Particle Multiplicity of QCD jets:

~~vacuum KNO scaling~~ and medium decoherent quenching

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JET-SOFT CORRELATIONS
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- multiplicity of jet is related to the radiation and showering pattern (and hadronization) of high energy parton.
- Average multiplicity can be calculated by zero-th Mellin moment of fragmentation function.

$$\bar{n}(s) = \tilde{D}(j=1), \quad \tilde{D}(j) \equiv \int \mathrm{d}z \; z^{j-1} D(z) \tag{1}$$

- The evolution equation follows DGLAP

$$t\frac{\partial}{\partial t} D_a(z,t) = \int \frac{\mathrm{d}\xi}{\xi} \frac{\alpha_s}{2\pi} P(z) D(z/\xi, \xi^2 t)$$
$$t\frac{\partial}{\partial t} \tilde{D}_a(j,t) = \frac{\alpha_s}{2\pi} \int \mathrm{d}z \; z^{j-1} P(z) \tilde{D}(j, z^2 t) \tag{2}$$

- Assume the simple form

$$\tilde{D}(j,t) \propto t^{\gamma(j,\alpha_s)} \tag{3}$$

- The anomalous dimension satisfy the following

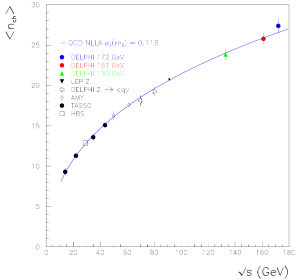
$$\gamma(j,\alpha_s) = \frac{\alpha_s}{2\pi} \int \mathrm{d}z \; z^{j-1+2\gamma(j,\alpha_s)} P(z) \tag{4}$$

- solving by taking soft approximation

$$\gamma(j,\alpha_s) = \frac{C_A \alpha_s}{\pi} \frac{1}{j-1} - 2\left(\frac{C_A \alpha_s}{\pi}\right)^2 \frac{1}{(j-1)^3} + \cdots \tag{5}$$

- plucking this back and promote the coupling to running, the average multiplicity can be represented as

$$\bar{n}(s) \propto \exp\left[\sqrt{\frac{2C_A}{\pi b} \ln \frac{s}{\Lambda^2}}\right] \tag{6}$$



- The generating function(al) (GF) can be written as [Bassetto, Ciafaloni, and Marchesini, NPB 163 (1980) 477]

$$\mathcal{Z}_a\left(u(k,\theta),Q\right) = \sum_{n=0} \int \mathrm{d}\Phi_n \ \mathcal{P}_a\left(\{(k_i,\theta_i)\},Q\right) \prod_{i=1}^{n} u(k_i,\theta_i) \tag{7}$$

$$Z_a(u,Q) = \sum_{n=0} P_a(n,Q)u^n = \langle u^n \rangle \tag{8}$$

- One can get the n-particle production probability be successive differentiation

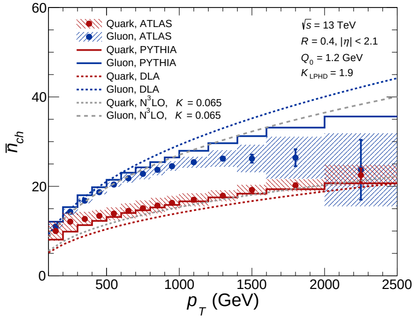
$$P_a(n,Q) = \frac{1}{n!}\left(\frac{\partial}{\partial u}\right)^n Z_a(u,Q)\bigg|_{u=0} \tag{9}$$

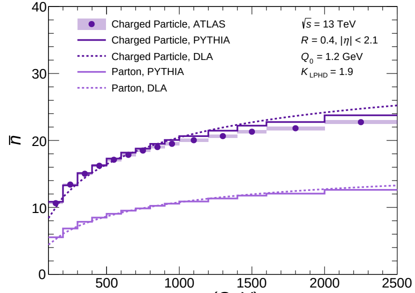
- The average multiplicity is simply the first u-differentiation

$$\bar{n}_a(Q) = \sum_{n=0} nP_a(n,Q) = \frac{\partial}{\partial u} Z_a(u,Q)\bigg|_{u=1} \tag{10}$$

- The GF follows the DGLAP evolution equation [Dokshitzer, Khoze, Mueller, and Troian, 1991]

$$\frac{\partial}{\partial \ln Q^2} Z_a(u,Q) = \int \mathrm{d}z \ \frac{\alpha_s}{2\pi} P_{a\to bc}(z) \left[Z_b(u,zQ)Z_c(u,(1-z)Q) - Z_a(u,Q)\right] \tag{11}$$





- In the double-logarithmic approximation (DLA)

$$\frac{\partial}{\partial \ln Q} Z_a(u,Q) = Z_a(u,Q) c_a \int \frac{dz}{z} \gamma_0^2 \left[Z_g(u,Q) - 1 \right] \qquad (12)$$

- One can solve the above (linear) [Dokshitzer, Khoze, Mueller, and Troian, 1991]

$$Z_a(u,Q) = u \exp\left\{ c_a \int_0^y dy' \ (y-y')\gamma_0^2 \left[Z_g(u,y') - 1 \right] \right\}, \qquad (13)$$

- The probability distribution can be calculated recursively [Duan, **LC**, Ma, Salgado, and Wu, PRD 112 (2025) 9, 094022]

$$P_a(1,Q) = \exp\left[-c_a \int_0^y dy' (y-y')\gamma_0^2 \right] \qquad (14)$$

$$P_a(n+1,Q) = c_a \sum_{k=1}^{n} \frac{k}{n} P_a(n+1-k,Q) \int_0^y dy' \ (y-y')\gamma_0^2 P_g(k,y')$$

- The average multiplicity is then [Dokshitzer et.al., ZPC 18 (1983) 37].

$$\bar{n}_g(Q) = z_1 \left[I_1(z_1) K_0(z_2) + K_1(z_1) I_0(z_2) \right] \qquad (15)$$

$$\lambda = \ln(Q_0/\Lambda), \ z_1 \sim \sqrt{y+\lambda}, \ z_2 \sim \sqrt{\lambda} \qquad (16)$$

- We begin with the LO cross section in pp

$$\frac{\mathrm{d}\sigma_i^{pp}}{\mathrm{d}p_T} = 2p_T \sum_{a,b,c,d} (\delta_{ic}+\delta_{id}) \int \mathrm{d}y_c \mathrm{d}y_d \; x_a f_{a/p}(x_a,\mu^2) x_b f_{b/p}(x_b,\mu^2) \frac{\mathrm{d}\hat{\sigma}_{ab\to cd}}{\mathrm{d}\hat{t}} \tag{17}$$

- where the $2 \to 2$ matrix element are calculated by

$$\hat{s} = x_a x_b s,\ \hat{t} = -x_a p_T \sqrt{s} e^{-y_c},\ \hat{u} = -x_b p_T \sqrt{s} e^{y_c},\ x_a = \frac{p_T}{\sqrt{s}}\left(e^{y_c}+e^{y_d}\right),\ x_b = \frac{p_T}{\sqrt{s}}\left(e^{-y_c}+e^{-y_d}\right) \tag{18}$$

- in the coherent picture, the AA cross section is written as

$$\frac{d\sigma^{AA}}{d^2\vec{b}\, dp_T} = \int d^2\vec{r}\, T_A(\vec{r}+\vec{b}/2) T_B(\vec{r}-\vec{b}/2) \int \frac{d\phi}{2\pi} \sum_i \int d\epsilon D_i(\epsilon) \left.\frac{d\sigma_i^{NN}}{dp'_T}\right|_{p'_T = p_T+\epsilon} \tag{19}$$

- in the decoherence picture, we have [Duan, **LC**, Ma, Salgado, and Wu, 2603.22014]

$$\begin{aligned} \frac{\mathrm{d}\sigma^{AA}}{\mathrm{d}^2\vec{b}\ \mathrm{d}p_T} &= \int d^2\vec{r}\ d^2\vec{b}\, T_A(\vec{r}+\vec{b}/2) T_B(\vec{r}-\vec{b}/2) \int \frac{d\phi}{2\pi} \\ &\times \sum_i \Bigg[P_i(1, p'_T R) \int d\epsilon_1 D_i(\epsilon_1) \left.\frac{d\sigma_i^{NN}}{dp'_T}\right|_{p'_T = p_T+\epsilon_1} \\ &+ \sum_{n=2}^{N} P_i(n, p'_T R) \int d\epsilon_1 D_i(\epsilon_1) \left(\prod_{m=2}^{n} \int d\epsilon_m D_g(\epsilon_m) \right) \left.\frac{d\sigma_i^{NN}}{dp'_T}\right|_{p'_T = p_T + \sum_{k=1}^{n} \epsilon_k} \Bigg], \end{aligned} \tag{20}$$

- The medium induced radiation spectrum [Baier, Dokshitzer, Mueller, and Schiff, NPB 531 403 (1998)]

$$\omega \frac{\mathrm{d}I_i}{\mathrm{d}\omega} = \frac{2\alpha_s C_i}{\pi} K_i(x) \ln \left| \cos \left[(1+i) \left(\frac{\omega_c}{2\omega} \frac{1-x+C_i x^2/N_c}{1-x} \right)^{1/2} \right] \right| \tag{21}$$

- with splitting function

$$K_i(x) = \begin{cases} \frac{1}{2}\left[1+(1-x)^2\right], & \text{for } i = q, \bar{q}, \\ x\left[\frac{x}{1-x} + \frac{1-x}{x} + x(1-x)\right], & \text{for } i = g \end{cases} \tag{22}$$

- The radiation probability is written as

$$D_i(\epsilon) = \sum_{n=0}^{\infty} \frac{1}{n!} \left[\prod_{k=1}^{n} \int \mathrm{d}\omega_k \frac{\mathrm{d}I_i(\omega_k)}{\mathrm{d}\omega} \right] \delta\left(\epsilon - \sum_{k=1}^{n} \omega_k \right) \exp\left[-\int \mathrm{d}\omega \, \frac{\mathrm{d}I_i(\omega)}{\mathrm{d}\omega} \right] \tag{23}$$

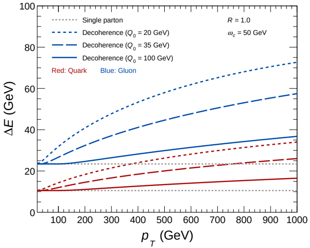
- Asymptotically, it can be solved [Baier, Dokshitzer, Mueller, and Schiff, JHEP 09 (2001) 033]

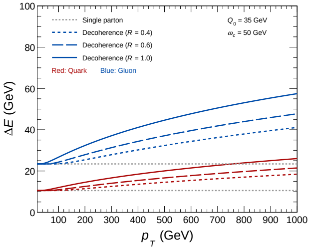
$$\epsilon D_i(\epsilon) = \alpha_i \sqrt{\frac{\omega_c}{2\epsilon}} \exp\left[-\frac{\pi \alpha_i^2 \omega_c}{\epsilon} \right], \quad \alpha_i \equiv \frac{2\alpha_s C_i}{\pi} \tag{24}$$

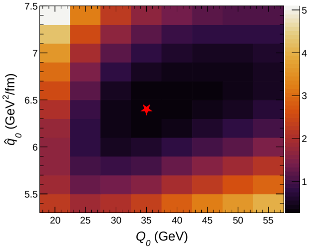
- We solve numerically to get the quenching weights [Salgado and Wiedemann, PRD 68 014008 (2003)]

$$\begin{aligned} D_i(\epsilon) &= \int_{-\infty}^{\infty} \frac{d\nu}{2\pi} \exp\left[i\nu\epsilon - \int_0^{\infty} d\omega \, \frac{dI_i(\omega)}{d\omega} \left(1 - e^{-i\nu\omega}\right) \right] \\ &= \int_0^{\infty} \frac{d\nu}{\pi} \exp\left\{ -\int_0^{\infty} d\omega \, \frac{dI_i(\omega)}{d\omega} \left[1 - \cos(\nu\omega)\right] \right\} \cos\left(\nu\epsilon - \int_0^{\infty} d\omega \, \frac{dI_i(\omega)}{d\omega} \sin(\nu\omega) \right) \end{aligned} \tag{25}$$

MULTIPLICITY - MEAN ENERGY LOSS





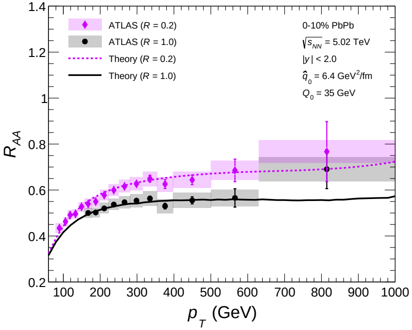


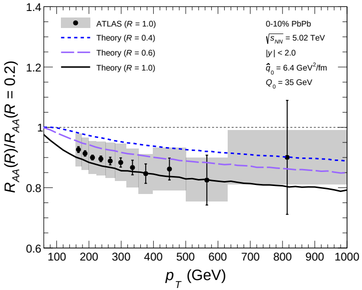
[Duan, **LC**, Ma, Salgado, and Wu, 2603.22014]

- The mean medium induced energy loss in the decoherent picture is

$$\Delta E_i = \sum_{n=1} P_i(n, p_T R) \int \mathrm{d}\epsilon_1 D_i(\epsilon_1) \prod_{m=2}^{n} \int \mathrm{d}\epsilon_m D_g(\epsilon_m) \left(\sum_{k=1}^{n} \epsilon_k \right) \tag{26}$$

- As Q_0 decrease, multiplicity increase, ΔE increase.
- As R increase, multiplicity also increase, ΔE increase.
- Fitting to the $R = 0.2$ and $R = 1.0$ ATLAS R_{AA} data, the best fit value for $\hat{q}_0$ and Q_0 is extracted from χ^2 analysis.





- A clear ordering of cone size dependence for $R = 0.2, 0.4, 0.6, 1.0$.
- At small-p_T, coherence dominate, ratio converge.
- At large-p_T, decoherence effect is more apparent for large radius jets.

[Duan, **LC**, Ma, Salgado, and Wu, 2603.22014]

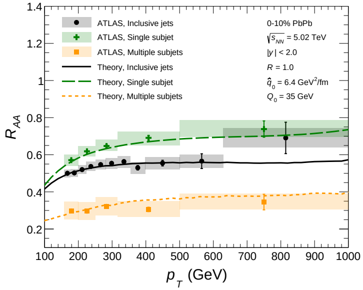
- Single subjet

$$R_{AA,S} = \frac{1}{T_{AB}(\vec{b})} \frac{d\sigma_S^{AA}/d^2\vec{b}\,dp_T}{d\sigma_S^{pp}/dp_T}, \qquad \frac{d\sigma_S^{pp}}{dp_T} = \sum_i P_i(1, p_T R) \frac{d\sigma_i^{pp}}{dp_T} \tag{27}$$

- Multiple subjets

$$R_{AA,M} = \frac{1}{T_{AB}(\vec{b})} \frac{d\sigma_M^{AA}/d^2\vec{b}\,dp_T}{d\sigma_M^{pp}/dp_T}, \qquad \frac{d\sigma_M^{pp}}{dp_T} = \sum_{n=2}^{N} \sum_i P_i(n, p_T R) \frac{d\sigma_i^{pp}}{dp_T} \tag{28}$$

- Overall good agreement with the ATLAS data.



NLO - CROSS SECTION AND MEASUREMENT

- The pp cross section [Nagy, PRD 68 (094002) 2003]

$$\frac{\mathrm{d}\sigma_{pp\to\mathrm{jet}}}{\mathrm{d}p_T^{\mathrm{jet}}\mathrm{d}y} = \sum_{a,b}\int \mathrm{d}x_a\mathrm{d}x_b\; f_{a/p}(x_a,\mu_F)f_{b/p}(x_b,\mu_F) \times \frac{\mathrm{d}\hat{\sigma}_{ab\to\mathrm{jet}}}{\mathrm{d}p_T^{\mathrm{jet}}\mathrm{d}y}(\mu_F,\mu_R) \qquad (29)$$

- It is useful to write the partonic cross section in terms of the measurement function

$$\mathrm{d}\hat{\sigma}_{2\to n} = \frac{1}{2\hat{s}}\left|\mathcal{M}_{2\to n}\right|^2 \mathrm{d}\Phi_n\; \mathcal{S}_n(\{p_i\},R) \qquad (30)$$

- Then the measurement is defined as

$$\mathcal{S}_2 = \sum_{i=1,2}\delta(p_T^{\mathrm{jet}} - p_T^i)$$

$$\mathcal{S}_3 = \sum_i\left[\prod_{j\neq i}\Theta(\Delta R_{ij} > R)\right]\delta(p_T^{\mathrm{jet}} - p_T^i) + \sum_{i<j}\Theta(\Delta R_{ij} < R)\delta(p_T^{\mathrm{jet}} - p_T^{ij}), \qquad (31)$$

- The AA cross section from pp baseline

$$\frac{\mathrm{d}\sigma_{AA\to\mathrm{jet}}}{\mathrm{d}p_T^{\mathrm{jet}}\mathrm{d}y} = \sum_{a,b}\int_{\mathcal{C}} \mathrm{d}^2\vec{b}_\perp T_{AA}(\vec{b}_\perp)\int\frac{\mathrm{d}\phi}{2\pi} \times \int \mathrm{d}x_a\mathrm{d}x_b\; f_{a/A}(x_a,\mu_F)f_{b/A}(x_b,\mu_F) \times \sum_{n=2,3}\int \mathrm{d}\Phi_n\,\frac{1}{2\hat{s}}\left|\mathcal{M}_{2\to n}\right|^2 \tilde{\mathcal{S}}_n(\{p_i\},R) \qquad (32)$$

- The medium measurement function is

$$\tilde{\mathcal{S}}_2 = \sum_{i=1,2}\delta(p_T^{\mathrm{jet}} - \mathcal{E}(p_T^i)) \qquad (33)$$

$$\tilde{\mathcal{S}}_3 = \sum\Theta(\text{no-cluster})\delta(p_T^{\mathrm{jet}} - \mathcal{E}(p_T^i)) + \sum\Theta(\text{cluster})\delta(p_T^{\mathrm{jet}} - \mathcal{E}(p_T^{ij})), \qquad (34)$$

$$\mathcal{E}(p_T) = p_T - \Delta E(p_T) \qquad (35)$$

[Casalderrey-Solana, Mehtar-Tani, Salgado and Tywoniuk, PLB 725 (2013) 357]

NLO - cross section and measurement

- The pp cross section [Nagy, PRD 68 (094002) 2003]

$$\frac{\mathrm{d}\sigma_{pp\to\text{jet}}}{\mathrm{d}p_T^{\text{jet}}\mathrm{d}y} = \sum_{a,b}\int \mathrm{d}x_a\mathrm{d}x_b\; f_{a/p}(x_a,\mu_F)f_{b/p}(x_b,\mu_F) \times \frac{\mathrm{d}\hat{\sigma}_{ab\to\text{jet}}}{\mathrm{d}p_T^{\text{jet}}\mathrm{d}y}(\mu_F,\mu_R) \qquad (29)$$

- It is useful to write the partonic cross section in terms of the measurement function

$$\mathrm{d}\hat{\sigma}_{2\to n} = \frac{1}{2\hat{s}}\left|\mathcal{M}_{2\to n}\right|^2 \mathrm{d}\Phi_n\; \mathcal{S}_n(\{p_i\},R) \qquad (30)$$

- Then the measurement is defined as

$$\mathcal{S}_2 = \sum_{i=1,2}\delta(p_T^{\text{jet}} - p_T^i)$$

$$\mathcal{S}_3 = \sum_i \left[\prod_{j\neq i}\Theta(\Delta R_{ij} > R)\right]\delta(p_T^{\text{jet}} - p_T^i) + \sum_{i<j}\Theta(\Delta R_{ij} < R)\delta(p_T^{\text{jet}} - p_T^{ij}), \qquad (31)$$

- The AA cross section from pp baseline

$$\frac{\mathrm{d}\sigma_{AA\to\text{jet}}}{\mathrm{d}p_T^{\text{jet}}\mathrm{d}y} = \sum_{a,b}\int_{\mathcal{C}} \mathrm{d}^2\vec{b}_\perp T_{AA}(\vec{b}_\perp)\int\frac{\mathrm{d}\phi}{2\pi} \times \int \mathrm{d}x_a\mathrm{d}x_b\; f_{a/A}(x_a,\mu_F)f_{b/A}(x_b,\mu_F) \times \sum_{n=2,3}\int \mathrm{d}\Phi_n\;\frac{1}{2\hat{s}}\left|\mathcal{M}_{2\to n}\right|^2\tilde{\mathcal{S}}_n(\{p_i\},R) \qquad (32)$$

- The medium measurement function is

$$\tilde{\mathcal{S}}_2 = \sum_{i=1,2}\delta(p_T^{\text{jet}} - \mathcal{E}(p_T^i)) \qquad (33)$$

$$\tilde{\mathcal{S}}_3^{\text{dec.}} = \sum\Theta(\text{no-cluster})\delta(p_T^{\text{jet}} - \mathcal{E}(p_T^i)) + \sum\Theta(\text{cluster})\left\{\Theta(Q^2_{\text{med}} - Q^2_{ij})\delta\left[p_T^{\text{jet}} - \mathcal{E}(p_T^{ij})\right] + \Theta(Q^2_{ij} - Q^2_{\text{med}})\delta\left[p_T^{\text{jet}} - \mathcal{E}(p_T^i) - \mathcal{E}(p_T^j)\right]\right\} \qquad (34)$$

[Casalderrey-Solana, Mehtar-Tani, Salgado and Tywoniuk, PLB 725 (2013) 357]

- The HT radiation kernel [Deng and Wang, PRC 81 024902 (2010)] [He, Luo, Wang and Zhu, PRC 91 054908 (2015)]

$$\frac{dN_g^a}{dz dl_T^2 d\tau} = \frac{2\alpha_s C_A}{\pi} \frac{P_a(z)}{l_T^4} \hat{q}_a(\vec{r}_\perp, \tau) \sin^2\left(\frac{\tau - \tau_0}{2\tau_f}\right), \quad \hat{q}_a(\vec{r}_\perp, \tau) = C_a \frac{42\zeta(3)}{\pi} \alpha_s^2 T^3 \ln\left(\frac{s^*}{4\mu_D^2}\right) \tag{35}$$

- Medium induced transverse momentum diffusion [He, Luo, Wang and Zhu, PRC 91 054908 (2015)]

$$F(q_T) = \frac{1}{2\pi\langle q_T^2\rangle} \exp\left(-\frac{q_T^2}{2\langle q_T^2\rangle}\right), \quad \langle q_T^2\rangle = \int_{\tau_0} d\tau\ \hat{q}(\tau) \tag{36}$$

- jet algorithm constraint [Chang, Cao and Qin, PLB 781 423 (2018)] [Kang, Ringer and Vitev, JHEP 10 (2016) 125]

$$\Theta(R_c - k_T), \quad R_c \equiv 2z(1-z)E \tan\frac{R}{2}, \quad \vec{k}_T = \vec{l}_T + \vec{q}_T \tag{37}$$

- parton energy loss ratio [Hu, **LC**, Gao, Zhu and Zhang, in preparation]

$$\begin{aligned} \frac{\Delta E_a}{E} &= \int dz dl_T^2 d\tau \left(z \frac{dN_a}{dz dl_T^2 d\tau}\right) \int d^2\vec{q}_T\ F(q_T)\, \Theta(R_c - k_T) \\ &= \int dz dl_T^2 d\tau \left(z \frac{dN_a}{dz dl_T^2 d\tau}\right) \left[1 - Q_1\left(\frac{l_T^2}{\langle q_T^2\rangle}, \frac{R_c^2}{\langle q_T^2\rangle}\right)\right] \end{aligned} \tag{38}$$

- effective average inclusive jet energy loss [Han, Xie and Zhang, 2509.07842]

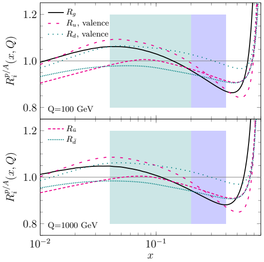
$$\Delta E(p_T) = f_q(p_T)\Delta E_q + f_g(p_T)\Delta E_g \tag{39}$$

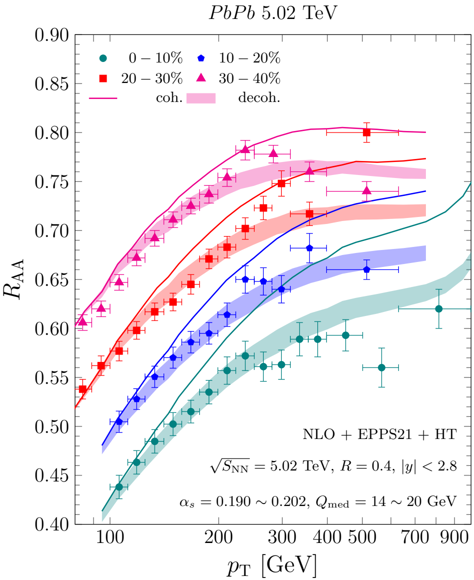
NLO - NUCLEAR MODIFICATION

- Coupling the NLO cross section and HT energy loss, we can plot the inclusive jet R_{AA}.
- Comparing with ATLAS data, $Q_{\text{med}} = 14 \sim 20$ GeV.
- $Q_{\text{med}} \to \infty$ corresponds to fully coherence, $Q_{\text{med}} \to \Lambda$ corresponds to fully decoherence.
- $\theta_c \propto 1/\sqrt{\hat{q}L^3}$, $Q^2_{\text{med}} \propto E^2\theta_c^2$.

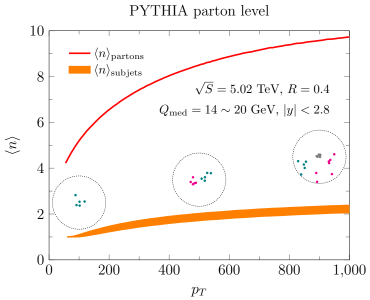
[Hu, **LC**, Gao, Zhu and Zhang, in preparation]

- nPDF effect is important in high energy jets.
- mild modification for $100 \sim 500$ GeV jets, correspond to $x = 0.04 \sim 0.1$.
- $500 \sim 1000$ GeV jets are suppressed, correspond to $x = 0.1 \sim 0.4$ (EMC).
- > 1000 GeV jets are enhanced.





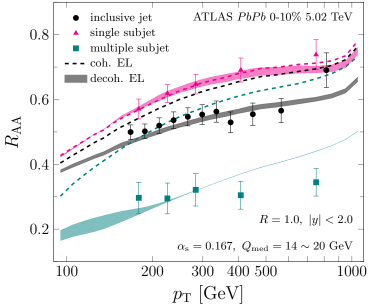
NLO - SUBJET MULTIPLICITY



- Q_{med} condition can be generalize as clustering algorithm.
  - sort $Q^2_{ij} = (p_i + p_j)^2$ for every possible pair.
  - if $Q^2_{ij,\min} < Q^2_{\text{med}}$ merge and repeat, otherwise stop.
- Applied algorithm to `PYTHIA` jets and plotted subjet multiplicity.
- mid-p_T (~ 100 GeV) jets consistent with coherent picture.
- high-p_T ($300 - 800$ GeV) jets favors decoherent picture.
- invariant mass merging automatically groups clusters.

[Hu, **LC**, Gao, Zhu and Zhang, in preparation]

- ATLAS measurement: first cluster $R = 0.2$, then $R = 1.0$.
- categorize whether contain single or multiple subjets.
- naturally correspond to decoherent clustering algorithm.
- $R = 0.2$ jets typically have invariant mass $< Q_{\text{med}}$, can be viewed as coherent objects.



Summary:

- Using the generating function method, particle multiplicity of QCD jets were calculated up to DLA.
- under the assumption of color decoherence, parton multiplicity that enters into the medium were calculated and its corresponding energy loss extracted by the R_{AA} spectrum.
- A similar calculation were done by using NLO partonic cross section with decoherent condition to extract the resolution scale of the medium.

Outlook:

- use MDLA instead of DLA to reduce multiplicity.
- NNLOjet for three-prong contribution.
- R_{AA} disagreement between ATLAS and CMS.

To be appear in the upcoming events:

- DIS2026 (talk), Bologna, Italy
- HP2026 (talk+poster), Nashville, USA
- ICHEP2026 (talk), Natal, Brazil