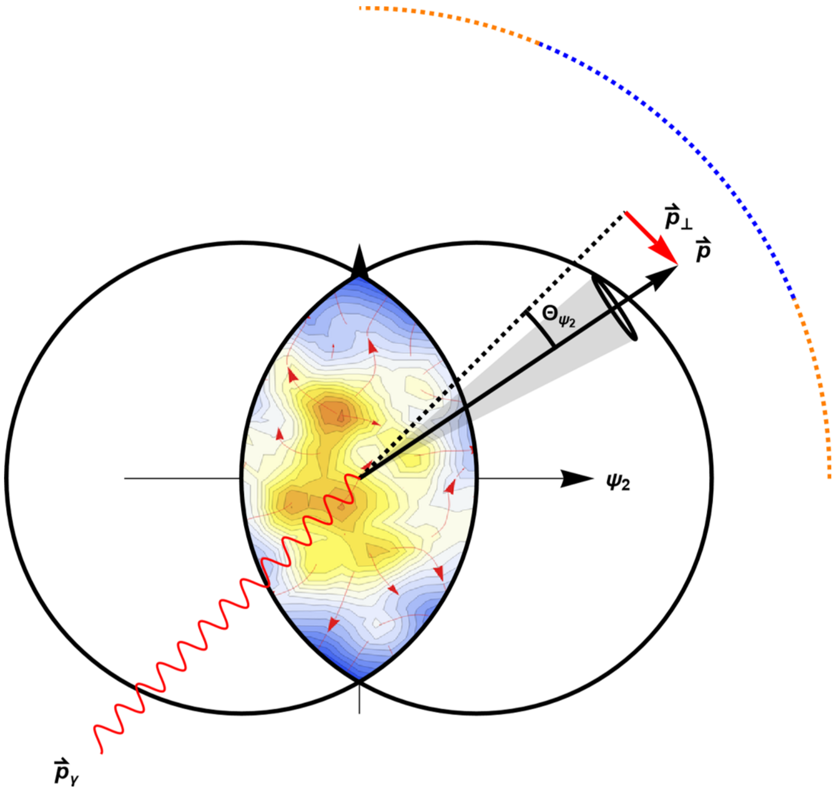
Must be near mid-rapidity



Unsigned version from
STAR: Photon-jet acoplanarity

Signed Jet Deflection

- Sensitive to absolute deflections towards the event plane
  - Can form similar measure for jet constituents
  - Most direct measurement of flow deflections
- Requires **azimuthal fencing**
- Rapidity difference for longitudinal flow coupling



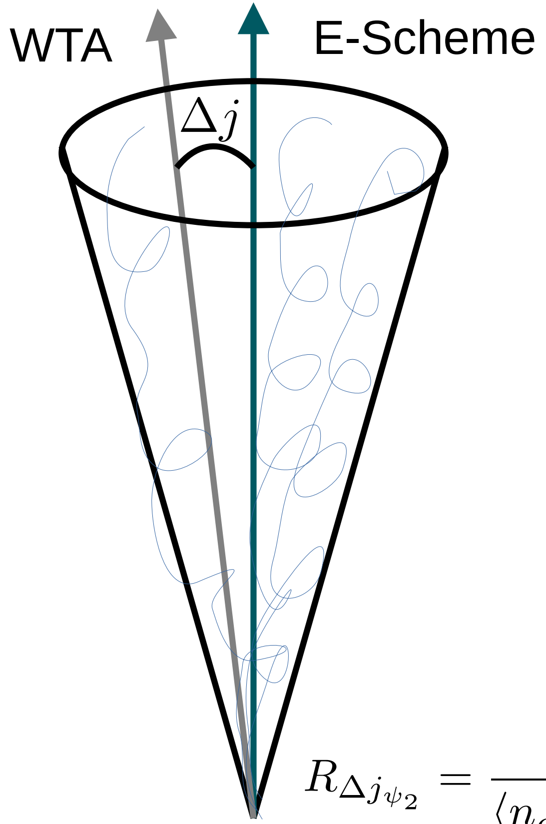
$$R_{\theta_{\psi_2}} = \frac{\frac{dN^{AA}}{d\Theta_{\psi_2}}}{\langle N_{coll}\rangle \frac{dN^{pp}}{d\Theta_{\psi_2}}}$$

For x-axis event plane

$$\Theta_{\psi_2} = (\mathrm{mod}(\phi_\gamma, \pi/2) - \mathrm{mod}(\phi_{jet}, \pi/2)) \begin{cases} +1, & \text{Quadrants I \& III} \\ -1, & \text{Quadrants II \& IV} \end{cases}$$

Signed Jet Axis Difference

Unsigned versions from
CMS: Dijets & Photon Tagged Jets
ALICE: D0 Tagged Jets



E-Scheme:
Four-vector sum of momenta at each step
- Sensitive to many deflected soft particles

Winner-Take-All (WTA):
Follows hardest prong at each step
- Sensitive to hard undeflected core of jet

$$\Delta j = \sqrt{(\eta_{WTA} - \eta_{E-scheme})^2 + (\phi_{WTA} - \phi_{E-scheme})^2}$$

$$R_{\Delta j_{\psi_2}} = \frac{\frac{dN^{AA}}{d\Delta j_{\psi_2}}}{\langle n_{coll} \rangle \frac{dN^{pp}}{d\Delta j_{\psi_2}}}$$

$$\Delta j_{\psi_2} = (\phi_{WTA} - \phi_{E-scheme}) \begin{cases} +1, & \text{Quadrants I \& III} \\ -1, & \text{Quadrants II \& IV} \end{cases}$$

For x-axis event plane

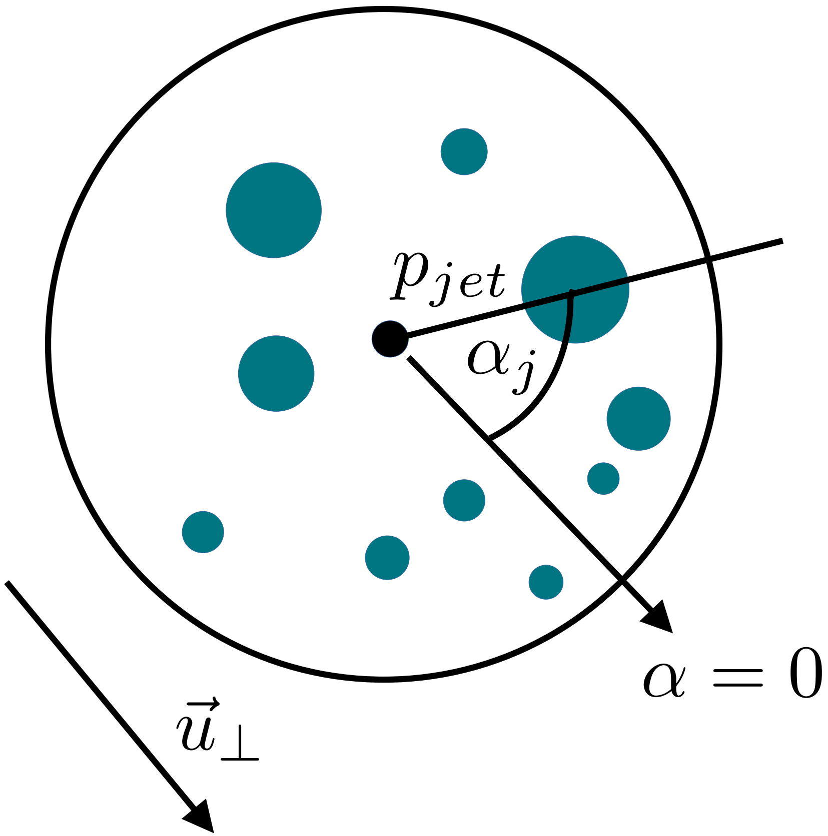
Jet Substructure Harmonics

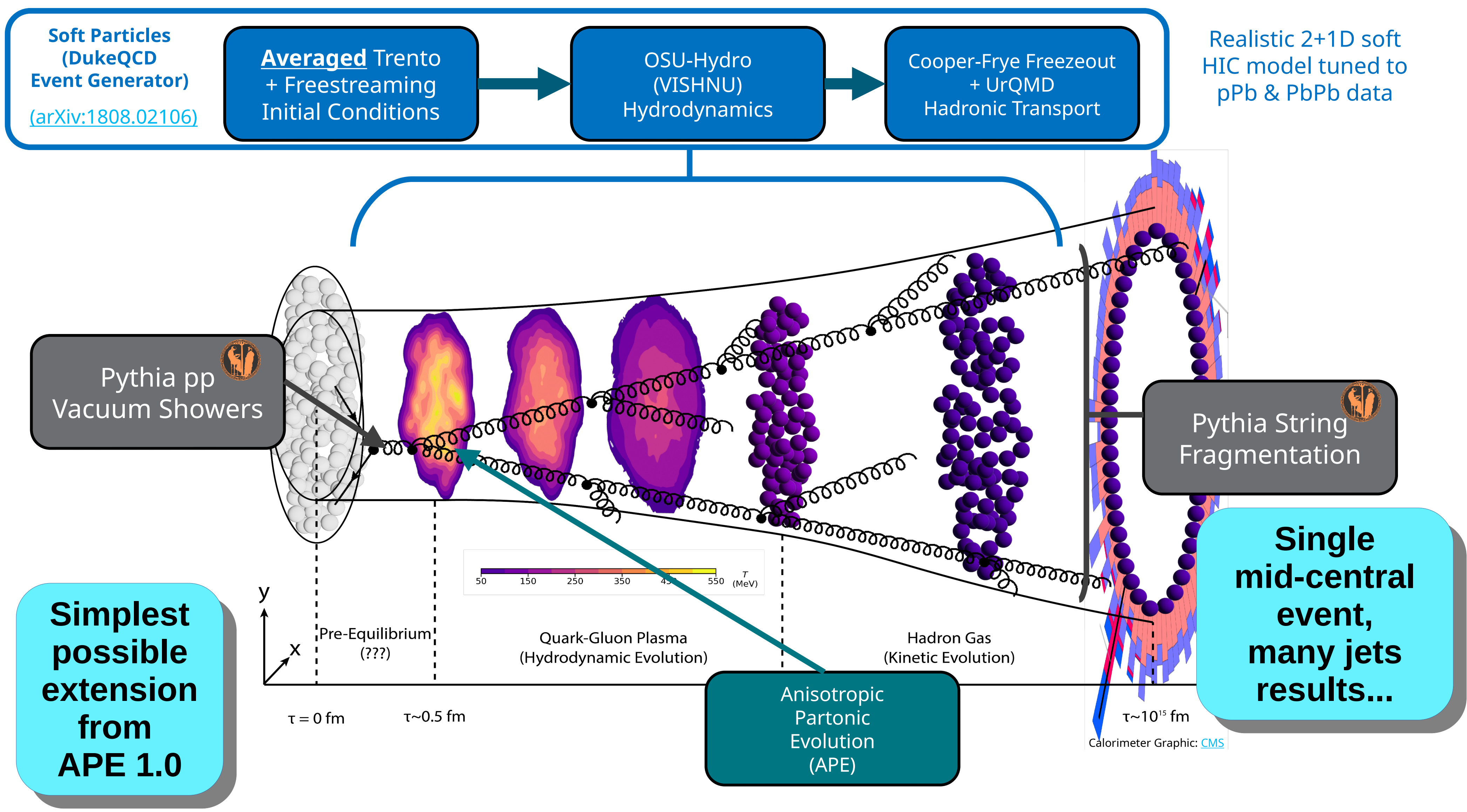
- Phase alignment with flow coupling direction
- Measures azimuthal distortion towards flow
- Highly dependent on your jet algorithm
  - E.g. anti-kT, enforces a circular jet on $y\phi$ cylinder
- We'll focus on this one for the moment...

$$v_n(E) = \left\langle \frac{\sum_j \cos(n\alpha_j)}{\sum_j} \right\rangle_{events}$$

To eliminate contribution from pp dist.:

$$\Delta v_n(E) = v_n^{AA}(E) - v_n^{pp}(E)$$



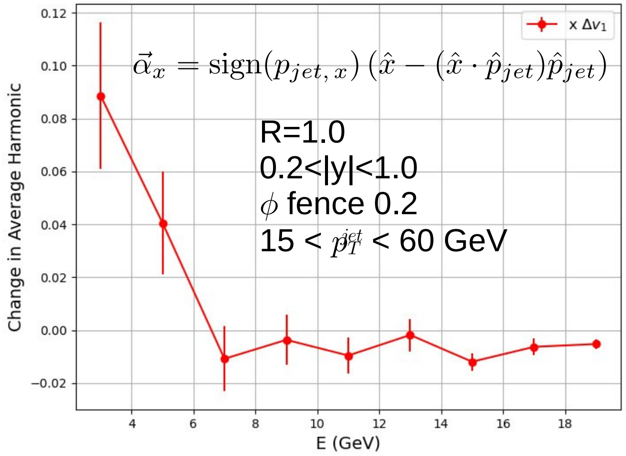


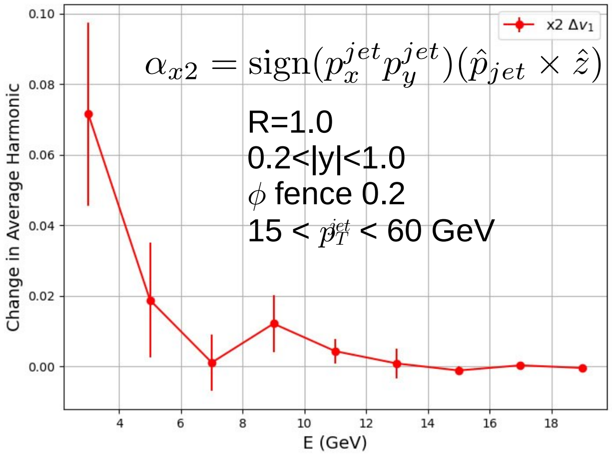
Soft Particles (DukeQCD Event Generator)
(arXiv:1808.02106)
Averaged Trento + Freestreaming Initial Conditions
OSU-Hydro (VISHNU) Hydrodynamics
Cooper-Frye Freezeout + UrQMD Hadronic Transport
Realistic 2+1D soft HIC model tuned to pPb & PbPb data
Pythia pp Vacuum Showers
Pythia String Fragmentation
50
150
250
350
550
T (MeV)
y
x
Pre-Equilibrium (???)
Quark-Gluon Plasma (Hydrodynamic Evolution)
Hadron Gas (Kinetic Evolution)
τ = 0 fm
τ~0.5 fm
τ~10^15 fm
Calorimeter Graphic: CMS
Anisotropic Partonic Evolution (APE)
Simplest possible extension from APE 1.0
Single mid-central event, many jets results...

Coupling to Transverse Flow

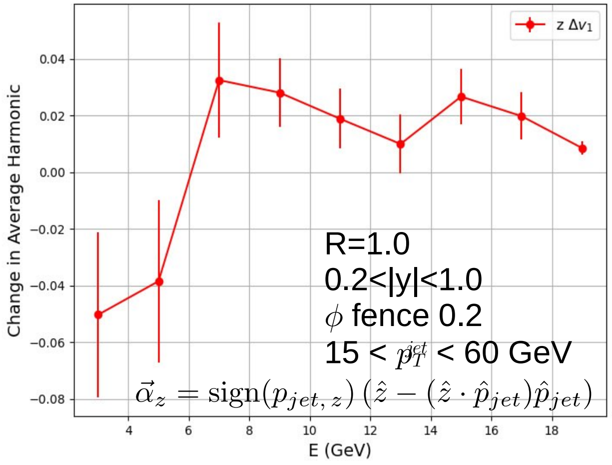
- Jets see substantial v1 enhancement below 5 GeV, but result negligible at intermediate pT
- Selection cuts on weak energy loss enhance the effect and interesting pT range

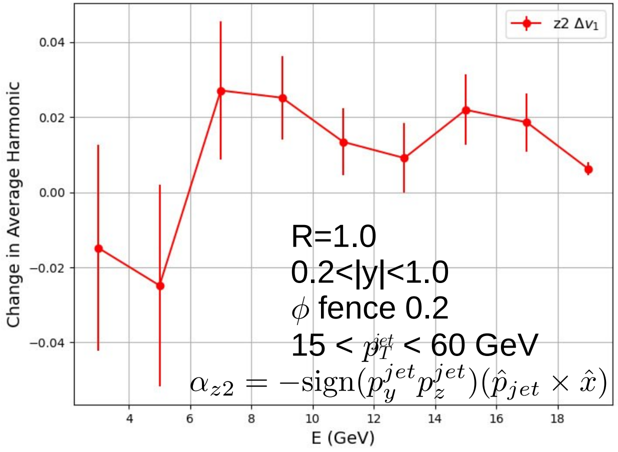
Apologies for the hard to read figures…





Coupling to Longitudinal Flow

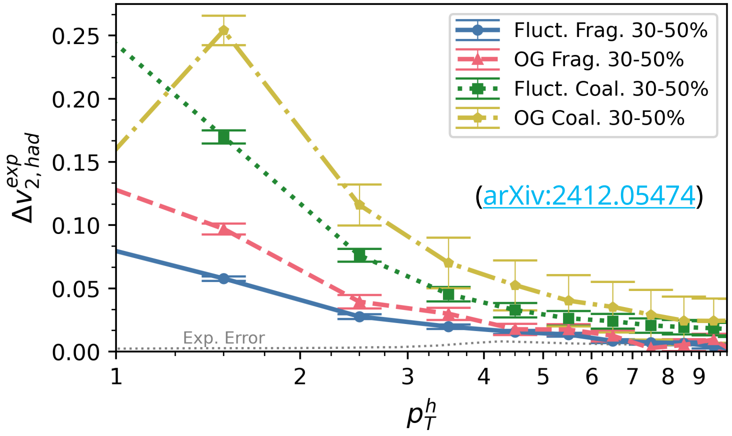


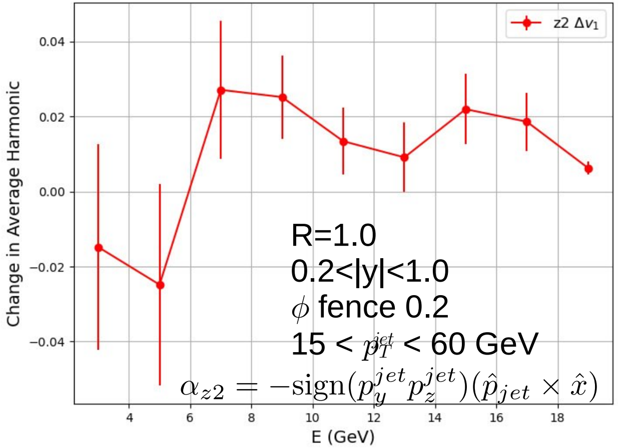


- Model longitudinal flow coupling much more consistent near mid-rapidity
  - Rare sign changes
- True observable would be sensitive to vorticity, which is difficult to sign.
  - Possible that short pathlengths in competing regions would be of little concern
  - Easy to perform model calculation

Intermediate pT Results?

- Why is the longitudinal effect nonzero at intermediate pT, for sub-eikonal effect?
  - Recombination-type hadronization of any sort dramatically enhances the effect – see old results
  - Lower pT partons “share” deflections to form deflected higher pT hadrons





Thanks a bunch!

See APE Phenomenology:
(arXiv:2412.05474)

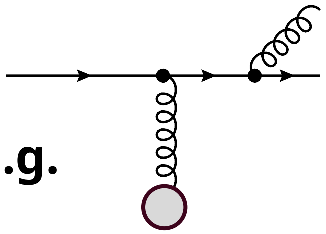
Future MC Plans:

- Flow x Gradient terms implementation
- Coupled evolution (virtuality + in-medium)
- EBE runs
  - Scanning new flow-sensitive observables and full phase space

Radiative Energy Loss

$$\frac{dE}{d\ell} = -\frac{d}{dL}\left(\frac{2C_R\alpha_s}{\pi}\frac{L}{\lambda}E\int_{k_{min}}^{k_{max}}\frac{dk}{k}\int_0^{q_{max}}dq\,q\int_0^{2\pi}d\phi \times \frac{\mu^2}{\pi(q^2+\mu^2)^2}\frac{2\mathbf{k}\cdot\mathbf{q}\,(\mathbf{k}-\mathbf{q})^2L^2}{16x^2E^2+(\mathbf{k}-\mathbf{q})^4L^2}\right)_{L=\ell}$$

E.g.



- Gyulassy, Levai, Vitev (2000) (arXiv:0006010)
- Single Emission GLV @ 1st order in opacity
  - APE 1.0: w/ finite kinematic bounds (q, k)
  - APE 2.0: Currently infinite kinematic bounds
    - Plans to implement anisotropy of radiation

Drift Deflection

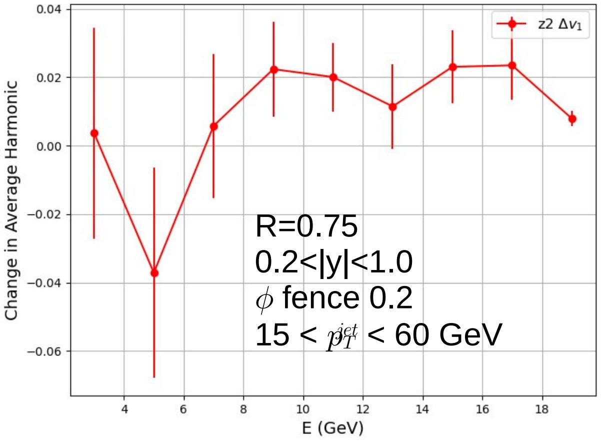
$$\langle \vec{q}_{drift}\rangle = \hat{e}_\perp \int d\tau\,\frac{3}{E(\tau)}\frac{\mu^2(\tau)}{\lambda(\tau)}\ln\frac{E(\tau)}{\mu(\tau)}\frac{u_\perp(\tau)}{1-u_\parallel(\tau)}$$

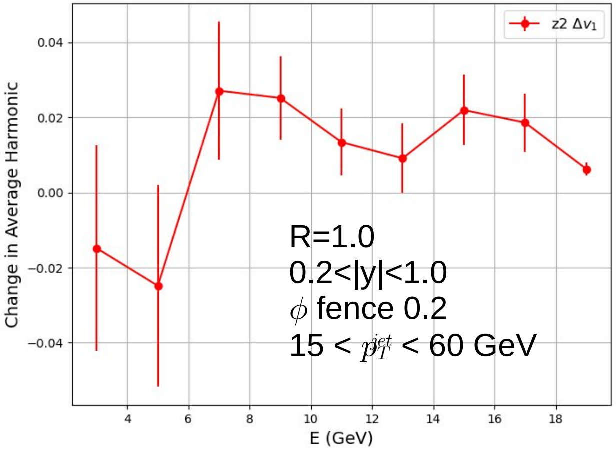
Model Choices & Selection Cuts

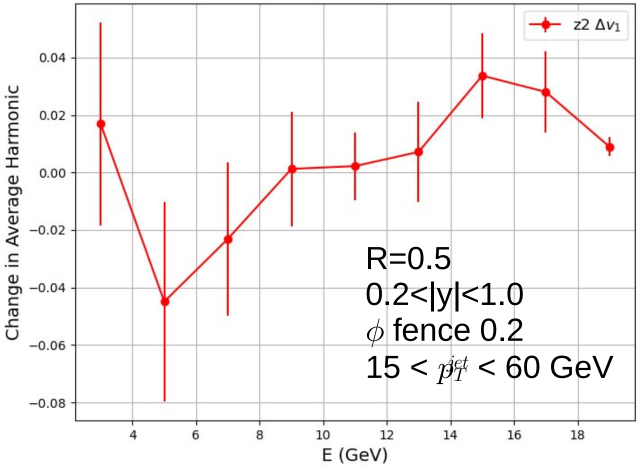
- Jet-jet particle hadronization only
  - Partons color is unchanged by medium – use Pythia color connections in hadronization
- Perfect background subtraction
- Vacuum shower completed before medium interaction
- No medium response
  - No wake, etc.
- Do not evolve particles y > 1
- Hard Particles “thermalize” at 0.75 GeV

- Hard scattering 10 < pThat < 100 GeV
- Hard scattering |Yhat| < 0.3
- Fastjet anti-kT, WTA-pT axis
  - arXiv:1111:6097
- Single event geometry – averaged Duke event generator IC @ 5.02 TeV PbPb

Shrinking Jet Cone

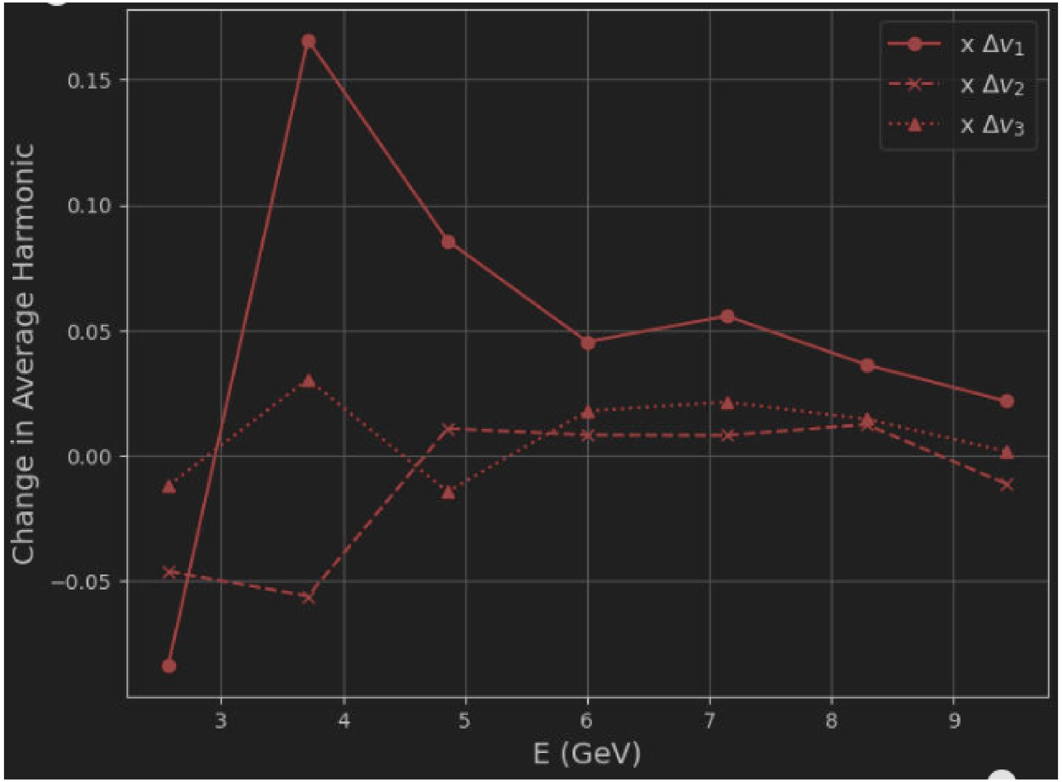




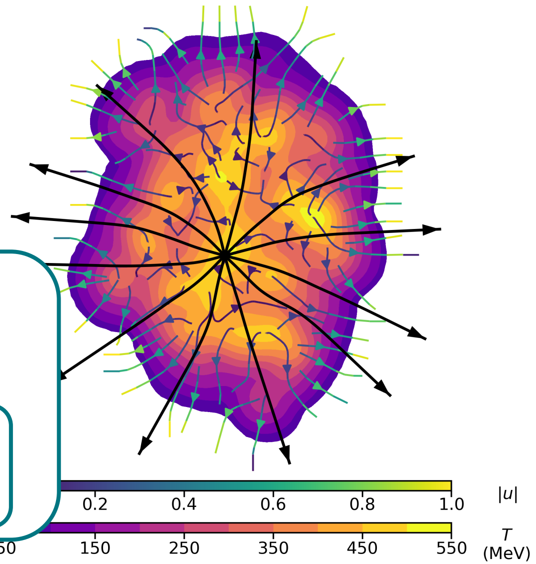


Selecting on Weak Energy Loss

- Hard scatterings set at pThat 25 GeV
- Select on 15 – 25 GeV Jets
- Biases towards small energy loss jets
- Anti-kT R=1.0, E-Scheme



Drift in APE 1.0



APE & Drift Pheno.
J. Bahder, H. Rahman, M. D. Sievert, & I. Vitev
Physical Review Research (2025/2026)
(arXiv:2412.05474)

Average effect as transport equation

Neglects longitudinal variation over $\Delta\tau$

Neglects transverse variation