Jet thermalization

A review from kinetic theory perspective

C3NT Workshop

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Jet quenching and thermalization

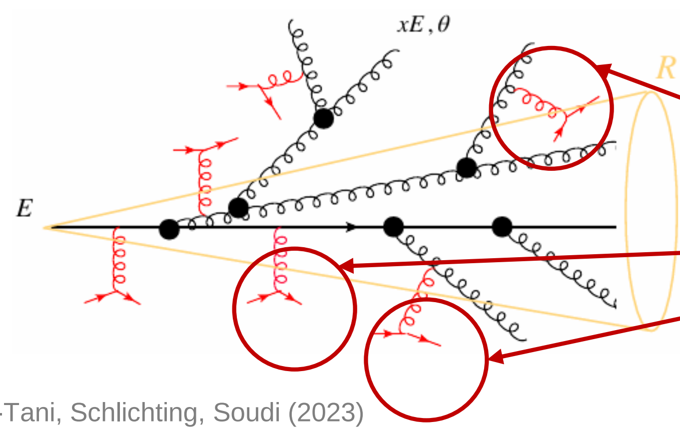
The overall physics picture

Introduction

Jet quenching and thermalization

High virtuality $\sim Q$ → Virtuality degrading $\tau \sim 1/Q$ → On-shell partons

Jets Radiation & parton shower in vacuum **Mini-jets**



Mehtar-Tani, Schlichting, Soudi (2023)

↓

Medium-induced radiation
(via small elastic kicks)

↓

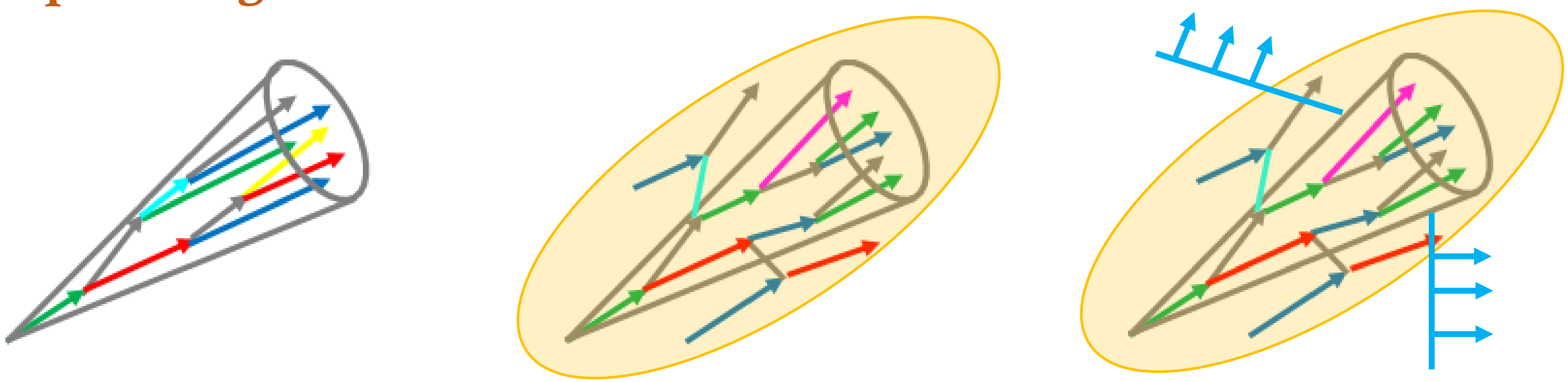
Long time evolution

QCD Effective Kinetic Theory ←

- Kinetic theory ← Dynamical on-shell particles
- Momentum for jet ← Path length dependence neglected
- Elastic collisions ← Soft part is also important
- Position for medium ← Affecting soft medium

Introduction

Jet quenching and thermalization



Highly energetic parton — **Traversing the medium** — **Jet wake**

Vacuum shower — Medium induced jet shower — Jet modified medium

Long time evolution

QCD Effective Kinetic Theory

Kinetic theory	• Dynamical on-shell particles
Momentum for jet	• Path length dependence neglected
Elastic collisions	• Soft part is also important
Position for medium	• Affecting soft medium

Turbulent cascade in jet thermalization

The “bottom-up thermalization” for jets

QCD effective kinetic theory

QCD effective kinetic theory (EKT)

Arnold, Moore, Yaffe (2003)

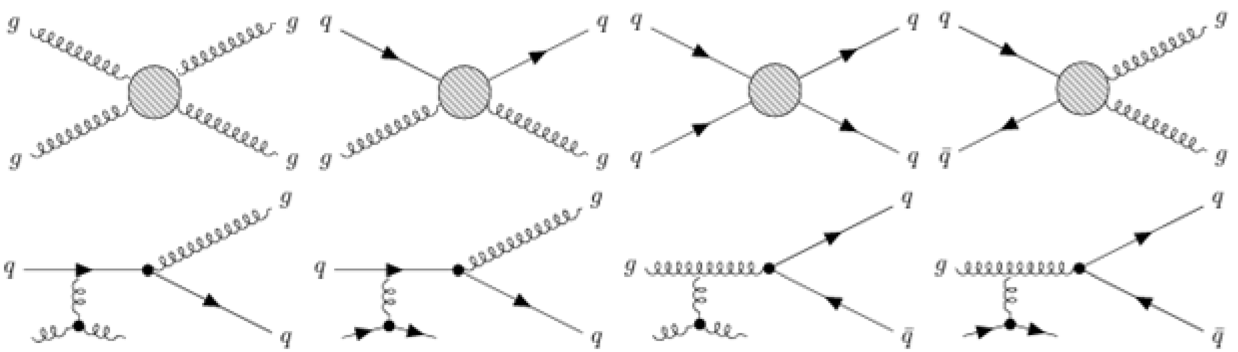
2-point correlations from the QCD $A_\mu, \psi_f \to f_g, f_q$

$$\mathcal{L}_{\mathrm{QCD}} = \sum_f^{N_f} \bar{\psi}_f \,(i\gamma^\mu D_\mu - m)\psi_f \; - \frac{1}{4} F^a_{\mu\nu} F^{\mu\nu}_a$$

Set of coupled Boltzmann equations for quarks and gluon distribution:

$$\frac{d}{d\tau} f_a(\tau, p) = C_a^{2\leftrightarrow 2}[f](\tau, p) + C_a^{1\leftrightarrow 2}[f](\tau, p) \qquad a = g, u, \bar{u}, d, \bar{d}, s, \bar{s}$$

Including both elastic and inelastic scatterings in the QCD:



$C_a^{2\leftrightarrow 2}$

$C_a^{1\leftrightarrow 2}$

Semi-classical:

- Classical: distribution level
- Quantum: quantum statistics (collision integral), amplitudes (QCD diagrams), …

QCD effective kinetic theory

QCD effective kinetic theory (EKT)

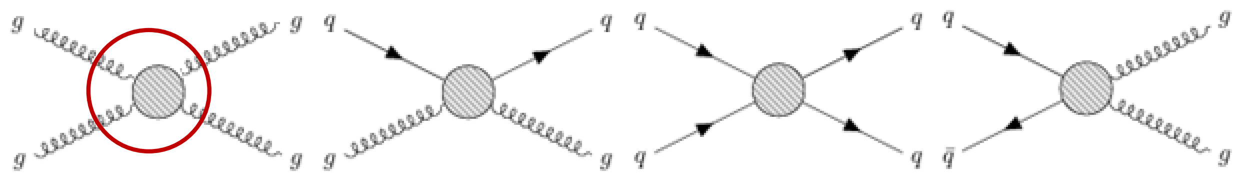
Arnold, Moore, Yaffe (2003)

Set of coupled Boltzmann equations for quarks and gluon distribution:

$$\frac{d}{d\tau} f_a(\tau, p) = C_a^{2\leftrightarrow 2}[f](\tau, p) + C_a^{1\leftrightarrow 2}[f](\tau, p) \qquad a = g, u, \bar{u}, d, \bar{d}, s, \bar{s}$$

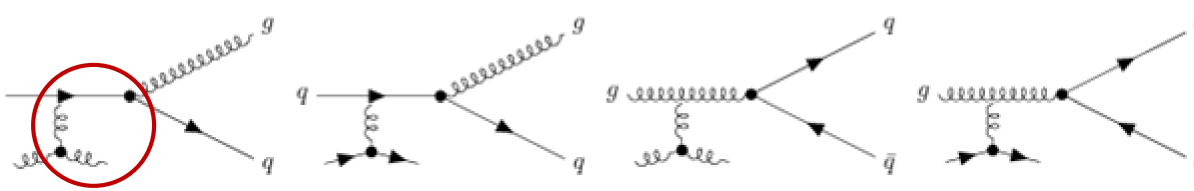
Including both elastic and inelastic scatterings in the QCD:

- Color-screened elastic scatterings



$$C_a^{2\leftrightarrow 2}[f](\tau, p) = \frac{1}{2\nu_a}\frac{1}{2E_p}\sum_{cd}\int d\Pi_{2\leftrightarrow 2}\left|\mathcal{M}_{cd}^{ab}(p, p_2|p_3, p_4)\right|^2 F_{cd}^{ab}(p, p_2|p_3, p_4)$$

- Medium-induced radiation

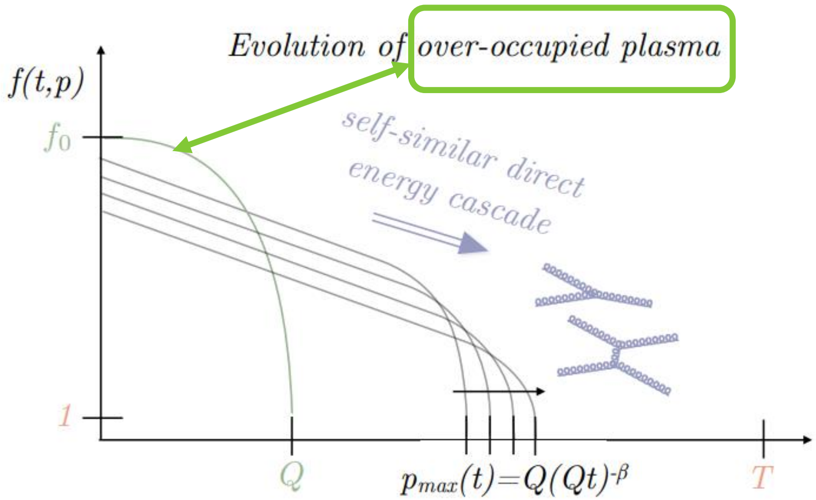


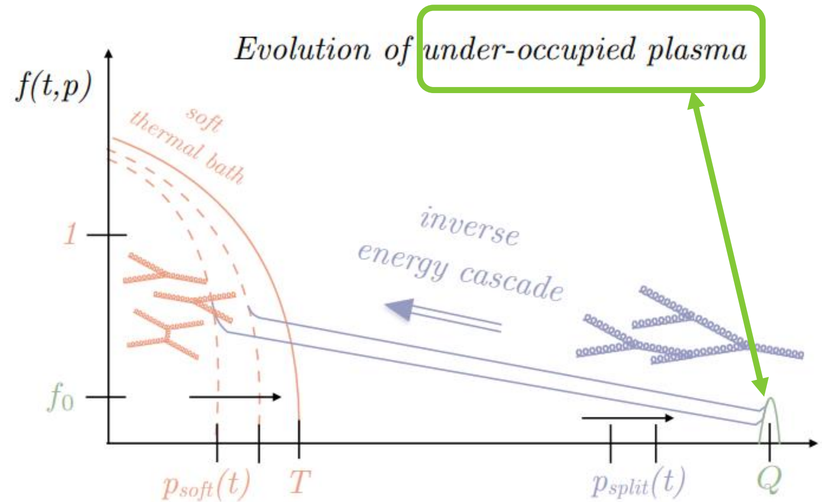
$$C_a^{1\leftrightarrow 2}[f](\tau, p)$$
$$= \frac{1}{2}\int_0^1 dz \left[\sum_{bc}\frac{d\Gamma_{bc}^a}{dz}(p, z)F_{bc}^a(p|zp, \bar{z}p) - \frac{1}{z^3}\frac{d\Gamma_{ab}^c}{dz}\left(\frac{p}{z}, z\right)F_{ab}^c\left(\frac{p}{z}|p, \frac{\bar{z}}{z}p\right) - \frac{1}{\bar{z}^3}\frac{d\Gamma_{ab}^c}{dz}\left(\frac{p}{\bar{z}}, \bar{z}\right)F_{ab}^c\left(\frac{p}{\bar{z}}|p, \frac{z}{\bar{z}}p\right)\right]$$

Bottom-up thermalization for jets

Thermalization in heavy-ion collisions

Over-occupied and under-occupied plasmas





Over-occupied plasma:

- Separation of scale
  $\langle p \rangle_0 \ll T$
- Direct energy cascade
  Low → High momentum
- Initial state in HICs

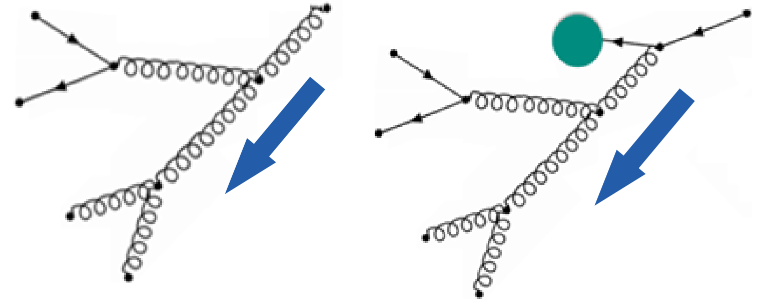
Under-occupied plasma:

- Separation of scale
  $\langle p \rangle_0 \gg T$
- Inverse energy cascade
  High → Low momentum
- **Jets in HICs**

Inverse energy cascade

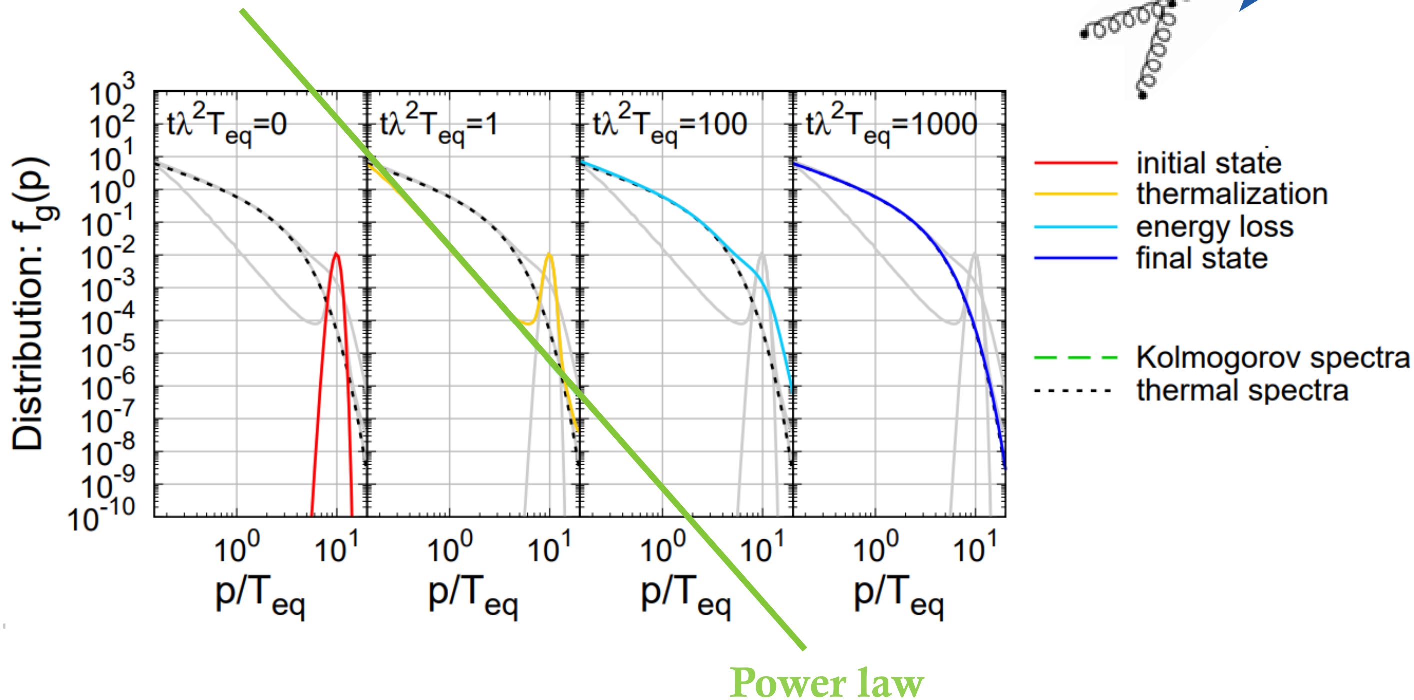
See Sergio's Talk on Bottom-up thermalization

Bottom-up thermalization for jets



Kolmogorov-Zakharov turbulent spectra

- Turbulence during jet thermalization



Kolmogorov-Zakharov spectra:

-7/2 power law in the QCD turbulence

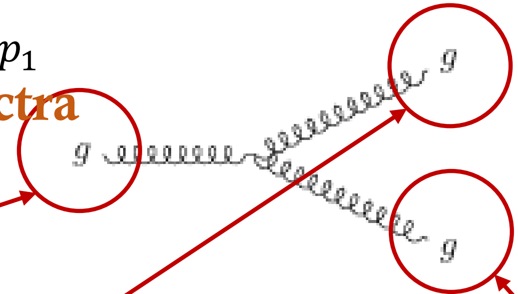
$$f_{KZ}(p,t) = K(t)\left(\frac{Q}{p}\right)^{\kappa}$$

Blaizot, Iancu, Mehtar-Tani (2013)
Iancu, Wu (2015)
Schlichting, Teaney (2019)
Du, Schlichting (2020)
Schlichting, Soudi (2020)
…

Bottom-up thermalization for jets

Kolmogorov-Zakharov turbulent spectra

- Dominated by inelastic radiation



p_1

$p_2 = zp_1$

$p_3 = (1-z)p_1$

$$C_g^{1\leftrightarrow 2}[f](\tau,p)$$
$$= \frac{1}{2}\int_0^1 dz \left[\frac{d\Gamma_{gg}^g}{dz}(p,z)F_{gg}^g(p|zp,\bar{z}p) - \frac{1}{z^3}\frac{d\Gamma_{gg}^g}{dz}\left(\frac{p}{z},z\right)F_{gg}^g\left(\frac{p}{z}|p,\frac{\bar{z}}{z}p\right) - \frac{1}{\bar{z}^3}\frac{d\Gamma_{gg}^g}{dz}\left(\frac{p}{\bar{z}},\bar{z}\right)F_{gg}^g\left(\frac{p}{\bar{z}}|p,\frac{z}{\bar{z}}p\right)\right]$$
$$= \int_0^1 dz \left[\frac{1}{2}\frac{d\Gamma_{gg}^g}{dz}(p,z)F_{gg}^g(p|zp,\bar{z}p) - \frac{1}{z^3}\frac{d\Gamma_{gg}^g}{dz}\left(\frac{p}{z},z\right)F_{gg}^g\left(\frac{p}{z}|p,\frac{\bar{z}}{z}p\right)\right]$$
$$= \int_0^1 dz \left[z\frac{d\Gamma_{gg}^g}{dz}(p,z)F_{gg}^g(p|zp,\bar{z}p) - \frac{1}{z^3}\frac{d\Gamma_{gg}^g}{dz}\left(\frac{p}{z},z\right)F_{gg}^g\left(\frac{p}{z}|p,\frac{\bar{z}}{z}p\right)\right]$$

For symmetric function $g \to gg$ process

- For an under occupied mini-jet $f \ll 1$

$$F_{gg}^g(p_1|p_2,p_3) = f(p_1)[1+f(p_2)][1+f(p_3)] - f(p_2)f(p_3)[1+f(p_1)] \approx f(p_1)$$

- In high energy limit, the LPM rate

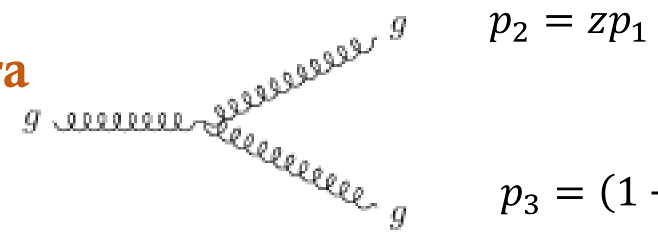
$$\frac{d\Gamma_{gg}^g}{dz}(p,z) \cong \frac{\alpha_s}{2\pi}\frac{2C_A\left(1-z(1-z)\right)^2}{z(1-z)}\sqrt{\frac{\hat{q}}{p}}\sqrt{\frac{\left(1-z(1-z)\right)C_A}{z(1-z)}} \Rightarrow \frac{d\Gamma_{gg}^g}{dz}\left(\frac{p}{z},z\right) \cong \sqrt{z}\frac{d\Gamma_{gg}^g}{dz}(p,z)$$

Bottom-up thermalization for jets

Kolmogorov-Zakharov turbulent spectra

- Inelastic integral for under-occupied mini-jet

p_1



$p_2 = zp_1$

$p_3 = (1-z)p_1$

$$C_g^{1\leftrightarrow 2}[f](\tau,p) = \int_0^1 dz\left[z\frac{d\Gamma_{gg}^g}{dz}(p,z)F_{gg}^g(p|zp,\bar{z}p) - \frac{1}{z^3}\frac{d\Gamma_{gg}^g}{dz}\left(\frac{p}{z},z\right)F_{gg}^g\left(\frac{p}{z}|p,\frac{\bar{z}}{z}p\right)\right]$$

$$F_{gg}^g(p_1|p_2,p_3) \approx f(p_1) \qquad \frac{d\Gamma_{gg}^g}{dz}\left(\frac{p}{z},z\right) \cong \sqrt{z}\frac{d\Gamma_{gg}^g}{dz}(p,z)$$

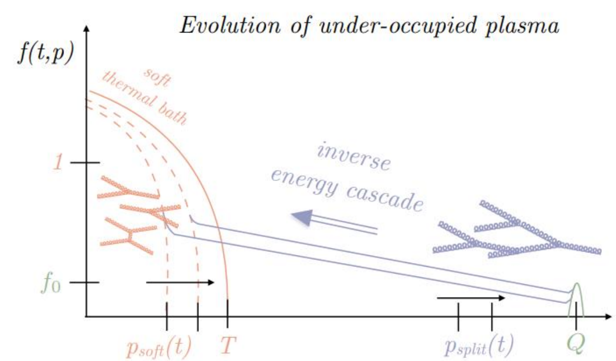
$$C_g^{1\leftrightarrow 2}[f](\tau,p) = \int_0^1 dz\left[z\frac{d\Gamma_{gg}^g}{dz}(p,z)f(p) - \frac{\sqrt{z}}{z^3}\frac{d\Gamma_{gg}^g}{dz}(p,z)f\left(\frac{p}{z}\right)\right]$$

- Effective kinetic theory with radiation dominated

$$\frac{\partial}{\partial t}f(p) + \int_0^1 zdz\frac{d\Gamma_{gg}^g}{dz}(p,z)\left[f(p) - z^{-\frac{7}{2}}f\left(\frac{p}{z}\right)\right]$$

- Stationary solution (Kolmogorov-Zakharov spectra)

$$f_{KZ}(p,t) \approx K(t)\left(\frac{Q}{p}\right)^{\frac{7}{2}}$$



Inverse energy cascade

2003 Dirac medal (ICTP)

Vladimir Zakharov

Transport coefficients in jet thermalization

Drag and diffusion coefficients for jets

QCD effective kinetic theory

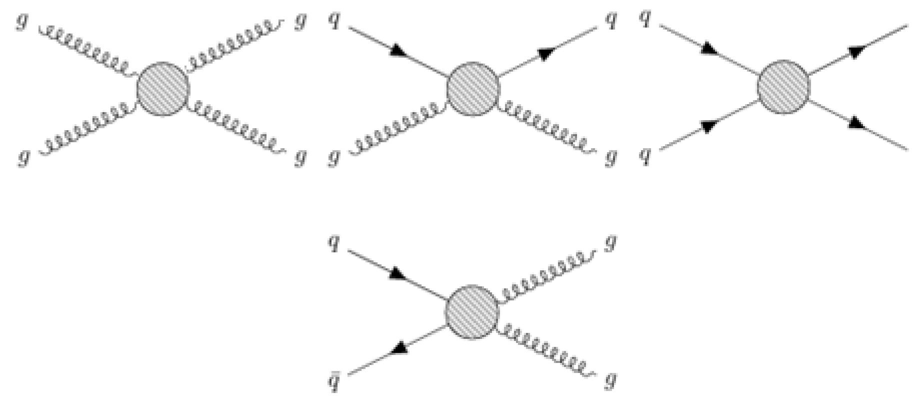
Small angle approximation

Simplified 2-2 Kernel

$$C_a^{2\leftrightarrow 2} = D_a + S_a$$

Momentum redistribution

$$D_a = \frac{1}{4} C_a \hat{q}(t) \nabla_p \cdot \left[\nabla_p f_a + \frac{v}{T^*(t)} f_a (1 + \epsilon f_a) \right]$$



Gluon-quark conversion

$$S_q = \frac{2\pi \alpha_s^2 C_F^2 \mathcal{L}}{p} \mathcal{I}_c [f_g (1 - f_q) - f_q (1 + f_g)]$$

$$S_g = \frac{N_f}{C_F} S_q$$

Coefficients

$$\hat{q} = 8\pi \alpha_s^2 \mathcal{L} \int \frac{d^3 p}{(2\pi)^3} [N_c f_g (1 + f_g) + N_f f_q (1 - f_q)]$$

$$\mathcal{I}_c = \int \frac{d^3 p}{(2\pi)^3} \frac{1}{p} [f_g + f_q]$$

$$T^* = \frac{C_A \hat{q}}{\alpha_s \mathcal{L} m_D^2}$$

$$m_D^2 = 16\pi \alpha_s \int \frac{d^3 p}{(2\pi)^3} \frac{1}{p} [N_c f_g + N_f f_q]$$

$$\mathcal{L} = ln \left(\frac{\langle p_T^2 \rangle}{m_D^2} \right)$$

Transport coefficients

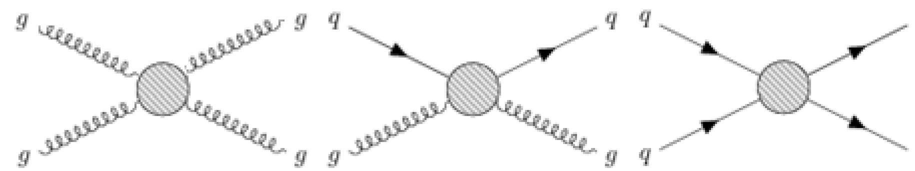
Drag and diffusion coefficients in small angle approximation

Simplified 2-2 Kernel

$$C_a^{2\leftrightarrow 2} = D_a + S_a$$

Momentum redistribution

$$D_a = \frac{1}{4} C_a \hat{q}(t) \nabla_p \cdot \left[\nabla_p f_a + \frac{v}{T^*(t)} f_a (1 + \epsilon f_a) \right]$$



$$D_a = \frac{1}{4} C_a \left[\hat{q}(t) \nabla_p^2 f_a + \eta(t) \nabla_p v f_a (1 + \epsilon f_a) \right]$$

Drag coefficient

$$\eta(t) = \frac{\hat{q}(t)}{T^*(t)} = \frac{\alpha_s \mathcal{L} m_D^2}{C_A} = 16\pi \alpha_s^2 \mathcal{L} \int \frac{d^3p}{(2\pi)^3} \frac{1}{p} \left[N_c f_g + N_f f_q \right]$$

Diffusion coefficient

$$\hat{q}(t) = 8\pi \alpha_s^2 \mathcal{L} \int \frac{d^3p}{(2\pi)^3} \left[N_c f_g (1 + f_g) + N_f f_q (1 - f_q) \right]$$

- In thermal equilibrium medium, one obtains analytical expression, and Einstein's relation

Fluctuation-dissipation relation

Stochastic picture of thermalization

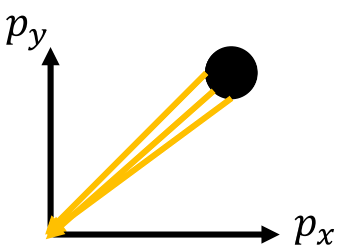
Why are drag and diffusion coefficients important?

Stochastic differential equation (SDE)

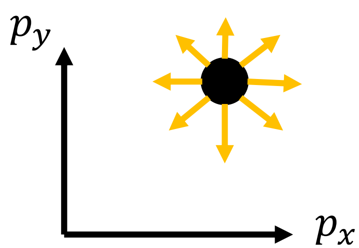
$$dp_i = -A_i dt + \sigma_{ij} dW_j$$

Stochastic term (elastic kicks)

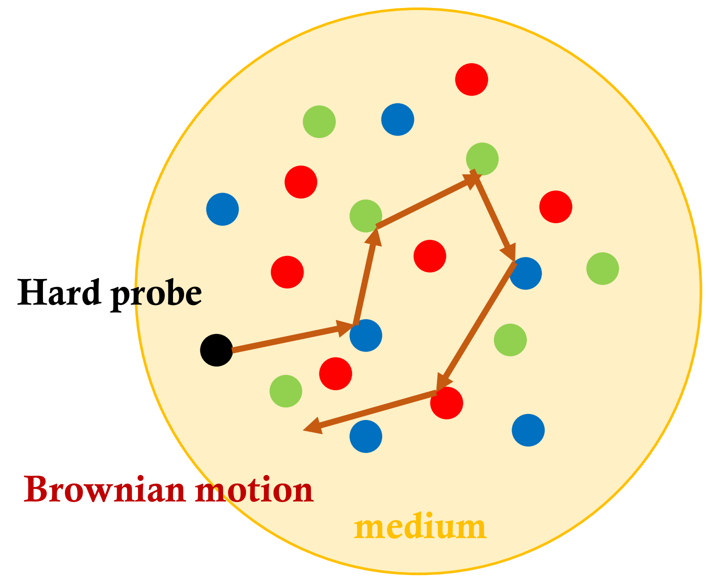
Drag Diffusion



Drag



Diffusion



$$B_{ij} = \sigma_{ik}\sigma_{kj}/2$$

From the SDE to partial differential equation (PDE)

(via Ito's lemma)

$$\partial_t f(p) = \partial_{p_i}[A_i f(p,t)] + \partial_{p_i}\partial_{p_j}[B_{ij} f(p,t)]$$

Drag — Dissipation/Energy loss

Diffusion — Momentum broadening

Thermalization

Similar in small angle approximation

Fokker-Planck Eq./Boltzman Eq. in diffusion approx. (BEDA)

Transport coefficients in this workshop

Drag and diffusion coefficients in non-equilibrium QCD plasma

See Andrey's Talk on Jet in anisotropic medium

See Daria's Talk on heavy flavor jet in Romatschke-Strickland medium

See Florian's Talk on EKT dynamically calculated coefficients and radiation rates

See Fabian's Talk on transport coefficients with opacity expansion

……

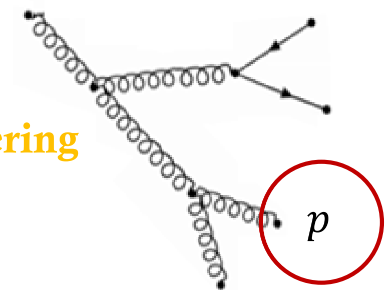
Parton shower in jet thermalization

Medium-induced parton shower and cone size broadening

QCD effective kinetic theory

Linearized QCD effective kinetic theory (LEKT)

Total jet energy E



Showering

Parton energy

Separate "medium" and "jet"

$$f_a(\tau,p) = n_a(\tau,p) + \delta f_a(\tau,p)$$

Medium — Jet as perturbation

$$\frac{d}{d\tau} n_a(\tau,p) = C_a^{2\leftrightarrow 2}[n](\tau,p) + C_a^{1\leftrightarrow 2}[n](\tau,p)$$

Medium evolution

$$\frac{d}{d\tau} \delta f_a(\tau,p) = \delta C_a^{2\leftrightarrow 2}[n,f](\tau,p) + \delta C_a^{1\leftrightarrow 2}[n,f](\tau,p)$$

Jet evolution/showering

In-medium "fragmentation function"

Energy fraction of parton: z

$$D_a(\tau,z) = z\frac{dN_a}{dz} = \nu_a \int \frac{d^3p}{(2\pi)^3} \frac{p}{E} \delta\left(\frac{p}{E} - z\right) \delta f_a(t,p)$$

$$\frac{d}{d\tau} D_a(\tau,z) = \delta C_a^{2\leftrightarrow 2}[n,D](\tau,z) + \delta C_a^{1\leftrightarrow 2}[n,D](\tau,z)$$

- Energy conservation/charge conservation

$$\sum_a \int dz\, D_a(\tau,z) = 1 \qquad \int \frac{dz}{z}\left[D_{q_f}(\tau,z) - D_{\bar{q}_f}(\tau,z)\right] = Q_f$$

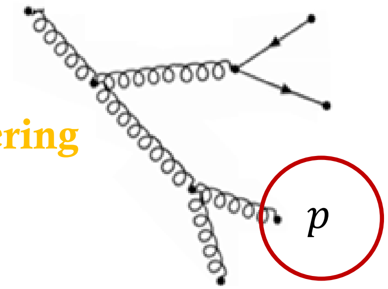
Parton shower

Scales in parton shower

Separate "medium" and "jet"

$$\frac{d}{d\tau} n_a(\tau,p) = C_a^{2\leftrightarrow 2}[n](\tau,p) + C_a^{1\leftrightarrow 2}[n](\tau,p)$$ Medium

$$\frac{d}{d\tau} D_a(\tau,z) = \delta C_a^{2\leftrightarrow 2}[n,D](\tau,z) + \delta C_a^{1\leftrightarrow 2}[n,D](\tau,z)$$ Jet



Arnold, Moore, Yaffe (2003)
Kurkela, Moore (2011)
Schlichting, Soudi (2020)
Zhou, Brewer, Mazeliauskas (2024)

Scales in collision integrals

- Thermal equilibrium medium background $\sim T$
- Typical scales for parton $\omega = zE$

Inelastic $1 \leftrightarrow 2$

$$\delta C_a^{\text{radiation}}[n,D](\tau,z) = g^4 T \sqrt{\frac{T}{zE}} D_a(\tau,z)$$

Elastic $2 \leftrightarrow 2$

$$\delta C_a^{\text{drag}}[n,D](\tau,z) = g^4 T \left(\frac{T}{zE}\right) z\partial_z D_a(\tau,z)$$

$$\delta C_a^{\text{diffusion}}[n,D](\tau,z) = g^4 T \left(\frac{T}{zE}\right)^2 (z\partial_z)^2 D_a(\tau,z)$$

$$\delta C_a^{\text{conversion}}[n,D](\tau,z) = g^4 T \left(\frac{T}{zE}\right) D_a(\tau,z)$$

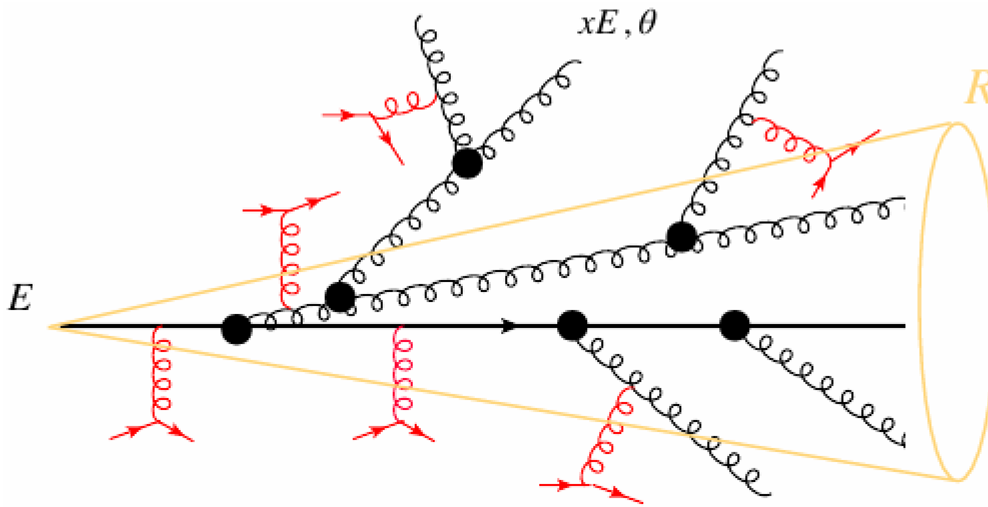
Early stage in parton showering:
High-momentum parton $\omega = zE \gg T$
Inelastic radiation dominates

Late stage in parton showering:
Parton close to thermal $\omega = zE \sim T$
Elastic/Inelastic comparable $\sim g^4 T$

Parton shower

What to expect in medium-induced parton shower

$$\frac{d}{d\tau}D_a(\tau,z) = \delta C_a^{2\leftrightarrow 2}[n,D](\tau,z) + \delta C_a^{1\leftrightarrow 2}[n,D](\tau,z)$$



Bottom-up thermalization for jets

- Start with a hard sector (mini-jet)
- Rapidly build a soft sector from energy of hard sector $\omega = zE \gg T$

  Collinear radiation
- Growth of the soft sector $\omega = zE \sim T$

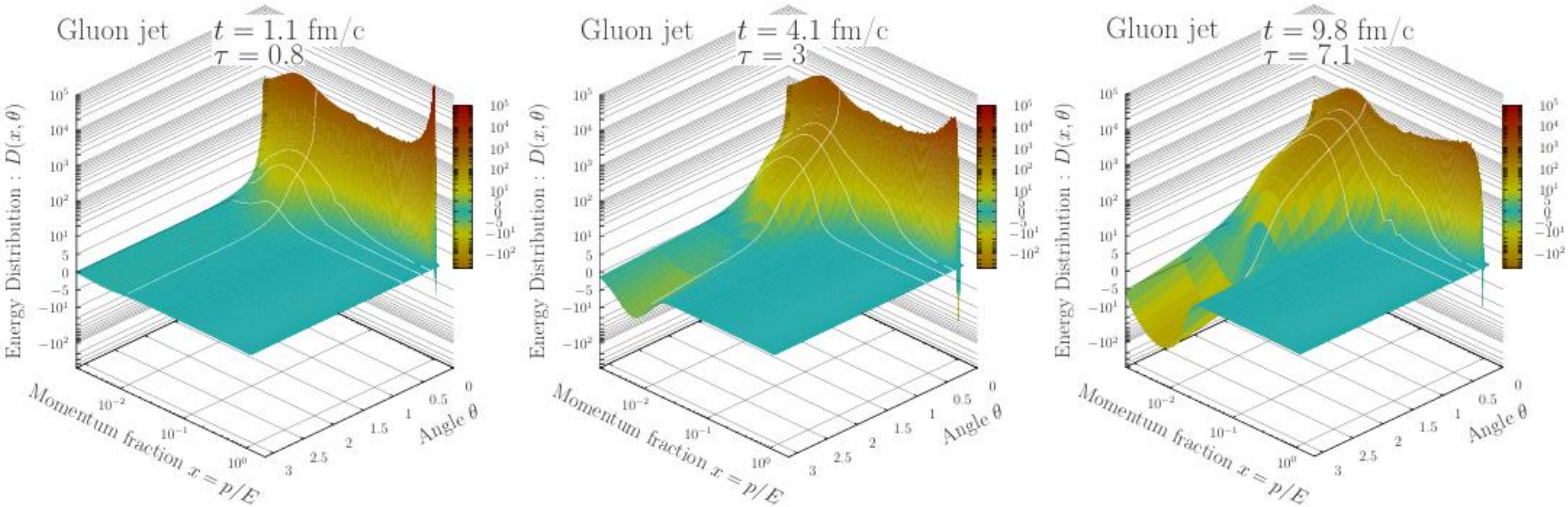
  Collinear radiation + elastic collisions (large angle)
- Turbulent cascade

  Stationary solution, constant energy flux

Parton shower

Jet cone size in parton shower

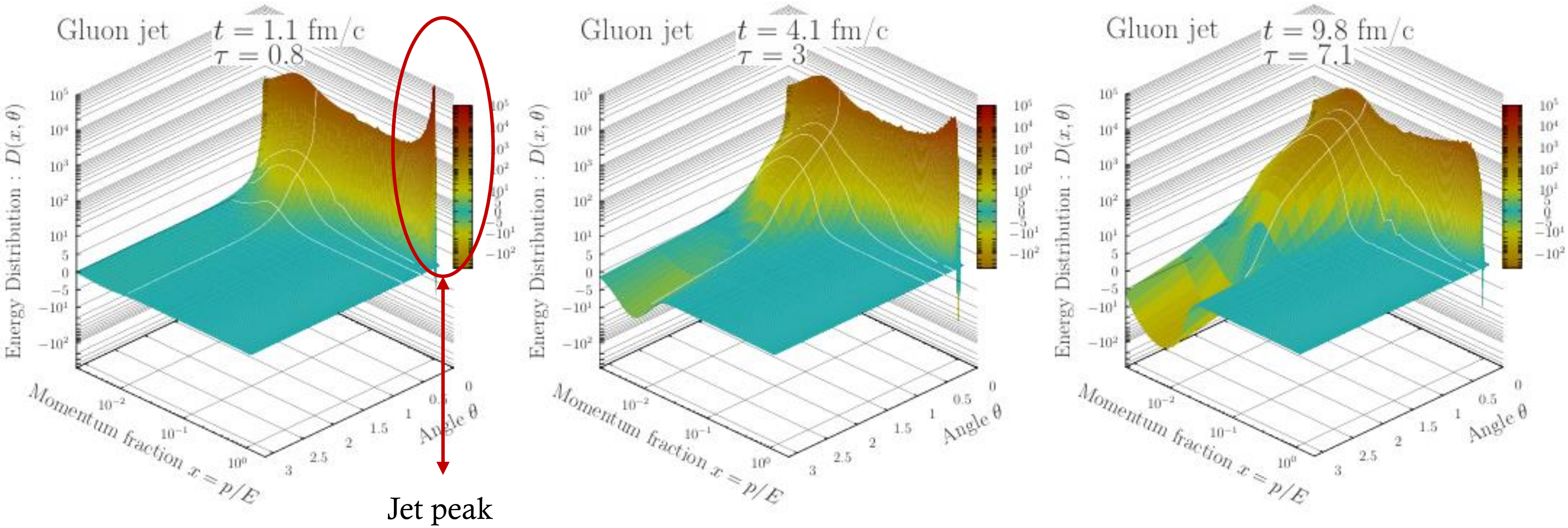
Mehtar-Tani, Schlichting, Soudi (2023)



Parton shower

Jet cone size in parton shower

Mehtar-Tani, Schlichting, Soudi (2023)



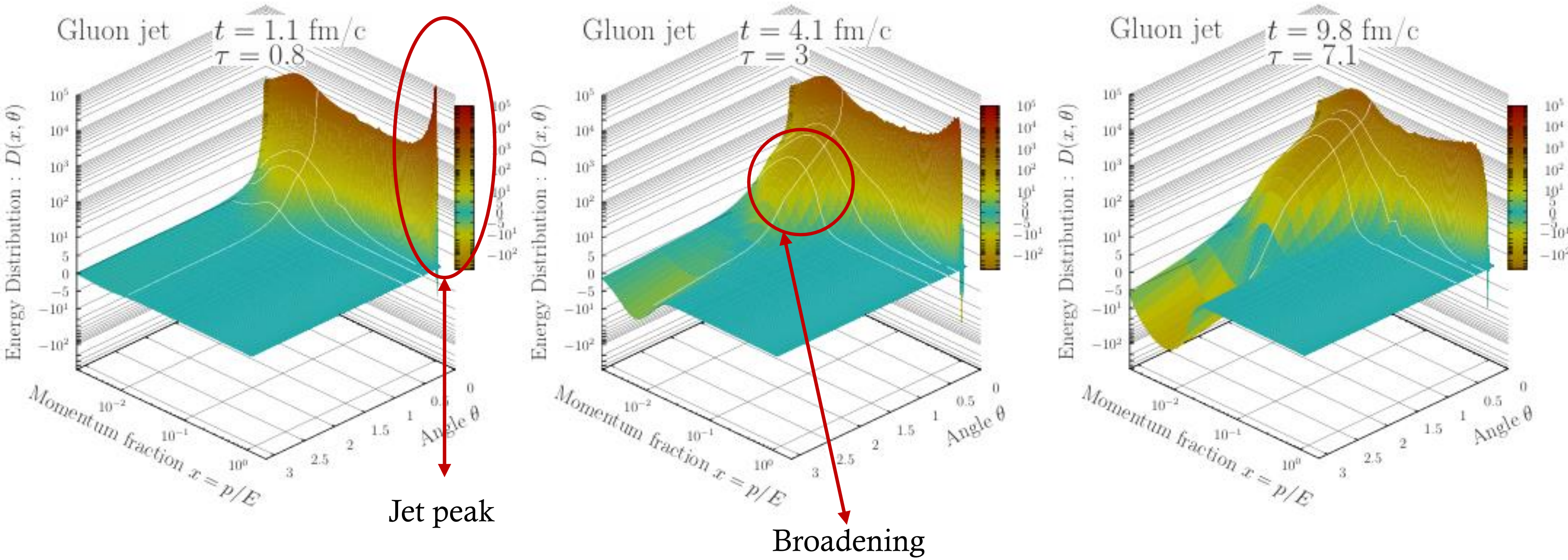
Early stage

- Collinear radiation
- Sizeable soft parton
- No visible broadening

Parton shower

Jet cone size in parton shower

Mehtar-Tani, Schlichting, Soudi (2023)



Early stage

- Collinear radiation
- Sizeable soft parton
- No visible broadening

Intermediate stage

- Hard sector:

Collinear radiation

- Soft sector:

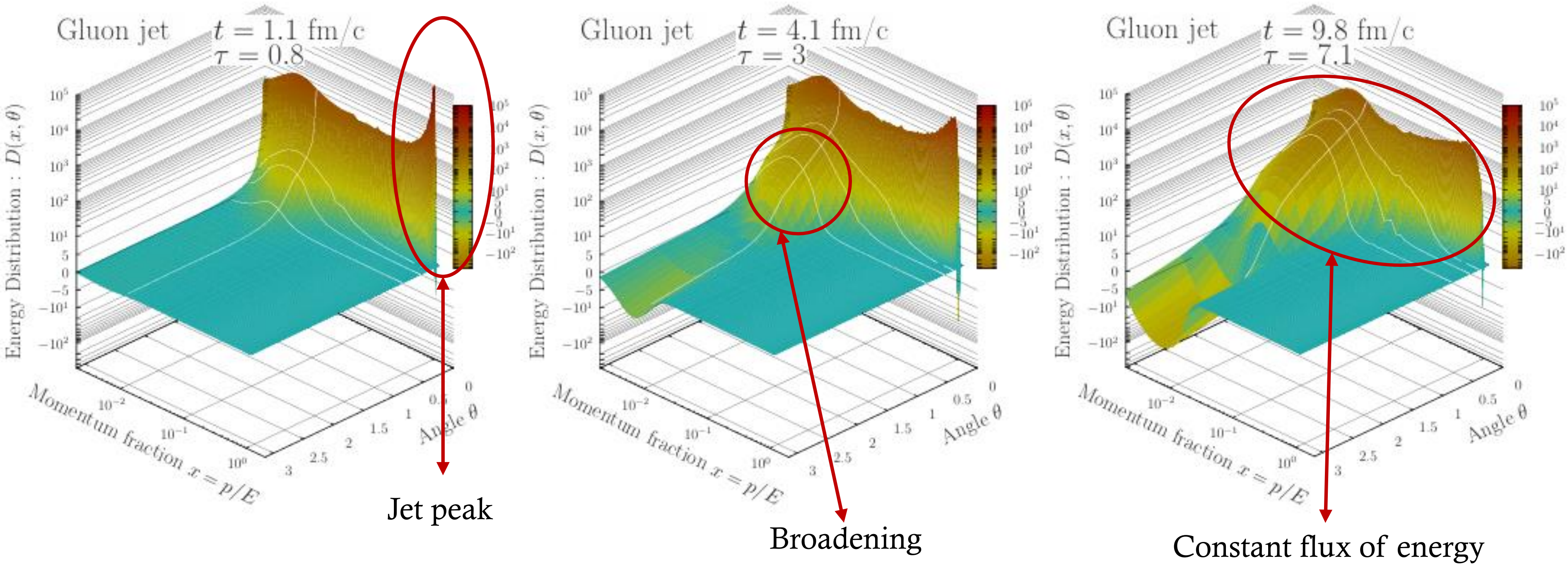
Collinear radiation+elastic collisions

- Visible broadening at soft sector

Parton shower

Jet cone size in parton shower

Mehtar-Tani, Schlichting, Soudi (2023)



Early stage

- Collinear radiation
- Sizeable soft parton
- No visible broadening

Intermediate stage

- Hard sector:

Collinear radiation

- Soft sector:

Collinear radiation+elastic collisions

- Visible broadening at soft sector

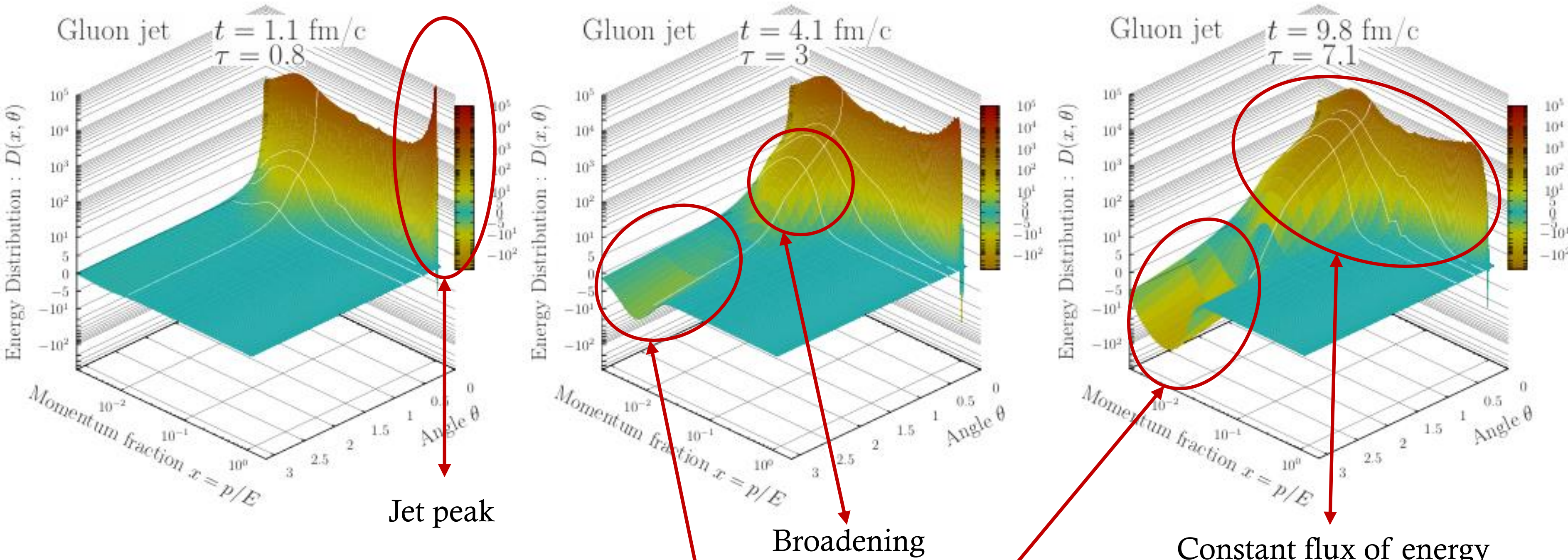
Late stage

- Jet peak disappeared
- Turbulent cascade from radiation

Parton shower

Jet cone size in parton shower

Mehtar-Tani, Schlichting, Soudi (2023)



Early stage

- Collinear radiation
- Sizeable soft parton
- No visible broadening

Intermediate stage

- Hard sector:

Collinear radiation

- Soft sector:

Collinear radiation+elastic collisions

- Visible broadening at soft sector

Late stage

- Jet peak disappeared
- Turbulent cascade from radiation

Diffusion wake?

Jet wake in jet thermalization

Medium response to highly energetic jet

QCD effective kinetic theory

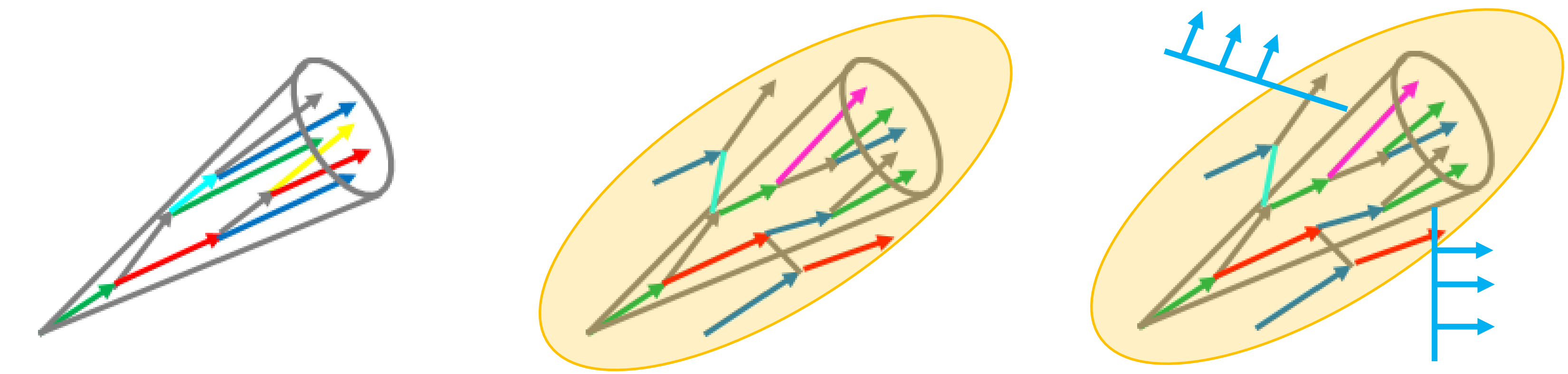
Linearized QCD effective kinetic theory (LEKT)

Separate "background medium" and "**wave perturbation/excitation**" from jet

$$f_a(\tau,p) = n_a(\tau,p) + \delta f_a(\tau,x,p)$$

$$\frac{d}{d\tau} n_a(\tau,p) = C_a^{2\leftrightarrow 2}[n](\tau,p) + C_a^{1\leftrightarrow 2}[n](\tau,p)$$ Background medium evolution

$$\frac{d}{d\tau} \delta f_a(\tau,x,p) = \delta C_a^{2\leftrightarrow 2}[n,f](\tau,x,p) + \delta C_a^{1\leftrightarrow 2}[n,f](\tau,x,p)$$ Wake stimulated by jet



Highly energetic parton **Traversing the medium** **Jet wake**

Medium induced jet shower **Jet modified medium**

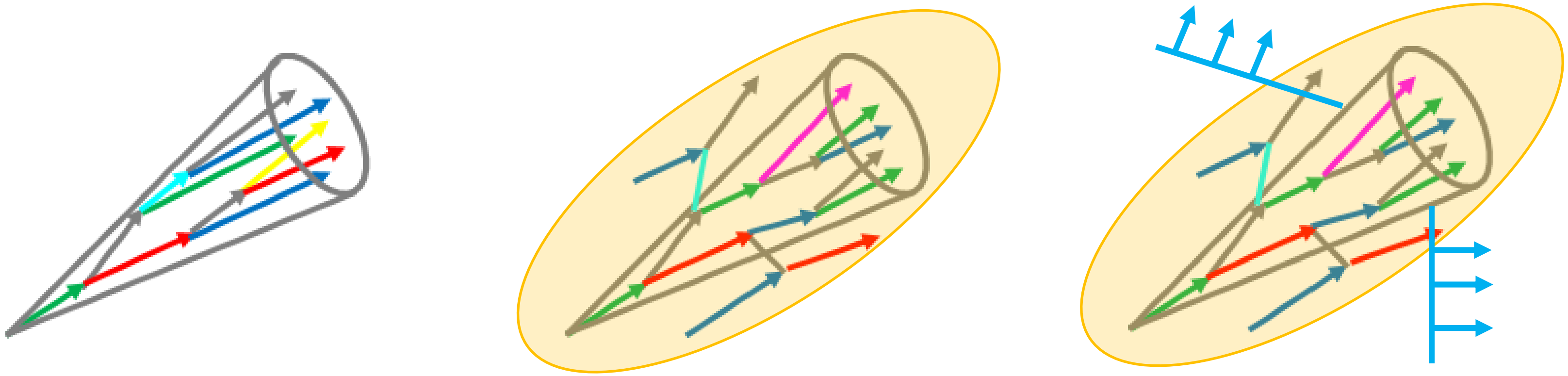
Energy/charge conservation for background & wake: Hydrodynamics fields

$$\partial_\mu T^{\mu\nu}(x) = 0$$
$$\partial_\mu J^{\mu}(x) = 0$$

$$J^\mu(x) = \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu}{p} \sum_a v_a f_a(t,x,p) \qquad T^{\mu\nu}(x) = \int \frac{d^3p}{(2\pi)^3} \frac{p^\mu p^\nu}{p} \sum_a v_a f_a(t,x,p)$$

Hydrodynamics

Jet wake in the hydrodynamic plasma



Highly energetic parton | Traversing the medium | Jet wake

Medium induced jet shower | Jet modified medium

Wake in hydrodynamics language

$$T^{\mu\nu}_{\text{total}}(t,x) = \boxed{\bar{T}^{\mu\nu}(t)} + \boxed{\delta T^{\mu\nu}(t,x)}$$

Homogeneous background — Perturbation (Wake)

$$\partial_\mu T^{\mu\nu}_{\text{total}}(t,x) = 0$$

⬇ Hydrodynamic equations for wake

$$\partial_\mu \delta T^{\mu\nu}(t,x) = 0$$

Linear response in kinetic theory

Energy-momentum tensor

- Background

$$\bar{T}^{\mu\nu}(t) = \int \frac{d^3p}{(2\pi)^3}\frac{p^\mu p^\nu}{p}\sum_a v_a \bar{f}_a(t,p)$$

- Perturbation

$$\delta T^{\mu\nu}(t,k) = \int \frac{d^3p}{(2\pi)^3}\frac{p^\mu p^\nu}{p}\sum_a v_a \delta f_a(t,k,p)$$

Fourier transform: **gradients → wavenumber k**

$$\delta f_a(t,\boldsymbol{k},p) = \int \frac{d^3x}{(2\pi)^3} e^{-ix\cdot k}\delta f_a(t,\boldsymbol{x},p)$$

Response function

- Response function in terms of time **t** and wavenumber **k**

$$G^{\mu\nu}_{\alpha\beta}(t,k) = \frac{\delta T^{\mu\nu}(t,k)}{\delta T^{\alpha\beta}(0,k)}$$

- Consider initial condition with scalar type perturbation

$$\bar{f}_a(t_0,p) = f_a^{eq}(T,p)$$ Background

Still fluid background

$$\delta f_a(t_0,k,p) = -\frac{\delta T}{T}\partial_p f_a^{eq}(T,p)$$ Perturbation

- Sound channel $G^{00}_{00}(t,k) = \frac{\delta T^{00}(t,k)}{\delta T^{00}(0,k)}$

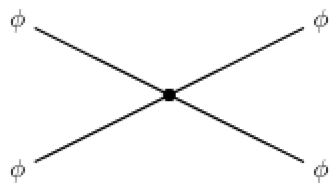
Universality among kinetic theories

- Relaxation time approximation (RTA)
- φ-4 scalar theory (SCL)
- Yang-Mills kinetic theory (YM)
- Quantum chromodynamic kinetic theory (QCD)

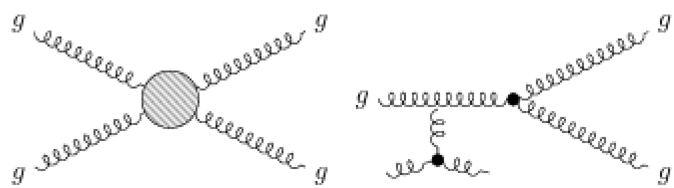
Collision integrals in kinetic theories

$$C_a^{RTA}[f](t,x,p) = \frac{p^\mu u_\mu}{\tau_R}\left[f(t,x,p) - f_{eq}(t,x,p)\right]$$

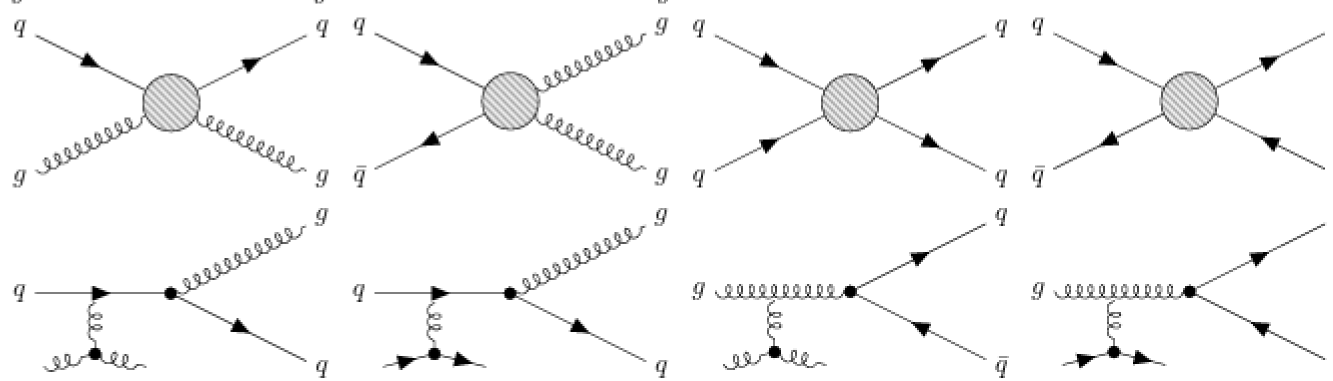
$C_a^{SCL}[f](t,x,p) =$



$C_a^{YM}[f](t,x,p) =$



$C_a^{QCD}[f](t,x,p) =$

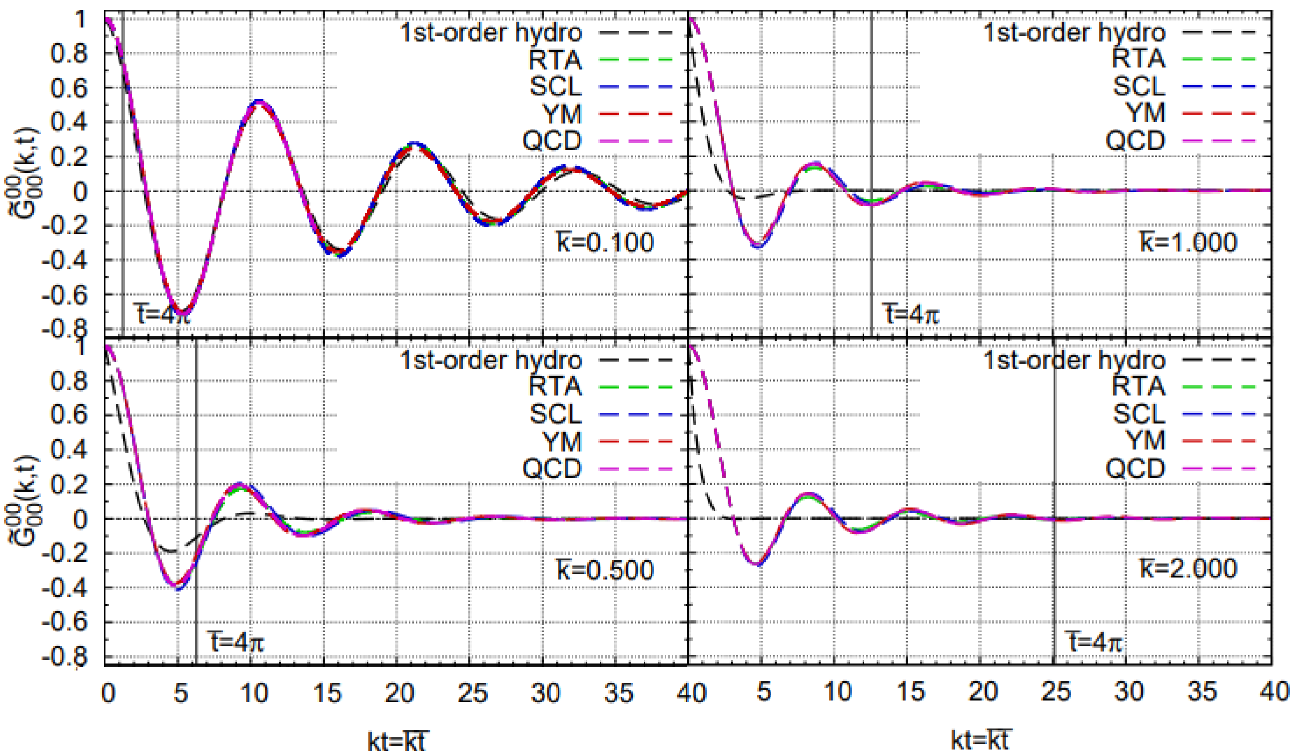


Universality among kinetic theories

- Relaxation time approximation (RTA)
- φ-4 scalar theory (SCL)
- Yang-Mills kinetic theory (YM)
- Quantum chromodynamic kinetic theory (QCD)

Response functions from kinetic theories

- Expected to reproduce hydrodynamics at small k (long wave-length limit)
- Universality among different kinetic theories even at large k and early time



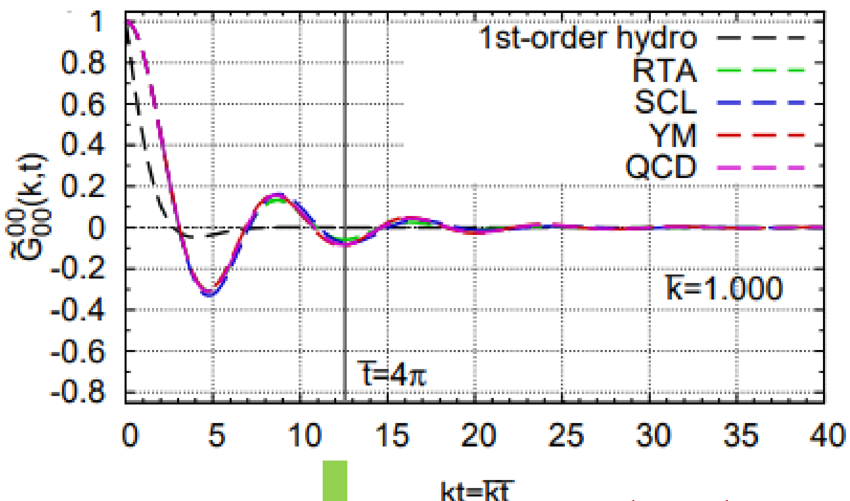
Sound and non-hydrodynamic modes

Fitting response functions in kinetic theory

- With real (dispersion/oscillation) and imaginary (damping) frequencies

$$G_{s,n}(t,k) \sim Z_k \exp[-i(\omega_k t + \phi_k)]$$

$$\omega_k = Re[\omega_k] + iIm[\omega_k]$$



non-hydro mode

$G_n(\bar{t} < \bar{t}_H, k)$

sound mode

$G_s(\bar{t} > \bar{t}_H, k)$

Hydrodynamization time

$\bar{t}_H = 4\pi$

Remarks

- Negative frequency gives the same mode as positive frequency

Sound modes appear in pair

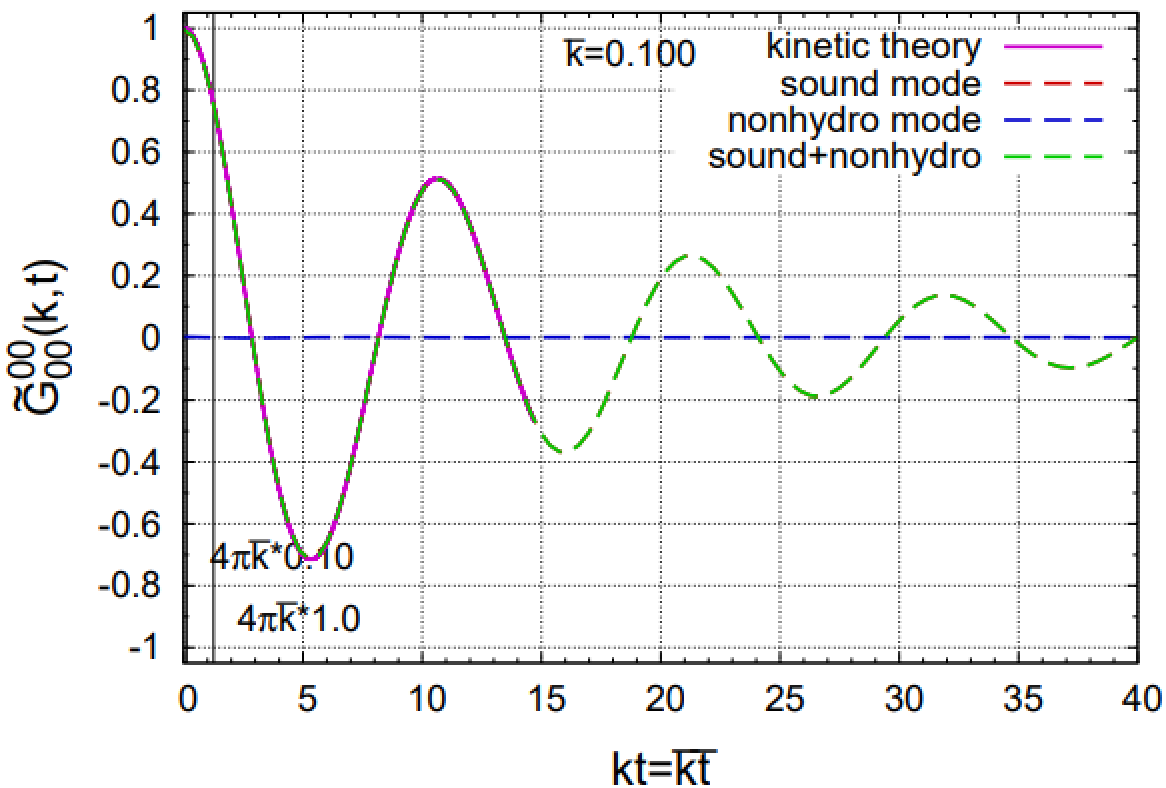
- Expected many/infinite number of non-hydrodynamic modes

We represent them with a single non-hydrodynamic mode

Sound and non-hydrodynamic modes

Fitting QCD response functions $(\bar{k}=0.1)$

- Sound mode dominates

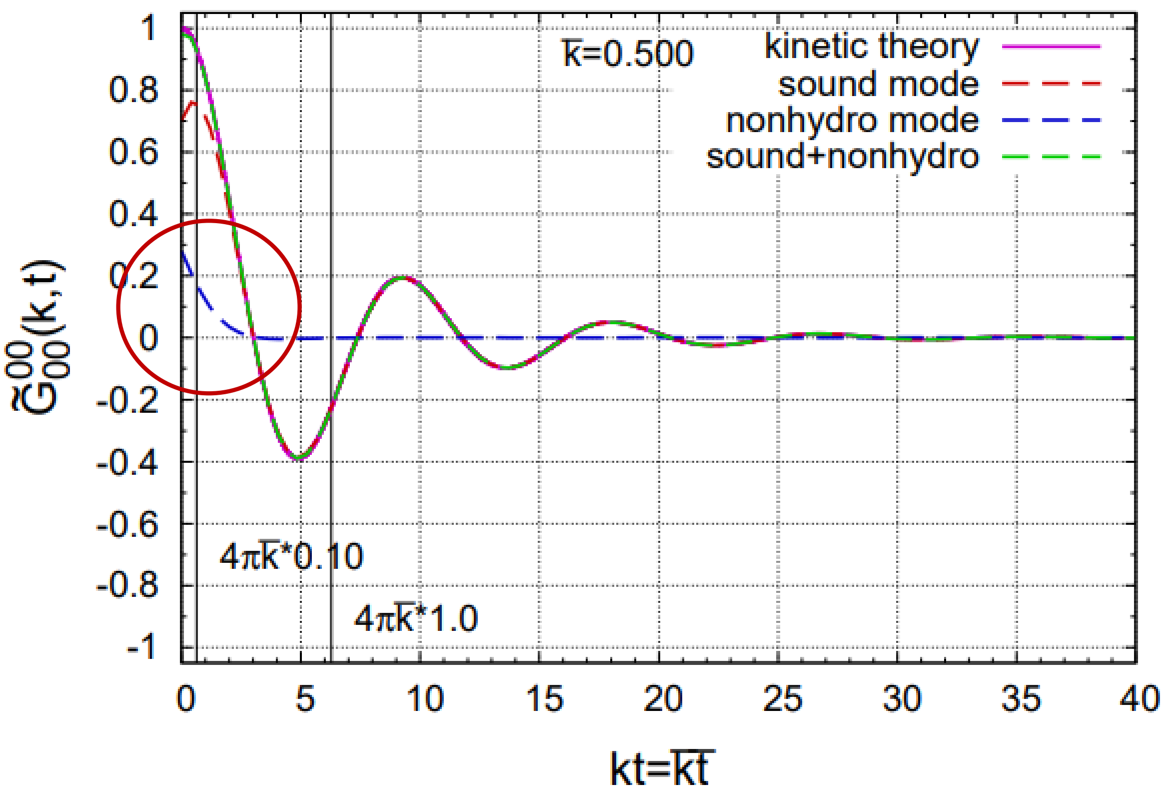


Sound and non-hydrodynamic modes

Fitting QCD response functions

$(\bar{k}=0.5)$

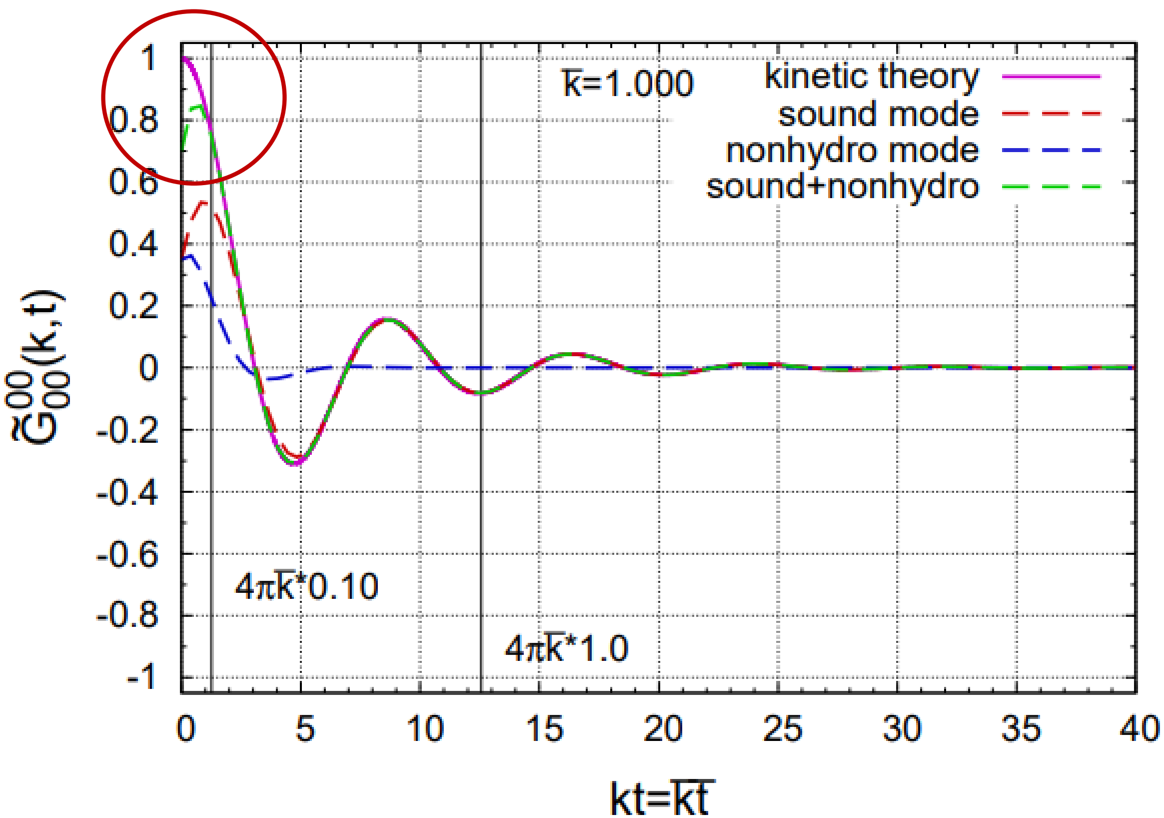
- Non-hydrodynamic mode appears



Sound and non-hydrodynamic modes

Fitting QCD response functions $(\bar{k}=1.0)$

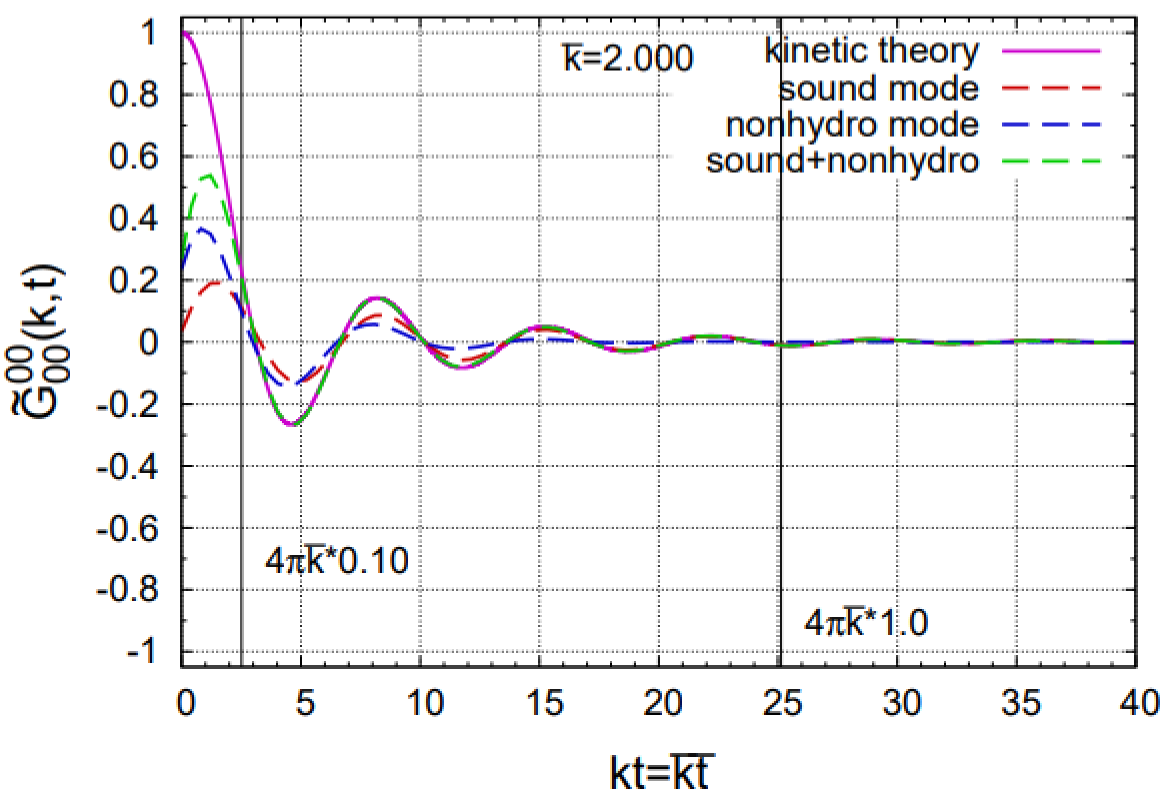
- More non-hydrodynamic modes appear at early time



Sound and non-hydrodynamic modes

Fitting QCD response functions $(\bar{k}=2.0)$

- Non-hydrodynamic mode takes over the domination by sound mode

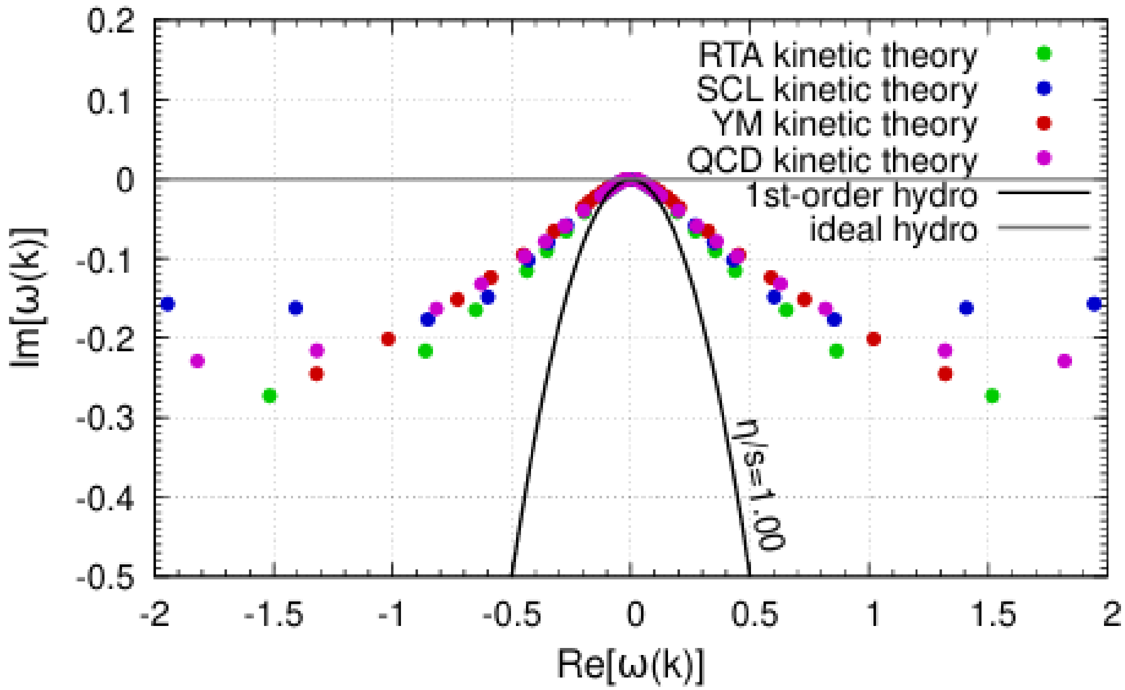


Complex frequencies

Poles in complex plane

- With complex frequencies

$$G_{s,n}(t,k) \sim Z_k \exp[-i(\omega_k t + \phi_k)] \qquad \omega_k = Re[\omega_k] + iIm[\omega_k]$$



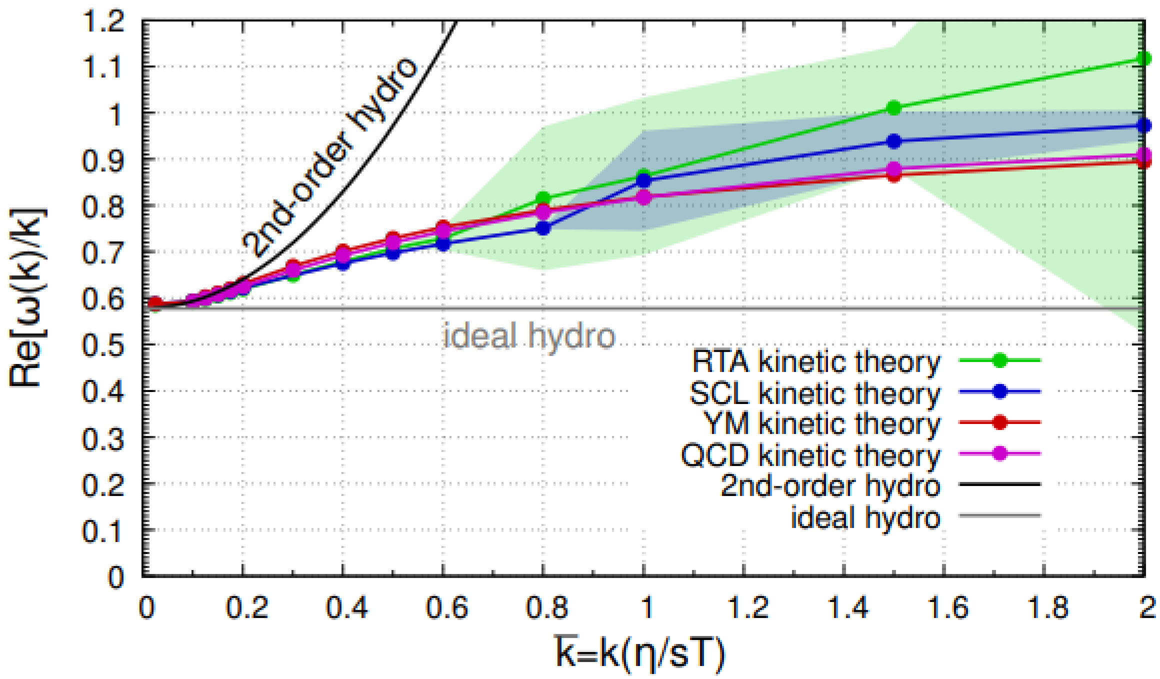
- Poles of sound modes converge to hydrodynamic limit

Dispersion relation

Dispersion relations among kinetic theories

- With real frequencies

$$G_{s,n}(t,k) \sim Z_k \exp[-i(\omega_k t + \phi_k)] \qquad \omega_k = Re[\omega_k] + iIm[\omega_k]$$



- Universality of sound modes among kinetic theories at various k
- Kinetic theories converge to 2nd-order hydrodynamics at small k

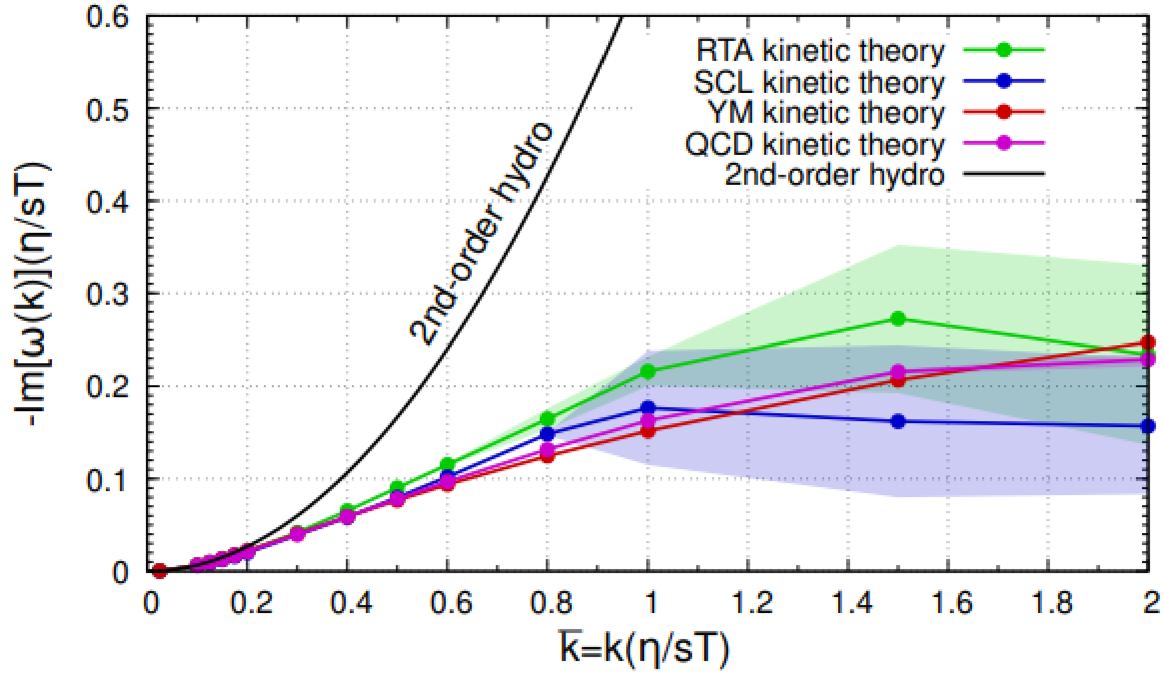
$$\omega_{hydro}^{2nd}(k) = c_s k - i\Gamma k^2 + \frac{\Gamma}{c_s}(c_s^2 \tau_\pi - \frac{\Gamma}{2})k^3 \qquad \text{with} \qquad \Gamma = \frac{2}{3}\frac{\eta}{sT}$$

Damping relation

Damping relations among kinetic theories

- With imaginary frequencies

$$G_{s,n}(t,k) \sim Z_k \exp[-i(\omega_k t + \phi_k)] \qquad \omega_k = Re[\omega_k] + iIm[\omega_k]$$



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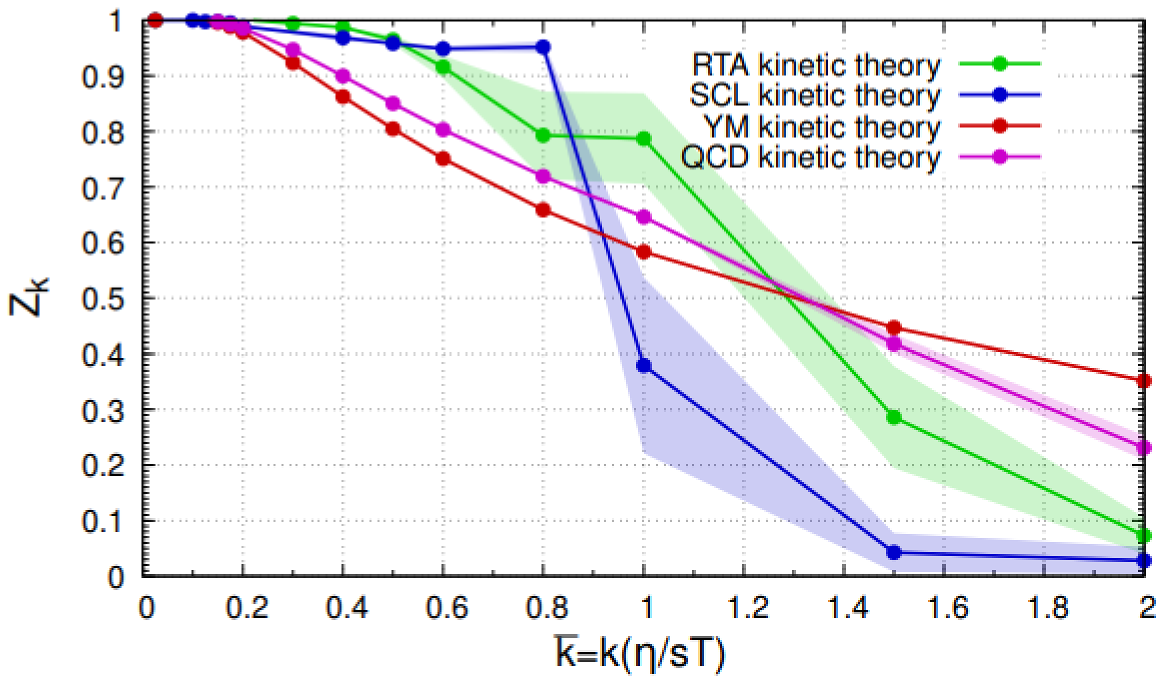
$$\omega^{2nd}_{hydro}(k) = c_s k - i\Gamma k^2 + \frac{\Gamma}{c_s}(c_s^2 \tau_\pi - \frac{\Gamma}{2})k^3 \qquad \text{with} \qquad \Gamma = \frac{2}{3}\frac{\eta}{sT}$$

Residue

Residue for sound & non-hydro modes among kinetic theories

- Non-equilibrium plasma described by sound mode + non-hydrodynamic mode

$$G_{s,n}(t,k) \sim Z_k \exp[-i(\omega_k t + \phi_k)]$$

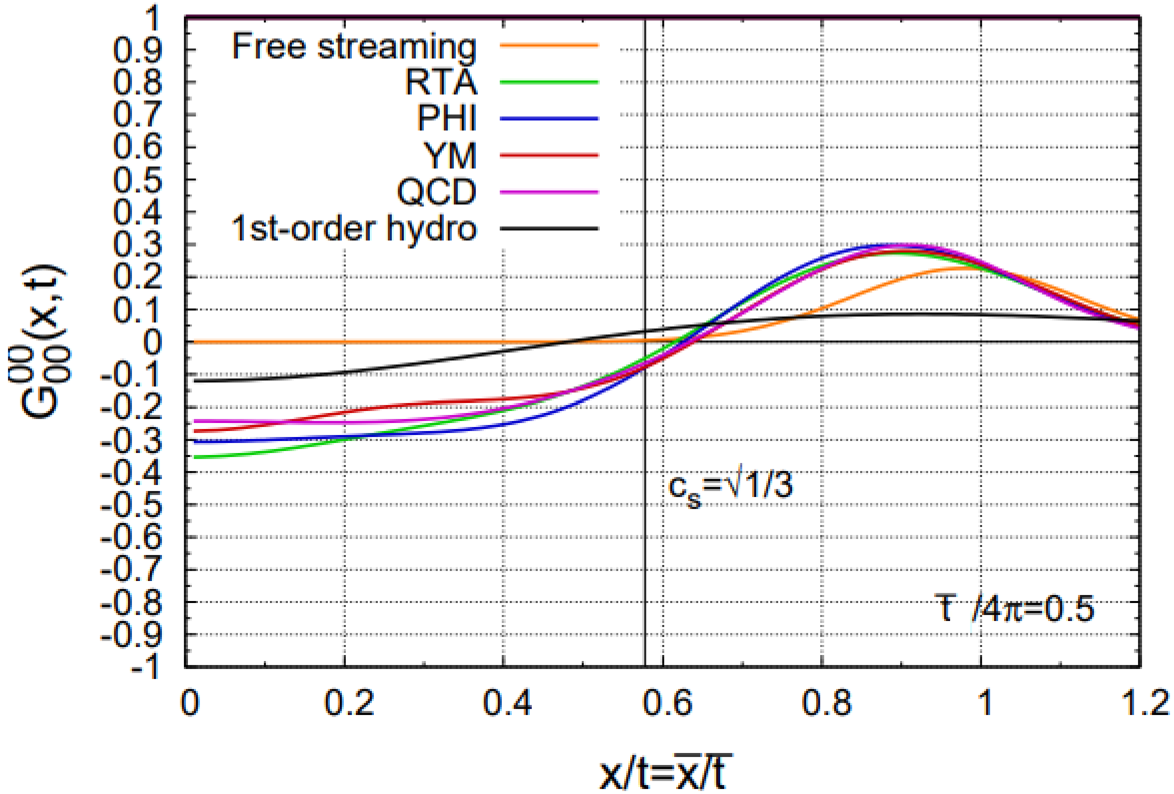


- Sound mode dominates at small k (non-hydro mode dominates at large k)
- Universality of residue at some degree

Response in position space

Response in position among kinetic theories

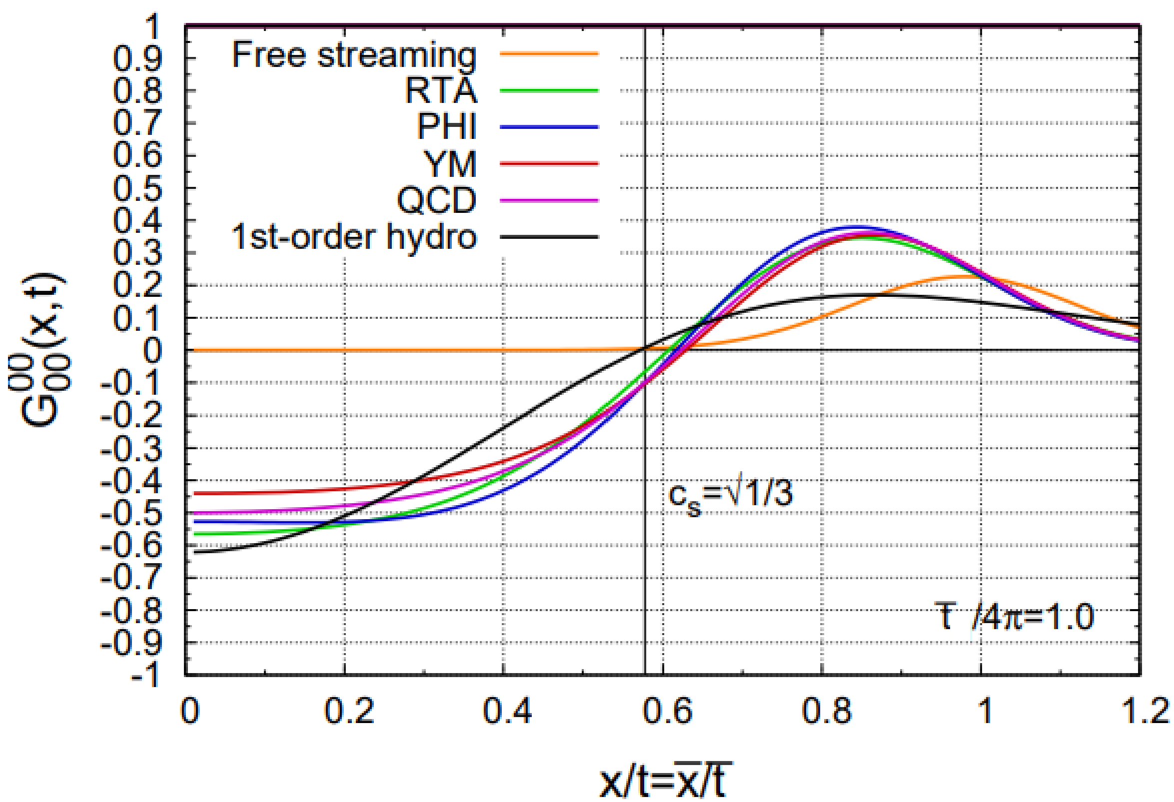
- Universality of position response



Response in position space

Response in position among kinetic theories

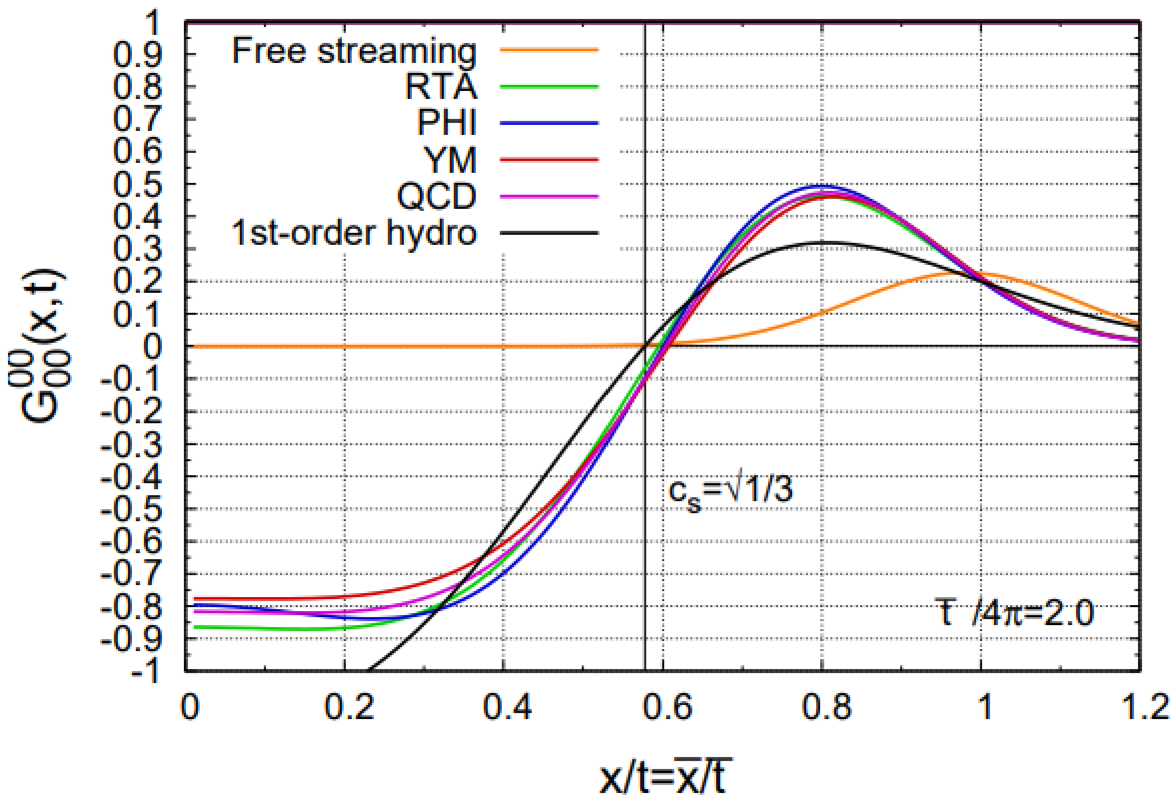
- Universality of position response



Response in position space

Response in position among kinetic theories

- Kinetic theories get closer to hydrodynamics at later time



Jet wake from kinetic theory

What we have leaned:

See also various papers by Cao, Wang et al. Pablo, Rajagopal et al. ...

- Jet wake stimulates non-hydrodynamic modes in fluid medium
- The sound modes of the wake is insensitive to medium chemistry

Jet wake in the hydrodynamic plasma → in non-equilibrium plasma

Wake in hydrodynamics

$$T^{\mu\nu}_{\text{total}}(t,x) = \boxed{\bar{T}^{\mu\nu}(t)} + \boxed{\delta T^{\mu\nu}(t,x)}$$

Homogeneous background — **Perturbation (Wake)**

Wake in QCD kinetic theory

$$f_{\text{total}}(t,x,p) = \boxed{f(t,x,p)} + \boxed{\delta f(t,x,p)}$$

Non-equilibrium background — **Perturbation (Wake)**

Summary

What we have learned and what we may expect to learn

Summary

What we have learned

Jet thermalization mainly from medium-induced reaction

- On-shell kinetic theory description applicable

Jet thermalization features inverse energy cascade

- Kolmogorov-Zakharov spectrum

Jet thermalization is similar to bottom-up thermalization

- Rapid buildup of soft thermal bath from collinear radiation
- Momentum/cone size broadening for soft parton from elastic collision + radiation

Jet wake excitation from kinetic theory perspective

- Jet wake stimulates non-hydrodynamic modes in fluid medium
- The sound modes of the wake is insensitive to medium chemistry

What we may expect to learn

Theory description

- Evolution including virtuality degrading + on-shell kinetic dynamics

Jet wake in non-equilibrium medium

- Convolution of Green's function in description wake (a.k.a KoMPoST framework)

Thanks!