# Flow of the Charm Hadrons to Constrain In-Medium Hadronization

Wei Dai

China University of Geosciences

April. 3, 2026

Wuhan

Central China
Center for Nuclear Theory
华中核理论中心

# Outline

# Motivation and Report Logic

- Heavy flavor provides a clean probe of in-medium hadronization; Flow observables provide excellent constraints.
- One common question: The impact of hadron formation evolved from vacuum-like fragmentation to medium-driven collective dynamics
- Three connected themes:
  - **Finding:** $D_s$ elliptic flow as evidence for sequential hadronization;
  - **DIscussion I:** Understanding the flow of heavy hadrons;
  - **DIscussion II:** The identification of sequential hadronization effect.
- The goal is to discuss what the flow can tell.

# Heavy Flavor as a Precision Probe

- Produced in initial hard scatterings, approximately Charm Quark Number Conservation.
- Hadron chemistry is sensitive to the competition between fragmentation, coalescence, and hadronic rescattering.
- Flow observables connect hadronization to the space-time structure of the QCD medium.

**Question.1**

Can one identify experimental signatures that distinguish a temperature-ordered, sequential mechanism from simultaneous hadronization?

**Question.2**

What exactly are the flow observables of heavy flavors related to?

# Model Ingredients

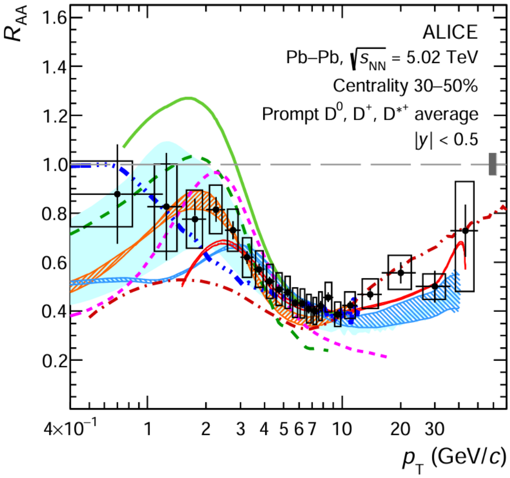
- Initial heavy-quark momentum: FONLL-based production.
- Initial geometry: Trento-Based.
- In-medium transport: Langevin evolution with collisional and radiative energy loss.
- Bulk medium: $(3+1)$-dimensional viscous hydrodynamics.
- Hadronization: simultaneous and sequential coalescence compared on the same footing, with fragmentation retained as a baseline channel.
- Hadronic phase: rescattering is included and tested explicitly as a competing explanation.

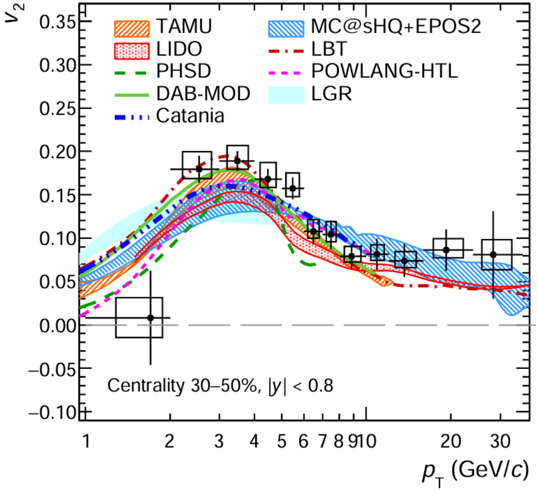
# Model comparison

## Model comparison — model description

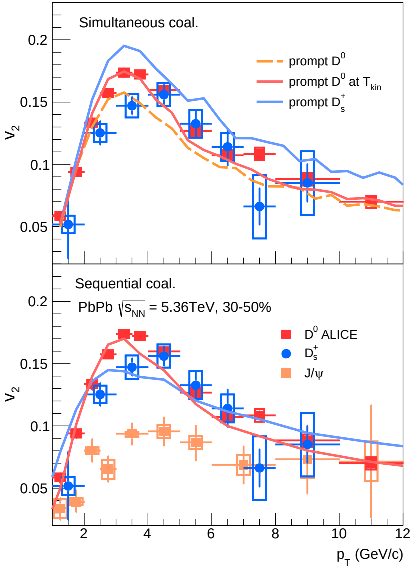
| | Frag. | Recom. | Recom. Form | Charmed hadrons involved |
|---|---|---|---|---|
| Catania | Peterson | Phase space Wigner function | $W(x,p)=\prod_{i=1}^{N_q-1} A_W \exp\left(-\frac{x_i^2}{\sigma_{ri}^2} - p_i^2\sigma_{ri}^2\right)$ | S-wave, D0,Ds, D*+,D*0,D*s,several excited states of \Lambda_c,\Sigma_c |
| Duke | Pythia 6.4/ Peterson | Momentum space Wigner function | $W(p)=g_h\frac{(2\sqrt{\pi}\sigma)^3}{V}e^{-\sigma^2p^2},$ | S-wave,D,D* |
| LBT | Pythia 6.4/ Peterson | Momentum space Wigner function | $W_s(p) = g_h \frac{(2\sqrt{\pi}\sigma)^3}{V} e^{-\sigma^2p^2},$ $W_p(p) = g_h\frac{(2\sqrt{\pi}\sigma)^3}{V}\frac{2}{3}\sigma^2p^2e^{-\sigma^2p^2}.$ | S-wave,P-wave,D,Ds,D*, \Lambda_c,\Sigma_c,\Xi_c. \Omega_c |
| Nantes | **HQET** | Phase space Wigner function | $W(x_Q,x_q,p_Q,p_q)=\exp\left(\frac{(x_q-x_Q)^2-[(x_q-x_Q)\cdot u_Q]^2}{2R_c^2} - \alpha_d^2(u_Q\cdot u_q -1)\right)$ | S-wave, D0 |
| PHSD | Peterson | Phase space Wigner function | $W_s(r,p) = \frac{8(2S+1)}{36} e^{-\frac{r^2}{\sigma^2}-\sigma^2p^2},$ $W_p(r,p)=\frac{2S+1}{36}\left(\frac{16}{3}\frac{r^2}{\sigma^2}+\frac{16}{3}\sigma^2p^2-8\right)e^{-\frac{r^2}{\sigma^2}-\sigma^2p^2},$ | S-wave, P-wave D+,D0,Ds, D*+,D*0,D*s |
| TAMU | thermal density correlated **HQET** | Resonance amplitude | $\frac{\gamma_M}{\Gamma} v_{rel} g_\sigma \frac{4\pi}{k^2}\frac{(\Gamma m)^2}{(s-m^2)^2+(\Gamma m)^2}$ | D+,D0,Ds and few excited states. Charm baryons+missing baryons |
| Turin | Pythia 6.4/ String fragmentation | Invariant mass criterion | $M_D < M_{Cluster} < M_{max.}$ | (prompt) D+,D0,Ds,\Lambda_c, \Xi_c,\Omega_c |
| Los Alamos | **HQET** | — | — | S-wave, D+,D0,Ds, charm-baryons |

# Model comparison



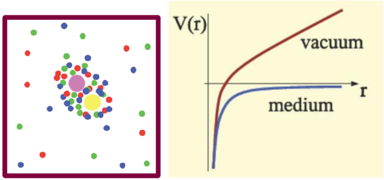
# The $D_s$ Elliptic-Flow Puzzle




Zi-Xuan Xu, WD, Ben-Wei Zhang, Jiaxing Zhao, Pengfei Zhuang (2510.16299)

- The key puzzle is the observed ordering of elliptic flow in the intermediate-$p_T$ region.
- Simultaneous coalescence tends to predict the wrong $D^0$ versus $D_s$ hierarchy.
- This points to a temperature-dependent hadronization sequence, ruling out hadronic phase interaction.
- Earlier production of $D_s$ can explain the experimental observation. $v2(J/\psi) < v2(D_s) < v2(D^0)$

# Theoretical origin of Sequential Coalescence I




In vacuum : $q\bar{q}$ interaction described by Coulomb + linear potential –

$$V(r) = -\frac{\alpha}{r} + kr$$

In medium: confinement term vanishes, Coulomb term is color screened beyond a Debye length –

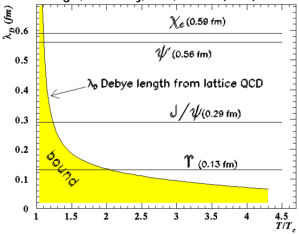
$$V(r) = -\frac{\alpha}{r} e^{-r/\lambda_D}$$

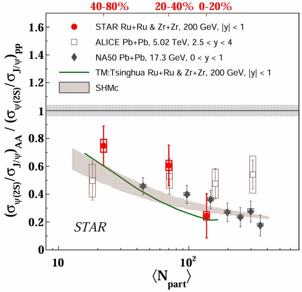
Quarkonia exhibit different binding energies and radii

Sequential hadronization inspired by charmonium:

R. Averbeck in Quark Matter 2025

Digal, Petreczky, Satz, PRD 64(2001)094015



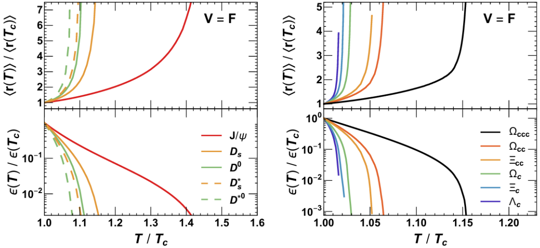




arXiv:2509.12842

# Theoretical origin of Sequential Coalescence II

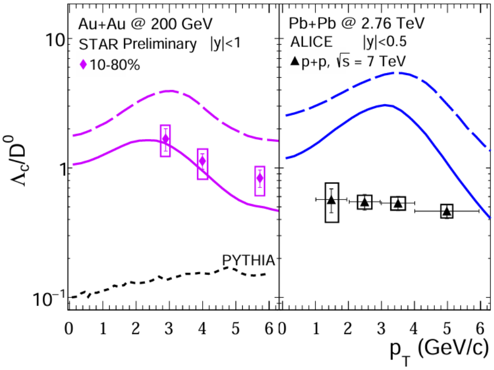
hadronization sequence ( by Zhuang et al. ) : Nucl. Phys. A 1005 (2021), 121898

$$T_d^{J/\psi} > T_d^{D_s} > T_d^{\Omega_{ccc}} > T_d^{D^0} > T_d^{\Omega_{cc},\Xi_{cc}} > T_d^{\Omega_c,\Xi_c,\Lambda_c} > T_d^{\pi,K,N} \approx T_c$$

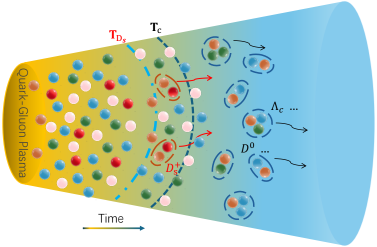


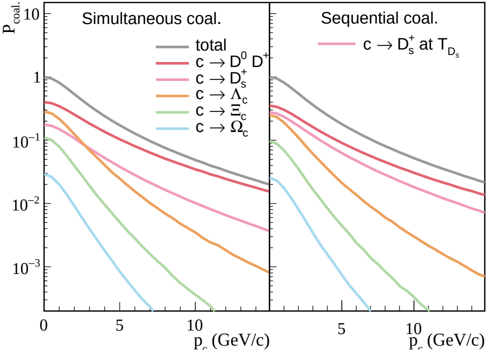

- Excited states.
- Gaussian widths.
- Coalescence & fragmentation fraction.
- ...
- A stronger signal remains to be found.

arXiv:1805.10858



# Sequential Coalescence Picture I

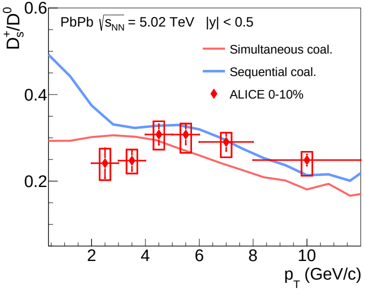


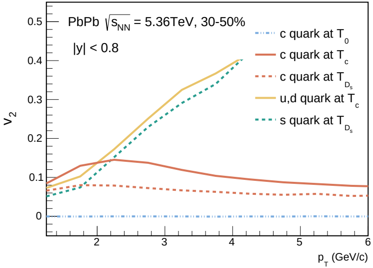




- Different charm hadrons ($D_s$) are assumed to hadronize on different hypersurfaces $1.2T_c$ .
- Charm Number Conservation: Earlier hadrons consume a larger proportion of the available charm-quark.

  charm quarks that hadronize via coalescence are 81.7%,
  in which $D^+$ 14.2%, $D^0$ 34.3%, $D_s^+$ 11.99%, $\Lambda_c$ 27.05%, $\Xi_c$ 12.03%, $\Omega_c$ 0.43%
  sequential case is 81.4%,
  in which $D^+$ 13.9%, $D^0$ 33.87%, $D_s^+$ 16.46%, $\Lambda_c$ 24.43%, $\Xi_c$ 10.94%, $\Omega_c$ 0.39%
- $D_s$ can therefore be enhanced at an earlier temperature.

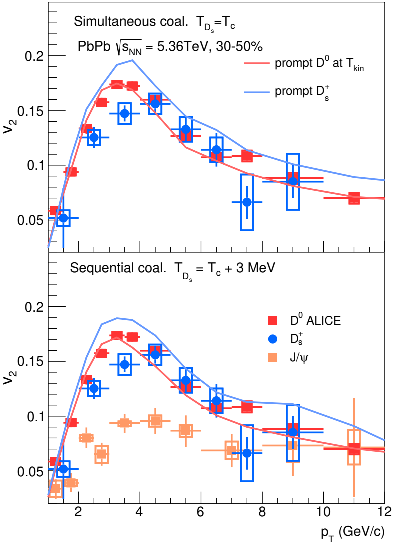
# Two Opposite Effects

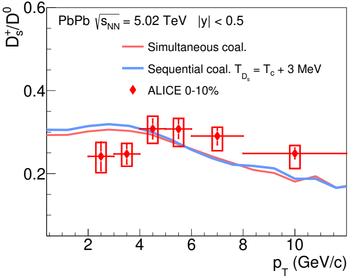



- Enhancement of $D_s/D^0$ at lower-$p_{\mathrm{T}}$, another evidence for sequential and Charm Number Conservation;
- Less interaction with medium, which reduces its $v_2$ .
- Reduction of $D_s$ $v_2$ unlikely arise from more strangeness at higher temperature.

# Clarification



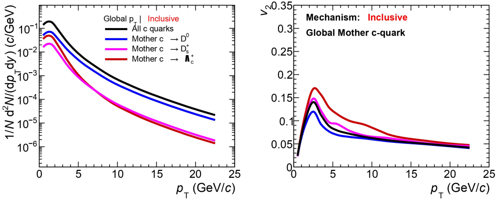



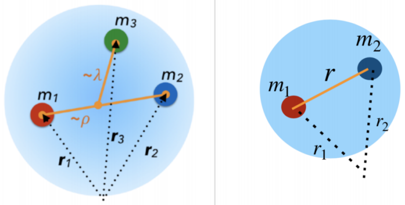

- How the higher amount of strange thermal quarks available plays a role.

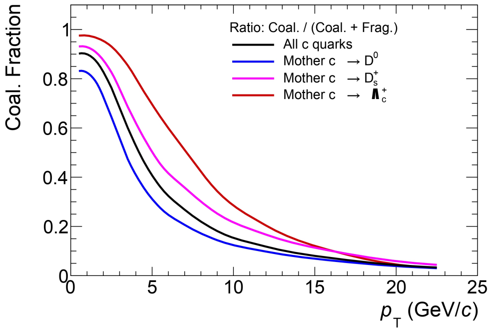
# Take-Home Messages

- $v_2$ hierarchy: its ordering is identified as a direct signature of sequential hadron formation effect, resulting $v_2$ hierarchy becomes consistent with the new ALICE trend. Different binding energies of hadrons imply a hierarchy in the elliptic flow: $v2(J/\psi) < v2(D_s) < v2(D^0)$
- Hadron chemistry: Early production of $D_s$ + Charm Number Conservation leads to more charm consumed for $D_s$, consequently enhanced $D_s/D^0$ in low-$p_{\mathrm{T}}$.
- Charm-hadron flow: Early production of $D_s$ leads to suppression of $D_s$ $v_2$ in the intermediate-$p_{\mathrm{T}}$.

# Hadronization will affect initial charm distribution!




The momentum distribution of hadrons produced from coalescence:

$$P_h = g_h \int \prod_{i=1}^{n} \frac{d^3x_i d^3p_i}{(2\pi)^3} f_i(x_i, p_i) \cdot W_h(x_1, \cdots, x_i, p_1, \cdots, p_i)$$

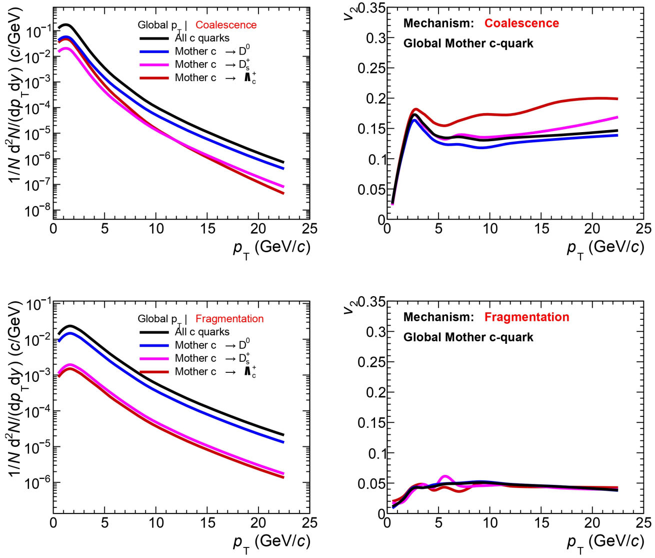
- Quark distribution function $f_i(x_i, p_i)$

  For light quark $f_i(p_i) = N_i/(e^{u_\mu p_i^\mu/T} + 1)$

- Wigner function $W_h(\boldsymbol{r}, \boldsymbol{p}) = \int d^3\boldsymbol{y} e^{-i\boldsymbol{p}\cdot\boldsymbol{y}} \psi\left(\boldsymbol{r} - \frac{\boldsymbol{y}}{2}\right) \psi^*\left(\boldsymbol{r} - \frac{\boldsymbol{y}}{2}\right)$

coal vs. frag change for different spices will play a role.

# Coal trigger vs. Frag trigger

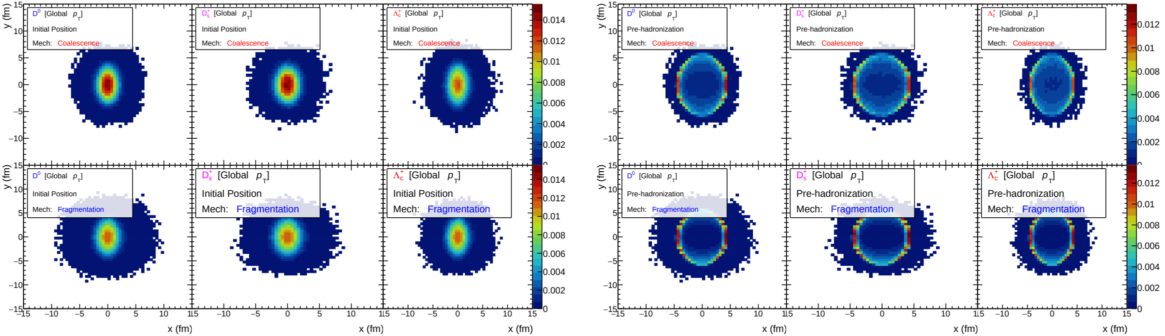



- Coal trigger: slope, $v_2$ $\Lambda_c > D_s > D^0$ .
- Frag trigger: slope, $v_2$ $\Lambda_c = D_s = D^0$ .
- mixture of Coal and Frag will still give $\Lambda_c > D_s > D^0$ for $v_2$ of pre-hadronized charm.

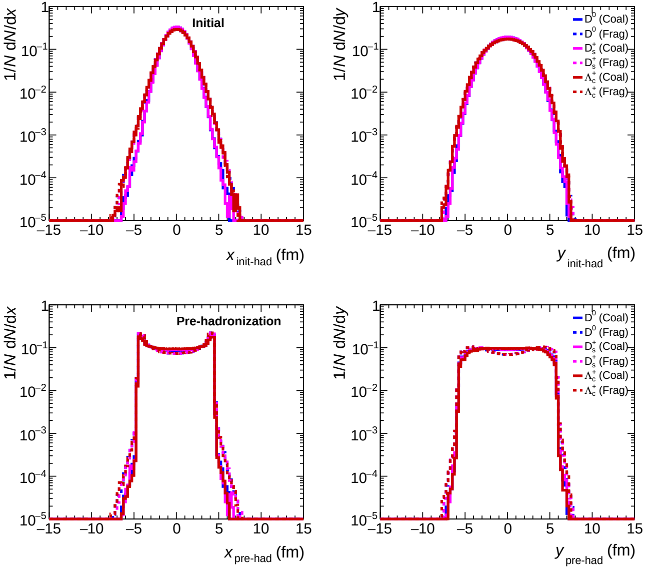
weidai@cug.edu.cn

# coordinate space: initial charm vs. Final charm

Hui Du@cug



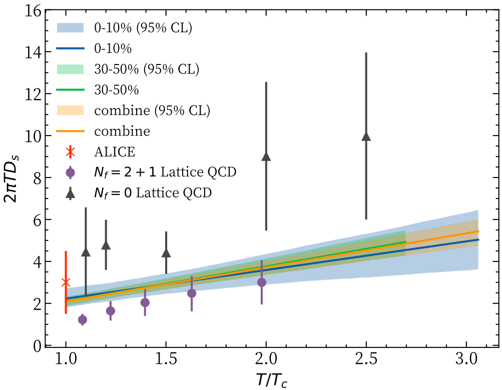
# coordinate space: initial charm vs. Final charm




- 30-50% profile
- hadronization surface: more heavy quark concentration in x-direction
- more outgoing in x-direction compared to y-direction
-

# Fix hadronization, constrain heavy quark transport

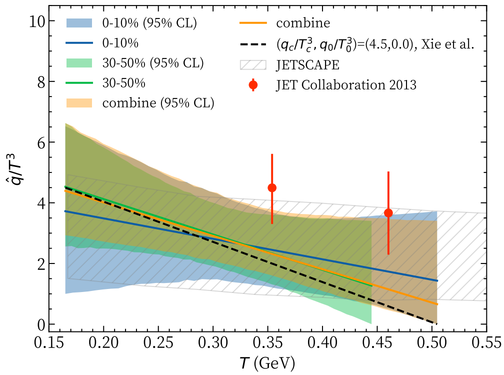
arXiv:2512.07169 Xu-Fei Xue, Zi-Xuan Xu, WD, Jiaxing Zhao, Ben-Wei Zhang




The relaxation of the assumption $\hat{q} = 2\kappa$ to infer.

$v_2$ is more important for constraining these transport parameters.

Sequential is not discussed. Data is updated to 2022.

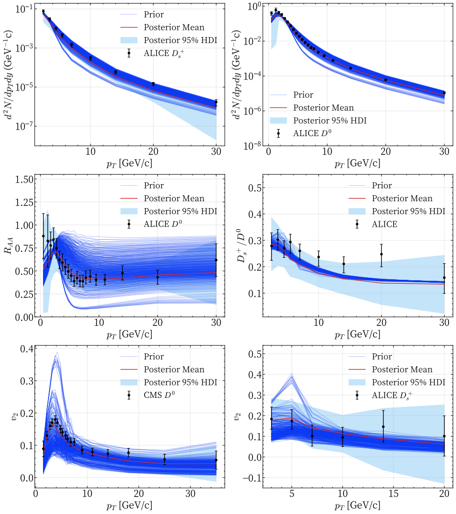


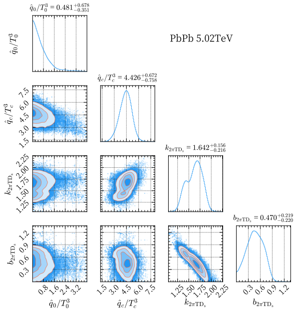

Simultaneously inferred $2\pi T Ds$ and $\hat{q}$ in D-meson observable, aligns well with light-flavor and heavy-flavor sectors.

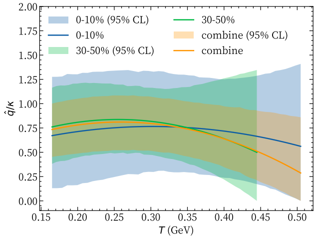
# Fix hadronization, constrain heavy quark transport

arXiv:2512.07169

PbPb 30-50% 5.02TeV



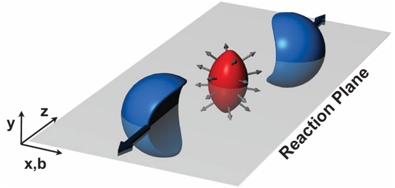



# Radial Flow: A Definition

- **Radial flow** is the isotropic, pressure-driven transverse expansion of the medium.
- **Hydrodynamic view:** pressure gradients create a radial boost and raise the event-wise mean transverse momentum $[p_T]$.
- **Caveat:** slope-based extractions mix collective outward transverse expansion with other effects, so less nonflow-sensitive observables are especially valuable for heavy flavor.

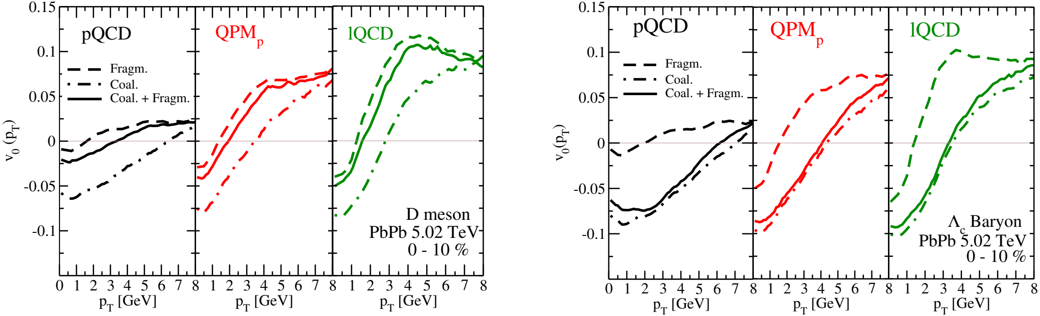
ALICE, Phys. Rev. C 88 (2013) 044910 / arXiv:1303.0737. ATLAS, CERN-EP-2025-068 / arXiv:2503.24125.




$$v_0(p) = \frac{1}{\langle N(p)\rangle} \frac{\left\langle \hat{\delta} N(p)\, \hat{\delta} P_{\mathrm{T}} \right\rangle}{\sigma_{\hat{P}_{\mathrm{T}}}}$$

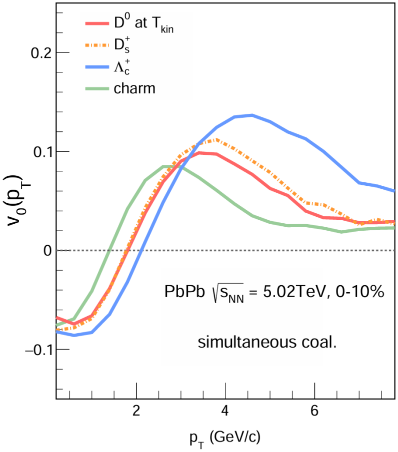
# $v_0(p_\mathrm{T})$: coalescence versus frag

arXiv:2510.19448

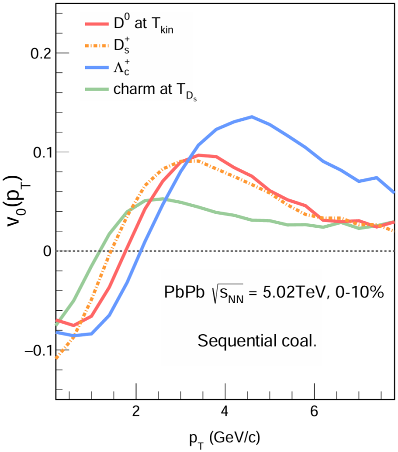



- heavy quarks exhibit collective behavior driven by the isotropic expansion of the QGP
- at low $p_\mathrm{T}$, it offers a marked signature of the heavy quark hadronization mechanism
- slope of $v_0(p_\mathrm{T})$ sensitive to different transport parameters

# $v_0(p_T)$: simultaneous versus sequential



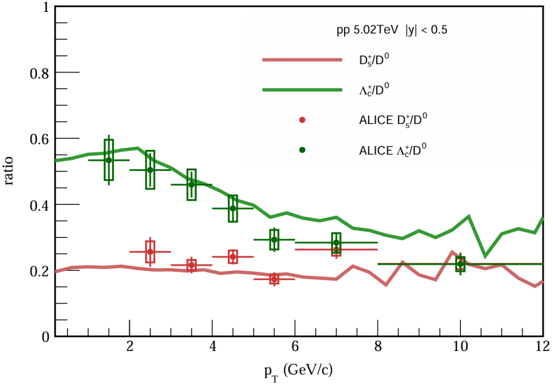

Zi-Xuan Xu, WD, Jiaxing Zhao in preparation



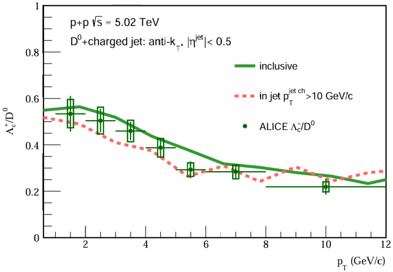

- same hierarchy as in $v_2$: $\Lambda_c > D_s > D^0$ for simultaneous.
- same kind of phenomenon for sequential hadronization mechanism: hierarchy break!

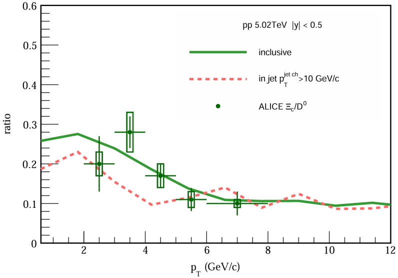
# Beyond Vacuum Fragmentation in Small Systems

Zi-Xuan Xu, WD, Ben-Wei Zhang, in preparation




- A “fireball” scenario adds a coalescence-like component on top of fragmentation.
- Even small systems may contain nontrivial hadronization dynamics.
- trying to build up a hadronchemistry in pp using new fragmentation fractions.



# Summary and Outlook

## Summary

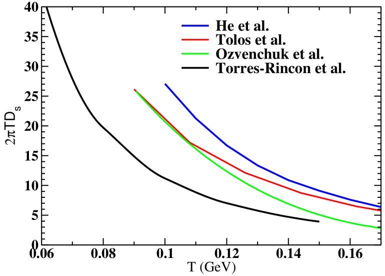
- Heavy-flavor flow can become a quantitative bridge between hadronization and transport.
- sequential hadronization effect will reveal itself more in many flow-related observables
- Precision data to constrain both the hadronization mechanism and the temperature dependence of heavy-quark transport.

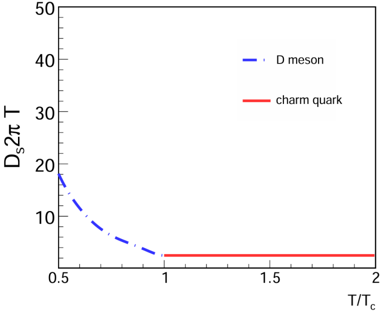
## Outlook

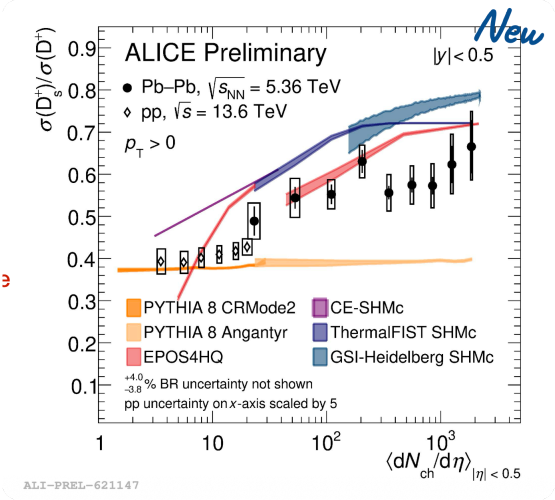
- Heavy-flavor flow can be used as a powerful tool to probe QGP properties.
- Extend the sequential picture to more observables (ESE, OO ...), more charm hadrons, and possibly small to large systems.
- multi-stage multi-messenger analysis for the nuclear structure, hydro description parameters, heavy-quark transport, and hadronization mechanism.

Thanks to the collaborators and students within SHELL+Tsinghua framework.

# Hadronic Rescattering Is Not Enough



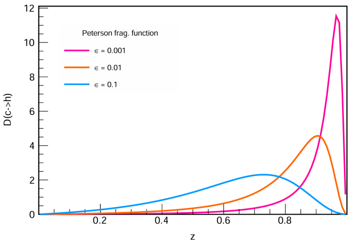



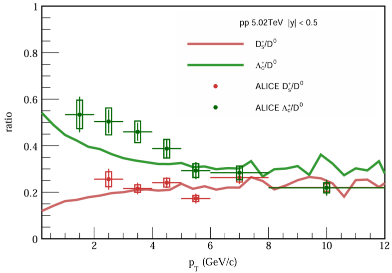



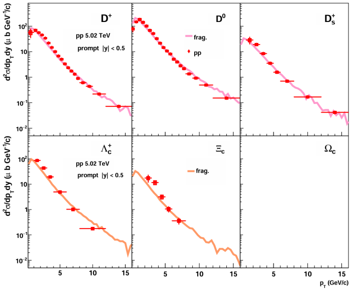

weidai@cug.edu.cn

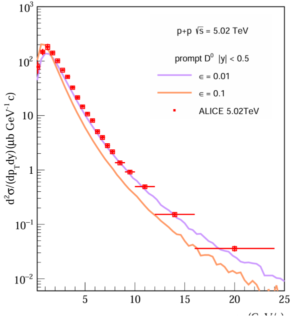
# Theme II: Beyond Vacuum Fragmentation I

Peterson: $\mathcal{D}_{c\to h} \propto \dfrac{1}{z\left(1-\frac{1}{z}-\frac{\epsilon}{1-z}\right)^2}$



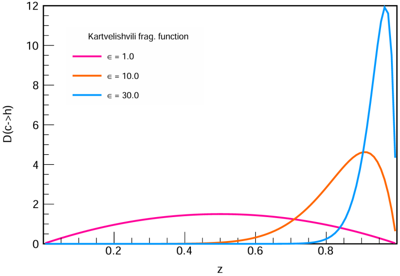



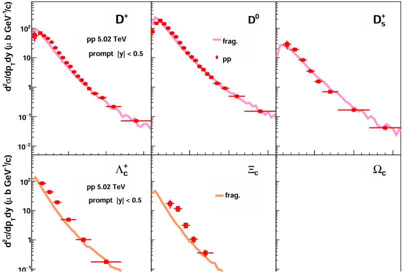



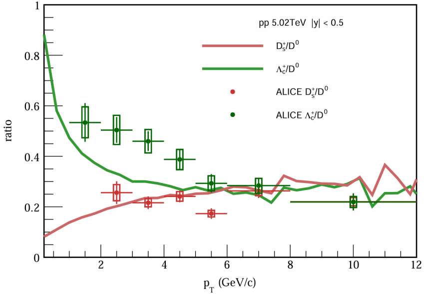



# fragmentation II

Kartvelishvili: $\mathcal{D}_{c\to h} \propto (1-z)z^{\epsilon}$




Charm cross section and fragmentation fractions in p–Pb collisions — ALICE Collaboration

**Table 2:** Fragmentation fractions $f(\mathrm{c}\to\mathrm{h_c})$ of charm hadrons in pp [4] and p–Pb collisions at $\sqrt{s_{\mathrm{NN}}} = 5.02$ TeV.

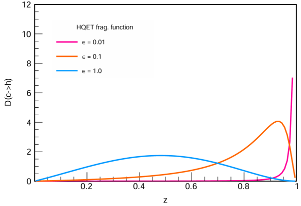
| $f(\mathrm{c}\to\mathrm{h_c})$ (%) | pp, $\sqrt{s} = 5.02$ TeV [4] | p–Pb, $\sqrt{s_{\mathrm{NN}}} = 5.02$ TeV |
|---|---|---|
| $\mathrm{D^0}$ | $39.6 \pm 1.7\,(\text{stat.})^{+2.6}_{-3.8}\,(\text{syst.})$ | $41.3 \pm 1.3\,(\text{stat.})^{+2.7}_{-3.2}\,(\text{syst.})$ |
| $\mathrm{D^+}$ | $17.5 \pm 1.8\,(\text{stat.})^{+1.7}_{-2.1}\,(\text{syst.})$ | $18.2 \pm 1.0\,(\text{stat.})^{+1.6}_{-1.6}\,(\text{syst.})$ |
| $\mathrm{D_s^+}$ | $7.4 \pm 1.0\,(\text{stat.})^{+1.9}_{-1.1}\,(\text{syst.})$ | $9.0 \pm 0.5\,(\text{stat.})^{+1.6}_{-1.2}\,(\text{syst.})$ |
| $\Lambda_c^+$ | $18.9 \pm 1.3\,(\text{stat.})^{+1.5}_{-2.0}\,(\text{syst.})$ | $17.1 \pm 1.1\,(\text{stat.})^{+1.5}_{-1.8}\,(\text{syst.})$ |
| $\Xi_c^0$ | $8.1 \pm 1.2\,(\text{stat.})^{+2.5}_{-2.5}\,(\text{syst.})$ | $7.0 \pm 1.1\,(\text{stat.})^{+2.5}_{-1.8}\,(\text{syst.})$ |
| $J/\psi$ | $0.44 \pm 0.03\,(\text{stat.})^{+0.04}_{-0.06}\,(\text{syst.})$ | $0.41 \pm 0.02\,(\text{stat.})^{+0.04}_{-0.04}\,(\text{syst.})$ |
| $\mathrm{D^{*+}}$ | $15.7 \pm 1.2\,(\text{stat.})^{+4.1}_{-1.9}\,(\text{syst.})$ | $12.9 \pm 0.6\,(\text{stat.})^{+1.1}_{-1.2}\,(\text{syst.})$ |

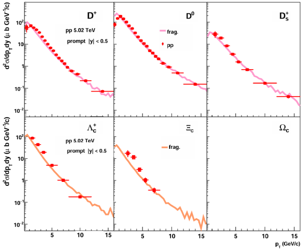
integrated cross sections of the ground-state charm hadrons: $\mathrm{D^0}$, $\mathrm{D^+}$, $\mathrm{D_s^+}$, $\Lambda_c^+$, $\Xi_c^0$, and $J/\psi$. The $\Xi_c^0$ contribution was considered twice to account for the $\Xi_c^+$ cross section, as discussed above. The rapidity shift between the measured charm hadrons and the originating charm-quark pairs was accounted for by multiplying the sum of the hadron species by the aforementioned correction factor of 1.03. The correlations of the systematic uncertainties between charm-hadron species were treated in the same way as for the fragmentation fractions discussed above. The resulting charm cross section is

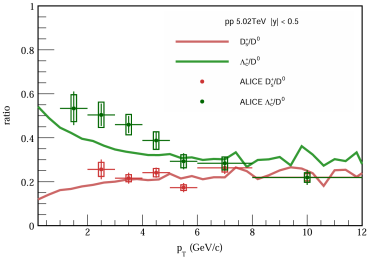
# fragmentation III

HQET:

$$\mathcal{D}_{Q\to H_P} \propto \frac{rz(1-z)^2}{[1-(1-r)z]^6}\Big[6-18(1-2r)z + (21-74r+68r^2)z^2 - 2(1-r)(6-19r+18r^2)z^3 + 3(1-r)^2(1-2r+2r^2)z^4\Big],$$

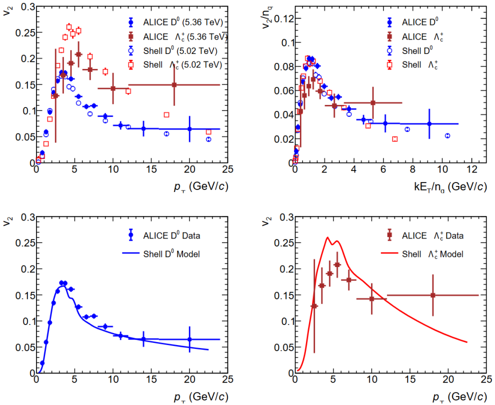



- A “fireball” scenario adds a coalescence-like component on top of fragmentation.
- Peterson, Kartvelishvili, and HQET-inspired forms.
- The main purpose is to quantify how momentum is transferred from the heavy quark to the final hadron.

# Tests: exploring NCQ scaling of $v_2$ I




- Test against simultaneous coalescence scenario.
- Charm Number Conservation: Earlier hadrons consume a larger proportion of the available charm-quark.
- $D_s$ can therefore be enhanced at an earlier temperature.

# From Slopes to Fluctuation-Sensitive Observables

For any observable O, the event-by-event deviation and variance are defined as

$$\delta O \equiv O - \langle O \rangle \quad , \qquad \sigma_O^2 = \langle (\delta O)^2 \rangle \tag{1}$$

We hope to find a quantity $\hat{\delta} O$ that measures the pure fluctuation of the observable O while keeping the multiplicity N fixed. For N,

$$\delta N \equiv N - \langle N \rangle \quad , \qquad \sigma_N^2 = \langle (\delta N)^2 \rangle \tag{2}$$

Suppose a linear form between O and N

$$O = \epsilon + \beta N \tag{3}$$

Using the least squares method, the regression coefficient $\beta$ is

$$\beta = \frac{\langle \delta O \delta N \rangle}{\sigma_N^2} \tag{4}$$

The residual fluctuation of O that cannot be explained by N is

$$\hat{\delta} O \equiv \delta O - \beta \delta N = \delta O - \frac{\langle \delta O \delta N \rangle}{\sigma_N^2} \delta N \tag{5}$$

Then the variance of O independent of N is

$$\begin{aligned} \hat{\sigma}_O^2 &\equiv \langle (\hat{\delta} O)^2 \rangle \qquad (6) \\ &= \langle (\delta O - \frac{\langle \delta O \delta N \rangle}{\sigma_N^2} \delta N)^2 \rangle \\ &= \langle (\delta O)^2 - 2 \frac{\langle \delta O \delta N \rangle}{\sigma_N^2} \delta O \delta N + \frac{(\langle \delta O \delta N \rangle)^2}{\sigma_N^4} (\delta N)^2 \rangle \end{aligned}$$

Using the linearity of expectation

$$\begin{aligned} &= \langle (\delta O)^2 \rangle - 2 \frac{\langle \delta O \delta N \rangle}{\sigma_N^2} \langle \delta O \delta N \rangle + \frac{(\langle \delta O \delta N \rangle)^2}{\sigma_N^4} \langle (\delta N)^2 \rangle \qquad (7) \\ &= \langle (\delta O)^2 \rangle - 2 \frac{(\langle \delta O \delta N \rangle)^2}{\sigma_N^2} + \frac{(\langle \delta O \delta N \rangle)^2}{\sigma_N^4} \sigma_N^2 \\ &= \langle (\delta O)^2 \rangle - \frac{(\langle \delta O \delta N \rangle)^2}{\sigma_N^2} \end{aligned}$$

We are interested in the fluctuations in the spectra at fixed multiplicity

$$\hat{\delta} \mathcal{N}(p^a) = \delta \mathcal{N}(p^a) - \frac{\langle \delta \mathcal{N}(p^a) \delta N \rangle}{\sigma_N^2} \delta N \tag{9}$$

The fluctuations in integrated $p_T$ at fixed multiplicity can be used to characterize the spectral fluctuations, one defined

$$\begin{aligned} P_T &\equiv \int_0^\infty dp p \mathcal{N}(p) \qquad (10) \\ \hat{\delta} P_T &= \int_0^\infty dp p \hat{\delta} \mathcal{N}(p) \qquad (11) \end{aligned}$$

Specifically it is a sum over particles and therefore characterizes the collective respose. One define the integrated $v_0$ via the variance in the integrated $P_T$ at fixed multiplicity

$$v_0^2 = \frac{\hat{\sigma}_{P_T}^2}{\langle P_T \rangle^2} \tag{12}$$

The momentum dependent $v_0(p)$ can be defined as

$$v_0(p) = \frac{1}{\langle \mathcal{N}(p) \rangle} \frac{\langle \hat{\delta} \mathcal{N}(p) \hat{\delta} P_T \rangle}{\hat{\sigma}_{P_T}} \tag{13}$$

It is evident from Eqs. (11), (12) and (13) that the integrated $v_0$ is determined by $v_0(p)$ according to

$$v_0 \equiv \frac{\int_0^\infty dp p \langle \mathcal{N}(p) \rangle v_0(p)}{\int_0^\infty dp p \langle \mathcal{N}(p) \rangle} \tag{14}$$

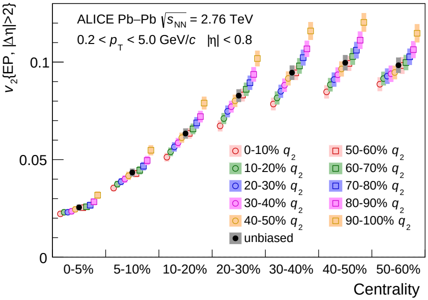
The integrated radial flow $v_0$ is the momentum-weighted average of $v_0(p_T)$.

Phys. Rev. C 102 (2020) no.3, 034905.

proposed a new observable $v_0(p_{\mathrm{T}})$, which quantifies the relative change in the $p_{\mathrm{T}}$ spectrum induced by a fluctuation.

# Event-shape Engineering (ESE)

classifies events according to their anisotropy within a centrality class




ALICE, Phys. Lett. B 777 (2018) 151–162

$$\vec{Q}_2 = \left( \sum_{i=1}^{M} w_i \cos(2\varphi_i), \sum_{i=1}^{M} w_i \sin(2\varphi_i) \right)$$

$$q_2 = \frac{|\vec{Q}_2|}{\sqrt{M}}$$

$\varphi_i$: azimuthal angle, $M$: multiplicity

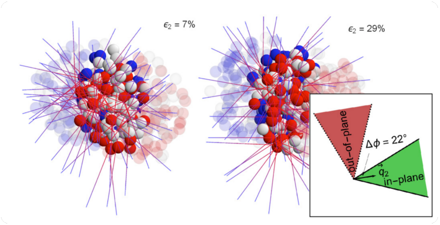



Figure: Beattie, Nijs, Sas, van der Schee, Phys. Lett. B 836 (2023) 137596