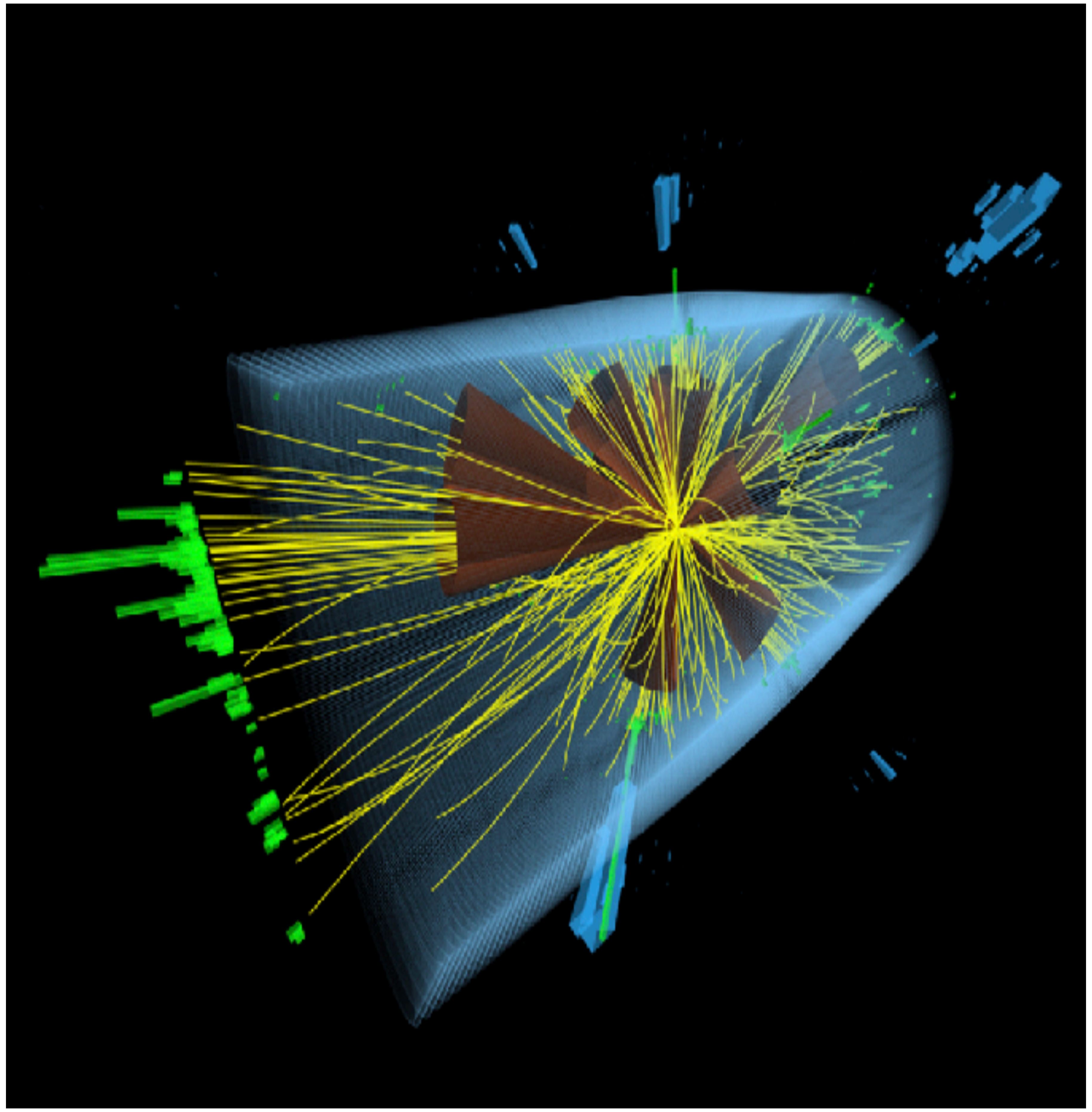
Energy correlators for heavy ion collisions



Balbeer Singh

LIP, Lisbon

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Collaborators: Ankita Budhraja, Raghav K Elayavalli, Varun Vaidya

Jet-soft dynamical medium interaction in high-energy heavy-ion collision C3NT, Wuhan

Correlation functions

- Correlation functions measure how different parts of a system are related to each other

$\langle O_1 O_2 \ldots . O_N \rangle$

Spatial correlations

-how a quantity varies across space at different points

$\langle O_1(x) O_2(y) \rangle$

Temporal correlations

-how a quantity evolves over time

$\langle O_1(t) O_2(t') \rangle$

Angular correlations

-how a quantity depends on direction

$\langle O_1(\theta) O_2(\phi) \rangle$

For a given system, the object O determines what aspect of it we want to probe

Temperature fluctuation

Question: How temperatures at different points related to each other?

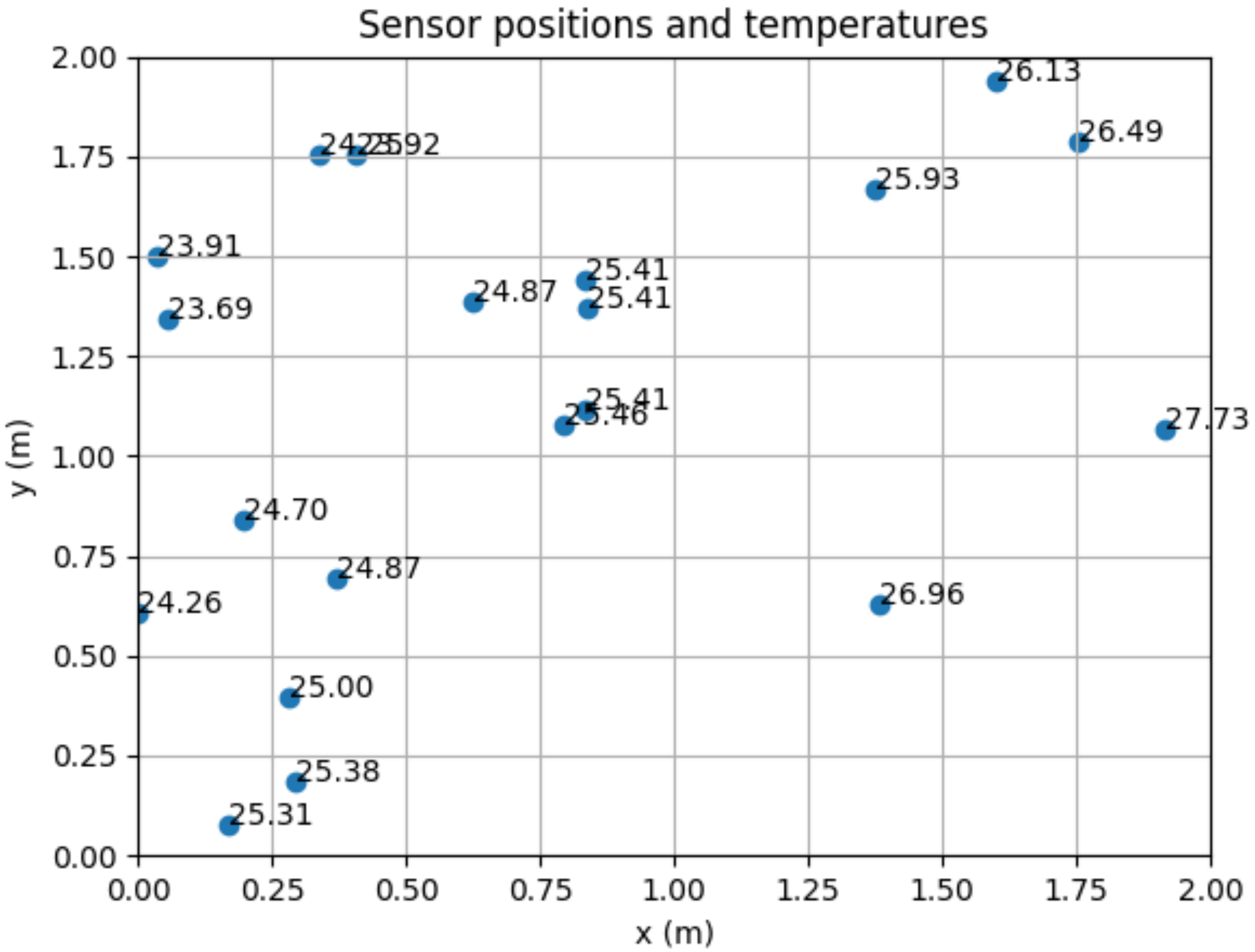
A sensor associated to each point measures the temperature of a 2D room

$$(x_i, y_i) \;\rightarrow\; T_i$$

We can think of temperature as a field $T(x_i, y_i)$ and we will focus on its fluctuations

$$\delta T(x, y) = T(x, y) - \bar{T}$$

$\bar{T}$ is mean temperature



Correlation of temperature fluctuations

We now want to calculate spatial correlation of temperature fluctuations and define the quantity

$$C(r) = \langle \delta T(x_i, y_i) \delta T(x_j, y_j) \rangle$$

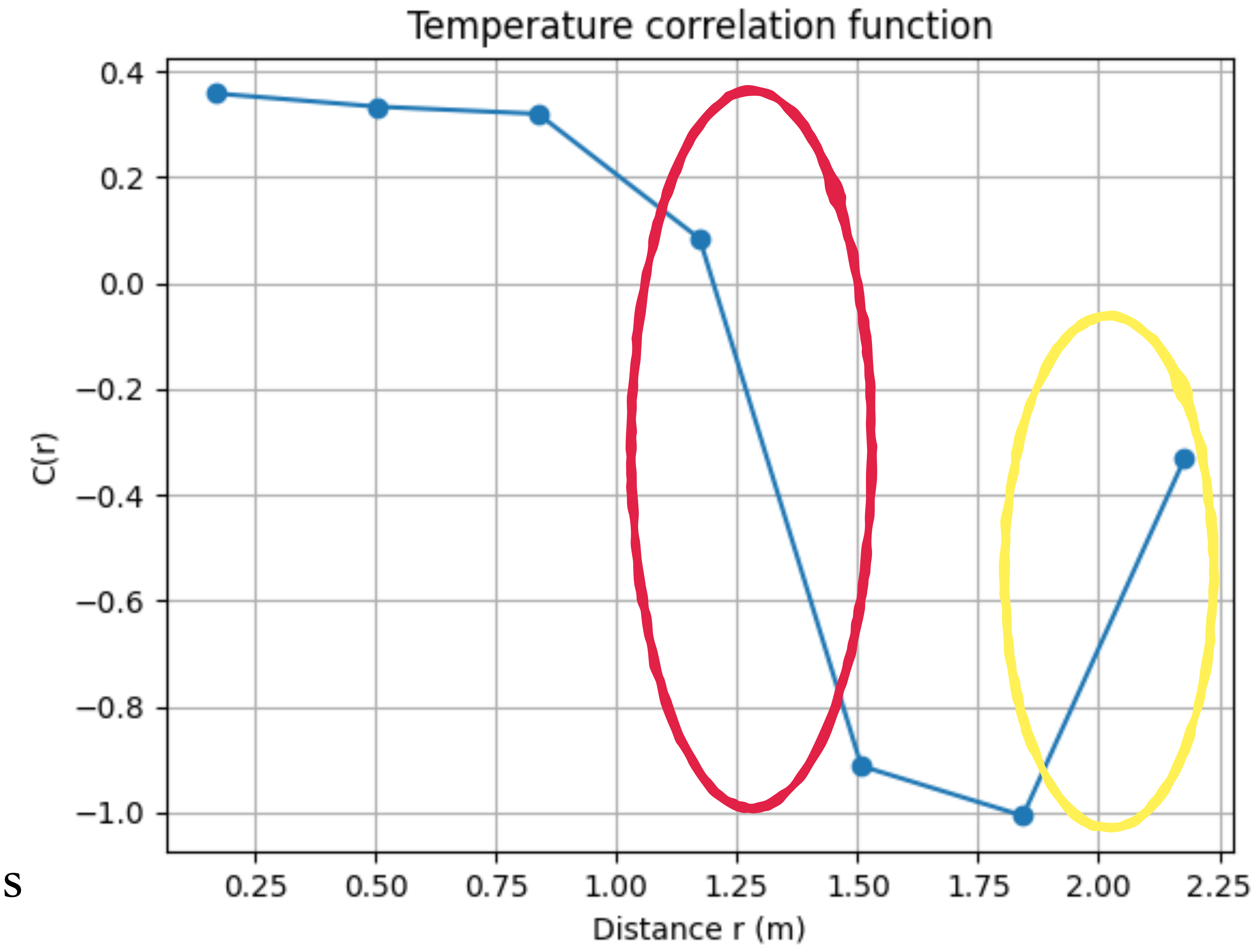
r is distance between the sensors defined as

$$r_{ij} = \sqrt{(x_i - x_j)^2 + (y_i - y_j)^2}$$

$C(r)$ measures how temperature fluctuation correlation vary as r

Can obtain some qualitative behavior of correlations

Not governed by underlying theory so not universal!



From temperature to energy correlations

- In high energy collision, particles carry energy flowing into the detector

$$\delta T_i \to E_i$$

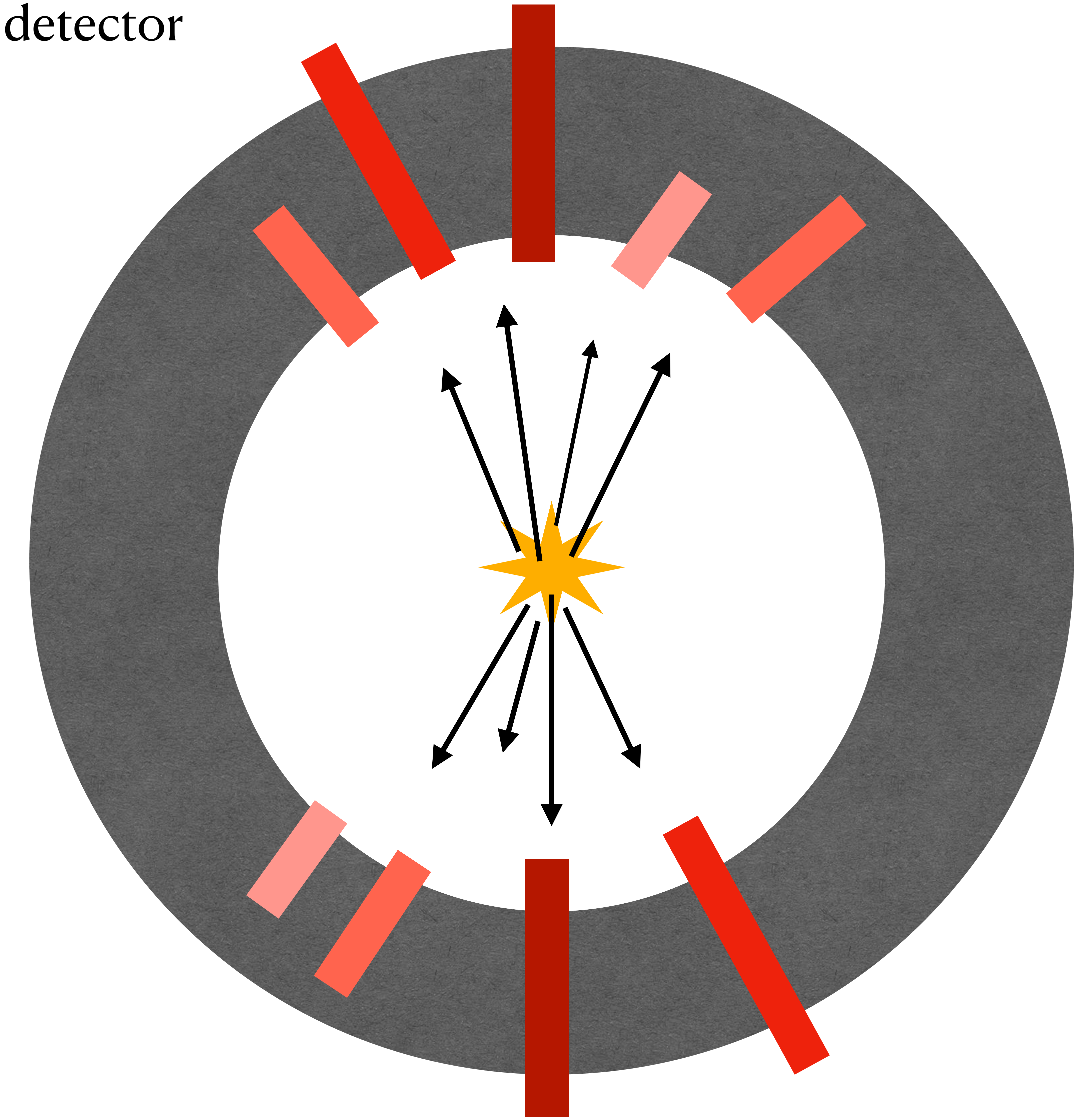
$$x_i, y_i \to \eta_i, \phi_i$$

$$r \to \theta$$

- Instead of temperature fluctuation, we study energy correlators

$$\langle \delta T_i \delta T_j \rangle \to \langle E_i E_j \rangle$$

- Particles and their interactions are governed by underlying theory
- Energy correlations probe the fundamental properties of underlying theory, universal scalings

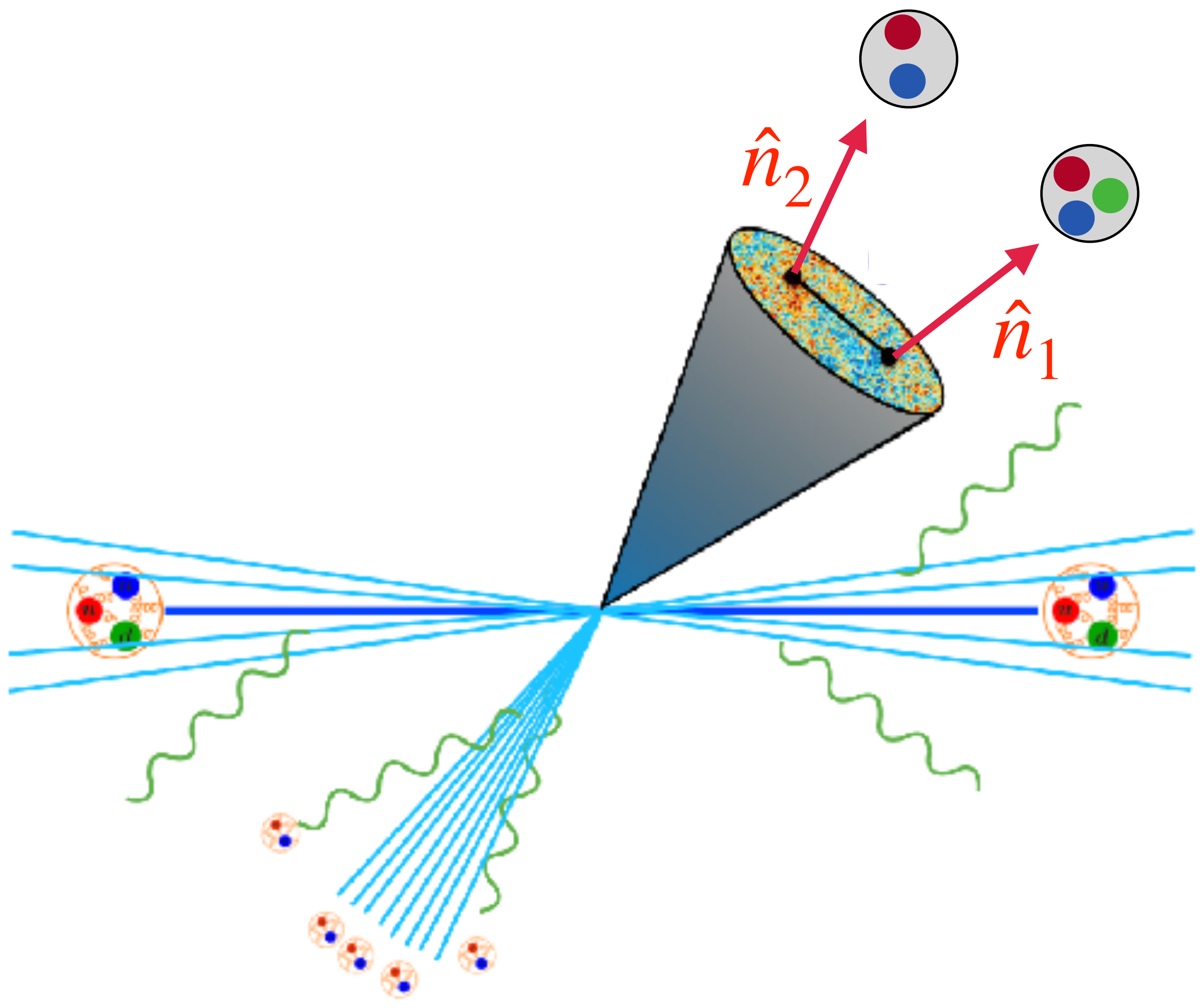


ECs in particle collisions

Energy correlators (ECs) characterize angular structure of energy flow into the detector

$$\frac{d\sigma}{dR_L} = \sum_{ij} \int d\sigma(R_L') \, \frac{p_{T,i} p_{T,j}}{p_{T,jet}^2} \, \delta(R_L' - R_L)$$

Phys. Rev. Lett. 41 (1978) 1585



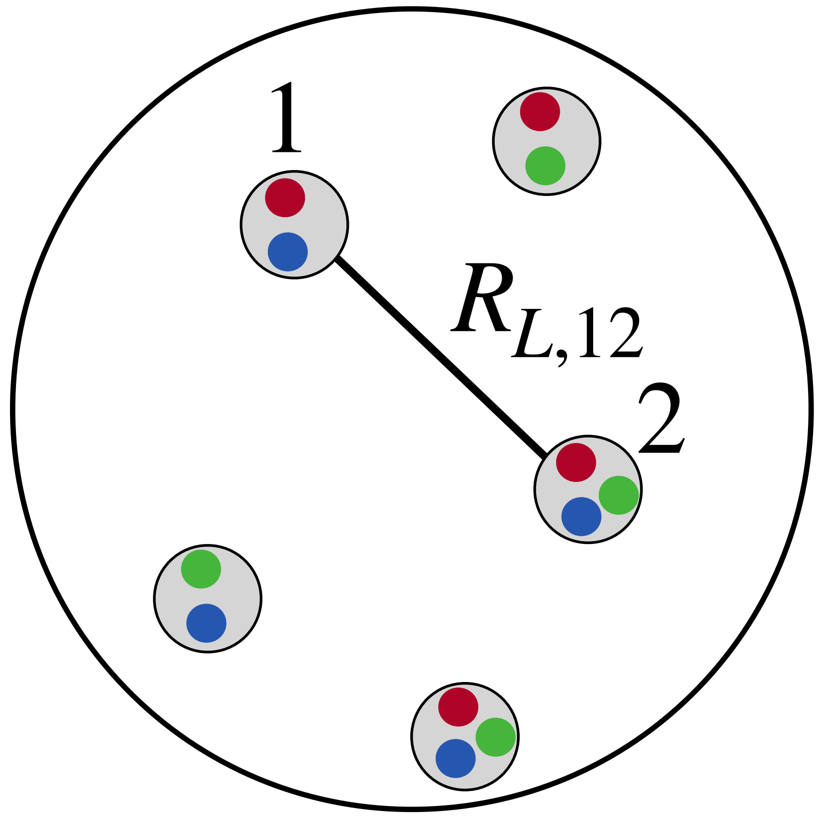
Energy weights

$$\frac{p_{T,1} p_{T,2}}{p_{T,jet}^2}$$

$$\mathscr{E}(\vec{n}) = \lim_{r\to\infty} \int_0^\infty dt \, r^2 \, n^i \, T_{0i}(t, r\vec{n})$$

Angular separation

$$R_L = \sqrt{\Delta\eta_{ij}^2 + \Delta\phi_{ij}^2}$$



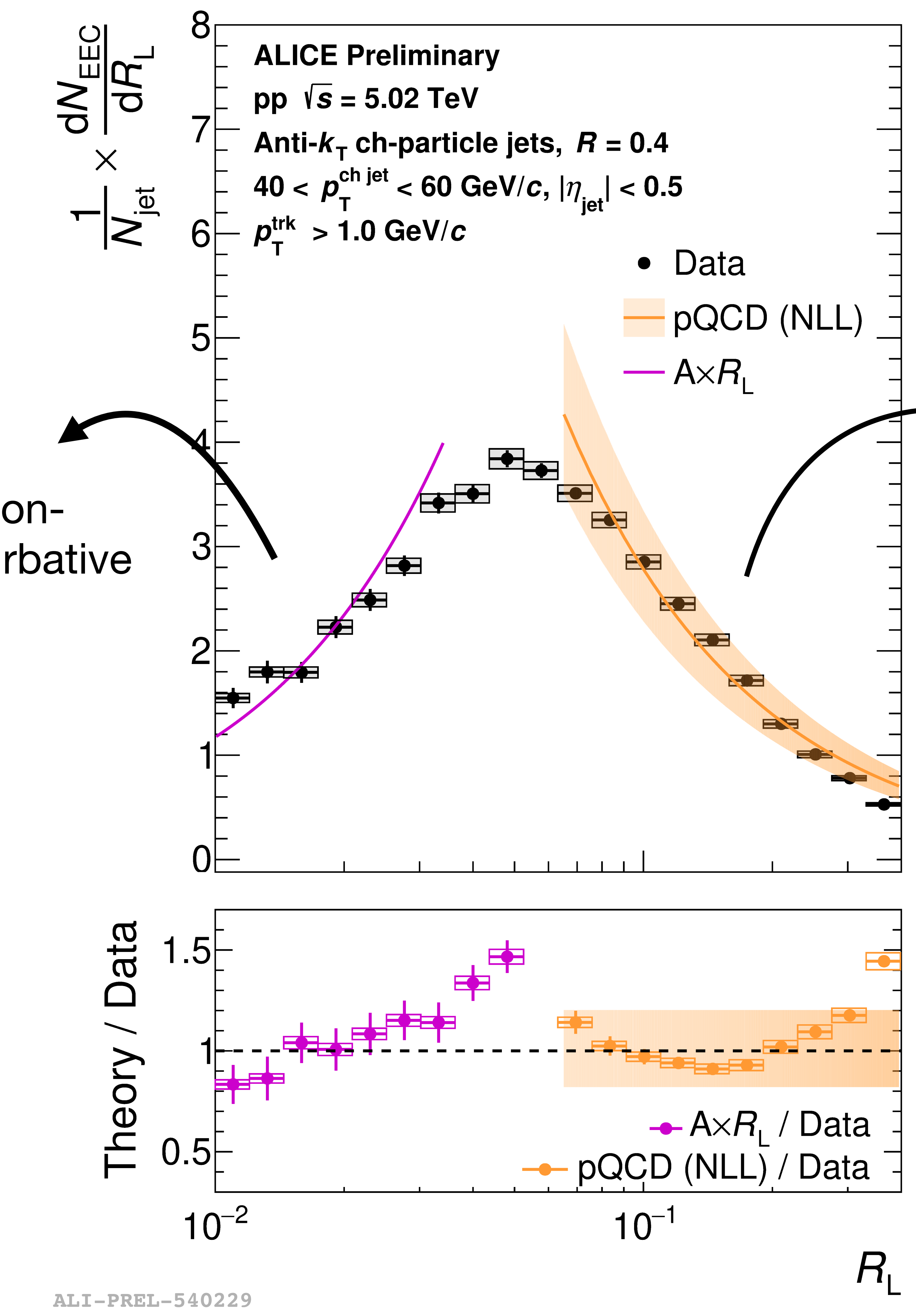
- IRC safe and well defined in QFT

Particle distribution inside the jet is defined by energy flow operators $\mathscr{E}(\vec{n})$

Energy correlators $\longrightarrow$ $\langle \Psi | \mathscr{E}(\vec{n}_1) \mathscr{E}(\vec{n}_2) \ldots \mathscr{E}(\vec{n}_k) | \Psi \rangle$

Talk by Andrey and Joao

Why ECs?



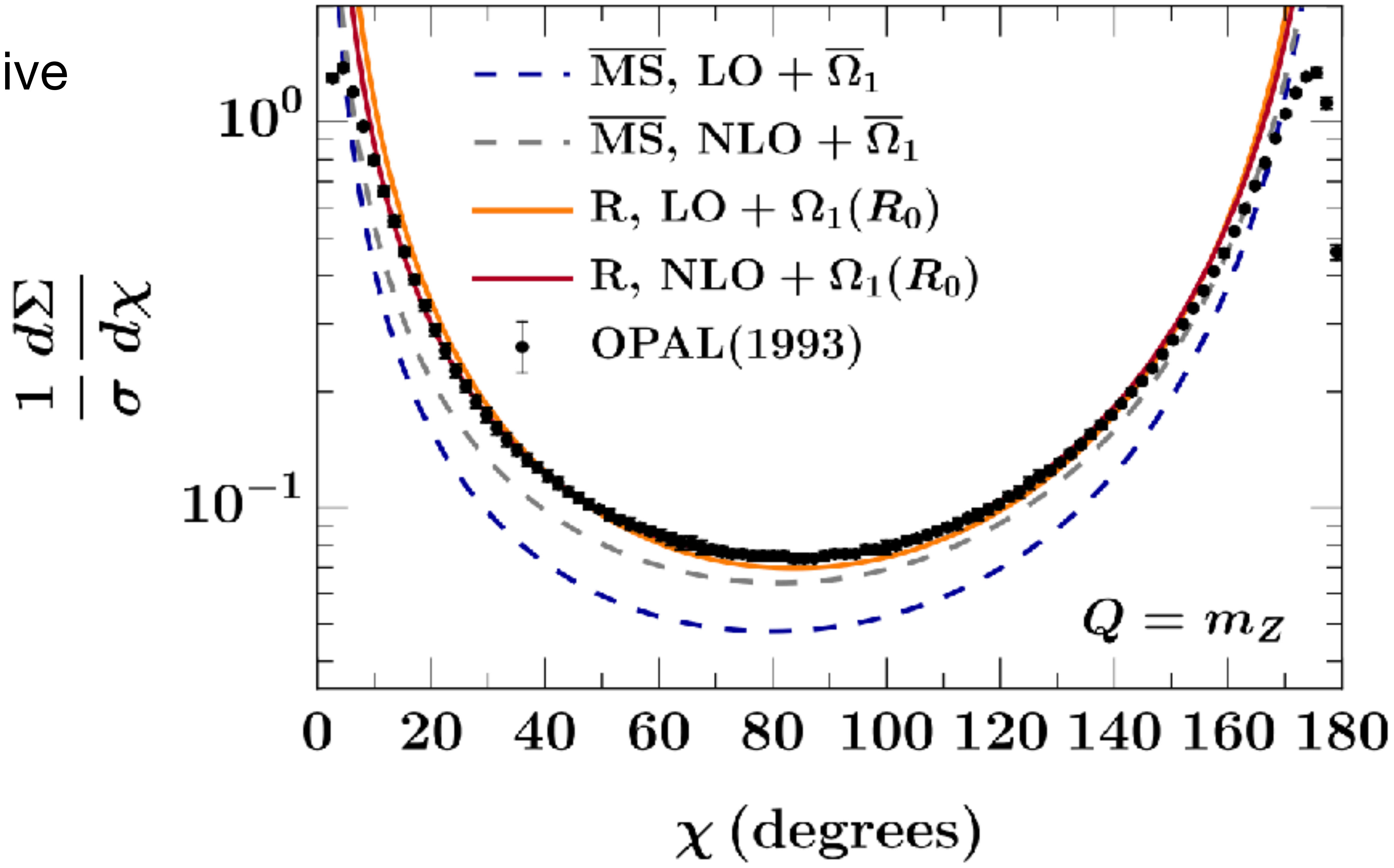
Non-perturbative

Perturbative

ALI-PREL-540229

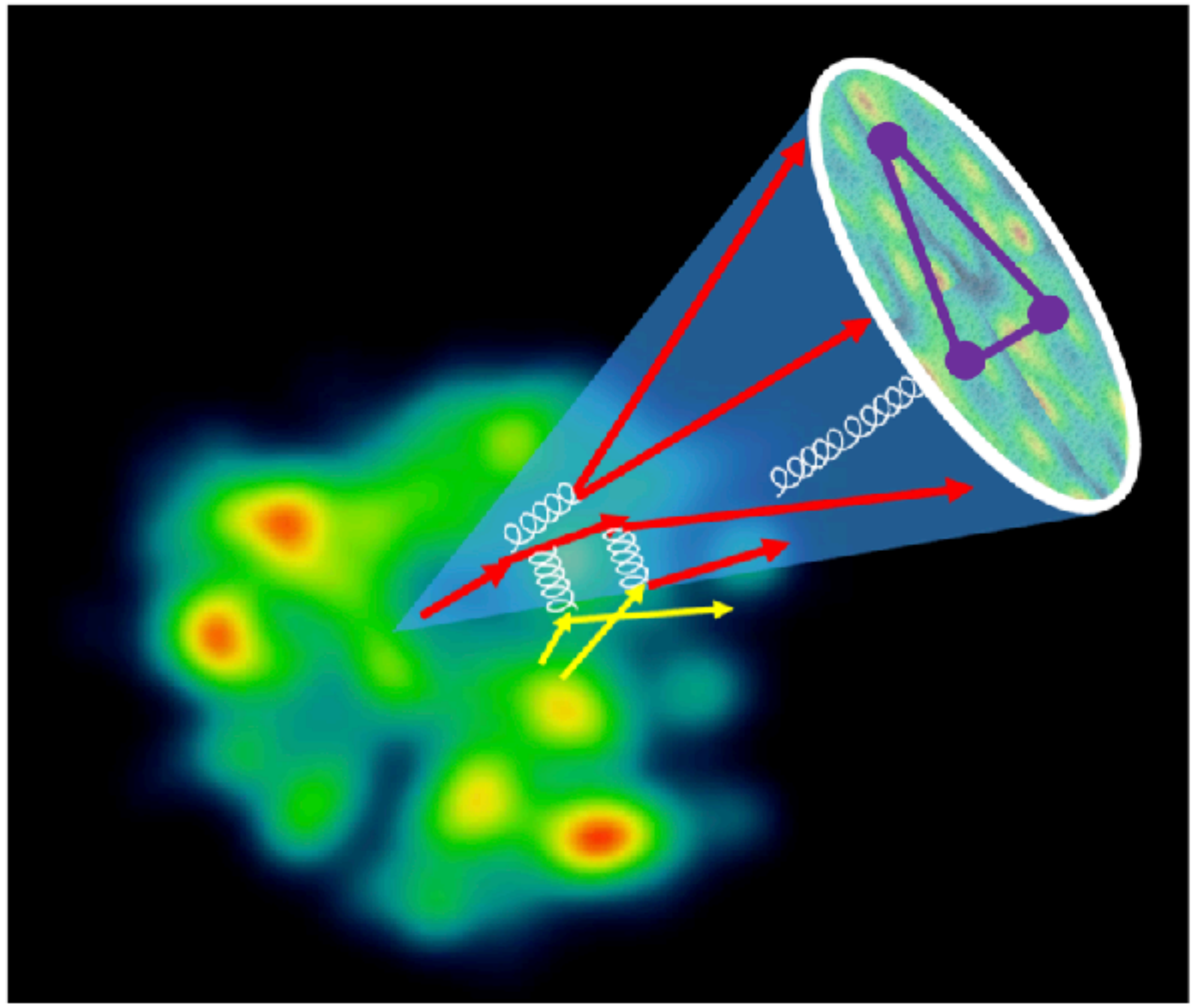
Two distinct scaling behaviors, additive

Impressive agreement with the data



Schindler, Stewart, Sun '23

ECs in HICs



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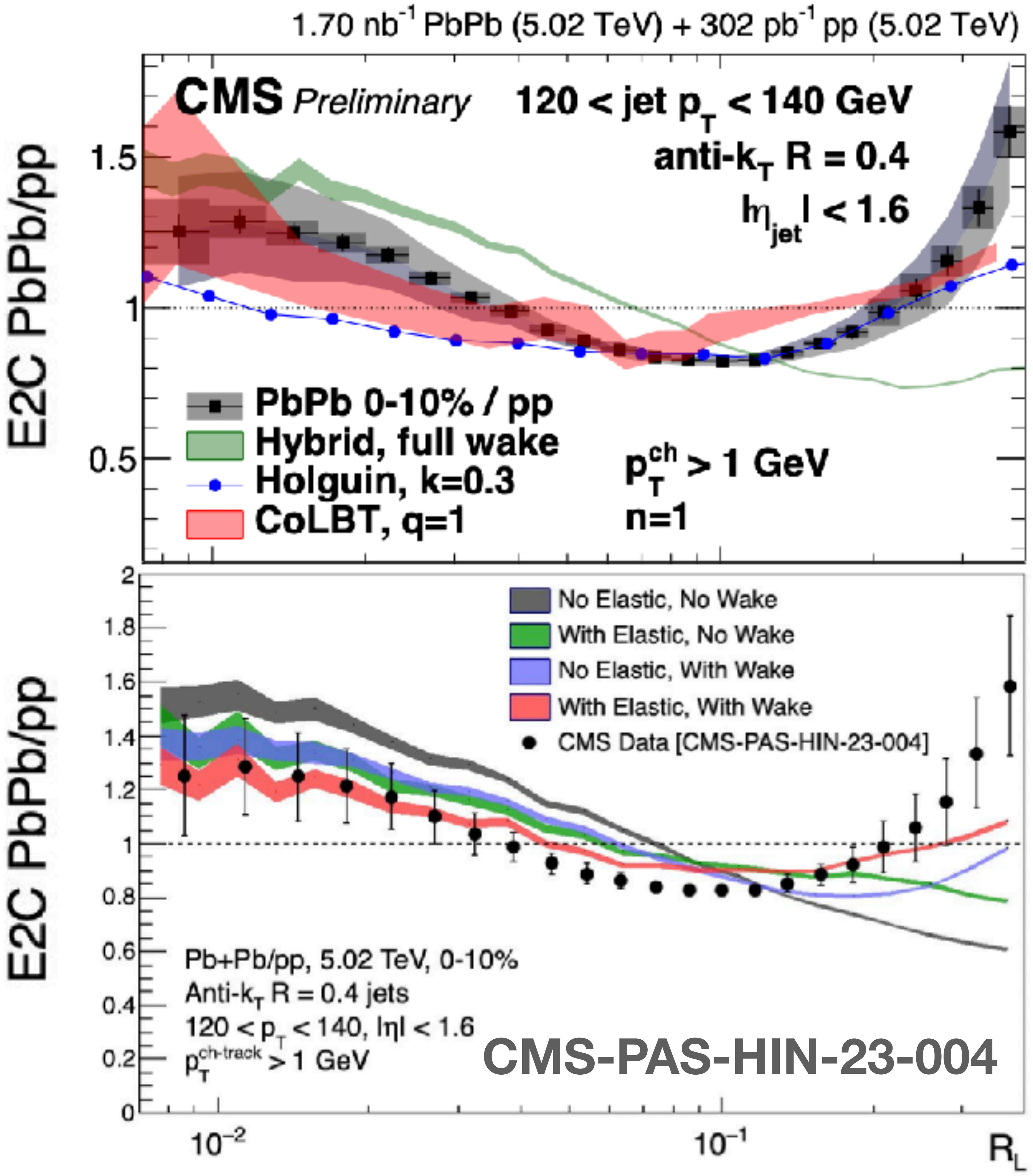
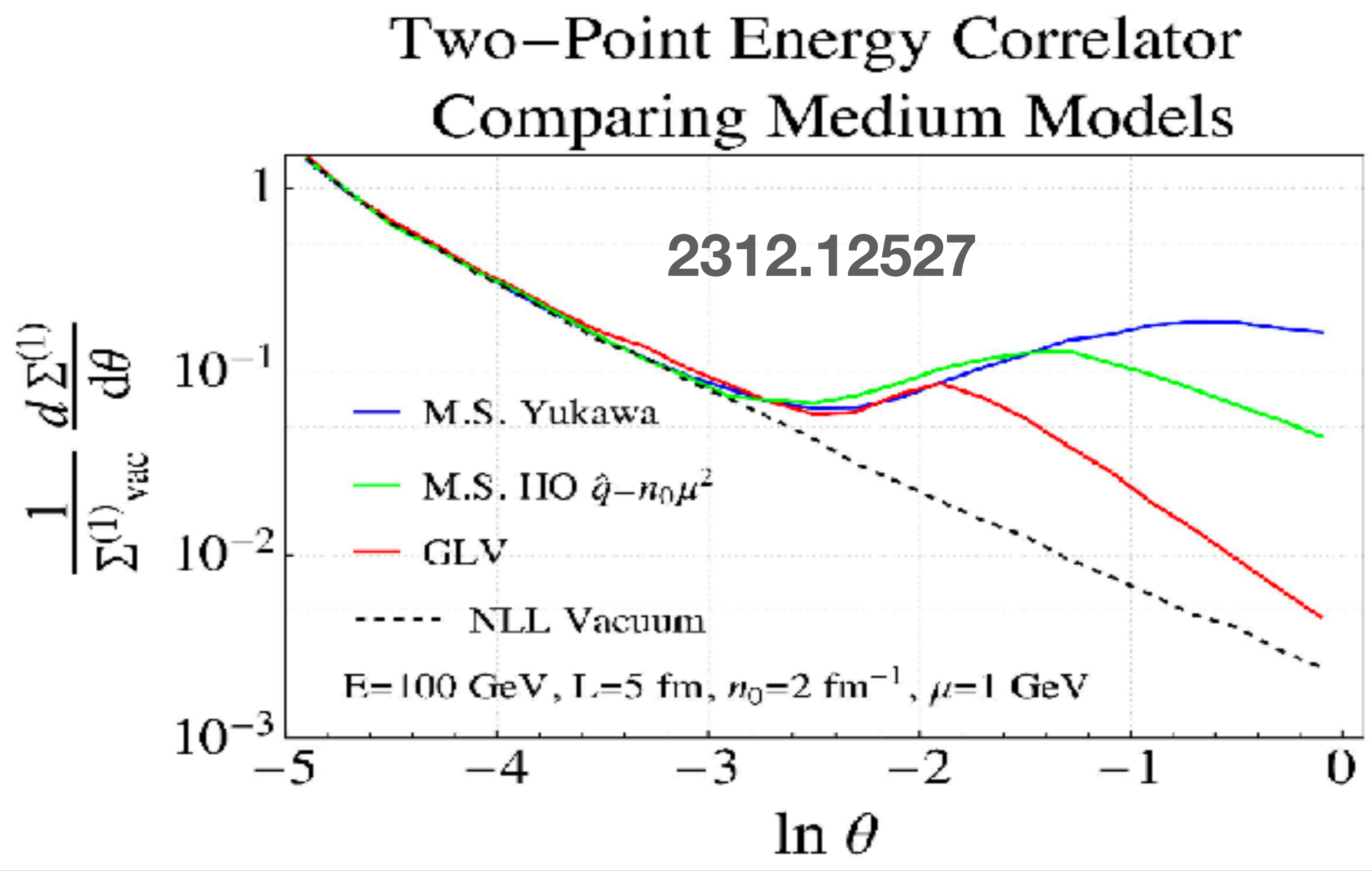
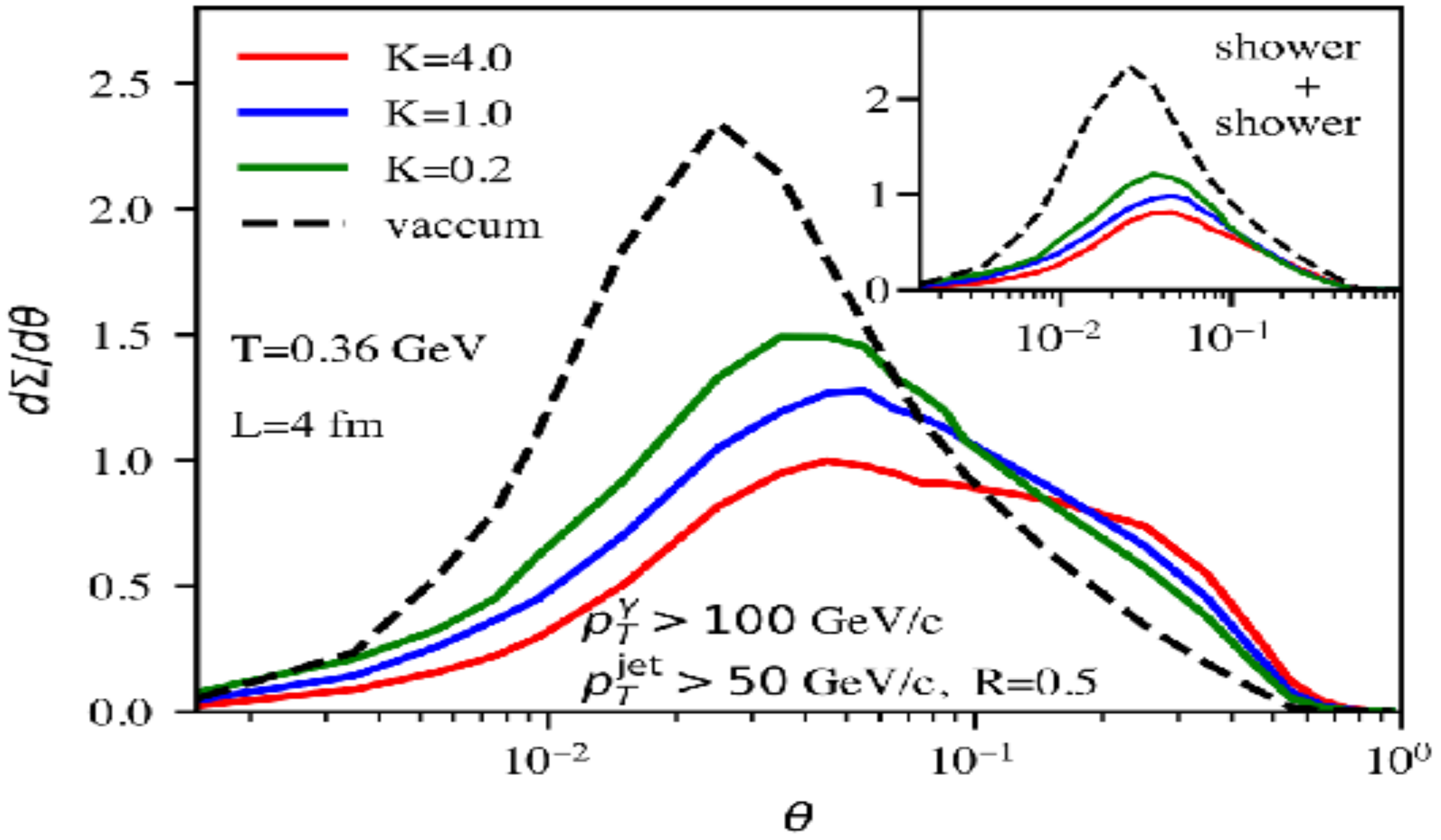


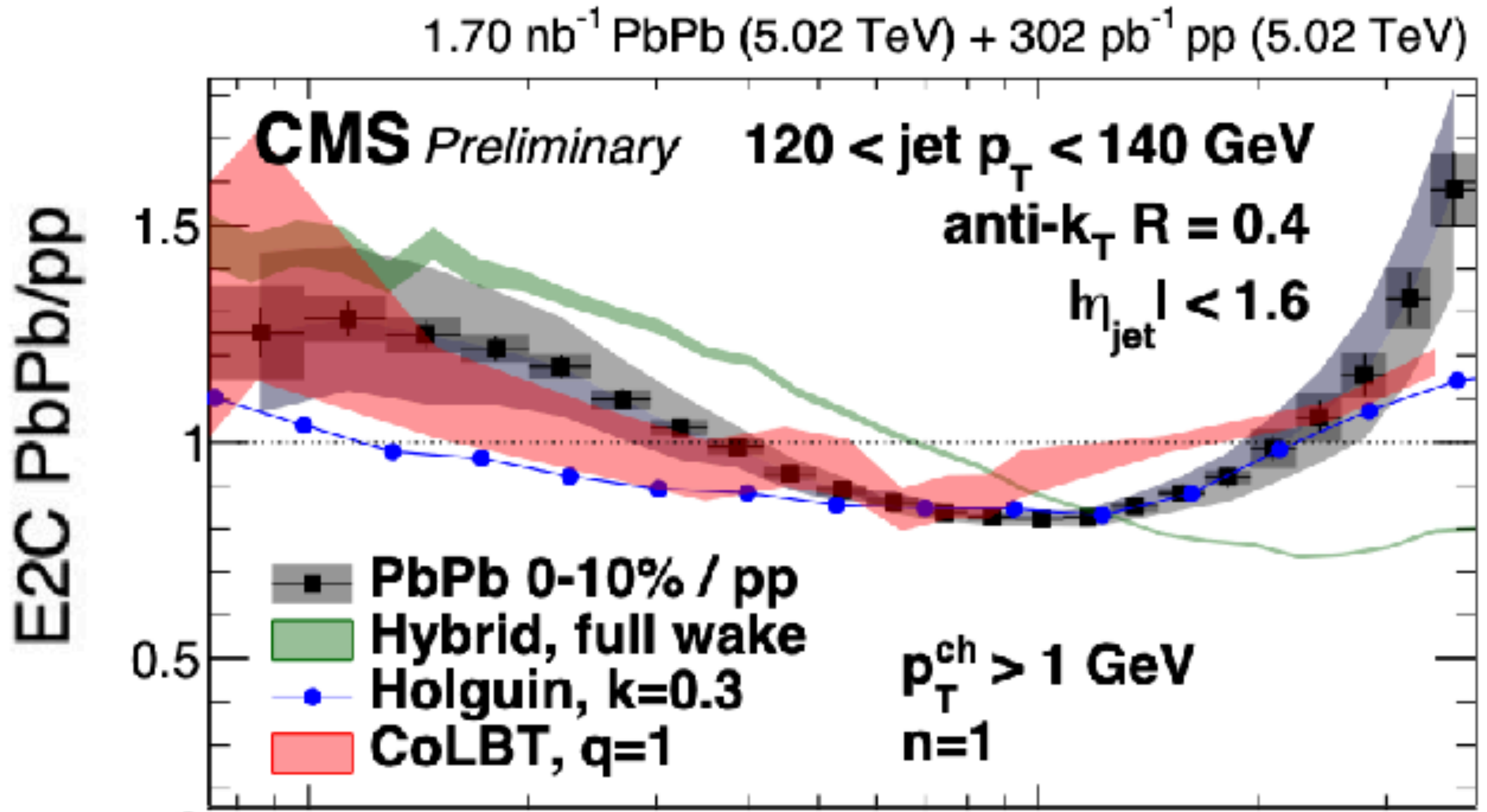
Imprints of medium interactions appears in the modification of scaling behavior

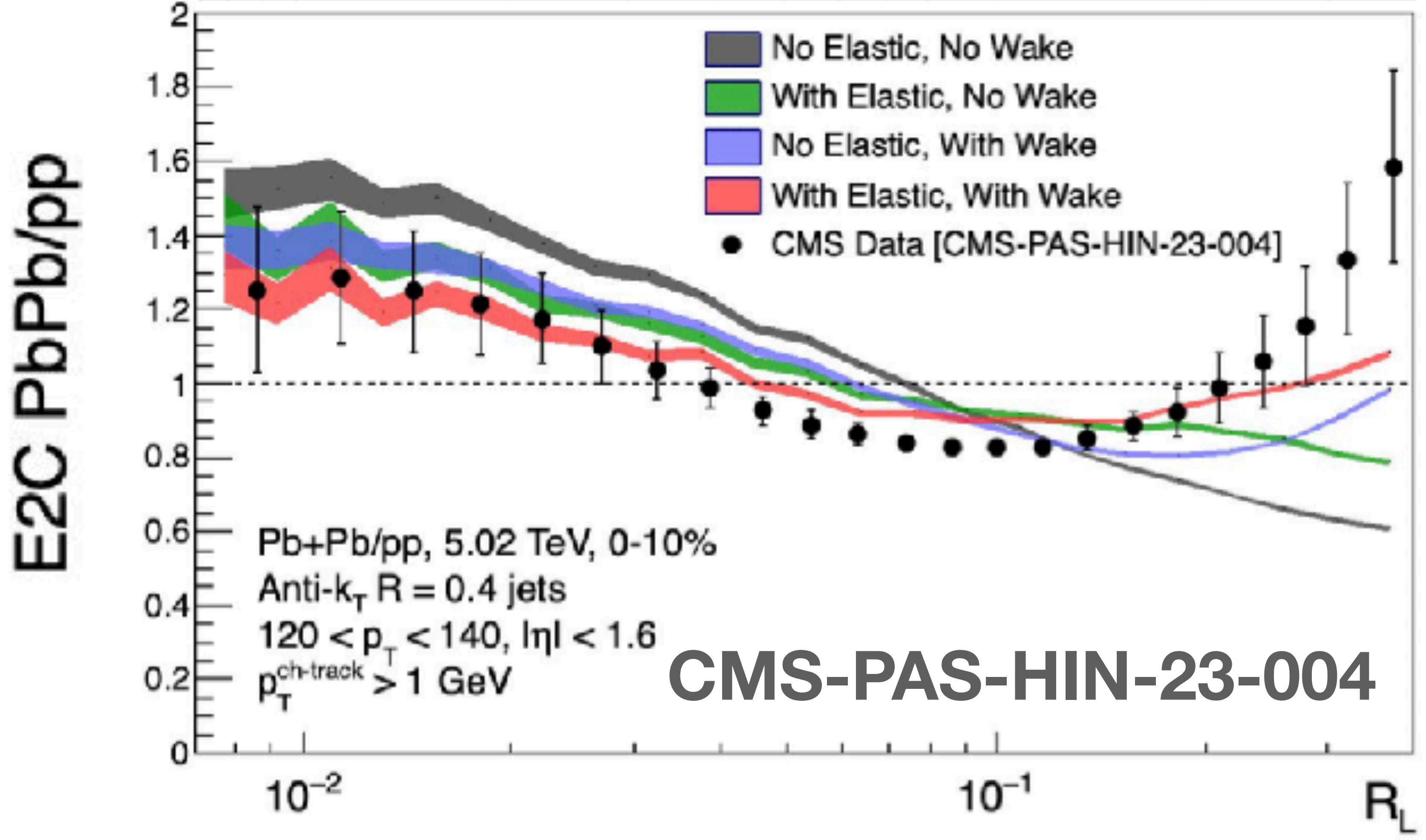
ECs in HICs



PRL 132, 011901 (2024)

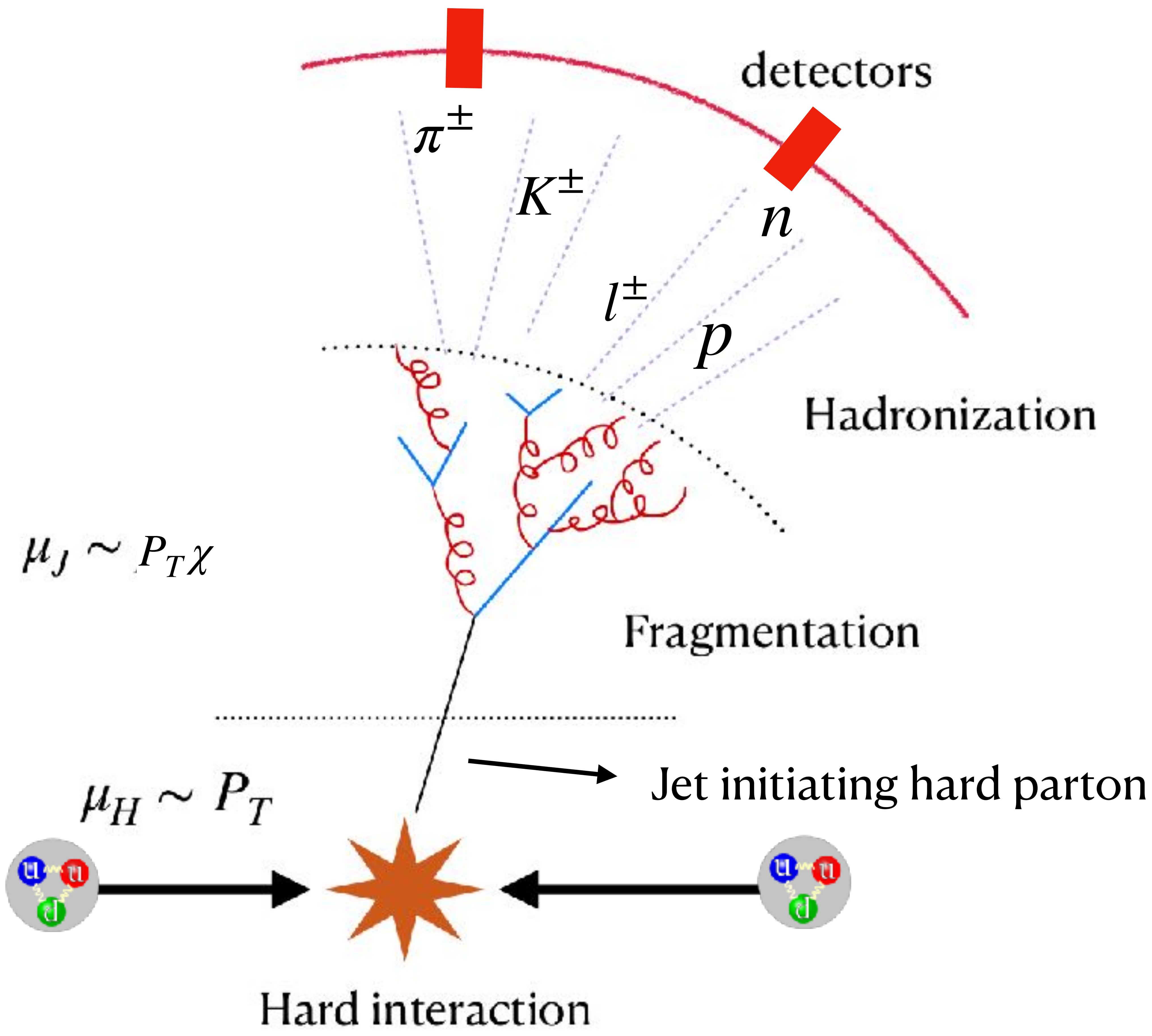




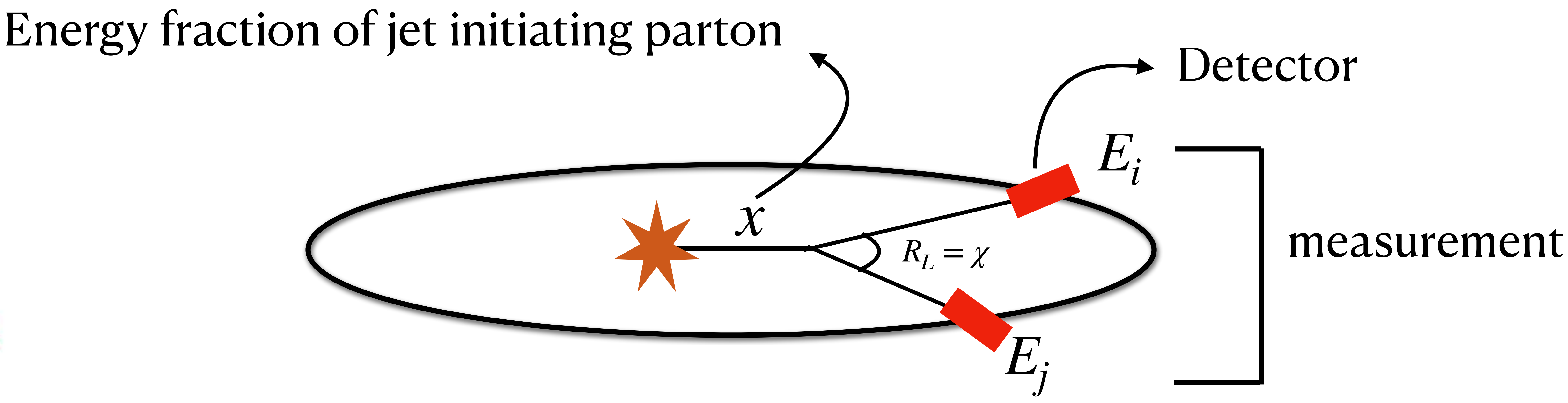


I will focus on factorization and general class of ECs

Factorization approach in pp collision



pp collision



- E_i and E_j are energies of two particles
- In pp collision environment only two scales
- For $\chi \ll 1$ two scales are widely separated

$$\mu_H \gg \mu_J$$

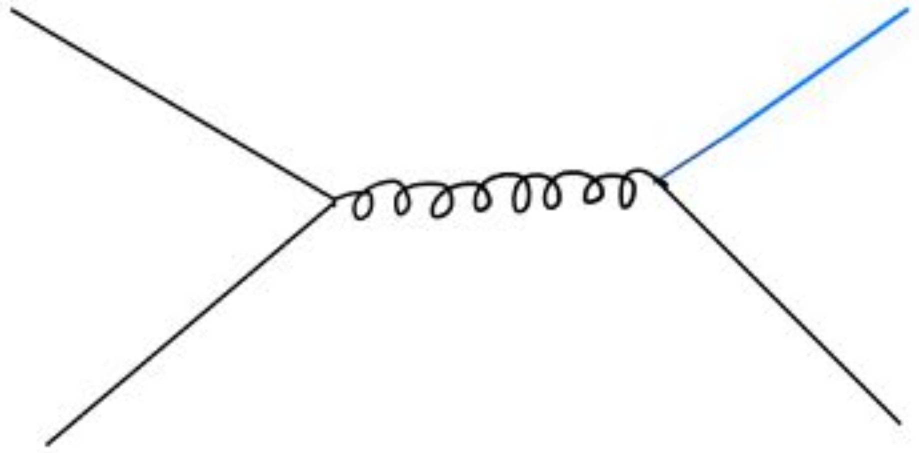
Factorization approach in pp collision

- Differential cross section can be factorized

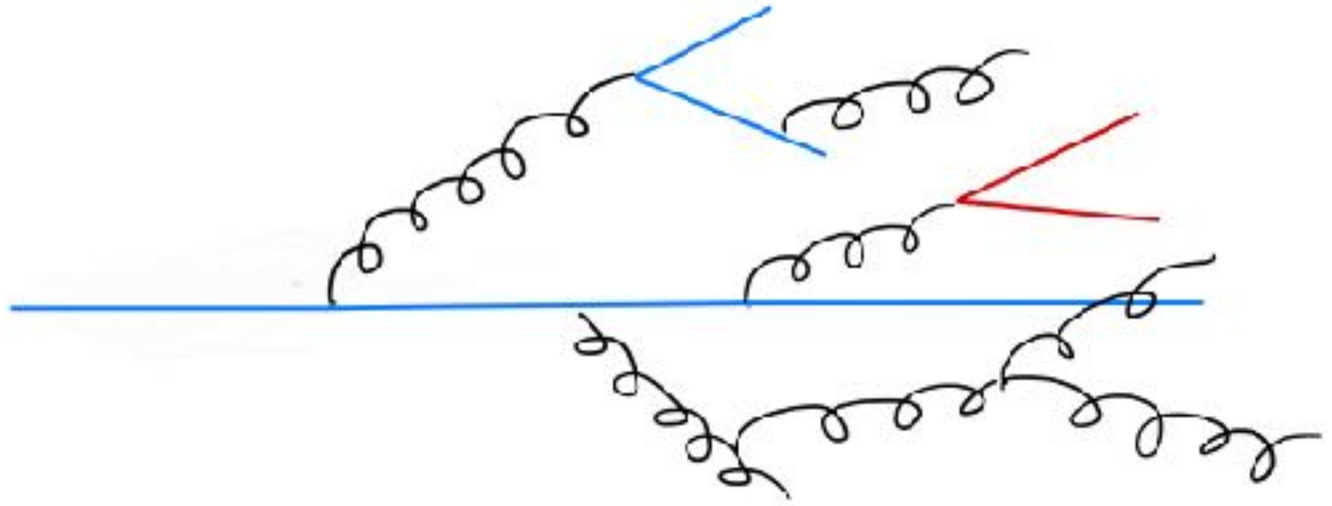
$$\frac{1}{\sigma_0}\frac{d\sigma}{d\chi} = \sum_{i\in\{q,\bar{q},g\}} \int dx\, x^2 H_i(xQ,\mu)\, J_i(xQ,\chi,\mu)$$

hard function

Jet function



Production mechanism

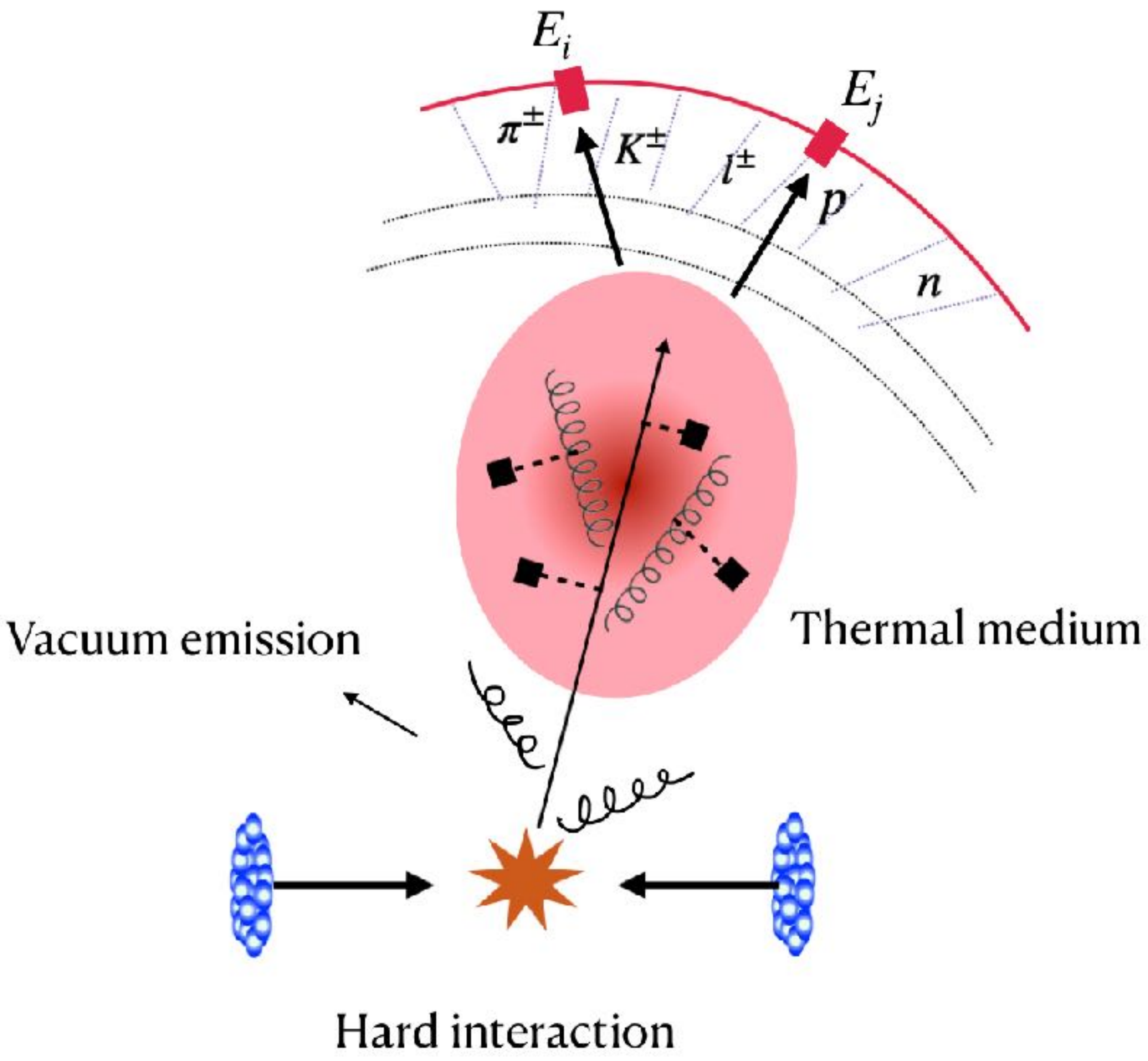


Subsequent evolution of jet

- **Hard function:** production of jet initiating parton, known to higher order
- **Jet function:** subsequent evolution in vacuum, known to higher orders
- Hard function does not depend on the measurement imposed on final state particles
- μ is factorization scale

Jets in a medium

- Presence of medium introduces many direct and indirect scales



$l_{\text{mfp}} \rightarrow$ mean free path

$T \sim m_D \rightarrow$ medium temperature

$\tau_f \sim \dfrac{\omega}{q_\perp^2} \rightarrow$ formation time

Direct scales

$\hat{q} \rightarrow$ jet quenching parameter

$\theta_c \sim \dfrac{1}{\sqrt{\hat{q}L^3}} \rightarrow$ critical angle

emergent scales

Scale hierarchy: $\mu_H \gg \mu_J \gg T \sim m_D \geq \Lambda_{QCD}$

We will use these scales to as a guide for the EFT

Soft Collinear Effective Theory (SCET)

- SCET is an EFT of QCD designed to study collinear and soft radiations

SCET 1410.1892

Systematically obtained from QCD

$$n^\mu = (1,0,0,1) \qquad \bar{n}^\mu = (1,0,0,-1)$$

$$n \cdot \bar{n} = 2$$

collinear: Boosted along jet direction $p^- \gg p_\perp \gg p^+$

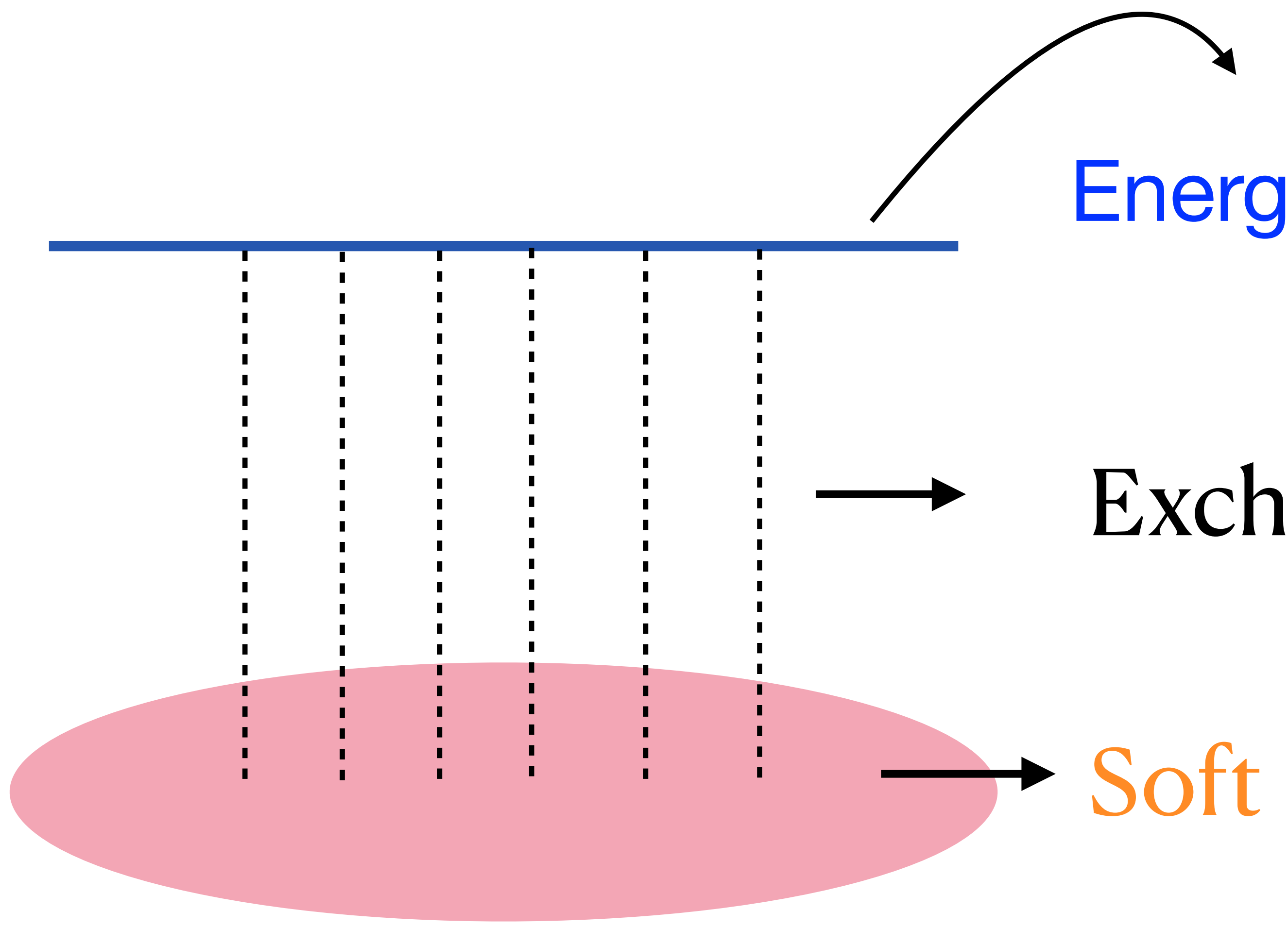
$$p^\mu = (p^-, p^+, \vec{p}_\perp)$$

soft : No preferred direction $p^- \sim p_\perp \sim p^+$

$$p^+ = n \cdot p \qquad p^- = \bar{n} \cdot p$$

$$\mathscr{L}^0_{\text{SCET}} = \mathscr{L}_s(\psi_s, A_s) + \sum_{n_i} \mathscr{L}^0_{n_i}(\xi_{n_i}, A_{n_i}) + \mathscr{L}_G(\xi_{n_i}, A_{n_i}, \psi_s, A_s)$$

- All hard emission $\mathscr{L}^0_n$
- Medium interactions $\mathscr{L}_s$
- Jet and medium interactions $\mathscr{L}_G$



Energetic jet parton

Exchange gluons

Soft thermal partons

EFT modes

- At least need four types of momentum modes

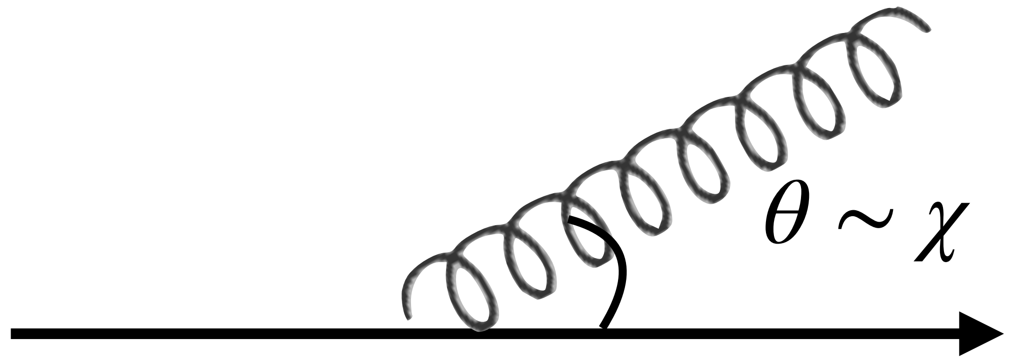
Production : hard mode $p_h \sim Q(1,1,1)$

Jet : collinear mode $p_c \sim Q(1,\lambda^2,\lambda)$

Medium : soft mode $p_s \sim Q(\lambda,\lambda,\lambda)$ with $Q\lambda = T \approx Q_{\rm med}$

Glauber : Scale such that interaction should not change the off-shellness of collinear or soft modes $k \sim Q(\lambda,\lambda^2,\lambda)$

Transverse momentum scaling of c mode is fixed by the measurement $p_{c\perp} = Q\chi$, which is χ



Region I : $Q \gg Q\chi \sim Q_{\rm med} = [2-3]$ GeV for $\hat{q} = [1-2]{\rm GeV}^2{\rm fm}^{-1}$ and $L = 5$ fm

Region II : $Q \gg Q\chi \gg Q_{\rm med}$, two stage EFT

Medium induced emissions : collinear soft $p_{cs} \sim Q\lambda(1,\lambda,\lambda^2)$

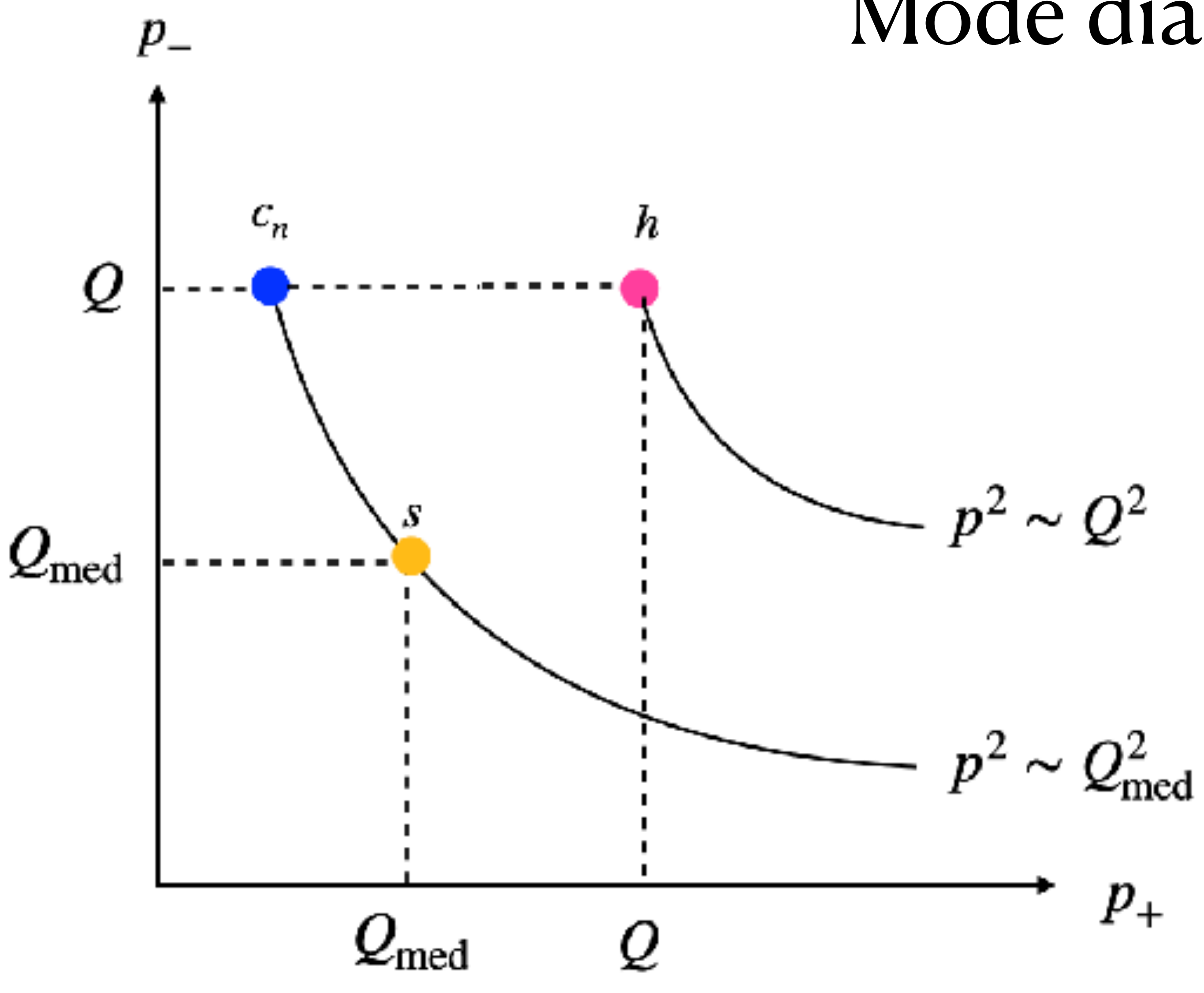
cs modes are also collinear modes but softer than collinear modes but energy larger than medium partons

Logs and runnings

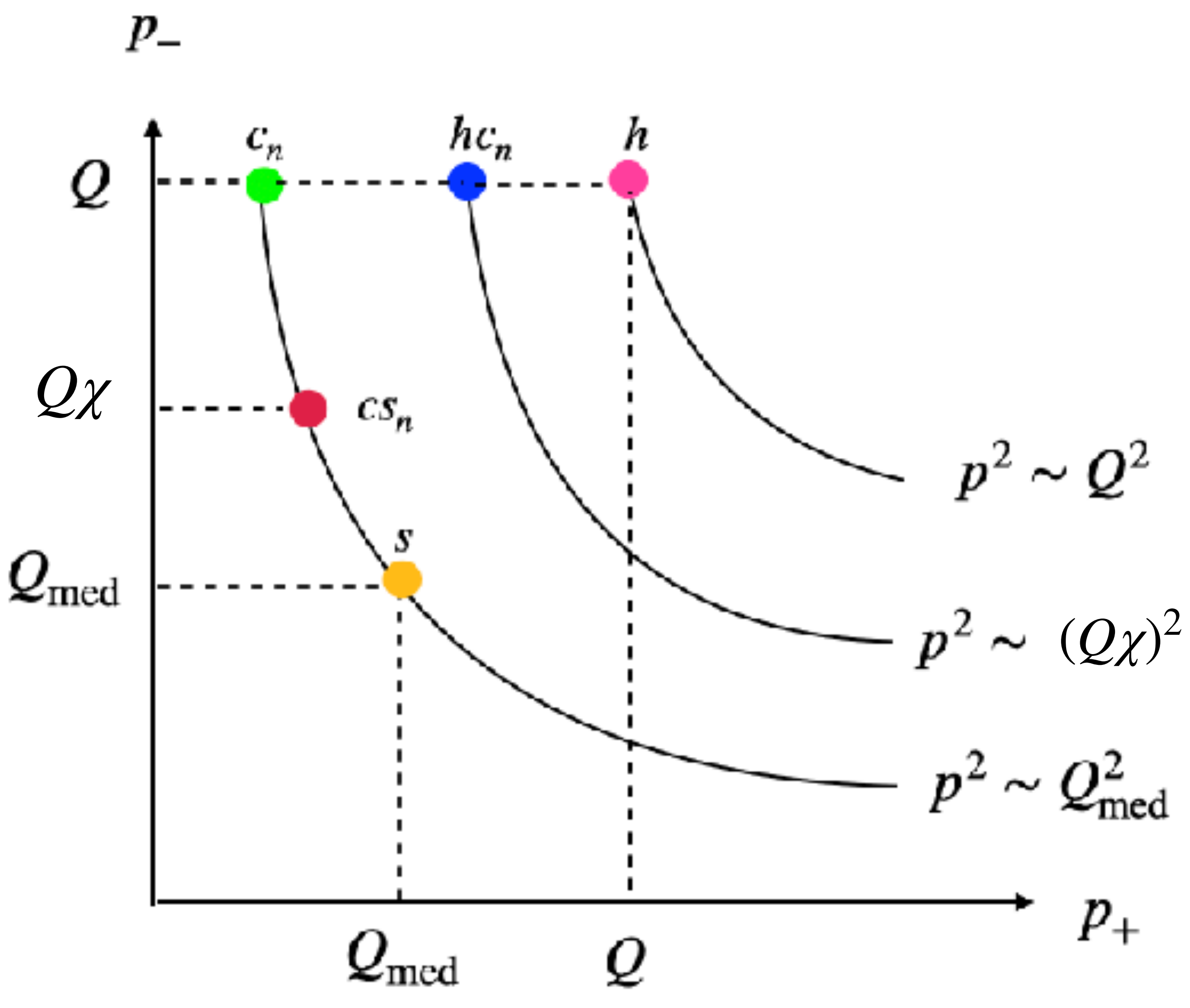
- Mode diagram with onshell condition $p^2 = p^+p^- - p_\perp^2 = 0$
- Soft and collinear soft modes sits on same mass hyperbola

RG runnings

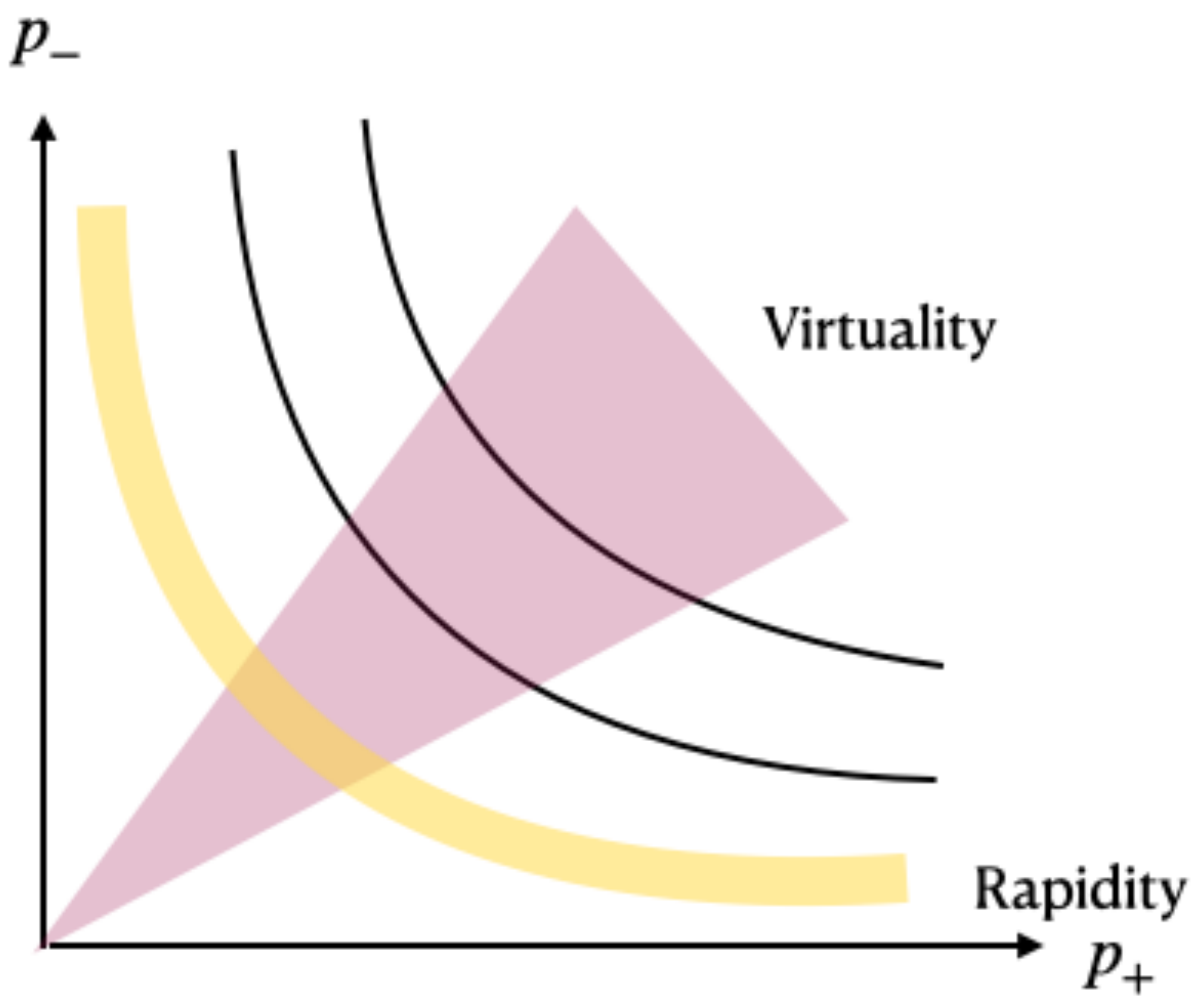
Mode diagrams



$Q\chi \sim Q_{\text{med}}$



$Q\chi \gg Q_{\text{med}}$

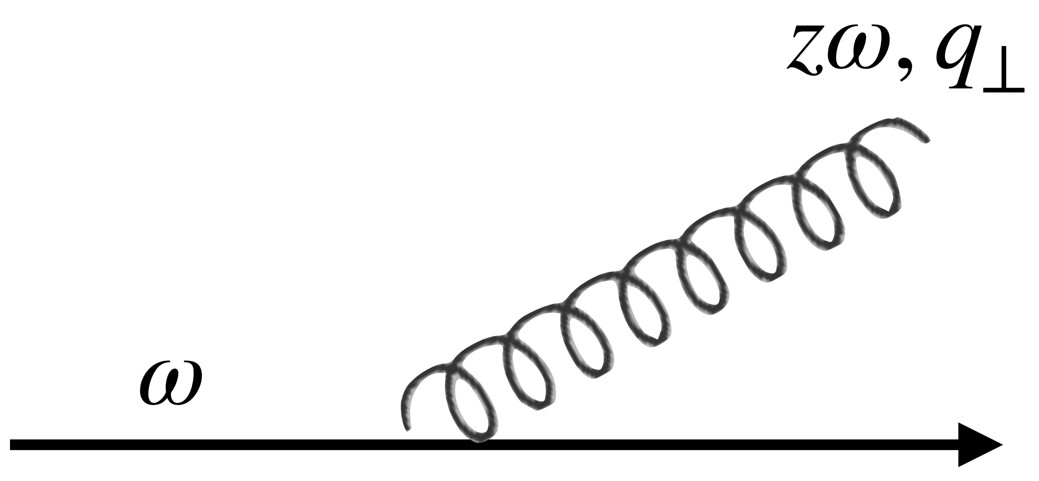


- Non-trivial running in both virtuality and rapidity, DGLAP and BFKL

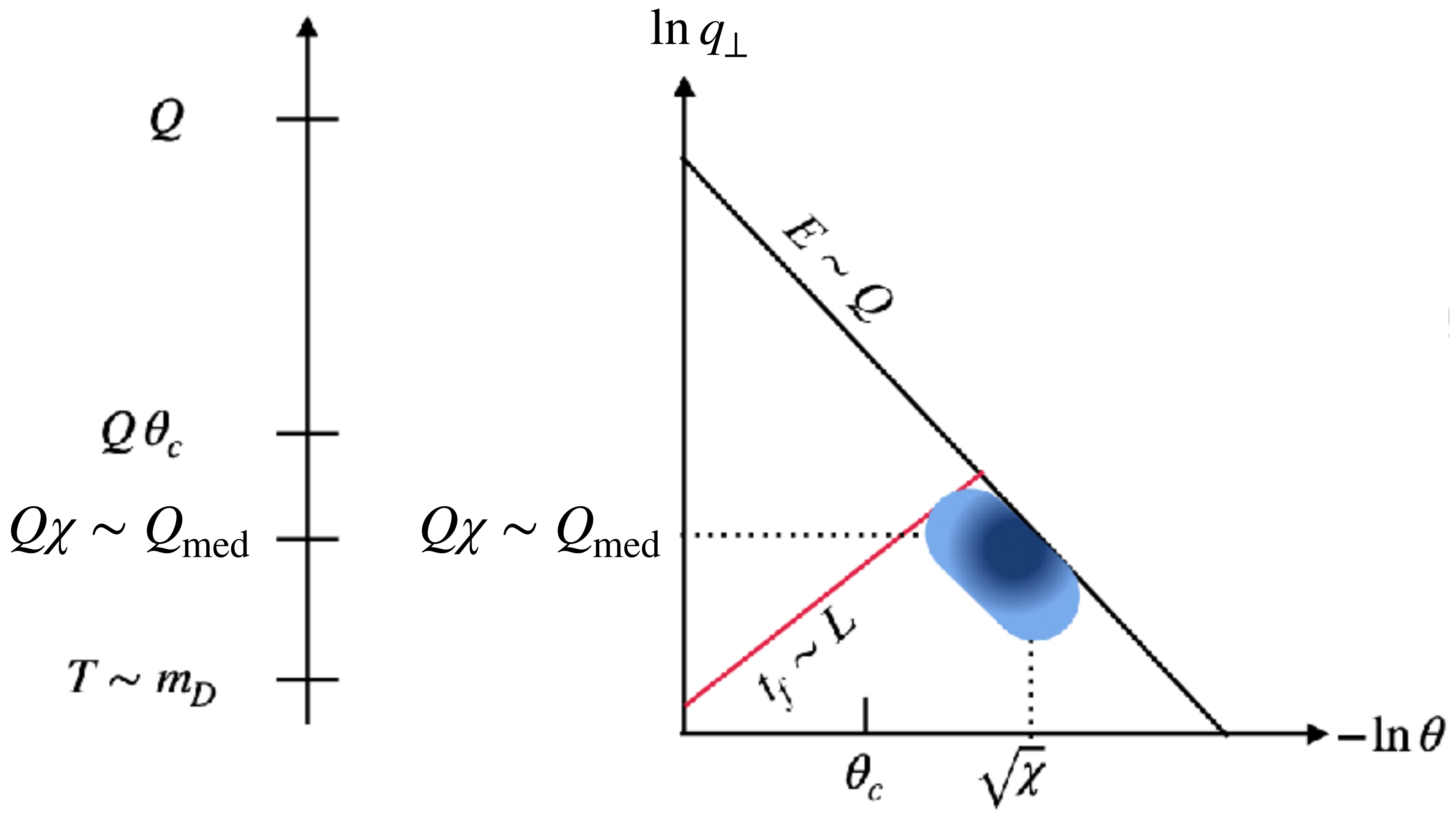
Lund plane representation

- Phase space region populated by each mode is assessed using Lund diagrams

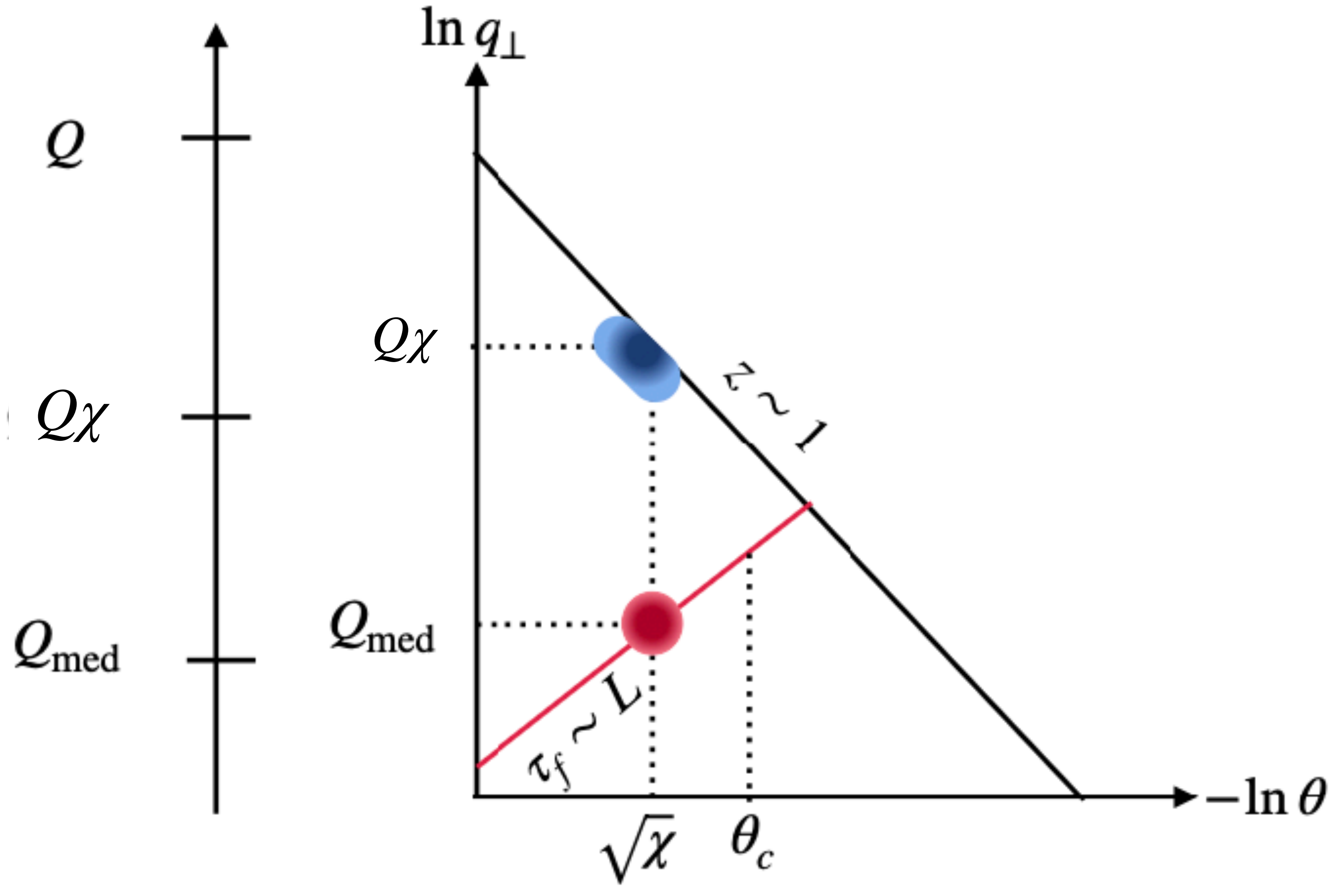
$$q_\perp = z\omega\theta \qquad z\omega = \tau_f q_\perp^2$$



- Emissions with $\theta > \theta_c$ are resolved by the medium



$Q\chi \sim Q_{\rm med}$

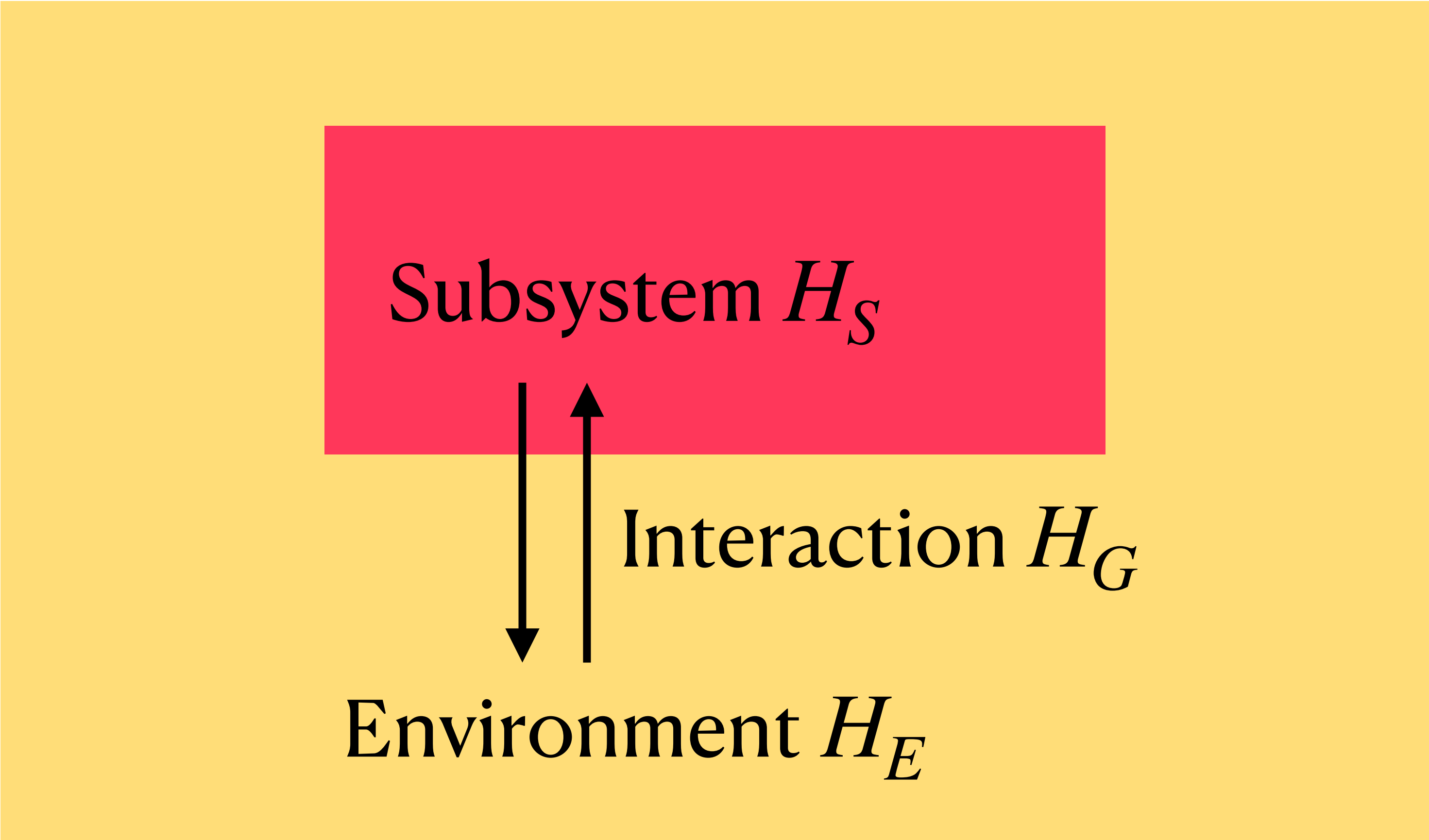


$Q\chi \gg Q_{\rm med}$

Jet as open quantum system

1. Factorized total initial density matrix

$$\rho(0) = |e^+e^-\rangle\langle e^+e^-| \otimes \rho_E(0) \qquad \rho_E = \frac{e^{-\beta H_E}}{Tr[e^{-\beta H_E}]}$$



Subsystem H_S

Interaction H_G

Environment H_E

2. Time evolution of the jet is defined through system density matrix evolution

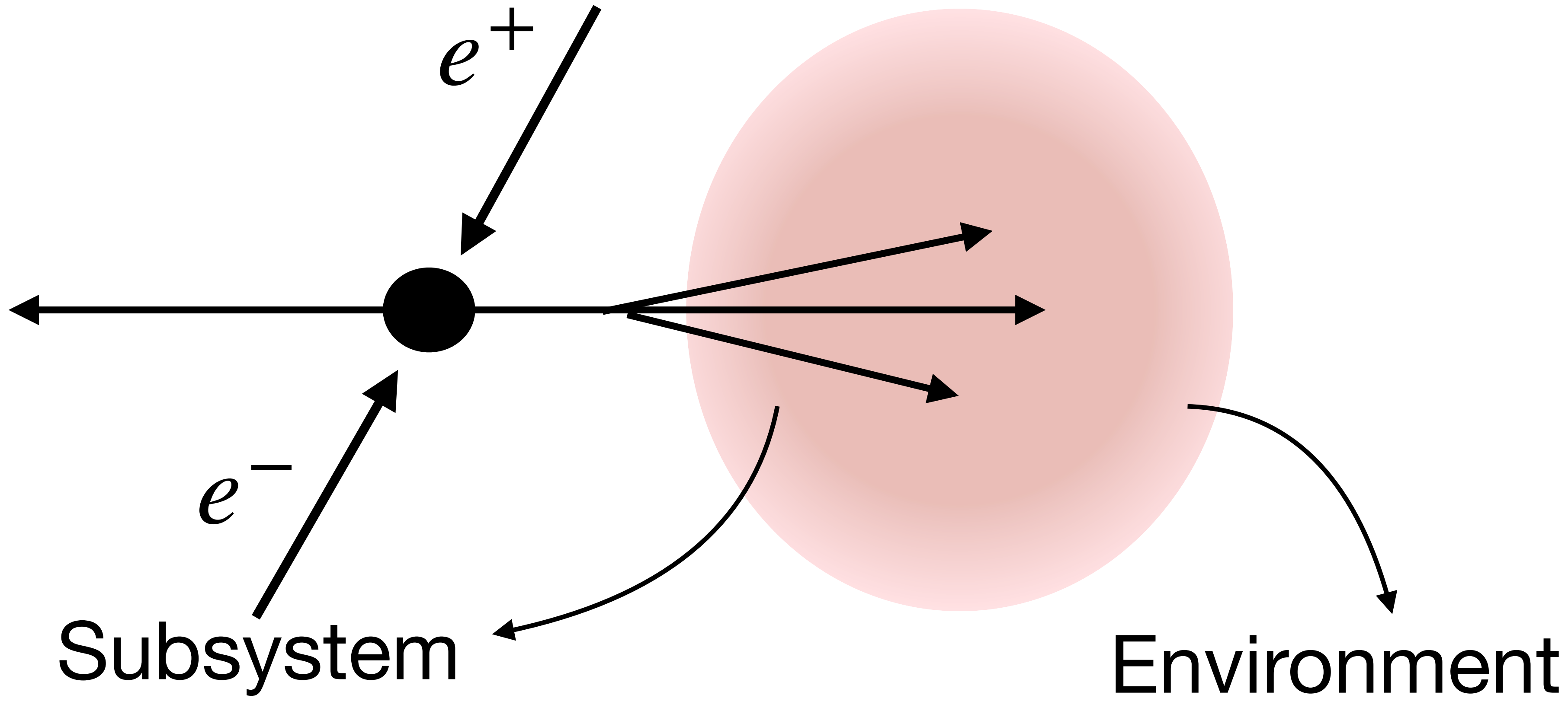
$$\rho(t) = e^{-iHt}\rho(0)e^{iHt} \longrightarrow$$

Can be solved to get Lindblad equation

$$H = H_S + H_E + H_G + C(Q)l^\mu j_\mu \equiv H_0 + \underset{\downarrow}{\mathcal{O}_H}$$

$$j^\mu = \bar{\chi}_n\gamma^\mu\chi_n$$

Hard interaction



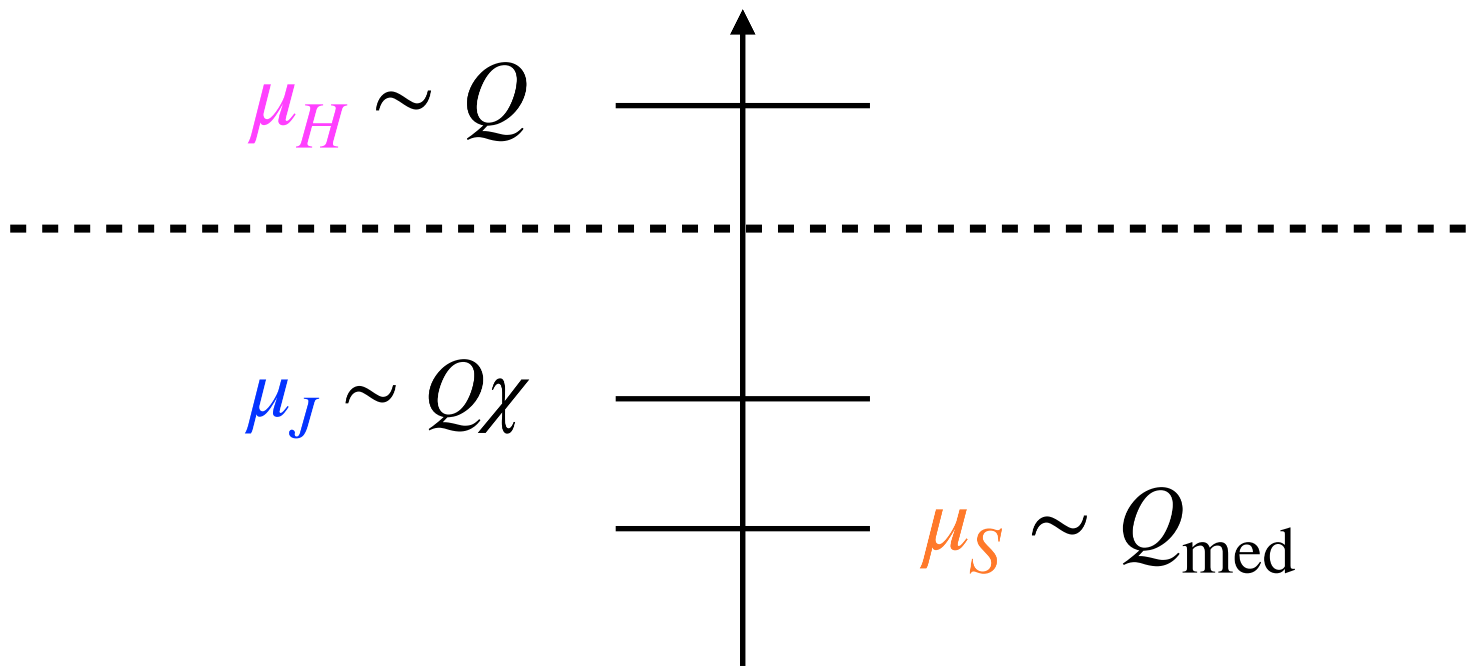
3. Hard operator creates hard scattering event that produces the jet

$$\frac{d\sigma}{d\chi} = \lim_{t\to\infty} \mathrm{Tr}[\rho(t)\mathcal{M}] = |C(Q)|^2 L_{\mu\nu} \lim_{t\to\infty} \int d^4x d^4y e^{iq\cdot(x-y)} \mathrm{Tr}[e^{-iH_0t} j^\mu(x)\rho(0)\mathcal{M} j^\nu(y) e^{iH_0t}]$$

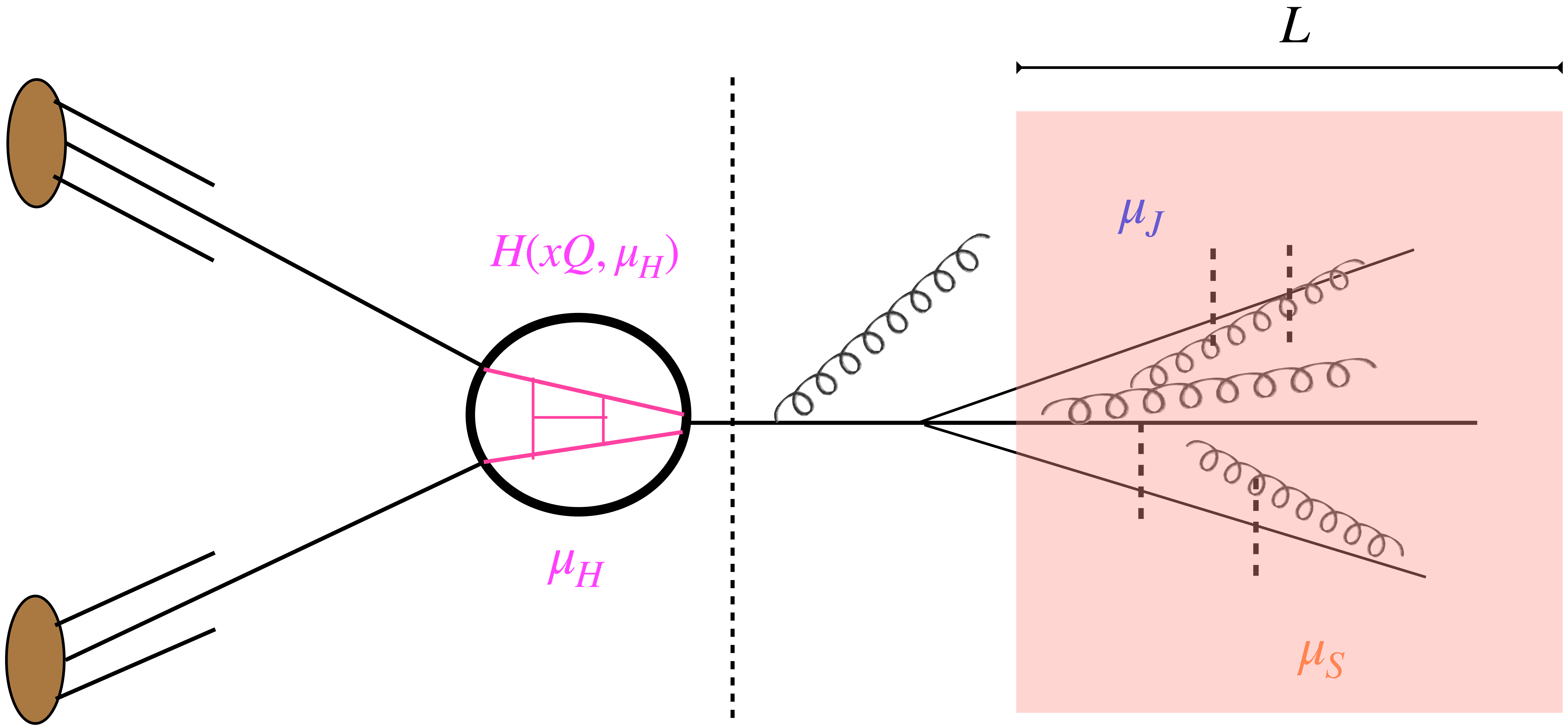
Factorizing hard function

- OPE for factorizing hard scales

$$\frac{d\sigma}{d\chi} = \sum_{i\in\{q,\bar{q},g\}} \int dx\, x^2 H_i(xQ,\mu)\, J_i(xQ,\chi,\mu)$$



- At this stage $J(xQ,\chi,\mu)$ contains both vacuum and medium physics



Soft scale depends only on the Glauber momentum which is transferred to the jet by the medium

Measurement factorization

$$J_q(\chi) = \frac{1}{2N_c}\sum_X \mathrm{Tr}\Big[\rho_E(0)\frac{\bar{n}}{2} e^{iH_{ns}t}\, \underbrace{\bar{\mathbf{T}}\Big\{e^{-i\int_0^t dt' H_{G,I}(t')}\chi_{n,I}(0)\Big\}}_{\text{Glauber interaction}}\, \mathscr{M}\,|X\rangle\langle X|\, \underbrace{\mathbf{T}\Big\{e^{-i\int_0^t dt' H_{G,I}(t')}\bar{\chi}_{n,I}(0)\Big\}}_{\text{Glauber interaction}}\, e^{-iH_{ns}t}\Big]$$

2004.11381

$$|X\rangle = |X_n\rangle \otimes |X_s\rangle$$

Factorizing measurement function $\mathscr{M} = \hat{E}(\chi)|X\rangle = \dfrac{1}{Q}\displaystyle\sum_{i\in\{X_n,X_s\}}\Big(E_{i,n}\Theta(\chi-\theta_{n,i}) + E_{i,s}\Theta(\chi-\theta_{s,i})\Big)|X_n\rangle|X_s\rangle$

- Soft contributions to the measurement are power suppressed

- Glaubers being off-shell modes do not contribute to the measurement

- Now we can separate vacuum and medium induced jet function in J_q

Vacuum jet function

$$J_q(\omega,\chi,\mu) = \sum_{i=0}^{\infty} J_q^{(i)}(\omega,\chi,\mu)$$

Expansion in Glauber Hamiltonian

$i = 0,$ vacuum

$i = 2,$ single scattering

$i \geq 4,$ multiple scattering

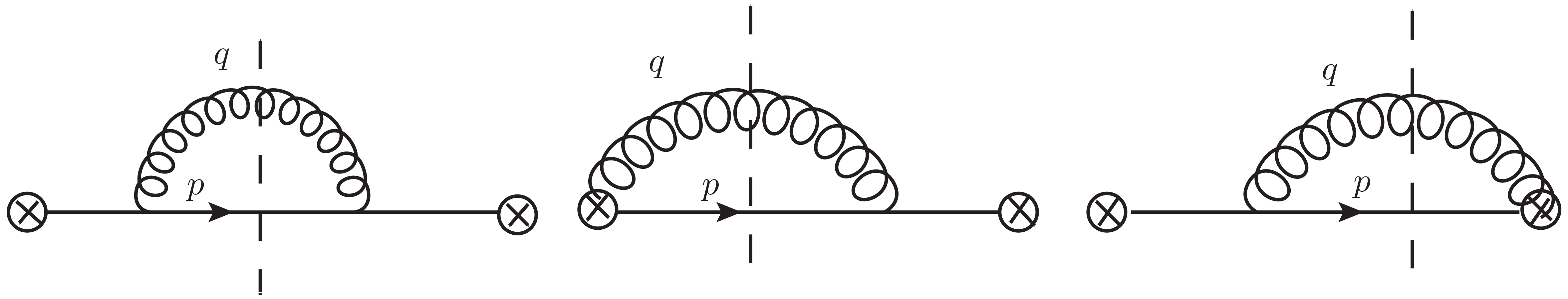
- Leading order

$$J_q(\omega,\chi,\mu) = J_q^{(0)}(\omega,\chi,\mu)$$

- Soft function does not depend on the measurement and becomes identity

$$J_q^{(0)}(\omega,\chi,\mu) = \frac{1}{2N_c}\sum_{X_n}\mathrm{Tr}\left[\frac{\bar{n}}{2}\langle 0|\chi_n(0)\mathscr{M}|X_n\rangle\langle X_n|\delta(\omega-\bar{n}\cdot\mathscr{P})\delta^2(\mathbb{P}_\perp)\bar{\chi}_n(0)|0\rangle\right]$$

Vacuum jet function diagrams



$\omega = xQ$

Log in cumulant

$$J_q^{(0)}(\omega,\chi) = \delta(\chi) + \frac{\alpha_s C_F}{\pi}\left(-\frac{3}{2\epsilon}\delta(\chi) + \frac{3}{2}\left[\frac{1}{\chi}\right]_+ - \frac{3}{2}\delta(\chi)\ln\left(\frac{\mu^2}{\omega^2}\right) - \frac{19}{3}\delta(\chi) + \mathscr{O}(\epsilon)\right)$$

$$\frac{3}{2}\left[\frac{\mu^2}{\omega^2\chi}\right]_+$$

Medium induced jet function

$$J_{q2}(\chi) = \frac{1}{2N_c}\sum_X \mathrm{Tr}\Big[\rho_E(0)\frac{\bar{n}}{2}e^{iH_{ns}t}\int_0^t dt' H_{G,I}(t')\,\chi_{n,I}(0)\mathscr{M}\,|X\rangle\langle X|\int_0^t dt' H_{G,I}(t')\,\bar{\chi}_{n,I}(0)e^{-iH_{ns}t}\Big] + c\,.c\,.$$

Collinear and soft operators act on their corresponding states so can be separated

Glauber Hamiltonian

$$H_G = c\sum_{i,j\in q,g}\int d^3\mathbf{y}\,\mathcal{O}_n^{ia}(\mathbf{y})\frac{1}{\mathcal{P}_\perp^2}\mathcal{O}_s^{ja}(\mathbf{y})$$

$$\mathcal{O}_n^{qA} = \bar{\chi}_n T^A\frac{\bar{\slashed{n}}}{2}\chi_n$$

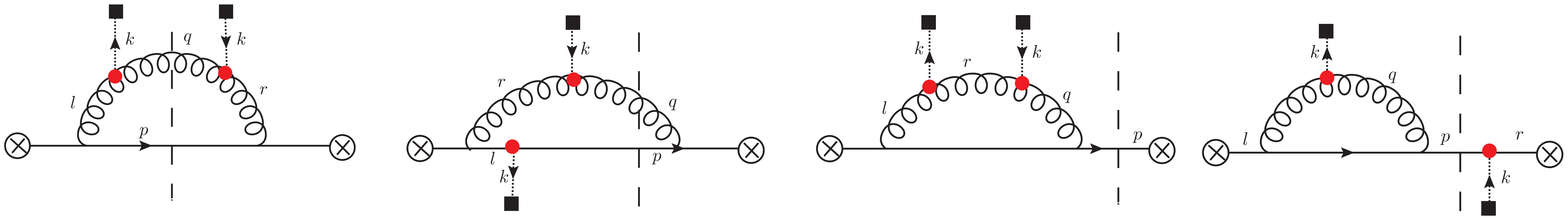
$$\mathcal{O}_s^{qA} = \bar{\chi}_s T^A\frac{\slashed{n}}{2}\chi_s$$

$$J_{q2}(\chi; L) = L\int\frac{d^2k_\perp}{(2\pi)^2}\mathbf{J}_{q2}(\chi, k_\perp, L)\otimes\mathbf{B}(k_\perp)$$

Production of medium induced emissions

Medium correlator

Medium induced jet function diagrams



Medium induced jet function

Medium induced jet function in the soft limit

$$\mathbf{J}_{q2}(\chi,\omega,k_\perp) = \frac{4C_F N_c g^2 L}{\pi}\int \frac{dz}{z}\int\frac{d^2q_\perp}{(2\pi)^2}\underbrace{\frac{\vec{q}_\perp\cdot\vec{k}_\perp}{\vec{q}_\perp^{\,2}\vec{\kappa}_\perp^{\,2}}}_{\vec{\kappa}=\vec{q}_\perp-\vec{k}_\perp}\left(1-\underbrace{\frac{z\omega}{\vec{\kappa}_\perp^{\,2}L}\sin\left[\frac{L\vec{\kappa}^2}{z\omega}\right]}_{\text{LPM}}\right)2z\,\delta\left(\chi^2-\frac{q_\perp^2}{z^2\omega^2}\right)$$

Agreement with GLV

Nucl. Phys. B 594 (2001) 371–419

Divergent if measurement is not energy

Comes from measurement function

Divergence require a regulator and in this set-up its a rapidity regulator and corresponding resummation resums the logs of energies

For two point energy correlator the divergence is expected to appear and NNLO accuracy

Medium function

- Thermal expectation value of soft operators

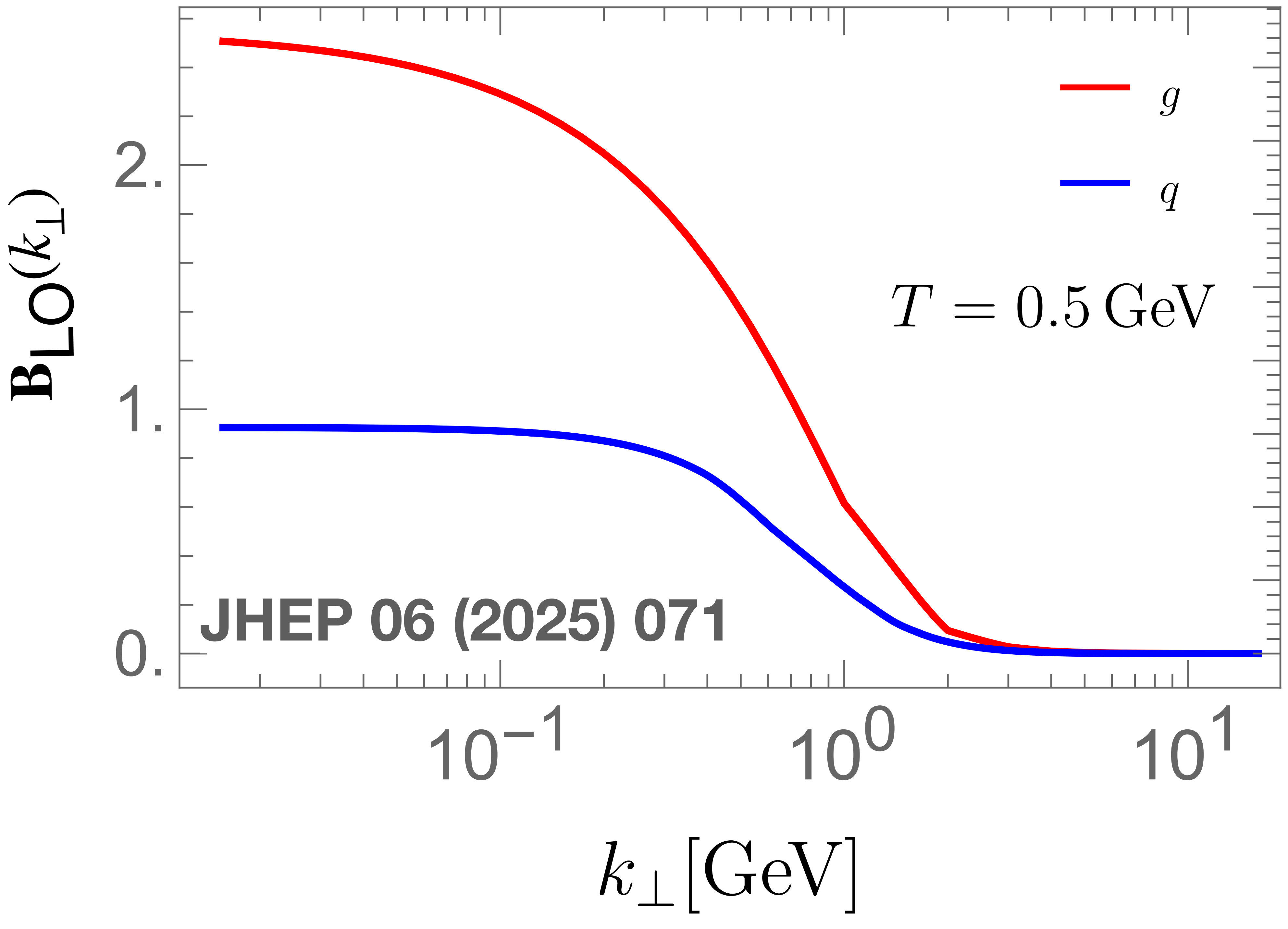
$$\mathbf{B}(k) = \int d^4r\, e^{ik\cdot r}\, \langle \rho_E O_s^a(r) O_s^a(0) \rangle$$

$$\mathbf{B}_{\mathrm{LO}}(k_\perp) \approx \frac{f(k_\perp)}{k_\perp^4} \equiv \frac{f(k_\perp)}{(k_\perp^2 + m_D^2)^2}$$

Weak dependence on transverse momentum

Soft loop correction on Glauber propagator

LO medium function



Medium function has BFKL anomalous dimension

- Allows for dynamic treatment of the medium through medium correlation function
- Medium function can be computed non-perturbatively using lattice gauge theory to give fully model independent description of jet-medium interactions

Resummed EEC jet function

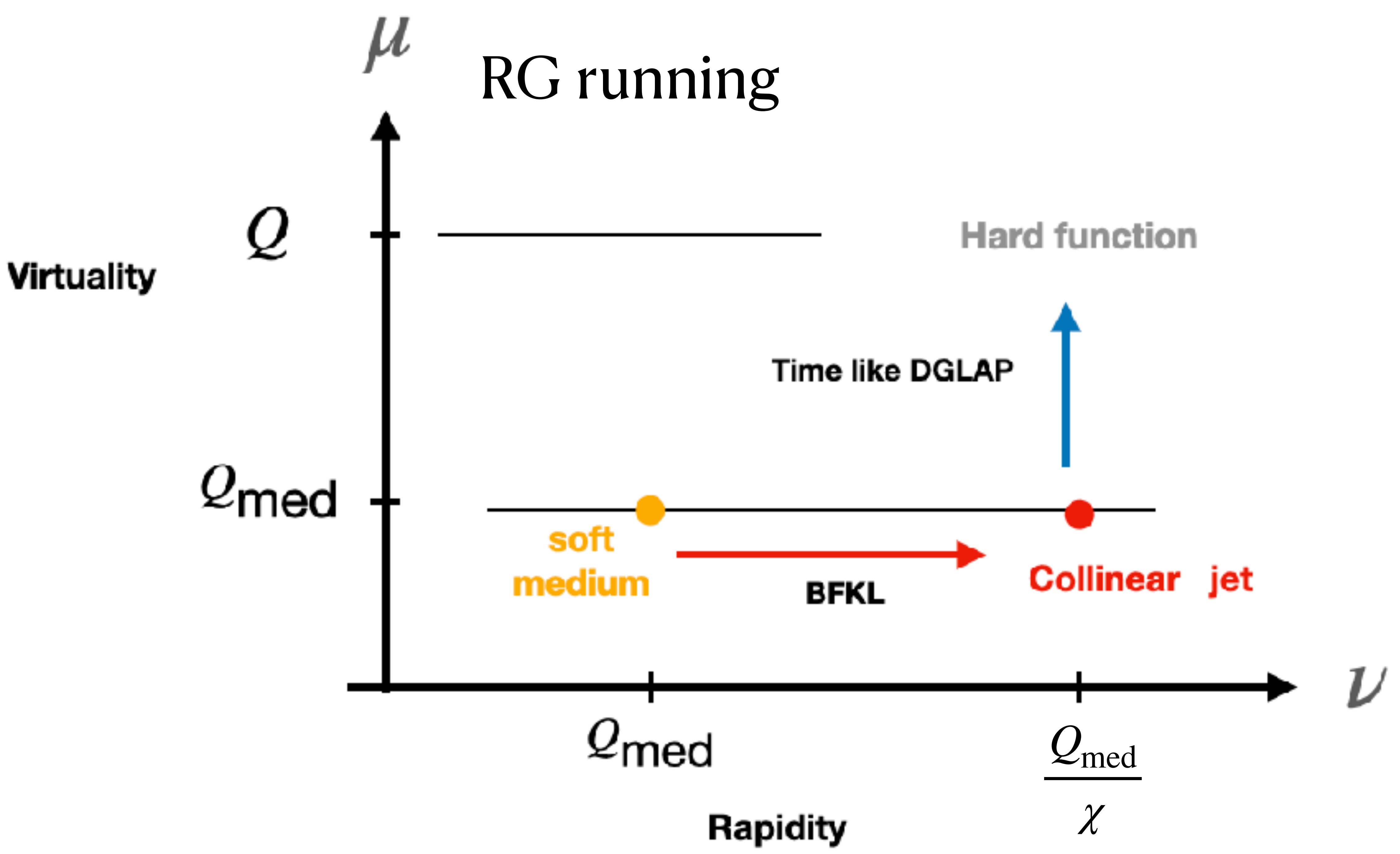
$$\mathbf{J}^R_{q2}(k_\perp,\mu,\nu_f) = \int d^2 l_\perp \mathbf{J}_{q2}(l_\perp,\mu,\nu_0) \int \frac{d\xi}{2\pi} k_\perp^{-1+2i\xi} l_\perp^{-1-2i\xi} e^{in(\phi_k-\phi_l)} e^{-\frac{\alpha_s(\mu)N_c}{\pi}\chi(n,r)\log\frac{\nu_f}{\nu_0}}$$

Scale for jet function

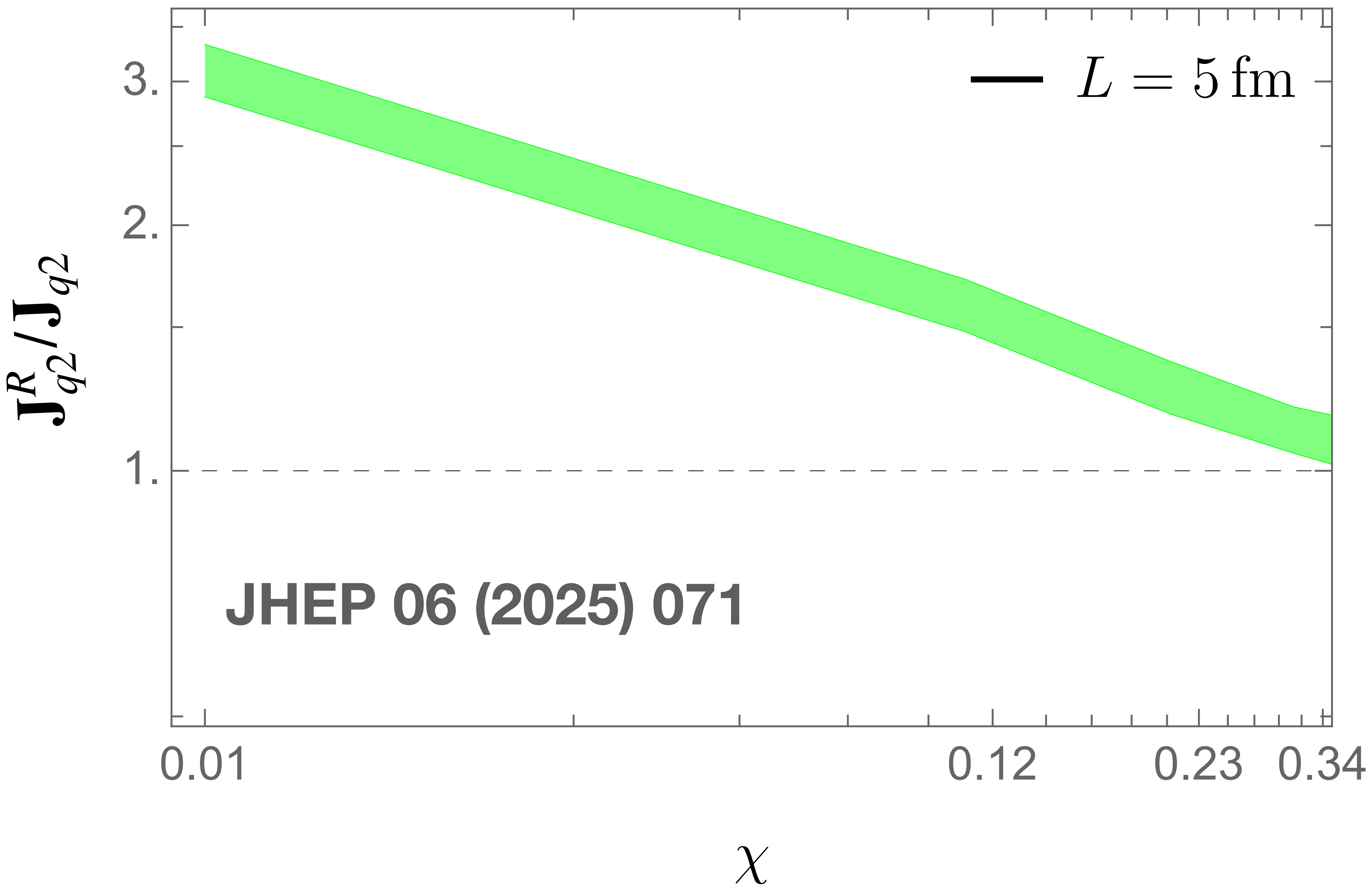
$$\nu_0 \sim \frac{Q_{\rm med}}{\chi}$$

Medium scale $\nu_f \sim Q_{\rm med}$

- Resums $\sim \alpha_s \log \chi$ terms which are relevant in small χ limit



Resummed and NLO ratio



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Projected energy correlators

- N-point projected energy correlators are defined for M number of particles

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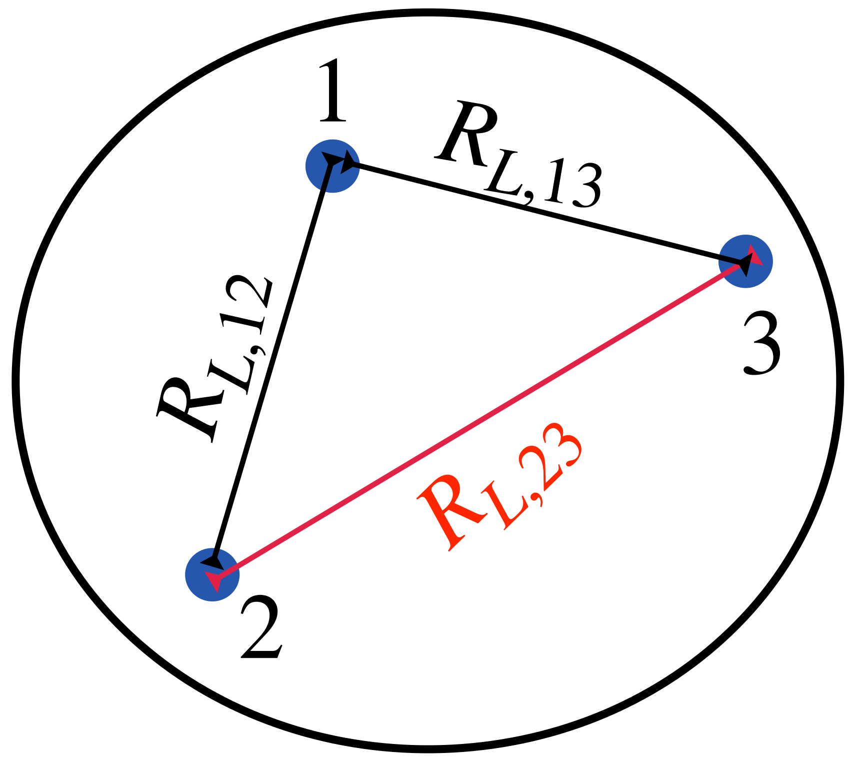


$$\frac{d\sigma^{[N]}}{d\chi} = \sum_{M} \sum_{1\le i_1,\ldots,i_N<M} \frac{1}{Q^N} \int d\sigma_{cd\to X_M} \left(\prod_{a=1}^{N} E_{i_a} \right) \delta(\chi - \max(\theta_{i_1 i_2}, \theta_{i_1 i_3,\ldots,i_{N-1},i_N}))$$

For M=1 $\dfrac{E_{i_1}^N}{Q^N}$ For M=2 $\dfrac{(E_{i_1}+E_{i_2})^N}{Q^N} - \dfrac{E_{i_1}^N}{Q^N} - \dfrac{E_{i_2}^N}{Q^N}$

- These are binomial expansions and can be analytically continued to non-integer values $N \to \nu$
  - ν can take any non-integer value
  - For IRC safe $\nu > 0$
  - Small ν values are sensitive to small-x physics

$$W_1^{[\nu]}(i_a) = \frac{E_{i_a}^{\nu}}{Q^{\nu}}$$

$$W_2^{[\nu]}(i_1, i_2) = \frac{(E_{i_1}+E_{i_2})^{\nu}}{Q^{\nu}} - \sum_{a=1,2} W_1^{[\nu]}(i_a)$$

Projected ν-correlators

$$\text{PENC}(\chi) \equiv \frac{d\sigma^{[N]}}{d\chi} = \sum_M \int d\sigma_X \Bigg[\sum_{1\le b_1 \le M} W_1^{[N]}(b_1)\,\delta(\chi) + \sum_{1\le b_1<b_2\le M} W_2^{[N]}(b_1,b_2)\delta(\chi - \Delta R_{b_1,b_2}) + \dots$$

$$+ \sum_{1\le b_1<..<b_M=N} W_M^{[N]}(b_1,\dots,b_M)\,\delta(\chi - \max\{\Delta R_{b_1,b_2},\dots,\Delta R_{b_{M-1,M}}\})\Bigg]$$

- ν-correlators are defined by analytic continuation of PENC with $N \to \nu$

$$W_1^{[\nu]}(i_a) = \frac{E_{i_a}^{\nu}}{\omega^{\nu}} \qquad\qquad W_2^{[\nu]}(i_1,i_2) = \frac{(E_{i_1}+E_{i_2})^{\nu}}{\omega^{\nu}} - \sum_{a=1,2} W_1^{[\nu]}(i_a)$$

- ν can take any value even a complex number
- For IRC safe $\nu > 0$
- Small ν values are sensitive to small-x physics

Factorization for ν-correlators

- Analytically continued energy weights do not change the power counting of the modes
  - Does not affect the scalings of the modes

$$\mu_H \sim Q$$

$$\mu_J \sim Q\chi$$

$$\mu_S \sim Q_{\text{med}}$$

Factorization structure

$$\frac{d\sigma^{[\nu]}}{d\chi} = \sum_{i\in\{q,\bar{q},g\}} \int dx\, x^{\nu} H_i(\omega = xQ, \mu)\, J_i^{[\nu]}(\omega, \chi, \mu)$$

$$J_{q2}^{[\nu]}(\chi) = \frac{1}{2N_c} \sum_X \mathrm{Tr}\Big[\rho_E(0) \frac{\bar{n}}{2} e^{iH_{ns}t}\, \underbrace{\bar{\mathbf{T}}\Big\{ e^{-i\int_0^t dt' H_{G,I}(t')} \chi_{n,I}(0) \Big\}}_{\text{Glauber interaction}} \mathscr{M}^{[\nu]} |X\rangle\langle X|\, \underbrace{\mathbf{T}\Big\{ e^{-i\int_0^t dt' H_{G,I}(t')} \bar{\chi}_{n,I}(0) \Big\}}_{\text{Glauber interaction}} e^{-iH_{ns}t}\Big]$$

$$J_q^{[\nu]}(\omega,\chi) = \underbrace{J_{q0}^{[\nu]}(\omega,\chi)}_{\text{vacuum}} + \underbrace{J_{q2}^{[\nu]}(\omega,\chi;L) + \dots}_{\text{medium induced}}$$

$$J_{q2}^{[\nu]}(\chi;L) = L \int \frac{d^2k_\perp}{(2\pi)^2} \mathbf{J}_{q2}^{[\nu]}(\chi, k_\perp, L) \otimes \mathbf{B}(k_\perp)$$

ν-correlator jet function

$$\mathbf{J}^{[\nu]}_{q2}(\chi,\omega,k_\perp) = \underbrace{\mathbf{J}^{[\nu]}_{qR}(\chi,\omega,k_\perp)}_{\text{real emission}} \;\mathcal{M}^{[\nu]} + \underbrace{\left[\mathbf{J}^{[\nu]}_{qR}(\chi,\omega,k_\perp) - \overbrace{\mathbf{J}^{[\nu]}_{qV}(\chi,\omega,k_\perp)}^{\text{virtual}}\right]}_{=0}\delta(\chi)$$

$$\mathbf{J}^{[\nu]}_{q2}(\chi,\omega,k_\perp) = \frac{4C_F N_c g^2 L}{\pi}\int\frac{dz}{z}\int\frac{d^2q_\perp}{(2\pi)^2}\underbrace{\frac{\vec{q}_\perp\cdot\vec{k}_\perp}{\vec{q}^{\,2}_\perp\vec{\kappa}^2_\perp}}_{\vec{\kappa}=\vec{q}_\perp-\vec{k}_\perp}\Bigg(1-\underbrace{\frac{z\omega}{\vec{\kappa}^2_\perp L}\sin\Big[\frac{L\vec{\kappa}^2}{z\omega}\Big]}_{\text{LPM}}\Bigg)\mathcal{M}^{[\nu]}$$

For $\nu = 2$ $\qquad \mathcal{M}^{[2]} = -\,2z\,[\delta(\chi)-\delta(\chi-\theta)]$

For $\nu = 0$ $\qquad \mathcal{M}^{[0]} = 0$ $\qquad$ Exact real and virtual cancellation

For $\nu \ll 1$ $\qquad \mathcal{M}^{[\nu]} = z^\nu\,[\delta(\chi)-\delta(\chi-\theta)]$

Measurement function is dominated by different terms

Scaling behavior in vacuum

$$J^{[\nu]}_0(\chi) \propto \frac{1}{\chi} \qquad J^{[\nu]}_0(\chi) \propto \frac{1}{\chi^{1-\gamma(\nu+1)}}$$

Measurement function

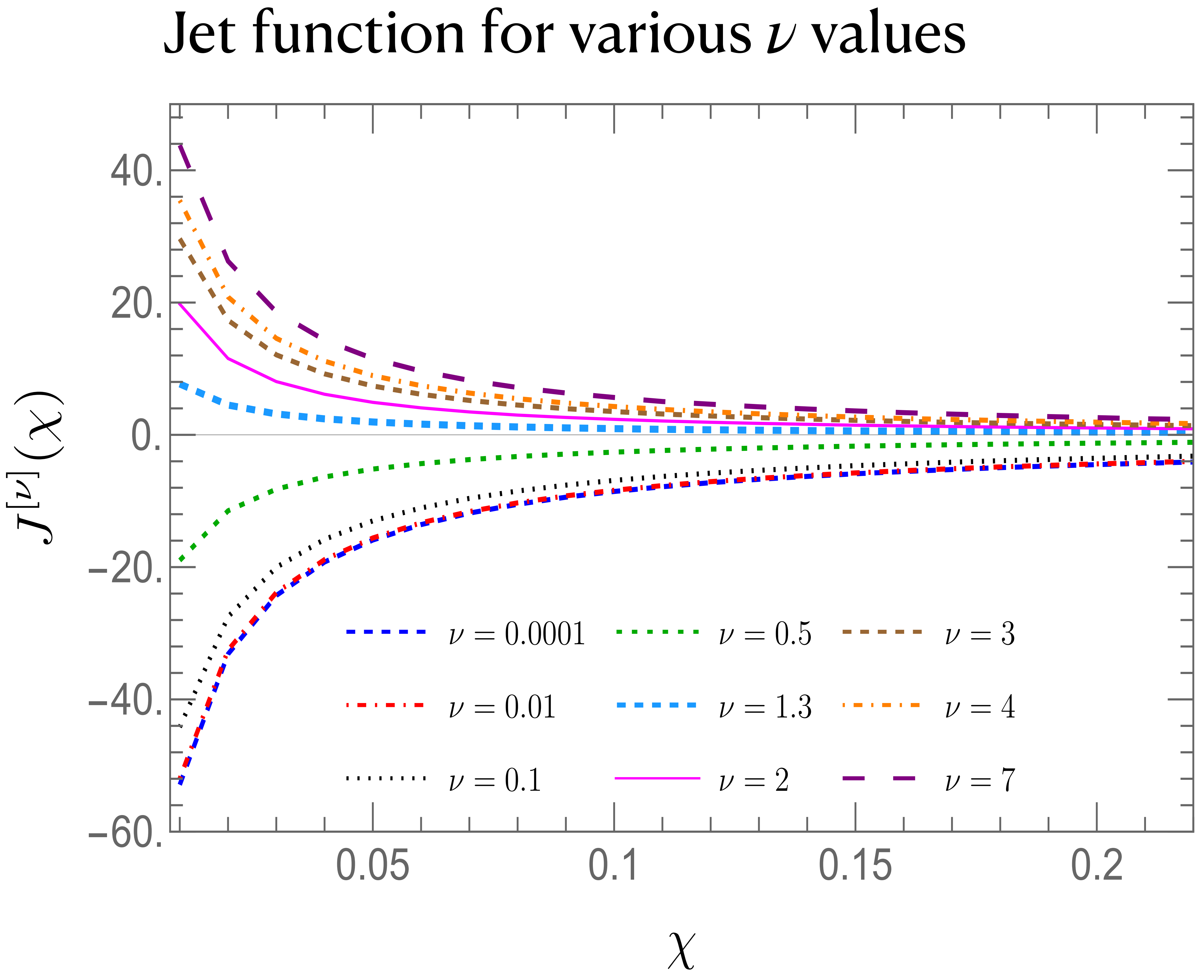
$$\mathcal{M}^{[\nu]} = (z^\nu - \nu z)\,[\delta(\chi)-\delta(\chi-\theta)]$$

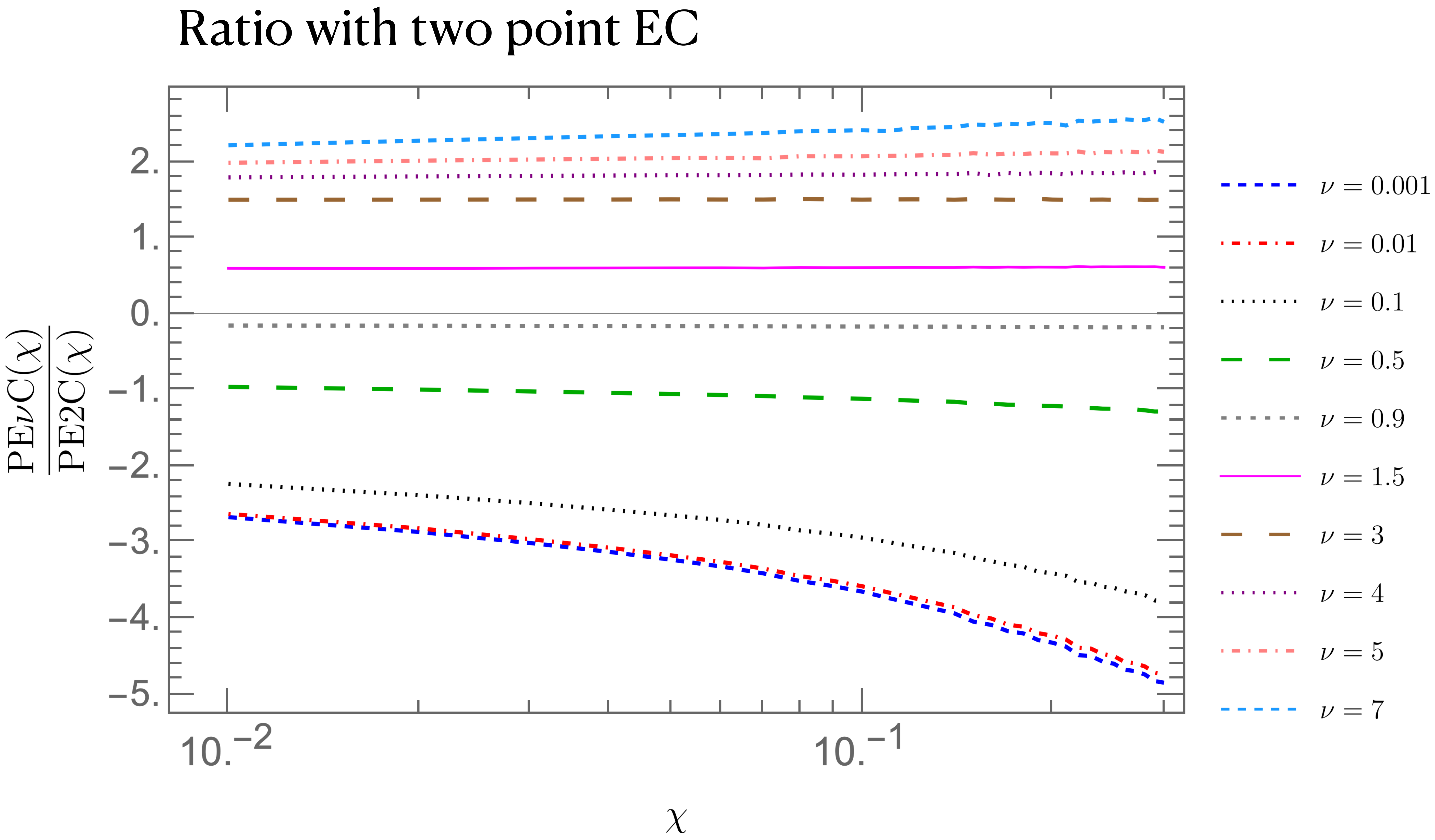
NLO jet function

Small ν point energy correlators are more sensitive to large angle radiations

For small ν values correlators saturate at small ν values

$L = 5\,\text{fm} \quad T = 0.5\,\text{GeV} \quad \alpha_s = 0.3$

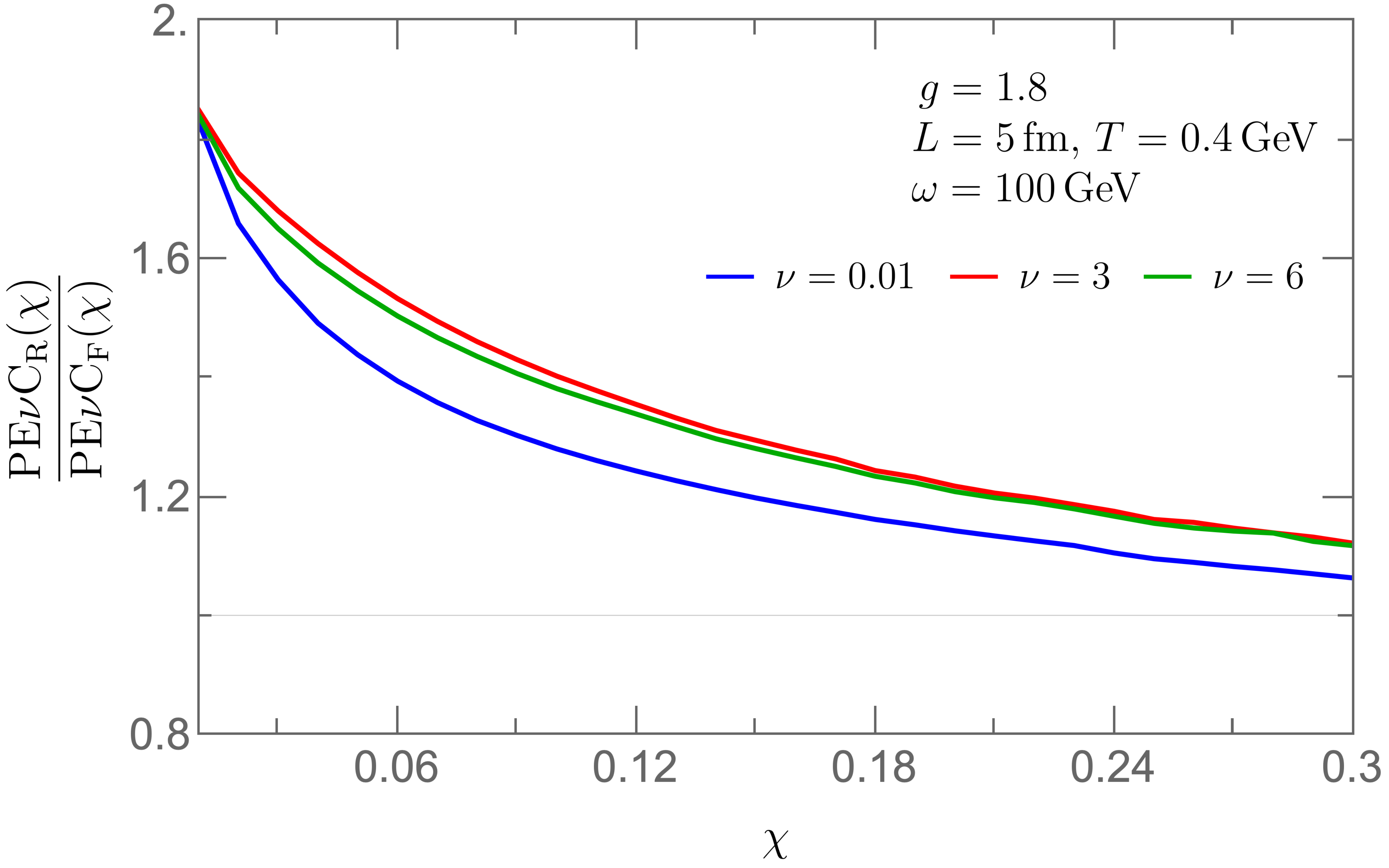




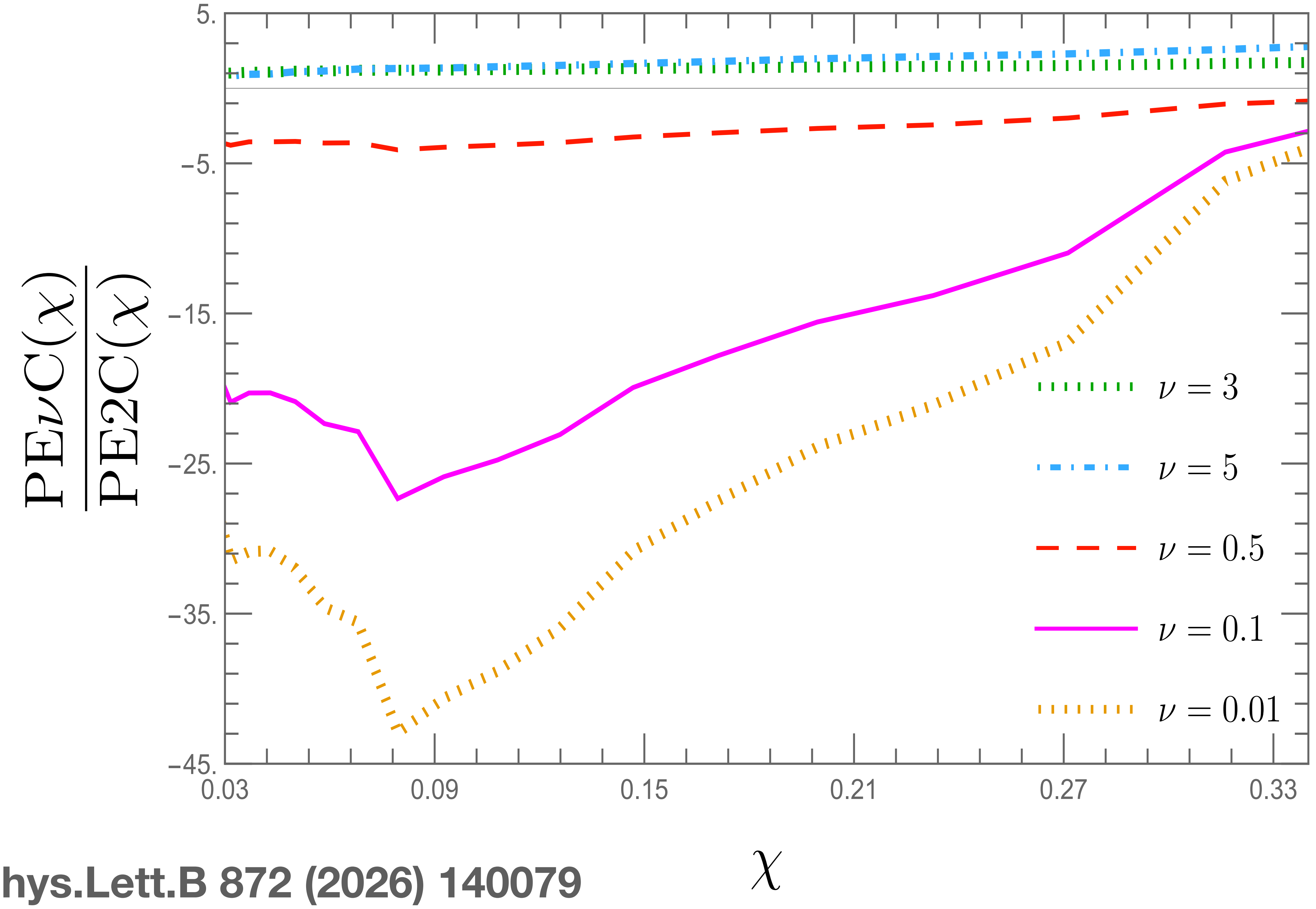
Resummed jet function

$$\mathbf{J}_R^{[\nu]}(\chi, k_\perp) = \frac{1}{\pi k_\perp}\sqrt{\frac{\pi}{14\zeta(3)\bar{\alpha}Y}}e^{(a_p-1)Y}\int d^2 l_\perp \frac{\mathbf{J}^{[\nu]}(\chi, l_\perp)}{l_\perp}e^{-\frac{\log^2(k_\perp/l_\perp)}{14\zeta(3)\bar{\alpha}Y}} \qquad Y = \log(\nu_0'/\nu_f') \qquad \nu_0' \sim \frac{Q_{\text{med}}}{\chi} \qquad \nu_f' \sim Q_{\text{med}}$$

Ratio of resummed and fixed order



Ratio with two point EC



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Less effect of resummation across different ν-values

JEWEL simulation

Vacuum vs vacuum like emissions

- Vacuum distribution scales as $1/\nu$

$$\frac{d\sigma^{[\nu]}_{\text{vle}}}{d\chi} = \sum_{i,j\in J}\int dz\, d\theta_{ij}\frac{dP}{dz\, d\theta_{ij}}\mathscr{M}^{[\nu]}(z,\theta_{ij},\chi)$$

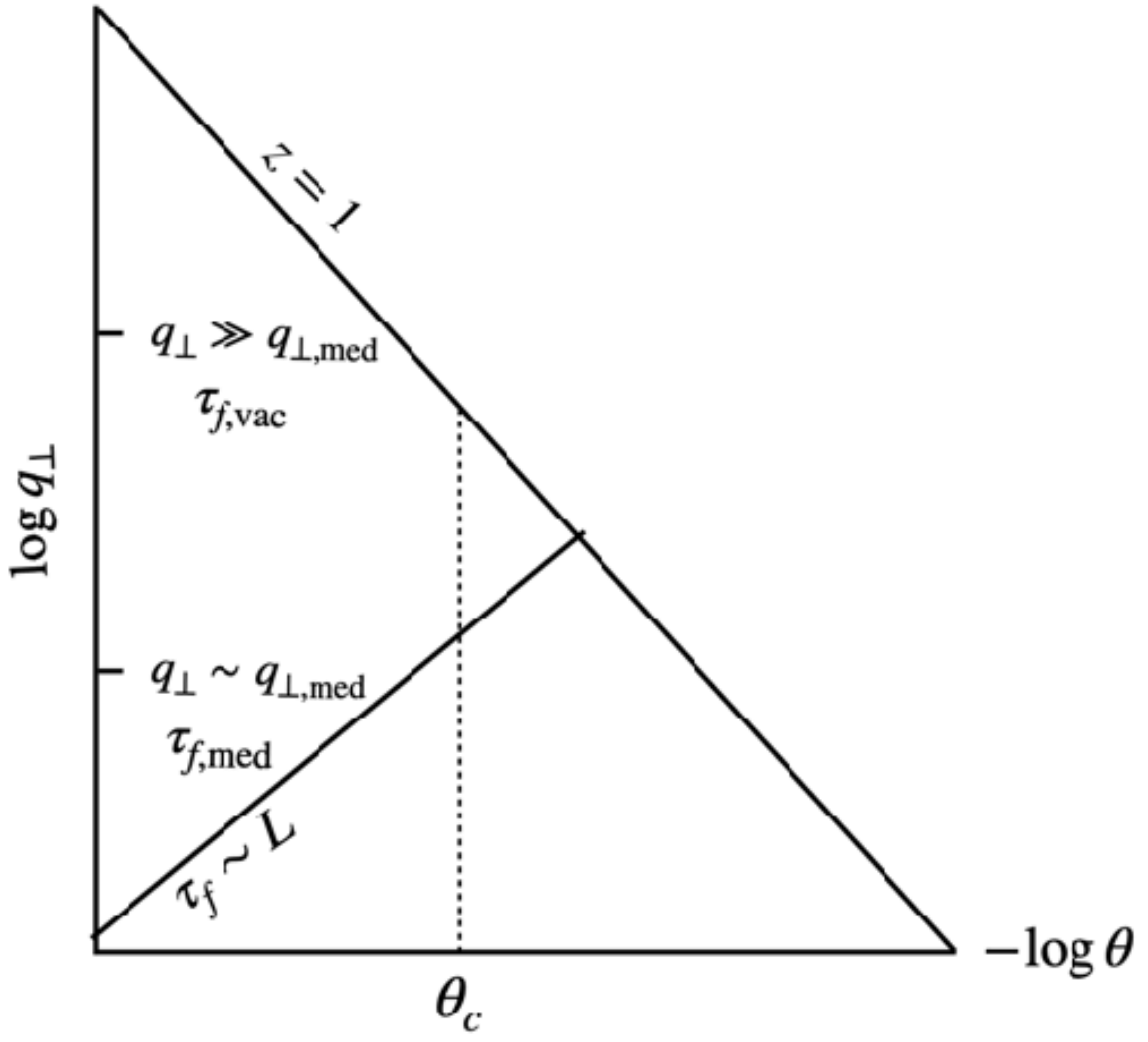
$$\nu > 1, \ \sim \frac{\nu\alpha_s(\mu)}{\chi}$$

Lund diagram

$$\frac{d\sigma^{[\nu]}_{\text{vle}}}{d\chi} \sim \frac{\alpha_s(\mu)}{\chi}\int dz(\nu - z^{\nu-1}) \equiv \frac{\alpha_s(\mu)}{\chi}\left(\nu - \frac{1}{\nu}\right)$$

$$\nu < 1, \ \sim \frac{\alpha_s(\mu)}{\nu\chi}$$

- In-medium emissions can be vacuum like and medium-induced emissions



Medium-induced emissions

$$q_\perp \sim q_{\perp,\text{med}} = Q_{\text{med}}$$

- Produced by interactions with medium
- larger formation time

$$\tau_{f,\text{med}} = \sqrt{\frac{z\omega}{\hat{q}}}$$

Vacuum like emissions

- Produced with large transverse momentum
- Short formation time

$$q_\perp \gg q_{\perp,\text{med}} = Q_{\text{med}}$$

$$\tau_{f,\text{vac}} \gg \tau_{f,\text{med}}$$

Vacuum like emissions

- Differential distribution of VLEs

$$\frac{d\sigma^{[\nu]}_{\text{vle}}}{d\chi} = \sum_{i,j\in J} \int dz\, d\theta_{ij} \frac{dP}{dz\, d\theta_{ij}} \mathscr{M}^{[\nu]}(z, \theta_{ij}, \chi)$$

$$\frac{d\sigma_{\text{vle}}}{d\chi} \sim \frac{\alpha_s(\mu)}{\chi} \int dz(\nu - z^{\nu-1})\Theta(\tau_{f,\text{med}} - \tau_{f,\text{vac}})\Theta(L - \tau_{f,\text{vac}})$$

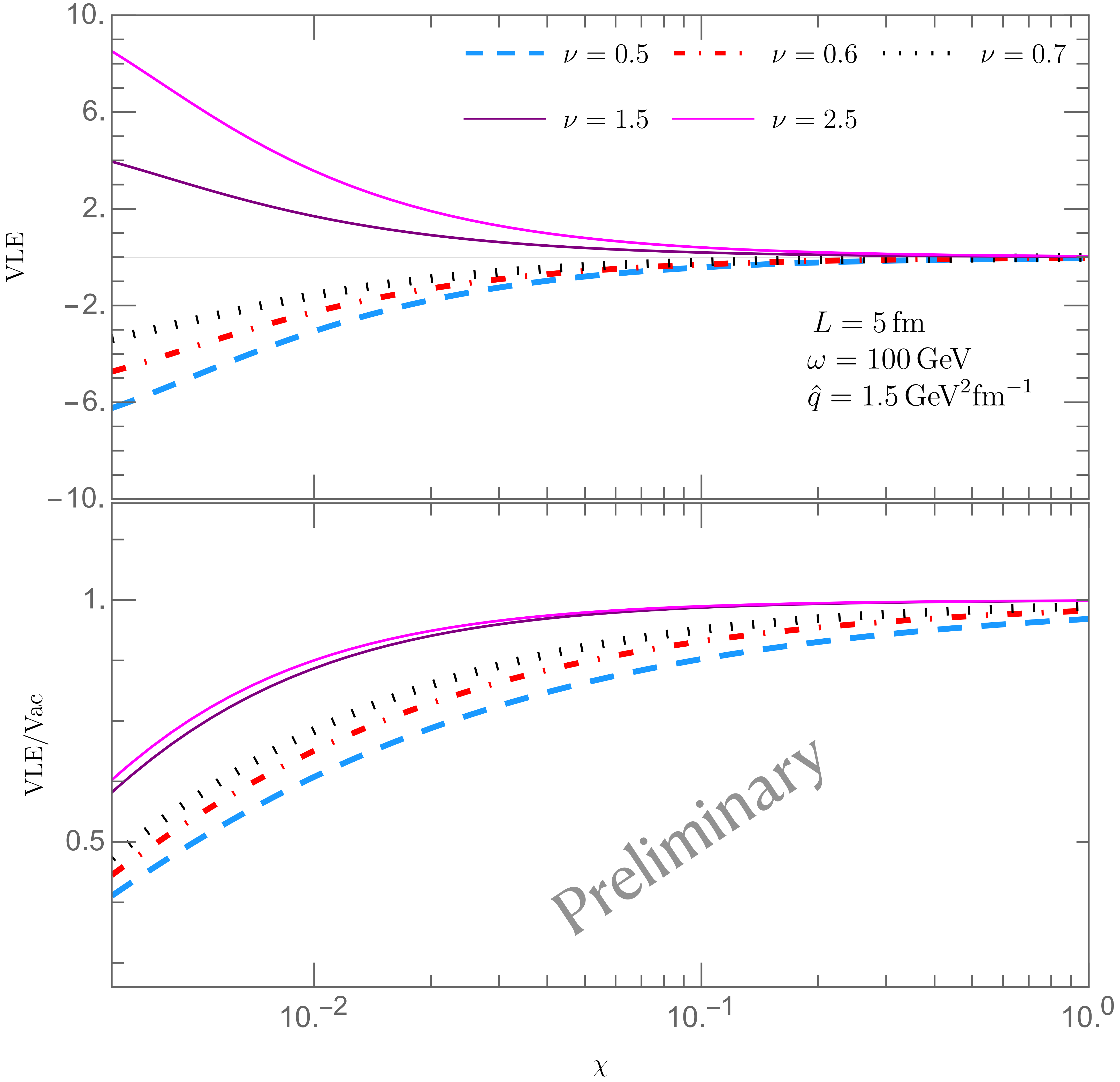
Restricts emissions inside the medium

$$\frac{d\Sigma_{\text{vle}}}{d\chi} \overset{\nu\to 0}{\equiv} \frac{\alpha_s(\mu)}{\chi} \log\left(\frac{1}{\chi\omega L}\right) + \mathcal{O}(\nu)$$

- Phase space kills $1/\nu$ behavior of VLEs

Higher values of ν seems to be better

VLE distribution and ratio with vac



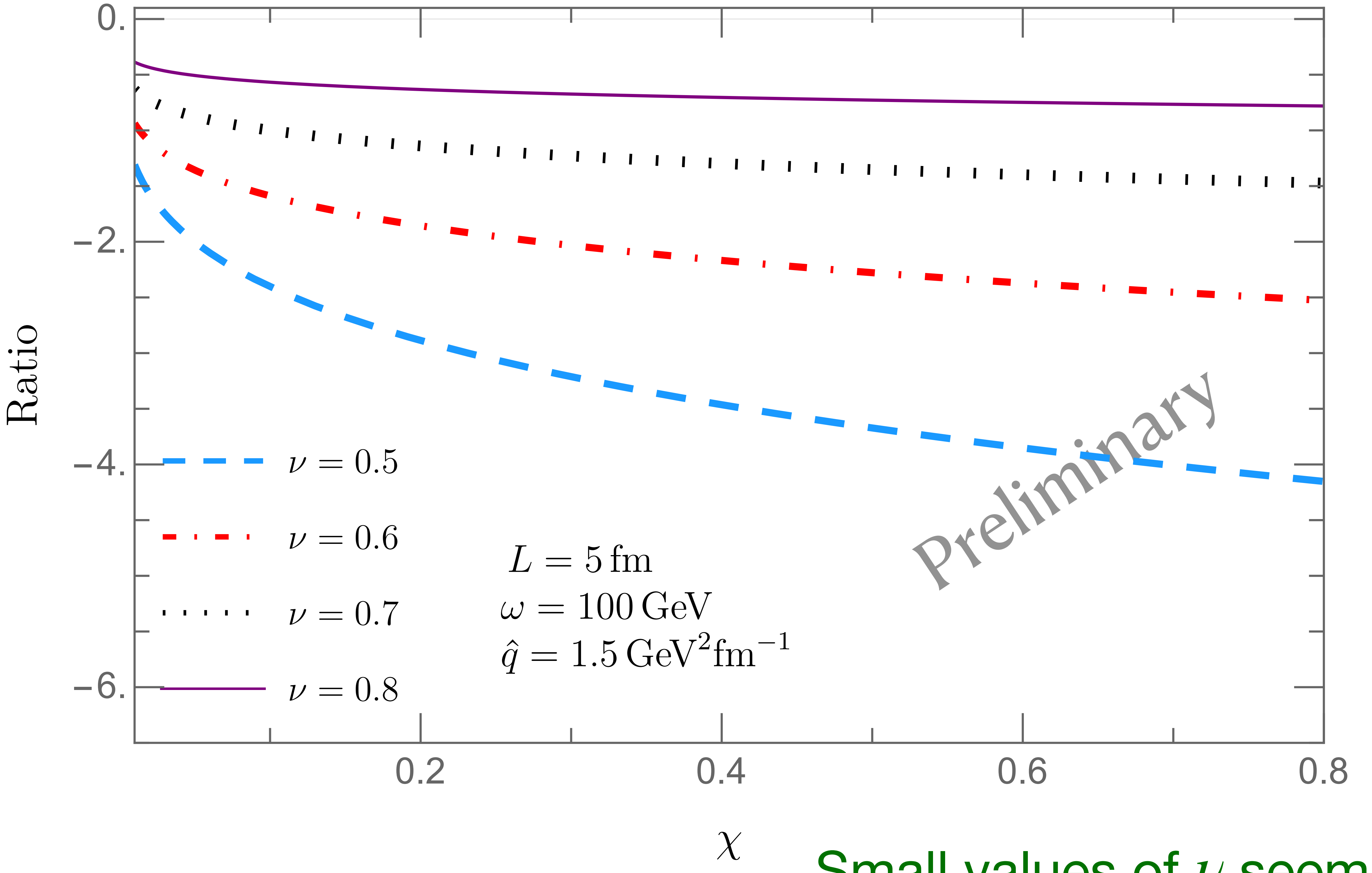
Medium induced emission

- BDMPS-Z splitting function

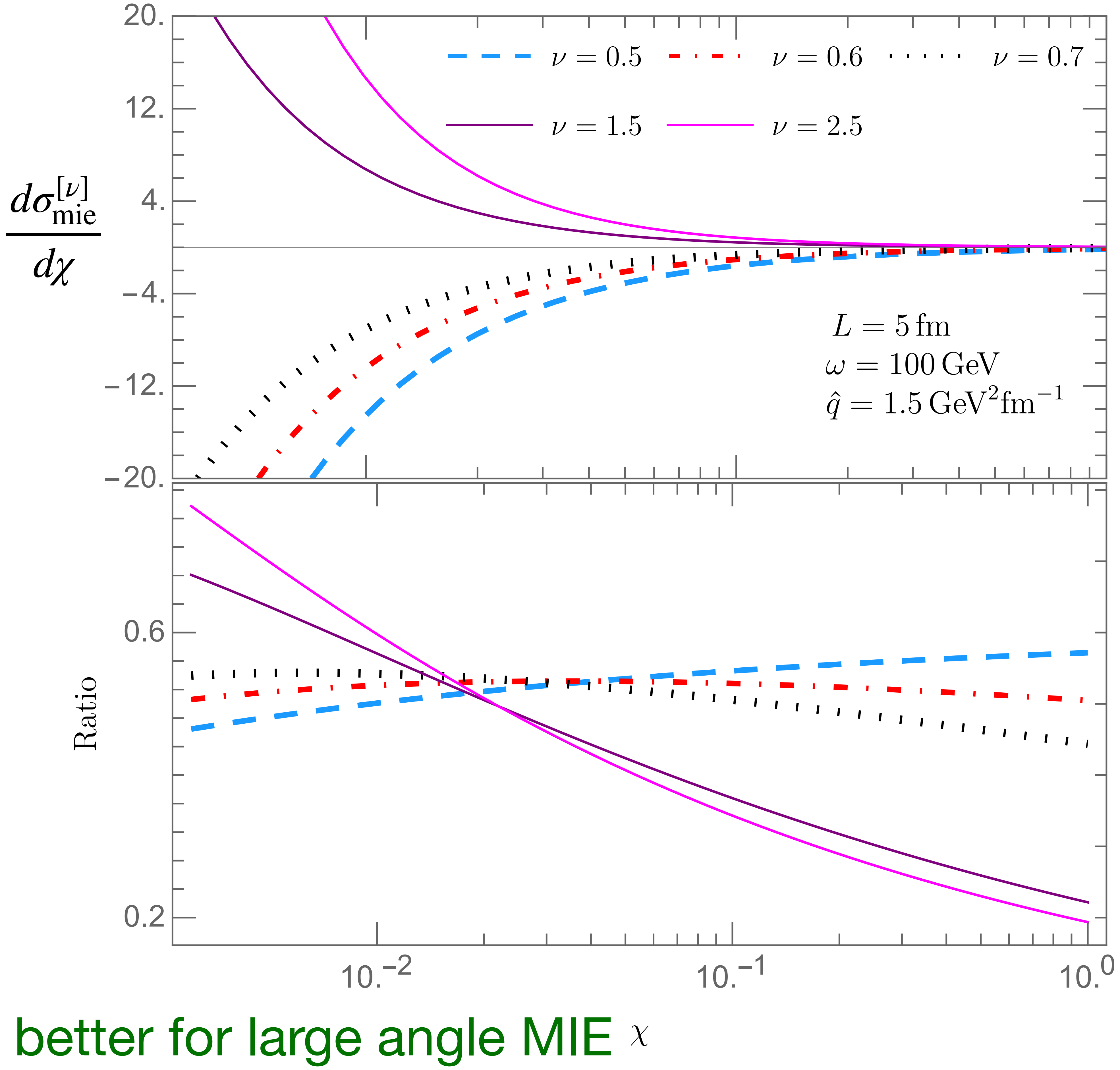
$$\frac{dP}{dz d\theta} = \frac{\alpha_{s,\text{med}}}{L}\sqrt{\frac{8\omega_c}{z^3\omega}}\Theta(\omega_c - z\omega)(\omega^2\theta)\int_0^L \frac{dt\, e^{-\frac{\omega^2\theta^2}{\hat{q}(L-t)}}}{\hat{q}(L-t)}$$

$L = 5$ fm $\quad \hat{q} = 1.5$ GeV^2 fm^{-1} $\quad \omega = 100$ GeV

Ratio with two point EC



MIE distribution and ration with vac

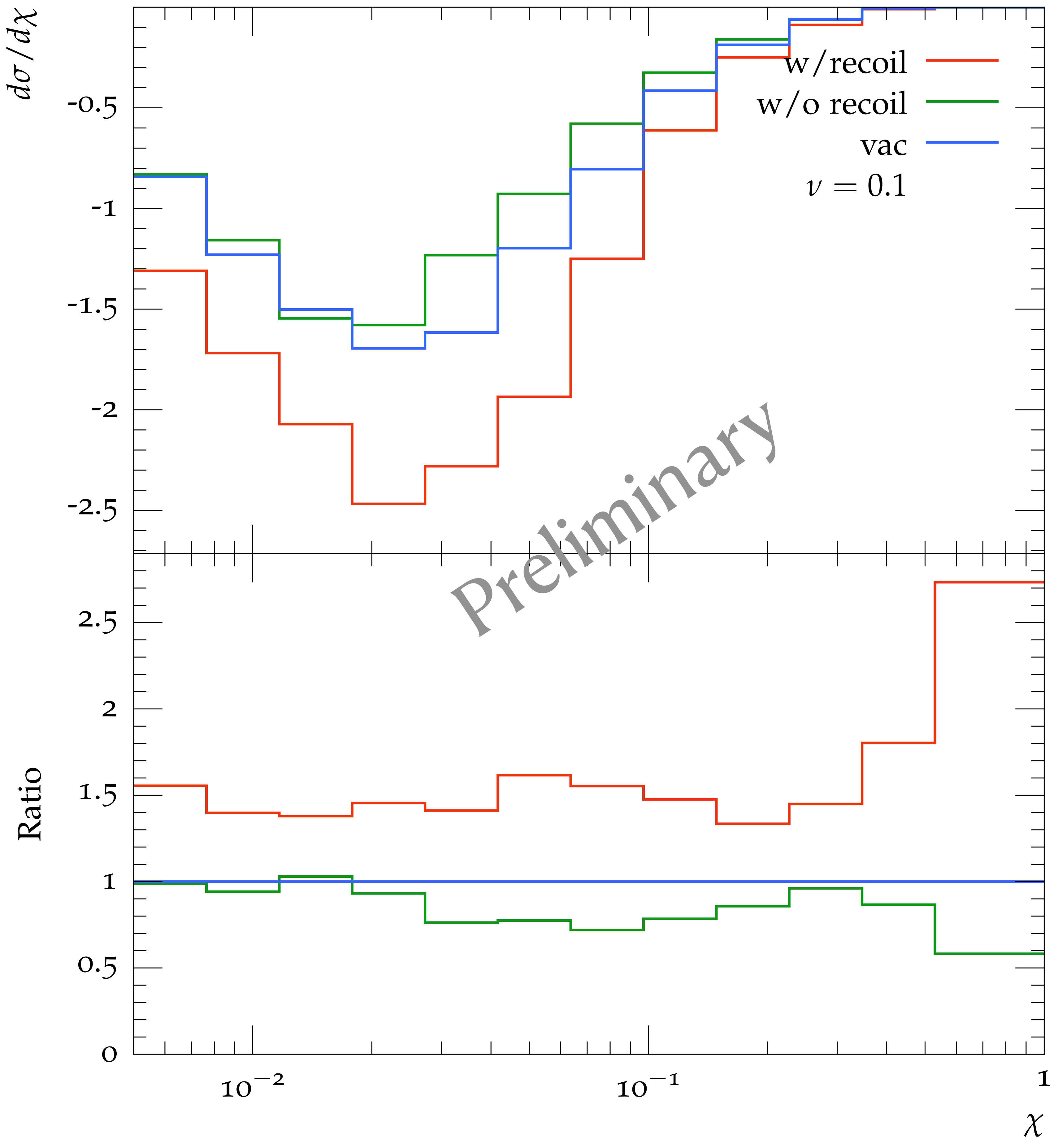


Small values of ν seems to be better for large angle MIE

JEWEL Simulations

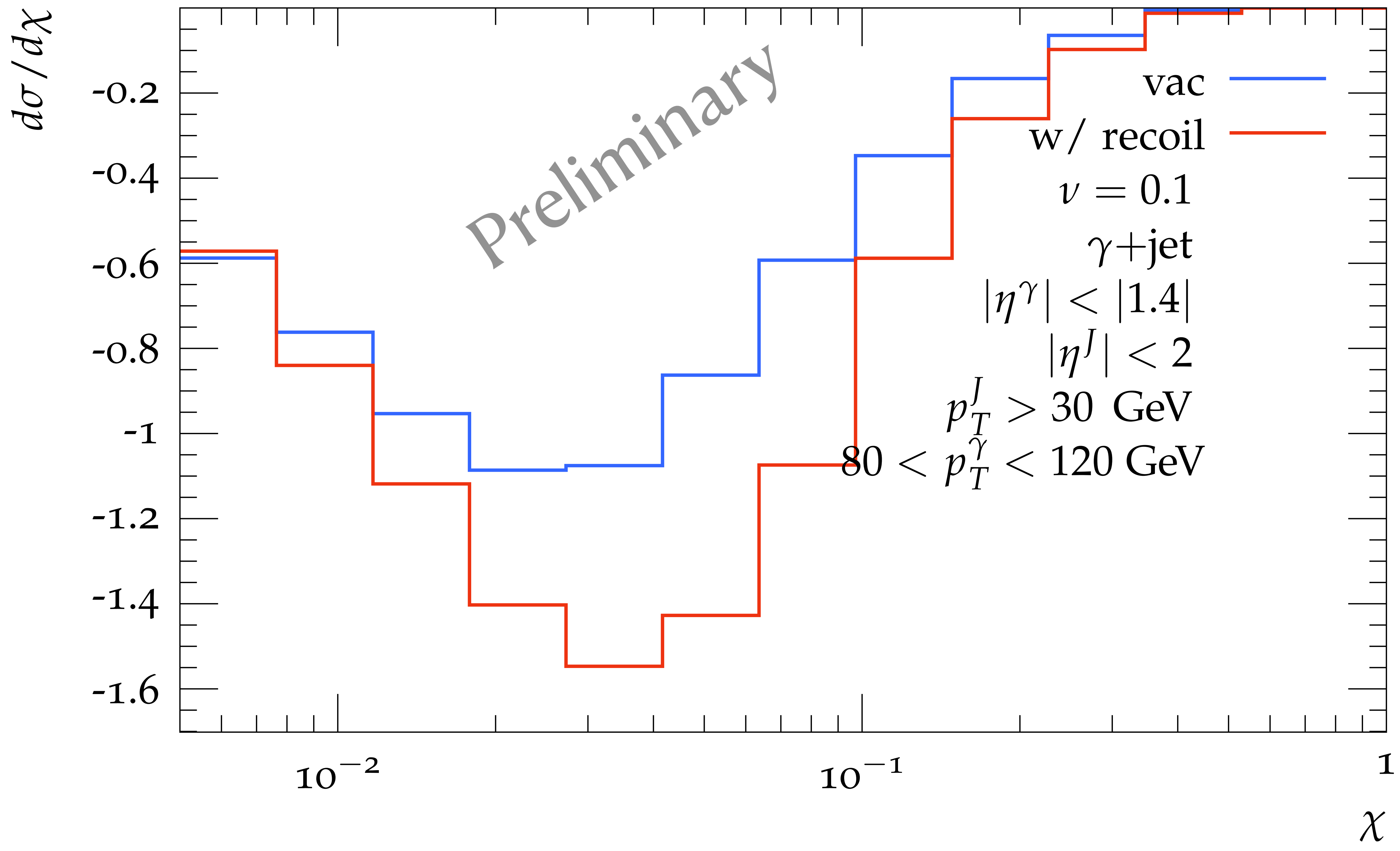
- Medium induced distribution is enhanced in all ranges of angular separations

Distribution inclusive



$100 < p_T < 120$ GeV $|\eta| < 1.9$

γ+ jet distribution



Small ν-values seems to be better for recoil

R Elyavalli, BS and V Vaidya, In preparation

Summary/conclusions

- EFT provides a systematically improvable framework for jet-medium interaction dynamics
- ν-correlators can be pivotal for separating the dynamics of large and small angle radiations.
- Smaller ν values are more sensitive to large angle scalings compared to $\nu > 1$
- General scaling of ν-correlators for large angle enhancement?
- Higher order jet function will be needed to fully understand the resummation effects on ν-correlators

Jet and Medium functions

- Soft/Medium function explicitly factors out

$$\mathbf{B}_{AB}(x,y) = \mathrm{Tr}\Big[\mathbf{T}\Big\{e^{-i\int dt_l H_{s,I}(t_l)}\Big(\frac{1}{\mathbb{P}_\perp^2}\mathcal{O}_s^A(x)\Big)\Big\}\rho_M(0)\bar{\mathbf{T}}\Big\{e^{-i\int dt_r H_{s,I}(t_r)}\Big(\frac{1}{\mathbb{P}_\perp^2}\mathcal{O}_s^B(y)\Big)\Big\}\Big]$$

SCET operators

$$\mathcal{O}_n^{qA} = \bar{\chi}_n T^A \frac{\bar{n}}{2}\chi_n$$

$$\mathcal{O}_s^{qA} = \bar{\chi}_s T^A \frac{n}{2}\chi_s$$

- sinc function leads to LPM terms

$$J_{qR}(\chi, k_\perp; L) = \frac{e^{-i\frac{L}{2}(\mathbb{P}_+^A - \mathbb{P}_+^B)}}{2N_c}\mathsf{sinc}\Big[\frac{L}{2}(\mathbb{P}_+^A - \mathbb{P}_+^B)\Big]\sum_X \mathrm{Tr}\Big[\frac{\bar{n}}{2}\bar{\mathbf{T}}\Big\{e^{-i\int dt H_n(t)}\Big[O_n^{qB}(0)\Big]\chi_n(0)\Big\}\mathcal{M}\,|X\rangle\langle X|$$
$$\mathbf{T}\Big\{e^{-i\int dt H_n(t)}\Big[O_n^{qB}(0)\Big]\Big[\bar{\chi}_n(0)\Big]\Big\}\Big] + \mathcal{O}(H_G^4)$$

- Sinc function leads to LPM terms

$$J_{qV}(\omega, \chi, k_\perp; L) = \frac{1}{2N_c}\frac{1}{2}e^{-i\frac{L}{2}(\mathbb{P}_+^A + \mathbb{P}_+^B)}\mathsf{sinc}\Big[\frac{L}{2}(\mathbb{P}_+^A + \mathbb{P}_+^B)\Big]\sum_X \mathrm{Tr}\Big[\frac{\bar{n}}{2}\langle 0|\,\bar{\mathbf{T}}\Big\{e^{-i\int dt H_{n,I}(t)}\chi_n(0)\Big\}\mathcal{M}\,|X\rangle\langle X|\,\mathbf{T}\Big\{e^{-i\int dt H_n(t)}\Big[O_n^A(0)\Big]$$
$$\times\Big[O_n^B(0)\Big]\Big[\bar{\chi}_n(0)\Big]\Big\}\,|0\rangle\Big]\delta^{AB} + \ c\,.\,c + \mathcal{O}(H_G^4)$$

Medium function

- Medium function can be obtained from spectral function which can computed in perturbatively and can also be evaluated on lattice

$$\mathbf{B}(k_\perp) = D^g_>(k) + D^q_>(k) \qquad D_>(k) = (1+f(k_0))\rho(k)$$

$D_>(k_\perp)$ is Weightman correlator in a thermal medium and depends on the properties of the medium

- In SCET framework spectral function is obtained from soft operators in the medium and also depends on the local properties of the plasma through soft operators

$$D_E^{AB}(K) = \int_0^\beta d\tau \int d^3x\, e^{iK\cdot X}\left\langle \frac{1}{\mathbb{P}_\perp^2} O_s^{g_n A}(X) \frac{1}{\mathbb{P}_\perp^2} O_s^{g_n B}(0)\right\rangle \propto \delta^{AB}\Big[\dots\Big]$$

SCET operators

$$\mathcal{O}_n^{qA} = \bar{\chi}_n T^A \frac{\bar{n}}{2}\chi_n$$

$$\mathcal{O}_s^{qA} = \bar{\chi}_s T^A \frac{n}{2}\chi_s$$

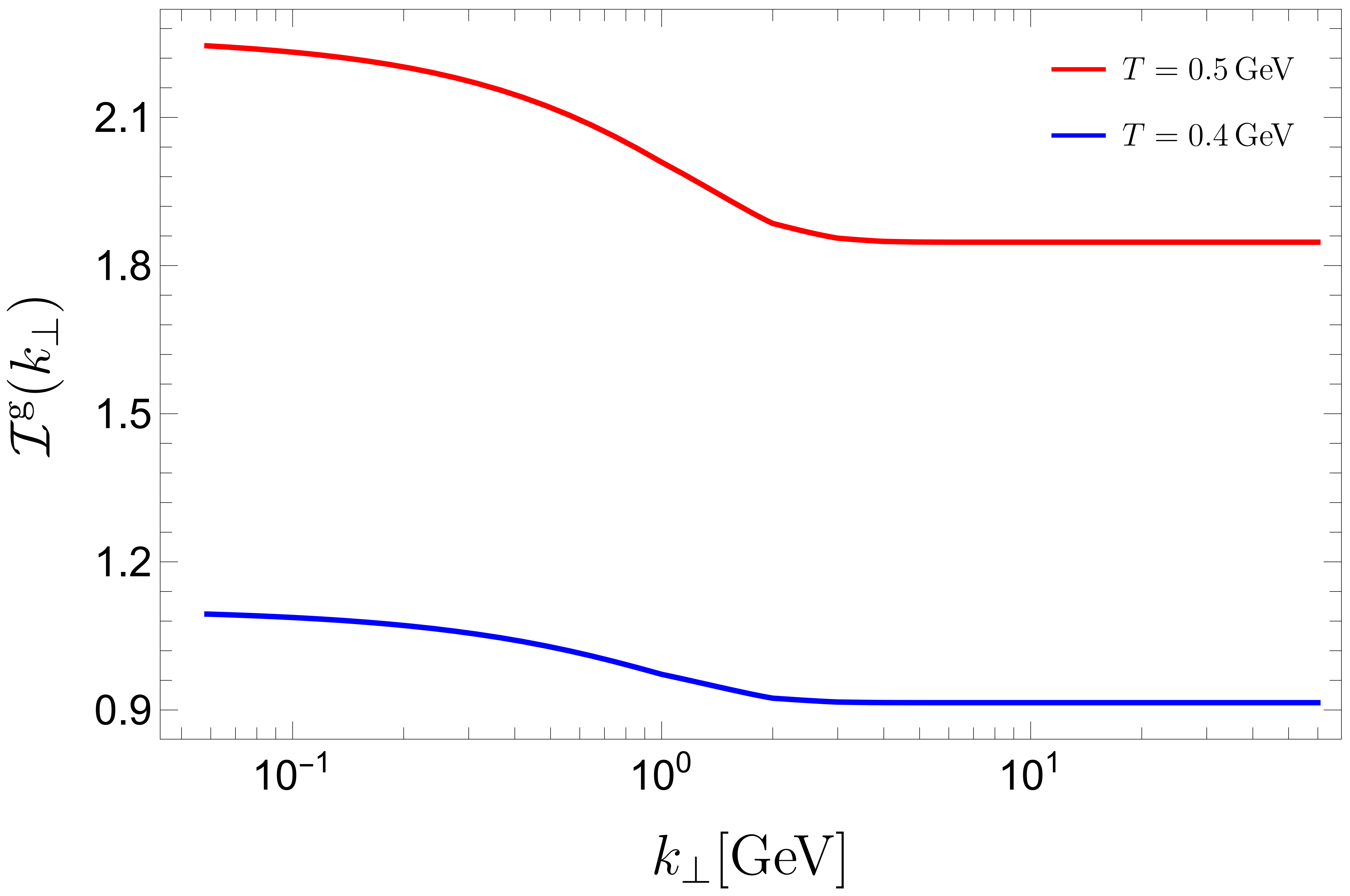
$$\mathcal{O}_s^{gA} = \frac{i}{2} f^{ACD} \mathcal{B}^C_{S\perp\mu} \frac{n}{2}\cdot(\mathcal{P}+\mathcal{P}^\dagger)\mathcal{B}^{D\mu}_{S\perp}$$

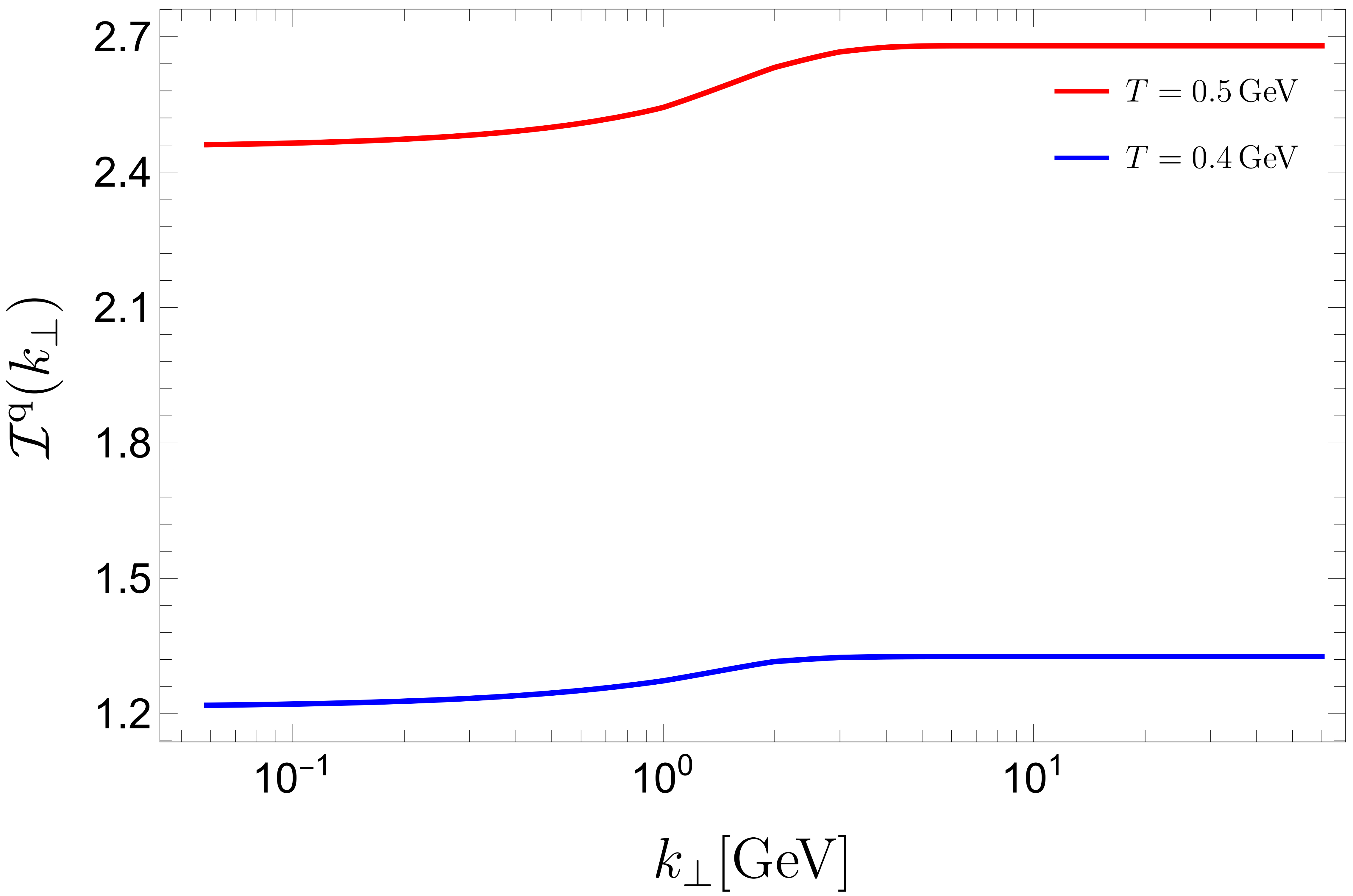
- Leading order medium function

$$\mathbf{B}_{\mathsf{LO}}(k_\perp) = (8\pi\alpha_s)^2\left(\frac{2\pi N_c^2}{16k_\perp^4}\mathcal{I}^g(k_\perp) + \frac{2\pi N_f}{k_\perp^4}\mathcal{I}^q(k_\perp)\right)$$

Medium function

- Bose enhance and Pauli blocking for quark and gluons
- At leading order medium function has somewhat weak dependence on Glauber momentum





BFKL evolution

- From RG consistency the jet function obeys BFKL evolution equation

$$\nu'\frac{d\mathbf{S}^{[\nu]}(k_\perp,\nu)}{d\nu'} = -\frac{\alpha_s(\mu)N_c}{\pi^2}\int d^2l_\perp\left[\frac{\mathbf{S}_1^{[\nu]}(l_\perp,\nu')}{(\vec{l}_\perp-\vec{k}_\perp)^2} - \frac{k_\perp^2\mathbf{S}_1^{[\nu]}(k_\perp,\nu')}{2l_\perp^2(\vec{l}_\perp-\vec{k}_\perp)^2}\right]$$

Scale for jet function

$$\nu_0 \sim \frac{Q_{\rm med}}{\sqrt{\chi}}$$

- Fixed order NLO jet function sets the boundary condition
- Running from jet scale to medium scale

Medium scale

$$\nu_f \sim Q_{\rm med}$$

$$\mathbf{S}_{\rm R}^{[\nu]}(k_\perp,\mu,\nu_f) = \int d^2l_\perp\mathbf{S}_1^{[\nu]}(l_\perp,\mu,\nu_0)\int\frac{d\xi}{2\pi}k_\perp^{-1+2i\xi}l_\perp^{-1-2i\xi}e^{in(\phi_k-\phi_l)}e^{-\frac{\alpha_s(\mu)N_c}{\pi}\chi(n,r)\log\frac{\nu_f}{\nu_0}}$$

- Resums $(a_p-1)\log(\nu_0/k_\perp)$

- Solution for $k_\perp \sim l_\perp$

$$\mathbf{S}_R^{[\nu]}(\chi,k_\perp) = \frac{1}{\pi k_\perp}\sqrt{\frac{\pi}{14\zeta(3)\bar{\alpha}Y}}e^{(a_p-1)Y}\int d^2l_\perp\frac{\mathbf{S}^{[\nu]}(\chi,l_\perp)}{l_\perp}e^{-\frac{\log^2(k_\perp/l_\perp)}{14\zeta(3)\bar{\alpha}Y}}$$

Medium induced jet function

- Real contribution with Glauber insertions

$$J_{q,o}(\chi, k_\perp; L) = \frac{1}{2N_c}\sum_X \int d^4x\Theta(L - x^-)\int d^4y\Theta(L - y^-)\mathrm{Tr}\Big[e^{iH_{n+s}t}\bar{\mathbf{T}}\Big\{H_{G,I}(x)\chi_{n,I}(0)\Big\}\rho_M(0)\frac{\bar{n}}{2}\mathbf{T}\Big\{H_{G,I}(y)\delta^2(\mathbb{P}_\perp)\delta(\omega - \bar{n}\cdot\mathscr{P})\bar{\chi}_{n,I}(0)\Big\}e^{-iH_{n+s}t}\mathscr{M}\Big] + \mathcal{O}(H_G^4)$$

- Medium and jet interaction constraint to medium length L

$$H_G(x) = \sum_{ij} C_{ij}O_{ns}^{ij}(x)$$

- $\mathbb{P}_\perp^2$ pulls out Glauber momentum

$$\mathcal{O}_{ns}^{qg} = O_n^{qA}\frac{1}{\mathbb{P}_\perp^2}O_s^{gA} \qquad \mathcal{O}_{ns}^{qq} = O_n^{qA}\frac{1}{\mathbb{P}_\perp^2}O_s^{qA}$$

- Order by order factorization for jet and medium functions

$\mathbf{B}(k_\perp, \mu, \nu) \to$ medium function

$$J_{q,o}(\omega, \chi; L) = L\int \frac{d^2k_\perp}{(2\pi)^2}J_{q,R}(\omega, \chi, k_\perp, \mu, \nu; L) \otimes \mathbf{B}(k_\perp, \mu, \nu)$$

$J_{q,R}(\omega, \chi, k_\perp, \mu, \nu; L) \to$ single scattering jet function

Medium induced jet function

- Real contribution with Glauber insertions

$$J_{q,o}(\chi, k_\perp; L) = \frac{1}{2N_c}\sum_X \int d^4x\,\Theta(L - x^-)\int d^4y\,\Theta(L - y^-)\mathrm{Tr}\Big[e^{iH_{n+s}t}\bar{\mathbf{T}}\Big\{H_{G,I}(x)\chi_{n,I}(0)\Big\}\rho_M(0)\frac{\bar{n}}{2}\mathbf{T}\Big\{H_{G,I}(y)\delta^2(\mathbb{P}_\perp)\delta(\omega - \bar{n}\cdot\mathscr{P})\bar{\chi}_{n,I}(0)\Big\}e^{-iH_{n+s}t}\mathscr{M}\Big] + \mathcal{O}(H_G^4)$$

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$$H_G(x) = \sum_{ij} C_{ij} O_{ns}^{ij}(x)$$

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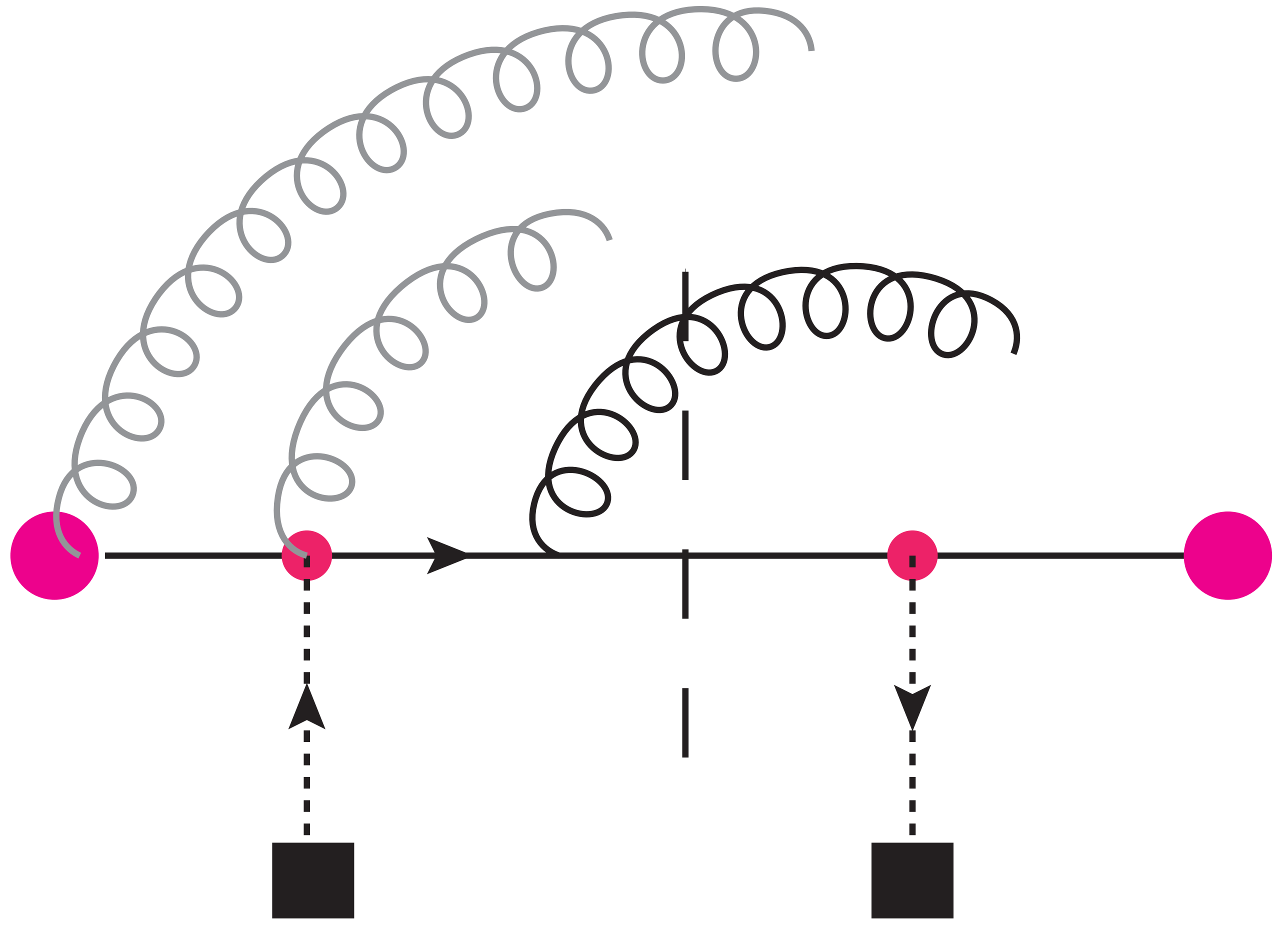
$$\mathcal{O}_{ns}^{qg} = O_n^{qA}\frac{1}{\mathbb{P}_\perp^2}O_s^{gA} \qquad \mathcal{O}_{ns}^{qq} = O_n^{qA}\frac{1}{\mathbb{P}_\perp^2}O_s^{qA}$$



- Order by order factorization for jet and medium functions

$\mathbf{B}(k_\perp, \mu, \nu) \to$ medium function

$$J_{q,o}(\omega, \chi; L) = L\int \frac{d^2k_\perp}{(2\pi)^2} J_{q,R}(\omega, \chi, k_\perp, \mu, \nu; L) \otimes \mathbf{B}(k_\perp, \mu, \nu)$$

$J_{q,R}(\omega, \chi, k_\perp, \mu, \nu; L) \to$ single scattering jet function

Medium induced jet function

- sinc function leads to LPM terms

$$J_{q,R}(\chi, k_\perp; L) = \frac{e^{-i\frac{L}{2}(\mathbb{P}_+^A - \mathbb{P}_+^B)}}{2N_c} \mathrm{sinc}\left[\frac{L}{2}(\mathbb{P}_+^A - \mathbb{P}_+^B)\right] \sum_X \mathrm{Tr}\left[\frac{\bar{n}}{2}\bar{\mathbf{T}}\left\{ e^{-i\int dt H_n(t)}\left[\delta(\mathscr{P}^-)\delta^2(\mathbb{P}_\perp - k_\perp)O_n^{qB}(0)\right]\chi_n(0)\right\}\mathscr{M}\,|X\rangle\langle X|\right.$$
$$\left.\mathbf{T}\left\{ e^{-i\int dt H_n(t)}\left[\delta(\mathscr{P}^-)\delta^2(\mathbb{P}_\perp + k_\perp)O_n^{qB}(0)\right]\left[\delta(\omega - \bar{n}\cdot\mathscr{P})\delta^2(\mathbb{P}^\perp)\bar{\chi}_n(0)\right]\right\}\right] + \mathscr{O}(H_G^4)$$

- Soft/Medium function explicitly factors out

$$\mathbf{B}_{AB}(x,y) = \mathrm{Tr}\left[\mathbf{T}\left\{ e^{-i\int dt_l H_{s,I}(t_l)}\left(\frac{1}{\mathbb{P}_\perp^2}\mathscr{O}_s^A(x)\right)\right\}\rho_M(0)\bar{\mathbf{T}}\left\{ e^{-i\int dt_r H_{s,I}(t_r)}\left(\frac{1}{\mathbb{P}_\perp^2}\mathscr{O}_s^B(y)\right)\right\}\right]$$

- $J_{q,R}(\omega, \chi, k_\perp; L)$ now depends on medium parameters

- $\mathbf{B}(x,y)$ does not depend on measurement

- $\mathbf{B}(x,y)$ depends only on medium parameters

SCET operators

$$\mathscr{O}_n^{qA} = \bar{\chi}_n T^A \frac{\bar{n}}{2}\chi_n$$

$$\mathscr{O}_s^{qA} = \bar{\chi}_s T^A \frac{n}{2}\chi_s$$

$$\mathscr{O}_s^{gA} = \frac{i}{2} f^{ACD}\mathscr{B}_{S\perp\mu}^C \frac{n}{2}\cdot(\mathscr{P} + \mathscr{P}^\dagger)\mathscr{B}_{S\perp}^{D\mu}$$

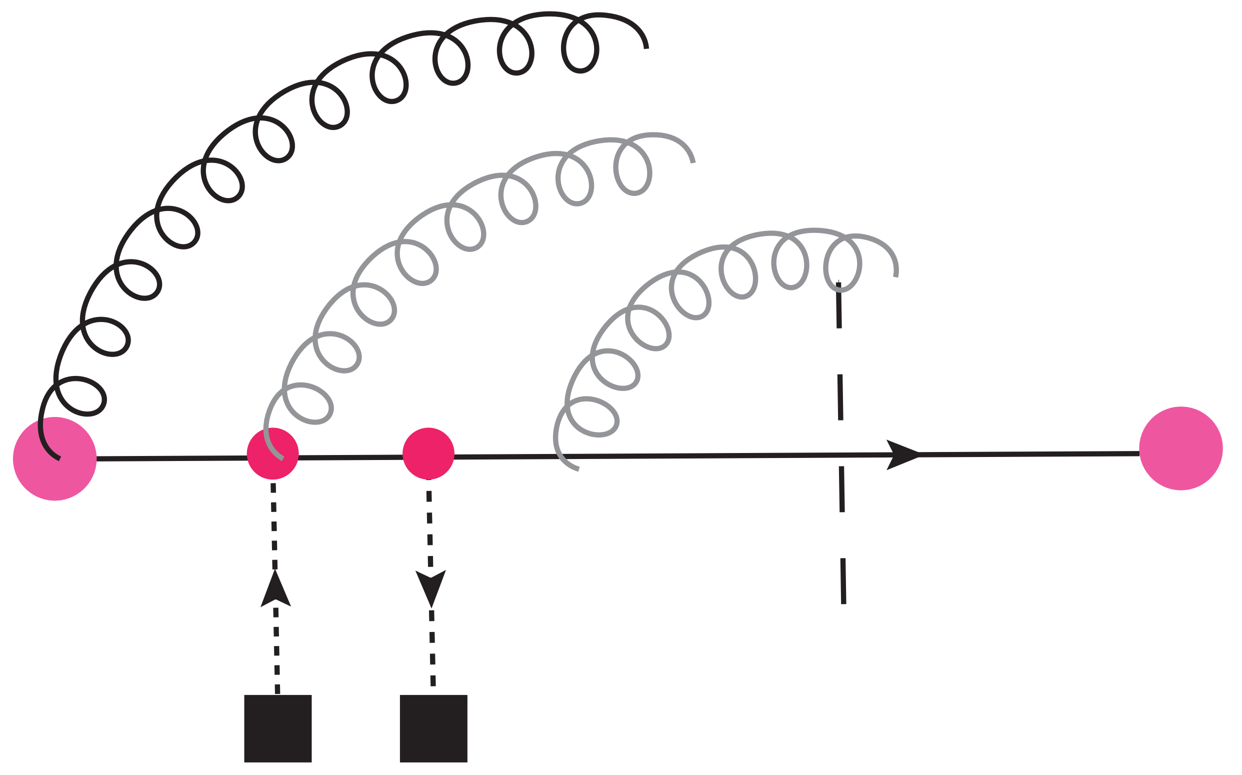
Medium induced jet function

- $\mathcal{O}(H_G^2)$ expansion at the same side

$$J_{q,s}(\chi, k_\perp; L) = \frac{1}{2N_c}\int d^4x\,\Theta(x^- - L)\int d^4y\,\Theta(y^- - L)\sum_X \mathrm{Tr}\Big[\frac{\bar{n}}{2}\langle 0|\,\bar{\mathbf{T}}\Big\{e^{-i\int dt H_n(t)}\chi_n(0)\mathscr{M}\,|X\rangle\langle X|$$
$$\mathbf{T}\Big\{e^{-i\int dt H_n(t)}\{H_{G,I}(x)H_{G,I}(y)\}\Big\}\Big[\delta(\omega - \bar{n}\cdot\mathscr{P})\delta^2(\mathbb{P}^\perp)\bar{\chi}_n(0)\Big]\Big\}\,|0\rangle\Big] + \text{c.c} + \mathcal{O}(H_G^4)$$

- Soft/Medium function explicitly factors out

$$J_{q,s}(\omega, \chi; L) = L\int\frac{d^2k_\perp}{(2\pi)^2}J_{q,V}(\omega, \chi, k_\perp, \mu, \nu; L)\otimes\mathbf{B}(k_\perp, \mu, \nu)$$



$+ \quad c.c$

- Sinc function leads to LPM terms

$$J_{q,V}(\omega, \chi, k_\perp; L) = \frac{1}{2N_c}\frac{1}{2}e^{-i\frac{L}{2}(\mathbb{P}_+^A + \mathbb{P}_+^B)}\mathrm{sinc}\Big[\frac{L}{2}(\mathbb{P}_+^A + \mathbb{P}_+^B)\Big]\sum_X \mathrm{Tr}\Big[\frac{\bar{n}}{2}\langle 0|\,\bar{\mathbf{T}}\Big\{e^{-i\int dt H_{n,I}(t)}\chi_n(0)\Big\}\mathscr{M}\,|X\rangle\langle X|\,\mathbf{T}\Big\{e^{-i\int dt H_n(t)}\Big[\delta^2(\overrightarrow{\mathbb{P}}_\perp + \vec{k}_\perp)\delta(\mathscr{P}^-)O_n^A(0)\Big]$$
$$\times\Big[\delta^2(\overrightarrow{\mathbb{P}}_\perp - \vec{k}_\perp)\delta(\mathscr{P}^-)O_n^B(0)\Big]\Big[\delta^2(\mathbb{P}_\perp)\delta(\omega - \bar{n}\cdot\mathscr{P})\bar{\chi}_n(0)\Big]\Big\}\,|0\rangle\Big]\delta^{AB} + c.c + \mathcal{O}(H_G^4)$$

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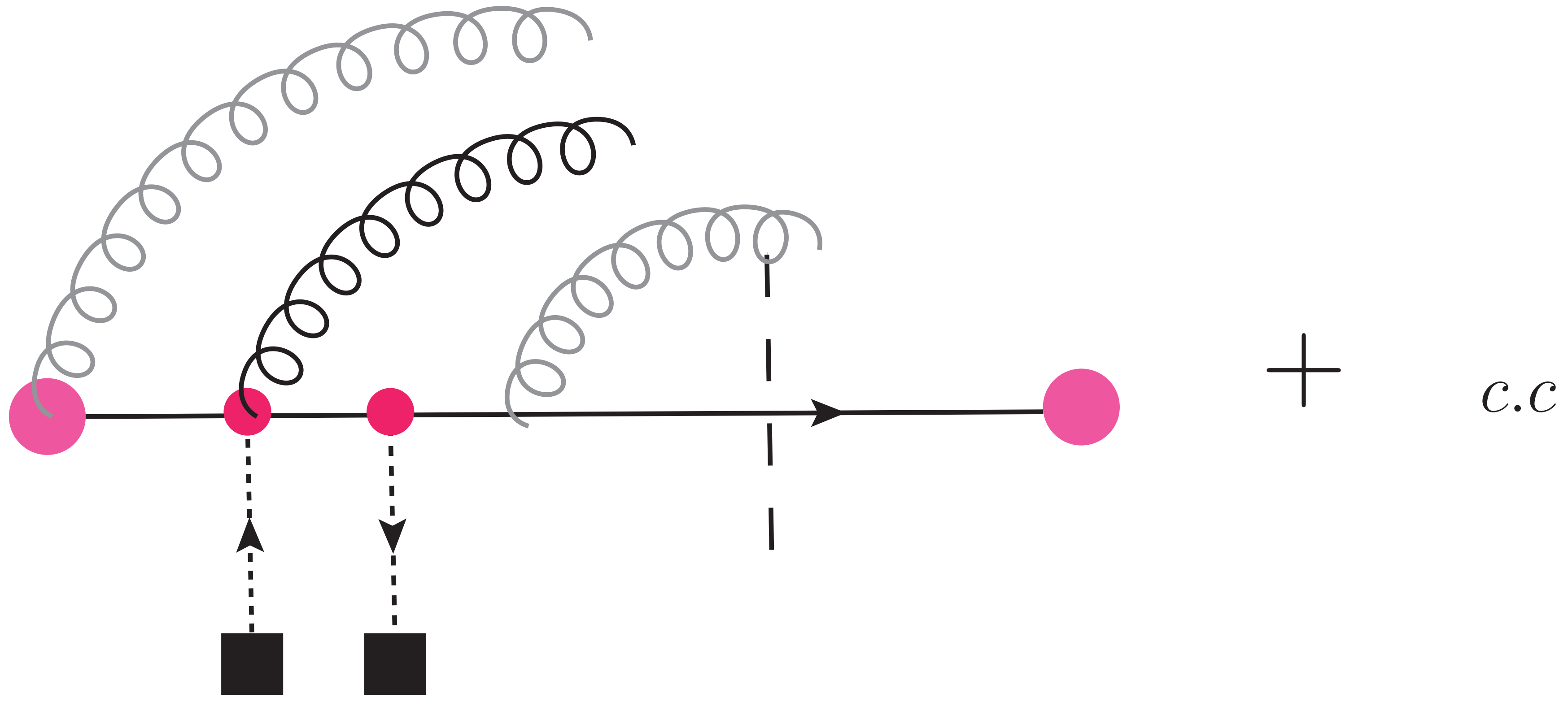

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$$J_{q,V}(\omega, \chi, k_\perp; L) = \frac{1}{2N_c} \frac{1}{2} e^{-i\frac{L}{2}(\mathbb{P}_+^A + \mathbb{P}_+^B)} \mathrm{sinc}\Big[\frac{L}{2}(\mathbb{P}_+^A + \mathbb{P}_+^B)\Big] \sum_X \mathrm{Tr}\Big[\frac{\bar{n}}{2} \langle 0 | \bar{\mathbf{T}} \Big\{ e^{-i\int dt H_{n,I}(t)} \chi_n(0) \Big\} \mathscr{M} | X \rangle \langle X | \mathbf{T}\Big\{ e^{-i\int dt H_n(t)} \Big[\delta^2(\overrightarrow{\mathbb{P}}_\perp + \vec{k}_\perp) \delta(\mathscr{P}^-) O_n^A(0) \Big] \\ \times \Big[\delta^2(\overrightarrow{\mathbb{P}}_\perp - \vec{k}_\perp) \delta(\mathscr{P}^-) O_n^B(0) \Big] \Big[\delta^2(\mathbb{P}_\perp) \delta(\omega - \bar{n}\cdot\mathscr{P}) \bar{\chi}_n(0) \Big] \Big\} |0\rangle \Big] \delta^{AB} + c.c + \mathcal{O}(H_G^4)$$

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$$\mathbf{T}\Big\{e^{-i\int dtH_n(t)}\{H_{G,I}(x)H_{G,I}(y)\}\Big\}\Big[\delta(\omega - \bar{n}\cdot\mathcal{P})\delta^2(\mathbb{P}^\perp)\bar{\chi}_n(0)\Big]\Big\}|0\rangle\Big] + \text{c.c} + \mathcal{O}(H_G^4)$$

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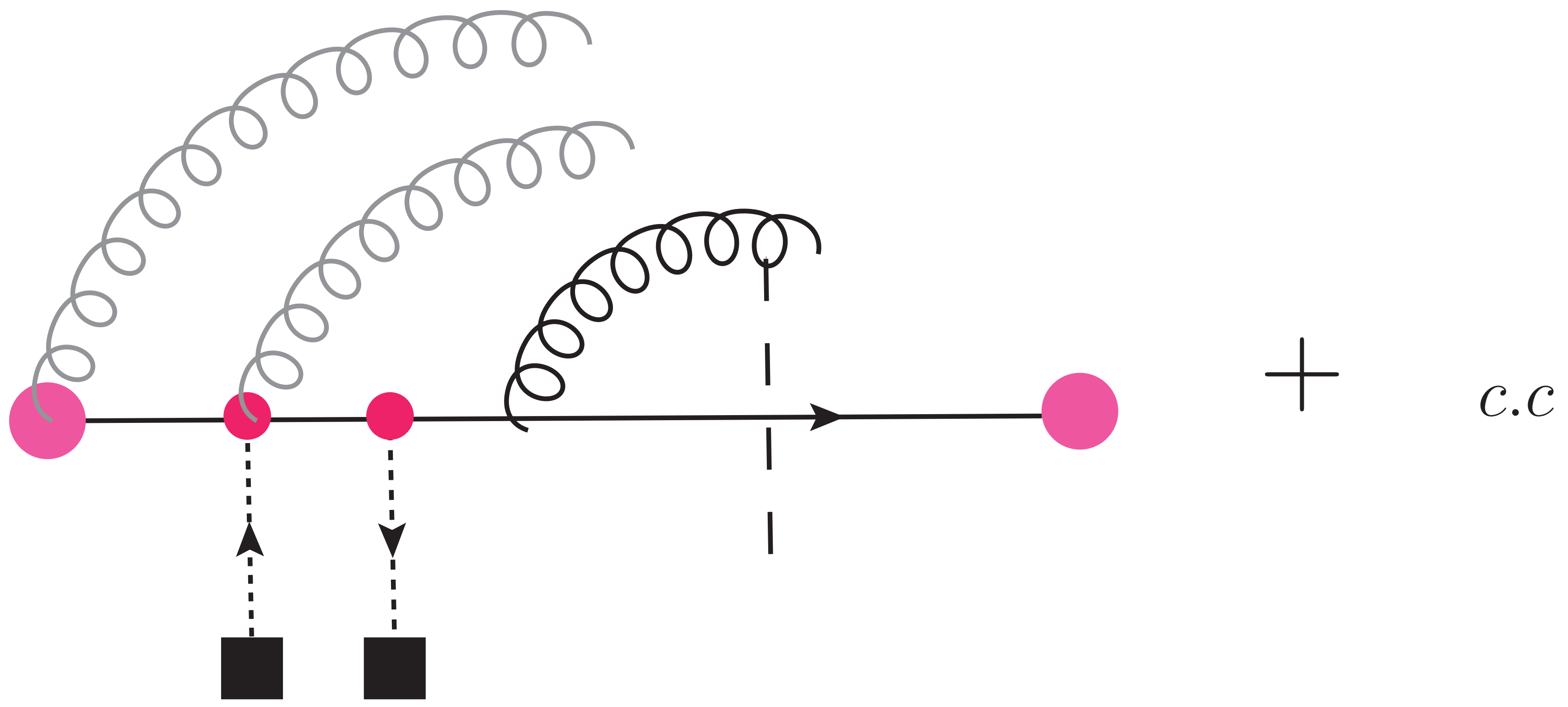

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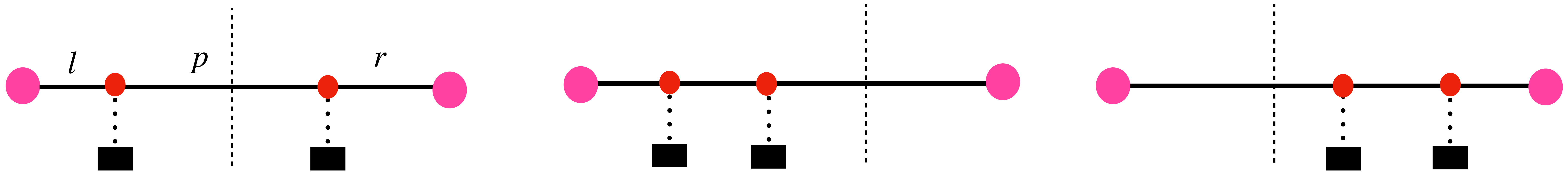
$$J_{q,V}(\omega, \chi, k_\perp; L) = \frac{1}{2N_c}\frac{1}{2}e^{-i\frac{L}{2}(\mathbb{P}_+^A + \mathbb{P}_+^B)}\mathrm{sinc}\Big[\frac{L}{2}(\mathbb{P}_+^A + \mathbb{P}_+^B)\Big]\sum_X \mathrm{Tr}\Big[\frac{\bar{n}}{2}\langle 0|\bar{\mathbf{T}}\Big\{e^{-i\int dtH_{n,I}(t)}\chi_n(0)\Big\}\mathcal{M}|X\rangle\langle X|\mathbf{T}\Big\{e^{-i\int dtH_n(t)}\Big[\delta^2(\overrightarrow{\mathbb{P}}_\perp + \vec{k}_\perp)\delta(\mathcal{P}^-)O_n^A(0)\Big]$$
$$\times\Big[\delta^2(\overrightarrow{\mathbb{P}}_\perp - \vec{k}_\perp)\delta(\mathcal{P}^-)O_n^B(0)\Big]\Big[\delta^2(\mathbb{P}_\perp)\delta(\omega - \bar{n}\cdot\mathcal{P})\bar{\chi}_n(0)\Big]\Big\}|0\rangle\Big]\delta^{AB} + c.c + \mathcal{O}(H_G^4)$$

Medium induced jet function

- At leading order jet parton gets kicks from the medium
- Both real and virtual contribution
- Measurement function is $\delta(\chi)$

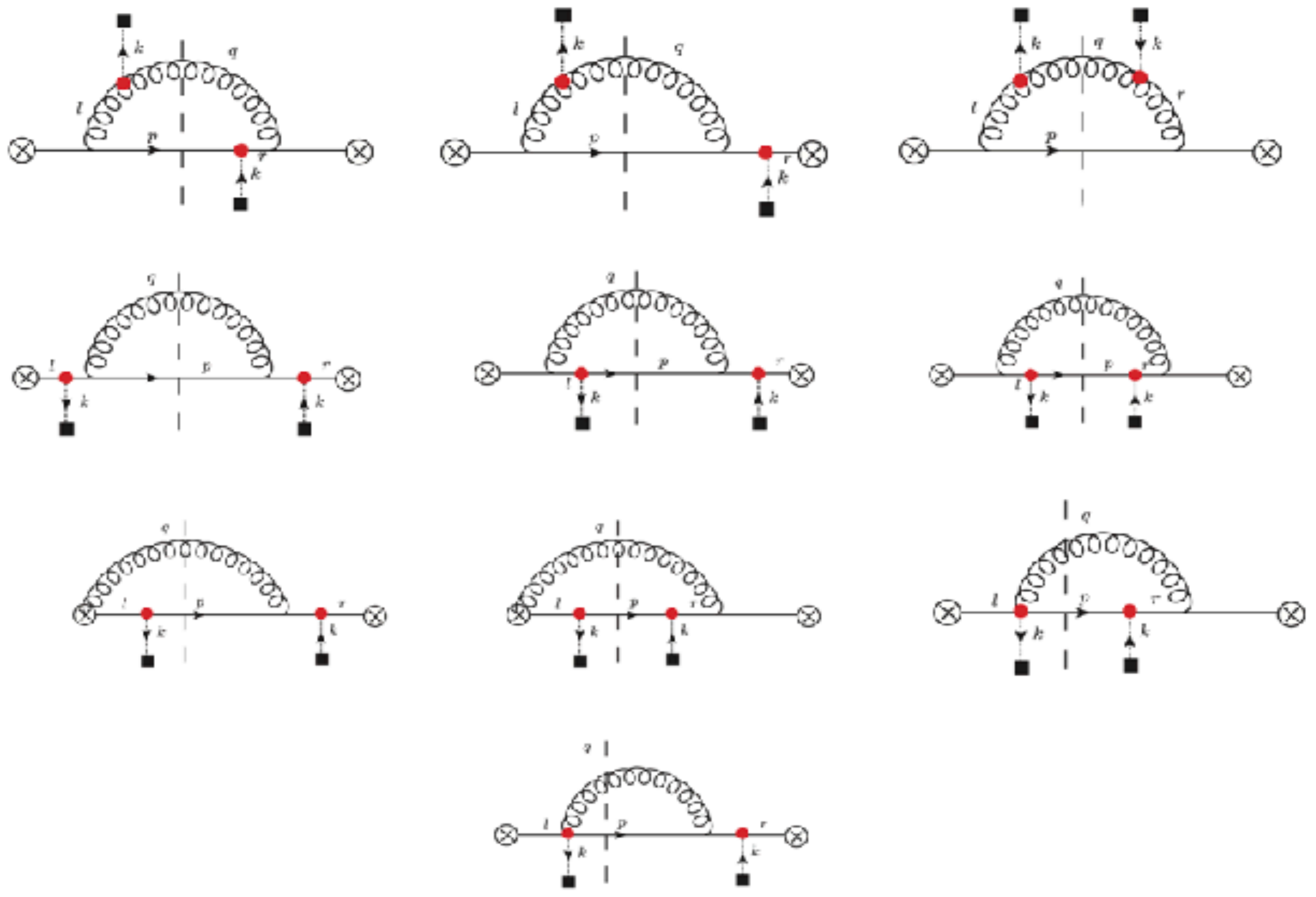


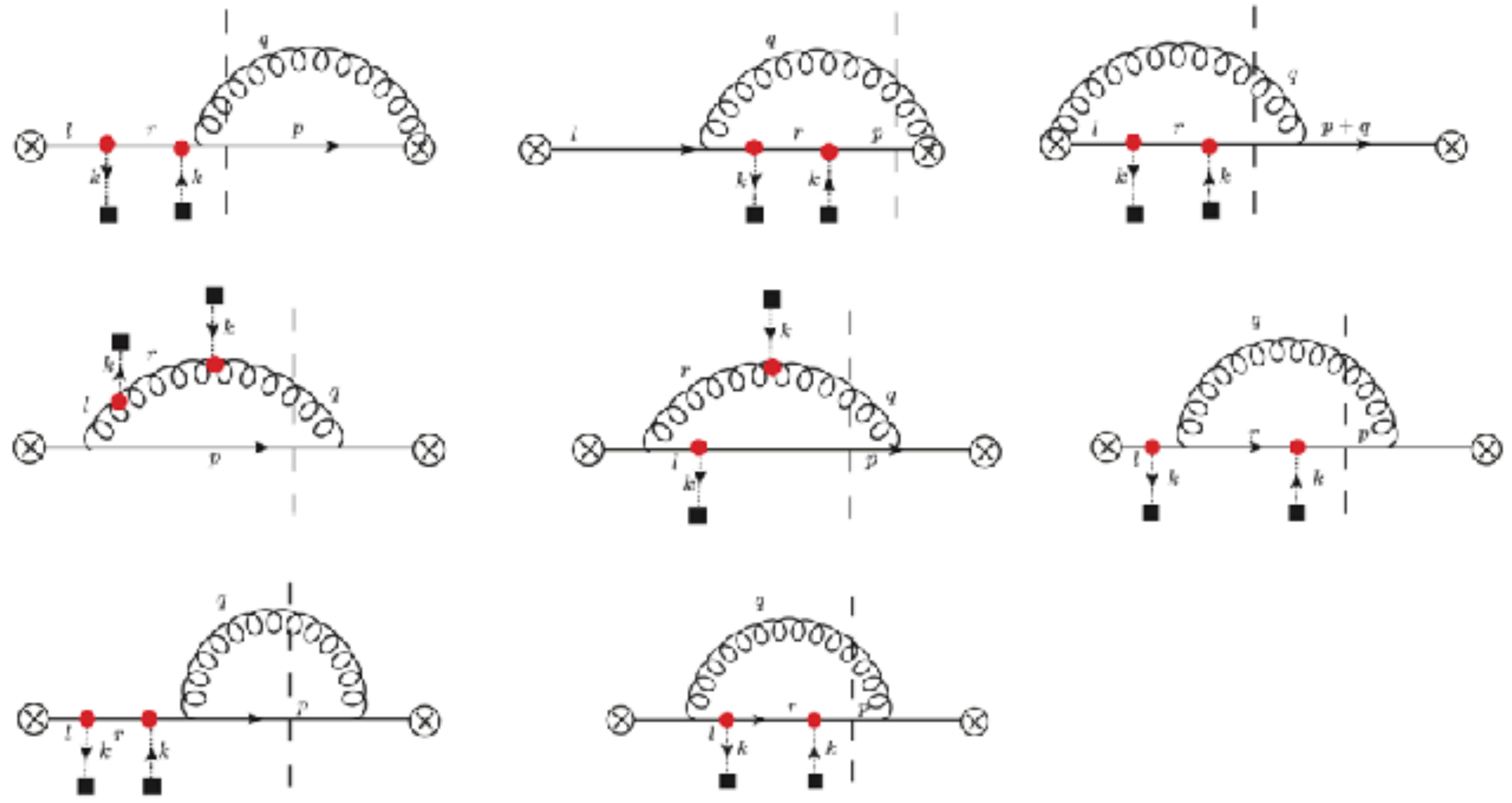
$$J_{R,\mathrm{LO}}(\omega,\chi) = -\,\delta_{AB}\int \frac{d^4p}{(2\pi)^4}\delta(p^2)\delta(p^- - \omega)\delta^2(\vec{p}_\perp - \vec{k}_\perp)\int \frac{dl^+}{2\pi}\int \frac{dr^+}{2\pi}\frac{\bar{n}\cdot l}{l^2 + i\epsilon}\frac{\bar{n}\cdot r}{r^2 - i\epsilon}\bar{n}\cdot p e^{-i(\frac{L}{2}(l^+ - r^+))}\mathrm{Sinc}\left[\frac{L}{2}(l_+ - r_+)\right]\delta(\chi)\,.$$

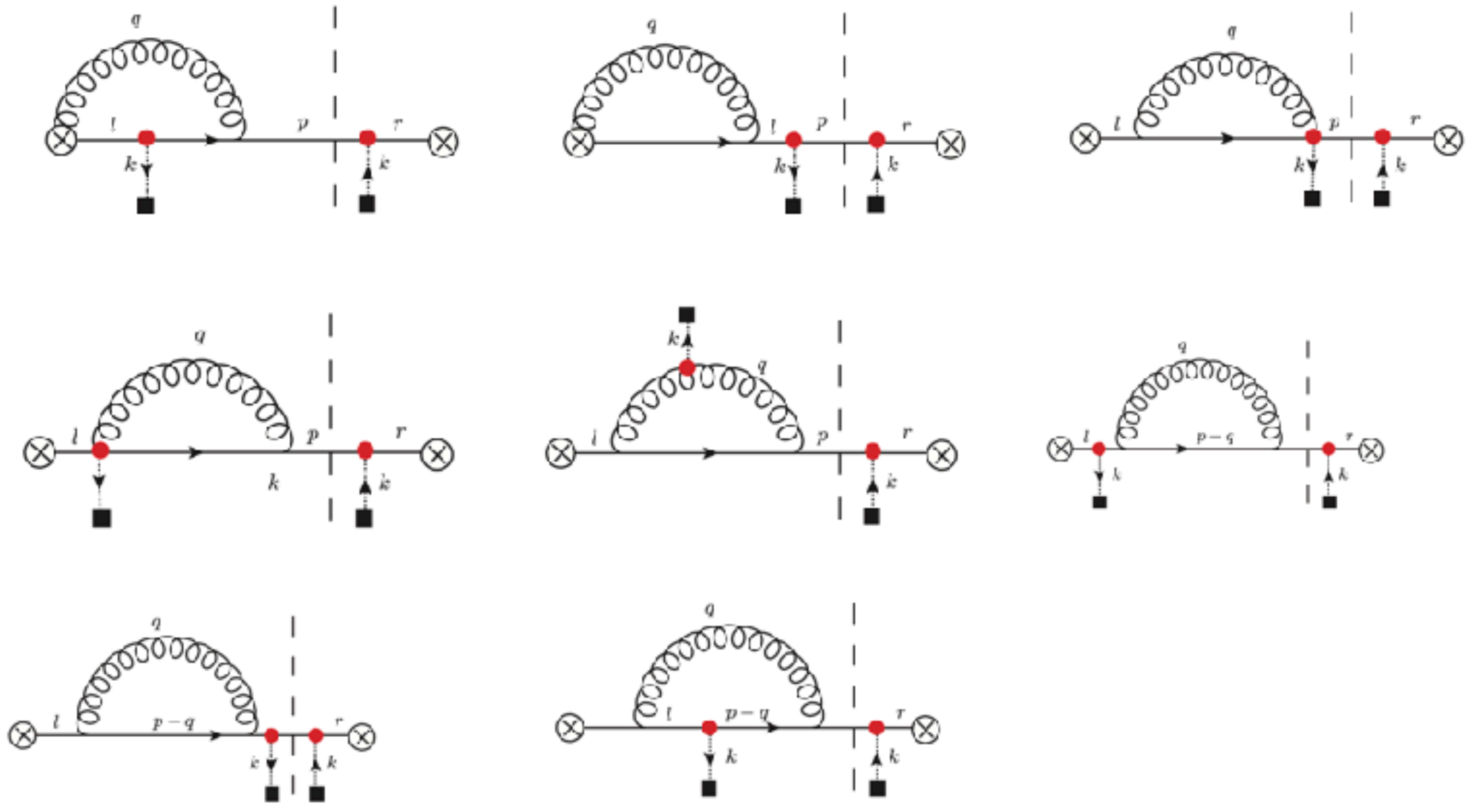
$$J_{R,\mathrm{LO}} = -\,4\delta_{AB}\delta(\chi) \qquad J_{V,\mathrm{LO}} = -\,4\delta_{AB}\delta(\chi)$$

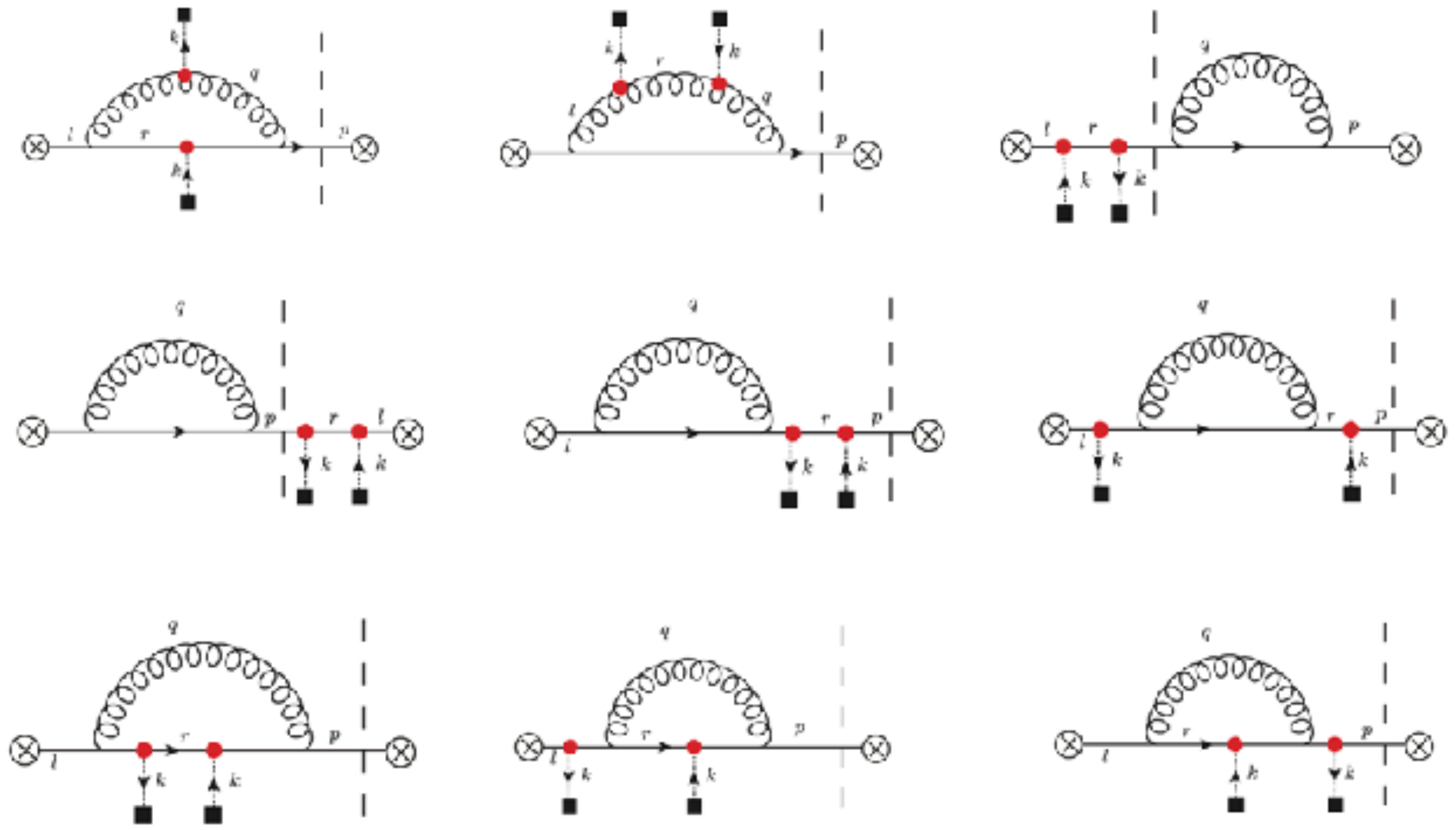
- Total contribution vanishes

Medium induced jet function









Full NLO quark jet function

- NLO jet function for quark

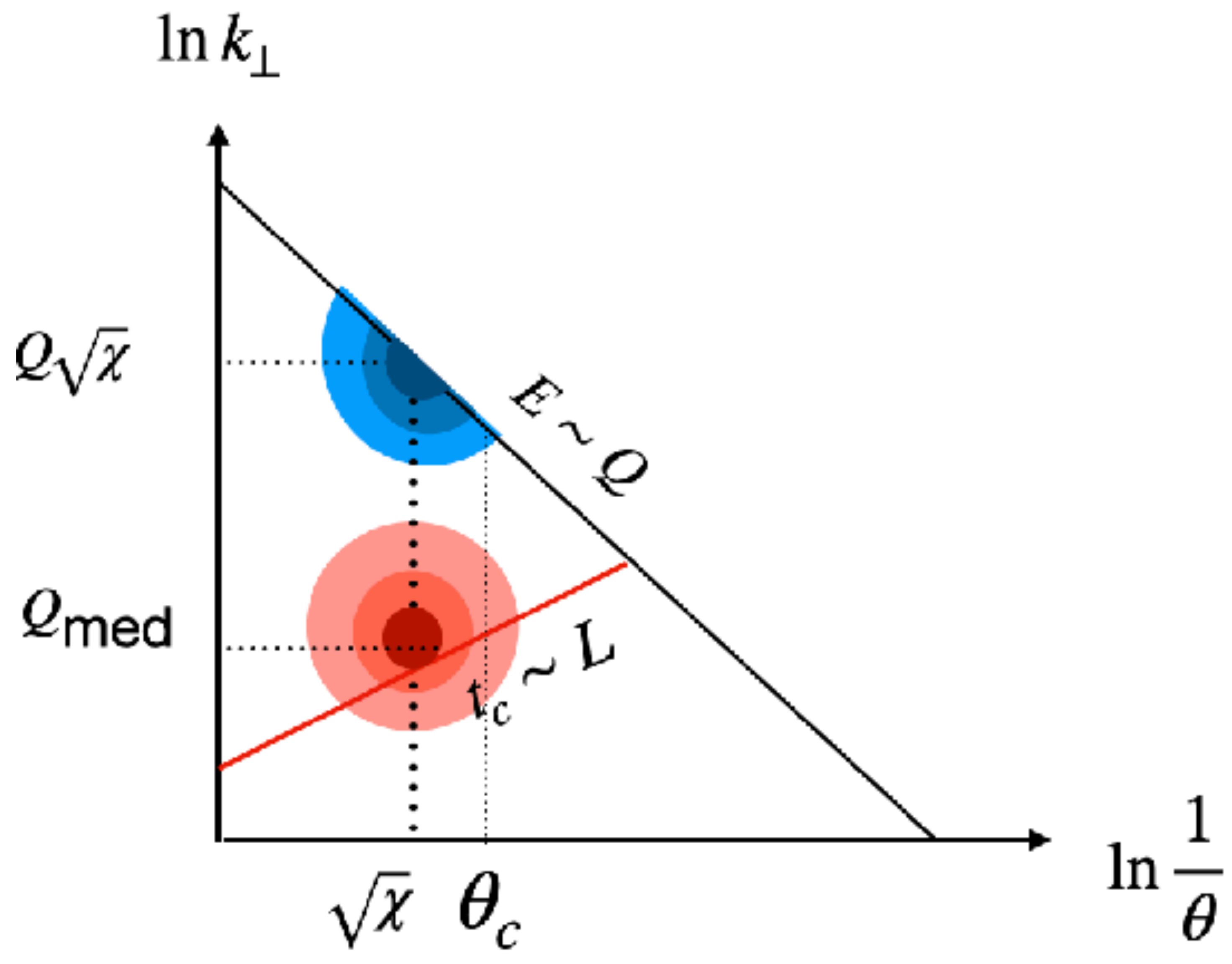
$$
\begin{aligned}
J_{1,\mathrm{NLO}}(\chi,\omega,k_\perp) = {}& \frac{\alpha_s C_F}{2\pi^2}\int dz z(1-z)\int d^2q_\perp \Bigg[\Bigg\{ -\frac{2N_c(\vec{q}_\perp\cdot\vec{\kappa}_\perp)}{q_\perp^2\kappa_\perp^2} f(z)\bigg(1-\frac{\sin\omega_1}{\omega_1}-\frac{\sin\omega_2}{\omega_2}+\frac{\sin(\omega_2-\omega_1)}{\omega_2-\omega_1}\bigg) - \frac{4N_c(1-z)^2}{\kappa_\perp^2 Q_\perp^2}\bigg(\frac{\vec{q}_\perp\cdot\vec{\kappa}_\perp}{z} \\
& + \frac{\kappa^2+\vec{q}_\perp\cdot\vec{\kappa}_\perp}{2(1-z)} + \frac{\vec{k}\cdot\vec{\kappa}_\perp}{2} + \frac{\kappa_\perp^2 z}{2(1-z)^2}\bigg)\bigg(1-\frac{\sin\omega_1}{\omega_1}\bigg) + \frac{4N_c f(z)}{\kappa_\perp^2}\bigg(1-\frac{\sin\omega_1}{\omega_1}\bigg) + \frac{4C_F z}{q_\perp^2}\bigg(1-\frac{\sin\omega_1}{\omega_1}\bigg) + \frac{2}{3}\frac{z(1-z)^2}{q_\perp^2 Q_\perp^2} \\
& \bigg(\frac{\vec{q}_\perp\cdot\vec{\kappa}_\perp}{(1-z)^2} + \frac{\vec{k}_\perp\cdot\vec{\kappa}_\perp}{1-z}\bigg)\bigg(1-\frac{\sin\omega_1}{\omega_1}\bigg) - 2C_F\frac{z(1-z)^2}{Q_\perp^2}\bigg(k_\perp^2+\frac{\kappa_\perp^2}{(1-z)^2}+\frac{\vec{k}_\perp\cdot\vec{\kappa}_\perp}{(1-z)}\bigg) + \frac{4C_F(1-z)}{q_\perp^2 z}\bigg(1-\frac{\sin\omega_1}{\omega_1}\bigg) \\
& -\frac{2}{3}\frac{(1-z)}{zQ_\perp^2}\frac{\sin\omega_1}{\omega_1} + \frac{2N_c(1-z)}{zq_\perp^2}\bigg(1-\frac{\sin\omega_1}{\omega_1}\bigg) + \frac{2N_c(1-z)}{zQ_\perp^2}\Bigg\}\bigg(\frac{(z(1-z)\omega)^2\delta(q_\perp-q_0)}{2|q_0-k_\perp z\cos\theta|} - \delta(\chi)\bigg) \\
& -\Bigg\{\frac{4C_F(1-z)}{q_\perp^2 z} - \frac{2N_c(\vec{q}_\perp\cdot\vec{\kappa}_\perp)}{q_\perp^2\kappa_\perp^2} f(z)\bigg(\frac{\sin(\omega_2-\omega_1)}{\omega_2-\omega_1}-\frac{\sin\omega_1}{\omega_1}\bigg) + \frac{2N_c}{q_\perp^2} f(z)\bigg(1-\frac{\sin\omega_1}{\omega_1}\bigg)\Bigg\} \\
& \bigg((z(1-z)\omega)^2\delta(q_\perp^2-[z(1-z)\omega]^2\chi) - \delta(\chi)\bigg)\Bigg]
\end{aligned}
$$

$$\omega_1 = \frac{L\kappa_\perp^2}{z(1-z)\omega} \qquad \omega_2 = \frac{Lq_\perp^2}{z(1-z)\omega} \qquad Q_\perp^2 = \omega(\kappa_\perp^2 q^- + q_\perp^2 p^-) - k_\perp^2 p^- q^- \qquad \vec{\kappa}_\perp = \vec{q}_\perp - \vec{k}_\perp$$

Refactorization

$$J_q = J_i^{(0)}(\omega,\chi,\mu) + \sum_{m=1}^{\infty}\sum_{j=1}^{m} \mathcal{J}^j_{i\to m}\underbrace{(\{\underline{m}\},\theta_c}_{\text{Matching function}},\omega,\mu) \otimes_\theta \ \mathcal{S}_{m,j}(\{\underline{m}\},\chi,\mu)$$

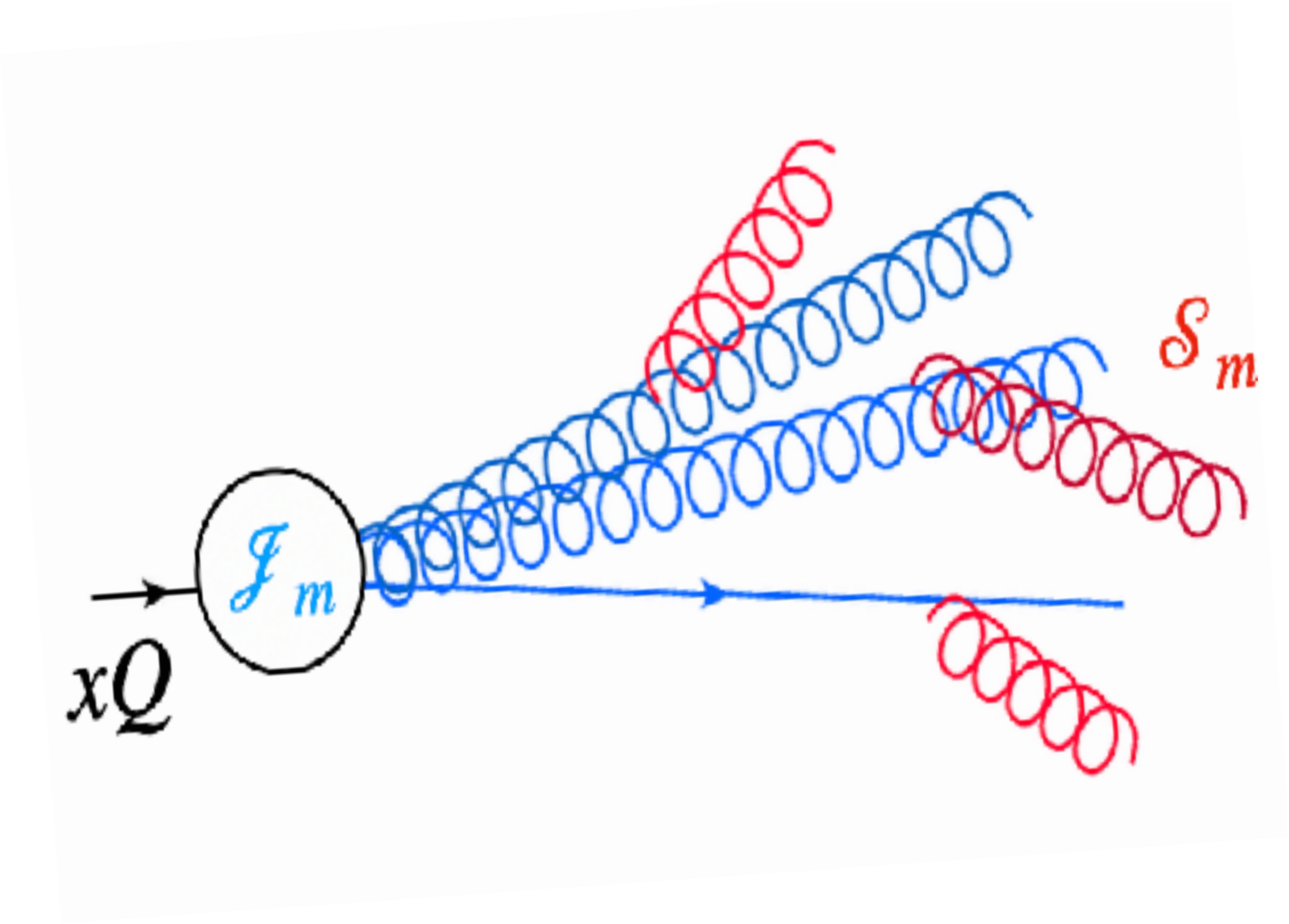
Collinear-soft function



- Matching function describes the production of m resolved hard partons from initial parton i

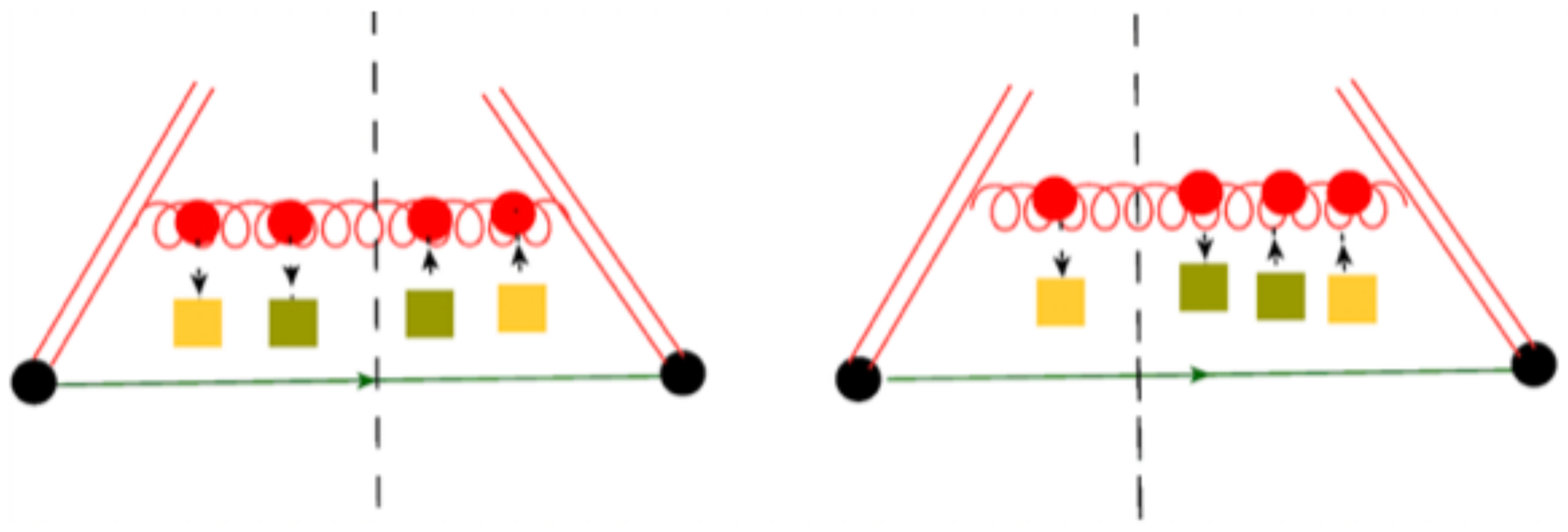
- Collinear soft function describes the production of medium induced radiation

- Perturbative matching coefficient $\mathcal{J}_{i\to m}$ starts at $\mathcal{O}(\alpha_s^{m-1})$

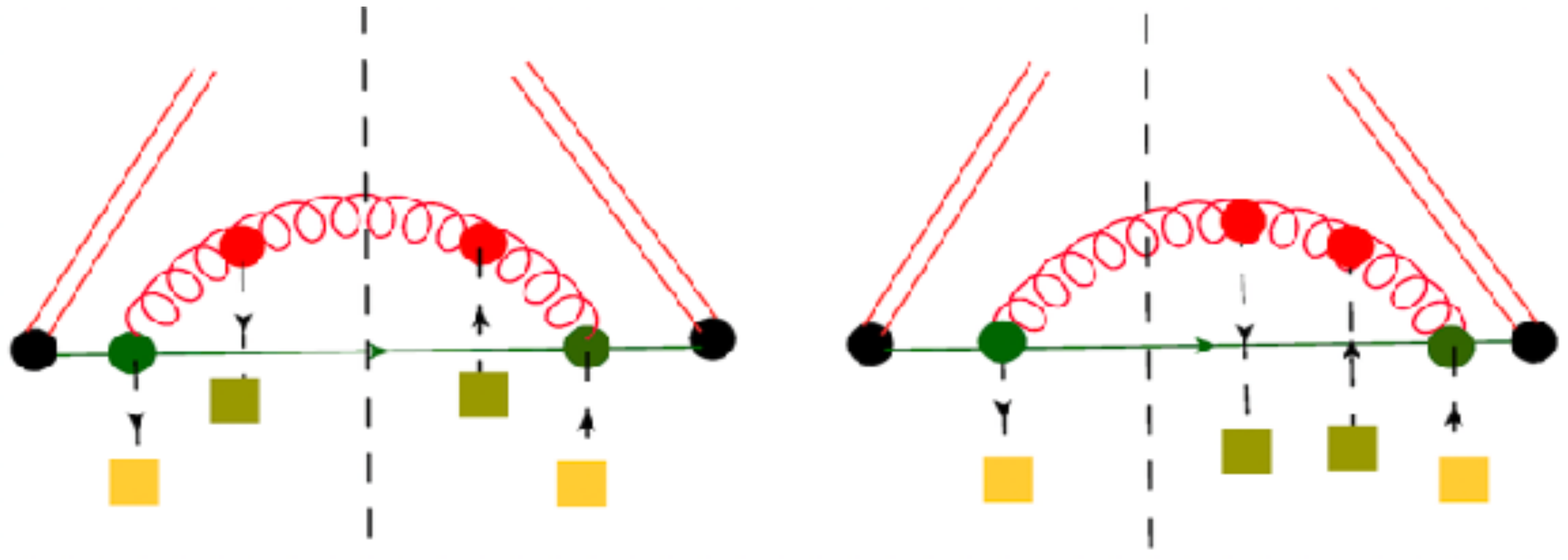


Multiple scatterings

Broadening of vacuum emission



Broadening of medium induced emission



Phase space constraint for the measurement is $q_{f\perp}/\omega \sim R$

- In the Markovian limit and $L \to \infty$ limit, i.e., $t_f \ll L$

$$\mathbf{S}_B^{(n)}(\epsilon_L, k_{1\perp}, k_{2\perp}, \ldots, k_{n\perp}) = \frac{\alpha_s(N_c^2-1)}{4\pi^2}\int \mathrm{d}^2q_\perp \int \frac{\mathrm{d}q^-}{q^-}\Theta\left(q_\perp - \frac{q^-R}{2}\right)\left[\delta(\epsilon_L)-\delta(q^- - \epsilon_L)\right]\int\frac{d^2p_\perp}{(2\pi)^2}\int\frac{d^2b\, e^{ip_\perp\cdot b}}{(p_\perp+q_\perp)^2}\prod_{i=1}^{n}\frac{\left(e^{-ik_{i\perp}\cdot b}-1\right)}{k_{i\perp}^2}$$

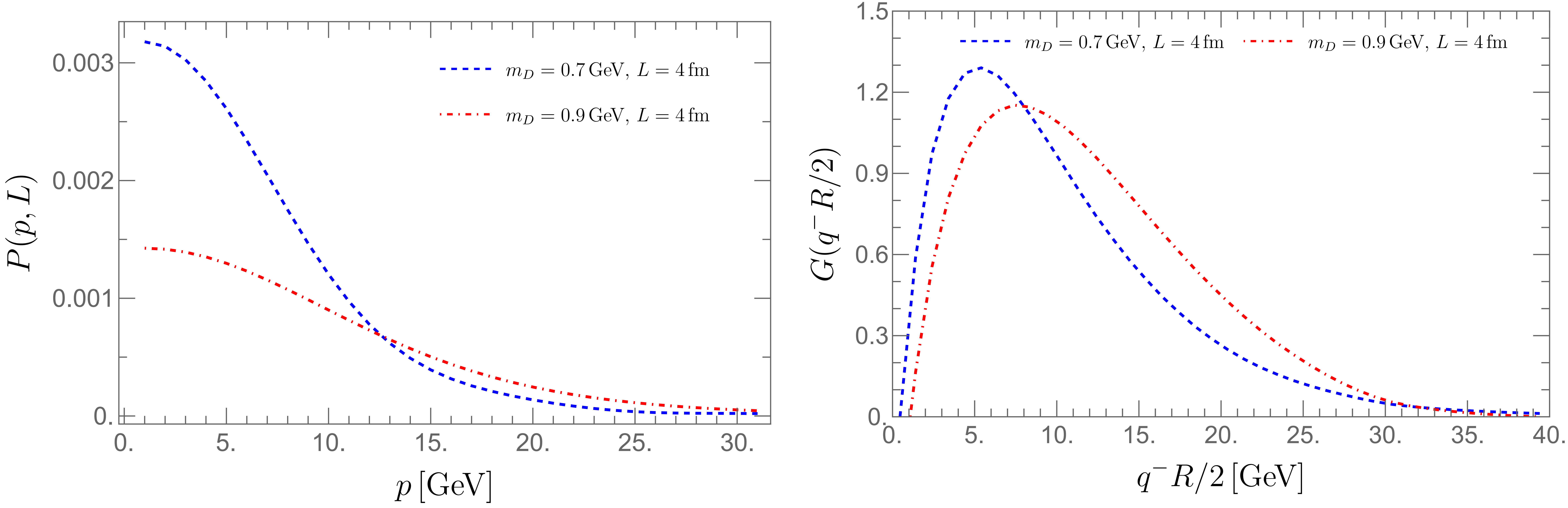
- The jet function can be written as a distribution in the impact parameter space

$$\mathcal{S}_{1,B}(\epsilon_L, \mu) \approx \frac{\alpha_s(N_c^2-1)}{4\pi^2}\int \frac{\mathrm{d}q^-}{q^-}\left[\delta(\epsilon_L)-\delta(q^- - \epsilon_L)\right]\int\frac{d^2p_\perp}{(2\pi)^2}\int\frac{\mathrm{d}^2q}{(q_\perp+p_\perp)^2}\Theta\left(q_\perp - \frac{q^-R}{2}\right)\left(P(p_\perp, L)-\delta^2(p_\perp)\right)$$

Q_{med} scale

Multiple scatterings lead to larger momentum transfer from medium to jet parton

B.S. and V.Vaidya, 2412.18967

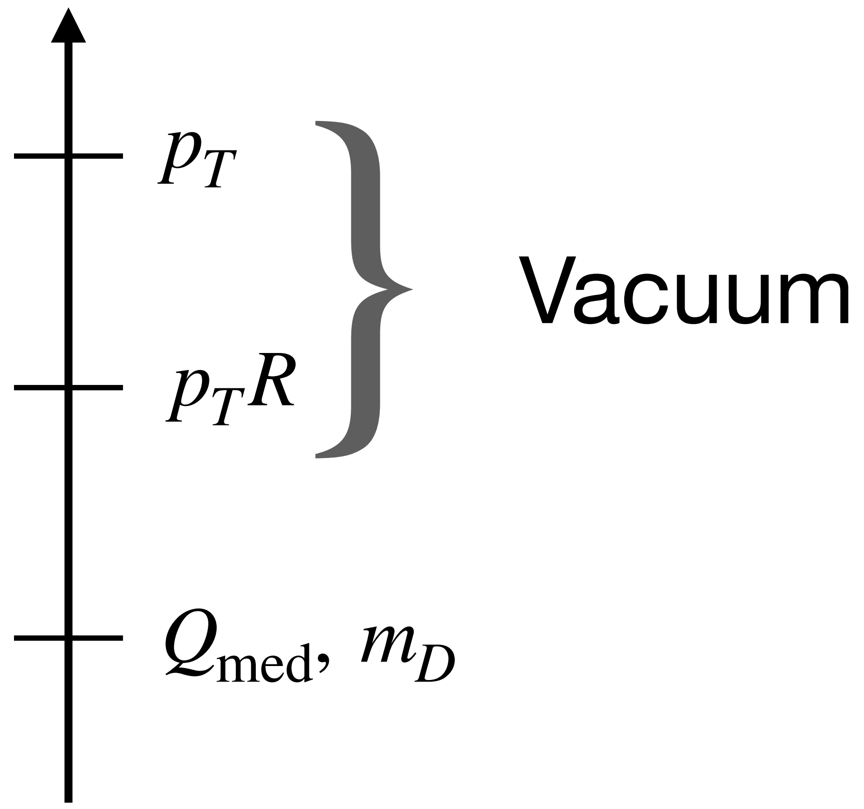


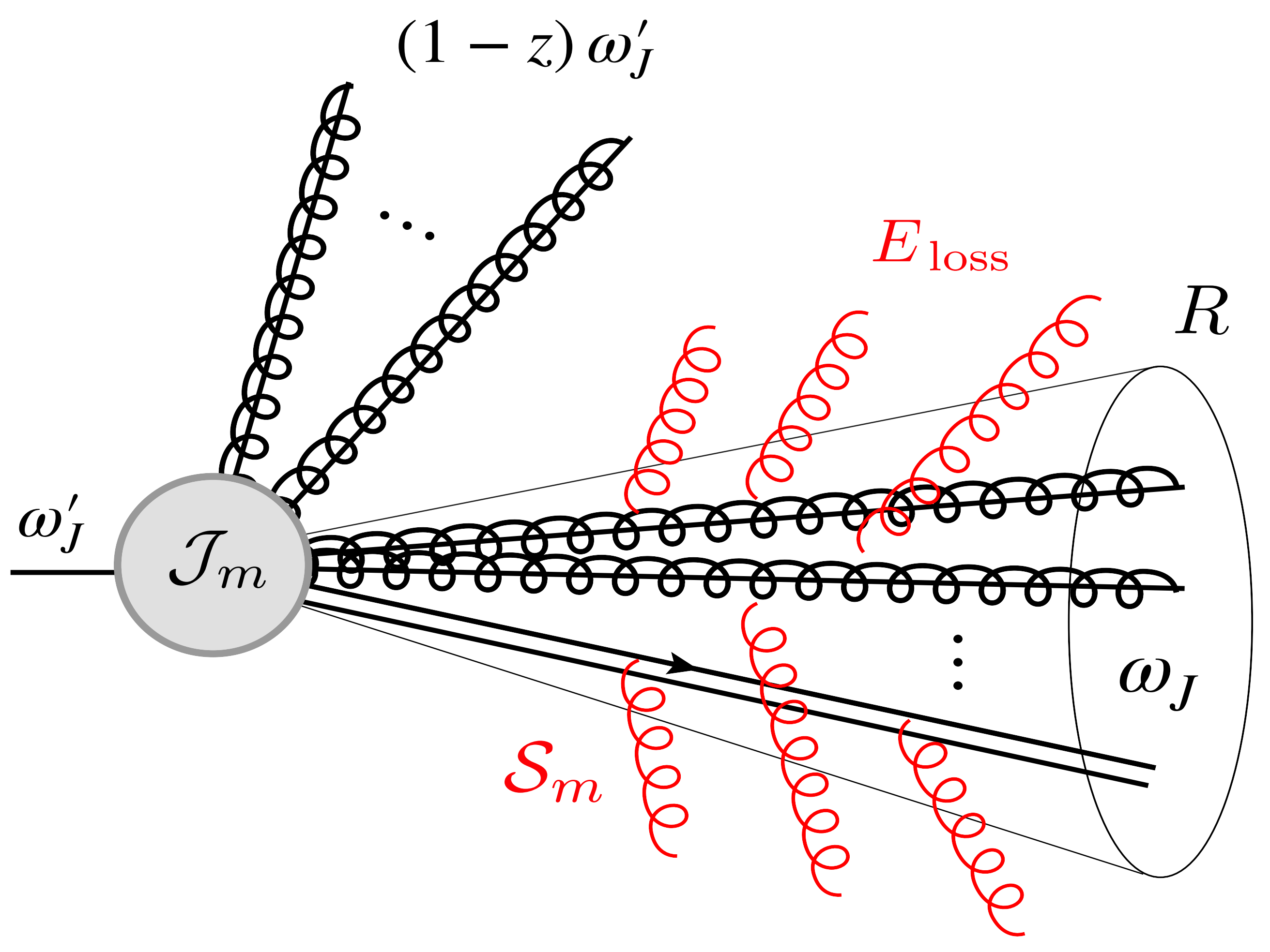
Peak in the distribution provides an estimate for the emergent scale Q_{med} through multiple scattering

- Exact value of the emergent scale depends on medium properties and parameters

Towards all order factorization

- For a dense medium multiple scatterings are important and can lead to non trivial evolution equations

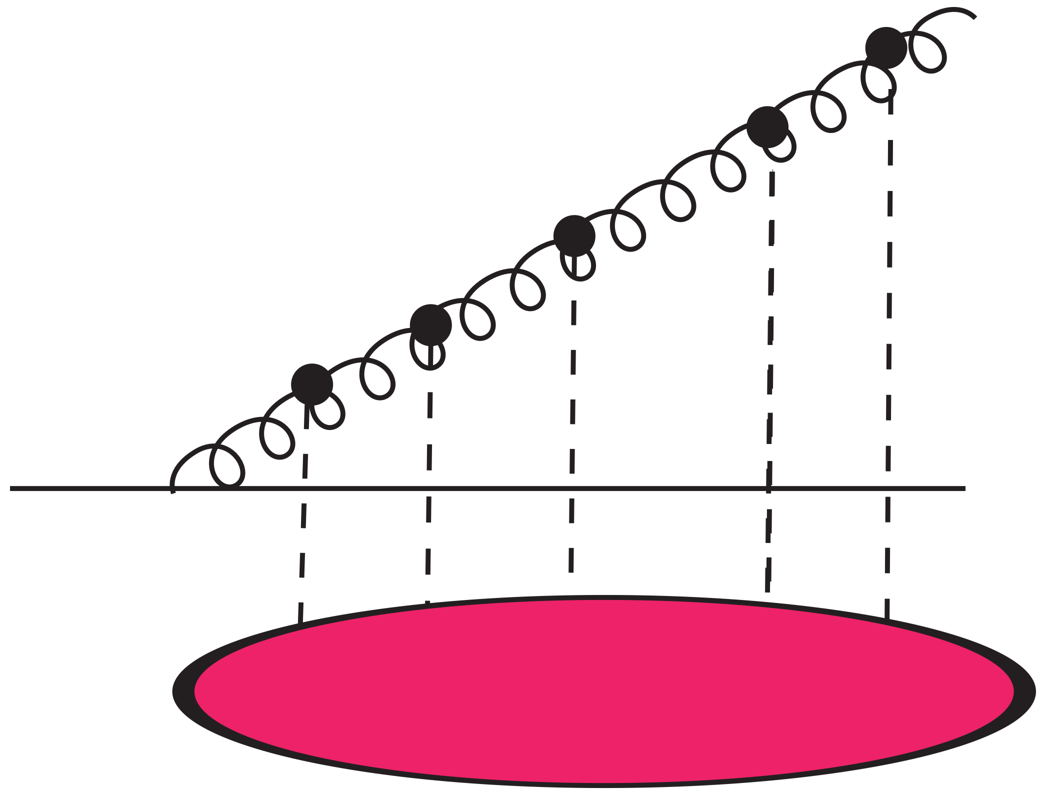




$$\frac{\mathrm{d}\sigma}{\mathrm{d}p_T\mathrm{d}\eta} = \sum_{i\in q,\bar{q},g}\int_0^1 \frac{\mathrm{d}z}{z} H_i\left(\omega = \frac{\omega_J}{z}, \mu\right) J_i(z,\omega_J,\mu)$$

$$J_i(z,\omega_J,\mu) = \int_0^1 dz' \int_0^\infty d\epsilon_L\, \delta(\omega'_J - \omega_J - \epsilon_L) \sum_m \prod_{j=2}^m \int \frac{d\Omega(n_j)}{4\pi} \mathcal{J}_{i\to m}\left(\{\underline{n}\}, z', \omega'_J = \frac{z'\omega_J}{z}, \mu, \mu_{cs}\right) \mathcal{S}_m(\{\underline{n}\}, \epsilon_L, \mu_{cs})$$

$$\mathcal{S}_1^{(n)}(\epsilon_L,\mu) = |\,C_G|^{2n}\left[\prod_{i=1}^n \int_0^L \mathrm{d}x_i^- \Theta(x_i^- - x_{i+1}^-) \int \frac{\mathrm{d}^2k_i}{(2\pi)^3} \mathbf{B}(k_{i\perp},\mu,\nu',x_i^-)\right] \mathbf{S}_1^{(n)}(\epsilon_L; k_{1\perp},\ldots,k_{n\perp}; x_1^-,\ldots x_n^-;\nu')$$



The factorization formula assumes that medium correlators are not correlated beyond the scattering length

Angularity distribution non-perturbative effects

$$\frac{d\sigma}{d\eta dp_T d\tau_a} = \int dk \, \frac{d\sigma^{\mathrm{pert}}}{d\eta dp_T d\tau_a}\left(\tau_a - \frac{k}{p_T R}\right) \mathcal{S}_{\mathrm{np}}(k) \, .$$