University of Regina

JETSCAPE

Workshop: Jet-soft dynamical medium interaction in high-energy heavy-ion collisions

Glauber quark and gluon contributions to jet energy loss at NLO within HT formalism

Amit Kumar
Postdoctoral Fellow
Department of Physics
University of Regina
Regina, Canada

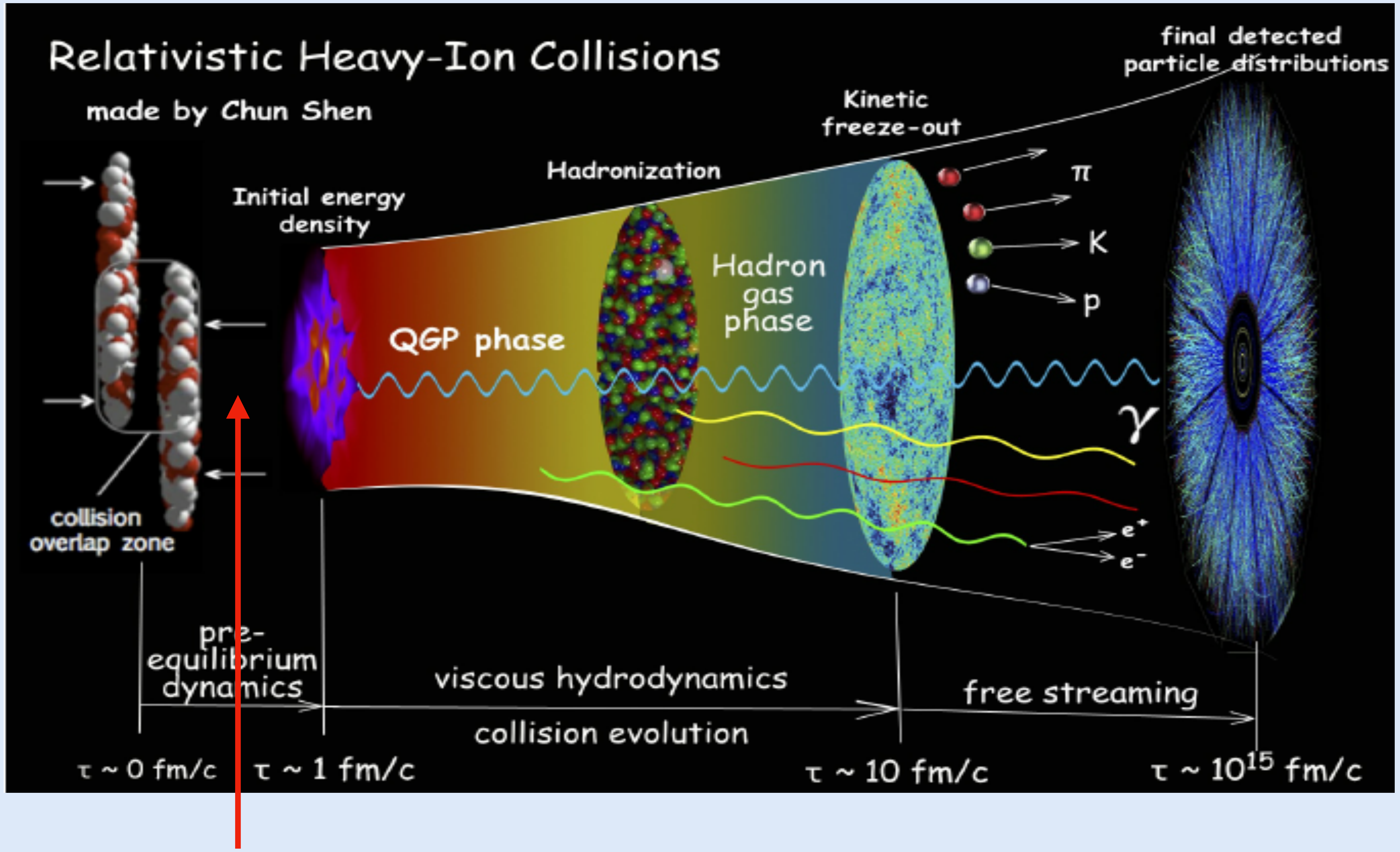
Based on

(1) A. Kumar, G. Vujanovic, arXiv:2509.10743 (2025)
(2) A. Kumar, A. Majumder, I. Soudi, J. Weber, arXiv:2602.22338 (2026)
(3) A. Kumar, A. Majumder, J. Weber, Phys. Rev. D 106, 034505 (2022)

April 1, 2026, CCNU, Wuhan, China

CANADA RESEARCH CHAIRS
CHAIRES DE RECHERCHE DU CANADA

Stages in relativistic heavy-ion collisions

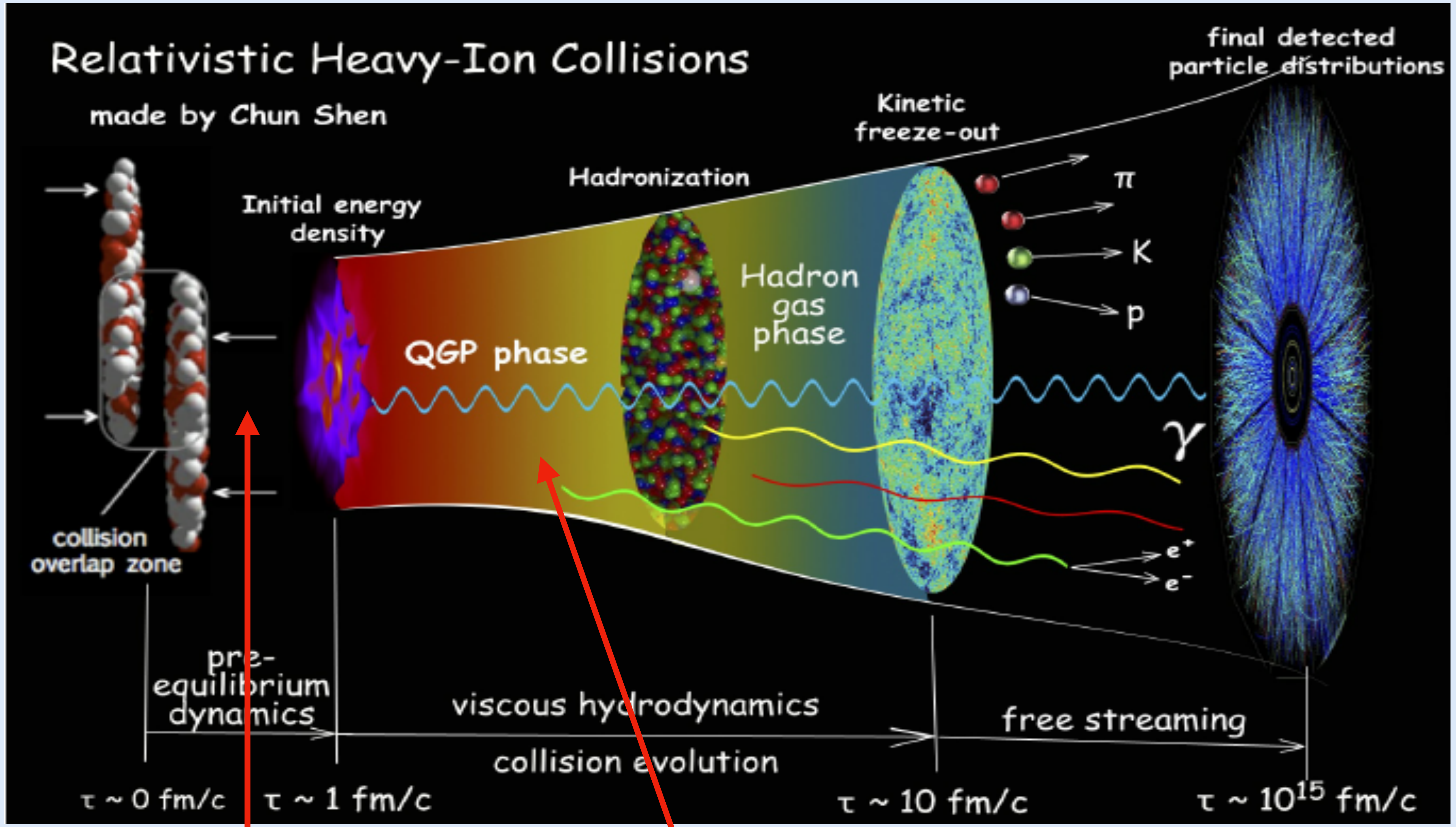


- ❑ **Pre-equilibrium dynamics**: Incoming nuclei undergo hard collisions to produce a hot/dense nuclear matter, also produces hard probes
  Generation of quarks in plasma

- ❑ **QGP is gluon dominant in early stage (Glasma).** & Quark density increases in evolution that drives the system to thermalization

How do we probe the pre-equilibrium dynamics and learn more about evolution of quark density at early times ? $\Longrightarrow$ Use jets as probe, Glauber quark interaction between jets and medium

Stages in relativistic heavy-ion collisions



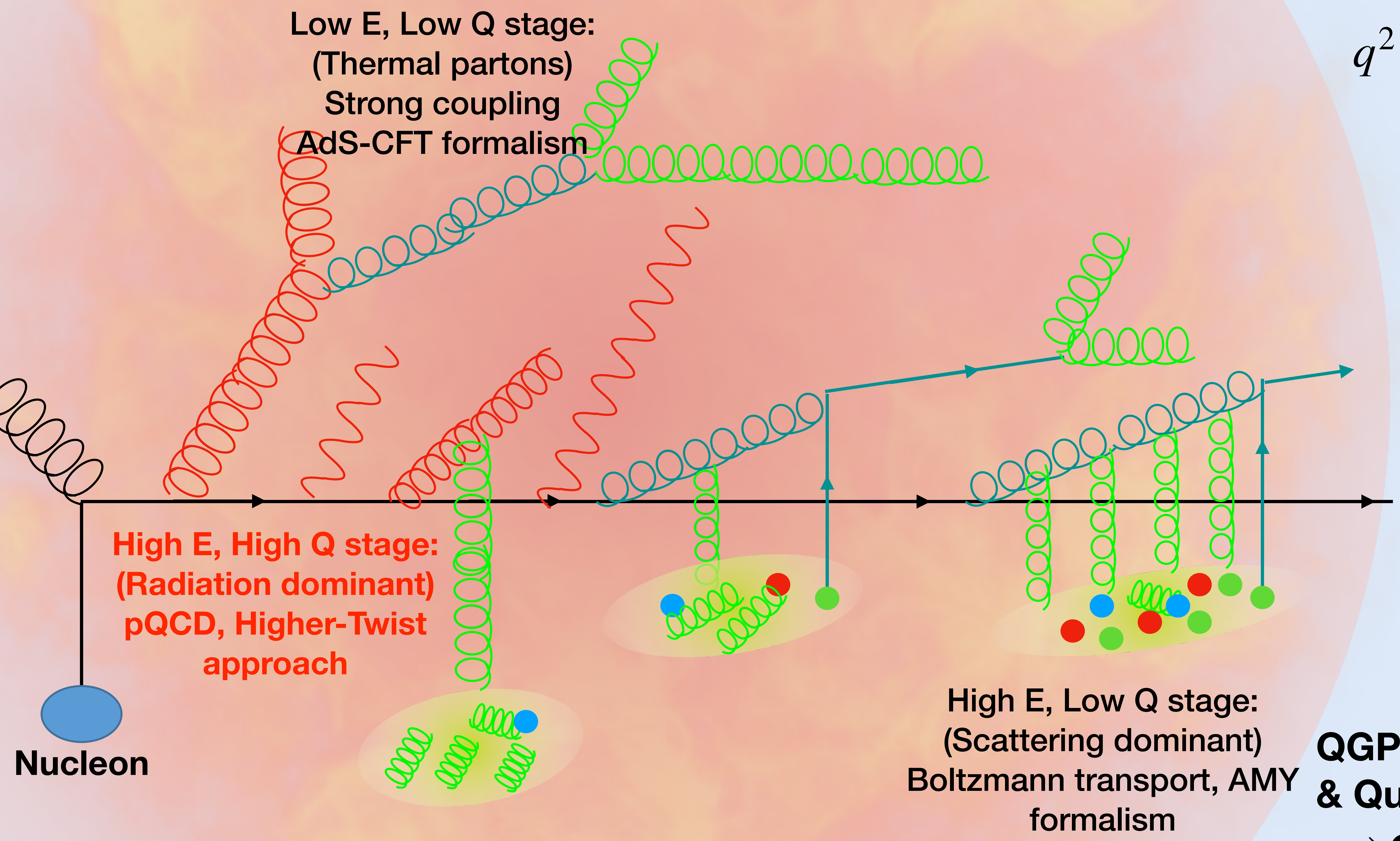
- ❑ **Pre-equilibrium dynamics**: Incoming nuclei undergo hard collisions to produce a hot/ dense nuclear matter, also produces hard probes
  Generation of quarks in plasma

- ❑ **QGP is gluon dominant in early stage (Glasma).** & Quark density increases in evolution that drives the system to thermalization

How do we probe the pre-equilibrium dynamics and learn more about evolution of quark density at early times ? $\Longrightarrow$ Use jets as probe, Glauber quark interaction between jets and medium

Also, in QGP phase, the interactions between the hard parton and Glauber quarks has not been studied thoroughly in Higher-twist formalism

Jet evolution in QGP a multi-scale phenomenon



Virtuality = Offshellness

$$q^2 = (q^0)^2 - |\vec{q}\,|^2 - m^2$$

Lifetime, $\tau_f = \dfrac{2E}{Q^2}$

Traditional approach:

Gluon exchanges with the medium

$\hat{q}$ as a dominant coefficient (Diffusion in transverse direction)

Extend the formalism to include:

Quark exchanges with the medium

Beyond $\hat{q}$, include contributions from parton drag and diffusion in longitudinal direction

QGP is gluon dominant in early stage. & Quark density increases in evolution

$\Longrightarrow$Quark exchange processes between jet & medium can be used to probe pre-equilibrium dynamics

Quark energy loss at NLO (All scattering kernels)

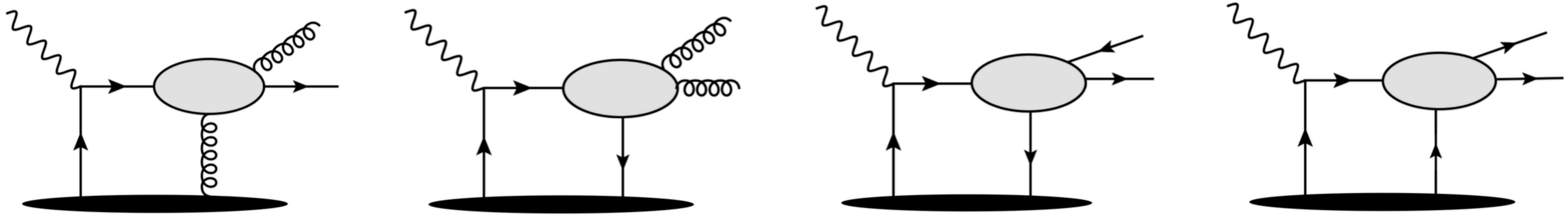
❑ Photon emission kernels at O($\alpha_s \alpha_{em}$)

(a) A photon and quark in the final state. (b) A photon and gluon in the final state.

Kumar and Vujanovic, "Bremsstrahlung photon contributions to parton energy loss at high virtuality with O($\alpha_s \alpha_{em}$)" Phys. Rev. C 112, 025204 (2025)

(Use perturbative QCD to evaluate Feynman Diagrams)

❑ Gluon/Quark emission kernels O(α_s^2)

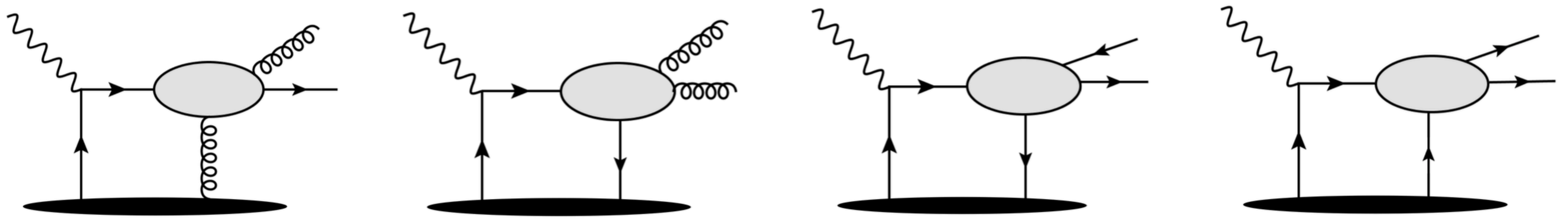


(a) A gluon and quark in the final state. (b) Two gluons in the final state. (c) Quark anti-quark in the final state. (d) Two quarks in the final state.

Kumar and Vujanovic, "Glauber quark and gluon contributions to quark energy loss at NLO", arXiv:2509.10743 (2025)

Keypoints to cover in this talk

❑ **Gluon/Quark emission kernels O(α_s^2)**



(a) A gluon and quark in the final state. (b) Two gluons in the final state. (c) Quark anti-quark in the final state. (d) Two quarks in the final state.

Kumar and Vujanovic, "Glauber quark and gluon contributions to quark energy loss at NLO", arXiv:2509.10743 (2025)

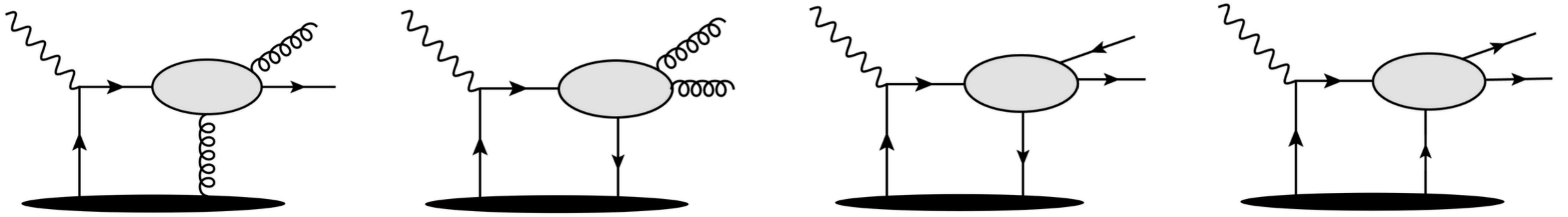
- ✦ **HT formalism is used to derive all possible medium-induced single-emission energy loss kernel at O(α_s^2)**
  - ✦ **Included contributions from transverse momentum ($k_\perp$) as well as longitudinal momentum (k^-)**
  - ✦ **Included Quark mass effects, contributions from Glauber quark and gluon exchange**
  - ✦ **Glauber gluon/quark exchange diagrams give rise to series of transport coefficients**

Two-point gauge field correlators: $\hat{\mathscr{A}}_0, \hat{\mathscr{A}}_{T,2}, \hat{\mathscr{A}}_{L,1},$

Two-point quark field correlators: $\hat{\mathscr{F}}_0, \hat{\mathscr{F}}_{T,2}, \hat{\mathscr{F}}_{L,1},$

Keypoints to cover in this talk

❑ **Gluon/Quark emission kernels O(α_s^2)**



(a) A gluon and quark in the final state. (b) Two gluons in the final state. (c) Quark anti-quark in the final state. (d) Two quarks in the final state.

Kumar and Vujanovic, "Glauber quark and gluon contributions to quark energy loss at NLO", arXiv:2509.10743 (2025)

✦ **HT formalism is used to derive all possible medium-induced single-emission energy loss kernel at O(α_s^2)**

- ✦ **Included contributions from transverse momentum ($k_\perp$) as well as longitudinal momentum (k^-)**
- ✦ **Included Quark mass effects, contributions from Glauber quark and gluon exchange**
- ✦ **Glauber gluon/quark exchange diagrams give rise to series of transport coefficients**

Two-point gauge field correlators: $\hat{\mathscr{A}}_0, \hat{\mathscr{A}}_{T,2}, \hat{\mathscr{A}}_{L,1},$

Two-point quark field correlators: $\hat{\mathscr{F}}_0, \hat{\mathscr{F}}_{T,2}, \hat{\mathscr{F}}_{L,1},$

✦ **Phase factor of the jet transport coefficients has hard transverse momentum dependence of radiated quark/gluon $\Longrightarrow$ Resembles TMD pdf**

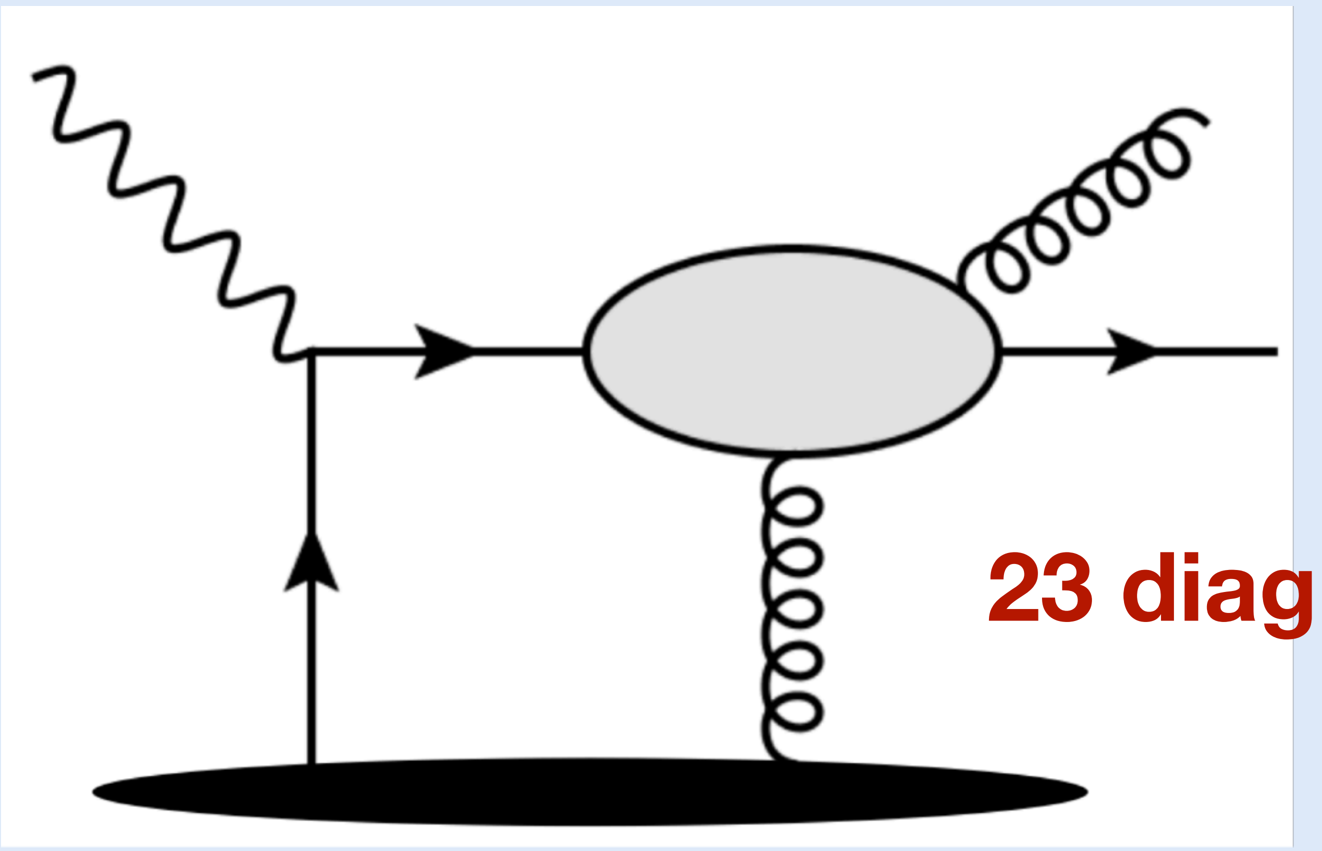
✦ **Lattice calculation of jet transport coefficient $\hat{q}$** A. Kumar A. Majumder, J. Weber, PRD 106, 034505 (2022)

✦ **Fluctuation-dissipation relation for parton drag and diffusion in non-perturbative plasma**

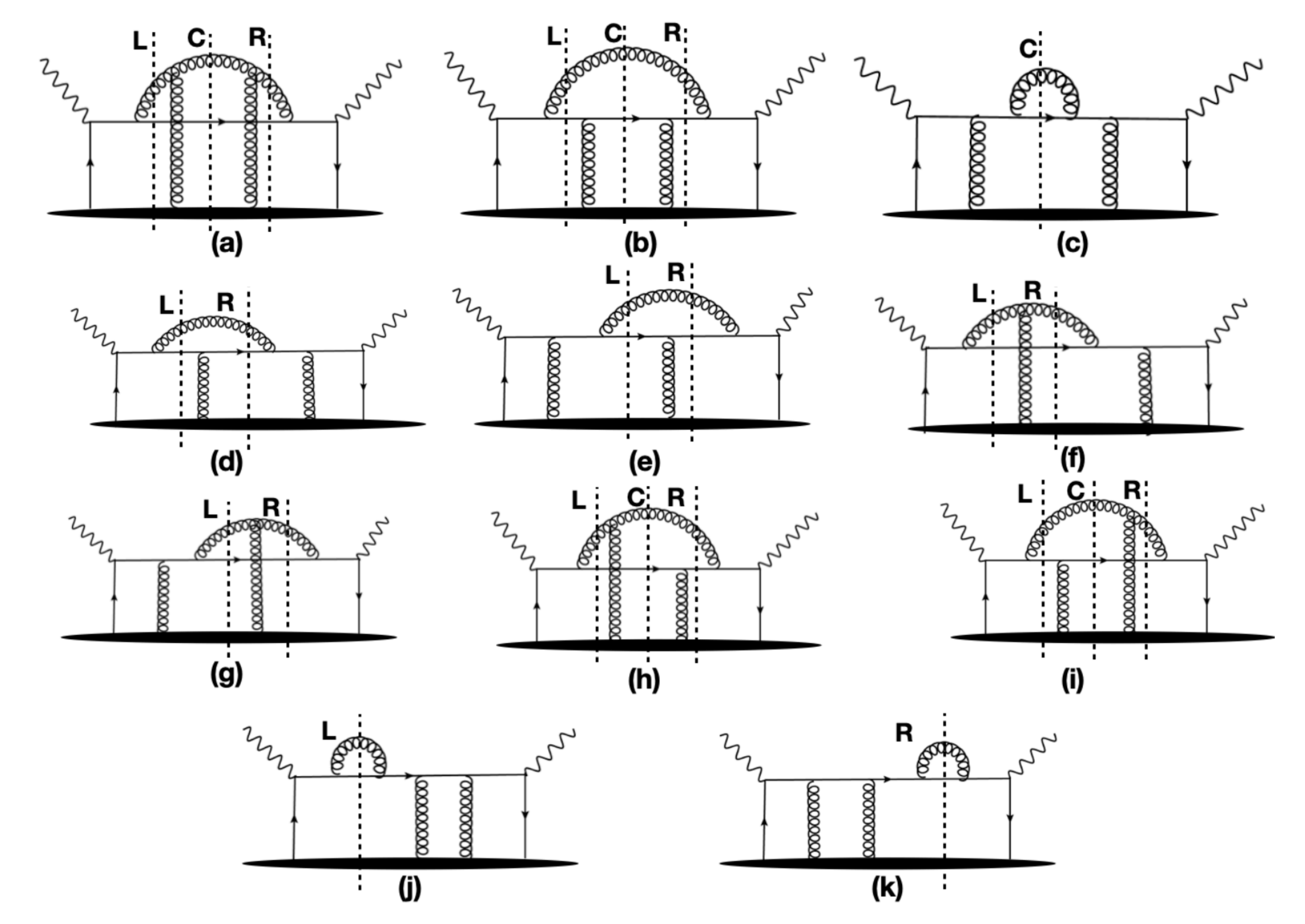
✦ **Connection to EIC physics** A. Kumar, A. Majumder, I. Soudi, J. Weber, arXiv:2602.22338 (2026)

Medium-induced single-gluon emission kernel

- A gluon and a quark final state



$\Longrightarrow$ 23 diagrams (α_s^2)



(Use perturbative QCD to evaluate Feynman Diagrams)

Kumar and Vujanovic
"Glauber quark and gluon contributions to quark energy loss at NLO"
arXiv:2509.10743 (2025)

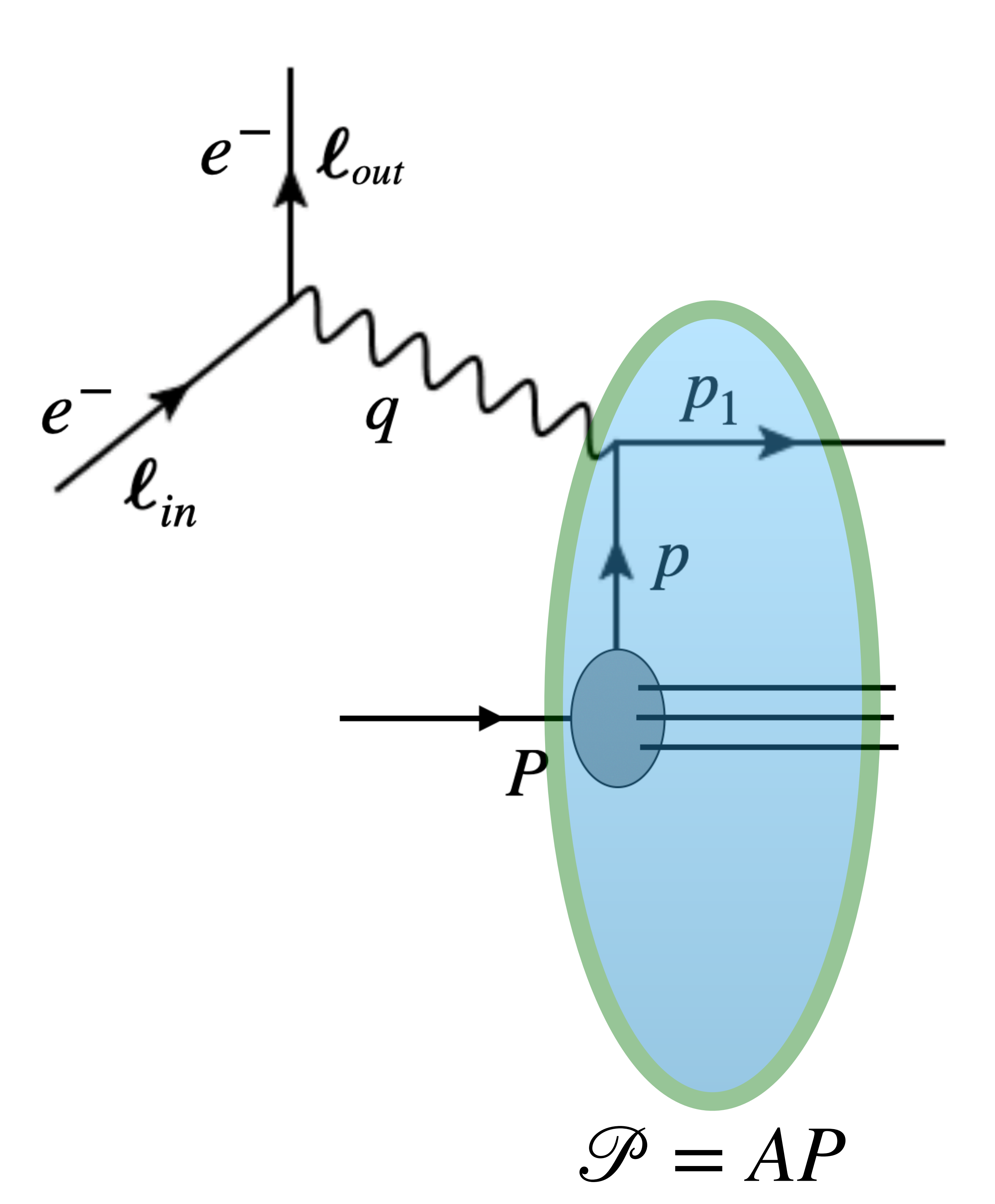
Evaluation of this scattering kernel and diagrams herein has a history of ~25 years

Deep inelastic scattering

- ❑ **DIS between an electron and a nucleon inside the nucleus (In Breit frame)**

$$e^-(\mathscr{L}_{in}) + A(\mathscr{P}) \to e^-(\mathscr{L}_{out}) + X$$

$$q^\mu = [q^+, q^-, q_\perp] = \left[\frac{-Q^2}{2q^-}, q^-, 0\right]; \qquad \mathscr{P} = AP = A[P^+, M^2/2P^+, 0]$$



- ❑ **The differential cross section of the reaction**

$$E_{l_{out}} \frac{d^3\sigma}{d^3 l_{out}} = \frac{\alpha_{em}^2}{2\pi s} \frac{L_{\mu\nu} W^{\mu\nu}}{Q^4}$$

- ❑ **Leptonic tensor**

$$L^{\mu\nu} = \frac{1}{2} \mathrm{Tr}\left[\not{\ell}_{\mathrm{in}} \gamma^\mu \not{\ell}_{\mathrm{out}} \gamma^\nu\right] = M_{leptonic} M^*_{leptonic}$$

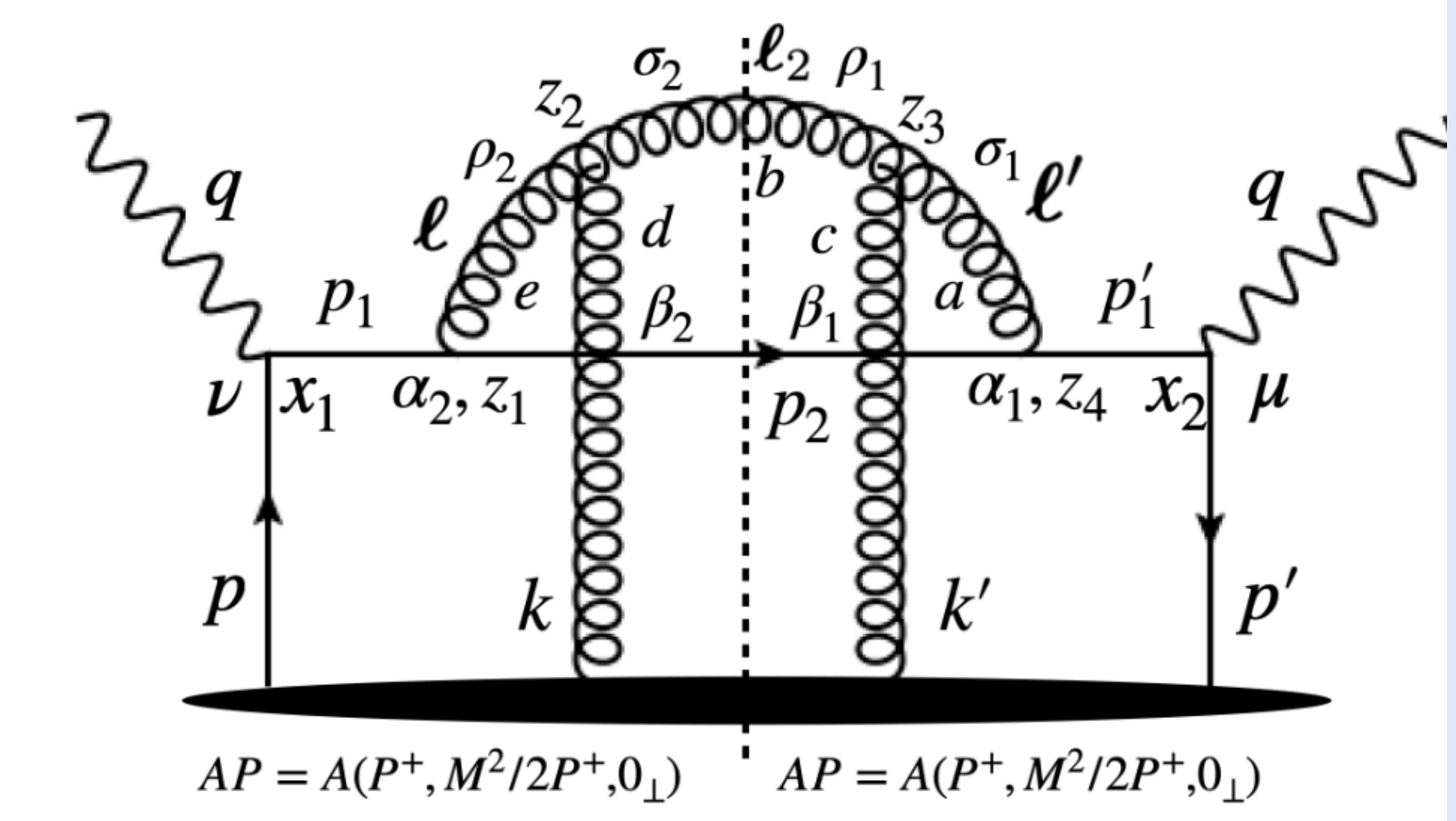
- ❑ **Hadronic tensor**

$$W^{\mu\nu} = \sum_X \delta^{(4)}(p_X - \mathscr{P} - q) \langle AP, S | j^\mu(0) | X \rangle \langle X | j^\nu(0) | AP, S \rangle = M_{hadronic} M^*_{hadronic}$$

Deep inelastic scattering

❑ **Optical theorem**

$$MM^* = 2\mathrm{Im}[T]; \qquad W^{\mu\nu} = 2\ \mathrm{Im}[T^{\mu\nu}]$$



Cutline: $\mathrm{Disc}\left[\frac{1}{\ell_2^2 + i\epsilon}\right] = -2\pi i \delta(\ell_2^2)$ $\qquad \mathrm{Disc}\left[\frac{1}{p_2^2 + i\epsilon}\right] = -2\pi i \delta(p_2^2)$

❑ **Assumptions:** In light-cone gauge: $A^- = 0$

❑ **Leading order term in quark PDF and gauge field correlator:**

$$\psi(x_1)\bar{\psi}(x_2) = \not{p}T(x_1, x_2) \approx \frac{1}{4}\gamma^- \mathrm{Tr}\left[\bar{\psi}(x_2)\gamma^+\psi(x_1)\right]; \qquad \ell_2^\mu A_\mu \approx \ell_2^- A_\mu^+$$

❑ **Factorization Nucleon quark PDF and gluon-gluon correlator:**

$$\langle AP | \mathrm{Tr}[\bar{\psi}(x_2)\gamma^+\psi(x_1)] A^{c+}(z_3) A^{d+}(z_2) | AP \rangle \approx \langle P | \mathrm{Tr}[\bar{\psi}(x_2)\gamma^+\psi(x_1)] | P \rangle \times \langle P_{A-1} | A^{c+}(z_3) A^{d+}(z_2) | P_{A-1} \rangle$$

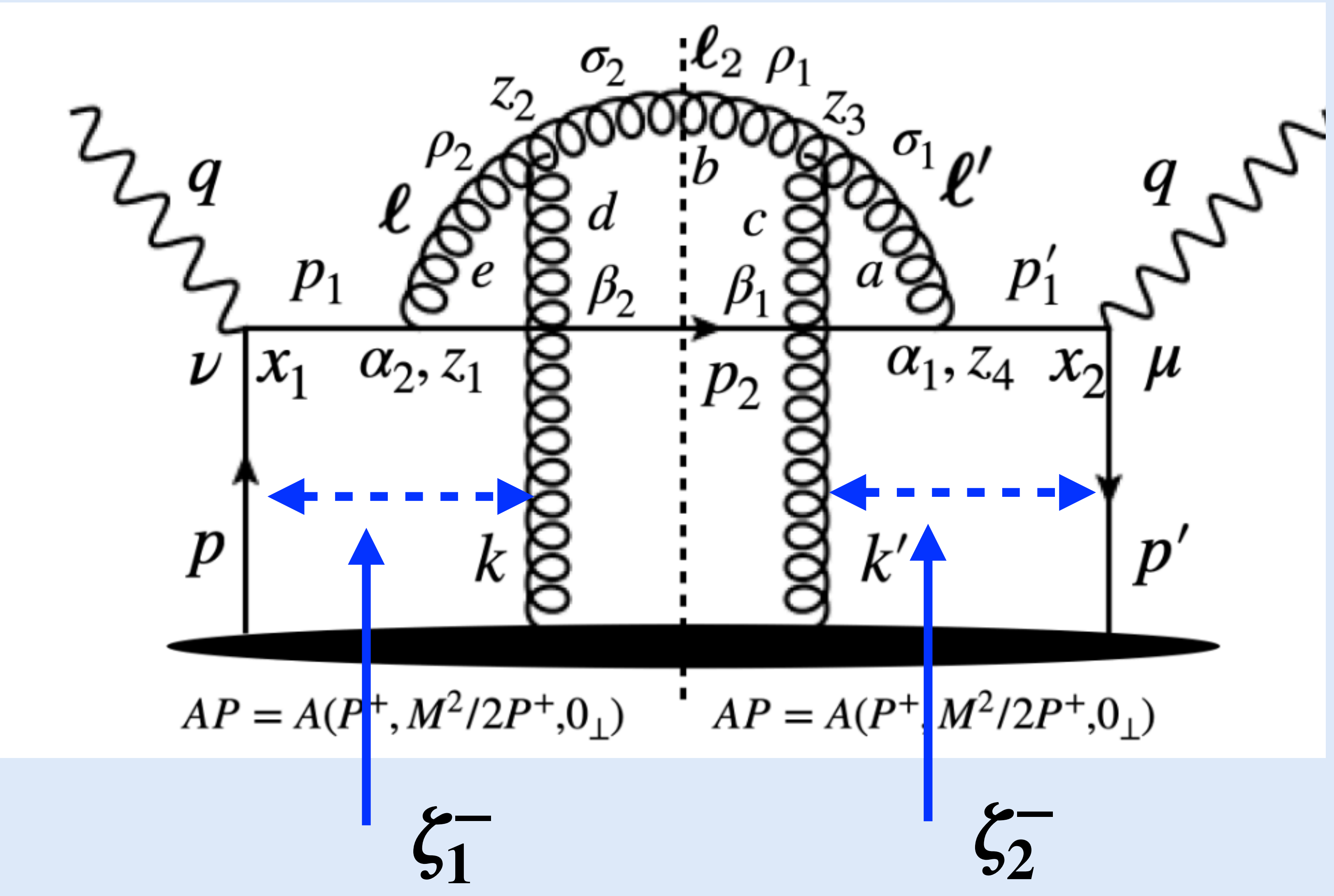
❑ **Glauber gluon momentum scaling:** $(k^+, k^- \ll k_\perp)$

Singularities in the amplitude of the diagram

❑ **Contour integration C1:**

$$C_1 = \oint \frac{dp^+}{2\pi} \frac{e^{ip^+\zeta_1^-}}{[(q+p)^2 - M^2 + i\epsilon][(q+p-p_2)^2 + i\epsilon]}$$

$$= \frac{(2\pi i)}{2\pi} \frac{\theta(\zeta_1^-)}{4q^-(q^- - p_2^-)} e^{i(-q^+ + M^2/2q^-)\zeta_1^-} \left[\frac{-1 + e^{iG\zeta_1^-}}{G}\right]$$



❑ **Contour integration C2:**

$$C_2 = \oint \frac{dp'^+}{2\pi} \frac{e^{-ip'^+\zeta_2^-}}{[(q+p')^2 - M^2 - i\epsilon][(q+p'-p_2)^2 - i\epsilon]}$$

$$= \frac{(-2\pi i)}{2\pi} \frac{\theta(\zeta_2^-)}{4q^-(q^- - p_2^-)} e^{i(-q^+ + M^2/2q^-)\zeta_2^-} \left[\frac{-1 + e^{-iG\zeta_2^-}}{G}\right]$$

where, $G = \dfrac{(\ell_2 - k_\perp)^2 + M^2 y^2 (1-\eta)^2}{2y(1-y+\eta y)(1-\eta)q^-}$

Phase Coherence effect in HT formalism

❑ Coherence effects:

$$\zeta_1^- \approx \zeta_2^- = \zeta^-$$

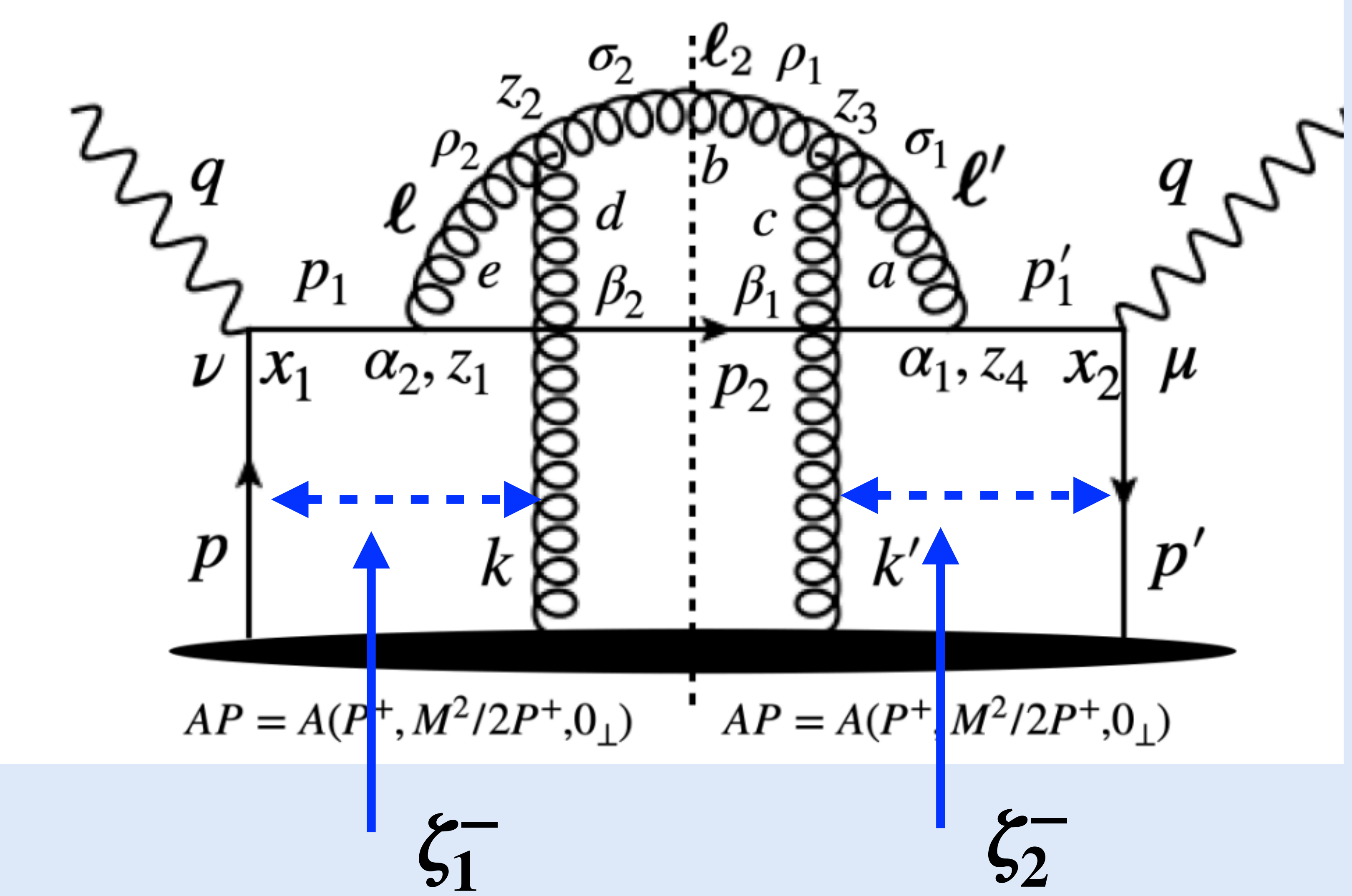
(Size of the nucleon is much smaller than the size of nucleus)

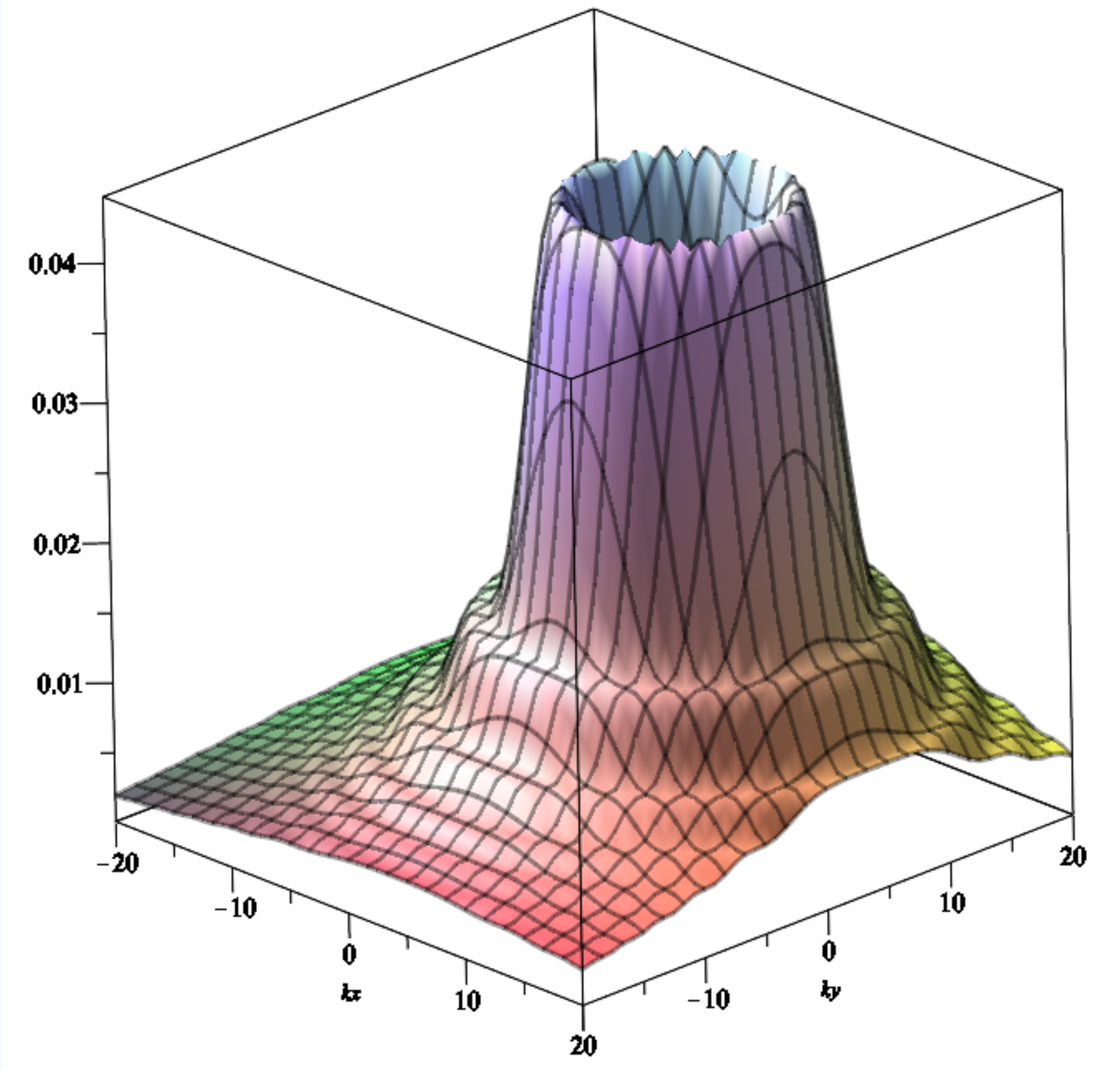
$$\left[\frac{-1+e^{iG\zeta_1^-}}{G}\right]\left[\frac{-1+e^{iG\zeta_2^-}}{G}\right] \approx C\frac{2-2\cos\left\{\frac{(\ell_2-k_\perp)^2+y^2(1-\eta)^2M^2}{2y(1-y+\eta y)(1-\eta)q^-}\zeta^-\right\}}{\left[(\ell_2-k_\perp)^2+y^2(1-\eta)^2M^2\right]^2}$$

✦Plot of length integrated function

$$\int_0^{2\tau_f^-} d\zeta^- F(\ell_2, k_\perp, \zeta^-, M=0, y=0.1, \eta=0)$$

Spectrum peaks in region $|\ell_{2\perp}| \approx |k_\perp|$

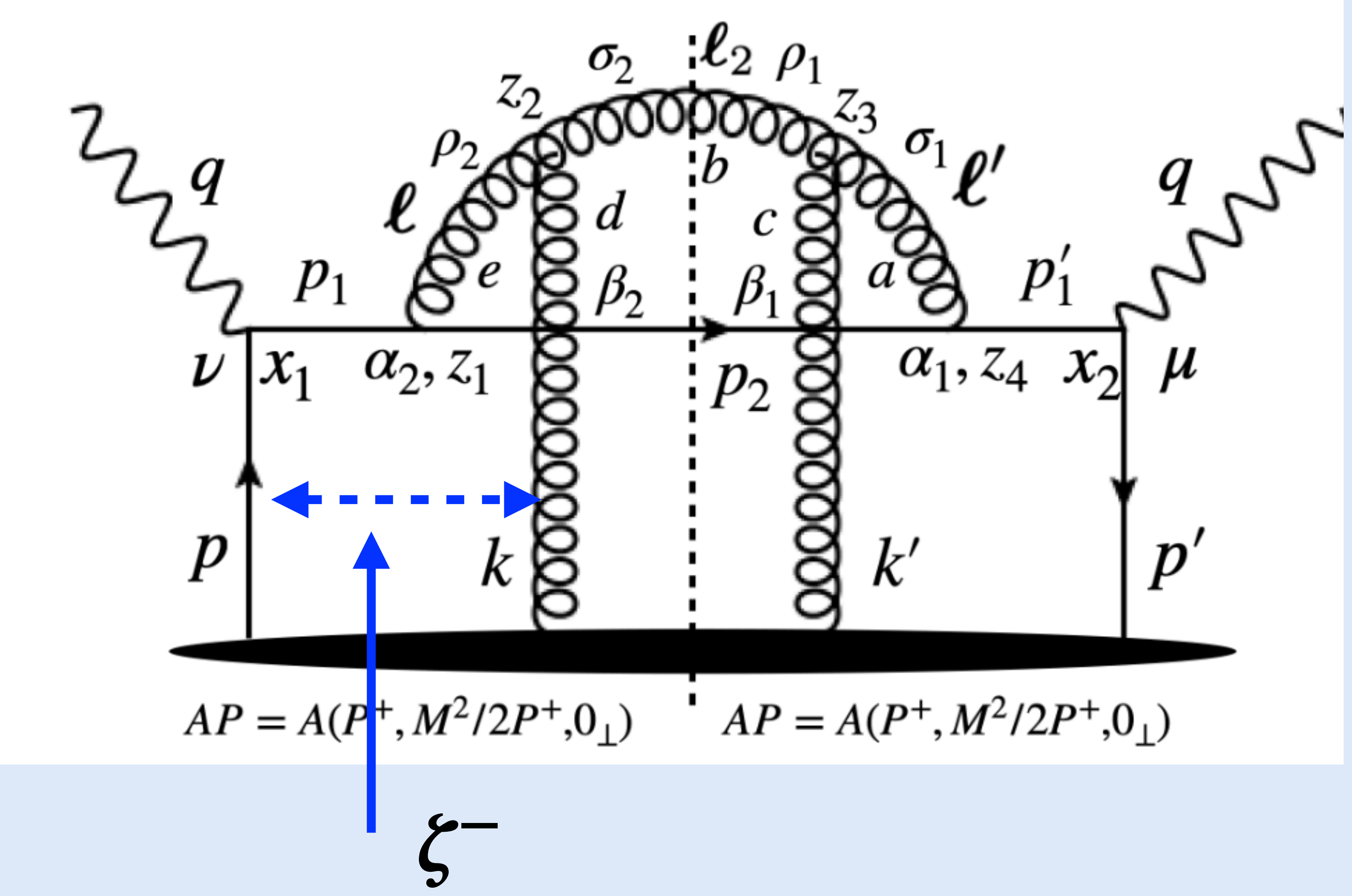




Kumar, Majumder, Sirimanna, and Tachibana, PRC 112, 014907 (2025)

Medium-induced single gluon emission scattering

$$W^{\mu\nu}_{1,c} = C_A \sum_f 2\left[-g^{\mu\nu}_{\perp\perp}\right] e_f^2 g_s^4 \int d(\Delta x^-) e^{i\Delta x^-\left(q^+ - \frac{M^2}{2q^-}\right)} \left\langle P \left| \bar{\psi}_f(\Delta x^-) \frac{\gamma^+}{4} \psi_f(0) \right| P \right\rangle$$
$$\times \int d(\Delta z^-) d^2\Delta z_\perp \frac{dy}{2\pi} \frac{d^2\ell_{2\perp}}{(2\pi)^2} \frac{d^2k_\perp}{(2\pi)^2} \left[\frac{1+(1-y)^2}{y}\right] \left[\frac{\left(1-\frac{\eta}{2}\right)^2}{(1-\eta)^2}\right] e^{-i\Delta z^- \mathcal{H}_M^{(\ell_2,p_2)}} e^{i\boldsymbol{k}_\perp \cdot \Delta\boldsymbol{z}_\perp}$$
$$\times \int d\zeta^- \theta(\zeta^-) \frac{\left[(\boldsymbol{\ell}_{2\perp} - \boldsymbol{k}_\perp)^2 + \kappa y^4 M^2\right] \left[2 - 2\cos\left\{\mathcal{G}_M^{(p_2)} \zeta^-\right\}\right]}{\left[(\boldsymbol{\ell}_{2\perp} - \boldsymbol{k}_\perp)^2 + y^2(1-\eta)^2 M^2\right]^2}$$
$$\times \langle P_{A-1} | \mathrm{Tr}\left[A^+(\zeta^-, \Delta z^-, \Delta z_\perp) A^+(\zeta^-, 0)\right] | P_{A-1} \rangle,$$



✦**Splitting Function** $\left[\frac{1+(1-y)^2}{y}\right]$

✦**Non-perturbative in-medium correlator**

$$\langle P_{A-1} | A^+(\zeta^-, \Delta z^-, \Delta z_\perp) A^+(\zeta^-, 0) | P_{A-1} \rangle$$

✦**Path length integration** $\left[2 - 2\cos\left\{\frac{(\ell_{2\perp} - k_\perp)^2 + y^2(1-\eta)^2 M^2}{2y(1-y+\eta y)(1-\eta)q^-}\zeta^-\right\}\right]$

✦**Glauber gluon:** $(k^+, k^- \ll k_\perp)$

✦**Collinear expansion around $k_\perp = 0$ is performed to decouple $k_\perp$ and $l_\perp$ integration**

Collinear ($k_\perp$) expanision of scattering kernel and non-perturbative correlators

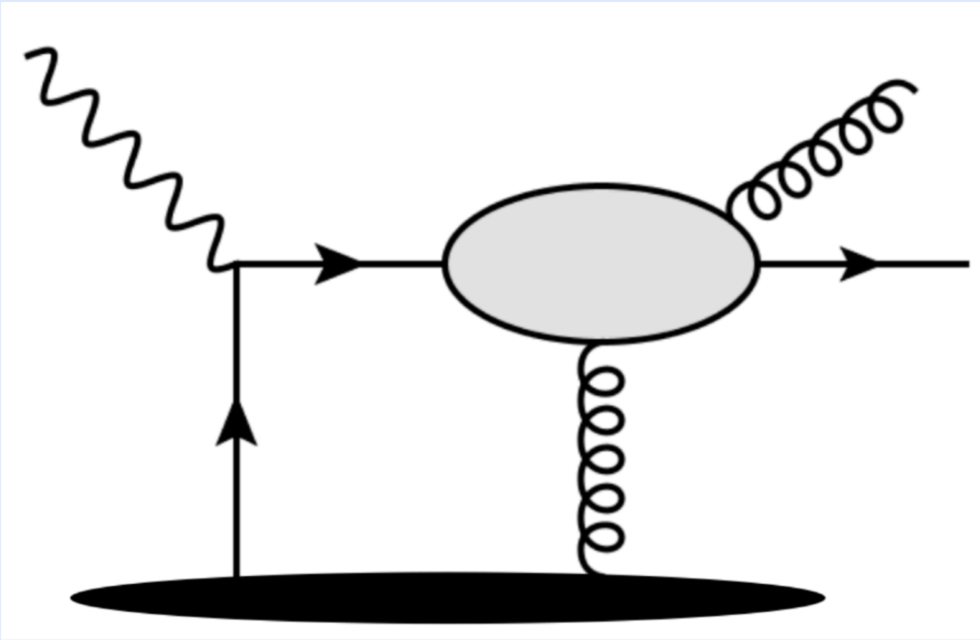
$$W^{\mu\nu}_{i,\text{full}} = 2\left[-g^{\mu\nu}_{\perp\perp}\right] e_f^2 \int d(\Delta x^-) e^{i\Delta x^-\left(q^+ - \frac{M^2}{2q^-}\right)} \langle P|\bar{\psi}(\Delta x^-)\frac{\gamma^+}{4}\psi(0)|P\rangle \times \mathcal{K}^{\text{eff}}_i$$

$$\mathcal{K}^{\text{eff}}_i = g_s^4 \int d(\Delta z^-) d^2\Delta z_\perp \frac{dy}{2\pi}\frac{d^2\ell_{2\perp}}{(2\pi)^2}\frac{d^2k_\perp}{(2\pi)^2} \; e^{-i\Delta z^- \mathcal{H}} e^{ik_\perp\cdot\Delta z_\perp}$$

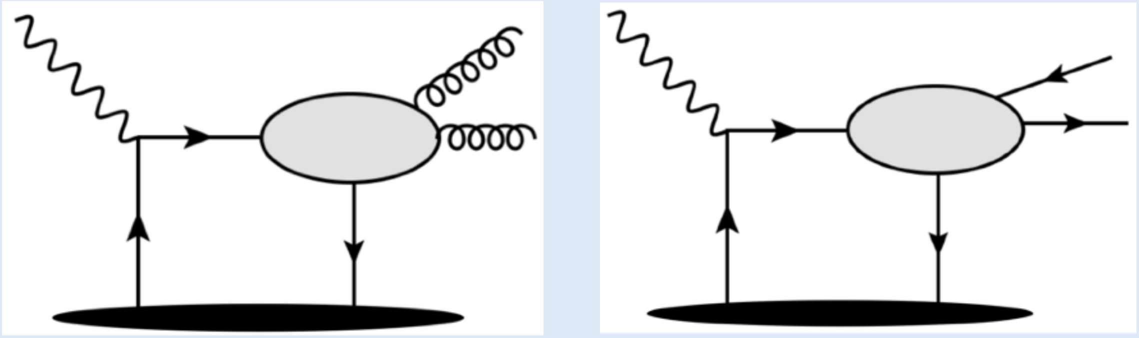
$$\times \int d\zeta^-\theta(\zeta^-)\, S_i\, \langle P_{A-1}|A^+(\zeta^-,\Delta z^-,\Delta z_\perp)\, A^+(\zeta^-,0)|P_{A-1}\rangle$$

or

$$\times \int d\zeta^-\theta(\zeta^-)\, S_i\, \langle P_{A-1}|\bar{\psi}(\zeta^-,\Delta z^-,\Delta z_\perp)\, \gamma^+\, \psi(\zeta^-,0)|P_{A-1}\rangle$$



Glauber gluon exchange



Glauber quark exchange

✦**Collinear expansion around $k_\perp = 0$ is performed to decouple $k_\perp$ and $l_\perp$ integration**

$$S_i(\ell_\perp, k_\perp, k^-) = S_i(\ell_\perp)\Big|_{k=0} + \frac{\partial S_i}{\partial k^\rho_\perp}\Big|_{k=0} k^\rho_\perp + \frac{1}{2!}\frac{\partial^2 S_i}{\partial k^\rho_\perp k^\sigma_\perp}\Big|_{k=0} k^\rho_\perp k^\sigma_\perp + \ldots\ldots\ldots + \frac{\partial S_i}{\partial k^-}\Big|_{k=0} k^- + \frac{1}{2!}\frac{\partial^2 S_i}{\partial k^{-2}}\Big|_{k=0} (k^-)^2 + \ldots\ldots\ldots$$

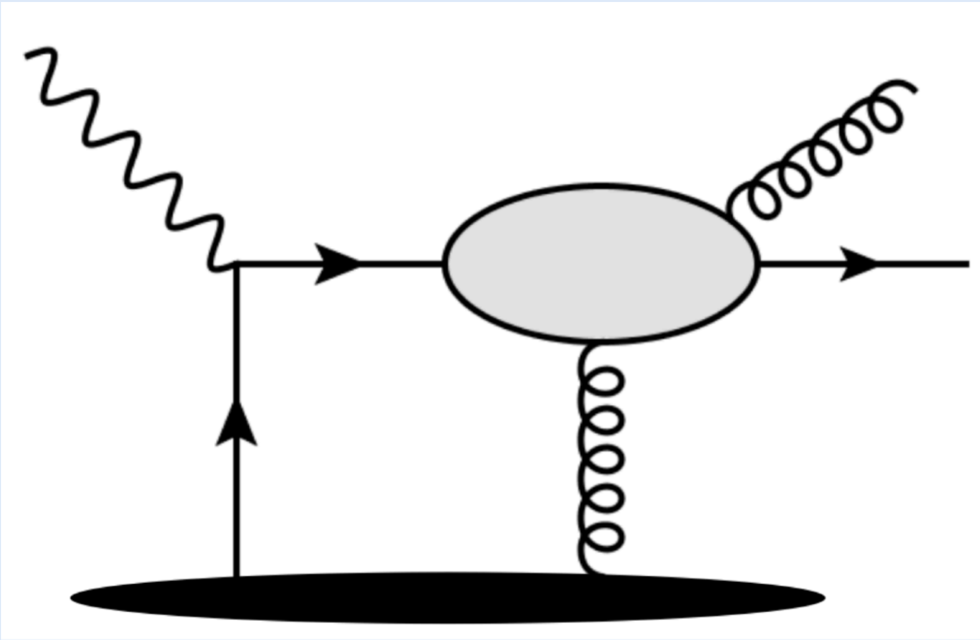
$\hat{q} = \langle k_\perp^2\rangle$ $\hat{e} = \langle k^-\rangle$ $\hat{e}_2 = \langle (k^-)^2\rangle$

Collinear ($k_\perp$) expanision of scattering kernel and non-perturbative correlators

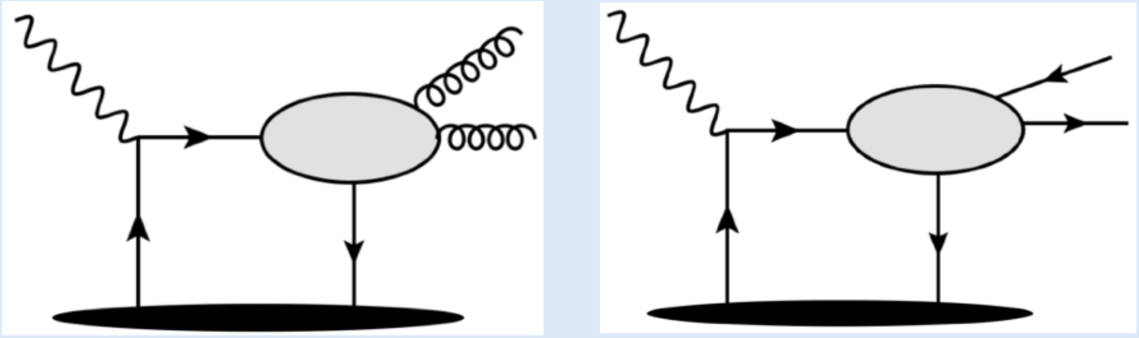
❑ Terms in Scattering kernel:

$$\frac{d^3\mathcal{K}_1^{eff}}{dyd^2\ell_{2\perp}} = \int d\zeta^- \left[\mathcal{R}_0^{(1)}\hat{\mathcal{A}}_0 + \mathcal{R}_{T2}^{(1)}\hat{\mathcal{A}}_{T2} + \mathcal{R}_{L1}^{(1)}\hat{\mathcal{A}}_{L1} + \ldots\right]$$

$$\frac{d^3\mathcal{K}_2^{eff}}{dyd^2\ell_{2\perp}} = \int d\zeta^- \left[\mathcal{R}_0^{(2)}\hat{\mathcal{F}}_0 + \mathcal{R}_{T2}^{(2)}\hat{\mathcal{F}}_{T2} + \mathcal{R}_{L1}^{(2)}\hat{\mathcal{F}}_{L1} + \ldots\right]$$



Glauber gluon exchange



Glauber quark exchange

✦ **Perturbative coefficients:** $\mathcal{R}_0^{(1)}$, $\mathcal{R}_{T2}^{(1)}$, $\mathcal{R}_{L1}^{(1)}$

✦ **Non-perturbative correlators:** $\hat{\mathcal{A}}_0$, $\hat{\mathcal{A}}_{T2}$, $\hat{\mathcal{A}}_{L1}$

$$\hat{\mathcal{A}}_0 = \langle M | A^+(\Delta z^-) A^+(0) | M\rangle \qquad \hat{\mathcal{F}}_0 = \langle M | \bar{\psi}(0)\gamma^+\psi(\Delta z^-) | M\rangle$$

$$\hat{\mathcal{A}}_{T2} = \langle M | \partial_\perp A^+(\Delta z^-)\partial_\perp A^+(0) | M\rangle \qquad \hat{\mathcal{F}}_{T2} = \langle M | \partial_\perp\bar{\psi}(0)\gamma^+\partial_\perp\psi(\Delta z^-) | M\rangle$$

$$\hat{\mathcal{A}}_{L1} = \langle M | i\partial^- A^+(\Delta z^-) A^+(0) | M\rangle \qquad \hat{\mathcal{F}}_{L1} = \langle M | i\partial^-\bar{\psi}(0)\gamma^+\psi(\Delta z^-) | M\rangle$$

✦ $\hat{\mathcal{A}}_0$ **is gauge corrections to PDF and does not contribute to energy loss**

✦ **Tranverse diffusion is controlled by** $\hat{\mathcal{A}}_{T2}$ **and** $\hat{\mathcal{F}}_{T2}$

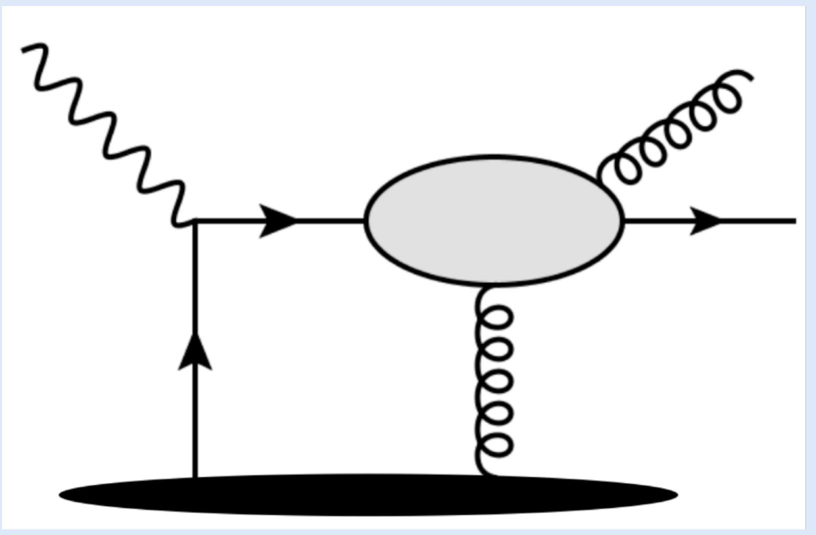
✦ **Longitudinal drag is controlled by** $\hat{\mathcal{A}}_{L1}$ **and** $\hat{\mathcal{F}}_{L1}$

Transverse momentum dependence in jet transport coefficients

❑ **Gluonic Non-Perturbative in-medium correlators**

$$\hat{\mathcal{A}}_{T,2} = g_s^2 \int d(\Delta z^-) d^2\Delta z_\perp \frac{d^2 k_\perp}{(2\pi)^2} \; e^{-i\Delta z^- \left[\frac{\vec{\ell}_{2\perp}^2 - yM^2}{2yq^-} + \frac{\left(\vec{\ell}_{2\perp} - \vec{k}_\perp\right)^2 + M^2}{2q^-(1-y+\eta y)} \right]} e^{i\vec{k}_\perp \cdot \Delta \vec{z}_\perp}$$

$$\times \langle P_{A-1} | \; \partial_\perp A^+(\zeta^-, \Delta z^-, \Delta z_\perp) \; \partial_\perp A^+(\zeta^-, 0) \; | P_{A-1} \rangle$$



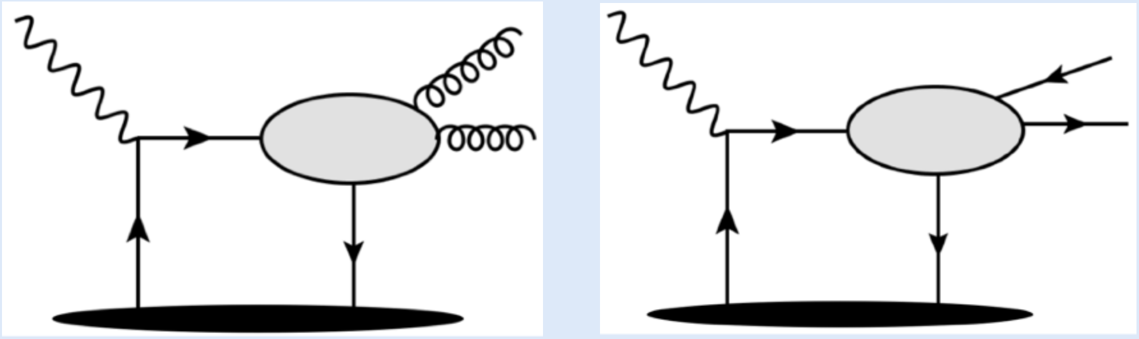
Glauber gluon exchange

- ✦ **Presence of explicit $\ell_{2\perp}$ dependence**
- ✦ **TMD-like in-medium gluonic distribution function**
- ✦ **Origin of scale depedence of jet transport coefficients is presence of explicit $\ell_{2\perp}$ dependence**

❑ **Fermionic Non-Perturbative in-medium correlators**

$$\hat{\mathcal{F}}_{T,2} = g_s^2 \int d(\Delta z^-) d^2\Delta z_\perp \frac{d^2 k_\perp}{(2\pi)^2} \; e^{-i\Delta z^- \left[\frac{\vec{\ell}_{2\perp}^2}{2yq^-} + \frac{\left(\vec{\ell}_{2\perp} - \vec{k}_\perp\right)^2}{2q^-(1-y+\eta y)} \right]} \; e^{i\vec{k}_\perp \cdot \Delta \vec{z}_\perp}$$

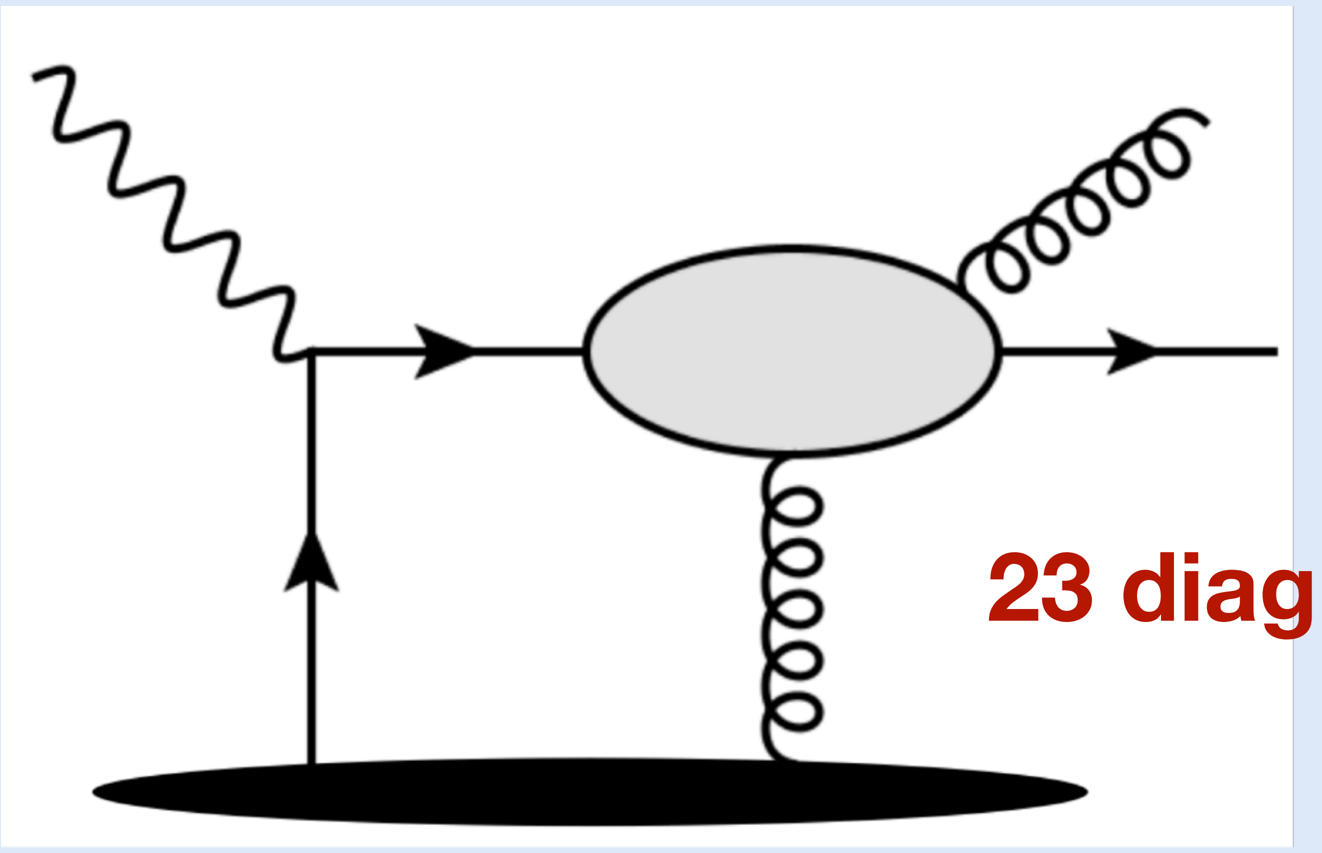
$$\times \langle P_{A-1} | \; \partial_\perp \bar{\psi}^+(\zeta^-, 0) \; \gamma^+ \; \partial_\perp \psi(\zeta^-, \Delta z^-, \Delta z_\perp) \; | P_{A-1} \rangle$$



Glauber quark exchange

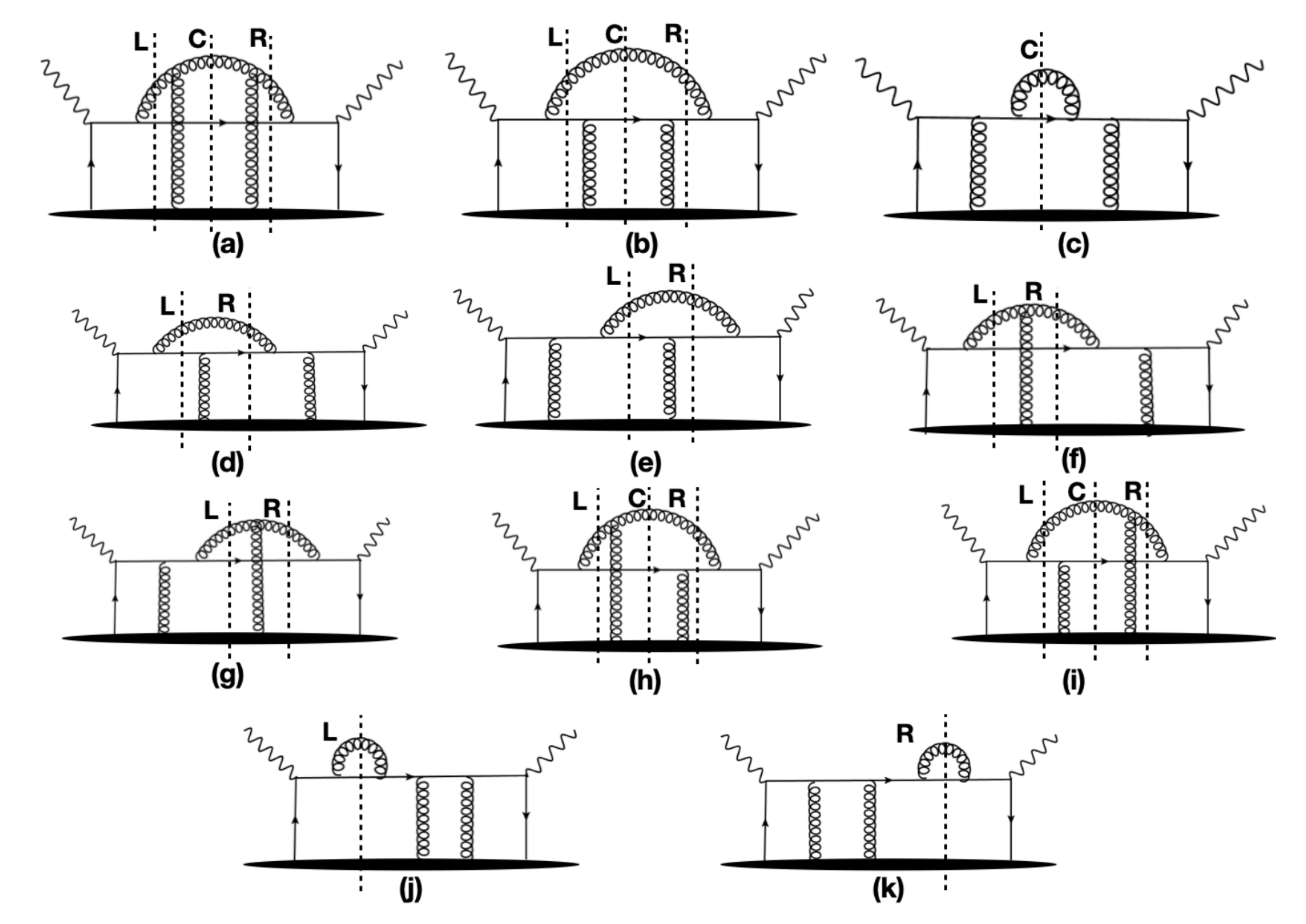
History of computing Medium-induced single-gluon emission kernel

- A gluon and a quark final state



⟹

23 diagrams (α_s^2)



Kumar and Vujanovic
"Glauber quark and gluon contributions to quark energy loss at NLO"
arXiv:2509.10743 (2025)

(Use perturbative QCD to evaluate Feynman Diagrams)

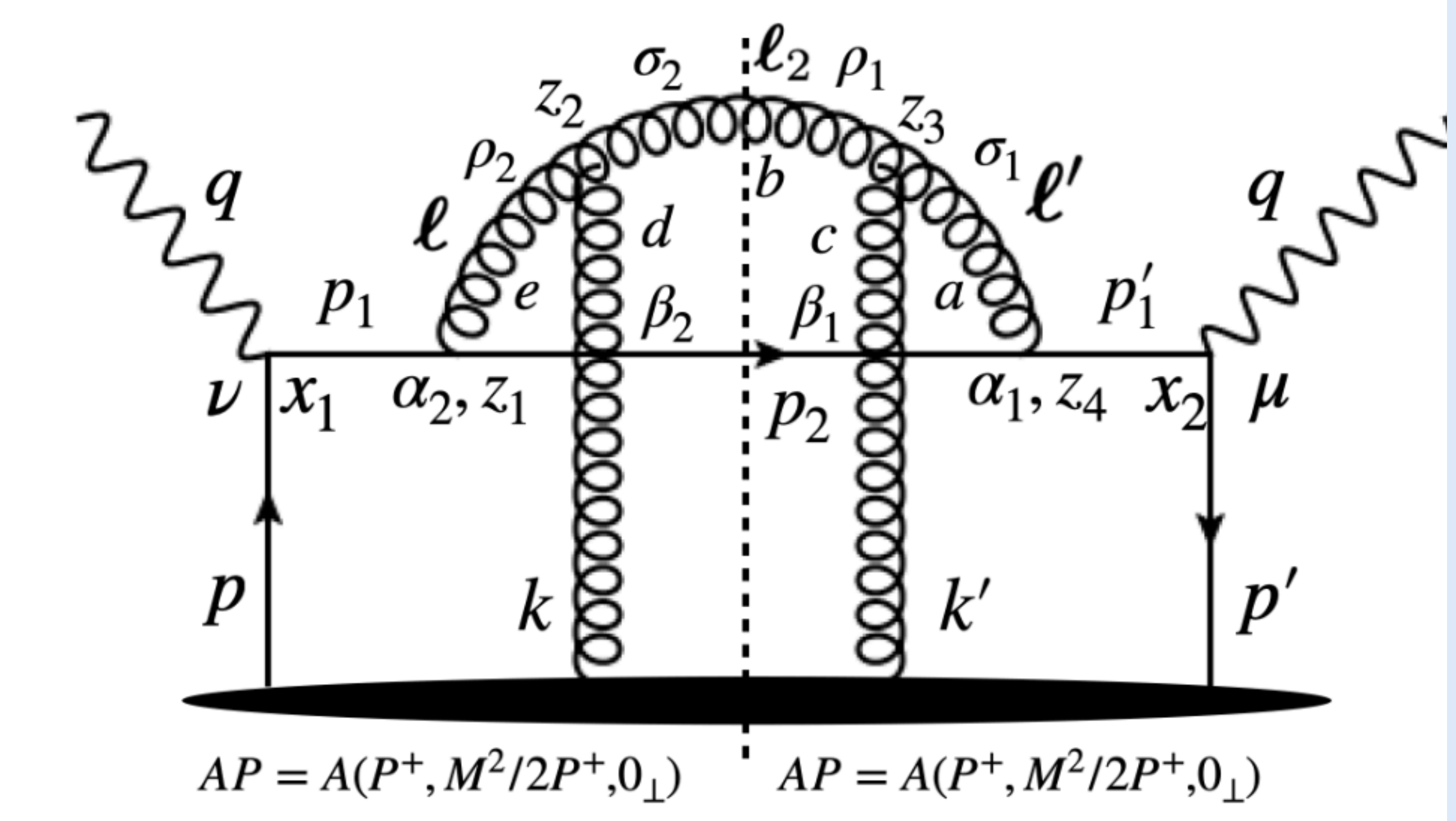
Medium-induced single gluon emission scattering

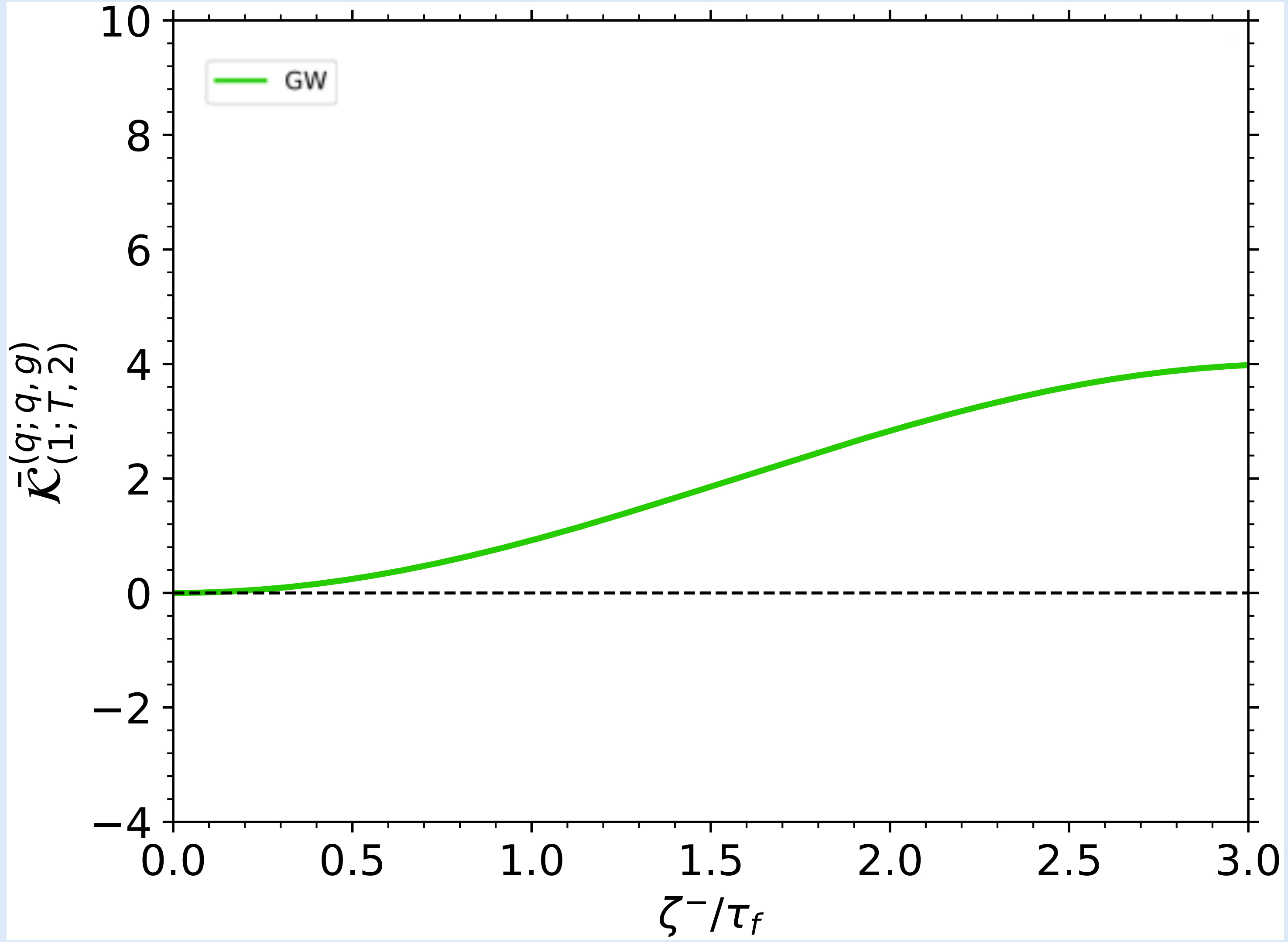
✦ **First calculated by GW (Xiao-feng Guo and Xin-nian Wang) (2000)**

$$\bar{K}_{GW} = 2 - 2\cos(x) \quad \text{; where } x = \zeta^-/\tau_f^-$$

(Concluded only one diagram contribute to energy loss)

✦**Glauber gluon:** $(k^+, k^- \ll k_\perp)$





Medium-induced single gluon emission scattering

✦ **First calculated by GW (Xiao-feng Guo and Xin-nian Wang) (2000)**

$$\bar{K}_{GW} = 2 - 2\cos(x) \quad \text{; where } x = \zeta^-/\tau_f^-$$

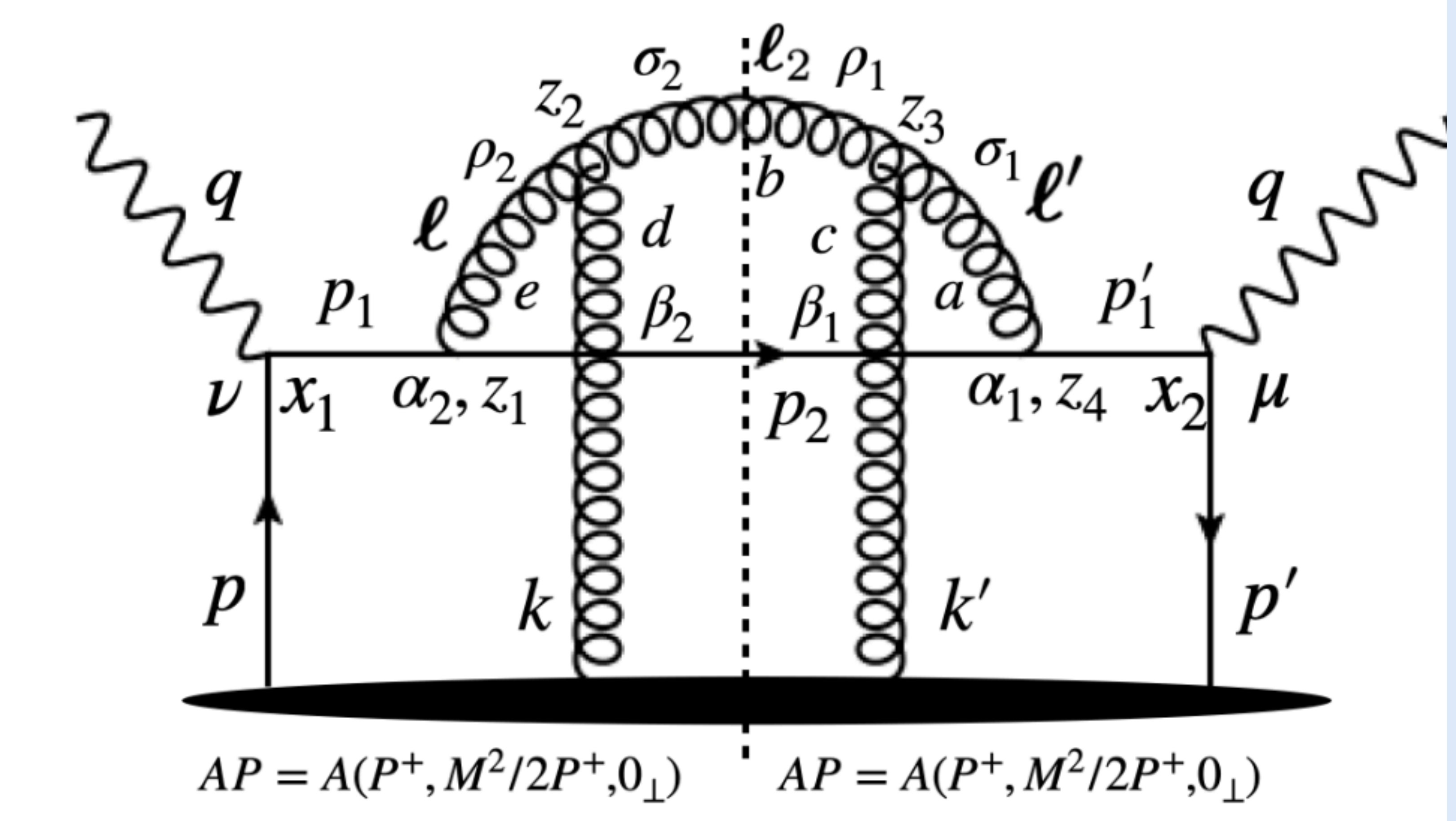
(Concluded only one diagram contribute to energy loss)

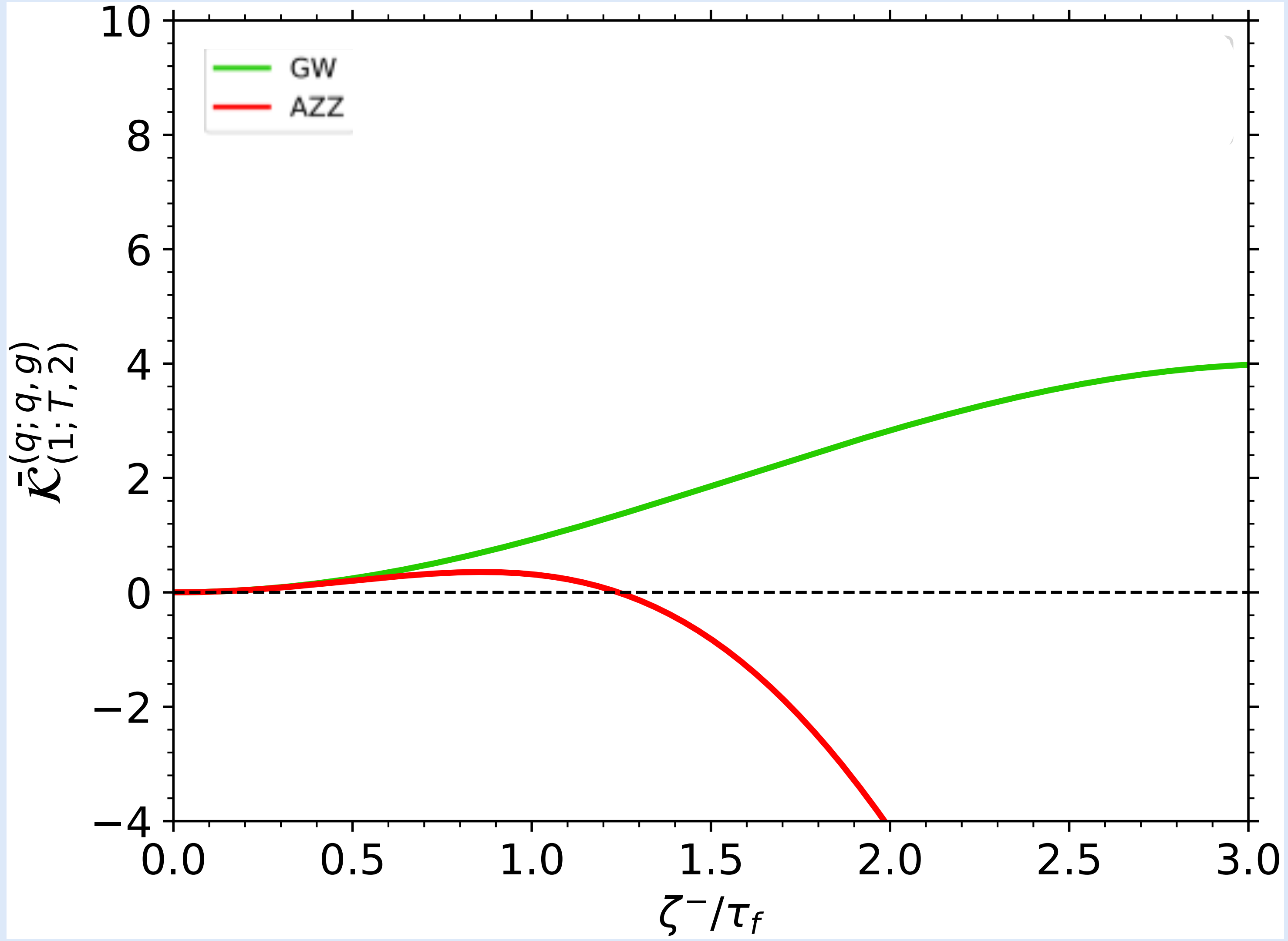
✦**AZZ (Aurenche, Zakharov and Zaraket) calculation (2008)**

$$\bar{K}_{AZZ} = 2 - 2\cos(x) - 2x\sin(x) + 2x^2\cos(x)$$

(Proper collinear expansion of first diagram)

✦**Glauber gluon:** $(k^+, k^- \ll k_\perp)$





Medium-induced single gluon emission scattering

✦ **First calculated by GW (Xiao-feng Guo and Xin-nian Wang) (2000)**

$$\bar{K}_{GW} = 2 - 2\cos(x) \quad \text{; where } x = \zeta^-/\tau_f^-$$

(Concluded only one diagram contribute to energy loss)

✦ **AZZ (Aurenche, Zakharov and Zaraket) calculation (2008)**

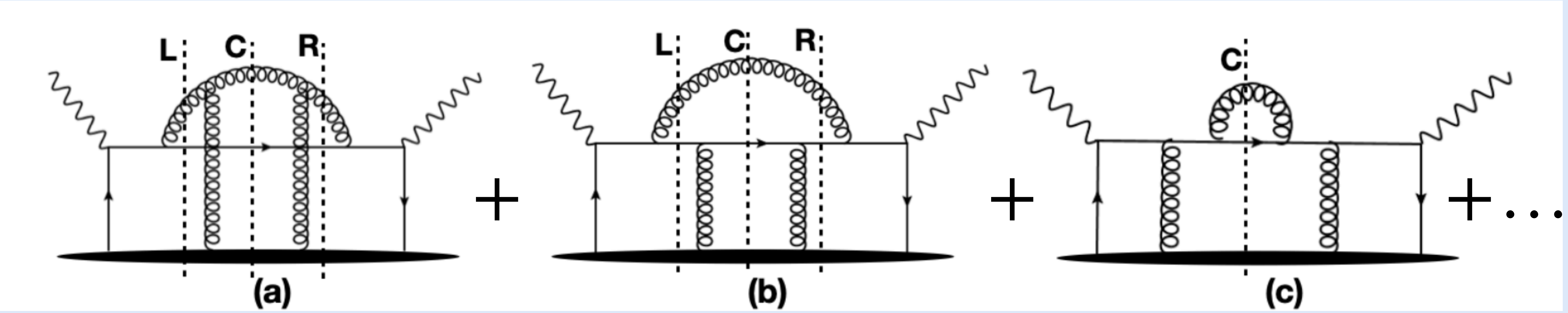
$$\bar{K}_{AZZ} = 2 - 2\cos(x) - 2x\sin(x) + 2x^2\cos(x)$$

(Proper collinear expansion of first diagram)

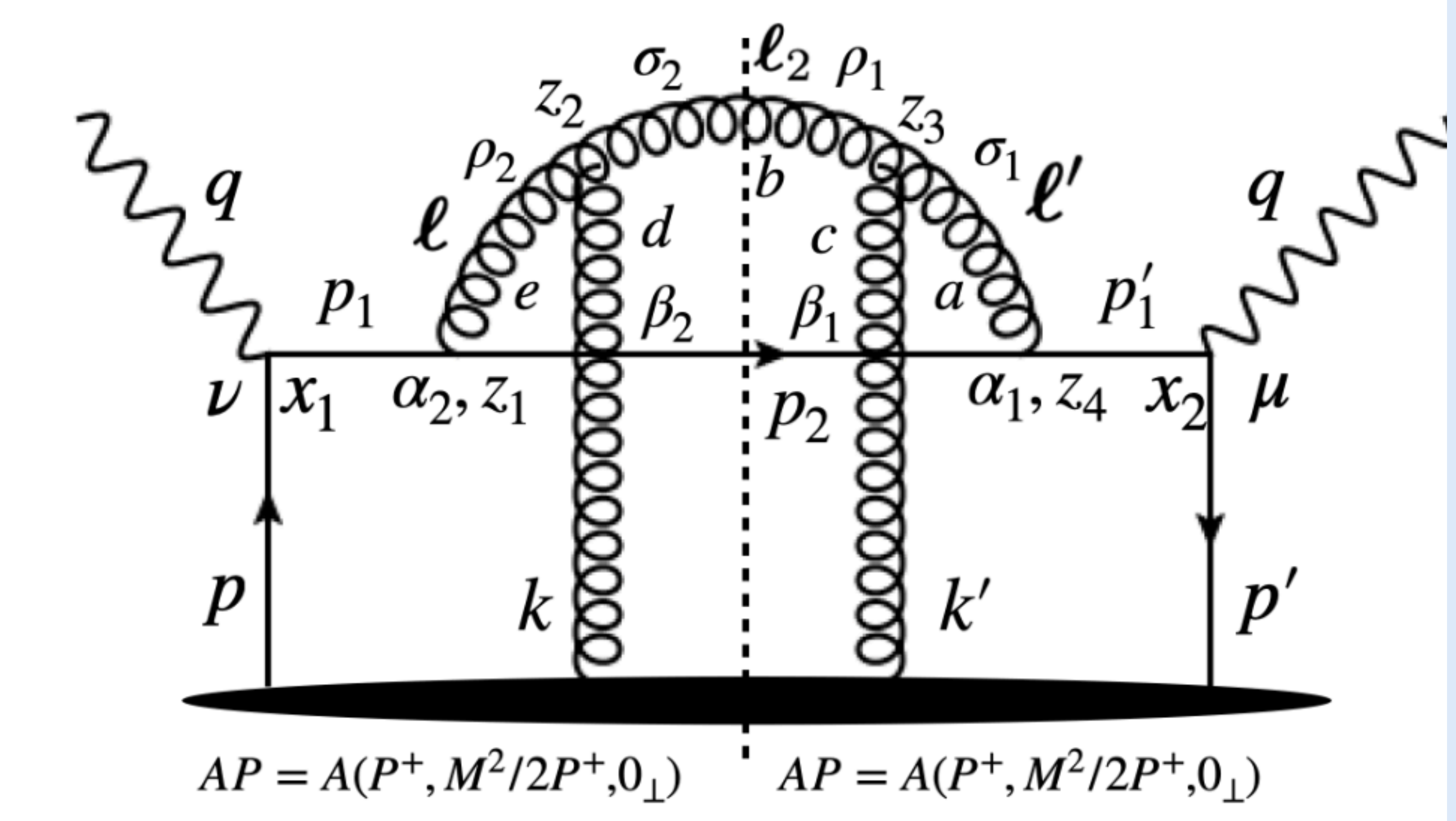
✦ **SCM (Sirimanna-Cao-Majumder) calculation (2022)**

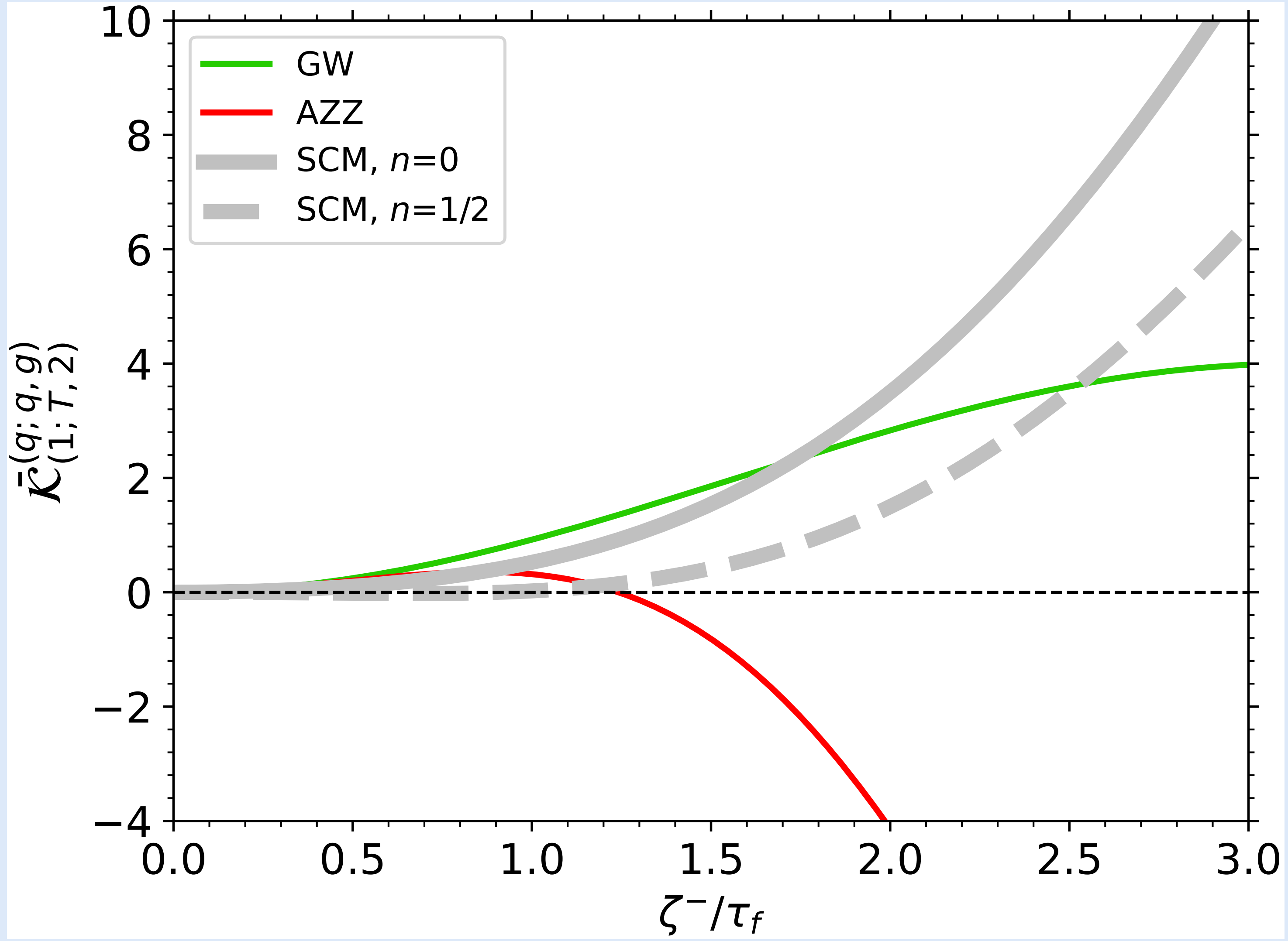
$$\bar{K}_{SCM} = \frac{2-y}{2}\left[2 - 2\cos(x) - 2x\sin(x)\right] + (1-n)x^2$$

(Found other diagrams to be contributing, ignored before)



✦ **Glauber gluon:** $(k^+, k^- \ll k_\perp)$





Medium-induced single gluon emission scattering

✦ **First calculated by GW (Xiao-feng Guo and Xin-nian Wang) (2000)**

$$\bar{K}_{GW} = 2 - 2\cos(x) \quad \text{; where } x = \zeta^-/\tau_f^-$$

(Concluded only one diagram contribute to energy loss)

✦ **AZZ (Aurenche, Zakharov and Zaraket) calculation (2008)**

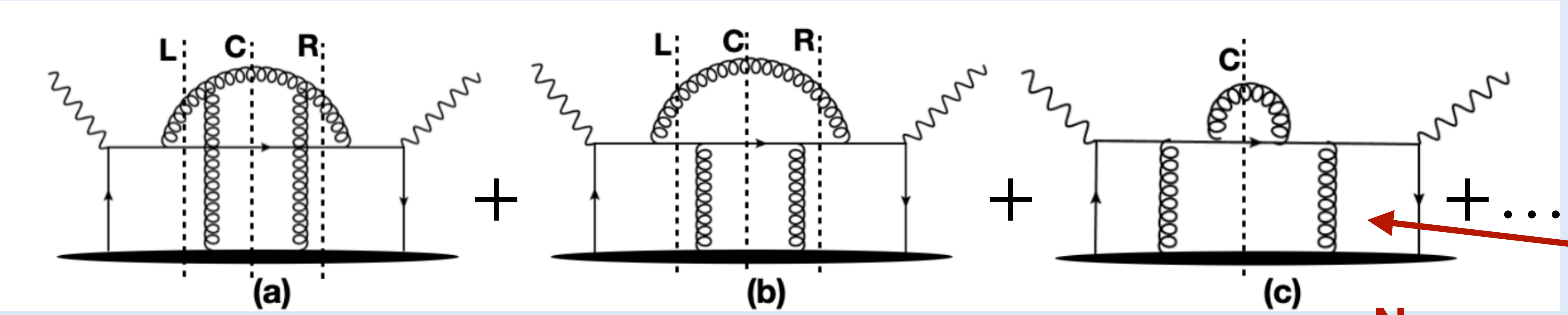
$$\bar{K}_{AZZ} = 2 - 2\cos(x) - 2x\sin(x) + 2x^2\cos(x)$$

(Proper collinear expansion of first diagram)

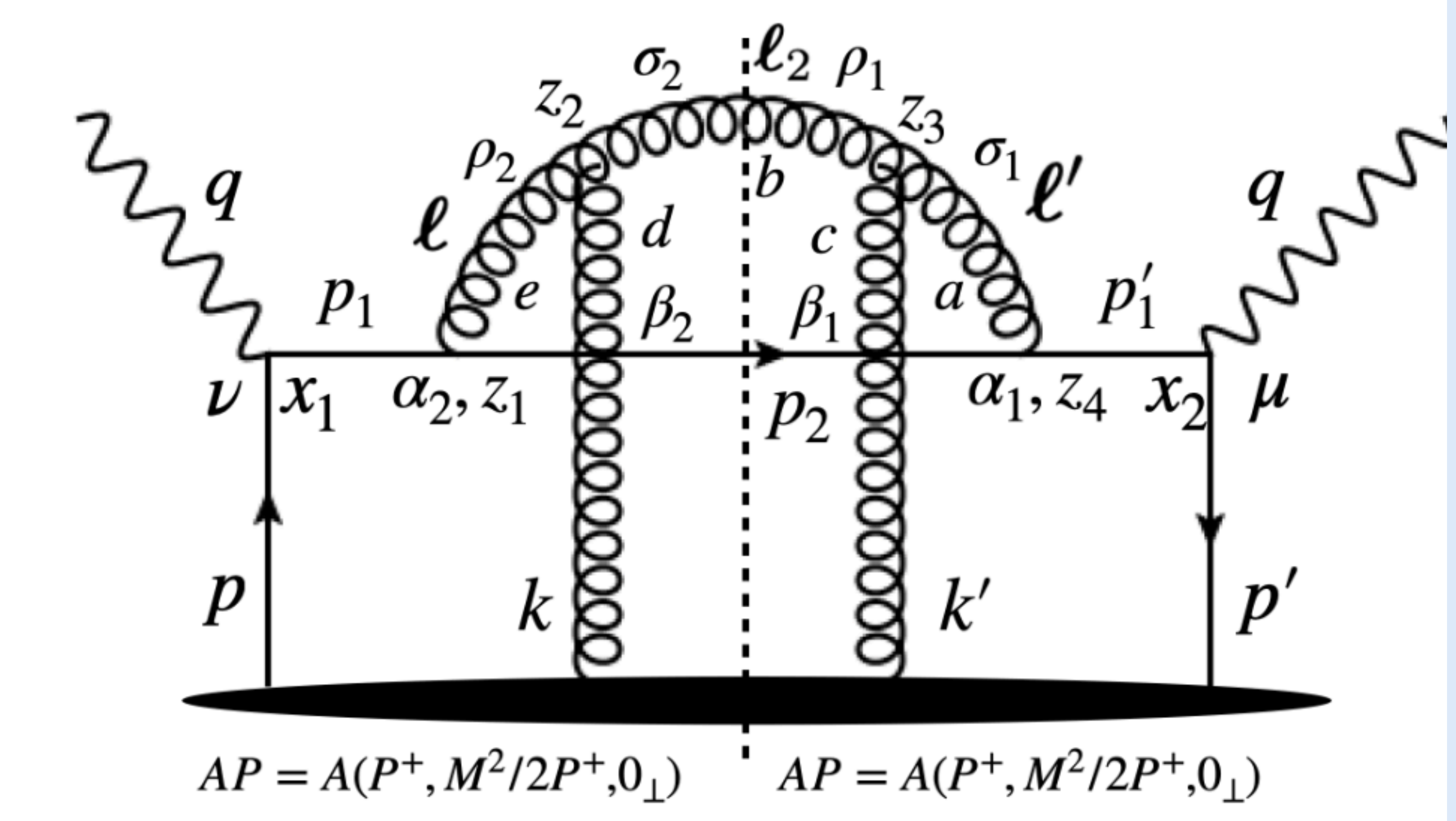
✦ **SCM (Sirimanna-Cao-Majumder) calculation (2022)**

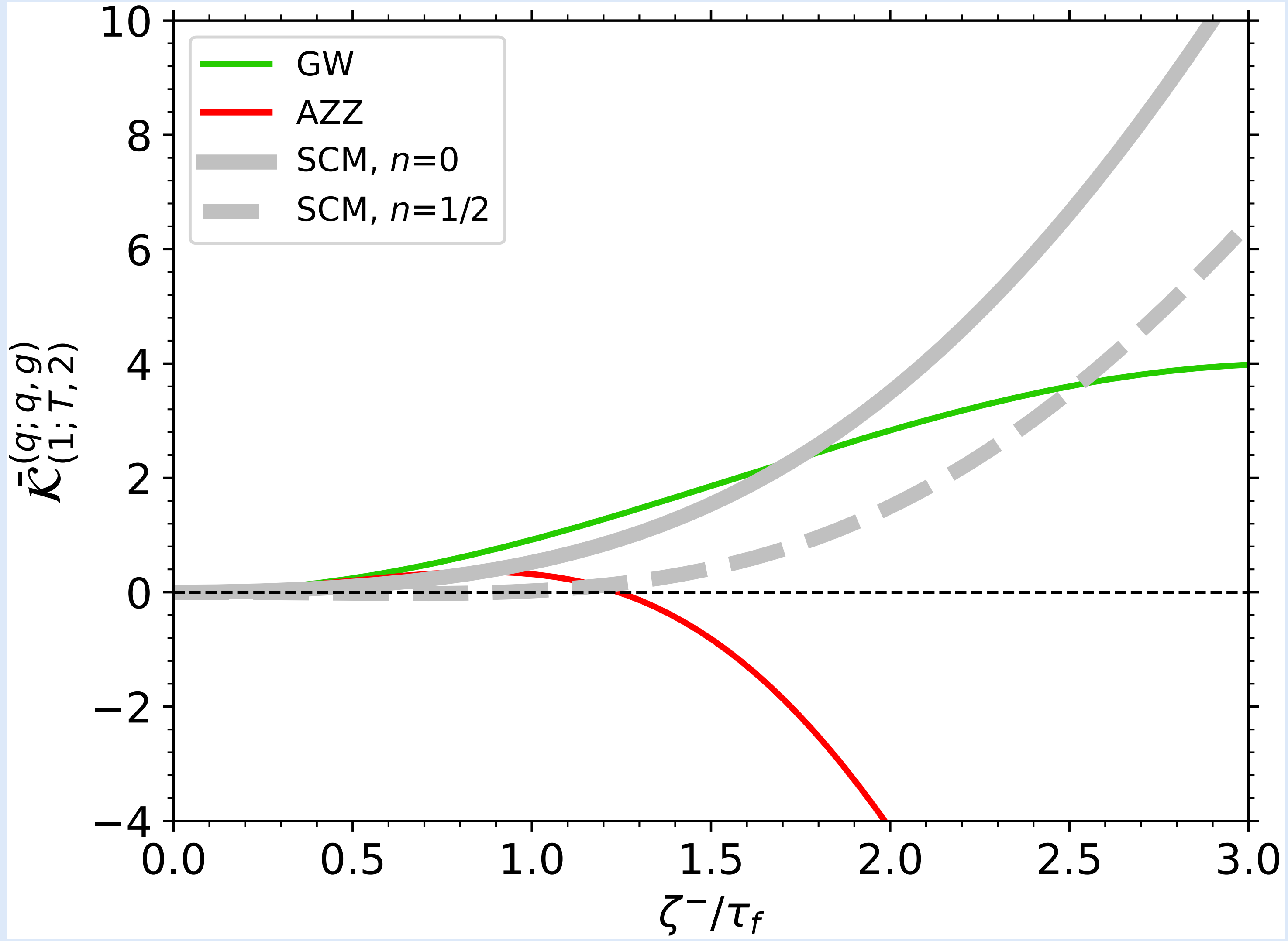
$$\bar{K}_{SCM} = \frac{2-y}{2}\left[2 - 2\cos(x) - 2x\sin(x)\right] + (1-n)x^2$$

(Found other diagrams to be contributing, ignored before)



✦ **Glauber gluon:** $(k^+, k^- \ll k_\perp)$





Non-zero, if massless limit is taken after collinear expansion

Medium-induced single gluon emission scattering

✦ **First calculated by GW (Xiao-feng Guo and Xin-nian Wang) (2000)**

$$\bar{K}_{GW} = 2 - 2\cos(x) \quad \text{; where } x = \zeta^-/\tau_f^-$$

(Concluded only one diagram contribute to energy loss)

✦ **AZZ (Aurenche, Zakharov and Zaraket) calculation (2008)**

$$\bar{K}_{AZZ} = 2 - 2\cos(x) - 2x\sin(x) + 2x^2\cos(x)$$

(Proper collinear expansion of first diagram)

✦ **SCM (Sirimanna-Cao-Majumder) calculation (2022)**

$$\bar{K}_{SCM} = \frac{2-y}{2}\left[2 - 2\cos(x) - 2x\sin(x)\right] + (1-n)x^2$$

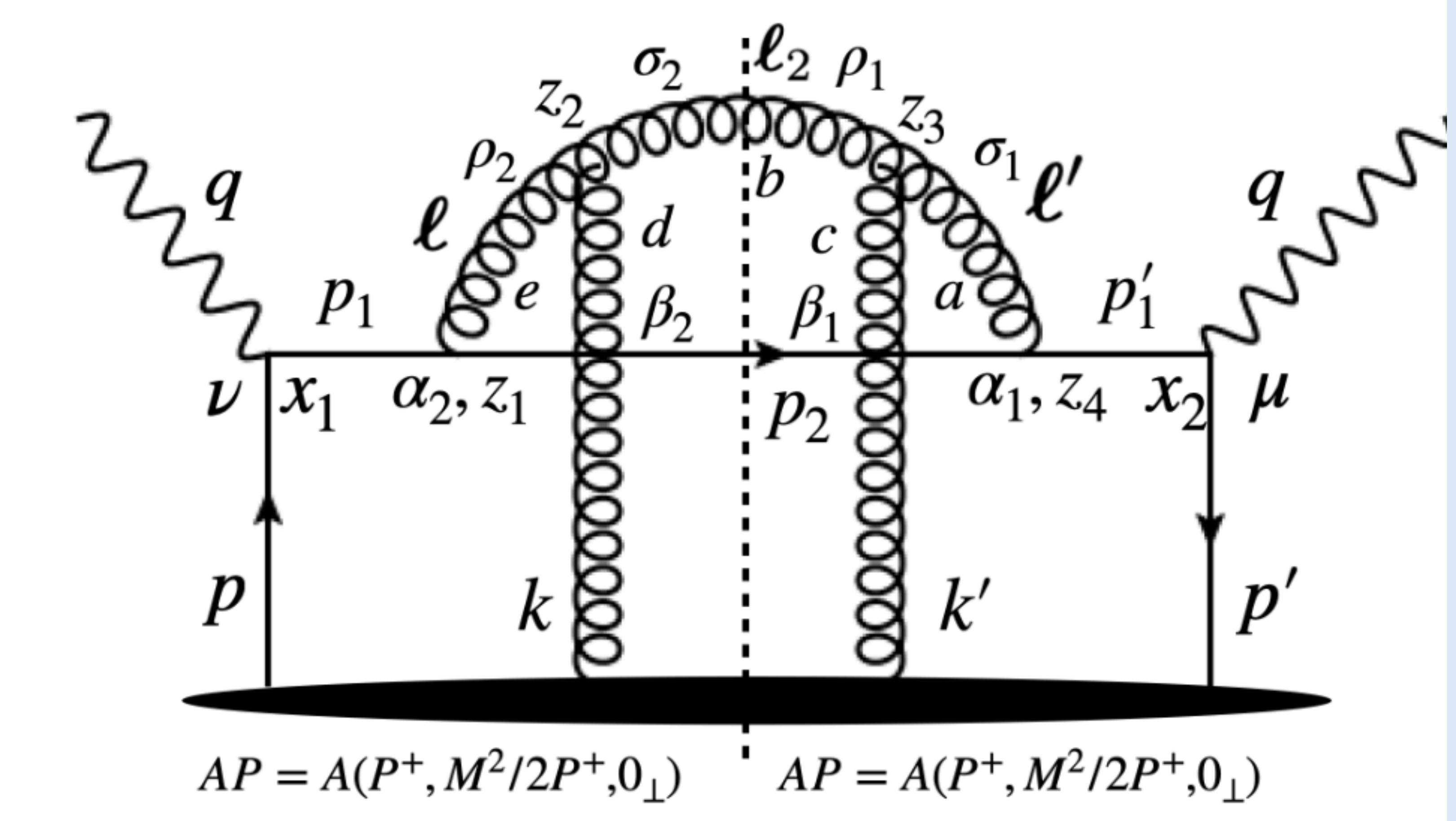
(Found other diagrams to be contributing, ignored before)

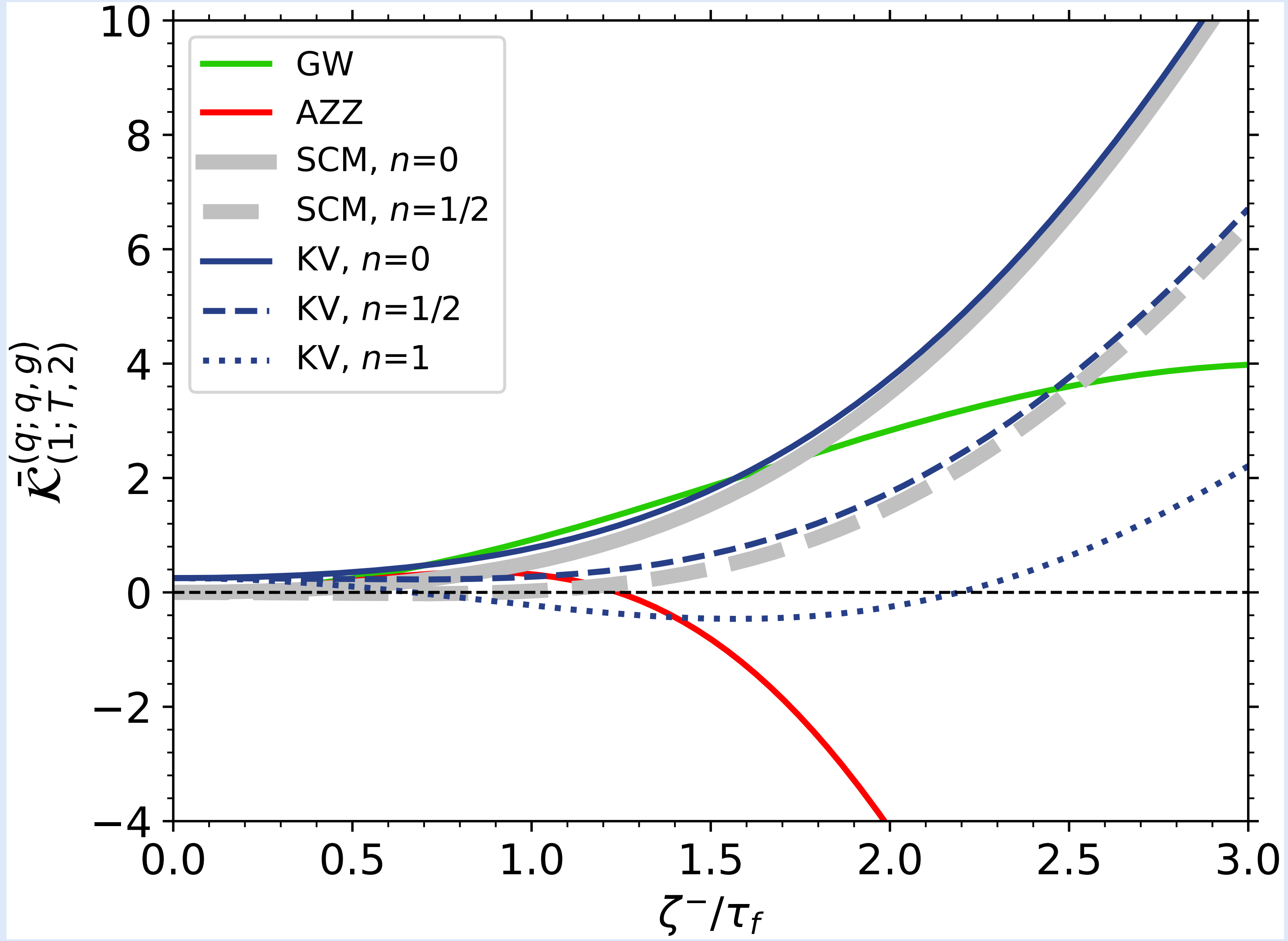
✦ **Kumar and Vujanovic calculation (2025)**

$$\bar{K}_{KV} = \frac{2-y}{2}\left[2 - 2\cos(x) - 2x\sin(x)\right] + (1-n)x^2 + \frac{C_F}{C_A}y^2$$

(Included quark mass effect, k^- momentum contribution, correct phases for NP operators)

✦ **Glauber gluon:** $(k^+, k^- \ll k_\perp)$





Quark energy loss at NLO (All scattering kernels)

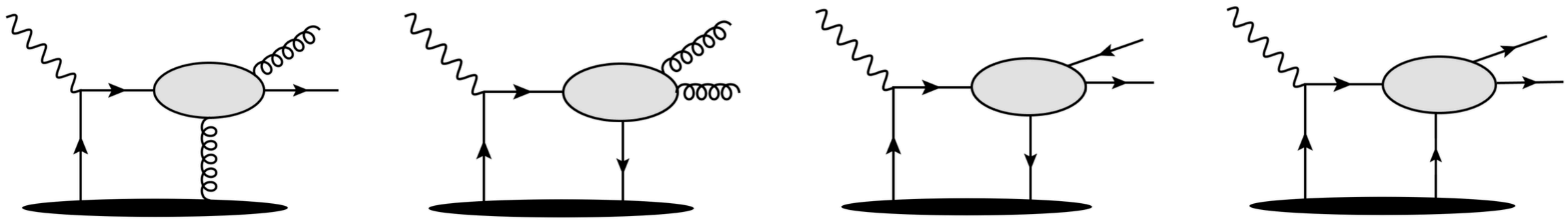
❑ **Photon emission kernels at O($\alpha_s\alpha_{em}$)**

(a) A photon and quark in the final state. (b) A photon and gluon in the final state.

Kumar and Vujanovic, ""Bremsstrahlung photon contributions to parton energy loss at high virtuality with O($\alpha_s\alpha_{em}$)" Phys. Rev. C 112, 025204 (2025)

❑ **Gluon/Quark emission kernels O(α_s^2)**

(Use perturbative QCD to evaluate Feynman Diagrams)



(a) A gluon and quark in the final state. (b) Two gluons in the final state. (c) Quark anti-quark in the final state. (d) Two quarks in the final state.

Kumar and Vujanovic, "Glauber quark and gluon contributions to quark energy loss at NLO", arXiv:2509.10743 (2025)

Quark energy loss at NLO (All scattering kernels)

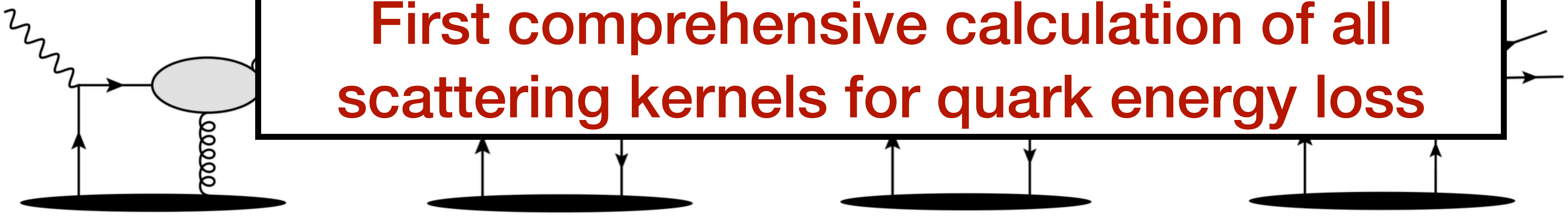
- Photon emission kernels at O($\alpha_s \alpha_{em}$)

(a) A photon and quark in the final state. (b) A photon and gluon in the final state.

Kumar and Vujanovic, ““Bremsstrahlung photon contributions to parton energy loss at high virtuality with O($\alpha_s \alpha_{em}$)” Phys. Rev. C 112, 025204 (2025)

(Use perturbative QCD to evaluate Feynman Diagrams)

- Gluon/Quark emission kernels O(α_s^2)



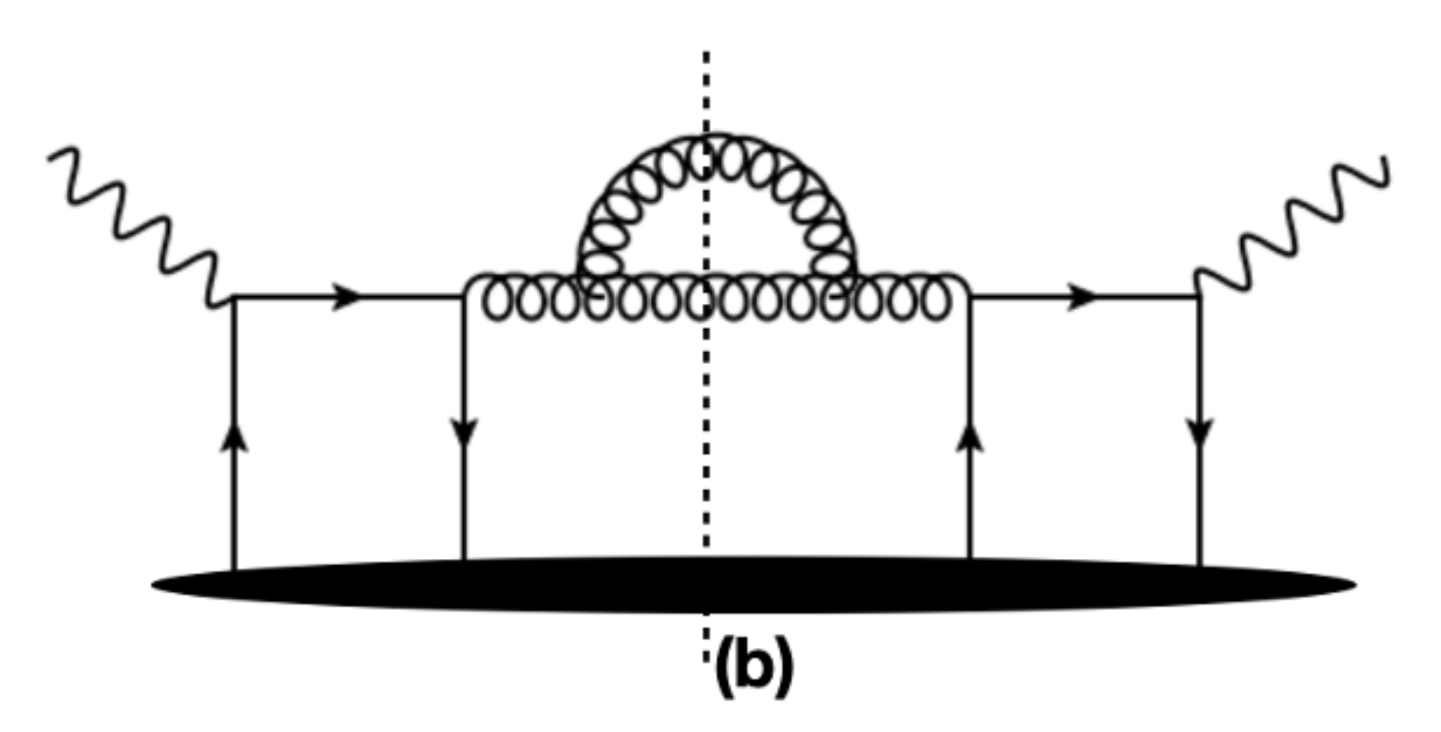
First comprehensive calculation of all scattering kernels for quark energy loss

(a) A gluon and quark in the final state. (b) Two gluons in the final state. (c) Quark anti-quark in the final state. (d) Two quarks in the final state.

Kumar and Vujanovic, “Glauber quark and gluon contributions to quark energy loss at NLO”, arXiv:2509.10743 (2025)

Jet parton chemistry: quark-gluon conversion processes

$$W_2^{\mu\nu} = \sum_f 2\left[-g_{\perp\perp}^{\mu\nu}\right] e_f^2 \int d(\Delta x^-)\, e^{iq^+\Delta x^-} \langle P|\bar{\psi}_f(\Delta x^-)\gamma^+\psi_f(0)|P\rangle \times \mathcal{K}_2^{(q;g,g)}$$



$$\mathcal{K}_2^{(q;g,g)} = g_s^4 \int d(\Delta z^-) d^2\Delta z_\perp \frac{dy}{2\pi}\frac{d^2\ell_{2\perp}}{(2\pi)^2}\frac{d^2k_\perp}{(2\pi)^2}\; e^{-i\Delta z^- \mathcal{H}} e^{ik_\perp\cdot\Delta z_\perp}$$

$$\times \int d\zeta^-\, \theta(\zeta^-)\, S_2^{(q;g,g)}\; \langle P_{A-1}|\bar{\psi}(\zeta^-,0)\,\gamma^+\,\psi(\zeta^-,\Delta z^-,\Delta z_\perp)|P_{A-1}\rangle$$

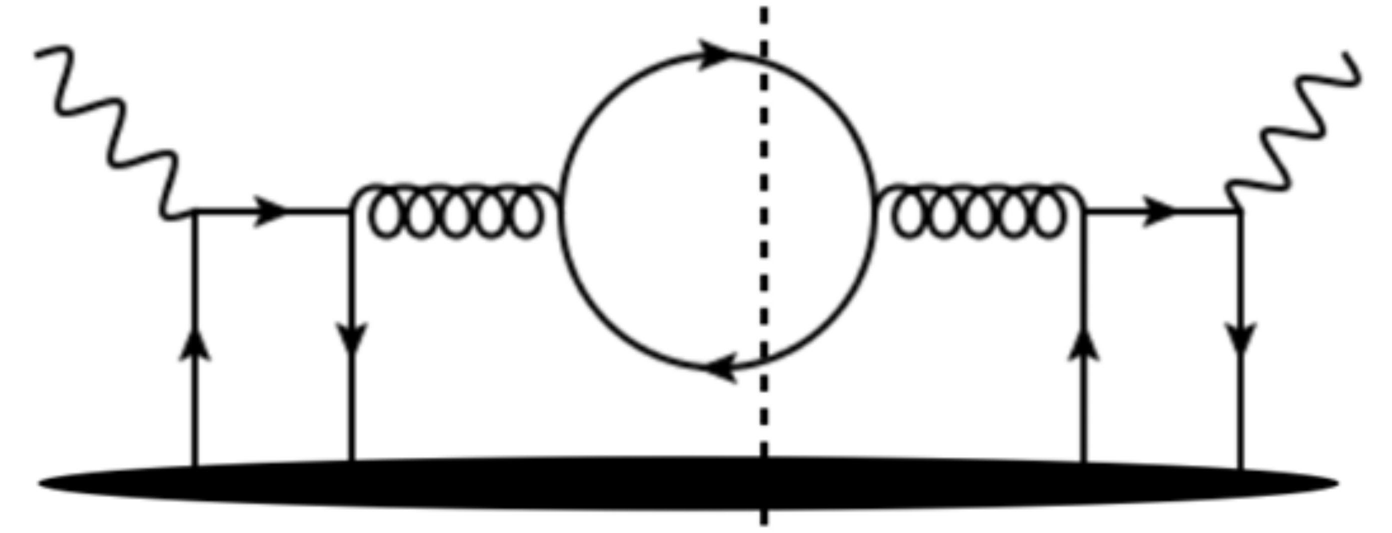
$$S_2^{(q;g,g)} = \frac{2C_A^2C_F}{(1+\eta y)^2}\left[\frac{y(1+\eta y)^2}{1-y+\eta y} + \frac{(1+2\eta y)(1-y+\eta y)}{y} + y(1+\eta^2)(1-y+\eta y)\right] \times \frac{1}{q^-}\frac{1}{[(1+\eta y)\ell_{2\perp} - yk_\perp]^2}$$

Medium modified splitting function for $g \to g + g$

Suppress by the hard parton energy

In 2->2 scattering limit at LO with no emission:
Phys.Rev.C 108, 014911 (2023)

Heavy-quarks ($c\bar{c}$, $b\bar{b}$, and $t\bar{t}$) production in QGP



$$W_3^{\mu\nu} = 2\left[-g_{\perp\perp}^{\mu\nu}\right] e_f^2 \int d(\Delta x^-)\, e^{iq^+\Delta x^-} \langle P|\bar{\psi}_f(\Delta x^-)\gamma^+\psi_f(0)|P\rangle \times \mathcal{K}_3^{(q;q',\bar{q}')}$$

$$\mathcal{K}_3^{(q;q',\bar{q}')} = g_s^4 \sum_{f'\neq f} \int d(\Delta z^-) d^2\Delta z_\perp \frac{dy}{2\pi} \frac{d^2\ell_{2\perp}}{(2\pi)^2} \frac{d^2k_\perp}{(2\pi)^2}\; e^{-i\Delta z^- \mathcal{H}} e^{ik_\perp\cdot\Delta z_\perp}$$

$$\times \int d\zeta^-\, \theta(\zeta^-)\, S_3^{(q;q',\bar{q}')}\; \langle P_{A-1}|\bar{\psi}_f(\zeta^-,0)\,\gamma^+\,\psi_f(\zeta^-,\Delta z^-,\Delta z_\perp)|P_{A-1}\rangle$$

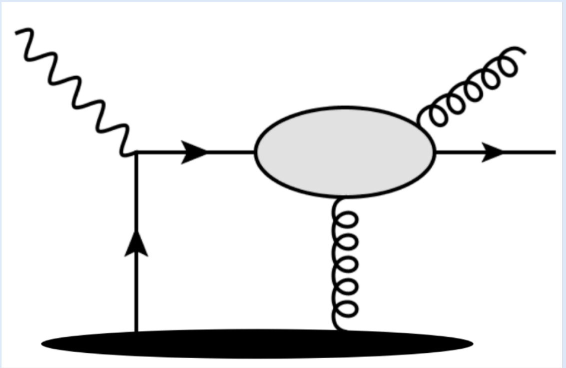
$$S_3^{(q;q',\bar{q}')} = \frac{C_A C_F}{2(1+\eta y)^2} \times \frac{\left[y^2 + (1-y+\eta y)^2\right]}{[\{(1+\eta y)\ell_{2\perp} - yk_\perp\}^2 + M_{f'}^2(1+\eta y)^2]} \times \frac{1}{q^-}$$

Medium modified splitting function for $g \rightarrow g + g$

Suppress by the hard parton energy

Jet energy loss transport coefficients

In-medium Glauber gluon exchange



- **Transport coefficient $\hat{q}$:**
  Average transverse momentum squared per unit length

$$\hat{q}(\vec{r},t) = \frac{\langle \vec{k}_\perp^2 \rangle}{L} \propto \langle M | F_\perp^{+}(y^-, y_\perp) F^{+\perp}(0) | M \rangle$$

- **Transport coefficient $\hat{e}$:** Change in longitudinal momentum

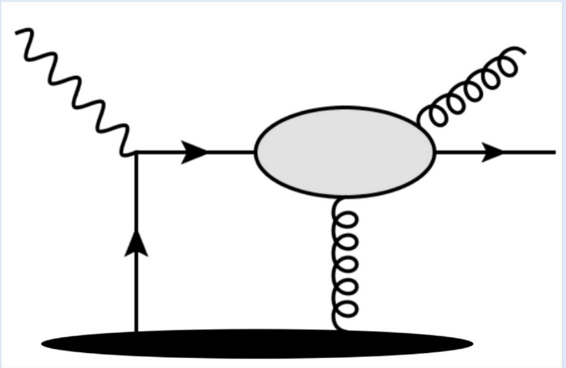
$$\hat{e}(\vec{r},t) = \frac{\langle k^- \rangle}{L} \propto \langle M | \partial^- A^+(y^-, y_\perp) A^+(0) | M \rangle$$

- **Transport coefficient $\hat{e}_2$:** Square of change in longitudinal momentum

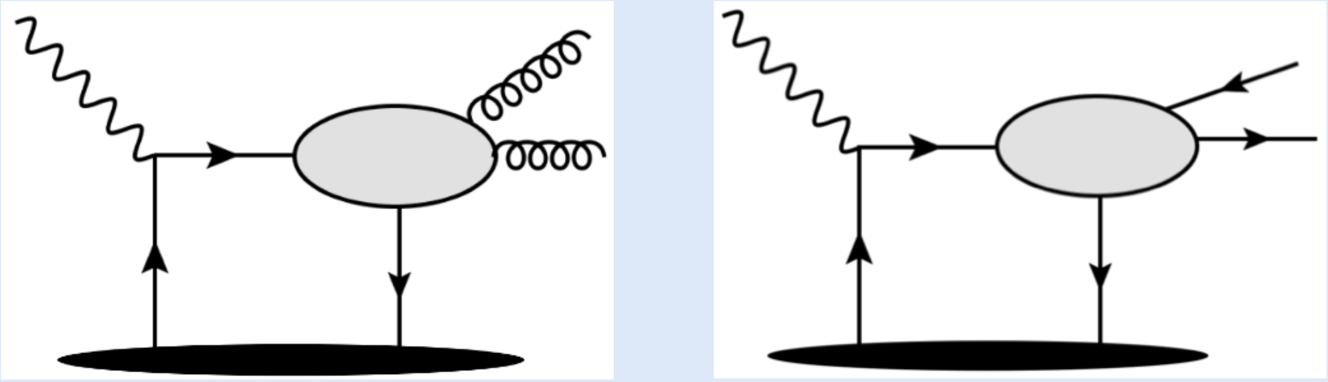
$$\hat{e}_2(\vec{r},t) = \frac{\langle (k^-)^2 \rangle}{L} \propto \langle M | F^{+-}(y^-, y_\perp) F^{+-}(0) | M \rangle$$

Jet energy loss transport coefficients

In-medium Glauber gluon exchange



In-medium Glauber quark exchange



- **Transport coefficient $\hat{q}$:**
  Average transverse momentum squared per unit length

$$\hat{q}(\vec{r},t) = \frac{\langle \vec{k}_\perp^2 \rangle}{L} \propto \langle M | F_\perp^{+}(y^-, y_\perp) F^{+\perp}(0) | M \rangle$$

- **Transport coefficient $\hat{e}$:** Change in longitudinal momentum

$$\hat{e}(\vec{r},t) = \frac{\langle k^- \rangle}{L} \propto \langle M | \partial^- A^+(y^-, y_\perp) A^+(0) | M \rangle$$

- **Transport coefficient $\hat{e}_2$:** Square of change in longitudinal momentum

$$\hat{e}_2(\vec{r},t) = \frac{\langle (k^-)^2 \rangle}{L} \propto \langle M | F^{+-}(y^-, y_\perp) F^{+-}(0) | M \rangle$$

- **Transport coefficient:**

$$\hat{F}_0 \propto \langle M | \bar{\psi}_f(0)\, \gamma^+\, \psi_f(y^-, y_\perp) | M \rangle$$

- **Transport coefficient : $\langle k_\perp^2 \rangle$**

$$\hat{F}_{T,2} \propto \langle M | \partial_\perp \bar{\psi}_f(0)\, \gamma^+\, \partial_\perp \psi_f(y^-, y_\perp) | M \rangle$$

- **Transport coefficient : $\langle k^- \rangle$**

$$\hat{F}_{L,1} \propto \langle M | i\partial^- \bar{\psi}_f(0)\, \gamma^+\, \psi_f(y^-, y_\perp) | M \rangle$$

$\hat{q}$ for pure gluon plasma and 2+1 flavor QCD

- **At high temperature:** $\hat{q} \propto T^3$

  1) $\hat{q}/T^3 \sim$ 1.25-2.5 **(2+1 flavor QCD plasma)**
  2) $\hat{q}/T^3 \sim$ 0.75-1.25 **(pure gluon plasma)**

- **At low temperature:**

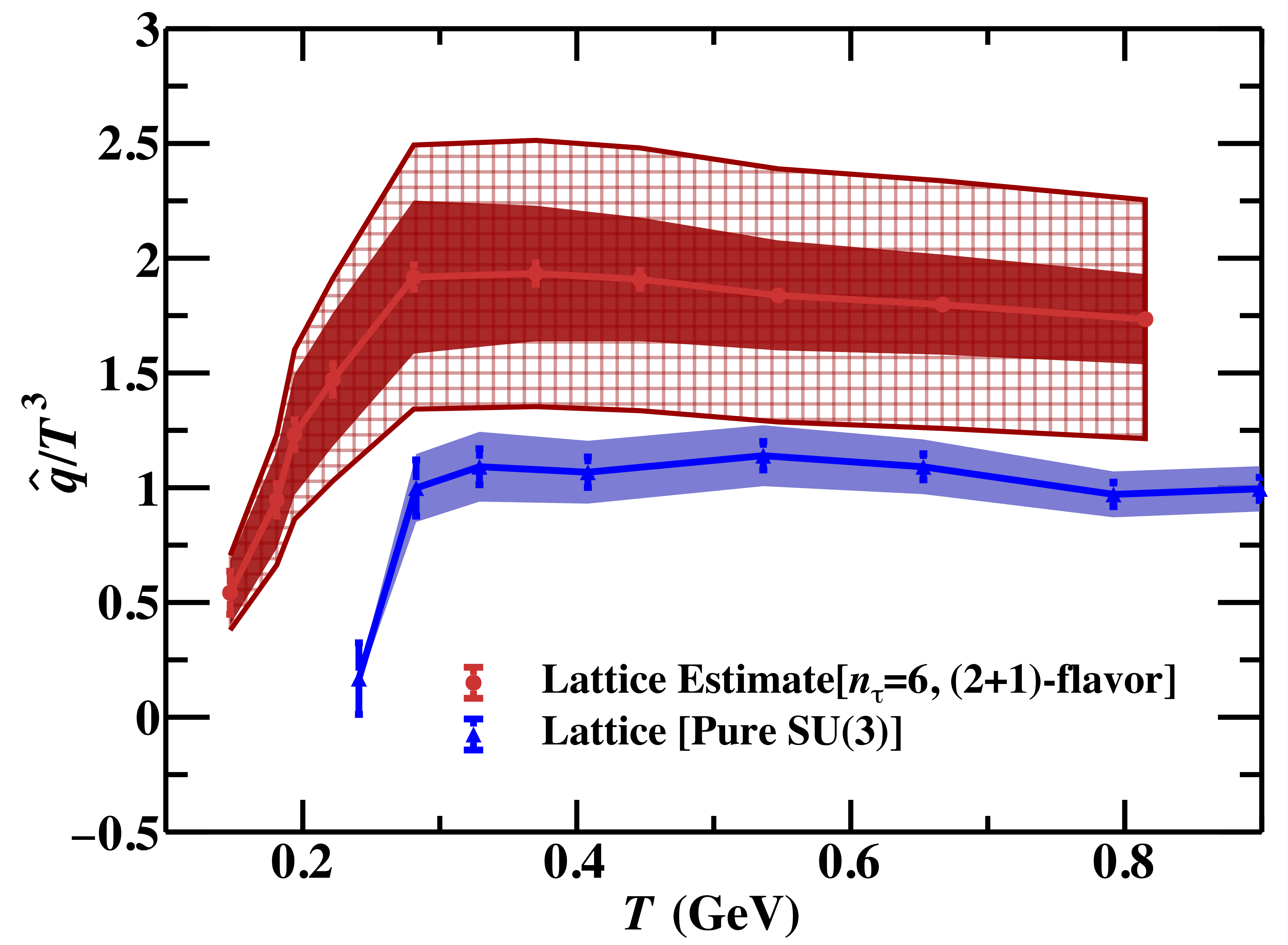
  $\hat{q}/T^3$ becomes smaller and No signature of log-dependence as in HTL form

  $$\text{(HTL formula)} \equiv \hat{q} \propto \alpha_s^2 T^3 \ln(E/T)$$

- **The qualitative behavior is similar to entropy density**
- **Uncertainty**

  Grid line: 30% error from renormalization (corrections from quark and gluon renormalization)

  Solid red and blue band: uncertainty in scale $2\pi T<\mu<4\pi T$ for strong coupling constant



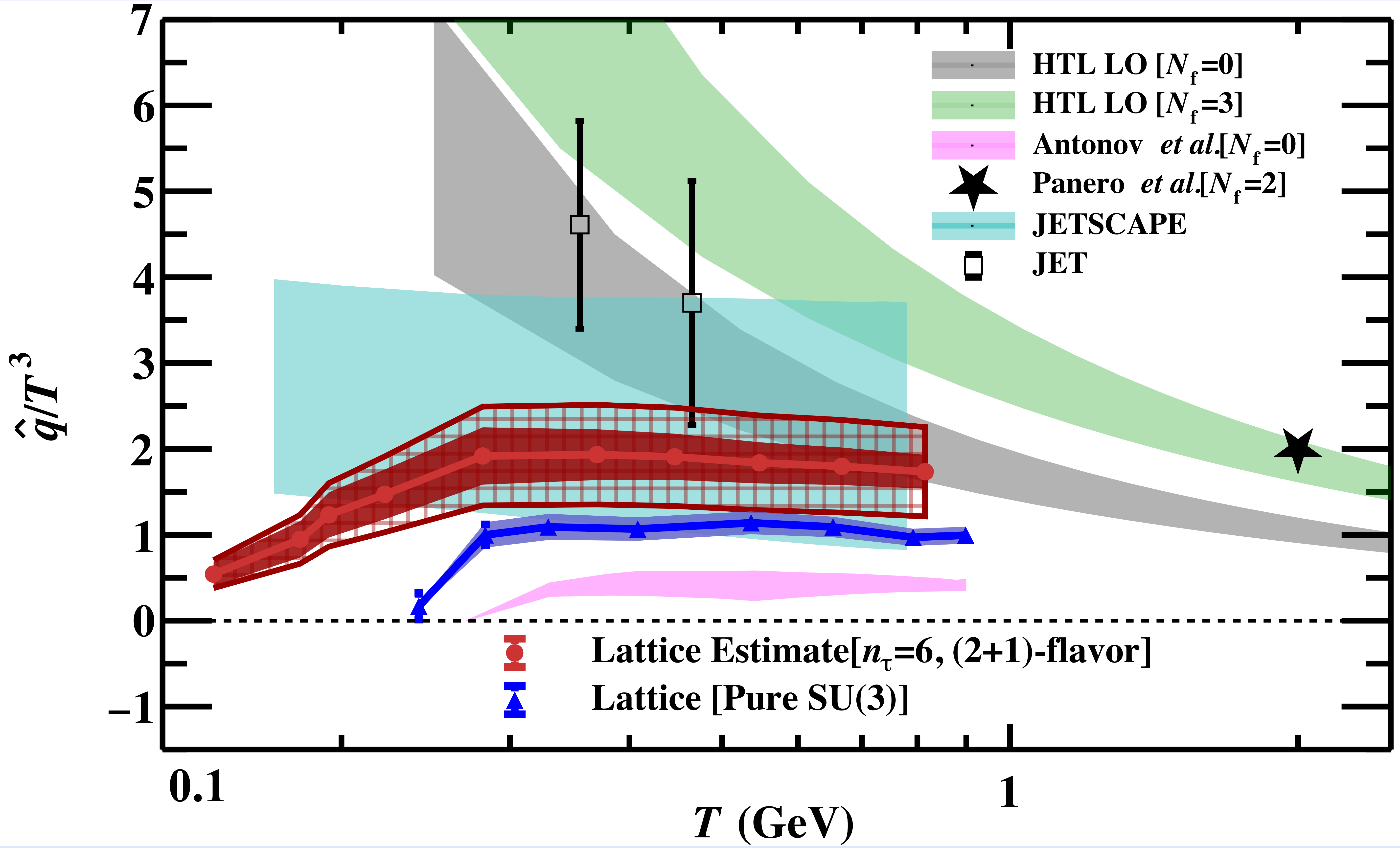
A. Kumar A. Majumder, J. Weber, PRD 106, 034505 (2022)

Comparison with previous work

- Lattice extractions are consistent with JET and JETSCAPE collaboration extractions!

- Lattice extracted $\hat{q}$ does not show a log-like behavior

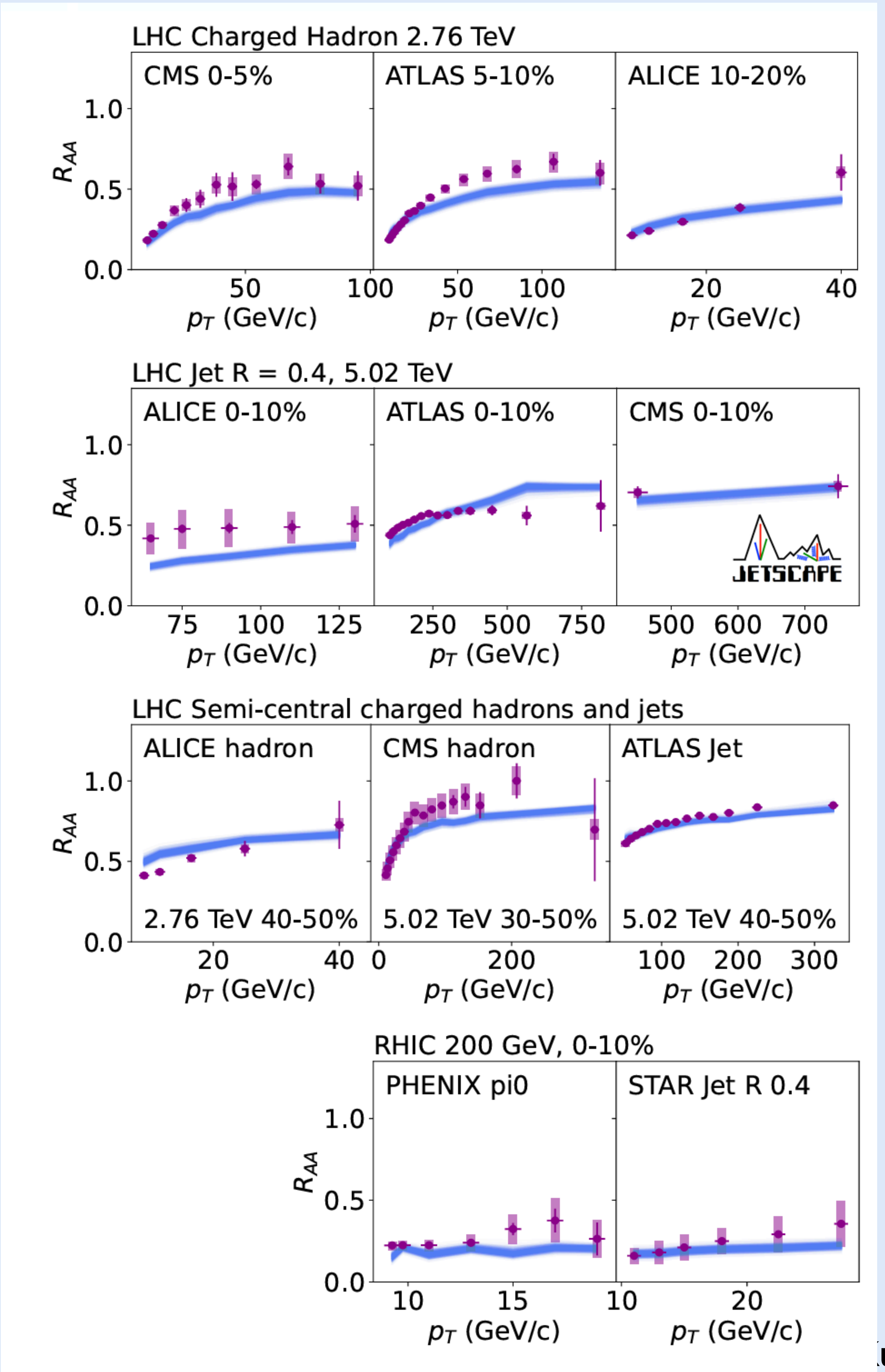
$$\hat{q} \propto \alpha_s^2 T^3 \ln\left(\frac{E}{T}\right) \text{ (HTL formula)}$$



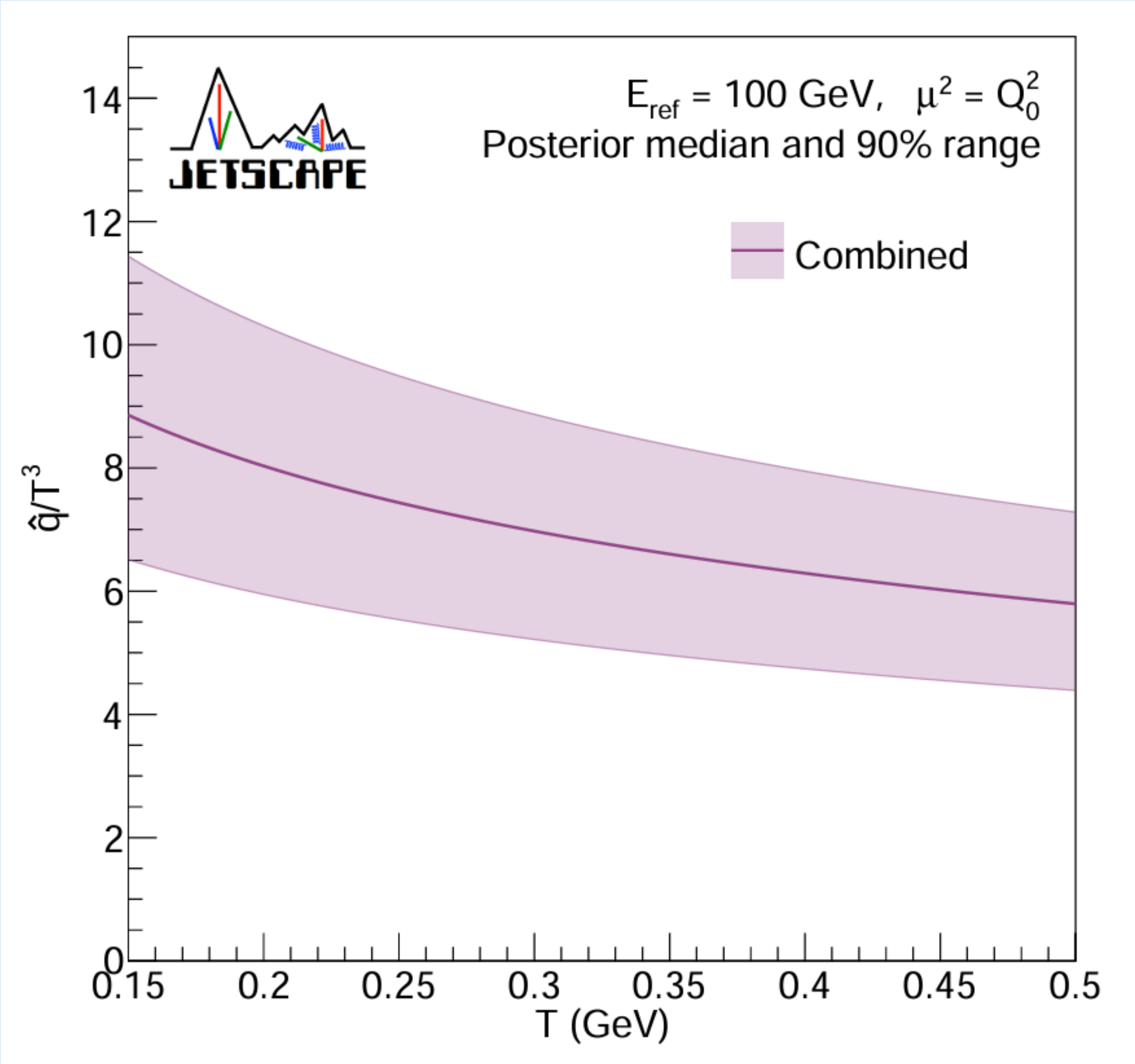
A. Kumar A. Majumder, J. Weber, PRD 106, 034505 (2022)

- Hard-thermal loop results with $q^- = 100$ GeV for $N_f = 0$ and $N_f = 3$
- Electrostatic QCD results for $N_f = 2$ at high temperature $T = 2$ GeV. (Panero, et al. PRL 112(2014))
- Stochastic vacuum model (Landau damping) for $N_f = 0$: similar shape. (Antonov, et al, EPJC55 439 (2008))

Bayesian extraction of $\hat{q}$ using RHIC and LHC data



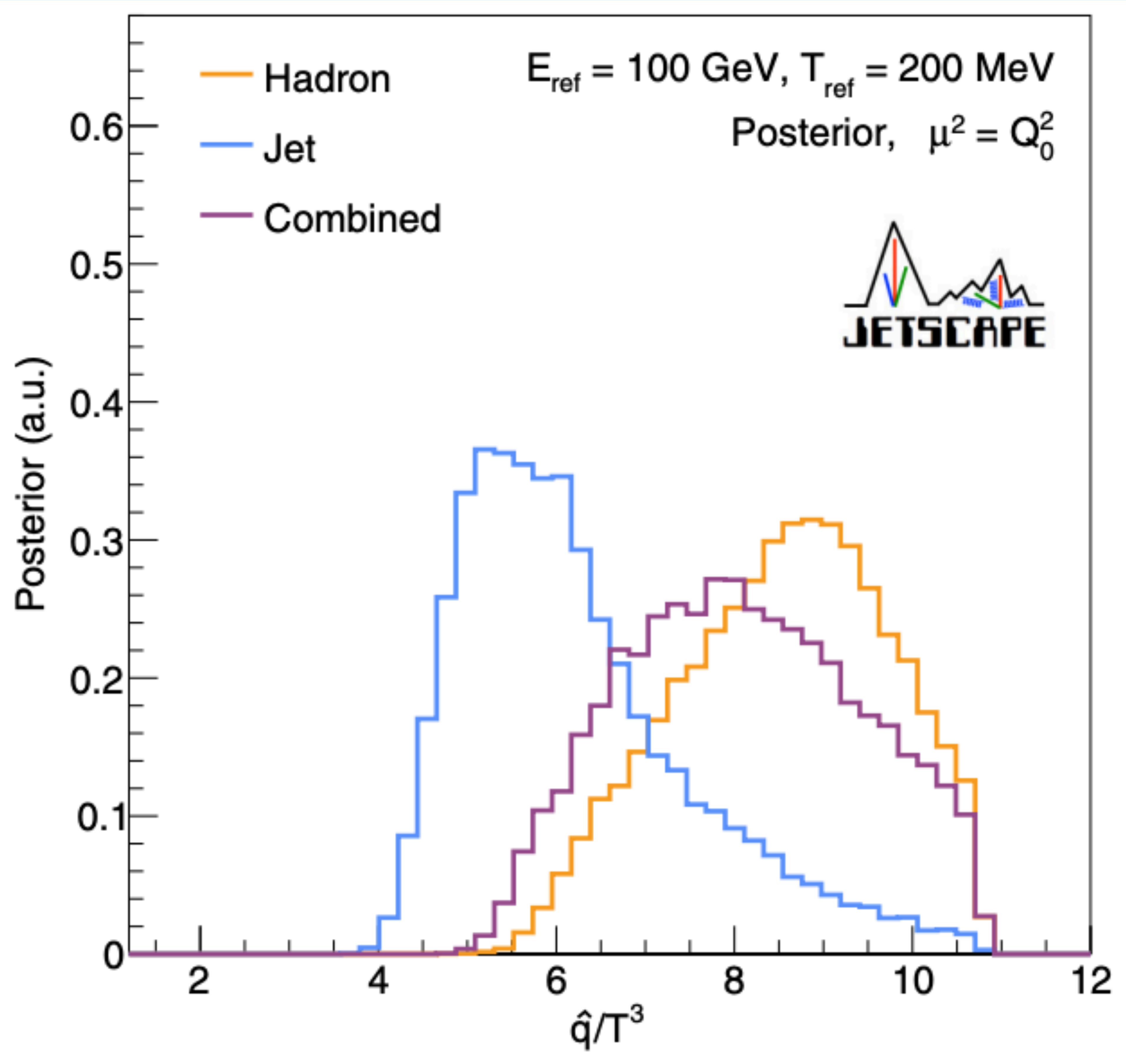
- ❑ Combined fit to charged-hadrons and inclusive jet spectrum
- ❑ Three collision energy ($\sqrt{s_{NN}} =$ 0.2, 2.76, 5.02 TeV)
- ❑ $\mathcal{O}(10^7)$ core hours used in Bayesian analysis

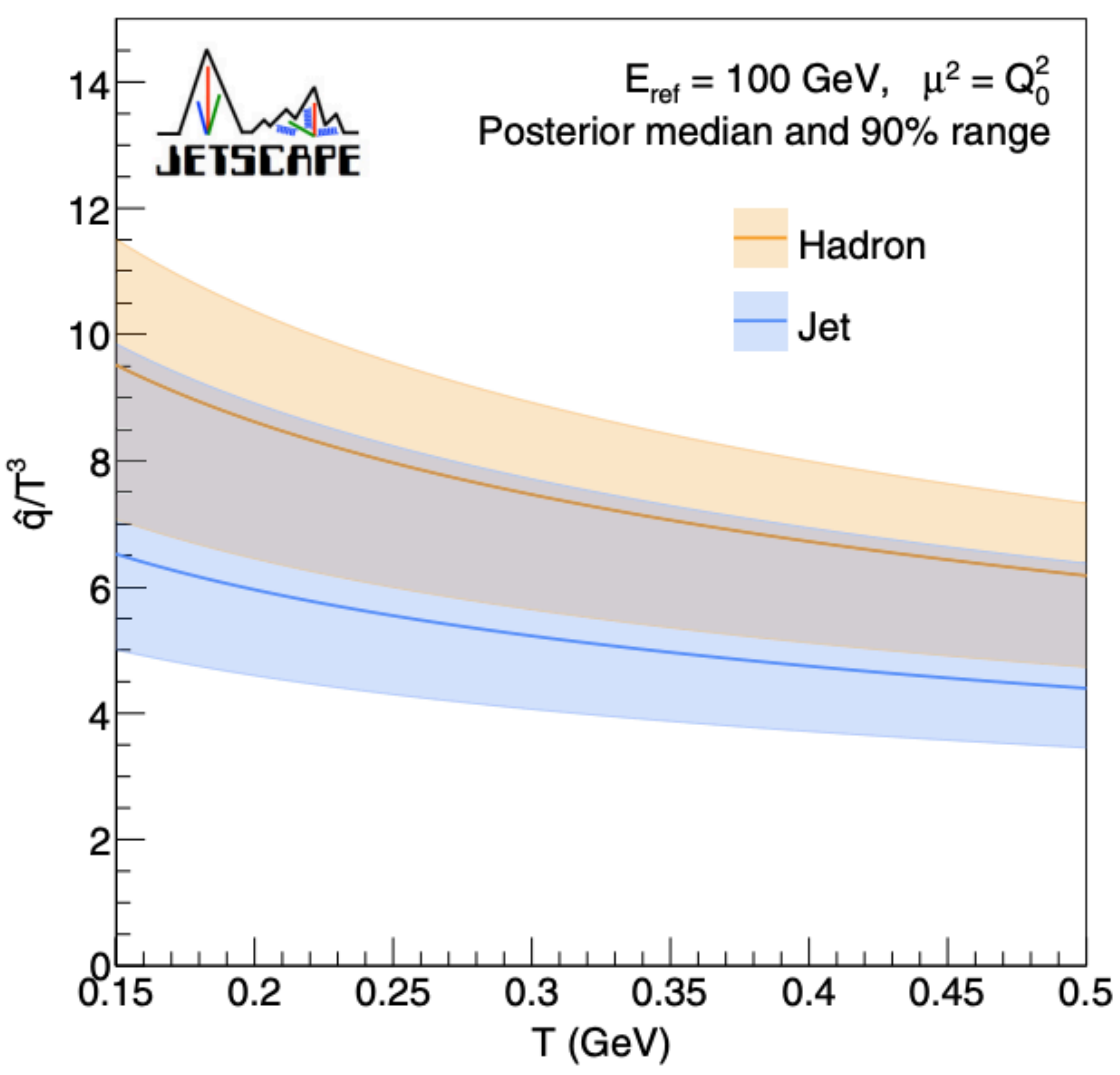


R. Ehlers et al., PRC 111, 054913 (2025)

- ❑ p_T dependence of spectrum can be improved

Bayesian extraction of $\hat{q}$ using RHIC and LHC data



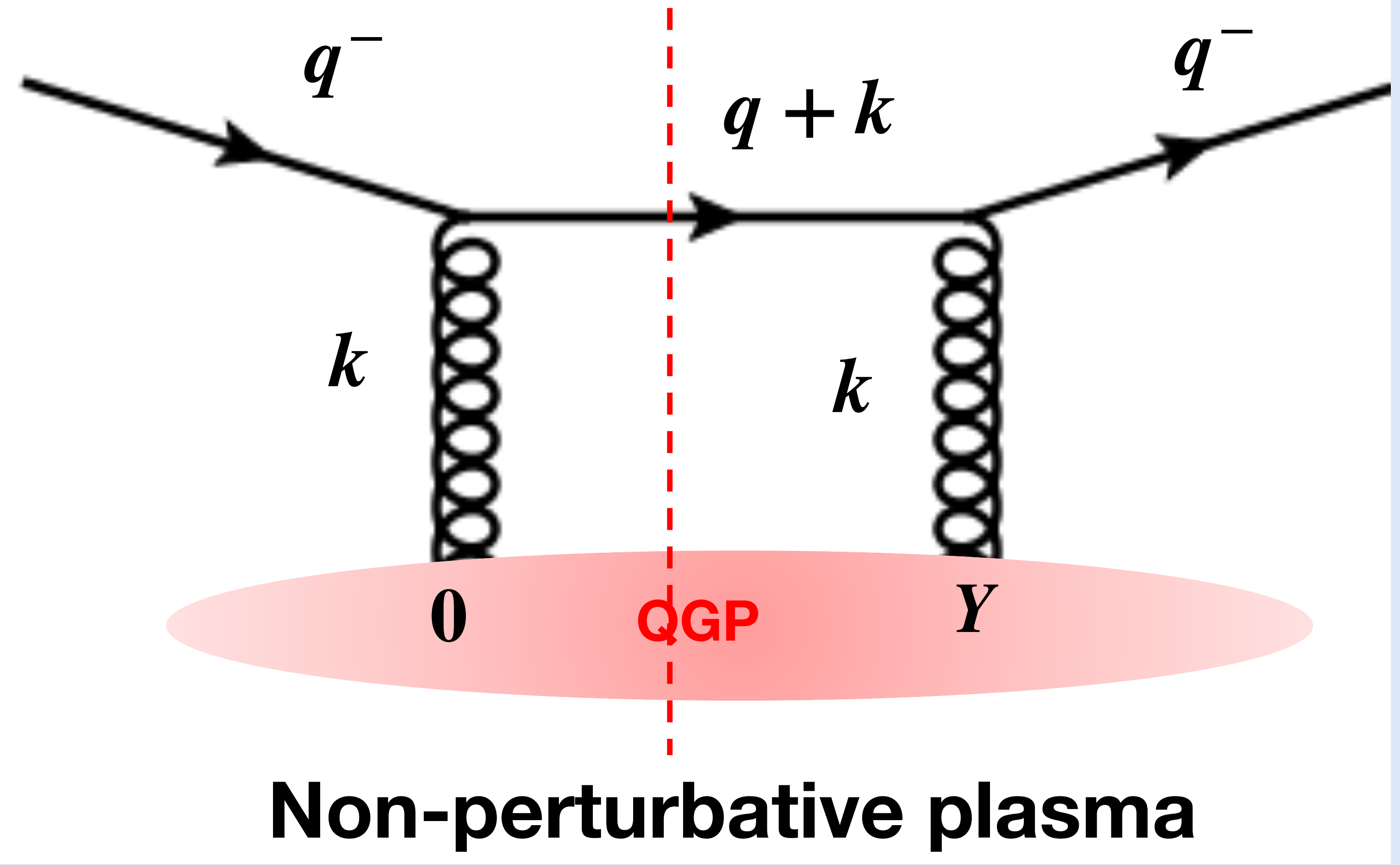


R. Ehlers et al., PRC 111, 054913 (2025)

- Hadron R_{AA} prefers higher $\hat{q}/T^3$ compared to jet R_{AA} data
- Inconsistency could be attributed to missing contributions from new jet transport coefficients
- Jet energy loss in hadronic phase seems to reduce the tension [Datta, Majumder, arXiv:2512.23692]

Fluctuation-dissipation relation (Parton drag/diffusion)

- ❑ **Leading order (LO) process:** A high energy quark propagating (along -ve z-dir) through plasma



- ❑ Assumptions:
  - ✦ The hard quark's energy is larger than the temprature of the plasma ($E > T$)
  - ✦ Plasma dynamics and gluon field-strength-correlator are assumed to be non-perturbative

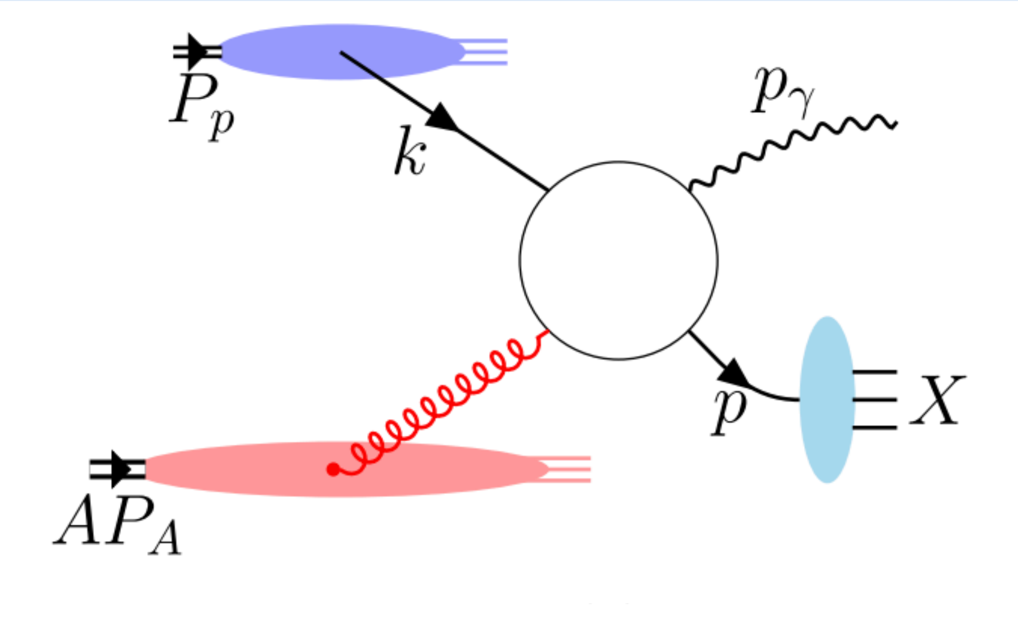
$$\hat{e} = -\frac{1}{q^-}\left[\hat{e}_2 - \frac{\hat{q}}{2} + \frac{c_0}{T}\langle M \,|\, \underbrace{\alpha_s}_{} F^{\mu\nu}F_{\mu\nu} \,|\, M\rangle\right]$$

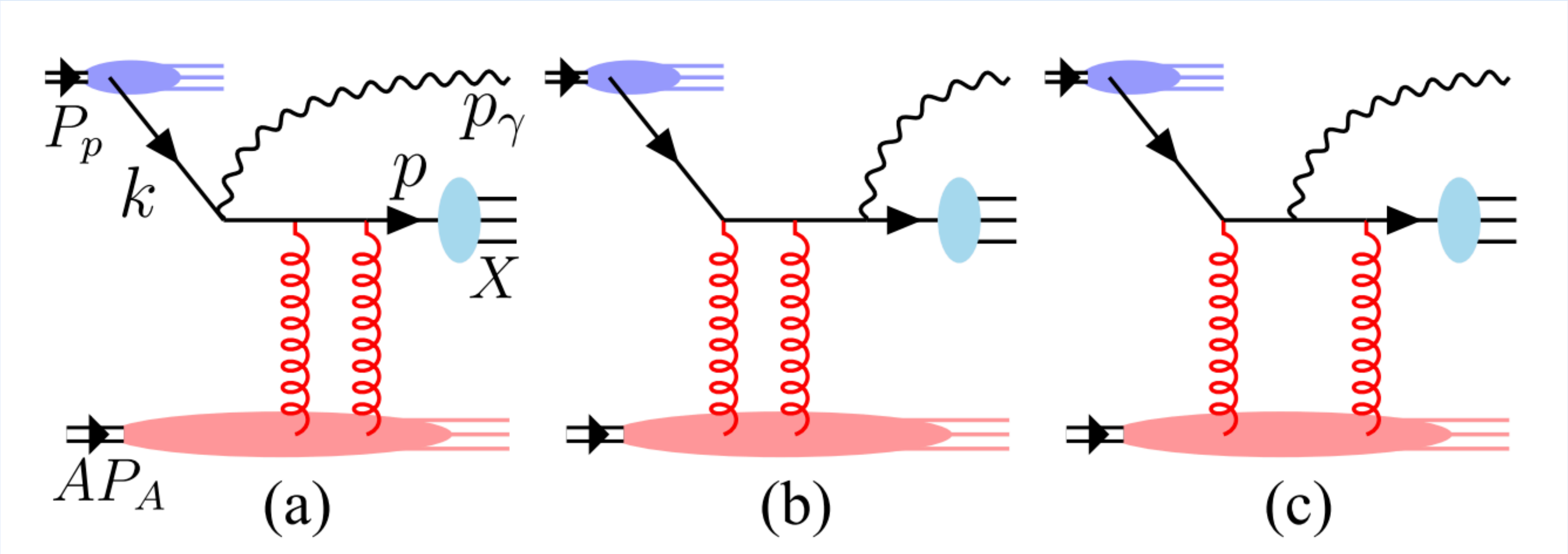
A. Kumar, A. Majumder, I. Soudi, J. Weber, arXiv:2602.22338 (2026)

Thermal-gluon-condensate (Vacuum subtracted)

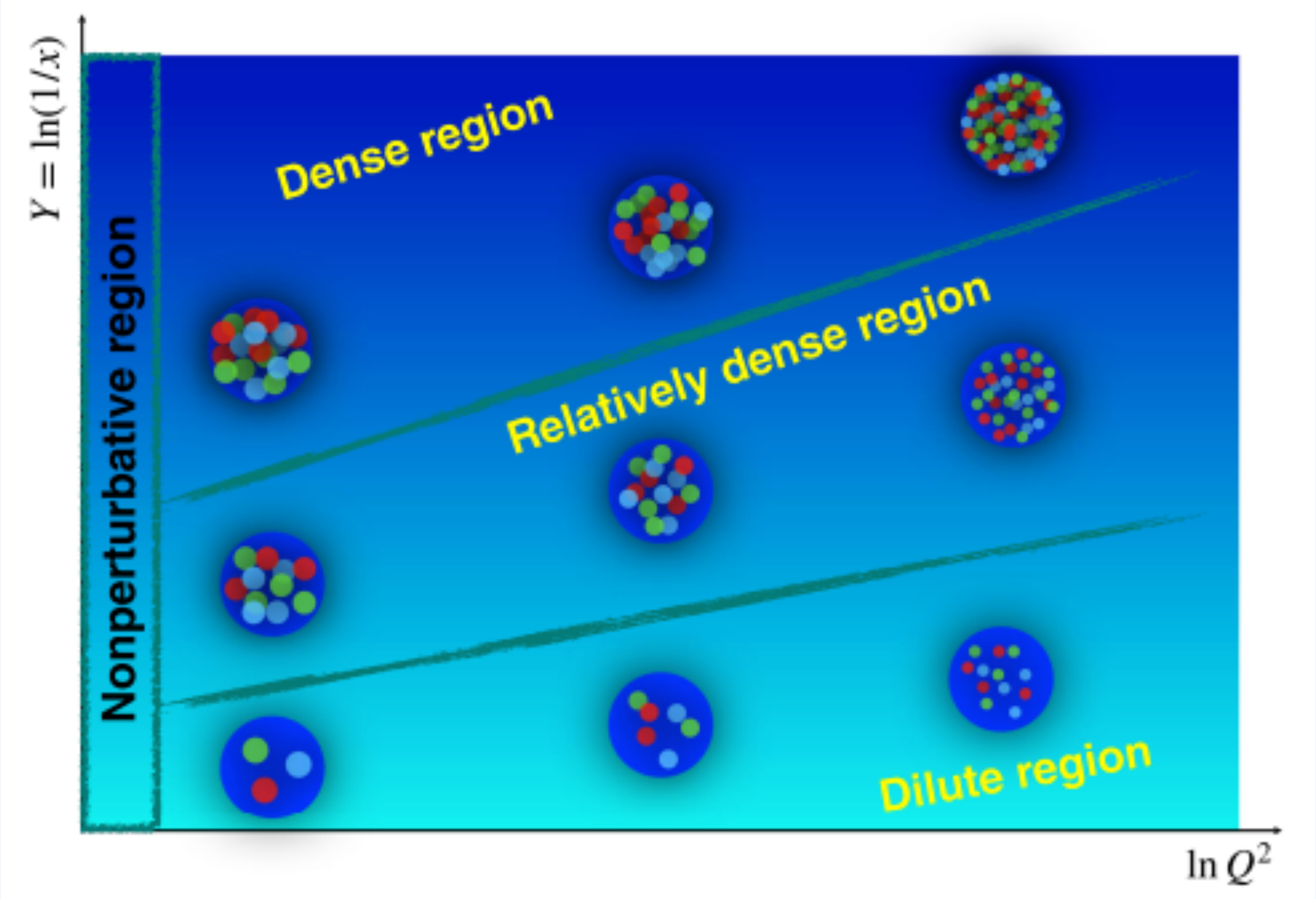
- ❑ There is a non-trivial non-perturbative contribution from thermal-gluon-condensate

Connection between HT formalism and CGC approach





Yu Fu et al., Phys. Rev. Letter. 135, 032301 (2025)

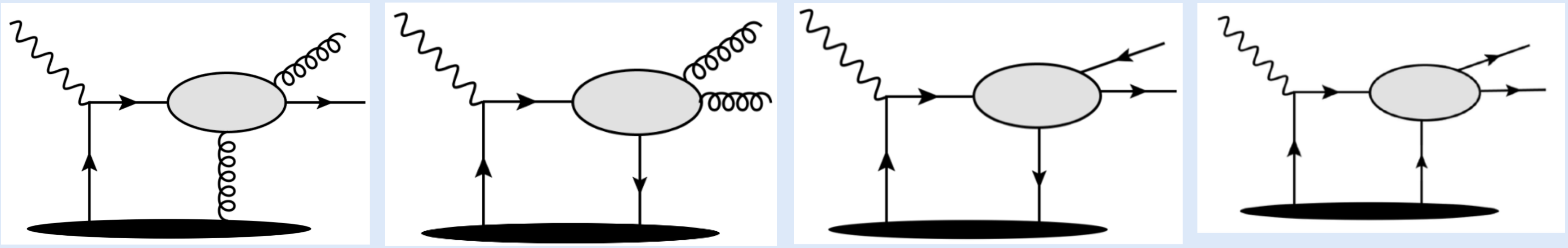


Yu Fu et al., Phys. Rev. Letter. 135, 032301 (2025)

- ❑ Photon production cross-section in p-A collisions:
  - ✦ CGC formalism and HT formalism results coincide at finite-x (intermediate values of x)

⟹ Results presented using HT formalism for medium-induced gluon/quark emission in DIS should be compatiable with CGC formalism at early times

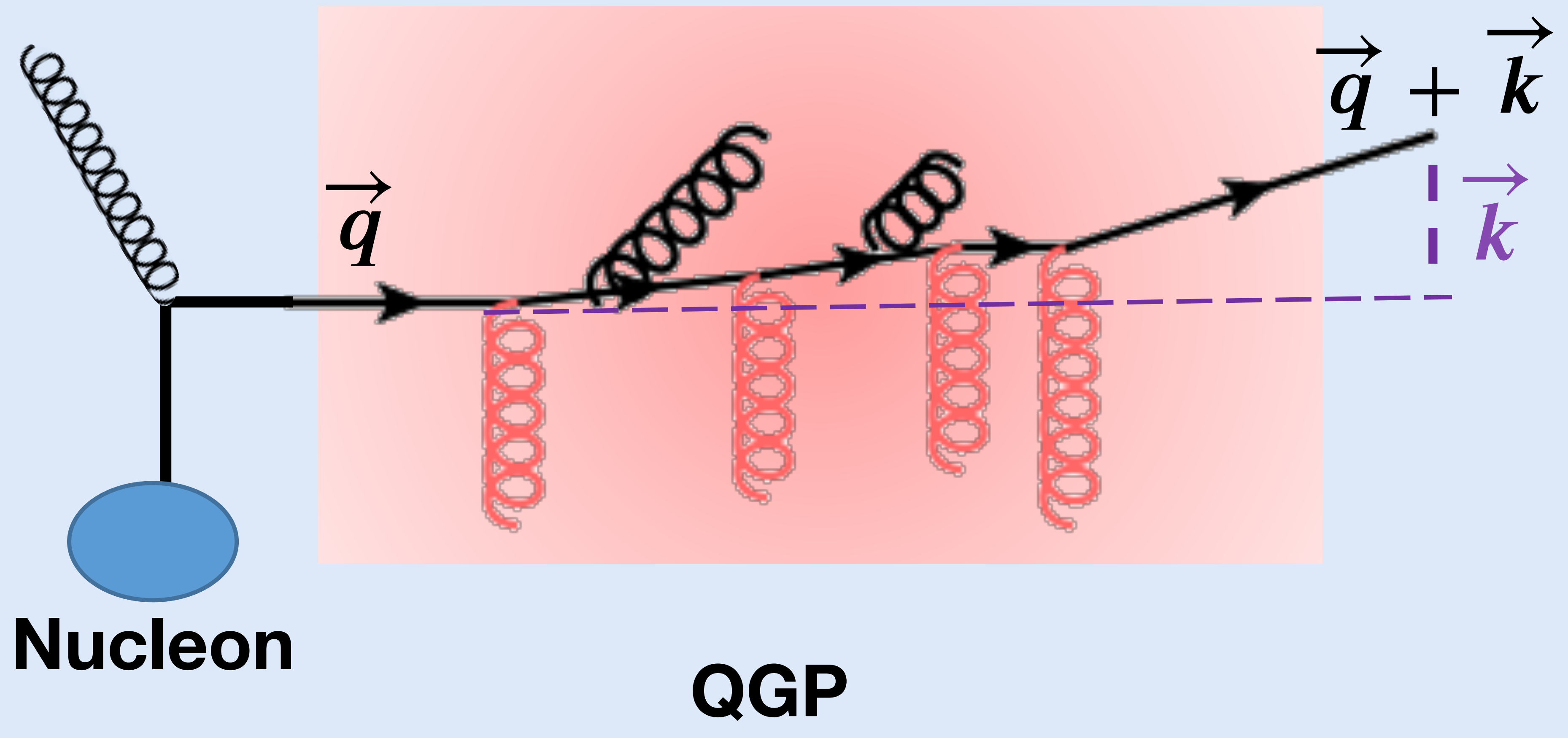
Kumar and Vujanovic, arXiv:2509.10743 (2025)

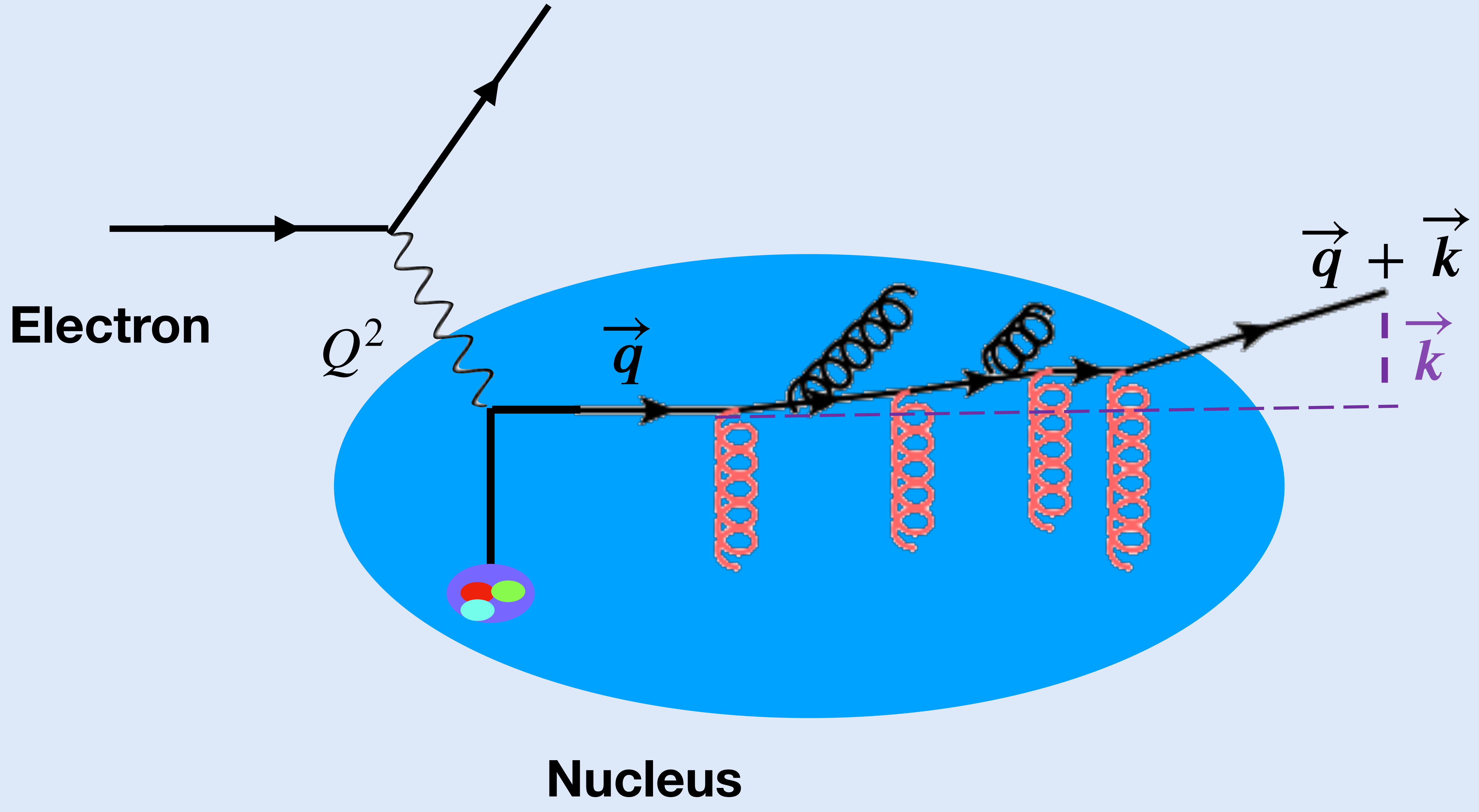


Glauber quark/gluon exchange

Jet transport coefficients in Electron-Ion collider

- Factorized approach to jet evolution





Kumar and Vujanovic, arXiv:2509.10743 (2025)

- Transport coefficient $\hat{q}$:

$$\hat{q}(\vec{r},t) = \frac{\langle \vec{k}_\perp^2 \rangle}{L} \propto \langle M | F_\perp^+(y^-, y_\perp) F^{+\perp}(0) | M \rangle$$

- Transport coefficient $\hat{e}$:

$$\hat{e}(\vec{r},t) = \frac{\langle k_z \rangle}{L} \propto \langle M | \partial^- A^+(y^-, y_\perp) A^+(0) | M \rangle$$

- Transport coefficient $\hat{e}_2$:

$$\hat{e}_2(\vec{r},t) = \frac{\langle k_z^2 \rangle}{L} \propto \langle M | F^{+-}(y^-, y_\perp) F^{+-}(0) | M \rangle$$

✦ Transport coefficient arise from pQCD based calculations

✦ Replace QGP thermal state with hadronic nuclear state

$\Longrightarrow$Energy loss calculation can be extened to Electron-Ion Physics

Summary

- ❑ First effort to derive all possible scattering kernels for quark energy loss including
  - ✦ Quark mass effects
  - ✦ k^- compoent contributions
  - ✦ Interference/coherence effects
  - ✦ Incorporated Fermion-to-Boson conversion processes
- ❑ Lattice QCD calculation of $\hat{q}$ for pure gluonic and (2+1)-flavor QCD plasma
- ❑ Bayesian extraction of $\hat{q}$ using JETSCAPE monte-carlo framework
- ❑ Fluctuation-dissipation relation for parton drag/diffusion in non-perturbative plasma
- ❑ Connection between CGC and HT formalism at finite-x and early times in HIC
- ❑ Compute longitudinal momentum coefficients $\hat{e}_2$ using lattice QCD
- ❑ Connections to Electron-Ion Collider (EIC) physics